



Optimal Transmit Strategies for Multi-antenna Systems with Joint Sum and Per-antenna Power Constraints

PHUONG LE CAO

Doctoral Thesis
Stockholm, Sweden 2019

Division of Information Science and Engineering
School of Electrical Engineering and Computer Science
KTH Royal Institute of Technology

TRITA-EECS-AVL-2019:21
ISBN 978-91-7873-134-3

SE-100 44 Stockholm
SWEDEN

Akademisk avhandling som med tillstånd av Kungl Tekniska högskolan framlägges till offentlig granskning för avläggande av teknologie doktorsexamen i electro- och systemteknik fredag den 26 April 2019 klockan 13:00 i Kollegiesalen, Brinellvägen 8, Stockholm.

© 2019 Phuong Le Cao, unless otherwise noted.

Tryck: Universitetsservice US AB

To my family

Abstract

Nowadays, wireless communications have become an essential part of our daily life. During the last decade, both the number of users and their demands for wireless data have tremendously increased. Multi-antenna communication is a promising solution to meet this ever-growing traffic demands. In this dissertation, we study the optimal transmit strategies for multi-antenna systems with advanced power constraints, in particular joint sum and per-antenna power constraints. We focus on three different models including multi-antenna point-to-point channels, wiretap channels and massive multiple-input multiple-output (MIMO) setups. The solutions are provided either in closed-form or efficient iterative algorithms, which are ready to be implemented in practical systems.

The first part is concerned with the optimal transmit strategies for point-to-point multiple-input single-output (MISO) and multiple-input multiple-output (MIMO) channels with joint sum and per-antenna power constraints. For the Gaussian MISO channels, a closed-form characterization of an optimal beamforming strategy is derived. It is shown that we can always find an optimal beamforming transmit strategy that allocates the maximal sum power with phases matched to the complex channel coefficients. An interesting property of the optimal power allocation is that whenever the optimal power allocation of the corresponding problem with sum power constraint only exceeds per-antenna power constraints, it is optimal to allocate maximal per-antenna power to those antennas to satisfy the per-antenna power constraints. The remaining power is distributed among the other antennas whose optimal allocation follows from a reduced joint sum and per-antenna power constraints problem with fewer channel coefficients and a reduced sum power constraint. For the Gaussian MIMO channels, it is shown that if an unconstrained optimal power allocation for an antenna exceeds a per-antenna power constraint, then the maximal power for this antenna is used in the constraint optimal transmit strategy. This observation is then used in an iterative algorithm to compute the optimal transmit strategy in closed-form.

In the second part of the thesis, we investigate the optimal transmit strategies for Gaussian MISO wiretap channels. Motivated by the fact that the non-secure capacity of the MISO wiretap channels is usually larger than the secrecy capacity, we study the optimal trade-off between those two rates with different power constraint settings, in particular, sum power constraint only, per-antenna power constraints only, and joint sum and per-antenna power constraints. To character-

ize the boundary of the optimal rate region, which describes the optimal trade-off between non-secure transmission and secrecy rates, related problems to find optimal transmit strategies that maximize the weighted rate sum with different power constraints are derived. Since these problems are not necessarily convex, equivalent problem formulation is used to derive optimal transmit strategies. A closed-form solution is provided for sum power constraint only problem. Under per-antenna power constraints, necessary conditions to find the optimal power allocation are provided. Sufficient conditions, however, are available for the case of two transmit antennas only. For the special case of parallel channels, the optimal transmit strategies can be deduced from an equivalent point-to-point channel problem. In this case, there is no trade-off between secrecy and non-secrecy rate, i.e., there is only a transmit strategy that maximizes both rates.

Finally, the optimal transmit strategies for large-scale MISO and massive MIMO systems with sub-connected hybrid analog-digital beamforming architecture, RF chain and per-antenna power constraints are studied. The system is configured such that each RF chain serves a group of antennas. For the large-scale MISO system, necessary and sufficient conditions to design the optimal digital and analog precoders are provided. It is optimal that the phase at each antenna is matched to the channel so that we have constructive alignment. Unfortunately, for the massive MIMO system, only necessary conditions are provided. The necessary conditions to design the digital precoder are established based on a generalized water-filling and joint sum and per-antenna optimal power allocation solution, while the analog precoder is based on a per-antenna power allocation solution only. Further, we provide the optimal power allocation for sub-connected setups based on two properties: (i) Each RF chain uses full power and (ii) if the optimal power allocation of the unconstrained problem violates a per-antenna power constraint then it is optimal to allocate the maximal power for that antenna.

The results in the dissertation demonstrate that future wireless networks can achieve higher data rates with less power consumption. The designs of optimal transmit strategies provided in this dissertation are valuable for ongoing implementations in future wireless networks. The insights offered through the analysis and design of the optimal transmit strategies in the dissertation also provide the understanding of the optimal power allocation on practical multi-antenna systems.

Sammanfattning

Trådlös kommunikation har idag kommit att bli en viktig del av våra dagliga liv. Under det senaste decenniet har både antalet användare och deras efterfrågan på trådlös data ökat enormt. Att utöka antalet antenner i sändare och mottagare är lovande strategier för att möta det ständigt ökande trafikbehovet. I den här avhandlingen studerar vi optimala transmissionsstrategier för multi-antennsystem med avancerade effektbegränsningar. Mer specifikt antas sammanlänkade begränsningar på total effekt och effekt per antenn. Vi fokuserar på tre olika modeller, nämligen multi-antenn punkt-till-punkt kanaler, wiretap-kanaler samt s.k. massiv MIMO (eng. multiple-input multiple-output) scenarier. Lösningar ges antingen i form av slutna matematiska uttryck, alternativt genom effektiva iterativa algoritmer redo att implementeras i praktiska system.

Den första delen av avhandlingen studerar optimala transmissionsstrategier för punkt-till-punkt MISO (eng. multiple-input single-output) samt MIMO-kanaler med sammanlänkade begränsningar på total effekt och effekt per antenn. För Gaussiska MISO-kanaler härleds en sluten karakterisering av en optimal 'beamforming'-strategi. Vi visar att det alltid går att hitta en optimal 'beamforming'-strategi som allokerar den maximala totaleffekten med faser matchade till de komplexa kanalkoefficienterna. En intressant egenskap hos den optimala effekttallokeringen är att närhelst den optimala effekttallokeringen med enbart total effektbegränsning endast överskrider de individuella begränsningarna för specifika antenner, erhålls en optimal lösning genom att allokera maximal per-antenn effekt till just dessa antenner. Den återstående effekten distribueras sedan över de övriga antennerna enligt ett ekvivalent men reducerat optimeringsproblem med färre kanalkoefficienter. För Gaussiska MIMO-kanaler visas att om en obegränsad optimal effekttallokering för en antenn överskrider den individuella, per antenn angivna, begränsningen så är maximal effekt allokerad för just dessa antenner i den optimala strategin. Denna observation används för att beskriva en iterativ algoritm som beräknar den optimala transmissionsstrategin.

I den andra delen av avhandlingen undersöker vi optimala transmissionsstrategier för Gaussiska MISO wiretap-kanaler. Motiverat av faktumet att den icke-säkrade kapaciteten över MISO wiretap-kanalen vanligtvis är större än den säkrade s.k. 'secrecy'-kapaciteten, studerar vi den optimala avvägningen mellan dessa två överföringshastigheter givet olika effektbegränsningar. Mer specifikt studeras total effektbegränsning enskilt, individuell effektbegränsning per antenn enskilt, samt

sammanlänkade begränsningar på båda dessa. För att hitta regionsgränsen för optimala hastigheter, vilken beskriver den optimala avvägningen mellan icke-säkrad sändning och 'secrecy'-hastighet, härleds lösningar till relaterade problem där vi söker optimala transmissionsstrategier som maximerar den viktade summan av hastigheter med olika effektbegränsningar. Ekvivalenta formuleringar av optimeringsproblemen används för att härleda optimala transmissionsstrategier eftersom ursprungsproblemen ej är konvexa. En optimal lösning för problemet med total effektbegränsning ges i slutet form. För individuell effektbegränsning per antenn tillhandahåller vi nödvändiga villkor för att finna en optimal effekttallokering. Tillräckliga villkor är endast tillgängliga i fallet av två sändarantennerna. För specialfallet av parallella kanaler kan transmissionsstrategier härledas från ett ekvivalent problem för en punkt-till-punkt kanal. I detta fall existerar ingen avvägning mellan icke-säkrade och 'secrecy' kapaciteten, endast en optimal strategi som maximerar båda kapaciteter.

Avslutningsvis studeras optimala strategier för storskaliga MISO samt massiva MIMO system med sammankopplad hybrid analog-digital 'beamforming'-arkitektur, radiofrekvens-kedja samt individuella effektbegränsningar per antenn. Studerat system är konfigurerat så att varje radiofrekvens-kedja matar en grupp av antenner. För det storskaliga MISO systemet tillhandahålls nödvändiga och tillräckliga villkor för att design av optimala analoga och digitala kodningsstrategier ska vara möjligt. Optimal strategi uppnås då fasförskjutningen i varje antenn är matchad till motsvarande kanal, varvid konstruktiv samverkan uppstår. För massiv MIMO ges dessvärre endast nödvändiga villkor. De nödvändiga villkoren för att designa digitala kodningsstrategier etableras baserat på en generaliserad s.k. 'water-filling' effekttallokeringsmetod med sammanlänkade begränsningar på total effekt och effekt per antenn, medan villkoren för de analoga kodningsstrategierna endast är baserade på effektbegränsningar per antenn. Vidare beskriver vi optimal effekttallokering för sammankopplade system baserat på två egenskaper: (i) Varje radiokedja utnyttjas till full effekt, samt (ii) i fallet då optimala effekttallokeringen i det obegränsade problemet överskrider specifika antenners begränsningar fås den optimala lösningen genom att allokera maximal effekt till motsvarande antenner.

Resultaten i denna avhandling visar att framtida trådlösa nätverk kan uppnå högre datahastigheter med lägre effektförbrukning. Den design av optimala transmissionsstrategier som beskrivs i denna avhandling är därför värdefulla i den pågående implementeringen av framtida trådlösa nätverk. De insikter som ges genom analys och design av optimala transmissionsstrategier i avhandlingen ger också förståelse inom optimal effekttallokering i praktiska implementeringar av multi-antennsystem.

List of Papers

The thesis is based on the following papers:

- [A] P. L. Cao, T. J. Oechtering, R. F. Schaefer and M. Skoglund, "Optimal transmit strategy for MISO channels with joint sum and per-antenna power constraints", *IEEE Transaction on Signal Processing*, May, 2016.
- [B] P. L. Cao and T. J. Oechtering, "Optimal Transmit Strategy for MIMO Channel with Joint Sum and Per-antenna Power Constraints", in *IEEE International Conference on Acoustic, Speech and Signal Processing (ICASSP)*, March 2017.
- [C] P. L. Cao and T. J. Oechtering, "Optimal Transmit Strategies for Gaussian MISO Wiretap Channels", submitted to *IEEE Transactions on Information Forensics and Security*, 2018.
- [D] P. L. Cao, T. J. Oechtering and M. Skoglund, "Transmit Beamforming for Single-user Large-scale MISO systems with Sub-connected Architecture and Power Constraints", *IEEE Communication Letter*, Oct, 2018.
- [E] P. L. Cao, T. J. Oechtering and M. Skoglund, "Precoding Design for Massive MIMO systems with Sub-connected Architecture and Per-antenna Power Constraints", in *The 22nd International ITG Workshop on Smart Antennas (WSA 2018)*, March 2018.

In addition to papers A-E, the following papers have also been (co)-authored by the author of this thesis:

- [1] P. L. Cao, T. J. Oechtering, R. F. Schaefer and M. Skoglund, "Optimal Transmission Rate for MISO Channels with Joint Sum and Per-antenna power Constraints", in *IEEE International Conference on Communications (ICC)*, June 2015.
- [2] N. Qi, M. Xiao, T. Tsiftsis, P. L. Cao, M. Skoglund and L. Li, "Energy-Efficient Cooperative Network Coding With Joint Relay Scheduling and Power Allocation", in *23rd International Conference on Telecommunication (ICT)*, May, 2016.
- [3] P. L. Cao, T. J. Oechtering, and M. Skoglund, "Optimal Transmission with Per-antenna Power Constraints for Multiantenna Bidirectional Broadcast Channels", in *The Ninth IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM)*, July 2016.
- [4] N. Qi, M. Xiao, T. Tsiftsis, M. Skoglund, P. L. Cao and L. Li, "Energy-Efficient Cooperative Network Coding With Joint Relay Scheduling and Power Allocation", *IEEE Transaction on Communications*, August 2016.
- [5] P. L. Cao and T. J. Oechtering, "Optimal Trade-off Between Transmission Rate and Secrecy Rate in Gaussian MISO Wiretap Channels", in *International ITG Workshop on Smart Antennas (WSA)*, March 2017.

Acknowledgements

Now that a five-year journey of my Ph.D study has come to an end. I would like to take this opportunity to thank all people who surrounded, encouraged and supported me during these years.

First and foremost, I would like to express my deepest gratitude to my main advisor Prof. Tobias Oechtering. He is the most important person during five years of my academic life. His advices, guidances, encouragement and positive attitude helped me to overcome the difficulties during my Ph.D study. His sound knowledge, enthusiasm, and the sense of responsibility make him a great supervisor. I am proud to be his student. Besides, I would like to express my sincere gratitude to Prof. Mikael Skoglund to be my co-advisor. Mikael has always been kind and supportive throughout my Ph.D study. I am grateful to Prof. Rafael Schaefer from TU Berlin for our collaboration from which I am greatly benefited.

I would like to thank Dr. Martin Schubert from Huawei Technologies - European Research Center, for travelling from Germany to act as the opponent on my thesis. I am grateful to Prof. Elisabeth de Carvalho from Aalborg University, Prof. Kimmo Kansanen from The Norwegian University of Science and Technology (NTNU) and Prof. Mikael Sternad from Uppsala University for acting on the grading committee. I am also grateful to Prof. Mats Bengtsson from KTH Royal Institute of Technology for being the advance reviewer and to Prof. Lars Kildehøj from KTH Royal Institute of Technology for chairing the public defence of my doctoral thesis.

I would like to thank my colleagues Dr. Antonios Pitarokoilis, Dr. Hieu Do, Dr. Kittipong Kittichokechai, Sebastian Schiessl and Dr. Guang Yang for proof-reading parts of the thesis, and Henrik Forssell for a nice Swedish abstract translation. Special thanks to Prof. Mats Bengtsson for his careful reading and insightful comments that help me to greatly improve the quality of the thesis.

It has been my pleasure to be a member of The Division of Information Science and Engineering. I would like to express my thanks to senior faculties Prof. Magnus Jansson, Prof. Ragnar Thobaben, Prof. Ming Xiao, Prof. Saikat Chatterjee, Prof. James Gross, Prof. Peter Händel, Prof. Joakim Jaldén, Prof. Markus for their every effort to make our division more fantastic. I am also thankful to all my colleagues in Plan 3 and 4, in particular, Dr. Antonios Pitarokoilis, Dr. Peter Larsson, Dr. Haopeng Li, Dr. Du Liu, Dr. Tai Do, Dr. Nan Li, Dr. Nan Qi, Dr. Iqbal Hussain, Sebastian Schiessl, Dong Liu, Yu Ye, Dr. Zhao Wang, Dr. Qiwen Wang, Dr. Zuxing Li, Dr. Arun Venkitaraman, Dr. Ehsan Olfat, Hasan Basri Celebi, Dr. Germán

Bassi, Dr. Satyam Dwivedi, Dr. Hadi Ghauch, Sahar Imtiaz, Dr. Guang Yang, Dr. Lin Zhang, Hanwei Wu, Shaocheng Huang, You Yang, Henrik Forssell, Minh Thanh Vu, Marie Maros, Pol del Aguila Pla, Baptiste Cavarec, for all the good times we shared together. I also wish to thank Raine Tiivel for providing administrative supports during my Ph.D study.

I would like to thank all my friends in Sweden, Vietnam and other places for making my life more colorful.

Most importantly, this long journey would not have been possible without encouragement, endless love and support from my family. I wish to thank my parents, my parents in law, my brother for everything they have done for me. I wish to thank my late grandparents for their encouragement. They passed away when I am far from home. But they are always in my heart. Finally, I wish to express my sincerest gratitude to my lovely wife for her love, understanding and support. Thank you for sharing your life with me.

Phuong Le Cao
Stockholm, April 2019

Contents

Abstract	v
Sammanfattning	vii
List of Papers	ix
Acknowledgements	xii
Contents	xiii
Acronyms	xvii

I Thesis Overview

1 Introduction	1
1.1 Challenges for Future Wireless Systems	1
1.2 Key Technologies for Future Wireless Systems	3
1.3 Roles of Advanced Power Constraints in Future Wireless Systems	4
2 Main Contributions	7
2.1 Problem I: Point-to-Point Channels with Joint Sum and Per- antenna Power Constraints	7
2.2 Problem II: Trade-off Between Transmission and Secrecy Rates in Wiretap Channels	15
2.3 Problem III: Precoding Design for Massive MIMO with sub-connected architecture	18
3 Conclusions	25

II Included Papers	27
A Optimal Transmit Strategy for MISO Channels with Joint Sum and Per-antenna Power Constraints	29
A.1 Introduction	31
A.2 System Model and Power Constraints	35
A.2.1 System Model	35
A.2.2 Power Constraints	36
A.3 Problem Formulations and Solutions	37
A.3.1 Review of Known Results	37
A.3.2 Optimization Problem 3 (OP3) - Maximum Transmission Rate with Joint Sum and Per-antenna Power Constraints	38
A.4 Algorithm for Optimal Transmit Strategy	42
A.5 Numerical Examples	43
A.6 Conclusions	48
A.7 Appendix	49
A.7.1 Proof of Proposition A.1	49
A.7.2 Proof of Proposition A.2	49
A.7.3 Proof of Proposition A.3	50
A.7.4 Proof of Theorem A.5	50
A.7.5 Proof of Corollary A.6	52
A.7.6 Proof of Proposition A.8	53
B Optimal Transmit Strategy for MIMO Channel with Joint Sum and Per-antenna Power Constraints	55
B.1 Introduction	57
B.2 Problem Formulation	58
B.3 Optimal Transmit Strategies	59
B.4 Iterative Algorithm	61
B.5 Numerical Example	64
B.6 Conclusions	65
C Optimal Transmit Strategies for Gaussian MISO Wiretap Channels	67
C.1 Introduction	69
C.2 Problem Formulation	72
C.2.1 System Model and Power Constraint	72
C.2.2 Trade-off Between Transmission Rate and Secrecy Rate	72
C.3 Equivalent Problem Formulations and Parametrizations of the boundary of rate region	75
C.4 Analytical Discussion and Solutions	77
C.4.1 Sum Power Constraint Only	77
C.4.2 Per-antenna Power Constraints Only	77
C.4.3 Special Case of Parallel Channels	81

C.5	Numerical Examples	82
C.6	Conclusions	82
C.7	Appendix	83
C.7.1	Proof of Proposition C.1	83
C.7.2	Proof of Theorem C.3	84
C.7.3	Proof of Theorem C.4	86
C.7.4	Proof of Theorem C.5	87
C.7.5	Proof of Theorem C.6	87
C.7.6	Proof of Theorem C.10	88
D	Transmit Beamforming Design for Single-user Large-Scale MISO Systems with Sub-connected Architecture and Power Constraints	89
D.1	Introduction	91
D.2	System Model	93
D.3	Transmit beamforming design	94
D.3.1	Analog precoder	95
D.3.2	Power allocation	96
D.4	Numerical results	98
D.5	Conclusions	99
E	Precoding Design for Massive MIMO Systems with Sub-connected Architecture and Per-antenna Power Constraints	101
E.1	Introduction	103
E.2	System model	105
E.3	Precoding Design	106
E.3.1	Analog precoder	106
E.3.2	Digital precoder	109
E.4	Numerical Results	112
E.5	Conclusions	113
E.6	Appendix	114
E.6.1	Newton's method	114
	References	115

Acronyms

3GPP	The 3rd Generation Partnership Project
4G, 5G	The 4 th , 5 th Generation
CSI	Channel State Information
CSIR	Channel State Information at the Receiver
CSIT	Channel State Information at the Transmitter
i.i.d.	Independent and Identically Distributed
IoT	Internet of Things
JSPC	Joint Sum and Per-antenna Power Constraints
KKT	Karush-Kuhn-Tucker
LTE	Long Term Evolution
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
MU-MIMO	Multiple-User Multiple-Input Multiple-Output
mmWave	millimeter-Wave
OP	Optimization Problem
PAPC	Per-antenna Power Constraints
RF	Radio Frequency
SDP	Semi-Definite Programming
SNR	Signal-to-Noise Ratio
SPC	Sum Power Constraints
SVD	Singular Value Decomposition

SISO Single-Input Single-Output

w.r.t With Respect To

Part I

Thesis Overview

Introduction

Wireless communications play an important role in our society. The development of the wireless technology is not only leading to the emergence of innovative business models, reshaping the fields of health care, banking services and education but also changing the norms of public behavior and social interaction. In the era of Internet of Things (IoT), besides a massive number of smart devices such as smartphones, tablets and portable devices, millions of machines and products will also be connected to the Internet. Following Cisco's white paper *Visual Networking Index: Global Mobile Data Traffic Forecast Update, 2016-2021*, data-based service demands and applications have significantly increased. The overall mobile data traffic is expected to grow to 49 exabytes (49 billion gigabytes) per month by 2021, a threefold increase over current traffic provision and a sevenfold increase over 2016. In order to satisfy such a huge data demand, future wireless networks have to be evolved.

1.1 Challenges for Future Wireless Systems

The increase of user traffic in upcoming years brings many technical challenges to future wireless systems. The first challenge could be the channel capacity. Channel capacity is a fundamental quantity of a communication channel that defines the channel's maximum possible data rate for error-free transmission. One of the most efficient way to enhance the capacity of a wireless channel is to allocate more bandwidth to communication channels. Unfortunately, this solution comes at a very high price since spectral resources are very limited, expensive and tightly regulated commodity. Hence, improving the spectral efficiency is important for future wireless networks and is still an active research area. A different approach is to develop techniques that maximize channel capacity in a given bandwidth [Gol01]. However, some factors that have high impact on the wireless channels such as the propagation loss and interference bring more challenges in reaching the channel capacity limits. A more feasible solution is to develop transmit strategies to achieve transmission rates that are close to the channel capacity limits.

One more important challenge of future wireless communications is the energy efficiency. The energy efficiency is particularly important for the trend of green communication, i.e., reducing the carbon footprint of future wireless networks. Energy efficiency means consuming less energy to accomplish the same tasks. In a wireless network, networking components such as the switching and transceiver systems consume a lot of energy during their operations. In particular, more than 50% of the total energy is consumed by the radio access part, in which the transmission power corresponds to the energy used by power amplifiers, RF chains and feeders take 50-80% [FJL⁺13]. Further, the energy challenge comes not only from the networking components but also from the mobile client devices for whom the battery lifetime is very limited. This comes from the fact that the development in battery technologies lags significantly with respect to the development of other technologies such as processor, storage and transceivers. As a result, the gap between the devices' energy demands and the battery capacity is exponentially increasing [FJL⁺13]. Without a breakthrough in battery technologies, we believe that the battery life will be one of the biggest barriers for users' experiences, in particular when using energy-hungry applications such as video games and data services on terminals and devices. Thus, the reduction in power consumption by devices is fundamentally important in emerging future wireless networks.

The evolution towards future wireless communications also poses new challenges for information security. The challenges arise due to the broadcast nature of the wireless links. Basically, the information security today relies on computation-based cryptographic techniques and associated protocols which contain some major drawbacks such as standardized protections within public wireless networks are not secure enough. Many of their weaknesses are well known [WKX⁺18]. These techniques will also be compromised if the eavesdroppers' devices have sufficient computational power to solve complex mathematical problems. Further, due to the decentralized structure of future wireless networks where devices may randomly join or leave at any time instants, the distribution and management of cryptographic keys becomes very challenging. An alternative approach for the security in future wireless networks is to focus on the secrecy capacity of communication channels. This approach is based on information theory and is referred as the physical layer security [Wyn75, KW10b, KW10a, OH08, LP10, LP09, SSC12, LHW⁺13, WKX⁺18]. In comparison to the cryptography-based security, the physical layer security does not rely on computational complexity and can be used as an additional level of protection on top of the existing security scheme. This means the physical layer security techniques can be used to either perform secure data transmission directly or generate the distribution of cryptography keys in the future wireless networks. Due to the importance of the information security in future wireless communications, we believe that not only a well-integrated security scheme but also a high security level at the physical layer deserves more attention from the research community.

In addition to aforementioned challenges, other issues such as coding, interference cancellation, low latency, full-duplex transmission, beamforming and even reducing the infrastructure and operating cost are important challenges for the

future wireless network research. Indeed, to address the problems above, several technologies have been taken into consideration, in which, some of them will be discussed in the following sections.

1.2 Key Technologies for Future Wireless Systems

In recent years, the race to commercialize future wireless networks, in particular 5G - the latest generation of wireless technology, is speeding up. Many new technologies have been proposed as promising candidates such as *mmWave*, *small cell*, *massive MIMO*, *beamforming* and *full-duplex* [WSNW17], in order to increase the data rates, bandwidth, coverage and connectivity, with a massive reduction in latency and energy consumption. Some of them have already been adopted by 3GPP. In this section, we briefly introduce two key technologies: mmWave and massive MIMO, that are intensively studied for future wireless networks.

Radio spectrum is a fundamental resource for wireless communication. As mentioned above, the two most effective ways to increase the traffic capacity in wireless networks are to allocate available spectrum in other frequency bands and to use the available spectrum more efficiently. Typically, current wireless systems operate in spectrum below 6 GHz. However, this spectrum has been heavily exploited by multiple services. As more devices try to access the same communication resources, we are going to experience slower services and more dropped connections. Therefore, to meet the demand for ever-increasing data rates, connections and traffic volumes, new spectral bands with very wide channels (usually over 100 MHz per user) have been proposed [YC06, CHSY07]. The use of spectrum in higher frequency bands, up to 300 GHz, i.e., the mmWave bands, would increase the spectrum availability significantly. The mmWave spectrum has not been used before and opening it up means the possibility of increasing data rates is far beyond what is possible today. Unfortunately, higher frequency bands suffer from more severe path loss. The mmWave spectrum has some drawbacks such as difficulty to penetrate walls, getting higher oxygen absorption at very high frequencies and even requiring a higher diversity of access resources in the form of very dense deployments. These issues limit the transmission range of the mmWave communication and restrict its application to line-of-sight communication scenarios. To compensate for these drawbacks, massive MIMO systems, which are equipped with a large number of antenna elements, have been employed [Mar10, LLS⁺14, GETL15, LLS⁺14]. Therefore, systems are able to form accurate beams and high antenna directivity.

In wireless communication systems, MIMO is a method to enhance the spectral efficiency using multiple transmit and receive antennas to exploit multipath propagation [VA87, Fos96, FG98, Tel99]. Multi-antenna technologies allow wireless networks to transmit and receive more than one data stream signal simultaneously over the same radio channel to achieve higher data rates. Currently, point-to-point MIMO with up to 8 antennas per terminal has become an essential technology and has been included in some wireless communication standards such as

Wi-Fi, WiMAX and Long Term Evolution (LTE 4G). In the future, advanced multi-antenna technologies will play even more important roles in wireless communication systems. A proper use of multi-antenna technologies will further boost the system capacity and the achievable data transmission rates without requiring a corresponding expansion of the network infrastructure. Recently, massive MIMO has been intensively studied [LLS⁺14, GETL15, LETM14]. As proposed by Marzetta [Mar10, Mar15], the basic idea behind massive MIMO is to equip terminals and base stations with tens or even hundreds of antennas. The main operating principle in massive MIMO is a base station with a massive number of antenna elements that serves a few, non-cooperative single antenna users simultaneously over the same time frequency resource, using simple linear processing techniques. This technology offers higher capacity and better spectral efficiency than current point-to-point MIMO technology. Further, the greater number of antennas in a wireless network using massive MIMO will also make it far more resistant to interference and intentional jamming than current systems that only utilise a handful of antennas [BDP⁺13]. Performance evaluations of massive MIMO system in real propagation environment have been illustrated in [GTER12, GETL15]. In these works, massive MIMO was studied under different outdoor propagation conditions. The authors showed that massive MIMO leads to better orthogonality among channels to different users and better channel stability over conventional MIMO. Further, a paradigm using a combination of massive MIMO and mmWave has received considerable attention and emerged as main technologies for future wireless networks. With beamforming to individual users, benefits from massive MIMO and mmWave greatly improve the link budget and thereby extend the coverage. However, since massive MIMO and mmWave themselves have some drawbacks such as high hardware cost and large power consumption when equipping a separate RF chain for each antenna in the massive MIMO, and higher propagation loss in the mmWave, the implementation of analog-digital transceivers is proposed.

In addition to mmWave and massive MIMO, other technologies have been considered as promising candidates for the future wireless networks, in particular 5G mobile communications, such as small cells [JMZ⁺14], 3D beamforming [RAL14] and full-duplex [ZLS17]. In fact, each technology has its own benefits and drawbacks. Therefore, it is possible to claim that they might not be used alone but in a combination of several or maybe all of them in the development of future wireless systems.

1.3 Roles of Advanced Power Constraints in Future Wireless Systems

The issue of energy efficiency has attracted a lot of attention in research recently. It plays an increasingly important role when future wireless networks become more and more dense. Indeed, several solutions have been proposed such as designing low-power circuits, having a better hardware integration, using advanced power

amplifier techniques and even applying more energy efficient network architectures [FJL⁺13]. One of the most-efficient methods to reduce the energy consumption is to design a better transmit power allocation for base stations and network devices.

In this dissertation, we study optimal transmit strategies and thereby optimal power allocations for multi-antenna systems with advanced power constraint settings, in particular with joint sum and per-antenna power constraints. In practical systems, the joint sum and per-antenna power constraints setting applies either to systems with multiple antennas or to distributed systems in which base-stations are connected via high-speed links so that they can cooperate in the downlink transmission or in the uplink where mobile users cooperate in the transmission and each user has a limited power budget. A sum power constraint can be motivated by radiation limits or green aspects to limit the energy consumption. On the other hand, a per-antenna power constraint limits the power in the RF chain of each antenna and therefore allows to operate the power amplifier in the RF chain at a more energy-efficient operating point. We believe that intensive studies and results on the optimal transmit strategies for multi-antenna systems with advanced power constraint settings provided in this dissertation are valuable for ongoing implementations in future wireless networks.

