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Classical Formation Patterns and Flanking Strategies as a Result of Utility Maximization

Edvards Scukins and Petter Ögren

Abstract—In this paper, we show how classical tactical formation patterns and flanking strategies, such as the line formation and the enveloping maneuver, can be seen as the result of maximizing a natural formation utility.

The problem of automatic formation keeping is extremely well studied within the areas of control and robotics, but the reasons for choosing a particular formation shape and position is much less so.

By analyzing a situation with two adversarial teams of agents facing each other, we show that natural assumptions regarding the target selection of the agents and decreasing weapon efficiency over distance, can be used to optimize a measure of utility over agent positions. This optimization in turn results in formations and positions that are very similar to the ones being used in practice. We present both analytical results for simple examples as well as numerical results for more complex situations.

Index Terms—Cooperative control, Game theory, Optimization

I. INTRODUCTION

FOR thousands of years, formations and relative positioning have played a central role in military tactics, from Roman Legionaries to modern tank units and special forces [1].

It is clear that the use of formations is beneficial from an organizational point of view, to be able to know where your troops are in a chaotic environment, and to avoid friendly fire. However, our investigation shows that there are additional benefits to these classical patterns and strategies, that stem from the geometry itself, together with a few reasonable assumptions.

Looking at the use of formations through history, illustrated in Figures 1a to 1c, one can make the following observations.

Observation 1. *Moving on a line, extended perpendicular to the motion direction, seems to be preferred in some situations, see Figure 1b.*

Observation 2. *If the opponent is moving in a line, advancing from the flank (i.e. from the side with motion direction along the line) seems to be preferred in some situations, see Figure 1a.*

Observation 3. *Moving in a group, seems to be preferred in some situations, see Figure 1c.*

In this paper we will show how the maximization of a formation utility, given a few natural assumptions, give rise to

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Fig. 1: (a) Flanking (envelope) tactics, in combination with line formations, were used at the battle of Marathon in 490 B.C. (b) Line formations were used at the battle of Nivelle in 1813. (c) Group formations are sometimes used in modern day police training. Note that this is not a line formation seen from the side, as e.g., the second officer from the left is almost behind the first one.

solutions that exhibit all features described in Observations 1-3, see Figures 8 and 9.

The main contribution of this paper is that we show why these formation patterns and flanking maneuvers make sense from a purely geometric perspective. Despite the fact that literally thousands of papers have addressed different aspects of formation keeping (papers [2]–[6] all have over 2000 citations), very few have addressed why formations are useful in the first place, and what formations to use. Finally, out of those few, to the best of our knowledge, none have investigated the problem using the assumptions on weapon performance and target selection proposed here.

II. RELATED WORK

In [7] the problem of finding a minimum risk path for a formation of agents travelling through a dangerous area is considered. The formation is selected from the four types column, line, wedge and diamond, but is not allowed to change its geometry, beyond translation and rotation.

In [8], the problem of moving four agents in formation, towards a goal point, through obstacle terrain was studied. The shape of the formations used were taken from [9].

A survey of results applicable to robot control in adversarial environments can be found in [10]. There, recent development in the areas of adversarial patrolling, coverage, formation motion and navigation is described. Within the area of formation motion, the main reference given is [7], which is described above.

One area where the shape of the formation has been considered is that of gradient climbing, where a group of robots collaboratively searches for the source (or local maximum) or a scalar field, such as an underwater pollutant or heat source. In [11] the formation shape that optimizes a trade-off between noise suppression and second order errors in the gradient estimate was proposed. Furthermore, in [12] the rationale for birds flying in V formations was studied.

There has also been a wealth of results, including [13], considering the motion of agents through obstacle terrain, keeping some preferred agent distance. The actual formation is then a result of the obstacles and the density of agents at a given location.

In [14] the problem of evolving team tactics in an adversarial setting was investigated. The solution exhibited intelligent group behavior in the sense that two members of a team of five initially storm the enemy and get killed in the process, while the other three have time to gather ammunition and weapons, and finally team up to defeat the enemy. The behaviors are described in the form of decision trees that are then combined by a genetic algorithm. The available nodes of the trees are taken from a given set of 39 options, including actions such as Attack, positions such as Nearest Ally and Behind enemy, and checks of current health, or distance to the enemy. The result is an interesting group coordination, but no proper formation movement is possible due to the constraints in available action nodes.

