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Optimal Speed Control of a Heavy-Duty Vehicle in the Presence of Traffic Lights

Manne Held^{1,2}, Oscar Flårdh¹ and Jonas Mårtensson²

Abstract—The fuel consumption of heavy-duty vehicles in urban driving is strongly dependent on the acceleration and braking of the vehicles. In intersections with traffic lights, large amount of fuel can be saved by adapting the velocity to the phases of the lights. In this paper, a heavy-duty vehicle obtains information about the future signals of traffic lights within a specific horizon. In order to minimize the fuel consumption, the driving scenario is formulated as an optimal control problem. The optimal control is found by applying a model predictive controller, solving at each iteration a quadratic program. In such problem formulation, the constraints imposed by the traffic lights are formulated using a linear approximation of time. Since the fuel-optimal velocity can deviate strongly from how vehicles normally drive, constraints on the allowed velocity are imposed. Simulations are performed in order to investigate how the horizon length of the information from the traffic lights influences the fuel consumption. Compared to a benchmark vehicle without knowledge of future light signals, the proposed controller using a control horizon of 1000 m saves 26 % of energy with similar trip time. Increasing the control horizon further does not improve the results.

I. INTRODUCTION

The European Union has committed to reducing greenhouse gas emissions to 80-95 % below 1990 levels by 2050 [1]. Since the transport sector accounts for as much as 22 % of the CO₂ emissions in the EU [2], the emissions in this sector must be reduced to comply with the commitment. This can be done by reducing the fuel-consumption of vehicles. One way of doing so is by controlling them using look-ahead information, i.e., information about the upcoming road and traffic. This is done for a heavy-duty vehicle (HDV) in highway driving in [3], with experimental results of 3.5 % fuel reduction. Such techniques are today used in commercial solutions, for instance Scania Active Prediction [4] with fuel savings of 3 %.

Fuel-efficient control strategies are less studied in the case of urban driving. In the papers that constitute [5], it is investigated how look-ahead control can be applied to driving scenarios with large variations in the velocity, such as in urban driving. Simulations are performed on a driving cycle where the vehicle frequently encounters intersections where it must stop. This paper uses a similar model and problem formulation, but with intersections controlled by traffic lights instead of mandatory stop. The situation can be seen in

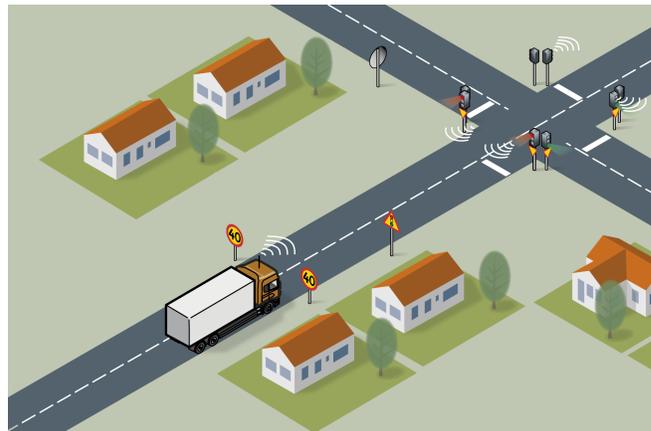


Fig. 1: A heavy-duty vehicle driving in an urban environment with intersections controlled by traffic lights.

Fig. 1, where an HDV approaches an intersection with traffic lights. The vehicle obtains signal phase and timing (SPaT) information, which is used to predict the control of the vehicle in a fuel-efficient way.

In recent years, there have been many different approaches to reducing the fuel consumption of vehicles driving in the presence of traffic lights. In [6], an electric car in an urban traffic network utilizes Infrastructure to vehicles (I2V) information to find energy-optimal velocity profiles. By first generating possible passing times of the traffic lights over the full length of the driving mission, Dijkstra's algorithm is then used to find the shortest path in terms of energy. The result by following the suggested trajectory is then compared to the optimal solution found by a much slower Dynamic programming (DP) algorithm.

In [7], the optimal velocity trajectory is first calculated by ignoring the traffic lights. These are then added, and if following the planned trajectory leads to passing a traffic light at red signal, the nearest green phase is calculated. Then the constraints for passing the traffic lights are used for solving the optimal control problem analytically.