Main Contributions

In this chapter, we discuss our contributions to optimal transmit strategies of multi-antenna systems for point-to-point, wiretap and massive MIMO channels with different power constraint setups. In the first part, we consider the problem of designing optimal transmit strategies for MISO and MIMO channels with joint sum and per-antenna power constraints. We have studied this problem in [COSS16, CO17b], which are included as Paper A and Paper B in the dissertation. In the second part, we investigate the trade-off between the transmission rate and the secrecy rate for wiretap channels under a sum power constraint and per-antenna power constraints. We have studied this problem in [CO18], which is included as Paper C. Finally, we consider the problem of designing the precoder for massive MIMO systems with sub-connected architecture with RF chain and per-antenna power constraints. We have studied this problem in [COS18a, COS18b], which are included as Paper D and Paper E. For each paper, we will briefly discuss the previous works, the system model and present the obtained main results.

2.1 Problem I: Point-to-Point Channels with Joint Sum and Per-antenna Power Constraints

Paper A: Optimal transmit strategy for MISO channels with joint sum and per-antenna power constraints [COSS16]

In this paper, we study an optimal transmit strategy design for point-to-point MISO Gaussian channels with joint sum and per-antenna power constraints. Under the non-trivial case where the sum of the per-antenna power constraints is larger than the sum power constraint, a characterization of an optimal transmit strategy is derived. The main result of the paper is a simple recursive algorithm to compute the optimal power allocation.

Background and Motivation

In recent decades, the problem of finding the optimal transmit strategy for Gaussian channels has been intensively studied. These studies are subjected to either specific power constraints such as a sum power constraint and per-antenna power constraints or more general power constraints such as arbitrary convex constraints [WSS06] and arbitrary linear power constraints [BJBO11]. For point-to-point Gaussian channels, a sum power constraint and per-antenna power constraints are mainly considered. Under a sum power constraint, the optimal transmit strategy is found by performing a singular value decomposition (SVD) and applying water-filling on the channel eigenvalues [PCL03, CT06, TV05, Tel99]. Under per-antenna power constraints, the problem has been studied for both point-to-point channels [Vu11a, Vu11b, Pi12, MDT14] and multi-user channels [YL07, SSB08, KYFV07, WESS08, BH06, Zha10, TCJ08, HPC10]. In spite of numerous studies on the design of optimal transmit strategy for a Gaussian channel subject to either a sum power constraint or per-antenna power constraints, a combination of both constraints has not been studied previously.

In practical systems, the joint sum and per-antenna power constraints setting applies either to systems with multiple antennas or to distributed systems with separated energy sources. A sum power constraint can be, for instance, motivated by radiation limits or green aspects to limit the energy consumption. On the other hand, a per-antenna power constraint can be motivated to limit the power in the RF chain of each antenna. This also allows the power amplifier in the RF chain to operate at a more energy efficient operating point. Since both aspects can be relevant in practical scenarios, it is reasonable to include them both in a classical MISO point-to-point setup.

Setup and Contributions

The main contribution of this paper is to characterize the optimal transmit strategy for the Gaussian point-to-point MISO channel with joint sum and per-antenna power constraints with the assumption of perfect channel state information at the transmitter (see Figure 2.1). The solution is developed from the two optimal transmit strategy design problems with a sum power constraints only (OP1) and with per-antenna power constraints only (OP2). It is shown that, for the joint sum and per-antenna power constraints problem (OP3), beamforming is optimal and the optimal transmit strategy can be obtained if the maximum sum power is allocated. The optimal solutions of OP3 are equal to the per-antenna power constraints on antennas to which OP1's optimal solutions violate those per-antenna power constraints. The remaining powers can be then found by solving a reduced optimization problem with a smaller number of channel coefficients and a smaller sum power constraint.

To obtain the results above, we first have shown that a Gaussian distributed input is capacity-achieving for the Gaussian MISO channel with joint average sum

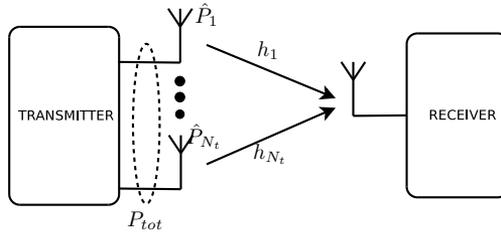


Figure 2.1: Point-to-point MISO channel

and per-antenna power constraints. This has been captured in Proposition A.1. This proposition allows us to formulate the problem to find the optimal transmit strategy subject to Gaussian distributed input with joint average sum and per-antenna power constraints (OP3).

Next, we focus on characterizing properties of the optimal transmit strategy. One of the most important properties is that the optimal transmit strategy for joint sum and per-antenna power constraints is beamforming, i.e., the rank of the transmit strategy has to be one at the optimum. The amplitudes of beamforming vectors' elements are then characterized such that at the optimum full transmit power is used and simultaneously the per-antenna power constraints are satisfied. These properties are captured in Proposition A.2 and Proposition A.3. The phases of beamforming vectors' elements are chosen to match the phase of the channel coefficients. The optimal beamforming vector corresponding to the optimal transmit strategy $\mathbf{Q}^{(3)}$ of OP3 therefore are captured and restated in the following lemma.

Lemma (Lemma A.4). Let $\mathbf{q}^{(3)}$ be the optimal beamforming vector corresponding to the optimal covariance matrix $\mathbf{Q}^{(3)}$, i.e., $\mathbf{Q}^{(3)} = \mathbf{q}^{(3)}\mathbf{q}^{(3)H}$. Then

$$\mathbf{q}^{(3)} \in \mathbb{Q} := \left\{ \mathbf{q} : \mathbf{q} = \left[\frac{\sqrt{P_1}h_1}{|h_1|}, \dots, \frac{\sqrt{P_{N_t}}h_{N_t}}{|h_{N_t}|} \right]^T, \mathbf{q}\mathbf{q}^H \in \mathcal{S}_3 \right\}$$

for some choices of P_i , $\forall i = \{1, \dots, N_t\}$.

In the paper, the optimal power allocation has been computed using a simple recursive algorithm. The algorithm is initialized by utilizing a property mentioned in Proposition A.3 that, since the capacity achieving transmit strategy always allocates full sum power, it is sufficient to find the optimal transmit strategy for the optimization problem with sum power constraint only (OP1). However, since the optimal power allocation solution of OP1 may result in a solution that violates some per-antenna power constraints, we have shown that if there exists any antenna for which the optimal power allocation of OP1 exceeds the per-antenna

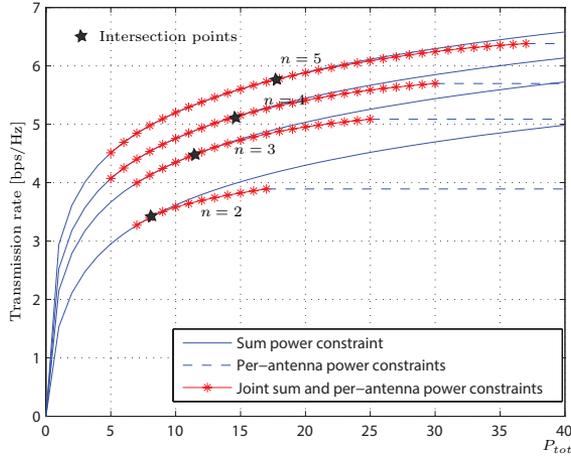


Figure 2.2: Transmission rate in different power constraint domains and different transmit antenna configurations.

power constraints of OP3, then the optimal powers for those antennas are equal to the per-antenna power constraints and the optimization problem reduces to a new optimization problem with a smaller total transmit power and a reduced number of channel coefficients. This property has been shown in Theorem A.5 and is restated in the following.

Theorem (Theorem A.5). Let $\mathcal{I} \subseteq \{1, \dots, N_t\}$ and $\mathcal{P}_V := \{i \in \mathcal{I} : P_i^{(1)} > \hat{P}_i\}$, if $\mathcal{P}_V = \emptyset$ then $P_i^{(3)} = P_i^{(1)} \forall i \in \mathcal{I}$, else $P_i^{(3)} = \hat{P}_i \forall i \in \mathcal{P}_V$ and the remaining optimal powers can be computed by solving a reduced optimization problem

$$\arg \max_{\mathbf{q}' \in \mathcal{Q}'} |\mathbf{h}'^H \mathbf{q}'|^2$$

where $\hat{P}_i \forall i \in \mathcal{I}$ are per-antenna power constraints, $\mathbf{h}' = [h_i]_{i \in \mathcal{P}_V^c}^T \in \mathbb{C}^{|\mathcal{P}_V^c| \times 1}$, $\mathcal{Q}' := \{\mathbf{q}' : \sum_{i \in \mathcal{P}_V^c} |q_i|^2 \leq P_{tot} - \sum_{i \in \mathcal{P}_V} \hat{P}_i, |q_i|^2 \leq \hat{P}_i, i \in \mathcal{P}_V^c\}$ and $\mathcal{P}_V^c = \mathcal{I} \setminus \mathcal{P}_V$. The notations $(\cdot)^{(1)}$ and $(\cdot)^{(3)}$ denote the corresponding optimal values of optimization problems according to the sum power constraint, and the joint sum and per-antenna power constraints.

The recursion finishes when all power constraints are satisfied. The number of iterations equals the times that the set of indices of optimal powers of the OP1 solution violating the per-antenna power constraints of OP3 is not empty. Thus, the maximum number of violated per-antenna power constraints is $N_t - 1$.

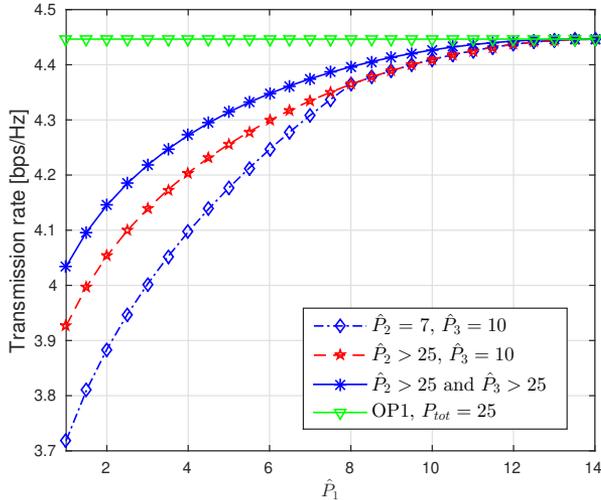


Figure 2.3: The impact of choice of power constraints on the optimal power allocation and the capacity of 3×1 MISO channel with $P_{tot} = 25$. The marker symbols correspond to the following power constraint settings: sum power constraint ($-\nabla-$), additional per-antenna power constraints on P_1 ($-*-$), P_1 and P_3 ($-*--$), and P_1 , P_2 and P_3 ($-\diamond-$).

In Figure 2.2, the theoretical result is illustrated. It is clear to see that by keeping a maximum sum transmit power while increasing the number of transmit antennas, the optimal allocated power of OP1 violating the per-antenna power constraints for a few antennas might not violate the per-antenna power constraints for a larger number of antennas since we have more alternatives to allocate the power. Therefore, the gap between the optimal transmission rate with joint sum and per-antenna power constraints and the optimal transmission rate with sum power constraint is decreased. The intersection point plays an important role since the power allocation behavior changes at this point and therewith the growth of the maximal achievable rate. We characterize the intersection point where the trajectory of the optimal power allocation for OP1 intersects a per-antenna power constraint when increasing the allowed sum power.

In Figure 2.3, the impact of choices of the power constraints on the optimal power allocation and the optimal transmission rate of the channel is illustrated. The curves in the figure are plotted by adjusting per-antenna power constraint for antenna 1 and setting per-antenna power constraint configurations for antenna 2 and 3 as follows: (i) the per-antenna power constraints for both antennas are active, (ii) the per-antenna constraints for antenna 3 is active only, and (iii) the per-antenna power constraints on both antennas are inactive. We can see from the figure that

the optimal transmission rate decreases if more per-antenna power constraints are added. In particular, when activating the per-antenna power constraints for all antennas, the capacity is always smaller or equal than the case of inactivating the per-antenna power constraints for one or more antennas. This happens because of the fact that adding constraints limits the optimization domain, i.e., we have less freedom to allocate the power.

Paper B: Optimal transmit strategy for MIMO channels with joint sum and per-antenna power constraints [CO17b]

In this paper, we propose an iterative algorithm to find the optimal transmit strategy in closed-form for a MIMO channel with joint sum and per-antenna power constraints using a generalized water-filling solution. The algorithm is based on a power allocation property that an optimal transmit strategy can be obtained if the maximal sum power is allocated and if an unconstrained optimal allocation for an antenna exceeds a per-antenna power constraint, then the maximal power for this antenna is used in the constrained optimal transmit strategy. This power allocation behavior also enables us to use the generalized water-filling and the closed-form solution from [XFZP15] in an iterative algorithm.

Background and Motivation

Depending on the per-antenna power constraints and the sum power constraint, we can identify three different cases as follows: The first case is when the per-antenna power constraints are never active. The second case is when the the sum power constraint is never active. The most interesting case is when both sum and per-antenna power constraints are active. The optimal transmit strategy problem with the joint sum and per-antenna power constraints for MISO channels has been considered in Paper A ([COSS16]). Previously, the optimal transmit strategy for MIMO channels has been studied with either sum power constraint [CT06, Tel99] or per-antenna power constraints [Vu11b, Pi12, COS16, YL07, SSB08, KYFV07]. Furthermore, in [XFZP15], the optimization problem with the assumption that several antenna subsets are constrained by a sum power constraint while the other antennas are subject to a per-antenna power constraint is studied and a closed-form solution is provided. Unfortunately, the results in that paper cannot be directly applied to the case where the transmit powers are jointly constrained by both sum and per-antenna power constraints. To make [XFZP15] applicable for the optimization problem with joint sum and per-antenna power constraints, we need to identify for each antenna which constraint is active, which is the key step in this paper.

Setup and Contributions

We have considered a MIMO channel (see Figure 2.4) with N_t transmit antennas and N_r receive antennas. The main task in the paper is to find the optimal

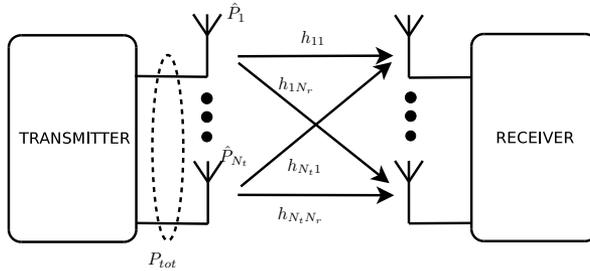


Figure 2.4: Point-to-Point MIMO channel

transmit covariance matrix $\mathbf{Q} = \mathbb{E}[\mathbf{x}\mathbf{x}^H]$ subject to the given joint sum and per-antenna power constraints such that the transmission rate of the MIMO channel is maximized. This optimization problem is denoted as OP- \mathcal{A} in the paper.

To approach the solution of the OP- \mathcal{A} , we first have shown that the maximum transmission rate can be achieved when the optimal transmit strategy uses the full power. Accordingly, it is sufficient for the optimization to consider only transmit strategies which allocate the full power, i.e., the sum power constraint is always active. An iterative algorithm to find the optimal transmit strategy in closed-form is then proposed. The algorithm relies on a sequence of optimization problems using the fact that, when the optimization domain is more restricted by adding more per-antenna power constraints, we have less freedom to allocate the optimal transmit power (see Figure 2.5). This can be described in the following sequence of optimization problems:

$$\begin{aligned}
 \max_{\mathbf{Q} \in \mathcal{S}(\emptyset)} f(\mathbf{Q}) &= \max_{\mathbf{Q} \in \mathcal{S}(\emptyset) \cap \{\mathbf{Q} |_{ii} \leq \hat{P}_i : \forall i \in \mathcal{P}(1)\}} f(\mathbf{Q}) \\
 &\geq \max_{\mathbf{Q} \in \mathcal{S}(\emptyset) \cap \{\mathbf{Q} |_{ii} \leq \hat{P}_i : \forall i \in \mathcal{P}(2)\}} f(\mathbf{Q}) \\
 &\dots \\
 &\geq \max_{\mathbf{Q} \in \mathcal{S}(\emptyset) \cap \{\mathbf{Q} |_{ii} \leq \hat{P}_i : \forall i \in \mathcal{P}(K-1)\}} f(\mathbf{Q}) \\
 &\geq \max_{\mathbf{Q} \in \mathcal{S}(\emptyset) \cap \{\mathbf{Q} |_{ii} \leq \hat{P}_i : \forall i \in \mathcal{P}(K)\}} f(\mathbf{Q}) \\
 &= \max_{\mathbf{Q} \in \mathcal{S}(\mathcal{A})} f(\mathbf{Q}), \tag{2.1.1}
 \end{aligned}$$

where $f(\mathbf{Q}) = \log \det(\mathbf{I}_m + \mathbf{H}\mathbf{Q}\mathbf{H}^H)$, $\mathcal{S}(\mathcal{A}) := \{\mathbf{Q} \succcurlyeq 0 : \text{tr}(\mathbf{Q}) \leq P_{tot}, P_i = \mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i \leq \hat{P}_i, \forall i \in \mathcal{A}\}$, $\mathcal{A} \subseteq \{1, \dots, N_t\}$ and $\mathcal{P}(k)$ is the set of indices of powers which violate the per-antenna power constraints in the k -th iteration with initialization $\mathcal{P}(1) = \emptyset$. The optimization problem in each iteration can be solved using the

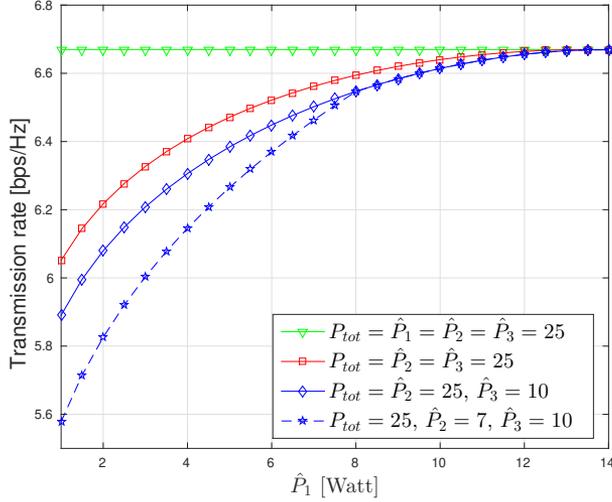


Figure 2.5: Capacity under different power constraint settings

closed-form solution in [XFZP15]. In detail, the optimal solution of the transmit strategy at the k -th iteration denoted by $\mathbf{Q}^*(k)$ is given by

$$\mathbf{Q}^*(k) = (\mathbf{D}^{-\frac{1}{2}}[\mathbf{U}]_{:,1:L}[\mathbf{U}]_{:,1:L}^H \mathbf{D}^{-\frac{1}{2}} - [\mathbf{U}]_{:,1:L} \mathbf{\Lambda}^{-1} [\mathbf{U}]_{:,1:L}^H)^+,$$

where diagonal matrix $\mathbf{\Lambda}$ and $L = \min(N_t, N_r)$ is the number of non-zero singular values of the channel coefficient matrix \mathbf{H} . The first L columns of a unitary matrix $[\mathbf{U}]_{:,1:L}$ are obtained from eigenvalue decomposition $\mathbf{H}^H \mathbf{H} = \mathbf{U} \begin{bmatrix} \mathbf{\Lambda} & 0 \\ 0 & 0 \end{bmatrix} \mathbf{U}^H$. The diagonal elements of $L \times L$ diagonal matrix $\mathbf{\Lambda}$ are positive real values in decreasing order.

The operation ‘+’ is to guarantee that the solution is positive-semi definite and the elements of the diagonal \mathbf{D} can be computed at high SNR as

$$[\mathbf{D}]_{i,i} = \frac{[[\mathbf{U}]_{:,1:L}[\mathbf{U}]_{:,1:L}^H]_{i,i}}{\hat{P}_i + [[\mathbf{U}]_{:,1:L} \mathbf{\Lambda}^{-1} [\mathbf{U}]_{:,1:L}^H]_{i,i}} \quad \text{if } i \in \mathcal{P}(k) \quad (2.1.2)$$

and

$$[\mathbf{D}]_{j,j} = d(k) = \frac{\sum_{j' \in \mathcal{P}^c(k)} [[\mathbf{U}]_{:,1:L}[\mathbf{U}]_{:,1:L}^H]_{j',j'}}{P_{tot}(k) + \sum_{j' \in \mathcal{P}^c(k)} [[\mathbf{U}]_{:,1:L} \mathbf{\Lambda}^{-1} [\mathbf{U}]_{:,1:L}^H]_{j',j'}}, \quad \text{if } j \in \mathcal{P}^c(k). \quad (2.1.3)$$

Each iteration in the algorithm can be related to an optimization problem with a total power constraint and a limited number of per-antenna power constraints,

which can be solved using the generalized water-filling solution. Since the optimal allocated powers of the optimal solution using full transmit power may exceed the maximum allowed power for some antennas, it is possible to distinguish the power allocation between two cases: (i) all per-antenna power constraints are satisfied, and (ii) at least one power exceeds the maximum allowed per-antenna power.

It can be seen from Lemma B.2 that the set of indices of antennas that would violate the power constraint might grow by adding new antenna indices in every iteration. The algorithm stops when no new per-antenna power constraint is violated in an iteration. It is also interesting to note that, similar to the MISO channel, the maximum number of iteration, which corresponds to the maximum number of violated per-antenna power constraints, is $N_t - 1$.

2.2 Problem II: Trade-off Between Transmission and Secrecy Rates in Wiretap Channels

Paper C: Optimal transmit strategy for Gaussian MISO wiretap channels [CO18]

In this paper, we have characterized the optimal trade-off between the secure and the non-secure transmission rate of the MISO wiretap channels with different power constraint settings, in particular, sum power constraint only (SPC), per-antenna power constraints only (PAPC) and joint sum and per-antenna power constraints (JSPC). The original optimization problem is non-convex. However, equivalent convex reformulations allows the characterization of the boundary of the rate region, on which the optimal rate pair can be found by a simple line search. Different solutions are derived depending on the spectral property of a channel matrix which includes the trade-off parameter t and the condition of the input channel coefficients. In more details, optimal transmit strategies have been characterized in closed-form for the sum power constraint only problem with an arbitrary number of transmit antennas. The closed-form solutions are also derived for the per-antenna power constraints only problems for a given power allocation. Necessary conditions to find the optimal power allocation are then derived. Sufficient conditions, however, are available for the parallel-channels case only.

Background and Motivation

Security is a critical aspect in wireless communication systems due to the open nature of wireless links. One of the pioneering works is the study of the secrecy capacity of the wiretap channel in [Wyn75]. Following Wyner's work, researchers in the physical-layer security area have extended and considered the wiretap channel in various aspects such as the extensions to the non-degraded case [CK78], MISO and MIMO Gaussian wiretap channels with a sum power constraints [LYCH78, KW10b, KW10a, OH08, LP10, LP09, SSC12, LHW⁺13].

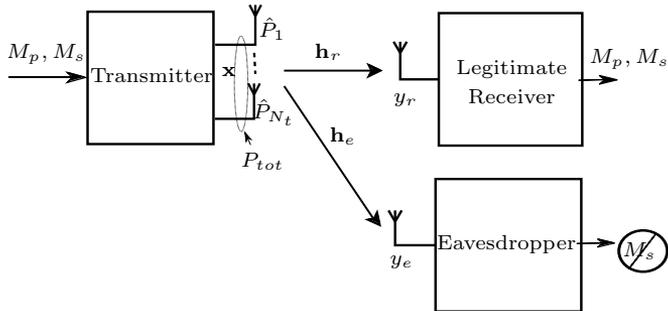


Figure 2.6: MISO wiretap channel with joint sum and per-antenna power constraints, public message M_p and secret message M_s

In this work we study MISO wiretap channels with different power constraint settings including sum power constraint only, per-antenna power constraints only, and joint sum and per-antenna power constraints. The optimal trade-off between communication rate and secrecy rate of MISO wiretap channels is motivated by the fact that the optimal coding strategy for the wiretap channel is using a two-layer codebook, i.e., the eavesdropper can only decode the public message on the public layer codebook, while the legitimate receiver can decode both the public and secret messages. Therefore, instead of sending some useless random messages on the public layer, a useful message can be communicated non-securely to the legitimate receiver [EU12b, EU12a, LLPS13] (see Figure 2.6). Since the maximal transmission rate and secrecy rate are, in general, achieved by different transmit strategies, we face a trade-off between both objectives.

Contributions

For a Gaussian MISO wiretap channel, the rate region \mathcal{R} describes the trade-off between the transmission rate and the secrecy rate with a given set of power constraints. This trade-off is controlled by optimal transmit strategy $\mathbf{Q} = \mathbb{E}[\mathbf{x}\mathbf{x}^H]$ and is given by

$$\begin{aligned} \mathcal{R}_{MISO}(\hat{\mathbf{p}}) &= \{(R, R_s) \in \mathbb{R}_+^2 : 0 \leq R_s \leq R_s(\mathbf{Q}), \\ & R = R_s + R_p \leq R(\mathbf{Q}) \text{ for some } \mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})\} \end{aligned}$$

with $R(\mathbf{Q}) = \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r)$ and $R_s(\mathbf{Q}) = \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r) - \log(1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e)$.

If this region is convex, then the set of weighted rate sum optimal rate pairs characterize the boundary of the rate region. If this region is non-convex, then the set of all weighted rate sum optimal rate pairs can be used to characterize the boundary of the convex hull of the rate region. In the latter case, we need to allow time-sharing between two rate pairs.

In the paper, we provide solutions to find optimal transmit strategies for the weighted rate sum optimization problem. Based on that, the rate region that describes the trade-off between the transmission rate and the secrecy rate has been characterized. We have shown an important property in the paper that for optimization it is sufficient to consider beamforming strategies, i.e., there exists always an optimal transmit strategy which has rank one. Since the weighted rate sum optimization problem is a non-convex optimization problem, we have reformulated it to an equivalent convex optimization problem that allows further analysis. Two reformulations have been derived, which then have been used to derive closed-form solutions and complexity efficient iterative algorithms. In particular, the use of equivalent convex reformulations allows the characterization of the boundary of the rate region, on which the optimal rate pair can be found by a simple line search.

With the reformulation, we have provided closed-form solutions of the optimal transmit strategies of the weighted rate sum optimization problem for the MISO wiretap channel for different power constraint settings as follows:

a). The SPC case is applied when the per-antenna power constraints are never active. The solution for the optimal transmit strategy with sum power constraint only and arbitrary number of antennas has been expressed in closed-form in Theorem C.5 and can be restated as

$$\mathbf{Q}_{SPC}^{(1)}(t) = P_{tot} \mathbf{v} \mathbf{v}^H$$

where \mathbf{v} is the eigenvector associated with the positive eigenvalue of $\mathbf{A} = \mathbf{h}_r \mathbf{h}_r^H - t \mathbf{h}_e \mathbf{h}_e^H$ for a given t . A negative eigenvalue in the matrix \mathbf{A} does not affect the procedure to compute the optimal solution for the sum power constraint only case. In particular, the total power is always allocated.

b). The PAPC case is applied when the sum power constraint is never active. It is important to note that when \mathbf{A} has a negative eigenvalue, it may not be optimal to allocate the full transmit power on all antennas. However, it is interesting to know that, for the per-antenna power constraints only problem, there is always at least one per-antenna power constraint active. The solutions in the following is provided under the assumption that the power allocation per antenna is given, i.e., the transmit strategy has diagonal elements $q_{kk} = \tilde{P}_k, \forall k \in \mathcal{I}$. Necessary conditions to find the optimal power allocation are provided using alternating optimality method. The sufficient condition to find the optimal power allocation, however, is available when transmitter is equipped with two antennas only. The remaining problem is to find off-diagonal elements of the optimal transmit strategy. The main difficulty in finding the off-diagonal elements of the optimal transmit strategy is the positive semi-definite constraint. To overcome this, we have considered a relaxed optimization problem involving the 2×2 principal minors of the transmit strategy similarly as done in [Vu11a].

Special case of parallel channel: The special case is considered when channel vectors are parallel, i.e., \mathbf{A} to be positive semi-definite. This case directly corresponds to a point-to-point MISO channel problem. When \mathbf{A} is positive semi-definite, we

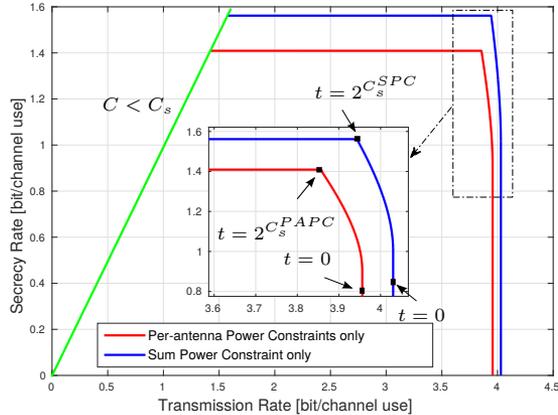


Figure 2.7: The optimal regions between the transmission rates and the secrecy rate with sum power constraint only $P_{tot} = 15$ and per-antenna power constraints only $\hat{P}_1 = 5$ and $\hat{P}_2 = 10$

can always find optimal transmit strategies that allocate full power. There is no trade-off between secrecy and non-secrecy rate, i.e., there is only transmit strategy that simultaneously maximizes both rates. The optimal transmit strategies can then be obtained from the solution of the point-to-point MISO channel problem: (i) [Vu11a] for the per-antenna power constraints only problem, (ii) [COSS16] for the joint sum and per-antenna power constraints problem, considering the channel $\mathbf{h}_A = \sqrt{|\mathbf{h}_r|^2 - t|\mathbf{h}_e|^2} \frac{\mathbf{h}_r}{|\mathbf{h}_r|}$.

To clarify the theoretical results, we have provided illustrative numerical examples for the optimization problems with sum power constraint only and per-antenna power constraint only with two antennas at the transmitter, and one antenna at the legitimate receiver and eavesdropper each (see Figure 2.7). The figure shows that the regions are fully characterized by the curved sections, which can be obtained from the derived optimal solutions.