In [15] the control of combating agents in a real time strategy game (RTS) is investigated. So-called influence maps, where agent positions and boundaries between areas of influence are indicated, were used to find target locations and to do path planning. This was combined with a flocking coordination scheme suggested by Reynolds, [16]. The authors made the following important observation regarding formations: “The bad results for the normal game behavior stem from the different speeds of the considered units. Here, these units reach their common target one after another, which enables the towers to kill them easily. Incorporating flocking, all units stuck together in a group and reached the target at the same time. In this situation, the flock is strong enough to destroy the towers.” Thus it was noted that group formations were preferable to arriving one at a time. We bring these ideas one

step further, and investigate what formations are optimal from a utility perspective.

RTS games were also investigated in [17]. Again, influence maps were used, but this time they were combined with potential fields. Using co-evolution, both sides of a game were evolved simultaneously to find optimal control parameters. Units of two types, ranged units and melee units were considered. Attacking, fleeing and so-called kiting, or hit and run tactics, were included, but no explicit formations were considered.

In [18], neuroevolution was used to evolve neural networks controlling movement and attack of agents in RTS games. The resulting agents were able to evolve a kiting strategy, see above, for dealing with opposing melee units, but again, no explicit formations were considered. In [19], a genetic algorithm was used to investigate similar problems. In [20], the problem of controlling entities in training simulators were studied. As above, influence maps and potential fields were used, in combination with hard-coded algorithms for a kiting behavior. The parameters above were then tuned using a genetic algorithm.

To conclude, the formation keeping problem itself has received enormous attention in the control and robotics communities [2]–[6], while the question of the utility of those formations has been studied by very few.

The benefit of some different formations in adversarial environments was investigated in [7] and the utility of grouping was observed in [15], but this paper is the first to investigate how classical formation patterns and flanking strategies can be motivated from a theoretical utility perspective.

III. PROBLEM FORMULATION

Let the agents be divided into two teams, P and Q of sizes N_p, N_q respectively, and let each agent be represented by its position in the plane $p_1, \dots, p_{N_p} \in \mathbb{R}^2$ and $q_1, \dots, q_{N_q} \in \mathbb{R}^2$. Before stating the main problem considered, we make some definitions and assumptions regarding damage and target selection.

Definition 1. (Damage Per Second, Single Target)

The damage each agent is able to inflict upon a target agent each time unit, i.e., the damage per second, is given by a function of the distance between the two agents $d_{ps} : \mathbb{R}_+ \rightarrow \mathbb{R}_+$. To account for differences in training and equipment, we allow for all agents having different functions, d_{psP_i} and d_{psQ_i} .

Assumption 1. *The functions d_{psP_i} and d_{psQ_i} are assumed to be monotonically decreasing.*

Definition 2. (Target Selection)

The target selected by p_i is denoted T_{p_i} , and the target selected by q_i is denoted T_{q_i} .

Assumption 2. (Best Shot)

Assume that each agent chooses its closest opponent as target. That is, let

$$T_{p_i} = \operatorname{argmax}_{q_j} d_{psP_i}(\|q_j - p_i\|), \quad (1)$$

$$T_{q_i} = \operatorname{argmax}_{p_j} d_{psQ_i}(\|p_j - q_i\|). \quad (2)$$

We can now formulate the main problem studied in this paper as follows.

Problem 1. (Optimal P-team positions)

Given the positions $q_1, \dots, q_{N_q}$, find positions $p_1, \dots, p_{N_p}$ so as to maximize the difference in damage per second inflicted by the two teams. That is, let

$$\Delta d_{ps} = \sum_{i=1}^{N_p} d_{psP_i}(\|T_{p_i} - p_i\|) - \sum_{i=1}^{N_q} d_{psQ_i}(\|T_{q_i} - q_i\|) \quad (3)$$

and choose positions $p_1 \dots p_{N_p}$ to maximize Δd_{ps} .

IV. ANALYTICAL RESULTS

We will start the analysis with the most simple cases, relying only on Assumption 1. In the following, let $r_{ij} = \|q_i - p_j\|$ be the distance between two agents from opposing teams.

Lemma 1. (1 vs 1)

With $N_p = 1$ and $N_q = 1$, $\Delta d_{ps} = d_{psP_1}(r_{11}) - d_{psQ_1}(r_{11})$, and the optimal distance r_{11} depends only on d_{psP_1} and d_{psQ_1} .