Rule-based algorithms includes [8] and [9], where an analysis of the optimality condition gives that the vehicle can be in a few different states such as acceleration, constant speed and coasting. The selection between these is based on the expected arrival time at the traffic lights. A statistical model of the fuel consumption is used in [10] to find the most fuel-efficient way of passing traffic lights.

A Model predictive controller (MPC) is developed in [11] where the cost function penalizes fuel consumption and

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¹Scania CV AB, 151 87 Södertälje, Sweden.

²Department of Automatic Control and the Integrated Transport Research Lab, KTH Royal Institute of Technology, 100 44 Stockholm, Sweden.
manneh@kth.se, oscar.flardh@scania.com,
jonas1@kth.se

deviation from a reference speed, which is calculated based on the current signal of the traffic light. A similar cost function is used in [12], where the time of passing the traffic lights is set as the intersection between the allowed speed based on legal restrictions and the set of speed for reaching the traffic light at a green signal. Another approach using an MPC is [13], where two different controllers are developed for an HEV. One MPC is used when the probability of passing a traffic light at red is very low, and it minimizes fuel consumption and deviation from reference speed etc. Another MPC is used when it is likely to arrive at the traffic light at red signal, in which case the time and distance from the traffic light is also included in the cost function.

A cooperative adaptive cruise controller is developed for a car-following scenario in the presence of traffic lights in [14]. The future speed of the preceding vehicles is not fully known, but estimated based on measurements and on I2V information from the traffic lights. As noted in [15], getting access to signal timing plans can be difficult due to the many different entities managing the traffic lights. SPaT information from fixed-time traffic lights that are not influenced by traffic can be derived by measuring and analyzing the phases. However, it is noted in the same paper that the significant drift of the clock in the traffic lights makes this analysis hard.

A pre-study for this paper was performed in [16], but with a linear program (LP) formulation and without the lower velocity constraint.

A. Contributions

The two main contributions of this paper and the way it differs from the work described above is:

- The formulation of the problem of minimizing the energy consumption of an HDV driving in the presence of traffic lights as a Quadratic program (QP).
- An algorithm to find the energy-optimal timing of passing the traffic lights when the vehicle is constrained by a minimum allowed velocity.

The optimal control problem on QP-form is used in two different ways: (a) to find the optimal way of passing the traffic lights when a new traffic light appears at the end of the horizon and (b) as an MPC to find the optimal control at all other positions. The MPC in (b) is needed in addition to (a), because it handles new road grade information at each position and because linearizations in the QP-model make the state prediction differ from the real value.

II. VEHICLE AND TRAFFIC LIGHT MODELS

The model of the HDV is given in section II-A using the same parametric values as in [5]. The model of the traffic lights is given in section II-B.

A. Vehicle model

Kinetic energy K is used as the state variable as a function of position s . Position is chosen as the independent variable since the road grade and the traffic lights are given as

functions of position. The dynamics of the vehicle are given by

$$\frac{dK(s)}{ds} = F_t(s) + F_b(s) + F_a(K(s)) + F_r(s) + F_g(s) \quad (1)$$

where F_t and F_b are the controllable tractive and braking forces respectively and the air resistance, rolling resistance, and gravitation are given by

$$F_a(K(s)) = -\frac{\rho A_f c_d K(s)}{m} \quad (2a)$$

$$F_r(s) = -mg c_r \cos(\alpha(s)) \quad (2b)$$

$$F_g(s) = -mg \sin(\alpha(s)) \quad (2c)$$

where ρ is the air density, A_f is the vehicle frontal area, c_d is the air drag coefficient, m is the vehicle mass, c_r is the coefficient for the rolling resistance, g is the gravitational constant, and α is the road slope.

B. Traffic light model

The signal switching of the traffic lights is modeled such that g_{np} denotes the start of phase p of the traffic light at the position with index n . The start of the succeeding red signal is denoted r_{np} . The condition for the time t when the vehicle is allowed to pass the traffic light with position index n is then given by

$$\bigcup_{p=1}^{\infty} g_{np} \leq t_n \leq r_{np}. \quad (3)$$

The length of the green and red phases of each traffic lights are drawn randomly from uniform distributions similar to the values used in [11]. For traffic light n , the length of the green phase is drawn from [15, 30] s and the length of the red phase is drawn from [23, 44] s. The initial time offset of each traffic light is drawn uniformly from its total period [0, 74] s.