2.3 Problem III: Precoding Design for Massive MIMO with sub-connected architecture

Paper D: Transmit beamforming for single-user large-scale MISO systems with sub-connected architecture and power constraints [COS18a]

In this work, we have considered a single-user large-scale MISO system with sub-connected architecture (see Figure 2.8). The system is configured such that each

RF chain serves a group of antennas. Power dividers are used to split the output powers to its connected antennas. The optimal transmit beamforming has been studied under the case where the power constraint on the RF chain is smaller than the sum of the corresponding per-antenna power constraints. It is shown that, in the optimum, the optimal phase shift at each antenna has to match the phase of the channel coefficient and the phase of the digital precoder. The optimal power has been found using an iterative algorithm based on two properties: Each RF chain uses the full power and if the optimal power allocation of the unconstrained problem violates a per-antenna power constraint, then it is optimal to allocate the maximal power for that antenna.

Background and Motivation

In recent years, large-scale multiple-input multiple-output (massive MIMO) wireless communication has received much attention due to its envisioned application in 5G wireless systems. The motivation for massive MIMO is to use a very large number of antennas, which enhances the spectral efficiency significantly [Mar10, LETM14, Mar15, LLS⁺14], jointly with higher frequencies (mmWave), which reduce the antennas' size and radiated energy [GDH⁺16, SY16, LXD14]. From a hardware perspective, we distinguish between two configurations of the large-scale antenna system, namely fully-connected and sub-connected architectures [GBKS15, SY16, LXD14, LWY⁺17]. In comparison to the fully-connected architecture, a sub-connected architecture has a reduced complexity, where each RF chain is connected to a subset of transmit antennas. Since this sub-connected architecture requires no adder and less phase shifters, it is less expensive to implement than the fully-connected one but results in less freedom for signalling. In this work, motivated from the physical limitations on the RF chain and the splitting ratio of the power divider, we have focused on studying the optimal transmit strategy for a single-user large-scale MISO system with sub-connected architecture, per RF chain and per-antenna power constraints.

Contributions

The main contribution of this paper is to design the optimal digital precoder, the optimal analog precoder and the optimal power allocation for a single-user large-scale MISO system with sub-connected architecture and power constraints (see Figure 2.8). For the hybrid beamforming, it is interesting to study the case where the number of RF chains is strictly smaller than the number of antennas. The digital precoder, which can be shown to be rank one at the optimum, is designed under the assumption that an analog precoder and a power allocation matrix are given. In hybrid precoding, the analog precoder controls the phase for each antenna. Since it is sufficient to consider a digital precoder of rank one, the phase of the digital precoder can be merged with the analog precoder or simply chosen to be equal to zero. The amplitude of the optimal digital precoder is designed such that the

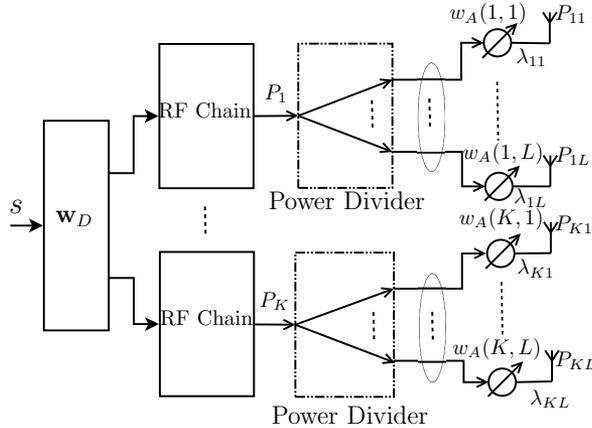


Figure 2.8: Transmitter architecture for large-scale antenna system with sub-connected architecture and power constraints.

RF chain power constraints are always active, i.e., we allocate full power on all RF chains.

The analog precoder is designed under the assumption of fixing the digital precoder and the power allocation. The necessary condition for the optimal analog precoder has been given in Proposition D.1 and can be restated as follows:

Proposition (Proposition D.1). Let $e^{i\angle h_{kl}} = \frac{h_{kl}}{|h_{kl}|} \forall k, l$. Then the optimal analog precoder has elements given as

$$w_A^*(k, l) = e^{-i\angle h_{kl}}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}.$$

This proposition shows that it is optimal to match the phase at each antenna to the channel coefficient. Therefore, it is sufficient to design the optimal analog precoder by aligning the phases of the phase shifters with the channel coefficient such that the signal coherently adds up at the receiver.

Next, we have designed the optimal power allocation under the assumption that the optimal analog and digital precoders are given. Since the power allocation for one RF chain is independent from the allocation at all other RF chains, the problem to find the optimal power allocation reduces to the problem to find the optimal power allocation for one RF chain only. This problem is exactly the same as the optimization problem to find the optimal transmit strategy for MISO channels with joint sum and per-antenna power constraints in Paper A. Therefore, the solution of the optimization problem to find the optimal power allocation for one RF chain only can be solved by utilizing the solutions of the sum power constraint only and per-antenna power constraints only problems as done in Paper A. First, we do not include per-antenna power constraints. Then the optimal power allocation for a

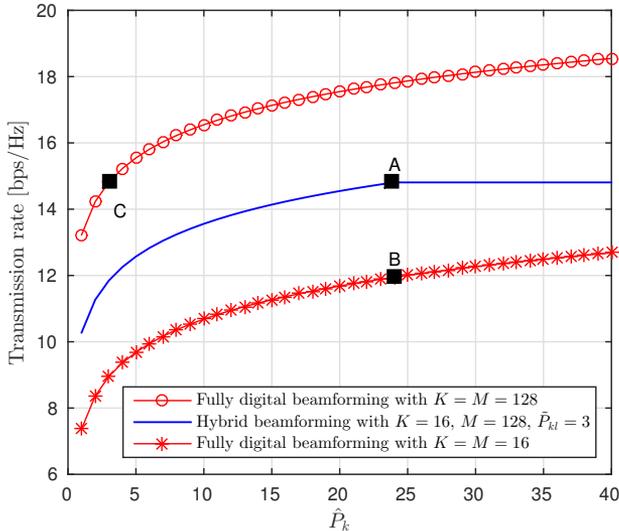


Figure 2.9: Transmission rate of the large-scale antenna system with different RF chains and antennas configurations.

group of antennas connecting to one RF chain is given by a waterfilling solution. However, the optimal powers of this relaxed optimization problem may violate the per-antenna power constraints. In this case, it is optimal to set those powers equal to the per-antenna power constraints and the remaining power allocations can then be obtained by solving a reduced optimization problem with a smaller total RF chain power. If the waterfilling solution of the reduced optimization problem again violates a per-antenna power constraint, then the procedure above has to be applied again until the waterfilling solution of the reduced optimization problem does not violate any per-antenna power constraints.

To illustrate the theoretical results, we have considered a large-scale MISO system with different transmit antenna configurations (see Figure 2.9). We can see from the figure that for the hybrid beamforming, if an RF chain power constraint is more restrictive than the sum of all individual powers of the group of antennas connected to that RF chain (operating point A), then it is optimal to transmit with the maximal per RF chain power. After this value, the RF power constraint is never active and it is optimal to transmit with the maximal individual power on all antennas. Under the same total transmit power, we have observed that a lower cost hybrid setup with a smaller number of RF chains and the same number of antennas can achieve almost the same transmission rate as the one with fully digital beamforming.

Paper E: Precoding design for massive MIMO systems with sub-connected architecture and per-antenna power constraints [COS18b]

In this paper, we have provided the necessary conditions to design precoding matrices for massive MIMO systems with a sub-connected architecture, RF power constraints and per-antenna power constraints. The necessary condition to design the digital precoder is established based on a generalized water-filling and joint sum and per-antenna optimal power allocation solution, while the analog precoder is based on a per-antenna power allocation solution only.

Background and Motivation

In Paper D [COS18a], we have studied the single-user large-scale MISO system with a sub-connected architecture, RF power constraints and per-antenna power constraints. Our work aimed to overcome new technological challenges from a transceiver hardware perspective which caused by the increase of antennas in massive MIMO setups. We are interested in the sub-connected architecture since it can reduce the complexity, high hardware cost and power consumption. The sub-connected architecture has been configured such that each RF chain serves a group of antennas. Regarding the power constraints, since RF chain has a physical limitation, it is reasonable to impose a power constraint on each RF chain. Furthermore, since each RF chain serves more than one antenna, power dividers are used to split the output powers to its connected antennas. Since it will be optimal to use the maximal power per RF chain and since the splitting ratio of the power divider has a physical limitation, it is reasonable to apply a power constraint on each antenna to limit the energy available to each antenna.

Motivated from that, in this paper, we have considered the massive MIMO setup with sub-connected architecture, RF chain and per-antenna power constraints. Our contributions are summarized in the next section.

Contributions

The main contributions of this paper are necessary conditions that lead to the optimal design of the digital and analog precoders for the massive MIMO system with sub-connected architecture and power constraints. The investigated model is depicted in Figure 2.10. We provide the necessary condition to design the optimal analog precoder under the assumption that the optimal digital precoder is given. The elements of the optimal analog precoder have to satisfy the following conditions

$$w_A^*(k, l) = e^{i(\theta_{m1} - \gamma_{m1})} = \dots = e^{i(\theta_{mN} - \gamma_{mN})}, \text{ for all } m \in \{1, \dots, M_t\}.$$

θ_{mn} and γ_{mn} are the optimal phases of the element $\tilde{w}_D^*(mn)$ of the digital precoder and an element v_{mn}^* of the MIMO precoding matrix \mathbf{V}^* . The optimal value \mathbf{V}^* can

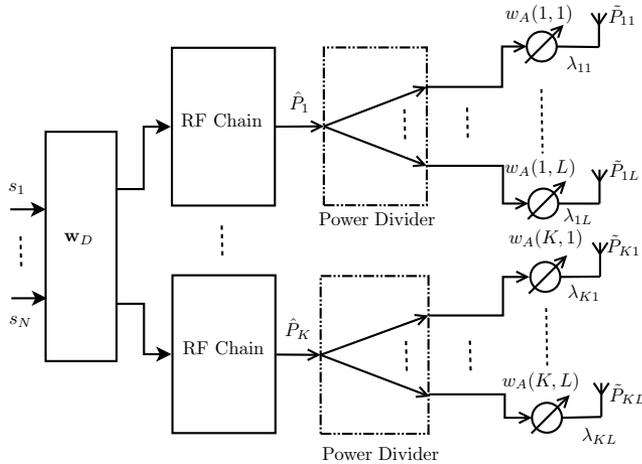


Figure 2.10: Sub-connected architecture for Massive MIMO with RF chain and per-antenna power constraints.

be obtained by solving the optimization problem

$$\begin{aligned} \mathbf{V}^* &= \arg \max_{\mathbf{V}} \log |\mathbf{I} + \mathbf{V}\mathbf{H}^H \mathbf{H}\mathbf{V}^H|, \\ \text{s. t. } \|\mathbf{v}(m)\|^2 &\leq P_m^* \quad \forall m = 1, \dots, M_t, \end{aligned}$$

using techniques in [Pi12].

The necessary condition to design the optimal digital precoder has been derived under the assumption that the optimal analog precoder is given. Since there exists always an optimal solution which allocates full power on each RF chain, it is sufficient for the optimization to consider only transmit strategies which allocate full power on all RF chains. This implies that, by using the generalized waterfilling, we can find the optimal digital precoder and its corresponding optimal power allocation subject to RF chain power constraints only. However it may happen that the optimal powers allocated under RF chain power constraints only problem may exceed the maximum allocated per-antenna powers. Following Paper A and Paper B, if an antenna has an optimal power allocation that violates the per-antenna power constraint, then it is optimal to allocate the maximal per-antenna power on that antenna. The remaining optimal power allocation can be found by considering a reduced optimization problem. This process is repeated again and again until all power constraints are satisfied.

To illustrate the theoretical results, we have considered numerical examples for a massive MIMO system with different transmit antenna configurations. We observe from Figure 2.11 that for the hybrid beamforming, the transmission rate

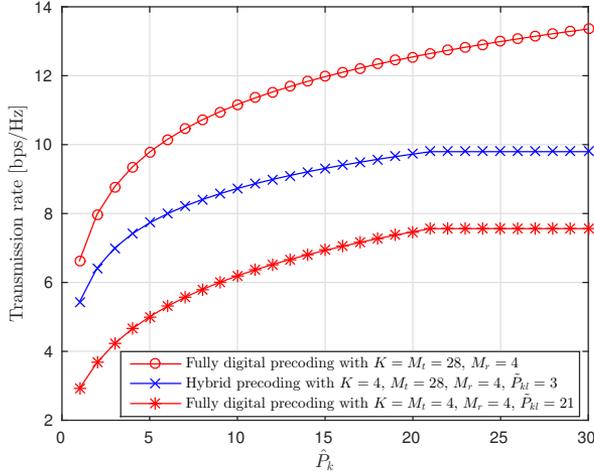


Figure 2.11: Transmission rate of the sub-connected architecture massive MIMO system with different RF chain and antenna configurations.

increases together with the increase of the RF chain power if the RF chain power constraint is more restrictive than the sum of all individual powers of a group of antennas connected to that RF chain. Beyond that, the transmission rate remains constant with increasing RF chain power constraints since the allocated RF chain powers remain constant due to the per-antenna power constraints. In comparison to the fully connected architecture, we observe that the sub-connected architecture provides significant performance gains if a hardware with a fixed number of RF chains should be used, but performs worse if a fully connected system with many antennas and equally many RF chains is available. The gap between the fully digital and the hybrid precodings is due to the per-antenna power constraints.

Conclusions

This dissertation aims to push the frontier of the knowledge of multi-antenna systems subject to different power constraint settings, in particular the joint sum and per-antenna power constraints. We strongly believe that the problems with advanced power constraints will play important roles in future wireless networks. In this dissertation, optimal multi-antenna transmissions for point-to-point channels, optimal trade-off between transmission and secrecy rates in wiretap channels and optimal precoding for massive MIMO with sub-connected architecture under advanced power constraint settings are investigated. The solutions are either provided as closed-form solutions or effective iterative algorithms, which allow their implementations in practical systems.

The problems to find the optimal transmit strategies for multi-antenna point-to-point channels with joint sum and per-antenna power constraints have been studied in Paper A and Paper B. The setup of the joint sum and per-antenna power constraints is relevant in practical systems, where we have to limit the average power in each RF chain for each antenna and the total radiated average power across all transmit antennas due to regulation or other reasons. For Gaussian MISO channels, it is shown that beamforming is the optimal transmit strategy and the capacity can be achieved when the maximum sum power is used in the optimal transmit strategy. It is optimal to set the power allocation for an antenna equal to the per-antenna power constraints if their optimal values in sum power constraint only problem violate those per-antenna power constraints. The remaining powers can then be found by solving a reduced optimization problem. We have also shown that the optimal transmit strategy for the joint sum and per-antenna power constraints problem can be derived from the sum power constraint only and the per-antenna power constraints only problems. For the Gaussian MIMO channels, we have provided an iterative algorithm to find the optimal transmit strategy in closed-form for a MIMO channel with joint sum and per-antenna power constraints using a generalized waterfilling solution. The algorithm exploits the fact that, again, if an unconstrained optimal power allocation of an antenna exceeds a per-antenna power constraint, then it is optimal to allocate the maximal power in the constraint optimal transmit strategy that includes the per-antenna power constraints. This enables us to design

an iterative algorithm to compute the optimal transmit strategy satisfying both sum and per-antenna power constraints.

In Paper C, we have studied the optimal trade-off between transmission and secrecy rates in wiretap channels considering different power constraint settings. The problem of the optimal trade-off between transmission and secrecy rates is formed based on the approach of using two-layers codebook in the coding strategy. The optimization problem is non-convex. However, the use of equivalent convex reformulations allows the characterization of the boundary of the rate region on which then the optimal rate pair can be found by a simple line search. In particular, for the optimization problem with a sum power constraint only, the optimal transmit strategy is characterized by a simple closed-form solution. For the optimization problem with per-antenna power constraints only, if channel vectors are not parallel, then it may be optimal not to allocate full power if there are per-antenna power constraints. For the general case, necessary conditions for optimality have been derived that have been used in an iterative person-by-person algorithm for the per-antenna power constraints only problems. Sufficient conditions, however, are provided for the special case of two transmit antennas only.

We are convinced that studies on advanced power constraint settings are highly relevant for massive MIMO setups. We have investigated the optimal precoder design for large-scale antenna setups in the last two papers, Paper D and Paper E. These papers studied a specific hardware setup called sub-connected architecture. We are interested in this setup since it results in lower hardware complexity which is less expensive and consumes less energy. In Paper D, we have provided necessary and sufficient conditions to design the hybrid beamforming for a single-user large-scale MISO system with a sub-connected architecture, RF chain and per-antenna power constraints. We have shown that phase matching is optimal and provided an algorithm to compute the optimal power allocation in closed-form. Unfortunately, when extending the study to the massive MIMO case in Paper E, only necessary conditions are available. In both papers, it has been shown that precoders can be found by applying similar techniques from the optimization problems of the point-to-point MISO and MIMO systems with per-antenna power constraints only and with joint sum and per-antenna power constraints. The numerical results illustrate that we obtain a significantly higher capacity by increasing number of antennas while keeping the number of RF chains fixed.

Increasing transmission rate and reducing power consumption are critical requirements for future wireless networks. We believe that the extensive studies and results on the optimal transmit strategies for multi-antenna systems with advanced power constraint settings provided in this dissertation are valuable for ongoing implementations of future wireless networks.

Part II

Included Papers

Optimal Transmit Strategy for MISO Channels with Joint Sum and Per-antenna Power Constraints

Phuong Le Cao, Tobias J. Oechtering, Rafael F. Schaefer and Mikael Skoglund

Revised version of refereed article published in
IEEE Transaction on Signal Processing, vol. 64, no. 16, pp. 4296-4306, Aug 2016.

©2016 IEEE

Layout has been revised for thesis consistency

Optimal Transmit Strategy for MISO Channels with Joint Sum and Per-antenna Power Constraints

Phuong Le Cao, Tobias J. Oechtering, Rafael F. Schaefer
and Mikael Skoglund

Abstract

In this paper, we study an optimal transmit strategy for multiple-input single-output (MISO) Gaussian channels with joint sum and per-antenna power constraints. We study in detail the interesting case where the sum of the per-antenna power constraints is larger than sum power constraint. A closed-form characterization of an optimal beamforming strategy is derived. It is shown that we can always find an optimal beamforming transmit strategy that allocates the maximal sum power with phases matched to the complex channel coefficients. The main result is a simple recursive algorithm to compute the optimal power allocation. Whenever the optimal power allocation of the corresponding problem with sum power constraint only exceeds per-antenna power constraints, it is optimal to allocate maximal per-antenna power to those antennas to satisfy the per-antenna power constraints. The remaining power is divided amongst the other antennas whose optimal allocation follows from a reduced joint sum and per-antenna power constraints problem of smaller channel coefficient dimension and reduced sum power constraint. Finally, the theoretical results are illustrated by numerical examples.

A.1 Introduction

For the last two decades, there has been a huge interest in vector-valued transmit strategies in wireless communications. The optimization problem of finding the optimal transmit strategy for a Gaussian channel has been extensively studied subject to either sum power constraint or per-antenna power constraints, but, to the best of our knowledge, a combination of both constraints surprisingly has not been considered yet. While a sum power constraint limits the total power of the transmitter, a per-antenna power constraint limits the used power on each transmitter chain of each antenna. Both constraints have reasonable physical motivations. For instance, the former constraint may be imposed by regulations or to limit the energy consumption, while the latter may be imposed by hardware limitations of each RF chain. Thus, it is reasonable to consider both constraints simultaneously.

Under a sum power constraint, when the channel state is known at both transmitter and receiver, the maximum transmission rate is obtained by performing

singular value decomposition and applying water-filling on the channel eigenvalues [CT06, TV05, Tel99]. In contrast, the per-antenna power constraints problem results in a different power allocation mechanism because the power can not be arbitrarily allocated among the transmit antennas. The per-antenna power constraints problem has received considerable attention recently [Vu11a, Vu11b, Pi12, MDT14, Tun14, YL07, SSB08, KYFV07, WESS08, BH06, Zha10]. Particularly, the problem of finding the capacity of point-to-point channels with per-antenna power constraints is well studied in [Vu11a, Vu11b, Pi12, MDT14]. In [Vu11a], the closed-form solution of the capacity and the optimal signaling scheme for MISO channels has been established for two separate cases assuming a constant channel which is known by both the transmitter and receiver, and also assuming Rayleigh fading where the channel coefficient is known at the receiver only. In addition, the optimal transmission schemes for point-to-point multiple-input multiple-output (MIMO) channels with per-antenna power constraints are studied in [Vu11b] and [Pi12]. In these works, the authors derived necessary and sufficient conditions for the optimal MIMO transmission schemes and developed an iterative algorithm that converges to the optimal solution. The ergodic capacity of the MISO channel with per-antenna power constraints is considered in [Vu11a] and [MDT14]. In [MDT14], the authors characterize the ergodic capacity of the fading MISO channel subject to long-term average per-antenna power constraints with perfect channel state information at all nodes. Then, they consider an application to the fading two-user cognitive interference channel.

The optimization problem with per-antenna power constraints for multi-user channels is studied in [YL07, SSB08, KYFV07, BH06, Zha10, TCJ08, HPC10, WESS08]. In [YL07], the problem of transmitter optimization for the multi-antenna downlink is considered. That work mainly focuses on the minimum-power beamforming design and the capacity-achieving transmitter design. It is shown that the solution to the per-antenna power constraints problem arises from a new interpretation of the uplink-downlink duality. In [SSB08], the authors focus on the discussion of linear signal processing strategies dealing with two optimization problems: maximizing the sum rate subject to per-antenna power constraints and maximizing the minimum user rate under per-antenna power constraints. Also, an iterative algorithm is proposed for solving the problem of maximizing the weighted rate sum for multi-user systems with per-antenna power constraints. In [KYFV07], the optimal zero-forcing beamforming in multiple antenna broadcast channels (BC) with per-antenna power constraints is considered. The results show that an optimization problem subjects to per-antenna power constraints for the broadcast channel may improve the rate considerably when the number of transmit antennas is larger than the number of receive antennas. The problem of linear zero-forcing precoder design is investigated in [WESS08]. The authors proved that under the assumption of a sum power constraint, precoders based on the pseudo-inverses are optimal among the generalized inverses. However, this is not necessarily true under the assumption of per-antenna power constraints.

The optimal power allocation problems for the MIMO broadcast channels with

per-antenna power constraints are studied in [BH06] and [Zha10]. In [BH06], the optimal power allocation to maximize the weighted rate sum assuming a zero-forcing precoder and a per-antenna power constraint are determined. In their work, the precoding vectors for the sum power constraint are adapted and then the power allocation is optimized to maximize the sum rate under per-antenna power constraints. In [Zha10], the author focuses on the block diagonalization based downlink precoding for a fully cooperative multi-cell system with per-base-station power constraints. To meet the per-base-station power constraints, a suboptimal heuristic method is proposed which combines the block diagonalization precoder design with an optimized power allocation scheme. In particular, the proposed solution in [Zha10] can be reduced to the optimal zero-forcing precoder design for weighted rate sum maximization with per-antenna power constraints if single-antenna base-stations and a mobile-station are used. Optimization problems whose classical formulations have been extended by adding unconventional constraints have been considered as well. For instance, in [TCJ08], the authors focus on designing linear multi-user MIMO transceivers subject to the different quality of service constraints per user and per-antenna power constraints. In [HPC10], the transmitter optimization problem for a MISO channel subject to general linear constraints is considered. Algorithms that solve this problem with both the optimal dirty-paper coding and simple sub-optimal linear zero-forcing beamforming are provided. The general linear constraints in their work include the sum power constraint, per-antenna power constraints and “forbidden interference direction” constraints.

Combinations of several power constraints have been considered in a range of other scenarios, for instance in the context cognitive radio channels [ZXL09, HS10] or wiretap channels [LHHT⁺13]. These works build on known results considering sum power, individual power or per-antenna power constraints, and extend them with additional power constraints to limit the received power at a third node. In particular for the cognitive radio channel, Zhang *et al.* studied in [ZXL09] the weighted rate sum maximization problem in which the secondary users have not only the sum power constraint but also interference constraints. The sum power constraint and interference constraints are also considered in [HS10], where the authors used the idea of antenna selection to jointly satisfy interference constraints at primary users while improving the rates of secondary users. In addition, the optimization problem with joint power and interference constraints has also received much attention in green radio setups [XQ13, MXL13, HU13]. The key difference of green radio is to focus more on the optimization of the energy efficiency instead of the transmission rate. In [XQ13], authors designed an effective multi-user MIMO transmission strategy to maximize the system energy efficiency defined as the ratio of the rate sum to the total power consumption. The energy efficiency optimization problem for MIMO broadcast channels subject to a sum power constraint, interference constraints, and a minimum throughput constraint was studied in [MXL13]. The solutions for the optimization problems in [XQ13, MXL13, HU13] are based on the duality between multiple access and broadcast channel as well as dirty paper coding.

In practical systems, the joint sum and per-antenna power constraints setting applies either to systems with multiple antennas or to distributed systems with separated energy sources. A sum power constraint can be, for instance, motivated by radiation limits or green aspects to limit the energy consumption. On the other hand, a per-antenna power constraint can be motivated to limit the power in the RF chain of each antenna. This also allows operating the power amplifier in the RF chain at a more energy efficient operating point. Since both aspects can be relevant in practical scenarios, it is reasonable to include them both in a classical MISO point-to-point setup. However, the joint sum and per-antenna power constraints are also relevant for future wireless systems in which base-stations are connected via high-speed links so that they can cooperate in the downlink transmission or in the uplink where mobile users cooperate in the transmission and each user has a limited power budget. Since the sum power constraint is not active if the allowed sum power is larger than the sum of the per-antenna power constraints, the problem is only interesting if the sum power constraint is smaller than the sum of the individual power constraints as illustrated in Figure A.1. Thus, the main purpose of this paper is to characterize the optimal transmit strategy for the point-to-point MISO channel with joint sum and per-antenna power constraints with the assumption of perfect channel state information at the transmitter. The solution is developed from the two original problems considering a sum power constraint or per-antenna power constraints only. A special case with two transmit antennas has been considered in [COSS15], where we have shown that if the sum power constraint only optimal power violates a per-antenna power constraint then the optimal power allocation of the considered joint power constraints is at the intersection of the sum power constraint and the per-antenna power constraint.

The organization of this paper is as follows. In the next section, we introduce the system model and the power constraints including the sum power constraint, the per-antenna power constraint and the joint sum and per-antenna power constraint. In Section A.3 we briefly recapitulate the known results for the problems of sum power constraint and per-antenna power constraints only. Then the properties of the optimal transmit strategy and power allocation for the joint sum and per-antenna constraints are discussed. The algorithm to find the optimal transmit strategy for joint sum and per-antenna power constraints is given in Section A.4. Then, the results and numerical examples are discussed in the next section. Finally, we provide some remarks and conclusions.

Notation

We use bold lower-case letters for vectors, bold capital letters for matrices. The superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$ stand for transpose, conjugate, and conjugate transpose; the superscripts $(\cdot)^{(1)}$, $(\cdot)^{(2)}$, $(\cdot)^{(3)}$, and $(\cdot)^{(\mathcal{S})}$ denote the corresponding optimal values of optimization problems according to the sum power constraint, the per-antenna power constraints, the joint sum and per-antenna power constraints, and a given optimization domain \mathcal{S} . We use \succcurlyeq for positive semi-definite relation,

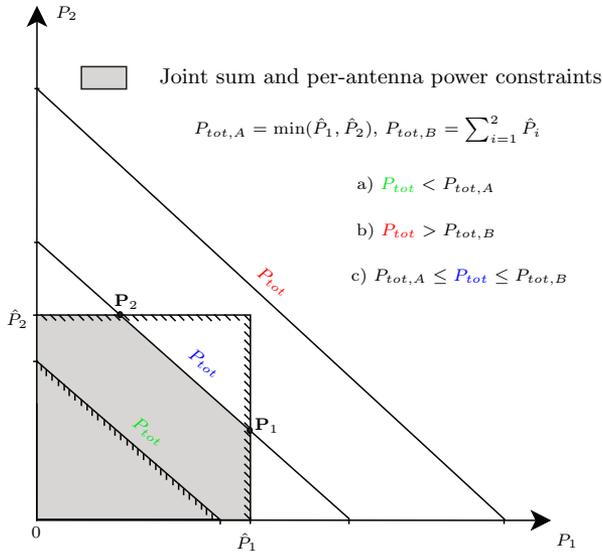


Figure A.1: Feasible power allocation region with joint sum and per-antenna power constraints with a) per-antenna power constraints are inactive, b) sum power constraint is inactive, c) sum and power antenna power constraints are all active.

$\text{tr}(\cdot)$ for trace, $\text{rank}(\cdot)$ for rank, and $\text{diag}\{\cdot\}$ for diagonal matrix. The expectation operator of a random variable is given by $\mathbb{E}[\cdot]$. \mathbb{N} , \mathbb{R}_+ , and \mathbb{C} are the sets of non-negative integers, non-negative real, and complex numbers.

A.2 System Model and Power Constraints

A.2.1 System Model

We consider a point-to-point MISO channel with N_t transmit antennas and one receive antenna. Further, we assume that channel state information (CSI) is available at both transmitter and receiver. The channel input-output relation of this transmission model can be written as

$$y = \mathbf{x}^H \mathbf{h} + z \quad (\text{A.2.1})$$

where $\mathbf{x} = [x_1, \dots, x_{N_t}]^T \in \mathbb{C}^{N_t \times 1}$ is the complex transmit signal vector, $\mathbf{h} = [h_1, \dots, h_{N_t}]^T = [h_i]_{i \in \{1, \dots, N_t\}}^T \in \mathbb{C}^{N_t \times 1}$ is the channel coefficient vector and z is zero-mean scalar additive white complex Gaussian noise with power σ^2 . Without loss of generality, we assume that $|h_i| > 0, \forall i \in \{1, \dots, N_t\}$, because otherwise we

can consider a corresponding MISO channel with a reduced number of antennas. In the following, we focus on achievable rates using Gaussian distributed input. Let $\mathbf{Q} = \mathbb{E}[\mathbf{x}\mathbf{x}^H]$ be the transmit covariance matrix of the Gaussian input \mathbf{x} , then the achievable transmission rate is

$$R = f(\mathbf{Q}) = \log \left(1 + \frac{1}{\sigma^2} \mathbf{h}^H \mathbf{Q} \mathbf{h} \right). \quad (\text{A.2.2})$$

There are two questions which we are going to answer in the upcoming sections. First, we show that Gaussian distributed input is capacity-achieving for the channel (A.2.1) with joint sum and per-antenna power constraints. Second, we identify the optimal transmit strategy \mathbf{Q} such that the transmission rate in (A.2.2) is maximized.