Proof.

$$\begin{aligned} \Delta d_{ps} &= \sum_{i=1}^{N_p} d_{psP_i}(\|T_{p_i} - p_i\|) - \sum_{i=1}^{N_q} d_{psQ_i}(\|T_{q_i} - q_i\|) \quad (4) \\ &= d_{psP_1}(\|T_{p_1} - p_1\|) - d_{psQ_1}(\|T_{q_1} - q_1\|) \\ &= d_{psP_1}(r_{11}) - d_{psQ_1}(r_{11}). \end{aligned}$$

□

Lemma 2. (2 vs 1)

With $N_p = 2$ and $N_q = 1$ the optimal formation is such that $r_{12} = \|q_1 - p_1\| = \|q_1 - p_2\| = r_{11}$, i.e., both agents on the P team keep the same distance to the agent on the Q team, see the illustration in Figure 2.

Proof. Since there is only one target on the Q-team we have that $T_{p_1} = T_{p_2} = q_1$. Without loss of generality we assume that the agents are labeled such that $r_{11} \leq r_{12}$, as illustrated in Figure 2. This implies that $T_{q_1} = p_1$, which gives

$$\begin{aligned} \Delta d_{ps} &= \sum_{i=1}^{N_p} d_{psP_i}(\|T_{p_i} - p_i\|) - \sum_{i=1}^{N_q} d_{psQ_i}(\|T_{q_i} - q_i\|) \quad (5) \\ &= d_{psP_1}(\|T_{p_1} - p_1\|) + d_{psP_2}(\|T_{p_2} - p_2\|) \\ &\quad - d_{psQ_1}(\|T_{q_1} - q_1\|) \\ &= d_{psP_1}(\|q_1 - p_1\|) + d_{psP_2}(\|q_1 - p_2\|) \\ &\quad - d_{psQ_1}(\|p_1 - q_1\|) \\ &= d_{psP_1}(r_{11}) + d_{psP_2}(r_{12}) - d_{psQ_1}(r_{11}). \end{aligned}$$

As d_{ps} is monotonically decreasing by Assumption 1, and $r_{11} \leq r_{12}$, the maximum must be achieved when $r_{12} = r_{11}$, as stated above. The optimal value of r_{11} then depends on the nature of d_{psP_i} and d_{psQ_i} . □

Note that when ϕ is very small this would result in a tight group, as in Observation 3, and when ϕ is somewhat bigger, but still small this would correspond to a small line, facing the opponent, as in Observation 1.

To address general cases with two agents on the Q team and an arbitrary number of agents on the P team, we make the following definition, inspired by the results above.

Fig. 2: Illustration of Lemma 2. Dashed circles are arbitrary agent positions with corresponding notation, and solid circles are the optimal positions. In 2 vs 1 situations, $r_{12} = r_{11}$ and an arbitrary ϕ is optimal.

Fig. 3: Illustration of the three translations to equal or better positions, *I, II, III* from Definition 3. *I* is a reflection in the y-axis (only needed when x-coordinate is positive), *II* is a motion at fixed distance to q_1 , and *III* is a motion to the same position as p_1 .

Definition 3. (Translations to equal or better positions)

Given two Q agents and an arbitrary number of P agents, let p_1 be the P agent with smallest distance to any Q agent and let q_1 be the Q agent that is closest to p_1 . Define a coordinate system such that both Q agents are on the x-axis with $x = 0$ in the middle of them, and the direction of the x-axis such that p_1 has a non-positive x-coordinate. This is illustrated in Figure 3. For $p_i \neq p_1$ with positive x-coordinate, Translation *I* is a reflection in the y-axis. For p_i with non-positive x-coordinate, Translation *II* is a motion at fixed distance to q_1 to the position on the x-axis that maximizes the distance to q_2 . For $p_i \neq p_1$ having done *II*, Translation *III* is a motion to the same position as p_1 .

Lemma 3. (Translations *I, II, III*)

With arbitrary N_p and $N_q = 2$, Translations *I, II, III* of Definition 3 does indeed lead to either equal or improved values of Δd_{ps} .

Proof. In general we have that

$$\Delta d_{ps} = \sum_{i \in N_p} d_{psP_i}(\|T_{p_i} - p_i\|) - \sum_{i \in N_q} d_{psQ_i}(\|T_{q_i} - q_i\|) \quad (6)$$

$$= \sum_{i \in N_p} d_{psP_i}(\|T_{p_i} - p_i\|) \quad (7)$$

$$+ d_{psP_j}(\|T_{p_j} - p_j\|) \quad (8)$$

$$- d_{psQ_1}(\|T_{q_1} - q_1\|) \quad (9)$$

$$- d_{psQ_2}(\|T_{q_2} - q_2\|) \quad (10)$$

For all translations *I, II, III* we only move p_j , so (7) is the same. Now consider Translation *I*. T_{p_j} changes from q_2 to q_1 ,

but the distance is the same, which makes (8) the same. We know that p_1 is closest to q_1 , so (9) is the same. Finally, the distance to q_2 increased, so (10) is either improved, if q_2 was aiming at p_j or the same.