III. PROBLEM FORMULATION

Time is used in two ways in the problem formulation: (a) to constrain the vehicle to pass the traffic lights at green signal, and (b) as a term in the cost function, which is mainly important when no traffic light is within sight. The relation between position, kinetic energy and time is given by

$$dt = ds \sqrt{\frac{m}{2}} K^{-1/2}. \quad (4)$$

The driving mission is formulated as an optimal control problem on QP-form in subsection III-A. Due to the nonlinear relation in (4), t is approximated by Taylor approximations of the kinetic energy around a reference K_r .

$$K^{-1/2} = K_r^{-1/2} - \frac{1}{2} K_r^{-3/2} (K - K_r) + \frac{3}{8} K_r^{-5/2} (K - K_r)^2 + \dots \quad (5)$$

Passing the traffic lights at green signal is formulated as inequality constraints on the time approximation. In a QP, the inequality constraints must be affine in the optimization variables, and the linear term of the Taylor approximation is

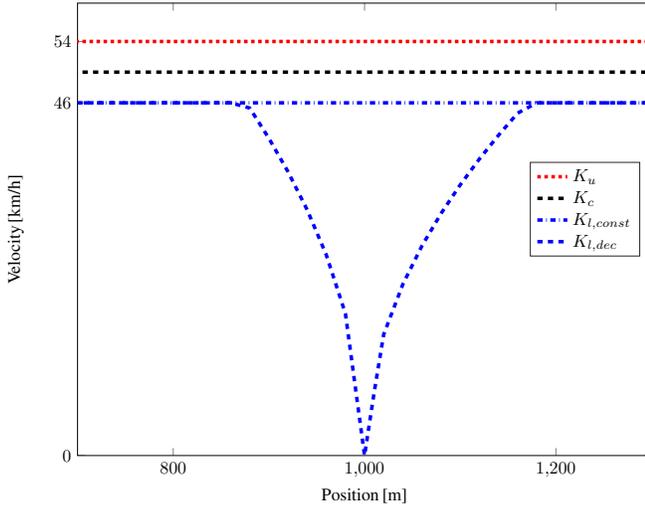


Fig. 2: The driving corridor near a traffic light at position 1000m when additional traffic lights can be seen further in the horizon. Either $K_{l,dec}$ or $K_{l,const}$ is used in the optimal control problem formulation depending on whether the vehicle must slow down at this traffic light or not.

therefore the highest that can be included. In this case the time approximation is given by

$$dt_k \approx ds (\phi_{0,k} + \phi_{1,k} K_k) \quad (6)$$

where k is the position index and

$$\phi_{0,k} = \frac{3}{2} \sqrt{\frac{m}{2}} K_{r,k}^{-1/2} \quad (7a)$$

$$\phi_{1,k} = -\frac{1}{2} \sqrt{\frac{m}{2}} K_{r,k}^{-3/2} \quad (7b)$$

from (5). The cost function however, may include quadratic terms, and the trip time can therefore be approximated by a second order Taylor approximation of the kinetic energy

$$dt_k \approx ds (\theta_{0,k} + \theta_{1,k} K_k + \theta_{2,k} K_k^2) \quad (8)$$

where (5) gives

$$\theta_{0,k} = \frac{15}{8} \sqrt{\frac{m}{2}} K_{r,k}^{-1/2} \quad (9a)$$

$$\theta_{1,k} = -\frac{10}{8} \sqrt{\frac{m}{2}} K_{r,k}^{-3/2} \quad (9b)$$

$$\theta_{2,k} = \frac{3}{8} \sqrt{\frac{m}{2}} K_{r,k}^{-5/2}. \quad (9c)$$

A. Optimal control problem

The problem is discretized using zero order hold. The problem can then be formulated as a QP as:

$$\min_{F_t, F_b} \sum_{j=k}^{k+N_H-1} \Delta s F_{t,j} - K_{k+N_H} \quad (10a)$$