A.2.2 Power Constraints

In this part, we formally introduce the sum power, the per-antenna power and the joint sum and per-antenna power constraints problems.

a). Sum Power Constraint

If we consider a sum power constraint [CT06, TV05, Tel99, Lo99, YC04, LTV06, WSS06], the total transmit power from all antennas is limited by P_{tot} . This power can be allocated arbitrarily among the transmit antennas, and the input covariance matrix has to satisfy the condition $\text{tr}(\mathbf{Q}) \leq P_{tot}$. Let \mathcal{S}_1 denote the set of all power allocations which satisfy the sum power constraint, then \mathcal{S}_1 can be represented as

$$\mathcal{S}_1 := \{\mathbf{Q} \succcurlyeq 0 : \text{tr}(\mathbf{Q}) \leq P_{tot}\}.$$

b). Per-antenna Power Constraints

In the per-antenna power constraints case [Vu11a, YL07, SSB08, MDT14, KYFV07, CTJLa07, LML12, TCJ07], each individual transmit antenna has its own average power limitation \hat{P}_i , $\forall i \in \{1, \dots, N_t\}$. In fact, there is no flexibility in allocating the transmit power over all transmit antennas. However, the antennas can fully cooperate with each other for the transmission. Thus, for the per-antenna power constraints, the input covariance matrix \mathbf{Q} has diagonal elements satisfy $q_{ii} = \mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i \leq \hat{P}_i$ with \mathbf{e}_i is the i^{th} Cartesian unit vector. Let \mathcal{S}_2 denote the set of all power allocations which satisfy the per-antenna power constraints, then \mathcal{S}_2 can be represented as

$$\mathcal{S}_2 := \{\mathbf{Q} \succcurlyeq 0 : \mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i \leq \hat{P}_i, i = 1, \dots, N_t\}.$$

c). Joint Sum and Per-antenna Power Constraints

In this case, we combine the sum power and per-antenna power constraints. This means that each transmit antenna has a maximum individual transmit power budget of \hat{P}_i , $\forall i \in \{1, \dots, N_t\}$ as in the per-antenna power constraints problem. Additionally, the sum power condition P_{tot} has to be satisfied as well. Let \mathcal{S}_3 denote

the set of all power allocations which satisfy the joint sum and per-antenna power constraints, then \mathcal{S}_3 can be represented as

$$\begin{aligned}\mathcal{S}_3 &= \mathcal{S}_1 \cap \mathcal{S}_2 \\ &= \{\mathbf{Q} \succcurlyeq 0 : \text{tr}(\mathbf{Q}) \leq P_{tot}, \mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i \leq \hat{P}_i, i = 1, \dots, N_t\}.\end{aligned}$$

In Figure A.1, the power constraint domains are illustrated with individual power constraints for two antennas and increasing sum power. Let $P_{tot,A} = \min(\hat{P}_1, \hat{P}_2)$ and $P_{tot,B} = \sum_{i=1}^2 \hat{P}_i$, then we have three different cases of power domains as follows:

- Case 1 - Sum power constraint domain: This domain occurs when the per-antenna power constraints are always inactive, i.e., $P_{tot} < P_{tot,A}$ [Tel99].
- Case 2 - Per-antenna power constraints domain: This domain occurs when the sum power constraint is inactive, i.e., $P_{tot} > P_{tot,B}$ [Vu11a].
- Case 3 - Joint sum and per-antenna power constraints domain: This domain (gray area in Figure A.1) is considered when the power relations satisfy $P_{tot,A} \leq P_{tot} \leq P_{tot,B}$, i.e., both sum and power antenna power constraints can be active.

A.3 Problem Formulations and Solutions

In this section, we derive our main result on the optimal transmit strategy that achieves the capacity of the Gaussian MISO channel with joint sum and per-antenna power constraints. We first review the known results of the optimization problems with a sum power constraint only and per-antenna constraints only. After that, the optimization problem with joint sum and per-antenna power constraints will be studied.

A.3.1 Review of Known Results

a). Optimization Problem 1 (OP1) - Maximum Transmission Rate with Sum Power Constraint

This problem aims to find the maximum transmission rate in (A.2.2) under the set of sum power constraint \mathcal{S}_1 . The optimization problem for our given MISO channel, in this case, can be written as

$$\begin{aligned}\text{maximize} \quad & \log \left(1 + \frac{1}{\sigma^2} \mathbf{h}^H \mathbf{Q} \mathbf{h} \right) \\ \text{subject to} \quad & \mathbf{Q} \in \mathcal{S}_1.\end{aligned}\tag{A.3.1}$$

The transmit strategy for the MISO channel is to send the information only in the direction of the channel vector \mathbf{h} [CT06], [TV05]. The optimal solution is to

perform beamforming using full power P_{tot} in the direction of the channel, i.e., $\mathbf{Q}^{(1)} = P_{tot} \mathbf{u}_1 \mathbf{u}_1^H$ with $\mathbf{u}_1 = \mathbf{h} / \|\mathbf{h}\|$. The MISO channel capacity with a sum power constraint P_{tot} is

$$R^{(1)} = \log \left(1 + \frac{P_{tot}}{\sigma^2} \sum_{i=1}^{N_t} |h_i|^2 \right) = \log \left(1 + \frac{P_{tot}}{\sigma^2} \|\mathbf{h}\|^2 \right). \quad (\text{A.3.2})$$

b). Optimization Problem 2 (OP2) - Maximum Transmission Rate with Per-antenna Power Constraints

In [Vu11a], Vu established the closed-form expression of the capacity and optimal transmit strategy for the single-user MISO channel with per-antenna power constraints. The capacity for this case can be found by solving the optimization problem

$$\text{maximize } \log \left(1 + \frac{1}{\sigma^2} \mathbf{h}^H \mathbf{Q} \mathbf{h} \right) \quad (\text{A.3.3})$$

subject to $\mathbf{Q} \in \mathcal{S}_2$.

The problem in (A.3.3) can be solved by relaxing the semi-definite constraint, reducing the problem to be solvable in closed-form, and then showing that the optimal solution to the relaxed problem is also the optimal solution to the original problem [Vu11a]. In the per-antenna power constraints case, there is no power allocation among the antennas. Therefore, the transmit power from the i^{th} antenna is fixed to be \hat{P}_i . The optimal covariance matrix $\mathbf{Q}^{(2)}$ has rank one with $\mathbf{Q}^{(2)} = (\sum_{i=1}^{N_t} \hat{P}_i) \mathbf{v}_1 \mathbf{v}_1^H$ where the beamforming vector \mathbf{v}_1 has the elements given as

$$v_k = \frac{\sqrt{\hat{P}_k}}{\sqrt{\sum_{i=1}^{N_t} \hat{P}_i}} \frac{h_k}{|h_k|}, k = 1, \dots, N_t. \quad (\text{A.3.4})$$

The capacity with per-antenna power constraints is then given as

$$\begin{aligned} R^{(2)} &= \log \left(1 + \frac{1}{\sigma^2} \sum_{i=1}^{N_t} \hat{P}_i |\mathbf{h}^H \mathbf{v}_1|^2 \right) \\ &= \log \left[1 + \frac{1}{\sigma^2} \left(\sum_{i=1}^{N_t} |h_i| \sqrt{\hat{P}_i} \right)^2 \right]. \end{aligned} \quad (\text{A.3.5})$$

A.3.2 Optimization Problem 3 (OP3) - Maximum Transmission Rate with Joint Sum and Per-antenna Power Constraints

In the following proposition, we show that Gaussian distributed input is optimal for OP3.

Proposition A.1. *Gaussian distributed input \mathbf{x} with covariance $\mathbf{Q} = \mathbb{E}[\mathbf{x}\mathbf{x}^H]$ is capacity-achieving for the Gaussian MISO channel (A.2.1) with joint average sum and per-antenna power constraints, i.e., $\mathbf{Q} \in \mathcal{S}_3$.*

Proof. The proof of Proposition A.1 can be found in Appendix A.7.1. The achievability and converse proofs of this proposition can be derived from [CT06] and [EGK12, Ash90, WOB⁺08, Tho87]. \square

Next, we are going to characterize the optimal transmit strategy, i.e., the optimal $\mathbf{Q} \in \mathcal{S}_3$. The optimization problem to find the MISO channel capacity with joint sum and per-antenna power constraints is a convex optimization problem [BV09] given as follows

$$\begin{aligned} & \text{maximize} \quad \log \left(1 + \frac{1}{\sigma^2} \mathbf{h}^H \mathbf{Q} \mathbf{h} \right) & (\text{A.3.6}) \\ & \text{subject to} \quad \mathbf{Q} \in \mathcal{S}_3. \end{aligned}$$

The objective function of problem (A.3.6) is concave while both constraints $\text{tr}(\mathbf{Q}) \leq P_{tot}$ and $\mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i \leq P_i \forall i \in \{1, \dots, N_t\}$ are linear in \mathbf{Q} .

Furthermore, since $\log \left(1 + \frac{1}{\sigma^2} \mathbf{h}^H \mathbf{Q} \mathbf{h} \right)$ is an increasing function in $\mathbf{h}^H \mathbf{Q} \mathbf{h}$, we can express the optimization problem in (A.3.6) as

$$\max_{\mathbf{Q} \in \mathcal{S}_3} \log \left(1 + \frac{1}{\sigma^2} \mathbf{h}^H \mathbf{Q} \mathbf{h} \right) = \log \left(1 + \frac{1}{\sigma^2} \max_{\mathbf{Q} \in \mathcal{S}_3} \mathbf{h}^H \mathbf{Q} \mathbf{h} \right). \quad (\text{A.3.7})$$

Thus, we can equivalently focus on the following convex optimization problem

$$\begin{aligned} & \text{maximize} \quad \mathbf{h}^H \mathbf{Q} \mathbf{h} & (\text{A.3.8}) \\ & \text{subject to} \quad \mathbf{Q} \in \mathcal{S}_3. \end{aligned}$$

The results in the following propositions will show that the optimal transmit strategy for joint sum and per-antenna power constraints is beamforming. The optimal transmission method is to transmit with the maximal sum power while the per-antenna power constraints have to be satisfied, i.e., at the optimum full transmit power is used. The phase is chosen to match the phase of the channel coefficient.

Proposition A.2. *For OP3 with $P_{tot} < \sum_{i=1}^{N_t} \hat{P}_i$ and a given channel $\mathbf{h} \in \mathbb{C}^{N_t \times 1}$ with $h_i \neq 0, \forall i \in \{1, \dots, N_t\}$, beamforming is the optimal transmit strategy.*

Proof. The proof of Proposition A.2 can be found in Appendix A.7.2. The key idea is to use Lagrange multiplier and slackness conditions of the necessary Karush-Kuhn-Tucker (KKT) conditions to show that the rank of $\mathbf{Q}^{(3)}$ has to be one at the optimum. \square

In the following, let \mathbf{q} denote a beamforming vector of a rank one transmit strategy \mathbf{Q} , i.e., $\mathbf{Q} = \mathbf{q}\mathbf{q}^H$.

Proposition A.3. For OP3 with $P_{tot} < \sum_{i=1}^{N_t} \hat{P}_i$ and a given channel $\mathbf{h} \in \mathbb{C}^{N_t \times 1}$ with $h_i \neq 0, \forall i \in \{1, \dots, N_t\}$, the maximum transmission rate $R^{(3)}$ can be achieved when the optimal transmit strategy $\mathbf{Q}^{(3)}$ uses full sum power P_{tot} , i.e., $\text{tr}(\mathbf{Q}^{(3)}) = P_{tot}$.

Proof. The proof of Proposition A.3 can be found in Appendix A.7.3. The proof follows from the monotonicity of the rate function in terms of \mathbf{Q} . \square

Next, we focus on characterizing properties of the optimal beamforming vector $\mathbf{q}^{(3)}$.

Lemma A.4. Let $\mathbf{q}^{(3)}$ be the optimal beamforming vector corresponding to the optimal covariance matrix $\mathbf{Q}^{(3)}$. Then

$$\mathbf{q}^{(3)} \in \mathbb{Q} := \left\{ \mathbf{q} : \mathbf{q} = \left[\frac{\sqrt{P_1} h_1}{|h_1|}, \dots, \frac{\sqrt{P_{N_t}} h_{N_t}}{|h_{N_t}|} \right]^T, \mathbf{q} \mathbf{q}^H \in \mathcal{S}_3 \right\}. \quad (\text{A.3.9})$$

for some choices of $P_i, \forall i = \{1, \dots, N_t\}$.

Proof. Consider optimization problem (A.3.8) with the optimization domain \mathcal{S}_3 , we have

$$\begin{aligned} \max_{\mathbf{Q} \in \mathcal{S}_3} \mathbf{h}^H \mathbf{Q} \mathbf{h} &\stackrel{(a)}{=} \max_{\mathbf{q} : \mathbf{q} \mathbf{q}^H \in \mathcal{S}_3} |\mathbf{h}^H \mathbf{q}|^2 \\ &\stackrel{(b)}{=} \max_{\mathbf{q} : \mathbf{q} \mathbf{q}^H \in \mathcal{S}_3} \left| \sum_{i=1}^{N_t} |h_i| \sqrt{P_i} e^{j(\varphi_i - \varphi_{h_i})} \right|^2 \\ &\leq \max_{\mathbf{q} : \mathbf{q} \mathbf{q}^H \in \mathcal{S}_3} \left(\sum_{i=1}^{N_t} |h_i| \sqrt{P_i} \right)^2 \\ &= \max_{\mathbf{q} \in \mathbb{Q}} \left(\sum_{i=1}^{N_t} |h_i| \sqrt{P_i} \right)^2 \\ &\stackrel{(c)}{\leq} \max_{\mathbf{q} : \mathbf{q} \mathbf{q}^H \in \mathcal{S}_3} |\mathbf{h}^H \mathbf{q}|^2 \end{aligned} \quad (\text{A.3.10})$$

where

(a) follows from Propositions A.1 and A.2,

(b) follows from the definition $h_i = |h_i| e^{j\varphi_{h_i}}$, $q_i = \sqrt{P_i} e^{j\varphi_i}$ with $\varphi_{h_i}, \varphi_i \in [0, 2\pi]$, and

(c) follows from the fact that $\mathbb{Q} \subseteq \{\mathbf{q} : \mathbf{q} \mathbf{q}^H \in \mathcal{S}_3\}$.

From (A.3.10) it follows that $\max_{\mathbf{Q} \in \mathcal{S}_3} \mathbf{h}^H \mathbf{Q} \mathbf{h} = \max_{\mathbf{q} \in \mathbb{Q}} |\mathbf{h}^H \mathbf{q}|^2$, i.e., the optimal beamforming vector $\mathbf{q}^{(3)}$ is in \mathbb{Q} . \square

In the joint sum and per-antenna power constraints problem, Proposition A.3 states that the capacity achieving transmit strategy always allocates full sum power P_{tot} . However, the optimal power allocation solution of OP1 may result in violating certain per-antenna power constraints.

In the following theorem, we will show how to allocate the powers for the MISO channel for the general case with an arbitrary number of transmit antennas. We will show that if there exists any antenna for which the optimal power allocation of OP1 exceeds the per-antenna power constraints of OP3, then for those the optimal power allocation is equal to the per-antenna power constraints and (A.3.8) reduces to a new optimization problem with a smaller total transmit power and a reduced number of channel coefficients.

Theorem A.5. *Let $\mathcal{I} \subseteq \{1, \dots, N_t\}$ and $\mathcal{P}_V := \{i \in \mathcal{I} : P_i^{(1)} > \hat{P}_i\}$, if $\mathcal{P}_V = \emptyset$ then $P_i^{(3)} = P_i^{(1)} \forall i \in \mathcal{I}$, else $P_i^{(3)} = \hat{P}_i \forall i \in \mathcal{P}_V$ and the remaining optimal powers can be computed by solving a reduced optimization problem*

$$\arg \max_{\mathbf{q}' \in \mathcal{Q}'} |\mathbf{h}'^H \mathbf{q}'|^2 \quad (\text{A.3.11})$$

where $\mathbf{h}' = [h_i]_{i \in \mathcal{P}_V^c}^T \in \mathbb{C}^{|\mathcal{P}_V^c| \times 1}$, $\mathcal{Q}' := \{\mathbf{q}' : \sum_{i \in \mathcal{P}_V^c} |q_i|^2 \leq P_{tot} - \sum_{i \in \mathcal{P}_V} \hat{P}_i, |q_i|^2 \leq \hat{P}_i, i \in \mathcal{P}_V^c\}$ and $\mathcal{P}_V^c = \mathcal{I} \setminus \mathcal{P}_V$.

Proof. The proof of Theorem A.5 can be found in Appendix A.7.4. Theorem A.5 is proved in two steps. In the first step, we prove that $P_i^{(3)} = \hat{P}_i \forall i \in \mathcal{P}_V$ by pointing out that the per-antenna power constraint is not active if the optimal power of OP1 solution on the i^{th} antenna does not exceed \hat{P}_i . After that, it is shown that the remaining problem can be reformulated as a reduced optimization problem using the properties in propositions and lemmas above. \square

It can be seen from Theorem A.5 that if there exists an optimal power allocation of the OP1 solution which violates the per-antenna power constraint, then it is optimal to allocate power equal to the per-antenna power constraint for the corresponding antenna.

When more power constraints are active, we have less freedom to allocate the power. In addition, Theorem A.5 also shows a recursive process which leads to an efficient optimization algorithm. After satisfying the per-antenna power constraint on the power violated antenna, a reduced optimization problem with the smaller size of channel coefficient and total transmit power is formulated. The remaining optimal power allocation can be computed by solving that reduced optimization problem. The recursion finishes when all power constraints are satisfied. The number of iterations equals the times that the set of indices of optimal powers of the OP1 solution violating the per-antenna power constraints of OP3 is not empty. The following corollary states how the set of powers which violated constraints can be computed.

Corollary A.6. Let $\mathcal{P}_V^{(3)} := \{i \in \{1, \dots, N_t\} : P_i^{(3)} = \hat{P}_i < P_i^{(1)}\}$ and K be the number of total iterations, then the set of violated power constraints is the union of such a set at each iteration,

$$\mathcal{P}_V^{(3)} = \bigcup_{k=1}^K \mathcal{P}_V(k), \quad (\text{A.3.12})$$

where $\mathcal{P}_V(k) = \{i \in \mathcal{I}(k) : P_i^{(1)} > \hat{P}_i\}$, $\mathcal{I}(k+1) = \mathcal{I}(k) \setminus \mathcal{P}_V(k)$ and $\mathcal{I}(1) = \mathcal{I}$ with $\mathcal{I} \subseteq \{1, \dots, N_t\}$.

Proof. The proof of Corollary A.6 can be found in Appendix A.7.5. \square

Remark A.7. For a MISO channel with N_t transmit antennas and $P_{tot} \leq \sum_{i=1}^{N_t} \hat{P}_i$, the maximum number of violated per-antenna power constraints is $N_t - 1$, which also corresponds to the maximal number of iterations, i.e., $K \leq N_t - 1$.

We discuss optima in the interesting joint sum and per-antenna power constraints domain, i.e., we have $P_{tot,A} \leq P_{tot} \leq P_{tot,B}$. In this domain, we can identify an intersection point where the trajectory of the sum power constraint only optimal power allocation intersects a per-antenna power constraint when increasing the allowed sum power for both setups. This point plays an important role since the power allocation behavior crossing this point changes and therewith the growth of the maximal achievable rate.

Proposition A.8. Let \bar{P}_{tot} denote the sum transmit power at the intersection point. Then

$$\bar{P}_{tot} = \frac{\sum_{i \in \mathcal{I}} |h_i|^2}{\sum_{j \in \mathcal{P}_V} |h_j|^2} \sum_{j \in \mathcal{P}_V} \hat{P}_j, \quad (\text{A.3.13})$$

where $\mathcal{I} \subseteq \{1, \dots, N_t\}$ and $\mathcal{P}_V := \{i \in \mathcal{I} : P_i^{(1)} > \hat{P}_i\}$.

Proof. The proof of Proposition A.8 can be found in Appendix A.7.6. The proof idea of this proposition follows from the property that at the intersection point $P_j^{(3)} = \hat{P}_j$ for $j \in \mathcal{P}_V$ and $R^{(3)} = R^{(1)}$. \square

In the next section, we propose an algorithm for optimal transmit strategy of MISO channels with joint sum and per-antenna power constraints derived from the analysis above.

A.4 Algorithm for Optimal Transmit Strategy

We use OP1, Theorem A.5, and Lemma A.4 from the previous sections to provide an algorithm to compute the optimum power allocation and optimal transmit strategy for a MISO system with a given channel $\mathbf{h} = [h_1, \dots, h_{N_t}]^T \in \mathbb{C}^{N_t \times 1}$, and joint sum

and per-antenna power constraints where the per-antenna power constraints are denoted as $\hat{\mathbf{P}} = [\hat{P}_1, \dots, \hat{P}_{N_t}]$ and the sum power constraint is denoted as $P_{tot} < \sum_{i=1}^{N_t} \hat{P}_i$.

In Algorithm 1, we start with computing the optimal power allocation $\mathbf{P}^{(1)}$ of optimization problem OP1 with sum power constraint P_{tot} only. Since $P_{tot} < \sum_{i=1}^{N_t} \hat{P}_i$, we know from the Proposition A.3 that the optimal power allocation of OP3 always allocates full sum power. Therefore, when all powers satisfy the constraints, the optimal transmit strategy of OP1 and the optimal transmit strategy of OP3 are the same. In this situation, the optimal transmission rate is $R^{(3)} = R^{(1)}$. Otherwise, we have $R^{(3)} < R^{(1)}$. From Theorem A.5, it follows that for any optimal transmit powers $P_i^{(1)}$ of OP1 which violates the maximum per-antenna power constraint, the optimal transmit power $P_i^{(3)}$ is set equal to \hat{P}_i . The number of antennas violating the per-antenna power constraints is $|\mathcal{P}_V|$ with $\mathcal{P}_V = \{i \in \mathcal{I} : P_i^{(1)} > \hat{P}_i\}$. In the next step, we need to find the optimal power allocation for the remaining antennas while their total power budget has reduced to $P_{tot} - \sum_{i \in \mathcal{P}_V} \hat{P}_i$. To do this, we simply repeat the computation of optimum power allocation of OP1 with a new total power $P_{tot} - \sum_{i \in \mathcal{P}_V} \hat{P}_i$ and a reduced channel defined as a new $\mathbf{h} \leftarrow [h_i]_{i \in \mathcal{I}}^T$ with $\mathcal{I} \leftarrow \mathcal{I} \setminus \mathcal{P}_V$. The algorithm stops when there is an OP1 solution with no per-antenna power constraint violated.

The optimal beamforming vector $\mathbf{q}^{(3)}$ and optimal transmit strategy $\mathbf{Q}^{(3)}$ of OP3 are then calculated using Lemma A.4. The details of the algorithm are shown in Algorithm A.4.1.

A.5 Numerical Examples

In this section, we give some numerical examples to illustrate the theoretical results. We first show the power allocation behavior and the feasible power domains when fixing the per-antenna power constraints. After that, numerical examples to show the trends of the transmission rate in different power constraint domains with different transmit antenna configurations are discussed. The unit of power and transmission rate using in all examples in this paper are Watt and bps/Hz.

a). Power constraint domains

For the first numerical example, we consider a MISO 2×1 system with complex channel $h = [1.0984 + 0.7015i, -0.2779 - 2.0518i]^T$, noise variance $\sigma^2 = 1$ and two per-antenna power constraints $\hat{P}_1 = 7$, $\hat{P}_2 = 20$ as shown in Figure A.2. Therewith, we have $P_{tot,A} = \min(\hat{P}_1, \hat{P}_2) = 7$ and $P_{tot,B} = \hat{P}_1 + \hat{P}_2 = 27$. In our simulation, we start to increase the total transmit power P_{tot} from 0 to 30 gradually. For any total transmit power $P_{tot} < P_{tot,A}$, the optimal solution of OP3 is the same as the optimal solution of OP1. Similarly, when $P_{tot} > P_{tot,B}$, the optimal solution of OP3 is the same as the optimal solution of OP2. For the case of two transmit antennas, the intersection point can be identified at $\bar{P}_{tot} = ((|h_1|^2 + |h_2|^2)/|h_1|^2)\hat{P}_1$

Algorithm A.4.1: Optimal transmit strategy for $P_{tot} < \sum_{i=1}^{N_t} \hat{P}_i$

Input : $\mathbf{h}, P_{tot}, \hat{\mathbf{p}}$
Output: $\mathbf{Q}^{(3)}$

- 1 Set of indices $\mathcal{I} := \{1, \dots, N_t\}$.
- 2 Compute optimum power allocation $\mathbf{P}^{(1)}$ with the elements $P_i^{(1)}, i \in \mathcal{I}$ of OP1(\mathbf{h} and P_{tot}).
- 3 Denote $\mathcal{P}_V = \{i \in \mathcal{I} : P_i^{(1)} > \hat{P}_i\}$ as a set of indices of powers violating the per-antenna power constraints.
- 4 **if** $\mathcal{P}_V = \emptyset$ **then**
- 5 $P_i^{(3)} \leftarrow P_i^{(1)}$ for all $i \in \mathcal{I}$.
- 6 **Go to** 16.
- 7 **else**
- 8 **for** $i \in \mathcal{P}_V$ **do**
- 9 $P_i^{(3)} \leftarrow \hat{P}_i$.
- 10 **end for**
- // Formulate the reduced problem
- 11 $\mathcal{I} \leftarrow \mathcal{I} \setminus \mathcal{P}_V$,
- 12 $P_{tot} \leftarrow P_{tot} - \sum_{k \in \mathcal{P}_V} \hat{P}_k$,
- 13 $\mathbf{h} \leftarrow [h_i]_{i \in \mathcal{I}}^T$.
- 14 **end if**
- 15 **Return to** 2.
- 16 Compute optimal beamforming vector $\mathbf{q}^{(3)}$ from Lemma A.4 using $[P_i^{(3)}]_{i=1}^{N_t}$, $[h_i]_{i=1}^{N_t}$.
- 17 Compute optimal transmit strategy $\mathbf{Q}^{(3)} = \mathbf{q}^{(3)}\mathbf{q}^{(3)H}$.

if $[P_1^{(1)} = \hat{P}_1, P_2^{(1)} \leq \hat{P}_2]$ or $\bar{P}_{tot} = ((|h_1|^2 + |h_2|^2)/|h_2|^2)\hat{P}_2$ if $[P_1^{(1)} \leq \hat{P}_1, P_2^{(1)} = \hat{P}_2]$. Regarding the optimal power allocation behavior in this domain, we obtain from Figure A.2 that if $P_{tot,A} \leq P_{tot} < \bar{P}_{tot}$, then the optimal power allocation satisfies $\mathbf{P}^{(3)} = \mathbf{P}^{(1)}$. Otherwise, the optimal power $P_1^{(1)}$ of the sum power constraint only problem violates the per-antenna power constraint \hat{P}_1 and $P_1^{(3)}$ is set equal to \hat{P}_1 .

The plot in Figure A.3 shows the power allocation behavior in the case of three antennas. In this example, we consider a MISO 3×1 system with channel $\mathbf{h} = [0.7 + 0.3i, 0.6 - 0.8i, -0.4 + 0.5i]^T$ and noise variance $\sigma^2 = 1$. The maximum transmit power on each antenna is set as $\hat{P}_1 = \hat{P}_2 = \hat{P}_3 = 10$ and total sum transmit power is $P_{tot} = 25$. The optimal power allocation region of OP3 is a polytope defined by $\{\mathbf{p} : P_1 \leq 10, P_2 \leq 10, P_3 \leq 10, P_1 + P_2 + P_3 \leq 25\}$. The optimum point of OP1 is found at the transmission rate $R^{(1)} = 5.0123$ with $P_1^{(1)} = 7.3, P_2^{(1)} = 12.6$, and $P_3^{(1)} = 5.1$. In this case, $P_2^{(1)} > 10$ violates the per-antenna power constraint

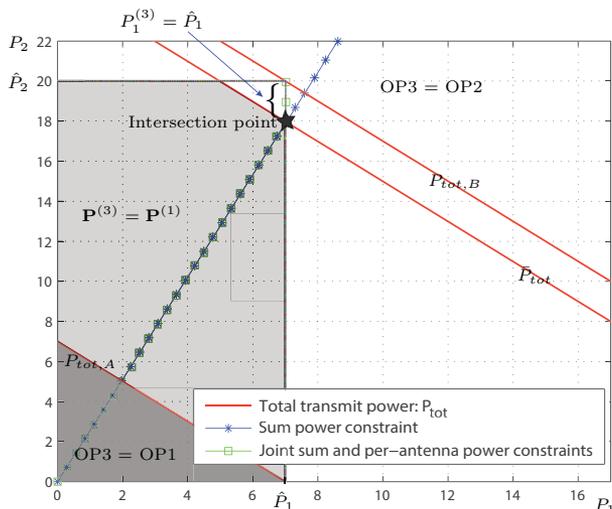


Figure A.2: Power allocation behavior with $\hat{P}_1 = 7$, $\hat{P}_2 = 20$.

\hat{P}_2 so that following Theorem A.5, to satisfy the power constraint of OP3, $P_2^{(3)}$ is set as $P_2^{(3)} = \hat{P}_2$. Therewith, the optimal power allocation of OP3 is determined as $P_1^{(3)} = 8.8$, $P_2^{(3)} = 10$, and $P_3^{(3)} = 6.2$ at rate $R^{(3)} = 5.0017$. This point is allocated on the boundary of the joint sum and per-antenna power constraints region.

b). Optimal transmission rate examples

In this part, we illustrate the trend of the transmission rate in different optimization domains and with various number of antennas. The examples in Figure A.4 consider 2 to 5 antennas respectively. The setup of channel coefficient \mathbf{h}_n and per-antenna power constraints $\hat{\mathbf{P}}_n$ corresponding to the number of antennas $N_t = 2, \dots, 5$ are configured by taking the first N_t elements of $\mathbf{h} = [0.9572 + 0.8003i, 0.4854 + 0.1419i, 0.6759 + 0.5236i, 0.5231 + 0.2563i, 0.2254 + 0.6225i]^T$ and $\hat{\mathbf{P}} = [\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4, \hat{P}_5] = [7, 10, 8, 5, 7]$. For instance, when we use two antennas, i.e., $N_t = 2$, then $\mathbf{h}_2 = [0.9572 + 0.8003i, 0.4854 + 0.1419i]^T$ and $\hat{\mathbf{P}}_2 = [\hat{P}_1, \hat{P}_2] = [7, 10]$. The noise variance is $\sigma^2 = 1$. The total transmit power increases from 0 to 40.