Now consider Translation *II*. This is the only translation that can also be applied to p_1 . T_{p_j} is now q_1 , and the distance is the same, which makes (8) the same. Similarly, no distances to q_1 changes, so (9) is the same. Finally, unless p_j was already on the x -axis and therefore did not move, we moved away from q_2 , so (10) is either improved, if q_2 was aiming at p_j or the same.

Finally consider Translation *III*. Being behind p_1 from the perspective of both q_1 and q_2 , no enemy is aiming at p_j , thus (9) and (10) are the same. $T_{p_j} = q_1$ and the distance to q_1 is reduced, which makes (8) improve, unless p_j is already at p_1 and no motion occurs.

Thus we conclude that for all the translations *I, II, III*, all the terms (7), (8), (9) and (10), and therefore also Δd_{ps} , either improves or stays the same. $\square$

Fig. 4: Illustration of Lemma 4. In N vs 2 situations, the optimal solution is a flanking maneuver, as in Observation 2 and Observation 3.

Lemma 4. (N vs 2)

With N_p arbitrary and $N_q = 2$ the optimal formation is such that all P agents are located at the same position, on the x -axis of Definition 3, with r_{11} depending on the nature of d_{psPi} and d_{psQi} . Thus the P agents are positioned on the flank of the Q team, as illustrated in Figure 4.

Proof. Assume that some given configuration is optimal. By Lemma 3 we can first move p_1 , the agent closest to any Q agent, down to the x -axis, using translation *II*, with equal or improved Δd_{ps} . We can then translate all other agents p_j to p_1 , using *I, II, III* if they are on the right of the y -axis, or *II, III* if they are on the left of the y -axis, with equal or improved Δd_{ps} . Thus the described formation is indeed optimal. The value of r_{11} depends on the nature of d_{psPi} and d_{psQi} . $\square$

Note that this corresponds to a flanking maneuver, as in Observation 2, in the form of a tight group, as in Observation 3.

So far we have seen both flanking maneuvers and grouping in the optimal solutions to Problem 1, but we have only seen vague examples of the line formation in Observation 1, in the case with small, but non-zero ϕ in Lemma 2. The reason for spreading out in a line, instead of forming a tight group, is that a tight group presents a target that is easier to hit, especially with weapons that can affect more than one target. In the next section we will add an assumption on area damage to account for this effect, allowing agents to take damage from attacks that are not aimed at them.

V. NUMERICAL RESULTS

In this section we will present numerical optimization results that verify and extend the analytical results of the

Fig. 5: Illustration of d_{psPi} and d_{psQi} . Note the advantage of the P -agents at distances between 2 and 3 kilometers.

Fig. 6: Illustration of how Δd_{ps} varies as a function of p_1 , with p_2 is fixed at the optimal position. The optimal positions of both are indicated in the lower part of the figure, and corresponds to the solution found in Figure 4.

previous section. First we present explicit examples of d_{psPi} and d_{psQi} , and use them to compute solutions corresponding to the (1 vs 2) and (2 vs 2) cases. Then we add the possibility of area damage, in Definition 4, and show how the solution resemble the line formation seen in Observation 1. Finally we compute solutions for larger scenarios, (4 vs 4), and dynamic scenarios where the agent move in a series of short time steps to optimize Δd_{ps} .

All numerical results were created using algorithms from the SciPy package [21].

So far, we only assumed that d_{psPi} and d_{psQi} are decreasing (Assumption 1). To enable numerical computations we choose the following functions

$$d_{psPi}(x) = 1 - \frac{1}{1 + e^{-3.8x+10}} \quad (11)$$

$$d_{psQi}(x) = 1 - \frac{1}{1 + e^{-3.8x+8.5}}. \quad (12)$$

The functions are chosen to roughly model tank guns, with high (expected) damage for ranges up to 1.5 kilometers and very low damage at over 3 kilometers. As can be seen in Figure 5, the functions are a bit different, with the P -agents having slightly better performance at distances of 2 to 3 kilometers. The reason for this is to give the P -agents a small advantage which can be exploited by the proper positioning, to avoid optimal solutions that just maximize the distance to the Q -agents.

Fig. 7: Illustration of how Δd_{ps} varies as a function of p_1 , using the area damage of Definition 4.