$$+\beta \Delta s (\theta_{0,j} + \theta_{1,j} K_j + \theta_{2,j} K_j^2)$$

$$\text{s.t.} \quad K_{j+1} = AK_j + BF_j + v_j \quad (10b)$$

$$K_{l,j} \leq K_j \leq K_{u,j} \quad (10c)$$

$$F_{t,j} \leq a_1 + a_2 K_j \quad (10d)$$

$$0 \leq F_{t,j} \leq F_{t,max} \quad (10e)$$

$$-F_{b,max} \leq F_{b,j} \leq 0 \quad (10f)$$

$$g_{np} \leq \Delta s \sum_{j=k}^n \phi_{0,j} + \phi_{1,j} K_j \leq r_{np} \quad (10g)$$

$$K_k \text{ given.} \quad (10h)$$

The terms in the cost function (10a) represents the energy converted by the tractive force, the kinetic energy at the end of the horizon, and the approximated trip time over the horizon weighted by a parameter β . Furthermore, the state dynamics are given in (10b), the lower and upper constraints on the kinetic energy in (10c), velocity dependent and constant constraints on the tractive force in (10d) and (10e) respectively, the constraint on the braking force in (10f), the constraint for passing the traffic lights at position index n at green signal in (10g), and the known current kinetic energy in (10h). In the expression above,

$$a_1 = \frac{3}{2} P_{max} \sqrt{\frac{m}{2}} K_{r,j}^{-1/2} \quad (11a)$$

$$a_2 = -\frac{1}{2} P_{max} \sqrt{\frac{m}{2}} K_{r,j}^{-3/2}. \quad (11b)$$

Furthermore, A, B and v_j are given by

$$A = e^{A_c \Delta s}, \quad (12a)$$

$$B = \int_0^{\Delta s} e^{A_c s} B_c ds \quad (12b)$$

$$v_j = \left(\frac{e^{A_c \Delta s} - 1}{A_c} \right) mg (\sin \alpha_j + c_r \cos \alpha_j) \quad (12c)$$

where

$$A_c = -\frac{\rho A_f c_d}{m} \quad (13a)$$

$$B_c = [1 \quad 1]. \quad (13b)$$

The parameter β has the effect that the optimal control in the absence of traffic lights and large road grade is to keep constant speed v_c . The relation between them, derived in [5], is given by $\beta = \rho A_f c_d v_c^3$. The reference trajectory K_r is the kinetic energy given by the state prediction from the previous iteration. The kinetic energy at the start is $K_0 = K_c = v_c^2/2$, and the reference trajectory for the very first iteration is set to this value for the full horizon.

B. The driving corridor

The position dependent upper and lower constraint on the kinetic energy (10c) are referred to as the driving corridor [5]. The idea of the lower constraint is to constrain the vehicle to drive in a way that does not deviate too much from a normal way of driving. Coasting for a long distance ahead of a red signal in order to arrive at green might be optimal

from an energy perspective, but could be disturbing both to the driver and to surrounding vehicles. The constraint is set based on a statistical analysis of how similar vehicles normally decelerate [5].

The scenario in this paper is a road with reference speed K_c corresponding to 50 km/h with K_u corresponding to 54 km/h and $K_l = K_{l,const}$ corresponding to 46 km/h if not in the proximity of a traffic light. When in the proximity of a traffic light, $K_l = K_{l,dec}$, which allows the vehicle to decelerate and stop. The last constraint is based on the statistical analysis on average deceleration rates in [5]. These constraints can be seen in Fig. 2.

C. Idling during red signal

Due to the lower constraint on the velocity, the vehicle might have to stop at red signals. However, this cannot be taken into account by the constraint (10g). Therefore, a variable representing the idle time at the traffic light at position n is introduced as τ_n . Before the controller is called, it is checked whether or not there is a feasible solution of (10) that does not have to idle at red. If not, the following modifications are performed: The idle time is added to the cost function such that (10a) becomes

$$\min_{F_t, F_b} \sum_{j=k}^{k+N_H-1} \Delta s F_{t,j} - K_{k+N_H} + \beta (\tau_n + \Delta s (\theta_{0,k} + \theta_{1,k} K_k + \theta_{2,k} K_k^2)), \quad (14)$$

the idle time is added to the time constraint such that (10g) becomes

$$g_{np} \leq \tau_n + \Delta s \sum_{j=k}^l \phi_{0,j} + \phi_{1,j} K_j \leq r_{np} \quad (15)$$

and finally, in order to only allow the vehicle to idle at standstill, (10c) is modified for index n to

$$K_n = 0. \quad (16)$$

IV. OPTIMAL PASSING TIMES

When a traffic light appears at the end of the horizon, the optimal times for passing the traffic lights within sight are found. This can be seen as finding the best path between red signals. First, the alternatives for the possible paths are found by a pathfinding algorithm. Next, the optimal path among these is found by a path selection algorithm. The following two subsections describe these algorithms.