Since for cases 1 and 2 the optimal transmit strategies follow directly from OP1 and OP2, the case $P_{tot,A} \leq P_{tot} \leq P_{tot,B}$ is the most interesting. In Figure A.4, the optimal transmission rates are considered with respect to $\min(\hat{P}_i, \forall i = \{1, \dots, N_t\}) \leq P_{tot,N_t} \leq \sum_{i=1}^{N_t} \hat{P}_i$ for $N_t = 2, \dots, 5$. We observe that in these ranges of total transmit power, the optimal transmission rates of OP3 have similar trend as the optimal transmission rates of OP1, but the growth is slightly smaller.

The intersection points are identified when $P_{tot,N_t} = \bar{P}_{tot,N_t}$ where $\bar{P}_{tot,N_t} =$

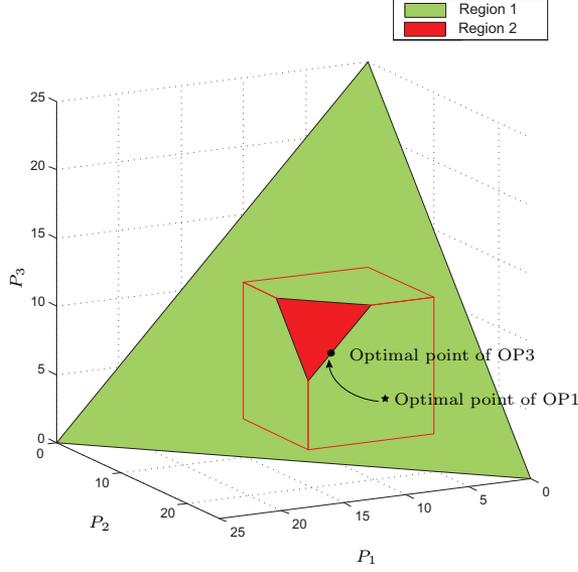


Figure A.3: MISO 3×1 optimal power allocation with joint sum and per-antenna power constraints where 'Region 1' is sum power constraint only region, and 'Region 2' is joint sum and per-antenna power constraints region when using full sum transmit power P_{tot} .

$(\sum_{i=1}^{N_t} |h_i|^2 / \sum_{j \in \mathcal{P}_V} |h_j|^2) \sum_{j \in \mathcal{P}_V} \hat{P}_j$ and j is summed over the indices of transmit antennas that violate the per-antenna power constraints. For instance, considering $N_t = 4$ transmit antennas, for which two antennas, antennas 2 and 3, violate per-antenna power constraints, then $\bar{P}_{tot,4} = (\sum_{i=1}^4 |h_i|^2 / (|h_2|^2 + |h_3|^2))(\hat{P}_2 + \hat{P}_3)$. We see that \bar{P}_{tot,N_t} changes with N_t . In this example, the intersection points are found as $\bar{P}_{tot,2} = 8.2$, $\bar{P}_{tot,3} = 11.5$, $\bar{P}_{tot,4} = 14.7$ and $\bar{P}_{tot,5} = 17.8$.

In Figure A.4, it is clear to see that when we keep a maximum sum transmit power while increasing the number of transmit antennas, it happens that $P_i^{(1)}$ violating \hat{P}_i for a few antennas might not violates \hat{P}_i for a larger number of antennas since we have more alternatives to allocate the power. Therefore, the gap between the optimal transmission rate with joint sum and per-antenna power constraints $R^{(3)}$ and the optimal transmission rate with sum power constraint $R^{(1)}$ is decreased.

c). Choices of power constraints

In this numerical example, we focus to clarify the impact of the choices of the power constraints on the optimal power allocation and the optimal transmission

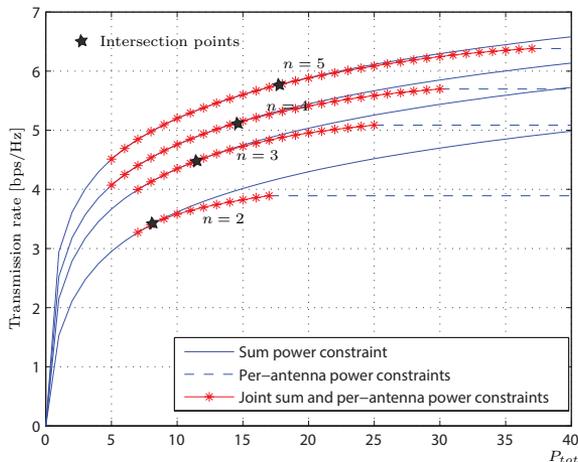


Figure A.4: Transmission rate in different power constraint domains and different transmit antenna configurations.

rate of the channel. The optimal transmission rate is shown with joint sum and per-antenna power constraints switching of a 3×1 MISO channel with $\mathbf{h} = [-1.2507 - 0.5078i, -0.9480 - 0.3206i, -0.7411 + 0.0125i]^T$. The total transmit power is set as $P_{tot} = 25$.

The curves in Figure A.5 are plotted by adjusting per-antenna power constraint on antenna 1 from 0 to 14 and setting per-antenna power constraint configurations on antenna 2 and 3 as follows: (i) $\hat{P}_2 = 7, \hat{P}_3 = 10$, (ii) $\hat{P}_2 > 25, \hat{P}_3 = 10$, (iii) $\hat{P}_2 > 25, \hat{P}_3 > 25$. For those settings, it turns out that at the optimum, both per-antenna power constraints on antennas 2 and 3 are active in case (i), and only per-antenna power constraint on the antenna 3 is active in case (ii). For the last case, per-antenna powers on the antennas 2 and 3 are not restricted. In Figure A.5, the OP1 solution is also shown, which corresponds to the case when all per-antenna power constraints are not active. The OP1 optimal power allocation is $P_1^{(1)} = 13.51, P_2^{(1)} = 7.42, P_3^{(1)} = 4.07$. We can see from the figure that the optimal transmission rate decreases if more per-antenna power constraints are added. In particular, we can see from Figure A.5 that the capacity when all per-antenna power constraints are included is always smaller or equal than the others. For instance, when $\hat{P}_1 = 6$, the capacity of the case (i) reaches $R = 4.25$. This value is smaller than the capacity of case (ii) with $R = 4.3$; and both are smaller than the capacity of case (iii) with $R = 4.35$. This happens because of the fact that adding constraints limits the optimization domain, i.e., we have less freedom to allocate the power.

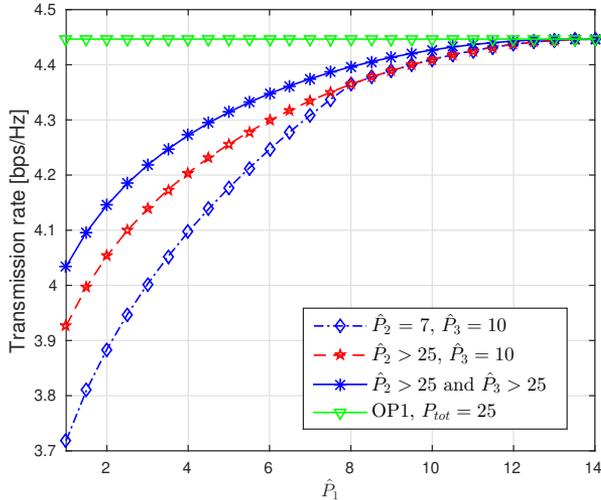


Figure A.5: The impact of choice of power constraints on the optimal power allocation and the capacity of 3×1 MISO channel with $P_{tot} = 25$. The marker symbols correspond to the following power constraint settings: sum power constraint ($-\nabla-$), additional per-antenna power constraints on P_1 ($-*-$), P_1 and P_3 ($-*-\$), and P_1 , P_2 and P_3 ($-\diamond-$).

A.6 Conclusions

In this paper, we derived the optimal power allocation for Gaussian MISO channels under joint sum and per-antenna power constraints. We further presented an iterative algorithm for this. The setup of the joint sum and per-antenna power constraints is relevant in practical systems where we have to limit the power in each RF chain for each antenna and the total radiated power across all transmit antennas due to regulation or other reasons. It is shown that beamforming is the optimal transmit strategy and the capacity is achieved when maximum sum power is used in the optimal transmit strategy. In the joint sum and per-antenna power constraints problem, the optimal powers are set equal to the per-antenna power constraints if their optimal values in sum power constraint only problem violate those per-antenna power constraints. The remaining powers can be found by solving a reduced optimization problem. Thus, we show that the optimal transmit strategy for the joint sum and per-antenna power constraints problem can be derived from the sum power constraint only and the per-antenna power constraints only problems.

A.7 Appendix

A.7.1 Proof of Proposition A.1

Proof of Achievability

We use $\mathcal{C}(n, \mathbf{Q}, R, \epsilon)$ to denote a codebook with codewords $\mathbf{x}^n(m)$ for messages $m \in \{1, \dots, M^{(n)}\}$ with $M^{(n)} = 2^{nR}$. This codebook is generated by selecting codewords of length n i.i.d. Gaussian with zero-mean and covariance $\mathbf{Q} - \rho \mathbf{I}$, where $\mathbf{Q} \in \mathcal{S}_3 := \{\mathbf{Q} \succcurlyeq 0 : \text{tr}(\mathbf{Q}) \leq P_{tot}, \mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i \leq \hat{P}_i, i = 1, \dots, N_t\}$. Following [CT06, Theorem 8.6.5] and the proof of achievability steps in [EGK12, Ash90, WOB⁺08] with $\mathbf{Q} \in \mathcal{S}_3$, we know that the channel capacity $R^{(3)}$ for the joint sum and per antenna power constrained channel must satisfy

$$R^{(3)} \geq \sup_{\mathbf{Q} \in \mathcal{S}_3} R. \quad (\text{A.7.1})$$

Proof of Converse

Following [CT06, EGK12, Ash90, WOB⁺08, Tho87], with a given codebook $\mathcal{C}(n, \mathbf{Q}, R, \epsilon)$, for a defined compact set \mathcal{S}_3 we obtain that $\frac{1}{n} \sum_{i=1}^n \mathbf{Q}_i \in \mathcal{S}_3$ since $\frac{1}{n} \sum_{i=1}^n \mathbf{Q}_i \succcurlyeq 0$, $\text{tr}(\frac{1}{n} \sum_{i=1}^n \mathbf{Q}_i) \leq P_{tot}$ and $\mathbf{e}_k^T \frac{1}{n} \sum_{i=1}^n \mathbf{Q}_i \mathbf{e}_k \leq \hat{P}_k, k = 1, \dots, N_t$ hold. This implies that there exists a subsequence $(n_l)_{l \in \mathbb{N}}$ of the codeword length such that $\frac{1}{n_l} \sum_{i=1}^{n_l} \mathbf{Q}_i \rightarrow \mathbf{Q}$ as $n_l \rightarrow \infty$ with $\mathbf{Q} \in \mathcal{S}_3$. Then,

$$\begin{aligned} R &\leq \limsup_{n_l \rightarrow \infty} \left\{ \log \left(1 + \frac{1}{\sigma^2} \mathbf{h}^H \left(\frac{1}{n_l} \sum_{i=1}^{n_l} \mathbf{Q}_i \right) \mathbf{h} \right) + \epsilon_{n_l} \right\} \\ &= \log \left(1 + \frac{1}{\sigma^2} \mathbf{h}^H \mathbf{Q} \mathbf{h} \right), \end{aligned} \quad (\text{A.7.2})$$

which proves the converse. \square

A.7.2 Proof of Proposition A.2

We denote $\mathbf{P} = \text{diag}\{\hat{P}_i\}$ as diagonal matrix of the per-antenna power constraints, P_{tot} as the total transmit power, $\mathbf{D} = \text{diag}\{\nu_i\}$ as diagonal matrix of Lagrangian multiplier for the per-antenna power constraints, μ as Lagrangian multiplier for the sum power constraint, and $\mathbf{K} \succcurlyeq 0$ as Lagrangian multiplier for the positive semi-definite constraint. Then the Lagrangian for problem (A.3.8) is given by

$$\mathcal{L} = \mathbf{h}^H \mathbf{Q} \mathbf{h} - \text{tr}[\mathbf{D}(\mathbf{Q} - \mathbf{P})] - \mu(\text{tr}(\mathbf{Q}) - P_{tot}) + \text{tr}(\mathbf{K}\mathbf{Q}). \quad (\text{A.7.3})$$

Taking the first derivative and set it equal to zero, we have

$$\frac{\partial \mathcal{L}}{\partial \mathbf{Q}} = \mathbf{h} \mathbf{h}^H - \mathbf{D} - \mu \mathbf{I} + \mathbf{K} \stackrel{!}{=} 0 \quad (\text{A.7.4})$$

or equivalently

$$\mathbf{h}\mathbf{h}^H = \mathbf{W} - \mathbf{K}, \quad (\text{A.7.5})$$

where $\mathbf{W} = \mathbf{D} + \mu\mathbf{I}$.

Using the slackness condition $\mathbf{K}\mathbf{Q} = \mathbf{0}$, we obtain

$$\mathbf{h}\mathbf{h}^H\mathbf{Q} = \mathbf{W}\mathbf{Q}. \quad (\text{A.7.6})$$

Since $\text{rank}(\mathbf{W}) = \text{rank}(\mathbf{D} + \mu\mathbf{I})$, which has full rank, at the optimum, we have

$$\text{rank}(\mathbf{Q}^{(3)}) \leq \text{rank}(\mathbf{h}\mathbf{h}^H) = 1. \quad (\text{A.7.7})$$

Obviously, since $h_i \neq 0, \forall i \in \{1, \dots, N_t\}$, $\text{rank}(\mathbf{Q}^{(3)}) = 0$ is not optimal. Therefore, the optimal rank of $\mathbf{Q}^{(3)}$ is one, i.e. beamforming is the optimal transmit strategy. \square

A.7.3 Proof of Proposition A.3

Given function $f : \mathbf{Q} \rightarrow \mathbb{R}_+$ as defined in (A.2.2). Following [HJ13] and [JB07], we obtain that $f(\mathbf{Q})$ is monotonic in terms of \mathbf{Q} . This implies that for any positive semi definite Hermitian matrices $\mathbf{Q}_1 \succcurlyeq \mathbf{Q}_2$, we have $f(\mathbf{Q}_1) \geq f(\mathbf{Q}_2)$.

Then, for OP3, if we suppose that $\mathbf{Q}^{(3)}$ is the optimal transmit strategy, the maximum transmission rate $R^{(3)} = f(\mathbf{Q}^{(3)})$ is achievable when $\mathbf{Q}^{(3)}$ allocates full sum power P_{tot} . \square

A.7.4 Proof of Theorem A.5

Given function $f : \mathbf{Q} \rightarrow \mathbb{R}_+$ as defined in (A.2.2). For the proof of Theorem A.5, we need following lemmas

Lemma A.9. *Let $\mathcal{A} \subseteq \mathcal{I}$, $\mathcal{B} := \{i \in \mathcal{I} \setminus \mathcal{A} : P_i^{(\mathcal{S}(\mathcal{A}))} > \hat{P}_i\}$, and $\mathcal{A}' = \mathcal{A} \cup \mathcal{B}$. If $\mathcal{B} \neq \emptyset$ then $P_i^{(\mathcal{S}(\mathcal{A}'))} = \hat{P}_i \forall i \in \mathcal{B}$, where $\mathcal{S}(\mathcal{A})$ and $\mathcal{S}(\mathcal{A}')$ are two given optimization domains defined as $\mathcal{S}(\mathcal{A}) := \{\mathbf{Q} \succcurlyeq 0 : \text{tr}(\mathbf{Q}) \leq P_{tot}, \mathbf{e}_j^T \mathbf{Q} \mathbf{e}_j \leq \hat{P}_j, j \in \mathcal{A}\}$.*

Proof (by contradiction). Since $\mathcal{S}(\mathcal{A}') \subseteq \mathcal{S}(\mathcal{A})$ we have

$$\max_{\mathbf{Q} \in \mathcal{S}(\mathcal{A})} f(\mathbf{Q}) \geq \max_{\mathbf{Q} \in \mathcal{S}(\mathcal{A}')} f(\mathbf{Q}). \quad (\text{A.7.8})$$

For every $\mathcal{B}' \subseteq \mathcal{B}$, $\mathcal{B}' \neq \emptyset$ we have

$$\max_{\mathbf{Q} \in \mathcal{S}(\mathcal{A})} f(\mathbf{Q}) > \max_{\mathbf{Q} \in \mathcal{S}(\mathcal{A} \cup \mathcal{B}')} f(\mathbf{Q}). \quad (\text{A.7.9})$$

If $\mathcal{B} \neq \emptyset$, suppose there exists $i \in \mathcal{B}$ for which $P_i^{(\mathcal{S}(\mathcal{A}'))} \neq \hat{P}_i$ is optimal. Since $P_i^{(\mathcal{S}(\mathcal{A}'))} \leq \hat{P}_i$ has to be satisfied for the optimization problem with domain $\mathcal{S}(\mathcal{A}')$,

this implies that $P_i^{(\mathcal{S}(\mathcal{A}'))} < \hat{P}_i$ and therefore the per-antenna power constraint is not active, i.e., $\max_{\mathbf{Q} \in \mathcal{S}(\mathcal{A})} f(\mathbf{Q}) = \max_{\mathbf{Q} \in \mathcal{S}(\mathcal{A} \cup \{i\})} f(\mathbf{Q})$. However, this contradicts with (A.7.9) with $\mathcal{B}' = \{i\}$. Thus, it follows that $P_i^{(\mathcal{S}(\mathcal{A}'))} = \hat{P}_i \forall i \in \mathcal{B}$. This proves Lemma A.9. \square

Lemma A.10. *Let $\mathbf{a} = [a_1, \dots, a_n]^T \in \mathbb{C}^{n \times 1}$, $\mathbf{x} = [x_1, \dots, x_n]^T \in \mathbb{D} \subseteq \mathbb{C}^{n \times 1}$ where \mathbb{D} has the property that if $\mathbf{x} \in \mathbb{D}$, then $\mathbf{D}\mathbf{x} \in \mathbb{D}$ with arbitrary $\mathbf{D} = \text{diag}\{e^{j\varphi_k}\} \in \mathbb{C}^{n \times n}$ and $\varphi_k \in (0, 2\pi] \forall k = 1, \dots, n$. For $\mathbf{a}^T \mathbf{x} \geq 0$ and $c \geq 0$, we have*

$$\arg \max_{\mathbf{x} \in \mathbb{D}} |\mathbf{a}^T \mathbf{x} + c|^2 = \arg \max_{\mathbf{x} \in \mathbb{D}} |\mathbf{a}^T \mathbf{x}|^2. \quad (\text{A.7.10})$$

Proof. We have

$$\begin{aligned} \arg \max_{\mathbf{x} \in \mathbb{D}} |\mathbf{a}^T \mathbf{x} + c|^2 &= \arg \max_{\mathbf{x} \in \mathbb{D}} |\mathbf{a}^T \mathbf{x}| + |c| \\ &= \arg \max_{\mathbf{x} \in \mathbb{D}} |\mathbf{a}^T \mathbf{x}| \\ &= \arg \max_{\mathbf{x} \in \mathbb{D}} |\mathbf{a}^T \mathbf{x}|^2. \end{aligned} \quad (\text{A.7.11})$$

This proves Lemma A.10. \square

Now, we prove Theorem A.5. Since $\mathcal{S}_3 \subseteq \mathcal{S}_1$ where $\mathcal{S}_1 := \{\mathbf{Q} \succcurlyeq 0 : \text{tr}(\mathbf{Q}) \leq P_{tot}\}$, we have $\max_{\mathbf{Q} \in \mathcal{S}_3} \mathbf{h}^H \mathbf{Q} \mathbf{h} \leq \max_{\mathbf{Q} \in \mathcal{S}_1} \mathbf{h}^H \mathbf{Q} \mathbf{h}$. The equality occurs when $P_i^{(1)} = P_i^{(3)} \forall i \in \mathcal{I}$, i.e., $P_i^{(1)} \leq \hat{P}_i, \forall i \in \mathcal{I}$. Otherwise there exists at least one power $P_i^{(1)}$ in the optimal power allocation of the OP1 solution that violates the per-antenna power constraints, i.e., $P_i^{(1)} > \hat{P}_i$ for some $i \in \mathcal{I}$, where the set of indices is defined as $\mathcal{P}_V := \{i \in \mathcal{I} : P_i^{(1)} > \hat{P}_i\}$.

Next, we will use Lemma A.9 with $\mathcal{A} = \mathcal{I} \setminus \mathcal{P}_V$ and $\mathcal{B} = \mathcal{P}_V$. First note that with this definition of \mathcal{A} and \mathcal{B} we have

$$\max_{\mathbf{Q} \in \mathcal{S}_1} f(\mathbf{Q}) = \max_{\mathbf{Q} \in \mathcal{S}(\mathcal{A})} f(\mathbf{Q}) \quad (\text{A.7.12})$$

and

$$\max_{\mathbf{Q} \in \mathcal{S}(\mathcal{A}')} f(\mathbf{Q}) = \max_{\mathbf{Q} \in \mathcal{S}_3} f(\mathbf{Q}). \quad (\text{A.7.13})$$

Then it follows from Lemma A.9 that $P_i^{(3)} = \hat{P}_i \forall i \in \mathcal{P}_V$.

Finally, we show that, in joint sum and per-antenna power constraints problem, the remaining optimal powers can be computed by solving a reduced optimization

problem. We have

$$\begin{aligned}
\arg \max_{\mathbf{Q} \in \mathcal{S}_3} f(\mathbf{Q}) &= \arg \max_{\mathbf{q}: \mathbf{q}\mathbf{q}^H \in \mathcal{S}_3} |\mathbf{h}^H \mathbf{q}|^2 \\
&\stackrel{(d)}{=} \arg \max_{\mathbf{q} \in \mathcal{Q}} \left| \sum_{i \in \mathcal{I}} |h_i| \sqrt{P_i^{(3)}} e^{j(\varphi_i - \varphi_{h_i})} \right|^2 \\
&\stackrel{(e)}{=} \arg \max_{\mathbf{q} \in \mathcal{Q}} \left(\sum_{i \in \mathcal{I}} |h_i| \sqrt{P_i^{(3)}} \right)^2 \\
&\stackrel{(f)}{=} \arg \max_{\mathbf{q} \in \mathcal{Q}} \left(\sum_{i \in \mathcal{P}_V^c} |h_i| \sqrt{P_i^{(3)}} + \sum_{i \in \mathcal{P}_V} |h_i| \sqrt{\hat{P}_i} \right)^2 \\
&\stackrel{(g)}{=} \arg \max_{\mathbf{q}' \in \mathcal{Q}'} |\mathbf{h}'^H \mathbf{q}' + \sum_{i \in \mathcal{P}_V} |h_i| \sqrt{\hat{P}_i}|^2 \\
&\stackrel{(h)}{=} \arg \max_{\mathbf{q}' \in \mathcal{Q}'} |\mathbf{h}'^H \mathbf{q}'|^2
\end{aligned} \tag{A.7.14}$$

where

(d) follows from the definition $h_i = |h_i| e^{j\varphi_{h_i}}$, $q_i = \sqrt{P_i^{(3)}} e^{j\varphi_i}$, $\forall i \in \mathcal{I}$ with $\varphi_{h_i}, \varphi_i \in [0, 2\pi]$,

(e) follows from Lemma A.4,

(f) follows from $P_i^{(3)} = \hat{P}_i \forall i \in \mathcal{P}_V$,

(g) follows from substituting variables with $\mathbf{q}' = [\sqrt{P_i^{(3)}} e^{j\varphi_i}]_{i \in \mathcal{P}_V^c}$ and changing the optimization domain to \mathcal{Q}' ,

(h) follows from Lemma A.10 with $\mathbf{a} \leftarrow \mathbf{h}'$ and $\mathbf{x} \leftarrow \mathbf{q}'$, and $\mathcal{Q}' := \{\mathbf{q}' : \sum_{i \in \mathcal{P}_V^c} |q_i|^2 \leq P_{tot} - \sum_{i \in \mathcal{P}_V} \hat{P}_i, |q_i|^2 \leq \hat{P}_i, i \in \mathcal{P}_V^c\}$ satisfies the condition of a set \mathbb{D} .

Thus, we have shown that if $\mathcal{P}_V \neq \emptyset$ then $P_i^{(3)} = \hat{P}_i \forall i \in \mathcal{P}_V$ and the remaining optimal powers can be allocated by solving $\arg \max_{\mathbf{q}' \in \mathcal{Q}'} |\mathbf{h}'^H \mathbf{q}'|^2$ which is equivalent to a reduced \mathcal{S}_3 problem. \square

A.7.5 Proof of Corollary A.6

Let $\mathcal{P}_V(k) = \{i \in \mathcal{I}(k) : P_i^{(1)} > \hat{P}_i\}$ be the set of indices of optimal powers of the OP1 solution violating the per-antenna power constraints of OP3 at the k^{th} iteration. Consider optimization problem (A.3.8) with the optimization domain \mathcal{S}_3 and the set of all indices is $\mathcal{I} \subseteq \{1, \dots, N_t\}$, the set of indices of optimal power allocations of the OP1 solution violating the per-antenna power constraints is given by

$$\mathcal{P}_V(1) = \{i \in \mathcal{I}(1) : P_i^{(1)} > \hat{P}_i\} \tag{A.7.15}$$

where $\mathcal{I}(1) = \mathcal{I}$. From Theorem A.5, we know that if $\mathcal{P}_V(1) \neq \emptyset$, then $P_i^{(3)} = \hat{P}_i \forall i \in \mathcal{P}_V(1)$. The reduced problem (A.3.11) with the set of all indices $\mathcal{I}(2) = \mathcal{I}(1) \setminus$

$\mathcal{P}_V(1) = \mathcal{I} \setminus \mathcal{P}_V(1)$ is considered instead of (A.3.8) with the set of all indices \mathcal{I} . To find the remaining optimal power allocation, we must solve (A.3.11) in the next iteration. The set of indices of violated power allocations in this iteration is given by

$$\mathcal{P}_V(2) = \{i \in \mathcal{I}(2) : P_i^{(1)} > \hat{P}_i\}. \quad (\text{A.7.16})$$

A new reduced problem can be formed if $\mathcal{P}_V(2) \neq \emptyset$.

The number of optimal powers of the OP1 solution violating the per-antenna power constraints of the OP3 in first two iterations is $|\mathcal{P}_V(1) \cup \mathcal{P}_V(2)|$.

Similarly, for the k^{th} iteration, the set of indices of violated power allocations is given by

$$\mathcal{P}_V(k) = \{i \in \mathcal{I}(k) : P_i^{(1)} > \hat{P}_i\} \quad (\text{A.7.17})$$

where $\mathcal{I}(k) = \mathcal{I}(k-1) \setminus \mathcal{P}_V(k) = \mathcal{I} \setminus \{\cup_{i=1}^{k-1} \mathcal{P}_V(i)\}$. The number of optimal powers of the OP1 solution violating the per-antenna power constraints of the OP3 solution in first k iterations is $|\mathcal{P}_V(1) \cup \mathcal{P}_V(2) \cup \dots \cup \mathcal{P}_V(k)|$.

Thus, assume that K is the number of iteration that the optimization problems have to reduce, it means that for any $l \geq K$, $\mathcal{P}_V(l) = \emptyset$, then the set of indices of total violated powers in joint sum and per-antenna power constraints is calculated as (A.3.12). \square

A.7.6 Proof of Proposition A.8

At the intersection point, we have:

$$\bar{P}_{tot} = \sum_{k \in \mathcal{I} \setminus \mathcal{P}_V} P_k^{(3)} + \sum_{j \in \mathcal{P}_V} \hat{P}_j, \quad (\text{A.7.18})$$

and it always holds

$$\sum_{i \in \mathcal{I}} |h_i|^2 = \sum_{k \in \mathcal{I} \setminus \mathcal{P}_V} |h_k|^2 + \sum_{j \in \mathcal{P}_V} |h_j|^2. \quad (\text{A.7.19})$$

Furthermore, at the intersection point, we also have $R^{(1)} = R^{(3)}$. This implies

$$\bar{P}_{tot} \sum_{i \in \mathcal{I}} |h_i|^2 = \left(\sum_{k \in \mathcal{I} \setminus \mathcal{P}_V} |h_k| \sqrt{P_k^{(3)}} + \sum_{j \in \mathcal{P}_V} |h_j| \sqrt{\hat{P}_j} \right)^2. \quad (\text{A.7.20})$$

Using (A.7.18) and (A.7.19), we can express (A.7.20) as the following equivalent equation

$$\frac{\sum_{k \in \mathcal{I} \setminus \mathcal{P}_V} P_k^{(3)}}{\sum_{j \in \mathcal{P}_V} \hat{P}_j} = \frac{\sum_{k \in \mathcal{I} \setminus \mathcal{P}_V} |h_k|^2}{\sum_{j \in \mathcal{P}_V} |h_j|^2}. \quad (\text{A.7.21})$$

This can be reformulated as in (A.3.13) by using (A.7.18) and (A.7.19) once again. This proves Proposition A.8. \square

Optimal Transmit Strategy for MIMO Channel with Joint Sum and Per-antenna Power Constraints

Phuong Le Cao and Tobias J. Oechtering

Revised version of referred paper published in
IEEE International Conference on Acoustic, Speech and Signal Processing
(*ICASSP*), Mar. 2017.

©2017 IEEE

Layout has been revised for thesis consistency

Optimal Transmit Strategy for MIMO Channel with Joint Sum and Per-antenna Power Constraints

Phuong Le Cao and Tobias J. Oechtering

Abstract

This paper studies optimal transmit strategies for multiple-input multiple-output (MIMO) Gaussian channels with joint sum and per-antenna power constraints. It is shown that if an unconstrained optimal allocation for an antenna exceeds a per-antenna power constraint, then the maximal power for this antenna is used in the constraint optimal transmit strategy. This observation is then used in an iterative algorithm to compute the optimal transmit strategy in closed-form. Finally, a numerical example is provided to illustrate the theoretical results.

B.1 Introduction

The optimization problem in maximizing the transmission rate for MIMO Gaussian channel has been extensively studied in the last two decades. Under the sum power constraint, the transmission rate is obtained by performing singular value decomposition and applying water-filling on channel eigenvalues [CT06, Tel99]. In contrast, the problem in finding maximal transmission rate with per-antenna power constraints results in a different mechanism since the power can not be arbitrarily allocated among the transmit antennas. This problem has been studied in [Vu11b, Pi12, COS16, YL07, SSB08, KYFV07]. In these works, the authors derived necessary and sufficient conditions for the optimal MIMO transmission schemes and developed an iterative algorithm that converges to the optimal solution. Besides iterative algorithms, a closed-form expression for the capacity of the static Gaussian MIMO channel under per-antenna power constraints is provided in [Tun14].