Computing the solution to the problem of 1 P -agent against 2 Q -agents, i.e., 1 vs 2, we get the solution in Figure 6. As can be seen, the results are in agreement with Lemma 4 and Figure 4. The optimal solution is $p_1 = (-2.51, 0)$, with the other agents at $q_1 = (0, 0)$ and $q_2 = (0.5, 0)$.

Adding one additional agent to the P -team, we obtain results for 2 vs 2 that are similar to Figure 6, with $p_1 = p_2 = (-2.30, 0)$, as expected from Lemma 4 and Figure 4.

Having verified the analytical results of the previous section we go on to study the effects of weapons with area damage. Thus we replace Definition 1 with Definition 4 below.

Definition 4. (Damage Per Second, Multiple Targets)

Agent q_i is not only inflicting damage on its target, T_{q_i} , but also on other opponents standing close to T_{q_i} , with a magnitude that decreases with both the distance between target and q_i , $\|T_{q_i} - q_i\|$ and the distance between the target and the nearby agent $\|T_{q_i} - p_j\|$. We now have a dependency on the position of both p and q and the target T_q , which might be different from p , thus: $d_{ps} : \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}_+$.

In the following examples we use

$$d_{psQi(q,p)} = \left(1 - \frac{1}{1 + e^{-3.8\|T_q - q\| + 8.5}}\right) e^{-81\|T_q - p\|^2}, \quad (13)$$

where the factor $e^{-81\|T_q - p\|^2}$ provides maximal damage to $p = T_q$ and additional damage to p -agents close to T_q .

Running the 2 vs 2 scenario with the new damage function, we see that it is no longer optimal to stay in a tight group, instead the solution is a small line formation, see Figure 7. The optimal positions are $p_1 = (-2.29, -0.17)$, $p_2 = (-2.29, 0.17)$.

We will now study the numerical solution to a number of 4 vs 4 cases.

The Q -formations employed will be the *Line*, *Vee*, *Wedge*, and *Coil*, as specified in [22], and the optimization will determine the optimal response from the P -team. Both the Q -formations listed above, and the resulting P -formations can be seen in Figure 8, with the corresponding values Δd_{ps} being 2.30 (*Line*), 2.21 (*Vee*), 2.13 (*Wedge*), 1.84 (*Coil*).

(a) Line formation

(b) Vee formation

(c) Wedge formation

(d) Coil formation

Fig. 8: Illustration of the optimal P -formation given four Q -agents in different formations.

In all cases the optimal response of the P -agents is a *Line*-formation performing a flanking maneuver, as seen in Observations 1 and 2, with the area damage effect making the agent prefer a line to a tight group. The spacing of the *Line*-formation is about the same, with all agent spread out across roughly 1km, but the direction from which to perform the flanking maneuver depends completely on the Q -formation.

Finally, from a Q -team perspective, it is interesting to note

that the optimal value of Δd_{ps} is worst for the *Line* and best for the *Coil*. Thus, when the enemy is allowed to choose formation and direction of attack, the omnidirectional *Coil* is less vulnerable. This observation makes perfect sense, given the instructions of [22], where the *Coil* is described as “a circular formation covering all suspected enemy avenues of approach”.

To conclude this section, we will illustrate how the proposed approach can be used to guide motion over a sequence of shorter time steps. Thus, we solve Problem 1 as described above, with the additional constraints that the *P*-agents are only allowed to choose new position within some bounded distance of their current positions. Then we move the agents to the newly computed positions and solve a new instance of Problem 1 with the same constraint of the next positions being close to the current ones. The solution can be found in Figure 9. As can be seen, the *P*-agents first form a line formation, then move towards the desired enemy distance and converge on the optimal flanking position.

Fig. 9: By constraining the new positions to be close to the previous ones, and solving a sequence of such problems, we can model the motion of the *P* agents over time. The initial positions are close to $(-3, -2)$ and dashed lines indicate concurrent positions of p_1 and p_2 . Note the similarity to step II and III of Figure 3. Step I is not needed since the agents start on the same side. The order of I, II, and the final target distance, is given by the choice of d_{ps} in Figure 5.

VI. CONCLUSIONS

In this paper we have demonstrated how classical formation patterns and flanking maneuvers can be seen in the solutions to optimization problems. Given a few natural assumptions on decreasing efficiency over range and greedy target selection we have shown that a flanking maneuver using a line formation is optimal against a set of target formations taken from the literature. We have provided general analytical results for smaller scenarios as well as explicit numerical results for larger scenarios, and also verified that these two predict similar solutions.

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