A. Path finding

When a new traffic light can be seen within the control horizon for the first time, the set of possible phases during which the vehicle can pass the n_{TL} lights within sight is derived. This is done using Algorithm 1. Since the vehicle is constrained by a minimum allowed velocity, there is a limited number of green phases to pass the traffic light at, given that it does not idle during green signal. To find the possible timings, the algorithm $TL = \text{PathCreator}(k_0, t_0, t_0)$ is called where k_0 is the index of the nearest traffic light and

t_0 is the current time. Each pair of two columns in the matrix TL consist of the start and end time of the possible passing times with the rows corresponding to the traffic lights within sight with the nearest on the first row.

Algorithm 1 $TL = \text{PathCreator}(k, t_{k,min}, t_{k,max})$

```

 $t_{min} \leftarrow t_{k,min} + \text{time when using } K = K_u$ 
 $t_{max} \leftarrow t_{k,max} + \text{time when using } K = K_{l,dec}$ 
find min  $i$  such that  $t_{min} \leq r_{ki}$ 
find min  $j$  such that  $t_{max} \leq g_{kj}$ 
if  $k == n_{TL}$  then
     $TL \leftarrow [g_{ki}, r_{ki}, \dots, g_{kj}, r_{kj}]$ 
else
     $TL \leftarrow []$ 
    for all  $l$  such that  $i \leq l \leq j$  do
         $t_{k,l,min} \leftarrow \text{first time to pass during phase } l$ 
         $t_{k,l,max} \leftarrow \text{last time to pass during phase } l$ 
         $TL_{l,k+1} \leftarrow \text{PathCreator}(k+1, t_{k,l,min}, t_{k,l,max})$ 
         $TL_l \leftarrow [t_{g,k}, t_{r,k}; TL_{l,k+1}]$ 
         $TL \leftarrow [TL, TL_l]$ 
    end for
end if
return  $TL$ 

```

B. Path selection

When all possible phases for when to pass the traffic lights have been found by algorithm 1, the cost for each of them is calculated. The costs are compared and the path with the lowest cost is chosen and used to set constraint (10g). In some cases, the cost can be calculated directly by solving (10) for a given path. However, due to the existence of a minimum allowed velocity, the vehicle might have to stop at a red signal. In these cases, the controller is called with the modifications discussed in section III-C.

A special situation that can occur when several traffic lights can be seen is that the vehicle can pass the first one without slowing down below $K_{l,const}$ but having to stop at a latter one. In such case, the vehicle could possibly save energy by slowing down already at the first traffic light. This would correspond to unnecessarily driving slower than $K_{l,const}$ around a traffic light with green signal. This situation is prohibited, if the vehicle can pass the first traffic light without slowing down, by setting $K_{l,const} \leq K_j$ in (10c). If the vehicle must slow down below $K_{l,const}$, it is said to *creep*. These constraints can be seen in Fig. 2. The algorithm for finding the value of a given path is summarized in algorithm 2.

C. Benchmark vehicle

To investigate the benefits with the MPC, a benchmark controller was developed to compare with. This controller represents a simplified model of an uninformed driver. Since such vehicle risks passing a light just as it turns red, the initial part of the red signal is considered amber with uniform distribution [3, 4] s. The controller was implemented in the following manner: Acceleration to cruising kinetic energy K_c

Algorithm 2 $cost = PathCost()$

$\mathcal{N} \leftarrow$ set of position indices of traffic lights within sight
for $n \in \mathcal{N}$ **do**
 calculate maximum time t_{max} by $K = K_l$
 calculate the time t_{low} by $K = K_{l, const}$
 $stop_n \leftarrow t_{max} \leq g_{np}$
 $creep_n \leftarrow t_{low} \leq g_{np}$
 if $stop_n$ **then**
 set $K_l = K_{l, dec}$ and $K_n = 0$
 else if $creep_n$ **then**
 set $K_l = K_{l, dec}$
 else
 set $K_l = K_{l, const}$
 end if
end for
if NONE($stop$) **then**
 $cost$ given by solving (10)
else
 $cost$ given by solving (10) modified by (14)-(16)
end if
return $cost$

using maximum power, cruising at constant kinetic energy K_c when it is reached, and coasting in downhills up to K_u where braking starts. If a red or amber signal of a traffic light is within sight, the vehicle uses constant deceleration to stop. If a green signal turns amber while within sight however, the vehicle continues if it is possible to pass before the signal turns red. The vehicle obtains information about the current signal when it is within 100 m.