In a practical system, when joint sum and per-antenna power constraints are considered, the transmitted energy should be limited in total and for each RF chain. The joint sum and per-antenna power constraints setting can be applied either to systems with multiple antennas or to distributed systems with separated energy sources. The optimal transmit strategy problem with the joint sum and per-antenna power constraints has been studied for MISO channels in [COSS16, Loy16]. Since the sum power constraint is not active if the allowed sum power is larger than the sum of the per-antenna power constraints, the problem is only interesting if the sum power constraint is smaller than the sum of the individual power constraints. In [XFZP15], the optimization problem with the assumption that several antenna

subsets are constrained by a sum power constraint while the other antennas are subject to a per-antenna power constraint is studied and a closed-form solution is provided. Unfortunately, the results in that paper can not be directly applied to the case where the transmit powers are jointly constrained by both sum and per-antenna power constraints. To make [XFZP15] applicable for the optimization problem with joint sum and per-antenna power constraints, we need to identify for each antenna which constraint is active, which is the key step in this paper.

The contributions of the paper can be summarized as follows: An optimal transmit strategy for MIMO channels with joint sum and per-antenna power constraints with the assumption of perfect channel knowledge at the transmitter is characterized. If an optimal power allocation of an antenna of the unconstrained problem exceeds a per-antenna power constraint, then it is optimal to allocate the maximal power in the constraint optimal transmit strategy. An iterative algorithm, which always converges to the optimum, to compute the optimal solution using [XFZP15] is proposed.

Notation

We use bold lower-case letters for vectors, bold capital letters for matrices. The superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$ stand for transpose, conjugate, and conjugate transpose. We use \succcurlyeq for positive semi-definite relation, $\text{tr}(\cdot)$ for trace. The expectation operator of a random variable is given by $\mathbb{E}[\cdot]$. \mathbb{N} , \mathbb{R}_+ , and \mathbb{C} are the sets of non-negative integers, non-negative real, and complex numbers.

B.2 Problem Formulation

We consider a MIMO channel with N_t transmit antennas and N_r receive antennas. We assume that channel state information (CSI) is known at both transmitter and receiver. The channel input-output relation of this transmission model can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z} \quad (\text{B.2.1})$$

where $\mathbf{x} \in \mathbb{C}^{N_t \times 1}$ is the complex transmit signal vector, $\mathbf{y} \in \mathbb{C}^{N_r \times 1}$ is the complex received vector. $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ is the channel coefficient matrix with complex elements. Finally, $\mathbf{z} \in \mathbb{C}^{N_r \times 1}$ is zero-mean scalar additive white complex Gaussian noise (AWGN), i.e., $\mathbf{z} \sim \mathcal{CN}(0, \sigma^2 \mathbf{I})$. In this paper, for simplicity, we assume that $\sigma^2 = 1$. Let $\mathbf{Q} = \mathbb{E}[\mathbf{x}\mathbf{x}^H]$ be the transmit covariance matrix of the Gaussian input, then the achievable transmission rate is

$$R = f(\mathbf{Q}) = \log \det \left(\mathbf{I}_m + \mathbf{H}\mathbf{Q}\mathbf{H}^H \right). \quad (\text{B.2.2})$$

The question is how to identify the transmit covariance matrix \mathbf{Q} subject to a given power constraint such that the transmission rate in (B.2.2) is maximized.

Let $\mathcal{A} \subseteq \{1, \dots, N_t\}$ and $\mathcal{S}(\mathcal{A}) := \{\mathbf{Q} \succcurlyeq 0 : \text{tr}(\mathbf{Q}) \leq P_{tot}, P_i = \mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i \leq \hat{P}_i, \forall i \in \mathcal{A}\}$ denote an index set and the set of transmit strategies with total transmit power P_{tot} and per-antenna power constraints $\hat{P}_i, \forall i \in \mathcal{A}$, where $\mathbf{e}_i = [0, \dots, 1, \dots, 0]^T$ is the i -th Cartesian unit vector. Let OP- \mathcal{A} be the optimization problem with optimization domain $\mathcal{S}(\mathcal{A})$ restricted by the per-antenna power constraints with index $i \in \mathcal{A}$. The optimization problem of transmission rate for MIMO channels then will be given as

$$\begin{aligned} & \text{maximize} && f(\mathbf{Q}) \\ & \text{subject to} && \mathbf{Q} \in \mathcal{S}(\mathcal{A}). \end{aligned} \tag{B.2.3}$$

If $\mathcal{A} = \emptyset$, then (B.2.3) corresponds to the sum power constraint only problem which has been well studied in [Tel99]. Therefore in the following we focus on the optimization problem when both sum and per-antenna power constraints are active, i.e., $\mathcal{A} \neq \emptyset$. In the following sections, we first investigate the properties of the power allocation of MIMO channels with joint sum and per-antenna power constraints. After that, an iterative algorithm to find the optimal transmit strategy is proposed.

B.3 Optimal Transmit Strategies

Depending on the per-antenna power constraints $\hat{P}_i, \forall i = 1, \dots, N_t$, and the sum power constraint P_{tot} , we can identify three different cases as follows: The first case is when the per-antenna power constraints are never active, i.e., $P_{tot} < \min_i(\hat{P}_i)$. The second case is when the the sum power constraint is never active, i.e., $P_{tot} > \sum_{i=1}^{N_t} \hat{P}_i$. The most interesting case is when both sum and per-antenna power constraints are active, i.e., $\min_i(\hat{P}_i) \leq P_{tot} \leq \sum_{i=1}^{N_t} \hat{P}_i$ [Paper A].

Proposition B.1 below shows that the capacity can be achieved with a transmit strategy which allocates full sum power if $\min_i(\hat{P}_i) \leq P_{tot} \leq \sum_{i=1}^{N_t} \hat{P}_i$.

Proposition B.1. *For OP- \mathcal{A} with a given channel $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ the maximum transmission rate R^* can be achieved when the optimal transmit strategy \mathbf{Q}^* uses full power P_{tot} , i.e., $\text{tr}(\mathbf{Q}^*) = P_{tot}$.*

Proof (by Contradiction). Suppose that it is not possible to achieve the maximum transmission rate using full power P_{tot} . This implies that for $\text{tr}(\mathbf{Q}^*) < P_{tot}$, the maximum transmission rate is

$$R^* = \log \det \left(\mathbf{I}_{N_r} + \mathbf{H} \mathbf{Q}^* \mathbf{H}^H \right) = \log \prod_{i=1}^{\min(N_t, N_r)} (1 + \lambda_i^2 \phi_i), \tag{B.3.1}$$

where λ_i and ϕ_i are the eigenvalues of \mathbf{H} and \mathbf{Q}^* .

Since $\text{tr}(\mathbf{Q}^*) < P_{tot} \leq \sum_{i=1}^{N_t} \hat{P}_i$, there exists a positive semi-definite Hermitian matrix $\mathbf{A} \succcurlyeq 0$ with $\mathbf{A} = \mathbf{A}^H$, such that $\mathbf{Q}^* + \mathbf{A} = \mathbf{Q}$, $\text{tr}(\mathbf{Q}) = P_{tot}$ and

$\mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i \leq \hat{P}_i, \forall i \in \mathcal{A}$. Following Weyl's theorem [HJ13], we have $\phi_i \leq \psi_i$ for each $i = 1, \dots, \min(N_t, N_r)$ where ψ_i are the eigenvalues of \mathbf{Q} . So that:

$$\begin{aligned} R &= \log \det \left(\mathbf{I}_{N_r} + \mathbf{H} \mathbf{Q} \mathbf{H}^H \right) = \log \prod_{i=1}^{\min(N_t, N_r)} (1 + \lambda_i^2 \psi_i) \\ &\geq \log \prod_{i=1}^{\min(N_t, N_r)} (1 + \lambda_i^2 \phi_i) = R^*. \end{aligned} \quad (\text{B.3.2})$$

This contradicts the assumption that when the transmit strategy uses full transmit power, the maximal transmission rate is not achievable. It follows that there always exists an optimal transmit strategy using full power. \square

Accordingly, it is sufficient for the optimization to consider only transmit strategy which allocate full power P_{tot} , i.e., the sum power constraint is always active. However, it may happen that the optimal transmit powers of the per-antenna unconstrained optimal solution using full transmit power may exceed the maximum allowed per-antenna powers. Therefore, we can distinguish between the two following cases: (i) All per-antenna power constraints are satisfied; (ii) There exists at least one power exceeds the maximum allowed per-antenna power. In the next lemma we study the properties of those two cases in more detail.

Lemma B.2. *Let $\mathcal{A}' \subseteq \mathcal{A} := \{1, \dots, N_t\}$, $\mathcal{S}(\mathcal{A}') := \{\mathbf{Q} \succcurlyeq 0 : \text{tr}(\mathbf{Q}) \leq P_{tot}, \mathbf{e}_j^T \mathbf{Q} \mathbf{e}_j \leq \hat{P}_j, j \in \mathcal{A}'\}$, and $\mathcal{P} := \{i \in \mathcal{A}^c : P_i^{S(\mathcal{A}')} > \hat{P}_i\}$ with $\mathcal{A}^c = \mathcal{A} \setminus \mathcal{A}'$ and $P_i^{S(\mathcal{A}')}$ is the corresponding power allocation at i -th antenna of the optimal transmit strategy of the optimization problem $\max_{\mathbf{Q} \in \mathcal{S}(\mathcal{A}')} f(\mathbf{Q})$. Then, for OP-A, the optimal power can be allocated as*

$$\begin{cases} P_i^* = P_i^{S(\mathcal{A}')} , \forall i \in \mathcal{A}^c & \text{if } \mathcal{P} = \emptyset, \\ P_i^* = \hat{P}_i, \quad \forall i \in \mathcal{P} & \text{otherwise,} \end{cases} \quad (\text{B.3.3})$$

with $P_i^* = \mathbf{e}_i^T \mathbf{Q}^* \mathbf{e}_i$.

Proof. The proof of this lemma can be divided into two parts.

First, we show that if $\mathcal{P} = \emptyset$ then $P_i^* = P_i^{S(\mathcal{A}')} , \forall i \in \mathcal{A}^c$. Since $\mathcal{S}(\mathcal{A}) \subseteq \mathcal{S}(\mathcal{A}')$, $\max_{\mathbf{Q} \in \mathcal{S}(\mathcal{A})} f(\mathbf{Q}) \leq \max_{\mathbf{Q} \in \mathcal{S}(\mathcal{A}')} f(\mathbf{Q})$. If $\mathbf{Q}^{S(\mathcal{A}')} \in \mathcal{S}(\mathcal{A})$ then $\mathbf{Q}^{S(\mathcal{A}')}$ is also the optimal transmit strategy for OP-A, i.e. $\mathbf{Q}^* = \mathbf{Q}^{S(\mathcal{A}')}$ or $P_i^* = P_i^{S(\mathcal{A}')} , \forall i \in \mathcal{A}^c$.

Next, we prove that if $\mathcal{P} \neq \emptyset$ then $P_i^* = \hat{P}_i, \forall i \in \mathcal{P}$. This part can be proved by applying [Paper A, Lemma A.9] directly to an optimization function $f(\mathbf{Q})$ given in (B.2.2). For a given $\mathcal{B} = \mathcal{A}' \cup \mathcal{P}$ with $\mathcal{A}' \subseteq \mathcal{A}$, if $\mathcal{P} \neq \emptyset$ then $P_i^{S(\mathcal{B})} = \hat{P}_i, \forall i \in \mathcal{P}$. Thus, by simply setting $\mathcal{A}' = \mathcal{A} \setminus \mathcal{P}$ we have $P_i^* = \hat{P}_i, \forall i \in \mathcal{P}$. \square

Note that, with Lemma B.2, if $\mathcal{A}' = \emptyset$, then for $\mathcal{P} = 0$, P_i^* is also the optimal power allocation of the optimization problem with sum power constraint only.

Lemma B.2 leads to an iterative algorithm to compute the optimal transmit strategy \mathbf{Q}^* in closed-form. To see this we consider the following sequence of optimization problems:

$$\begin{aligned}
\max_{\mathbf{Q} \in \mathcal{S}(\emptyset)} f(\mathbf{Q}) &= \max_{\mathbf{Q} \in \mathcal{S}(\emptyset) \cap \{\mathbf{Q} |_{ii} \leq \hat{P}_i : \forall i \in \mathcal{P}(1)\}} f(\mathbf{Q}) \\
&\geq \max_{\mathbf{Q} \in \mathcal{S}(\emptyset) \cap \{\mathbf{Q} |_{ii} \leq \hat{P}_i : \forall i \in \mathcal{P}(2)\}} f(\mathbf{Q}) \\
&\dots \\
&\geq \max_{\mathbf{Q} \in \mathcal{S}(\emptyset) \cap \{\mathbf{Q} |_{ii} \leq \hat{P}_i : \forall i \in \mathcal{P}(K-1)\}} f(\mathbf{Q}) \\
&\geq \max_{\mathbf{Q} \in \mathcal{S}(\emptyset) \cap \{\mathbf{Q} |_{ii} \leq \hat{P}_i : \forall i \in \mathcal{P}(K)\}} f(\mathbf{Q}) \\
&= \max_{\mathbf{Q} \in \mathcal{S}(\mathcal{A})} f(\mathbf{Q}), \tag{B.3.4}
\end{aligned}$$

where $\mathcal{P}(k)$ is the set of indices of powers which violate the per-antenna power constraints in the k -th iteration with initialization $\mathcal{P}(1) = \emptyset$. The update of $\mathcal{P}(k)$ in each iteration is done using Lemma B.2 and can be found in Section 4. The optimization problem in each iteration can be solved using the closed-form solution in [XFZP15] because every iteration in (B.3.4) can be related to an optimization problem with a total power constraint and a limited number of per antenna power constraints.

B.4 Iterative Algorithm

We now show in detail how to implement the iterative algorithm. If we assume that the iterative algorithm to find the optimal solution for OP- \mathcal{A} has K iterations in total, then at k -th iteration, $k = 1, \dots, K$, the set of powers that violate per-antenna power constraints can be calculated as

$$\mathcal{P}(k+1) = \mathcal{P}(k) \cup \{i \in \mathcal{P}^c(k) : \mathbf{e}_i^T \mathbf{Q}^*(k) \mathbf{e}_i > \hat{P}_i\} \tag{B.4.1}$$

with $\mathcal{P}^c(k) = \mathcal{A} \setminus \mathcal{P}(k)$.

Note that, $\mathcal{P}(1) = \emptyset$ and if we denote $\mathcal{P}^* := \{i \in \{1, \dots, N_t\} : P_i^* = \mathbf{e}_i^T \mathbf{Q}^* \mathbf{e}_i > \hat{P}_i\}$ as a set of all violated power of OP- \mathcal{A} , then $\mathcal{P}^* = \bigcup_{k=1}^K \mathcal{P}(k)$. Following this formulation, it is clear to obtain that if we consider an arbitrary index i , if $i \in \mathcal{P}(k)$ then $i \in \mathcal{P}(k+1)$.

Remark B.3. *The maximum number of violated per-antenna power constraints is $N_t - 1$, which also corresponds to the maximal number of iterations, i.e., $K \leq N_t - 1$.*

Therefore, we can, without loss of generality, re-assign the antenna coefficient order corresponding to the number of iteration such that the first $|\mathcal{P}(k)|$ coefficient

are in $\mathcal{P}(k)$, i.e., $\mathcal{P}(k) = \{1, \dots, |\mathcal{P}(k)|\}$. Therewith, the optimal transmit strategy at the k -iteration, we can be written as

$$\mathbf{Q}^*(k) = \begin{bmatrix} \mathbf{Q}_P(k) & \mathbf{q}^H(k) \\ \mathbf{q}(k) & \mathbf{Q}_S(k) \end{bmatrix}, \quad (\text{B.4.2})$$

with $\mathbf{Q}_P(k)$ is a $|\mathcal{P}(k)| \times |\mathcal{P}(k)|$ matrix which contains $P_i^* = \hat{P}_i$, $\forall i \in \mathcal{P}(k)$, and $\mathbf{Q}_S(k)$ is a $(N_t - |\mathcal{P}(k)|) \times (N_t - |\mathcal{P}(k)|)$ matrix which satisfies the condition that $\text{tr}(\mathbf{Q}_S(k)) = P_{tot}(k)$ where $P_{tot}(k) = P_{tot} - \sum_{i \in \mathcal{P}(k)} \hat{P}_i$. This implies that the diagonal elements of $\mathbf{Q}^*(k)$ can be formed as $P_i^*(k) = \mathbf{e}_i^T \mathbf{Q}_P(k) \mathbf{e}_i = \hat{P}_i$, $\forall i \in \mathcal{P}(k)$ and $P_j(k) = \mathbf{e}_j^T \mathbf{Q}_S(k) \mathbf{e}_j \forall j \in \mathcal{P}^c(k)$. To find the remaining optimal power allocation $P_j(k)$, $\forall j \in \mathcal{P}^c(k)$ in $\mathbf{Q}_S(k)$, we consider following reduced optimization problem

$$\begin{aligned} & \text{maximize} && f(\mathbf{Q}(k)) && (\text{B.4.3}) \\ & \text{subject to} && \mathbf{Q}(k) \in \mathcal{S}(\mathcal{P}(k)). \end{aligned}$$

where $f(\mathbf{Q}(k)) = \log \det(\mathbf{I}_m + \mathbf{H}\mathbf{Q}(k)\mathbf{H}^H)$, $\mathcal{S}(\mathcal{P}(k)) = \{\mathbf{Q}(k) \succcurlyeq 0 : P_i(k) = \hat{P}_i \forall i \in \mathcal{P}(k), \sum_{j \in \mathcal{P}^c(k)} P_j(k) \leq P_{tot}(k)\}$ with $P_i(k) = \mathbf{e}_i^T \mathbf{Q}(k) \mathbf{e}_i$ is the transmit power of i -th antenna at k -th iteration.

The Lagrangian of (B.4.3) is given as follows

$$\begin{aligned} \mathcal{L}(\mathbf{Q}(k), \mathbf{D}, \mathbf{M}) = & f(\mathbf{Q}(k)) + \sum_{i \in \mathcal{P}(k)} [\mathbf{D}]_{i,i} \hat{P}_i + d(k) P_{tot}(k) \\ & - \text{tr}(\mathbf{D}\mathbf{Q}(k)) + \text{tr}(\mathbf{M}\mathbf{Q}(k)), \end{aligned} \quad (\text{B.4.4})$$

where \mathbf{D} is a diagonal matrix with elements $[\mathbf{D}]_{i,i}$ if $i \in \mathcal{P}(k)$ and $[\mathbf{D}]_{j,j} = d(k)$ if $j \in \mathcal{P}^c(k)$ with $d(k)$ is Lagrange multiplier for the reduced sum power constraint. \mathbf{M} is Lagrange multiplier for the positive semi-definite constraint.

Following [XFZP15], the optimal transmit strategy $\mathbf{Q}^*(k)$ of the optimization problem (B.4.3) can be calculated in closed-form. For simplicity, we here assume that the number of nonzero singular values of \mathbf{H} is $L = \min(N_t, N_r)$. The detail about the more general case can be found in [XFZP15].

Lemma B.4 ([XFZP15]). *The optimal solution of the transmit strategy in (B.4.3) denoted by $\mathbf{Q}^*(k)$ is given by*

$$\mathbf{Q}^*(k) = (\mathbf{D}^{-\frac{1}{2}} [\mathbf{U}]_{:,1:L} [\mathbf{U}]_{:,1:L}^H \mathbf{D}^{-\frac{1}{2}} - [\mathbf{U}]_{:,1:L} \mathbf{\Lambda}^{-1} [\mathbf{U}]_{:,1:L}^H)^+, \quad (\text{B.4.5})$$

where diagonal matrix $\mathbf{\Lambda}$ and the first L columns of a unitary matrix $[\mathbf{U}]_{:,1:L}$ are obtained from eigenvalue decomposition $\mathbf{H}^H \mathbf{H} = \mathbf{U} \begin{bmatrix} \mathbf{\Lambda} & 0 \\ 0 & 0 \end{bmatrix} \mathbf{U}^H$. The diagonal elements of $L \times L$ diagonal matrix $\mathbf{\Lambda}$ are positive real values in decreasing order.

The operation ‘+’ is to guarantee that the solution is positive-semi definite, i.e., the negative eigenvalues of $\mathbf{Q}^*(k)$ are forced to be zero. At high SNR, the elements of the diagonal \mathbf{D} can be computed as

$$[\mathbf{D}]_{i,i} = \frac{[[\mathbf{U}]_{:,1:L}[\mathbf{U}]_{:,1:L}^H]_{i,i}}{\hat{P}_i + [[\mathbf{U}]_{:,1:L}\mathbf{\Lambda}^{-1}[\mathbf{U}]_{:,1:L}^H]_{i,i}} \text{ if } i \in \mathcal{P}(k) \quad (\text{B.4.6})$$

and

$$[\mathbf{D}]_{j,j} = d(k) = \frac{\sum_{j' \in \mathcal{P}^c(k)} [[\mathbf{U}]_{:,1:L}[\mathbf{U}]_{:,1:L}^H]_{j',j'}}{P_{tot}(k) + \sum_{j' \in \mathcal{P}^c(k)} [[\mathbf{U}]_{:,1:L}\mathbf{\Lambda}^{-1}[\mathbf{U}]_{:,1:L}^H]_{j',j'}}, \text{ if } j \in \mathcal{P}^c(k). \quad (\text{B.4.7})$$

From the result in Lemma B.4, the value of $\mathbf{Q}_S(k)$ and the optimal transmit power on j -th antennas at k -th iteration, $P_j(k)$ can be obtained as $P_j(k) = [\mathbf{Q}^*(k)]_{j,j}$ or $P_j(k) = \mathbf{e}_j^T \mathbf{Q}_S(k) \mathbf{e}_j$, $\forall j \in \mathcal{P}^c(k)$ respectively. However, it occurs that $P_j(k)$ may violate the per-antenna power constraint $\hat{P}_j(k)$ for some $j \in \mathcal{P}^c(k)$. According to the Lemma B.2, that power has to be set equal to the per-antenna power constraint. Consequently, for the OP- \mathcal{A} solution, a new iteration has to be performed with a new re-assigned covariance matrix which contains a $\mathbf{Q}_P(k+1)$ with diagonal elements contains powers that set equal to per-antenna power constraints and $\mathbf{Q}_S(k+1)$ with trace equal new reduced total power $P_{tot}(k+1)$.

The power allocations $\mathbf{P}^*(k) = [P_i^*(k), i \in \mathcal{A}]$, $\mathbf{P}_P(k) = [\hat{P}_i, i \in \mathcal{P}(k)]$, $\mathbf{P}_S(k) = [P_j(k), j \in \mathcal{P}^c(k)]$ which are corresponding to $\mathbf{Q}^*(k)$, $\mathbf{Q}_P(k)$ and $\mathbf{Q}_S(k)$ therefore are updated as follows:

$$\dots \mathcal{P}(k) \xrightarrow{(a)} \mathbf{P}_P(k) \xrightarrow{(b)} \mathbf{P}(k) \rightarrow \mathbf{P}_S(k) \xrightarrow{(c)} \mathcal{P}(k+1) \dots$$

where (a) follows Lemma B.2, (b) follows Lemma B.4 and (c) follows the limitation of per-antenna power on the antennas. This updated sequence stops when there is no per-antenna power constraint violated, i.e., $\mathcal{P}(k+1) = \mathcal{P}(k)$. The optimal transmit strategy of the optimization problem with joint sum and per-antenna power constraints then can be determined as $\mathbf{Q}^* = \mathbf{Q}^*(K)$.

From the discussion above, we can summarize on the iterative algorithm to compute the optimal solution of OP- \mathcal{A} in Algorithm B.4.1.

As a basic result of the algorithm, one can verify that $f(\mathbf{Q}^*(1)) \geq \dots \geq f(\mathbf{Q}^*(k)) \geq \dots \geq f(\mathbf{Q}^*(K))$. Since the reduce optimization problem is convex at every iteration, the global convergence of the proposed iteration method is guaranteed after at most $N_t - 1$ iterations [BV09].

Remark B.5. (B.4.5) is a generalized water-filling solution. In particular, if $\mathcal{P}^* = \emptyset$, \mathbf{D} is the proportional to an identity matrix, i.e., $\mathbf{D} = \mu \mathbf{I}$, and (B.4.5) reduces to the standard water-filling solution of the optimization problem with sum power constraint only [Tel99].

Algorithm B.4.1: Iterative algorithm with $P_{tot} \leq \sum_{i=1}^n \hat{P}_i$, $\mathcal{A} := \{1, \dots, N_t\}$.

Output: \mathbf{Q}^*

- 1 Initialize $k = 1$, $\mathcal{P}(1) = \emptyset$
 - 2 Compute $\mathbf{Q}^*(k)$ using Lemma B.4
 - 3 $\mathcal{P}(k+1) = \mathcal{P}(k) \cup \{i \in \mathcal{P}^c(k) : P_i(k) > \hat{P}_i\}$,
 - 4 **if** $\mathcal{P}(k+1) = \mathcal{P}(k)$ **then**
 - 5 $\mathbf{Q}^* \leftarrow \mathbf{Q}^*(k)$,
 - 6 **Break.**
 - 7 **else**
 - 8 **for** $i \in \mathcal{P}(k+1)$ **do**
 - 9 $k \leftarrow k+1$,
 - 10 **end for**
 - 11 $P_i(k) \leftarrow \hat{P}_i$ according to Lemma B.2,
 - 12 $P_{tot}(k) \leftarrow P_{tot} - \sum_{i \in \mathcal{P}(k)} \hat{P}_i$,
 - 13 **end if**
 - 14 **Return to 2.**
-

B.5 Numerical Example

For numerical example, we provide the transmission rate of the optimization problem OP- \mathcal{A} with a 3×3 complex channel $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3]$ with

$$\mathbf{h}_1 = [1.1356e^{0.9653j}, 0.9284e^{0.4658j}, 0.9553e^{-0.4193j}]^T,$$

$$\mathbf{h}_2 = [0.9640e^{-0.9996j}, 1.2905e^{-0.9527j}, 1.0384e^{-0.4533j}]^T,$$

$$\mathbf{h}_3 = [0.6110e^{-0.9156j}, 1.0559e^{-12103j}, 0.7126e^{-0.3535j}]^T.$$

Figure B.1 shows the choices of constraints result of the optimization problem OP- \mathcal{A} with $P_{tot} = 25$. We perform the example by adjusting per-antenna power constraint on antenna 1 from 0 to 14 and setting per-antenna power constraint configurations on antennas 2 and 3 as follows: (i) Per-antenna power constraints on antennas 2 and 3 are active; (ii) Only per-antenna power constraint on antenna 3 is active; (iii) Per-antenna power constraints on antennas 2 and 3 are not active. Setting $P_{tot} = \hat{P}_i$ in the numerical experiments illustrated in Figure B.1 denotes the case without a power constraint for the i -th antenna. The plot of the sum power constraint only solution, which corresponds to case when all per-antenna power constraints are never active, is also shown in the figure. We can see from the figure that the more restricted per-antenna power constraint, the less optimal transmission rate. This happens because of the fact we have less freedom to allocate the optimal transmit power when the optimization domain is limited by adding more per-antenna power constraints.

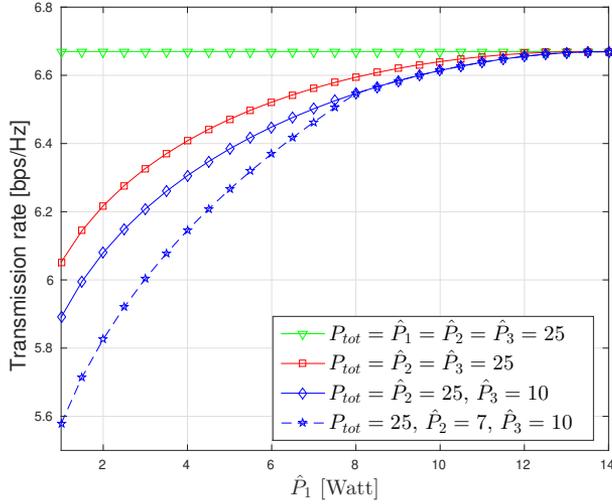


Figure B.1: Capacity under different power constraint settings

B.6 Conclusions

In this paper, we present an iterative algorithm to find the optimal transmit strategy for a MIMO channel with joint sum and per-antenna power constraints using generalized water-filling solution. The algorithm exploits the fact that if an unconstrained optimal power allocation of an antenna exceeds a per-antenna power constraint, then it is optimal to allocate the maximal power in the constraint optimal transmit strategy including the per-antenna power constraints which then enables us to use closed-form solution from [XFZP15] in an iterative algorithm to compute the optimal transmit strategy satisfying both sum and per-antenna power constraints.

Optimal Transmit Strategies for Gaussian MISO Wiretap Channels

Phuong Le Cao and Tobias J. Oechtering

Revised version of manuscript submitted to
IEEE Transactions on Information Forensics and Security.

©2018 IEEE

Layout has been revised for thesis consistency

Optimal Transmit Strategies for Gaussian MISO Wiretap Channels

Phuong Le Cao and Tobias J. Oechtering

Abstract

This paper studies the optimal tradeoff between secrecy and non-secrecy rates of the MISO wiretap channels for different power constraint settings: sum power constraint only, per-antenna power constraints only and joint sum and per-antenna power constraints. The problem is motivated by the fact that channel capacity and secrecy capacity are generally achieved by different transmit strategies. First, a necessary and sufficient condition to ensure a positive secrecy capacity is shown. The optimal tradeoff between secrecy rate and transmission rate is characterized by a weighted rate sum maximization problem. Since this problem is not necessarily convex, equivalent problem formulations are introduced to derive the optimal transmit strategies. Under sum power constraint only, a closed-form solution is provided. Under per-antenna power constraints, necessary conditions to find the optimal power allocation are provided. Sufficient conditions are provided for the special case of two transmit antennas. For the special case of parallel channels, the optimal transmit strategies can be deduced from an equivalent point-to-point channel problem. Lastly, the theoretical results are illustrated by numerical simulations.

C.1 Introduction

Security is a critical aspect in wireless communication systems due to the open nature of wireless links. To enhance the security, physical-layer secrecy methods have received much attention recently. One of the pioneer studies is the study of the secrecy capacity of the wiretap channel [Wyn75], where Wyner showed that a positive secrecy rate can be achieved when an eavesdropper's channel is a degraded version of the main channel. The maximal secrecy rate is given by the largest difference between the mutual information to the legitimate receiver and the mutual information to the eavesdropper. Csiszár and Körner extended the result to the non-degraded case in [CK78]. Following these works, researchers in the physical-layer security area have studied and extended the wiretap channel in various aspects.

The secrecy capacities for Gaussian multiple-input single-output (MISO) and multiple-input multiple-output (MIMO) wiretap channels with a sum power constraint have been studied in [KW10b, KW10a, OH08, LP10, LP09, SSC12, LHW⁺13]. In [KW10b] and [KW10a], the authors developed upper bounds that enable to characterize the secrecy capacities for MISO and MIMO wiretap channels. The proposed solutions are to reduce the wiretap system into a set of parallel channels based on

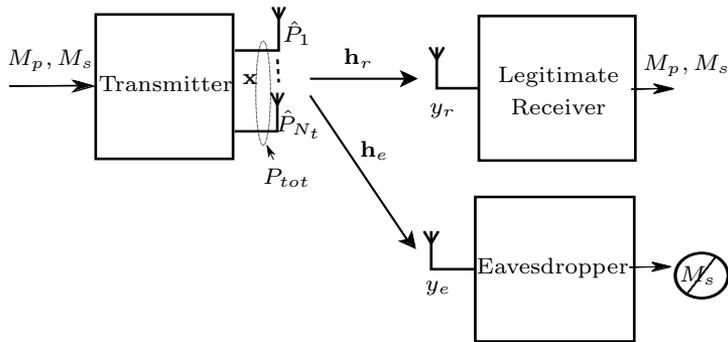


Figure C.1: MISO wiretap channel with joint sum and per-antenna power constraints, public message M_p and secret message M_s

the generalized singular value decomposition and using an independent Gaussian wiretap code books on those resulting channels. In [LP10], necessary conditions for the optimal input covariance matrix are derived. In particular, a closed-form expression of the MISO secrecy capacity has been shown. For the MIMO case an iterative algorithm is provided. In [SSC12] and [LHW⁺13], iterative optimization algorithms to find the secrecy capacity have been proposed based on the concave-convex alternating optimization procedure. Alternatively, indirect approaches such as using matrix analysis tools are also used to find bounds on the secrecy capacity of a MIMO Gaussian wiretap channel in [OH08, KW10b, KW10a].

In practice, each antenna has its own power amplifier, which means the power allocation at the transmitter is usually done under per-antenna power constraints. The problem of finding the channel capacity with average per-antenna power constraints has been investigated in both single-user [Vu11a, Pi12, MDT14, Tun14] and multi-user setups [SSB08, YL07, COS16]. Recently, the capacities of point-to-point channels with joint sum and per-antenna power constraints have been considered in [COSS15, COSS16, Loy16, CO17b]. An interesting aspect of the joint sum and per-antenna power constraints setting is that it can be applied to systems with multiple antennas as well as to distributed systems with separated energy sources. The optimal transmit strategy problem with joint sum and per-antenna power constraints has been studied first for MISO channel with two transmit antennas in [COSS15] and the general case in [COSS16]. In [COSS16], a closed-form characterization of an optimal beam-forming strategy is derived. Shortly after [COSS16], similar results have been published in [Loy16]. In [CO17b], the optimal transmit strategy problem for a point-to-point MIMO channel with joint sum and per-antenna power constraints has been studied.

In this work we study MISO wiretap channels with different power constraint settings including sum power constraint only, per-antenna power constraints only, and joint sum and per-antenna power constraints. The optimal trade-off between

the overall transmission rate and the secrecy rate of MISO wiretap channels is motivated by the fact that the optimal coding strategy for the wiretap channel is using a two-layer codebook [LLPS13]. The idea of the coding scheme is that the decoding capability of the eavesdropper is exhausted by the public message, while the legitimate receiver can decode both the public and secret messages. Therefore, instead of sending some useless random messages on the public layer, a useful message can be communicated non-securely to the legitimate receiver [LLPS13] (see Fig. C.1). Since for vector-valued transmission the maximal overall transmission rate and secrecy rate are, in general, achieved by different transmit strategies, we face a trade-off between both objectives which we will study in detail in the following. Some initial results of this paper have been presented in [CO17a]. The contributions of the paper can be summarized as follows:

- Properties and optimal trade-offs between the overall transmission rate and the secrecy rate of the Gaussian MISO wiretap channel under three different power constraint configurations including the sum power constraint only, the per-antenna power constraints only, and the joint sum and per-antenna power constraints are characterized and discussed.
- Parametrizations of the boundaries of the optimal rate regions of the transmission rate and the secrecy rate of the Gaussian MISO wiretap channel based on the weighted rate sum optimal rate pairs are derived.
- Closed-form solutions to compute optimal transmit strategies for general MISO wiretap channels with different power constraint settings are developed.

This paper is organized as follows. We start by briefly introducing the system model, sets of feasible transmit strategies considering different power constraint settings. After that equivalent formulations of the weighted rate sum maximization problem between the transmission rate and the secrecy rate are derived. Next, closed form solutions and iterative algorithms to find the optimal transmit strategies for different power constraint settings under different channel coefficient conditions. These solutions allow us to come up with a characterization of the boundary of the optimal region of transmission and secrecy rates. The results are then illustrated and discussed in numerical examples. Finally, we provide some remarks and conclusions.

Notation

We use bold lower-case letters for vectors, bold capital letters for matrices. The superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$ stand for transpose, conjugate, and conjugate transpose. We use \succ for positive semi-definite relation, $\text{tr}(\cdot)$ for trace, $\text{rank}(\cdot)$ for rank and $\text{diag}\{\cdot\}$ for diagonal matrix. The expectation operator of a random variable is given by $\mathbb{E}[\cdot]$. \mathbb{R}_+ and \mathbb{C} are sets of non-negative real and complex numbers.

C.2 Problem Formulation

C.2.1 System Model and Power Constraint

We consider a MISO wiretap channel with N_t antennas at the transmitter and single antenna at both legitimate receiver and eavesdropper. For each channel use, the received signals at the legitimate receiver and the eavesdropper are given as follows

$$y_r = \mathbf{h}_r^H \mathbf{x} + z_r, \quad (\text{C.2.1})$$

$$y_e = \mathbf{h}_e^H \mathbf{x} + z_e, \quad (\text{C.2.2})$$

where $\mathbf{x} = [x_1, \dots, x_{N_t}]^T \in \mathbb{C}^{N_t \times 1}$ is the random complex transmit signal vector, $\mathbf{h}_r = [h_{r1}, \dots, h_{rN_t}]^T \in \mathbb{C}^{N_t \times 1}$, $h_{ri} \neq 0 \forall i = 1, \dots, N_t$, and $\mathbf{h}_e = [h_{e1}, \dots, h_{eN_t}]^T \in \mathbb{C}^{N_t \times 1}$ are channel coefficient vectors between the transmitter and legitimate receiver, and between the transmitter and eavesdropper, which are perfectly known at the transmitter. In practical scenarios, the perfect channel state information at the transmitter corresponds to the case when the channel remains constant for sufficiently long time. The solution with perfect channel knowledge can be considered as an ideal case that serves as a benchmark. z_r and z_e are independent additive white complex Gaussian noise terms with $\sigma_r^2 = \sigma_e^2 = 1$. It will be optimal to use a zero-mean Gaussian distributed codebook generated with covariance $\mathbf{Q} = \mathbb{E}[\mathbf{x}\mathbf{x}^H]$, which also specifies the transmit strategy.

Let P_{tot} denote the maximal average sum transmit power and \hat{P}_k , $1 \leq k \leq N_t$, denotes the maximal average transmit power at the k -th antenna. Further, let $\mathcal{S}(\hat{\mathbf{p}})$, $\hat{\mathbf{p}} = [P_{tot}, \hat{P}_1, \dots, \hat{P}_{N_t}]$, denote the set of all transmit strategies satisfying the power constraints $\hat{\mathbf{p}}$, i.e.,

$$\mathcal{S}(\hat{\mathbf{p}}) := \{\mathbf{Q} \succeq 0 : \text{tr}(\mathbf{Q}) \leq P_{tot}, \mathbf{e}_k^T \mathbf{Q} \mathbf{e}_k \leq \hat{P}_k, \forall k \in \mathcal{I}\} \quad (\text{C.2.3})$$

where $\mathcal{I} := \{1, \dots, N_t\}$ and \mathbf{e}_k is the k -th Cartesian unit vector. Depending on the per-antenna power constraints \hat{P}_k and the sum power constraint P_{tot} , we can identify three different cases: (i) The *Sum power constraint only* case is considered when the per-antenna power constraints are never active, i.e., $P_{tot} < \min_k(\hat{P}_k)$, (ii) The *Per-antenna power constraints only* case is considered when the sum power constraint is never active, i.e., $P_{tot} > \sum_{k=1}^{N_t} \hat{P}_k$, and (iii) The *Joint sum and per-antenna power constraints* case are considered when the power constraints relations satisfy $\min_k(\hat{P}_k) \leq P_{tot} \leq \sum_{k=1}^{N_t} \hat{P}_k$, i.e., both sum and per-antenna power constraints can be active.

C.2.2 Trade-off Between Transmission Rate and Secrecy Rate

A wiretap channel consists of a legitimate receiver who wishes to receive messages of high rate from a transmitter in the presence of an eavesdropper. Csiszár and Körner have studied the general wiretap channel in [CK78].

They showed that for a discrete memoryless wiretap channel with finite alphabets, there exists a weakly-secure coding scheme with transmission rate R and equivocation rate $R_{eq} \leq \frac{1}{n}H(M|Y_e^n) + \epsilon$, for some $\epsilon > 0$. Let \mathcal{R} denote the set of all non-negative achievable rate pairs of transmission rate R and equivocation rate R_{eq} . Then $(R, R_{eq}) \in \mathcal{R}$ if and only if there exist auxiliary random variables U and V that satisfy the Markov chain $U - V - X - Y_r Y_e$ and rate constraints

$$0 \leq R_{eq} \leq I(V; Y_r|U) - I(V; Y_e|U) \quad (\text{C.2.4})$$

$$R_{eq} \leq R \leq I(V; Y_r|U) + \min[I(U; Y_r), I(U; Y_e)]. \quad (\text{C.2.5})$$

The secrecy capacity, which is defined as the maximum rate at which the message can be securely sent to the legitimate receiver, is then given as

$$C_s = \max_{p(u)p(v|u)p(x|v)} I(V; Y_r|U) - I(V; Y_e|U). \quad (\text{C.2.6})$$

It is known from [LYCH78, KW10b, KW10a] that for Gaussian channels, the secrecy capacity can be achieved with zero mean Gaussian distributed inputs. The MISO secrecy capacity can then be obtained from the following optimization problem

$$C_s(\hat{\mathbf{p}}) = \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} R_s(\mathbf{Q}), \quad (\text{C.2.7})$$

where $R_s(\mathbf{Q}) = \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r) - \log(1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e)$.

In the following proposition, we provide a condition for a positive secrecy capacity.

Proposition C.1. *A necessary and sufficient condition for a positive secrecy capacity of a Gaussian MISO wiretap channel, i.e., $C_s(\hat{\mathbf{p}}) > 0$, is that $\mathbf{h}_r \mathbf{h}_r^H - \mathbf{h}_e \mathbf{h}_e^H \in \mathbb{C}^{N_t \times N_t}$ has to have at least one positive eigenvalue.*

Proof. The proof of Proposition C.1 is in Appendix C.7.1. \square

In [LLPS13], the authors extended the problem in [CK78] to a problem of simultaneously transmitting public and secret messages¹. The coding schemes are designed such that the legitimate receiver can decode both the public and the secret messages while the eavesdropper might be able to decode the public message only. This means that the public layer can be used to transmit useful messages non-securely to the legitimate receiver (instead of broadcasting useless random messages in order to exhaust the capacity of the eavesdropper only). Accordingly, in our setting, $M = (M_p, M_s)$, where the public message M_p and the secret message M_s are uniformly distributed over $\{1, \dots, 2^{nR_p}\}$ and $\{1, \dots, 2^{nR_s}\}$, are transmitted from the transmitter to the legitimate receiver (see Fig. C.1). The secret message M_s needs to be kept secret from the eavesdropper, i.e., $R_s \leq \frac{1}{n}H(M_s|Y_e^n) + \epsilon$ for some

¹In [LLPS13], public and secret messages are correspondingly named private and confidential messages

$\epsilon > 0$, while there is no secrecy constraint applied on M_p . This means the eavesdropper may decode the public message M_p but does not have to. Accordingly, in our setup, we do not consider a common message which has to be decoded by the eavesdropper as well.

Following [LLPS13, Theorem 1], the region of the transmission and equivocation rates of the MISO Gaussian wiretap channel under the power constraint (C.2.3) given as follows:

$$\begin{aligned} \mathcal{R}_{MISO}(\hat{\mathbf{p}}) = \{ & (R, R_s) \in \mathbb{R}_+^2 : 0 \leq R_s \leq R_s(\mathbf{Q}), \\ & R = R_s + R_p \leq R(\mathbf{Q}) \text{ for some } \mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})\} \end{aligned} \quad (\text{C.2.8})$$

with $R(\mathbf{Q}) = \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r)$. In particular the maximal transmission rate is given by

$$C(\hat{\mathbf{p}}) = \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} R(\mathbf{Q}). \quad (\text{C.2.9})$$

Since the optimal transmit strategies to achieve the transmission capacity in (C.2.9) and the secrecy capacity in (C.2.6) do not have to be the same, a trade-off between the two objectives appears. The optimal trade-off between the overall transmission and the secrecy rates for a Gaussian MISO wiretap channel with power constraints $\hat{\mathbf{p}}$, is obtained by solving the following optimization problem

$$R_{\Sigma}(\hat{\mathbf{p}}, w) = \max_{\mathbf{Q}} R_{\Sigma}(\mathbf{Q}, w), \text{ s. t. } \mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}}), \quad (\text{C.2.10})$$

for a given weight $0 \leq w \leq 1$, where

$$R_{\Sigma}(\mathbf{Q}, w) = (1-w)R(\mathbf{Q}) + wR_s(\mathbf{Q}) = R(\mathbf{Q}) - wR_e(\mathbf{Q}) \quad (\text{C.2.11})$$

with $R_s(\mathbf{Q}) = R(\mathbf{Q}) - R_e(\mathbf{Q})$, $R(\mathbf{Q}) = \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r)$, and $R_e(\mathbf{Q}) = \log(1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e)$.

If the region $\mathcal{R}_{MISO}(\hat{\mathbf{p}})$ is convex (see Fig. C.2), then the set of weighted rate sum optimal rate pairs characterize the boundary of the rate region. If this region is non-convex, then the set of all weighted rate sum optimal rate pairs can be used to characterize the boundary of the convex hull of the rate region, i.e., $\mathcal{C}_{MISO}(\hat{\mathbf{p}}) = \text{ConvexHull}(\mathcal{R}_{MISO}(\hat{\mathbf{p}}))$. In this case, we would need to allow time-sharing between two rate pairs. Note that the secrecy rate is a fraction of the overall transmission rate, i.e., the condition $0 \leq R_s \leq R$ has to be satisfied. Therefore, the boundary of the rate region is also bounded by the line for which we have $R = R_s$.

In the following, we provide solutions for the optimization problem (C.2.10). These solutions also provide us a characterization of the rate region (C.2.8) that describes the trade-off between the overall transmission rate and the secrecy rate of the Gaussian MISO wiretap channel.

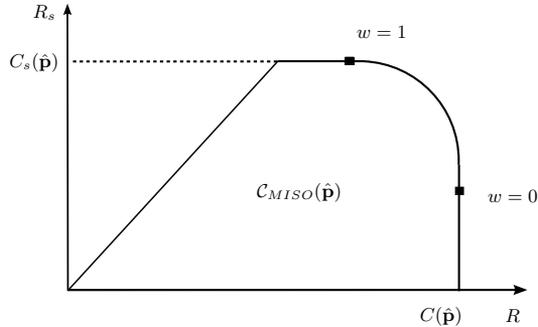


Figure C.2: Capacity region illustrating the trade-off between transmission rate R and secrecy rate R_s

C.3 Equivalent Problem Formulations and Parametrizations of the boundary of rate region

Since (C.2.10) is not a convex optimization problem, because $R_s(\mathbf{Q})$ is non-convex in \mathbf{Q} , we first reformulate (C.2.10) using the following lemma to an equivalent convex optimization problem that allows further analysis.

Lemma C.2 ([JPKR11], Lemma 2, scalar case). *Consider the function $f(D) = -DE + \log(D) + 1$ where $D, E \in \mathbb{R}, E > 0$. Then,*

$$\max_{D>0} f(D) = \log(E^{-1}), \quad (\text{C.3.1})$$

with the optimum value $D^* = E^{-1}$.

By applying Lemma C.2 with $\log E_i^{-1} = \max_{D_i>0} f_i(D_i)$ where $E_i = 1 + \mathbf{h}_i^H \mathbf{Q} \mathbf{h}_i$ and $f_i(D_i) = -D_i E_i + \log(D_i) + 1$ for $i \in \{r, e\}$, the optimization problem (C.2.10) can be expressed as

$$\begin{aligned} R_{\Sigma}(\hat{\mathbf{p}}, w) &= \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r) - w \log(1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e) \\ &= \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} (-\max_{D_r>0} f_r(D_r) + w \max_{D_e>0} f_e(D_e)) \\ &= \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} \min_{D_r>0} \max_{D_e>0} (-f_r(D_r) + w f_e(D_e)) \\ &= \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} \min_{D_r>0} \max_{D_e>0} D_r(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r) - \log(D_r) \\ &\quad - 1 + w(-D_e(1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e) + \log(D_e) + 1) \\ &= \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} \min_{D_r>0} \max_{D_e>0} D_r(\mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r + w \frac{D_e}{D_r} \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e) \end{aligned}$$

$$+ D_r - \log(D_r) - 1 + w(-D_e + \log(D_e) + 1). \quad (\text{C.3.2})$$

For a given w , let us define $t := w \frac{D_e}{D_r}$ and $\phi^{(1)}(\mathbf{Q}, t) = \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - t \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e = \text{tr}(\mathbf{A} \mathbf{Q})$ with $\mathbf{A} := \mathbf{h}_r \mathbf{h}_r^H - t \mathbf{h}_e \mathbf{h}_e^H$, then (C.3.2) can be written as

$$R_{\Sigma}(\hat{\mathbf{p}}, w) = \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} \min_{D_r > 0} \max_{D_e > 0} D_r \phi^{(1)}(\mathbf{Q}, t) + D_r - \log(D_r) + w(-D_e + \log(D_e) + 1) - 1. \quad (\text{C.3.3})$$

Although t is dependent on D_e , D_r and w , the optimization with respect to \mathbf{Q} only depends on t . Thus, we propose to find first the optimal transmit strategy $\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t)$ by solving

$$\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t) = \arg \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} \phi^{(1)}(\mathbf{Q}, t) \quad (\text{C.3.4})$$

for a given t and power constraints $\hat{\mathbf{p}}$. After having the optimal $\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t)$ for a given t , we can obtain the corresponding optimal D_e^* and D_r^* following Lemma C.2. The corresponding w is then given by $t \frac{D_r^*}{D_e^*}$. The following theorem shows that the previous procedure can be used to compute the optimal weighted rate sum $R_{\Sigma}(\hat{\mathbf{p}}, w)$.

Theorem C.3. *Let $t_{\max} = 2^{C_s(\hat{\mathbf{p}})}$, for the optimal solution of (C.2.10) we have the following properties:*

- (i) *For every $w \in [0, 1]$ there exists a $t \in [0, t_{\max}]$ such that $\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t)$ is an optimal transmit strategy, i.e., $R_{\Sigma}(\hat{\mathbf{p}}, w) = R_{\Sigma}(\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t), w)$.*
- (ii) *For every $t \in [0, t_{\max}]$ there exists a $w \in [0, 1]$ such that $\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t)$ is an optimal transmit strategy, i.e., $R_{\Sigma}(\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t), w) = R_{\Sigma}(\hat{\mathbf{p}}, w)$.*

Proof. The proof of Theorem C.3 is in Appendix C.7.2 □

As a result, the optimal region that describes the trade-off between the point-to-point transmission rate and the wiretap secrecy rate is equivalently described as $\mathcal{R}_{MISO}(\hat{\mathbf{p}}) = \{(R, R_s) : 0 \leq R_s \leq R \leq R(\mathbf{Q}(\hat{\mathbf{p}}, t)), R_s \leq R_s(\mathbf{Q}(\hat{\mathbf{p}}, t)), t \in [0, t_{\max}]\}$.

In the following, we derive solutions to find the optimal transmit strategies and characterize the optimal tradeoff between the transmission and secrecy rates of the Gaussian MISO wiretap channels with two different power constraint cases: (i) with a sum power constraint only; (ii) per-antenna power constraints only using the reformulation above. The solutions for the optimization problem with joint sum and per-antenna power constraints is discussed in a special case of parallel channels only. The optimal solution for a specific weight is then found after a simple line search over t .

C.4 Analytical Discussion and Solutions

Since for a given t the equivalent problem in (C.3.4) is convex, optimal solutions can be also found using standard convex optimization tools [BV09]. However, some results can be obtained in closed-form that lead to computationally efficient solutions, which are therefore interesting for practical applications.

The sufficiency of beamforming for optimality for a given set of power constraints $\hat{\mathbf{p}}$ is shown in the following theorem.

Theorem C.4. *For an optimal transmit strategy, it is sufficient to consider beamforming strategies, i.e., there exists always an optimal rank one solution.*

Proof. The proof of Theorem C.4 is in Appendix C.7.3. \square

C.4.1 Sum Power Constraint Only

Let \mathcal{S}_{SPC} denote the set of all transmit strategies which satisfy the sum power constraint P_{tot} only, i.e., $\mathcal{S}_{SPC} = \{\mathbf{Q} \succeq 0 : \text{tr}(\mathbf{Q}) \leq P_{tot}\}$. The equivalent problem of finding the weighted rate sum optimal transmit strategy for the MISO wiretap channel with sum power constraint only for a given t can be written as

$$\mathbf{Q}_{SPC}(t) = \arg \max_{\mathbf{Q} \in \mathcal{S}_{SPC}} \phi^{(1)}(\mathbf{Q}, t), \quad (\text{C.4.1})$$

where $\phi^{(1)}(\mathbf{Q}, t) = \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - t \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e = \text{tr}(\mathbf{A} \mathbf{Q})$ with $\mathbf{A} = \mathbf{h}_r \mathbf{h}_r^H - t \mathbf{h}_e \mathbf{h}_e^H$.

Theorem C.5. *The closed-form expression for the optimal transmit strategy of (C.4.1) is given by*

$$\mathbf{Q}_{SPC}^{(1)}(t) = P_{tot} \mathbf{v} \mathbf{v}^H \quad (\text{C.4.2})$$

where \mathbf{v} is the eigenvector associated with the positive eigenvalue of $\mathbf{h}_r \mathbf{h}_r^H - t \mathbf{h}_e \mathbf{h}_e^H$ for a given t .

Proof. The proof of Theorem C.5 is in Appendix C.7.4. \square

C.4.2 Per-antenna Power Constraints Only

Let \mathcal{S}_{PAPC} denote the set of all transmit strategies which satisfy all per-antenna power constraints $\hat{P}_k, \forall k \in \mathcal{I}$, i.e., $\mathcal{S}_{PAPC} = \{\mathbf{Q} \succeq 0 : \mathbf{e}_k^T \mathbf{Q} \mathbf{e}_k \leq \hat{P}_k, \forall k \in \mathcal{I}\}$. The equivalent problem of finding the weighted rate sum optimal transmit strategy for the MISO wiretap channel with per-antenna power constraints only for a given t can be written as

$$\mathbf{Q}_{PAPC}^{(1)}(t) = \arg \max_{\mathbf{Q} \in \mathcal{S}_{PAPC}} \phi^{(1)}(\mathbf{Q}, t), \quad (\text{C.4.3})$$

where $\phi^{(1)}(\mathbf{Q}, t) = \text{tr}(\mathbf{A} \mathbf{Q})$ with $\mathbf{A} = \mathbf{h}_r \mathbf{h}_r^H - t \mathbf{h}_e \mathbf{h}_e^H$.

From the definition of \mathbf{A} below (C.4.1) and (C.4.3) we know that this matrix may have a negative eigenvalue. A negative eigenvalue in the matrix \mathbf{A} does not affect the procedure to compute the optimal solution for the sum power constraint only case. In particular, the total power is always allocated. Interestingly this does not hold for the per-antenna power constraints only and joint sum and per-antenna power constraints problems. For instance, for the MISO wiretap channels with per-antenna power constraints, when \mathbf{A} has a negative eigenvalue, it may not be optimal to allocate full transmit power on all antennas. In this scenario, it is optimal to transmit with full individual power in the direction of the legitimate receiver but not in the direction of the eavesdropper. If \mathbf{A} is not positive semi-definite, then it may not be optimal to allocate full power on all antennas. In the following, we assume that the power allocation per antenna, which is denoted by $\tilde{P}_k \forall k \in \mathcal{I}$, is given. Later, we will discuss the power allocation problem. This assumption implies that $\mathbf{Q}_{PAPC}(t)$ has diagonal elements of $q_{kk} = \tilde{P}_k \forall k \in \mathcal{I}$.

The remaining problem is to find off-diagonal elements of $\mathbf{Q}_{PAPC}(t)$ for a given t . The main difficulty here is the positive semi-definite constraint of $\mathbf{Q}_{PAPC}(t)$. To overcome this, we consider a relaxed optimization problem involving the 2×2 principal minors of $\mathbf{Q}_{PAPC}(t)$ similarly as done in [Vu11a]. Let $\mathbf{X}_{k,l}(t)$ be a principal minor matrix which is obtained from \mathbf{Q} by removing $N_t - 2$ columns, except columns k and l , and the corresponding $N_t - 2$ rows except rows k and l . Then, $\mathbf{X}_{k,l}(t)$ is given as

$$\mathbf{X}_{k,l}(t) = \begin{bmatrix} \tilde{P}_k & q_{kl}^*(t) \\ q_{kl}(t) & \tilde{P}_l \end{bmatrix} \quad (\text{C.4.4})$$

where $k, l \in \mathcal{I}$, $k \neq l$. Therewith, we can formulate a relaxed optimization problem as follows

$$\begin{aligned} \max_{\mathbf{Q}} \phi^{(1)}(\mathbf{Q}, t), \quad \text{s. t. } q_{kk} = \tilde{P}_k, \forall k \in \mathcal{I} \\ \mathbf{X}_{k,l}(t) \succeq 0, \forall k, l \in \mathcal{I}, k \neq l. \end{aligned} \quad (\text{C.4.5})$$

The off-diagonal elements of the covariance matrix in (C.4.5) then can be obtained using the following theorem.

Theorem C.6. *The optimal transmit strategy $\mathbf{Q}_{PAPC-R}(t)$ of the relaxed optimization problem (C.4.5) has off-diagonal elements*

$$q_{kl}(t) = \frac{h_{rk}^* h_{rl} - t h_{ek}^* h_{el}}{|h_{rk}^* h_{rl} - t h_{ek}^* h_{el}|} \sqrt{\tilde{P}_k \tilde{P}_l}, \quad k, l \in \mathcal{I}, k \neq l. \quad (\text{C.4.6})$$

Proof. The proof of Theorem C.6 is in Appendix C.7.5. □

From Theorem C.6, we have the following conclusion and remarks.

Corollary C.7. *If there are only two transmit antennas, i.e., $N_t = 2$, then (C.4.6) always leads to a positive semi-definite solution with eigenvalues zero and $\tilde{P}_1 + \tilde{P}_2$, i.e., the optimal solution (C.4.6) of the relaxed optimization problem (C.4.5) is actually the optimal solution of (C.4.3).*

For $n > 2$, it is not clear if (C.4.6) always results in a positive semi-definite solution. Numerical experiments suggest this assumption, but a proof is missing. So that, we have only the following remark.

Remark C.8. *If the solution (C.4.6) leads to a positive semi-definite solution, then it is also an optimal solution of (C.4.3).*

Thus, it is a viable strategy to first compute the solution according to (C.4.6) and then test if it is positive semi-definite.

a). Optimal Power Allocation for the Per-antenna Power Constraints Only Problem: In this section, we discuss the optimal power allocation for the per-antenna power constraints only problem. We first establish a useful observation as follows.

Proposition C.9. *For the per-antenna power constraints only problem, there is always at least one per-antenna power constraint active.*

Proof. Assume that there exist an optimal beam forming vector which does not allocate full power on all antennas. Then we can scale it by a factor larger than one and achieve a larger rate. This implies that at least one antenna should allocate full power. \square

Next, a person-to-person optimality method is used to establish some necessary condition for optimality. In more details, we characterize the optimal beam-forming coefficient of the l -th antenna given a set of beamforming coefficients of all other antennas, $k \neq l$. Since it is sufficient to consider beam-forming strategy, i.e., $\mathbf{Q} = \mathbf{q}\mathbf{q}^H$ with $\mathbf{q} = [q_1(t), \dots, q_{N_t}(t)]^T$, $\phi^{(1)}(\mathbf{Q}, t)$ can be expressed as $\phi^{(1)}(\mathbf{Q}, t) = |\mathbf{q}^H \mathbf{h}_r|^2 - t|\mathbf{q}^H \mathbf{h}_e|^2$. Then, the optimal transmit power on the l -th antenna, i.e., $P_l^* = q_l q_l^*$, $l \in \mathcal{I}$, is then determined by solving the following optimization problem

$$\arg \max_{q_l \in \mathbb{C}} |q_l^* h_{rl} + \sum_{\substack{k \in \mathcal{I} \\ k \neq l}} q_k^* h_{rk}|^2 - t|q_l^* h_{el} + \sum_{\substack{k \in \mathcal{I} \\ k \neq l}} q_k^* h_{ek}|^2. \quad (\text{C.4.7})$$

The following proposition characterizes a condition when it is optimal to allocate full power on the l -th antenna.

Theorem C.10. *For given beamforming coefficients $q_k \forall k \in \mathcal{I}$, $k \neq l$, let α be the phase of $\sum_{\substack{k \in \mathcal{I} \\ k \neq l}} q_k (h_{rk}^* h_{rl} - t h_{ek}^* h_{el})$. Then the optimal beamforming coefficient*

q_l has phase $\varphi_l^ = \alpha$ and absolute value $|q_l^*| = \sqrt{\tilde{P}_l}$ if $|h_{rl}|^2 \geq t|h_{el}|^2$, else $|q_l^*| = \min\left\{\frac{|\sum_k q_k (h_{rk}^* h_{rl} - t h_{ek}^* h_{el})|}{|h_{rl}|^2 - t|h_{el}|^2}, \sqrt{\tilde{P}_l}\right\}$.*

Proof. The proof of Theorem C.10 is in Appendix C.7.6. \square

From Theorem C.10 we know that when $|h_{rl}|^2 < t|h_{el}|^2$ and $\frac{|\sum_k q_k (h_{rk}^* h_{rl} - t h_{ek}^* h_{el})|}{|h_{rl}|^2 - t|h_{el}|^2} < \sqrt{\hat{P}_l}$, then it is not optimal to allocate full power on the l -th antenna. In this case the optimal allocation for the l -th antenna is given by $\frac{|\sum_k q_k (h_{rk}^* h_{rl} - t h_{ek}^* h_{el})|}{|h_{rl}|^2 - t|h_{el}|^2}$. Note that this allocation depends on the assumed power allocation on all other antennas.