V. SIMULATION RESULTS

Simulations were performed in Matlab using the tool Yalmip [17], and the solver quadprog. The road grade was set using data from a drive cycle used at Scania CV AB as a reference cycle for testing urban driving. A section of 8.5 km was chosen from the drive cycle and traffic lights were placed every 500 m and the position space was discretized with $\Delta s = 10$ m. The simulation was repeated with 10 different sets of randomly drawn sequences of traffic light signals. The simulations were run with a laptop equipped with an Intel(R) Core(TM) i7-6820HQ CPU at 2.70 GHz and 16 GB of RAM. The average computational time for solving one iteration of (10) was 18 ms for the shortest horizon 100 m corresponding to 11 position steps and 197 ms for the longest horizon 1000 m corresponding to 101 position steps. With a velocity of 50 km/h, the vehicle drives the discretization distance 10 m in 720 ms. When used as an MPC, (10) converges in one or two iterations and the MPC thus works in real time even for the longest horizon. When a new traffic light appears at the horizon however, several paths must be checked and the convergence time is longer.

Simulations were performed in order to investigate the influence of the length of the control horizon on the energy consumption. The length of the horizon in relation to the distance between the traffic lights is of special interest. The

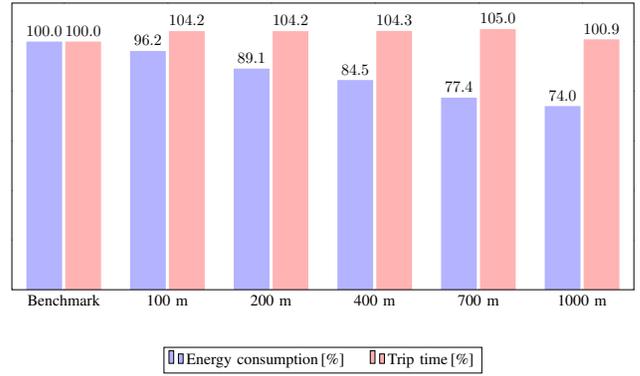


Fig. 3: Energy consumption and trip time for different control horizons normalized with the benchmark controller.

resulting energy consumption and trip time for different control horizons normalized by the benchmark controller can be seen in Fig. 3. Increasing the control horizon further than 1000 m does not improve the results.

For a qualitative comparison, results from one of the simulations with the benchmark controller and the MPC with horizon length 1000 m are shown in Fig. 4. It can be seen that the MPC keeps a more even velocity than the benchmark controller and uses both less tractive and braking force.

Two situations of extra interest in Fig. 4 are shown in Fig. 5. The first one is ahead of the traffic light at 1500 m shown in Fig. 5(a). Because of the downhill, the benchmark controller gains velocity, but has to brake as it approaches the red signal. The MPC however, keeps minimum allowed velocity in order to arrive at the traffic light as it turns green and can pass it without stopping. In the second situation, shown in Fig. 5(b), the MPC saves both time and energy compared to the benchmark controller. Before the traffic light at 7000 m, the MPC lowers the velocity slightly in order to pass the traffic light when it has turned green. The benchmark controller however, cannot foresee the green signal but brakes due to the red signal. By doing so, it gets behind the MPC and has to stop at the next light. The MPC controller consumes less energy in this case, since it only lowers the velocity at one traffic light, while the benchmark controller does so at three.

VI. CONCLUSIONS

An optimal controller in the form of an MPC has been developed for driving an HDV in the presence of traffic lights. Compared to a benchmark controller without information about future signals of the traffic lights, the MPC reduces the energy consumption by 26% while increasing the trip time by 1% by using a 1000 m control horizon. This horizon corresponds to always seeing two traffic lights ahead, and no further savings are found by using longer control horizons.

This paper assumes the availability of SPaT information within the control horizon and no external disturbances. Possible extensions include adding uncertainties to the SPaT information, and to use different horizon lengths for the SPaT information and the road grade.

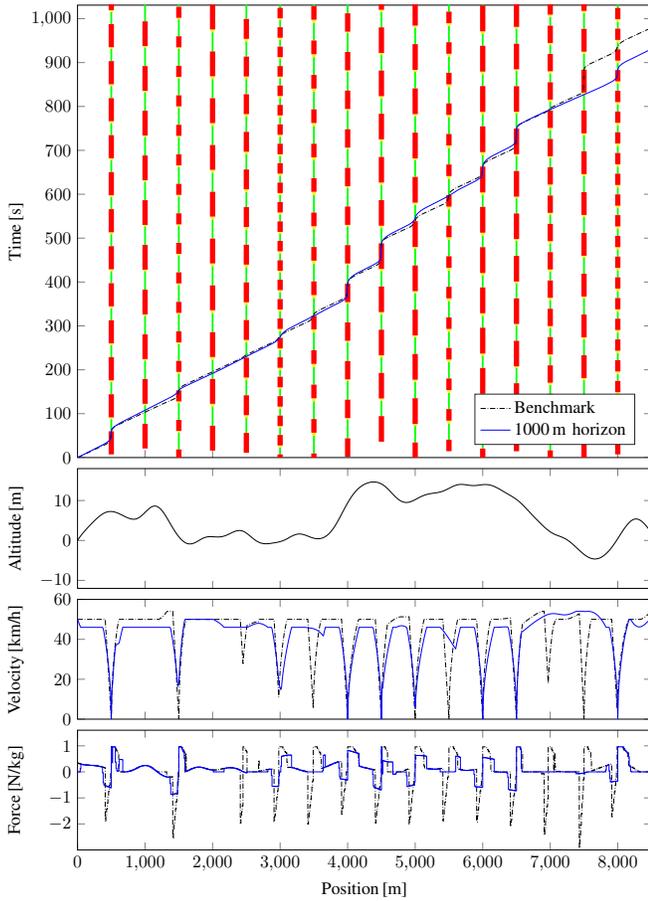


Fig. 4: The benchmark controller compared with the MPC with a control horizon of 1000m showing from above: time propagation together with the traffic light signals, road altitude, velocity, and control force.

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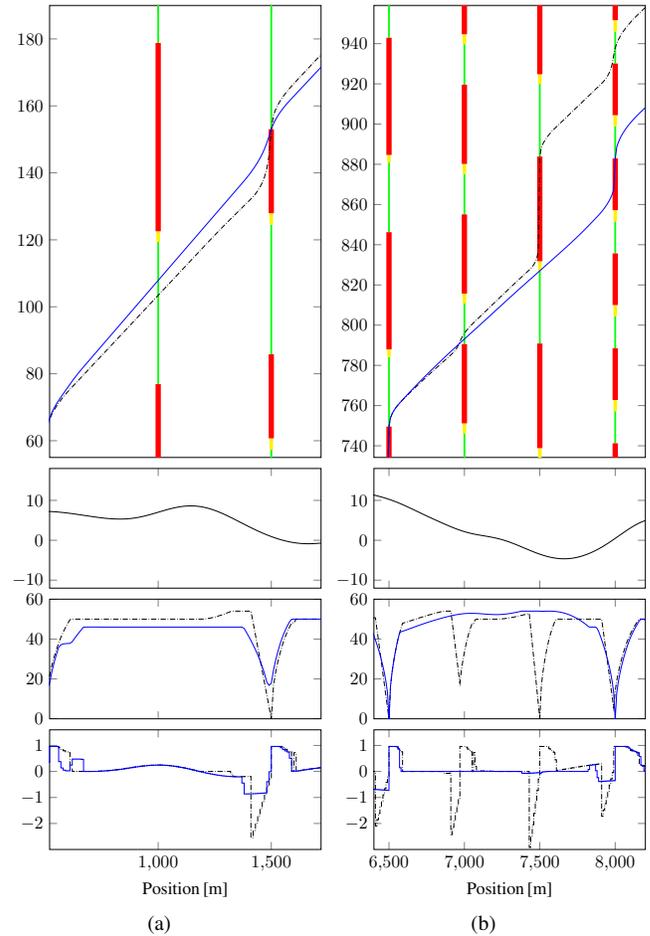


Fig. 5: Two situations of extra interest from Fig. 4.

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