Theorem C.10 suggests a simple algorithm to find the optimal power allocation for given $(t, \mathbf{h}_r, \mathbf{h}_e, \hat{\mathbf{P}})$, which is summarized in Algorithm C.4.1. The convergence of Algorithm C.4.1 is guaranteed because in every step it will improve and it is bounded by the sum power constraint optimization problem with $P_{tot} = \sum_{i=1}^{N_t} \hat{P}_i$. Further, Algorithm C.4.1 converges to the optimum since the problem is convex. Note that we can always pick one phase and therefore only $N_t - 1$ phases need to be optimized.

Algorithm C.4.1: Optimal Power Allocation

```

1 Set  $i = 1$ ,  $\mathbf{q}^{(i,1)} = [q_l]_{l=1}^{N_t} = [\sqrt{\hat{P}_1}, \dots, \sqrt{\hat{P}_{N_t}}]^T \in \mathbb{C}^{N_t}$ .
2 do
3   for  $l = 1$  to  $N_t$  do
4     Compute  $\alpha$ ,
5      $\varphi_l^*(\mathbf{q}^{(i,l)} \setminus q_l) = \alpha$ ,
6     if (i) or (ii) then
7        $|q_l^*(\mathbf{q}^{(i,l)} \setminus q_l)| = \sqrt{\hat{P}_l}$ ,
8     else
9        $|q_l^*(\mathbf{q}^{(i,l)} \setminus q_l)| = \frac{|\sum_k q_k (h_{rk}^* h_{rl} - t h_{ek}^* h_{el})|}{|h_{rl}|^2 - t|h_{el}|^2}$ ,
10    end if
11  end if
12  if  $l < N_t$  then
13     $\mathbf{q}^{(i,l+1)} = [q_1, \dots, q_{l-1}, q_l^*, \dots, q_{N_t}]^T$ ,
14  else
15     $\mathbf{q}^{(i+1,1)} = [q_1, \dots, q_{N_t-1}, q_{N_t}^*]^T$ .
16  end if
17  end if
18 end for
19  $i \leftarrow i + 1$ .
20 while  $|R(\mathbf{q}^{(i+1,1)}) - R(\mathbf{q}^{(i,1)})| \geq \epsilon$ ;

```

If the channels are parallel and the secrecy capacity is positive, then it is optimal to allocate full power on all antennas, which is discussed in more detail in Section C.4.3.

b). Two-Antenna Case: In this section, we provide solutions to find the optimal transmit power for a special case with two antennas at the transmitter. For the per-antenna power constraints optimization problem, Theorem C.6 in Section C.4.2 suggests that for a specific case with two antennas at the transmitter, the off-diagonal element of the covariance matrix can be computed as:

$$q_{kl}(t) = \frac{h_{rk}^* h_{rl} - t h_{ek}^* h_{el}}{|h_{rk}^* h_{rl} - t h_{ek}^* h_{el}|} \sqrt{P_k^* P_l^*}, k, l \in \mathcal{I}, k \neq l. \quad (\text{C.4.8})$$

Here, if we assume there is no phase on the l -th antenna, then the phase on the remaining antenna will be $e^{j\varphi_k} = \frac{h_{rk}^* h_{rl} - t h_{ek}^* h_{el}}{|h_{rk}^* h_{rl} - t h_{ek}^* h_{el}|}$, and vice versa.

The optimal power allocation for the optimization problem with per-antenna power constraints only can be found by applying following corollary.

Corollary C.11. *Let $k, l = \{1, 2\}$, $k \neq l$. For the per-antenna power constraint only problem, if $q_l^* = \sqrt{\hat{P}_l}$, then $e^{\varphi_k^*} = \frac{h_{rk}^* h_{rl} - t h_{ek}^* h_{el}}{|h_{rk}^* h_{rl} - t h_{ek}^* h_{el}|}$ and*

$$|q_k^*| = \begin{cases} \sqrt{\hat{P}_k}, & \text{if } |h_{rk}|^2 \geq t|h_{ek}|^2 \\ \min\left\{\frac{\sqrt{\hat{P}_l} |h_{rk}^* h_{rl} - t h_{ek}^* h_{el}|}{|h_{rk}|^2 - t|h_{ek}|^2}, \sqrt{\hat{P}_k}\right\}, & \text{if } |h_{rk}|^2 < t|h_{ek}|^2. \end{cases} \quad (\text{C.4.9})$$

In the following, we discuss and provide solutions for the special case that \mathbf{A} does not have a negative eigenvalue, i.e., \mathbf{A} is positive semi-definite. This case happens when channel vectors are parallel and the secrecy capacity is positive.

C.4.3 Special Case of Parallel Channels

In general, the function $R_{\Sigma}(\hat{\mathbf{p}}, w)$ is not a concave function. However, if we consider channel coefficients such that $\mathbf{A} = \mathbf{h}_r \mathbf{h}_r^H - t \mathbf{h}_e \mathbf{h}_e^H$ is a positive semi-definite matrix for a given t , then the secrecy rate function $R_{\Sigma}(\hat{\mathbf{p}}, w)$ is a concave function [SSC12, Proposition 2.1, MISO case]. In this case, \mathbf{A} has rank one with $\mathbf{A} = \mathbf{h}_A \mathbf{h}_A^H$ and $\mathbf{h}_A = \sqrt{|\mathbf{h}_r|^2 - t|\mathbf{h}_e|^2} \frac{\mathbf{h}_r}{|\mathbf{h}_r|}$. Then the objective function in the optimization problem (C.3.4) can be rewritten as

$$\phi^{(1)}(\mathbf{Q}, t) = \mathbf{h}_A^H \mathbf{Q} \mathbf{h}_A. \quad (\text{C.4.10})$$

It follows that the special case with \mathbf{A} to be positive semi-definite directly corresponds to a point-to-point MISO channel problem with channel \mathbf{h}_A . Moreover, since the arg max does not change with t , there is no trade-off between secrecy and non-secrecy rate, i.e., there is only transmit strategy that simultaneously maximizes both rates. Thus, we have the following remark.

Remark C.12. *If \mathbf{A} is positive semidefinite, then the optimal transmit strategies $\mathbf{Q}_{\text{opt}}^{(1)}(\hat{\mathbf{p}}, t)$ can be obtained from the solution of the point-to-point MISO channel problem:*

- (i) [Vu11a] for the per-antenna power constraints only problem,
- (ii) [COSS16] (Paper A) for the joint sum and per-antenna power constraints problem,

considering the channel \mathbf{h}_A .

In particular, this implies that in the case of \mathbf{A} positive semi-definite that we can always find optimal transmit strategies that allocate full power (more details can be found in [Vu11a], Paper A).

C.5 Numerical Examples

In this section, illustrative numerical examples for the optimization problems with sum power constraint only and per-antenna power constraint only with two antennas at the transmitter, and one antenna at legitimate receiver and eavesdropper each are shown. We first provide a MISO wiretap channel with two transmit antennas. The complex channel coefficients corresponds to legitimate receiver and eavesdropper are given as $\mathbf{h}_r = [0.3737 + 0.8912i, 0.9795 + 1.2926i]^T$ and $\mathbf{h}_e = [0.4387 + 0.7655i, 0.3816 + 0.7952i]^T$. The powers on maximum transmit power on antennas are set as $\hat{P}_1 = 5$ and $\hat{P}_2 = 10$. The sum power constraint $P_{tot} = 15$.

Figure C.3 depicts optimal regions between the transmission rate and the secrecy rate of the wiretap channel with two different sets of power constraints: sum power constraint only and per-antenna power constraints only. The figure shows that the regions are fully characterized by the curved sections which can be obtained from the optimal solutions. It also shows the optimal trade-off between the transmission rate and the secrecy rate. For instance, we can see that the strategies that maximize the secrecy rates, $t = 2^{C_s^{SPC}}$ with $C_s^{SPC} = 1.5783$ for the case with sum power constraint only and $t = 2^{C_s^{PAPC}}$ with $C_s^{PAPC} = 1.4182$ for the case with per-antenna power constraints only. These respectively correspond to $s_{\max}^{SPC} = 0.3349$ and $s_{\max}^{PAPC} = 0.3742$.

C.6 Conclusions

In this paper, we studied the tradeoff between the transmission rate and the secrecy rate of Gaussian MISO wiretap channels considering different power constraint settings. The original optimization problem is non-convex. However, using equivalent convex reformulations allow us to provide useful characterizations of the rate regions boundary. The optimal rate pair can be then found by a simple line search. In particular, for the optimization problem with sum power constraint only, the optimal transmit strategy is characterized by a simple closed-form solution. Next, it turns out that if channel vectors are not parallel, then it may be optimal not to allocate full power if there are per-antenna power constraints. For the general case necessary conditions for optimality have been derived that have been used in

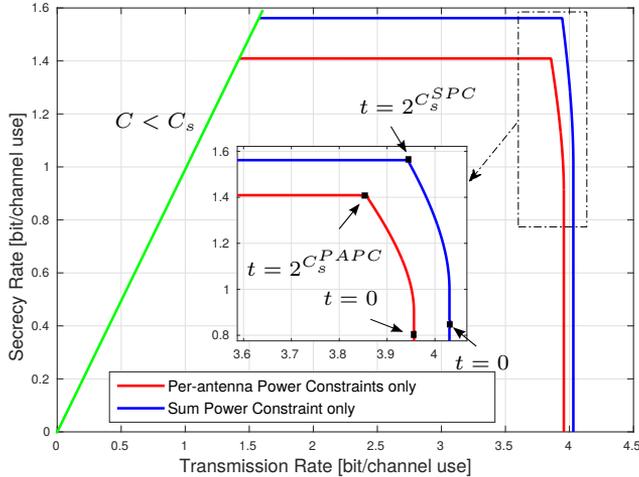


Figure C.3: The optimal regions between the transmission rates and the secrecy rate with sum power constraint only $P_{tot} = 15$ and per-antenna power constraints only $\hat{P}_1 = 5$ and $\hat{P}_2 = 10$

an iterative person-by-person algorithm for the per-antenna power constraints only problems. We believe that studies on optimal transmit strategies including more advanced power constraint settings are highly relevant for future wireless networks, in particular for massive MIMO setups, as used for instance in Paper D and Paper E.

C.7 Appendix

C.7.1 Proof of Proposition C.1

To prove the proposition, we need to show the necessity and sufficiency. For the necessary part, we need to show that for $R_s(\mathbf{Q}) > 0$, $\mathbf{h}_r \mathbf{h}_r^H - \mathbf{h}_e \mathbf{h}_e^H$ has at least one positive eigenvalue. The secrecy rate can be written as

$$\begin{aligned} R_s(\mathbf{Q}) &= \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r) - \log(1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e) \\ &= \log \left(1 + \frac{\text{tr}\{(\mathbf{h}_r \mathbf{h}_r^H - \mathbf{h}_e \mathbf{h}_e^H) \mathbf{Q}\}}{1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e} \right) > 0. \end{aligned} \quad (\text{C.7.1})$$

Since $1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e > 0$, it follows that $\text{tr}(\mathbf{A} \mathbf{Q}) > 0$ with $\mathbf{A} = \mathbf{h}_r \mathbf{h}_r^H - \mathbf{h}_e \mathbf{h}_e^H$. Since $\text{tr}(\mathbf{A} \mathbf{Q}) = \sum_{i=1}^{N_t} \lambda_i(\mathbf{Q}) \mathbf{v}_i^H \mathbf{A} \mathbf{v}_i > 0$, there must exist an $\hat{i} \in \{1, \dots, N_t\}$ such that

$\mathbf{v}_i^H \mathbf{A} \mathbf{v}_i > 0$. Thus, we have

$$\lambda_{\max}(\mathbf{A}) = \max_{\|\mathbf{v}_i\|=1} \mathbf{v}_i^H \mathbf{A} \mathbf{v}_i \geq \mathbf{v}_i^H \mathbf{A} \mathbf{v}_i > 0. \quad (\text{C.7.2})$$

For the sufficient part, we need to show that if $\mathbf{A} = \mathbf{h}_r \mathbf{h}_r^H - \mathbf{h}_e \mathbf{h}_e^H$ has a positive eigenvalue, then there exists $\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})$ such that $R_s(\mathbf{Q}) > 0$.

Since \mathbf{A} has a positive eigenvalue, there exist a vector $\mathbf{v} : \|\mathbf{v}\| = 1$ such that $\mathbf{v}^H \mathbf{A} \mathbf{v} > 0$. This implies that we can construct $\mathbf{Q} = \xi \mathbf{v} \mathbf{v}^H$, $\xi > 0$, such that $\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})$ and $\text{tr}(\mathbf{A} \mathbf{Q}) > 0$. Then we have

$$R_s(\mathbf{Q}) = \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r) - \log(1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e) = \log\left(1 + \frac{\text{tr}(\mathbf{A} \mathbf{Q})}{1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e}\right) > 0.$$

□

C.7.2 Proof of Theorem C.3

First, we show that for every $w \in [0, 1]$ there exists a $t \in [0, t_{\max}]$ with $t_{\max} = 2^{C_s(\hat{\mathbf{p}})}$ such that $\mathbf{Q}_{\text{opt}}^{(1)}(\hat{\mathbf{p}}, t)$ is an optimal transmit strategy, i.e., $R_{\Sigma}(\hat{\mathbf{p}}, w) = R_{\Sigma}(\mathbf{Q}_{\text{opt}}^{(1)}(\hat{\mathbf{p}}, t), w)$.

For a given $w \in [0, 1]$ we assume that there exist no $t \in [0, t_{\max}]$ such that $\mathbf{Q}_{\text{opt}}^{(1)}(\hat{\mathbf{p}}, t)$ is optimal. This implies that there exist a \mathbf{Q}^* so that $R_{\Sigma}(\hat{\mathbf{p}}, w) = R_{\Sigma}(\mathbf{Q}^*, w)$ and

$$R_{\Sigma}(\mathbf{Q}^*, w) > R_{\Sigma}(\mathbf{Q}_{\text{opt}}^{(1)}(\hat{\mathbf{p}}, t), w) \quad \forall t \in [0, t_{\max}]. \quad (\text{C.7.3})$$

Following Lemma C.2 we know that for an optimal \mathbf{Q}^* the corresponding values D_r^* and D_e^* are computed as

$$D_r^* = (1 + \mathbf{h}_r^H \mathbf{Q}^* \mathbf{h}_r)^{-1}, \quad (\text{C.7.4})$$

$$D_e^* = (1 + \mathbf{h}_e^H \mathbf{Q}^* \mathbf{h}_e)^{-1}. \quad (\text{C.7.5})$$

Then for $w \in [0, 1]$ we have:

$$\begin{aligned} R_{\Sigma}(\mathbf{Q}^*, w) &= D_r^*(1 + \mathbf{h}_r^H \mathbf{Q}^* \mathbf{h}_r) - \log(D_r^*) - 1 \\ &\quad + w(-D_e^*(1 + \mathbf{h}_e^H \mathbf{Q}^* \mathbf{h}_e) + \log(D_e^*) + 1) \\ &\leq \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} D_r^*(\mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - w \frac{D_e^*}{D_r^*} \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r) + D_r^* \\ &\quad - \log(D_r^*) - 1 - w D_e^* - w \log(D_r^*) - w. \end{aligned} \quad (\text{C.7.6})$$

Following (C.3.3) and (C.3.4) we know that the optimal solution for the latter of (C.7.6) is computed as

$$\mathbf{Q}^* = \arg \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - w \frac{D_e^*}{D_r^*} \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r = \mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, w \frac{D_e^*}{D_r^*}) = \mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t), \quad (\text{C.7.7})$$

where $0 \leq t = w \frac{D_e^*}{D_r^*} \leq t_{\max}$. This implies that

$$R_{\Sigma}(\mathbf{Q}^*, w) \leq R_{\Sigma}(\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t), w), \quad (\text{C.7.8})$$

for $t = w \frac{D_e^*}{D_r^*} \in [0, t_{\max}]$. However, this contradicts with (C.7.3). Thus, it follows that for every $w \in [0, 1]$ there exists a $t \in [0, t_{\max}]$ such that $\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t)$ is an optimal transmit strategy.

Next, we show that for every $t \in [0, t_{\max}]$ there exists a $w \in [0, 1]$ such that $\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t)$ is an optimal transmit strategy, i.e., $R_{\Sigma}(\hat{\mathbf{p}}, w) = R_{\Sigma}(\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t), w)$. Suppose that \mathbf{Q}^* is an optimal solution of (C.3.4), then from Lemma C.2 we know that for a given \mathbf{Q}^* the corresponding values $t \in [0, t_{\max}]$ is given by $t = w \frac{D_e^*}{D_r^*}$ with

$$D_r^* = (1 + \mathbf{h}_r^H \mathbf{Q}^* \mathbf{h}_r)^{-1}, \quad (\text{C.7.9})$$

$$D_e^* = (1 + \mathbf{h}_e^H \mathbf{Q}^* \mathbf{h}_e)^{-1}, \quad (\text{C.7.10})$$

and \mathbf{Q}^* must satisfy the KKT condition of (C.3.4) which is given as follows.

$$\begin{aligned} \frac{\partial}{\partial \mathbf{Q}} \phi^{(1)}(\mathbf{Q}, t) &= \mathbf{D} + \mu \mathbf{I} - \mathbf{M} & (\text{C.7.11}) \\ \text{tr}(\mathbf{Q}) &\leq P_{tot} & \mathbf{e}_k^T \mathbf{Q} \mathbf{e}_k &\leq \hat{P}_k, \forall k \in \mathcal{I} & \mathbf{Q} &\succeq 0 \\ \mu &\geq 0 & \mathbf{D} &\succeq 0 & \mathbf{M} &\succeq 0 \\ \mu(\text{tr}(\mathbf{Q}) - P_{tot}) &= 0 & \text{tr}(\mathbf{D}(\mathbf{Q} - \hat{\mathbf{P}})) &= 0 & \mathbf{M}\mathbf{Q} &= 0. \end{aligned}$$

On the other hand, we have

$$\begin{aligned} \frac{\partial}{\partial \mathbf{Q}} \phi^{(1)}(\mathbf{Q}, t) \Big|_{\mathbf{Q}=\mathbf{Q}^*, t=t} &= D_r^* \mathbf{h}_r \mathbf{h}_r^H - w D_e^* \mathbf{h}_e \mathbf{h}_e^H \\ &= \mathbf{h}_r (1 + \mathbf{h}_r^H \mathbf{Q}^* \mathbf{h}_r)^{-1} \mathbf{h}_r^H - w \mathbf{h}_e (1 + \mathbf{h}_e^H \mathbf{Q}^* \mathbf{h}_e)^{-1} \mathbf{h}_e^H \\ &= \frac{\partial}{\partial \mathbf{Q}} R_{\Sigma}(\mathbf{Q}, w) \Big|_{\mathbf{Q}=\mathbf{Q}^*, w=t \frac{D_r^*}{D_e^*}}. \end{aligned} \quad (\text{C.7.12})$$

Therefore, we can conclude from (C.7.11) and (C.7.12) that the optimal transmit strategy \mathbf{Q}^* is also an optimal solution of (C.2.10) with $0 \leq w = t \frac{D_r^*}{D_e^*} \leq 1$. \square

C.7.3 Proof of Theorem C.4

For the proof of Theorem C.4, we make use of a second equivalent reformulation where we apply Lemma C.2 to (C.2.10) with $E_e(\mathbf{Q}) = 1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e$ and $f_e(D_e, \mathbf{Q}) = -D_e E_e(\mathbf{Q}) + \log(D_e) + 1$ only. Thus, the optimization problem (C.2.10) can be alternatively expressed as

$$R_{\sum}(\hat{\mathbf{p}}, w) = \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} \max_{D_e > 0} \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r) + w(-D_e(1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e) + \log(D_e) + 1). \quad (\text{C.7.13})$$

If we define $s = wD_e$ and $\phi^{(2)}(\mathbf{Q}, s) = \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r) - s\mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e$, then (C.7.13) can be written as

$$R_{\sum}(\hat{\mathbf{p}}, w) = \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} \max_{D_e > 0} \phi^{(2)}(\mathbf{Q}, s) - s + w \log(D_e) + w. \quad (\text{C.7.14})$$

Then the problem of finding the optimal transmit strategy $\mathbf{Q}_{opt}^{(2)}(\hat{\mathbf{p}}, s)$ depends on s only and can be expressed as

$$\mathbf{Q}_{opt}^{(2)}(\hat{\mathbf{p}}, s) = \arg \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} \phi^{(2)}(\mathbf{Q}, s). \quad (\text{C.7.15})$$

Although s is dependent on D_e and w , the optimization with respect to \mathbf{Q} only depends on s . Thus, we propose to solve first (C.7.15) for a given s and power constraints $\hat{\mathbf{p}}$. Once $\mathbf{Q}_{opt}^{(2)}(\hat{\mathbf{p}}, s)$ is obtained, the corresponding D_e^* and w can be found using Lemma C.2 and formula $w = s/D_e^* = s(1 + \mathbf{h}_e^H \mathbf{Q}_{opt}^{(2)}(\hat{\mathbf{p}}, s) \mathbf{h}_e)$.

We are now ready to prove Theorem C.4. By replacing $t \in [0, t_{\max}]$ in Theorem C.3 by $s = [0, s_{\max}]$ where $s_{\max} = (1 + \mathbf{h}_e^H \mathbf{Q}^s \mathbf{h}_e)^{-1}$ with $\mathbf{Q}^s = \arg \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} R_{\sum}(\mathbf{Q}, w = 1) = \arg \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} R_s(\mathbf{Q})$, we obtained that every optimal transmit strategy obtained from (C.7.15) is the same as the optimal transmit strategy obtained from (C.3.4) and is the optimal transmit strategy of (C.2.10). This implies that, for a given w , at the optimum we have $s = wD_e^*$, $t = w \frac{D_e^*}{D_{r,*}}$ and

$$\mathbf{Q}_{opt}^{(2)}(\hat{\mathbf{p}}, s) = \mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t) = \mathbf{Q}_{opt}(\hat{\mathbf{p}}, w). \quad (\text{C.7.16})$$

Thus, it is sufficient to find the rank of the optimal transmit strategy by considering the following optimization problem

$$\max_{\mathbf{Q}} \phi^{(2)}(\mathbf{Q}, s), \text{ s. t. } \mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}}). \quad (\text{C.7.17})$$

The Lagrangian for problem (C.7.17) is given by

$$\mathcal{L} = \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r) - s\mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e - \text{tr}(\mathbf{D}(\mathbf{Q} - \hat{\mathbf{P}})) - \mu(\text{tr}(\mathbf{Q}) - P_{tot}) + \text{tr}(\mathbf{M}\mathbf{Q}), \quad (\text{C.7.18})$$

where $\mathbf{D} = \text{diag}\{\nu_k\}$ is a diagonal matrix of Lagrangian multiplier for the per-antenna power constraints, μ is the Lagrangian multiplier for the sum power constraint, \mathbf{M} is the Lagrangian multiplier for the positive semi-definite constraint, and $\hat{\mathbf{P}} = \text{diag}\{\hat{P}_k\}$, $\forall k \in \mathcal{I} = \{1, \dots, N_t\}$, is a diagonal matrix of the per-antenna power constraints.

Taking the first derivative of the Lagrangian above and set equal to zero, we have

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \mathbf{Q}} &= \mathbf{h}_r(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r)^{-1} \mathbf{h}_r^H - s \mathbf{h}_e \mathbf{h}_e^H - \mathbf{D} - \mu \mathbf{I} + \mathbf{M} \\ &= \mathbf{h}_r(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r)^{-1} \mathbf{h}_r^H - \mathbf{K} + \mathbf{M} \stackrel{!}{=} 0, \end{aligned} \quad (\text{C.7.19})$$

where $\mathbf{K} = s \mathbf{h}_e \mathbf{h}_e^H + \mathbf{D} + \mu \mathbf{I}$.

By using the slackness condition $\mathbf{M} \mathbf{Q} = 0$, we obtain $\mathbf{h}_r \mathbf{h}_r^H \mathbf{Q} = (1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r) \mathbf{K} \mathbf{Q}$ by multiplying (C.7.19) with \mathbf{Q} from the right. On the other hand, from the KKT condition of the convex optimization problem (C.7.17), we know that in the optimum we have either $\mu > 0$ or $\mathbf{D} \succ 0$. This implies that, in the optimum, \mathbf{K} has full rank and

$$\text{rank}(\mathbf{Q}_{opt}(\hat{\mathbf{p}}, w)) = \text{rank}(\mathbf{h}_r \mathbf{h}_r^H \mathbf{Q}) \leq \text{rank}(\mathbf{h}_r \mathbf{h}_r^H) = 1. \quad (\text{C.7.20})$$

Since $\text{rank}(\mathbf{Q}_{opt}(\hat{\mathbf{p}}, w)) = 0$ is not optimal, the optimal rank of $\mathbf{Q}_{opt}(\hat{\mathbf{p}}, w)$ is one. This proves Theorem C.4. \square

C.7.4 Proof of Theorem C.5

By using singular value decomposition, for a given t , we have $\mathbf{h}_r \mathbf{h}_r^H - t \mathbf{h}_e \mathbf{h}_e^H = \mathbf{V} \mathbf{\Lambda} \mathbf{V}^H$. Let $\tilde{\mathbf{Q}} = \mathbf{V}^H \mathbf{Q} \mathbf{V}$, we obtain $\tilde{\mathbf{Q}} \succeq 0$. Then

$$\begin{aligned} \phi^{(1)}(\mathbf{Q}, t) &= \text{tr}\{(\mathbf{h}_r \mathbf{h}_r^H - t \mathbf{h}_e \mathbf{h}_e^H) \mathbf{Q}\} = \text{tr}\{\mathbf{\Lambda} \tilde{\mathbf{Q}}\} \\ &= \text{tr}\{\mathbf{\Lambda} \text{diag}(\tilde{\mathbf{Q}})\} \leq \lambda_{\max} P_{tot} \end{aligned} \quad (\text{C.7.21})$$

with λ_{\max} is the positive, which is also the largest, entry of $\mathbf{\Lambda}$ and $\text{tr}(\tilde{\mathbf{Q}}) = \text{tr}(\mathbf{Q}) = P_{tot}$.

Equation (C.7.21) holds with equality if $\tilde{\mathbf{Q}}$ is diagonal and has a unique nonzero entry equal to P_{tot} corresponding to the positive entry of $\mathbf{\Lambda}$. This implies that \mathbf{Q} and $\mathbf{h}_r \mathbf{h}_r^H - t \mathbf{h}_e \mathbf{h}_e^H$ share the same eigenvectors and \mathbf{Q} has rank one. Therefore, we have $\mathbf{Q}_{SPC}^{(1)}(t) = P_{tot} \mathbf{v} \mathbf{v}^H$ where \mathbf{v} is the eigenvector associated with the positive eigenvalue of $\mathbf{h}_r \mathbf{h}_r^H - t \mathbf{h}_e \mathbf{h}_e^H$ for a given t . This proves Theorem C.5. \square

C.7.5 Proof of Theorem C.6

Consider an optimization problem (C.4.3). The Lagrangian for problem (C.4.5) is given by

$$\mathcal{L} = \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - t \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e - \sum_{k \neq l} \lambda_{kl} (|q_{kl}(t)|^2 - \tilde{P}_k \tilde{P}_l) - \sum_k \mu_k (q_{kk} - \tilde{P}_k), \quad (\text{C.7.22})$$

where λ_{kl} and μ_k are the Lagrange multipliers, and $k, l \in \mathcal{I}$. Taking the first derivative of (C.7.22) and set it equal to zero, we have

$$\frac{\partial \mathcal{L}}{\partial q_{kl}} = h_{rk}^* h_{rl} - t h_{ek}^* h_{el} - \lambda_{kl} q_{kl}(t) \stackrel{!}{=} 0, \quad (\text{C.7.23})$$

or equivalently

$$q_{kl}(t) = \frac{h_{rk}^* h_{rl} - t h_{ek}^* h_{el}}{\lambda_{kl}}. \quad (\text{C.7.24})$$

Similar to [COS16], the optimal value of q_{kl} in (C.7.24) is obtained when its constraint is satisfied with equality, i.e., $|q_{kl}(t)|^2 = \hat{P}_k \hat{P}_l$. By combining this condition with (C.7.24), we have the value of $q_{kl}(t)$ as in (C.4.6). \square

C.7.6 Proof of Theorem C.10

Consider the optimization problem (C.4.7). Equivalently, we can express (C.4.7) as

$$\arg \max_{q_l \in \mathbb{C}} |q_l^* h_{rl} + \sum_{\substack{k \in \mathcal{I} \\ k \neq l}} q_k^* h_{rk}|^2 - t |q_l^* h_{el} + \sum_{\substack{k \in \mathcal{I} \\ k \neq l}} q_k^* h_{ek}|^2 \quad (\text{C.7.25})$$

$$\begin{aligned} &= \arg \max_{q_l \in \mathbb{C}} \left| \sum_{\substack{k \in \mathcal{I} \\ k \neq l}} q_k^* h_{rk} \right|^2 + |q_l^* h_{rl}|^2 + 2 \operatorname{Re} \left\{ \sum_{\substack{k \in \mathcal{I} \\ k \neq l}} q_k h_{rk}^* h_{rl} q_l^* \right\} \\ &\quad - t \left| \sum_{\substack{k \in \mathcal{I} \\ k \neq l}} q_k^* h_{ek} \right|^2 - t |q_l^* h_{el}|^2 - 2t \operatorname{Re} \left\{ \sum_{\substack{k \in \mathcal{I} \\ k \neq l}} q_k h_{ek}^* h_{el} q_l^* \right\} \end{aligned} \quad (\text{C.7.26})$$

$$= \arg \max_{q_l \in \mathbb{C}} (|h_{rl}|^2 - t |h_{el}|^2) |q_l|^2 + 2 \operatorname{Re} \left\{ \sum_{\substack{k \in \mathcal{I} \\ k \neq l}} q_k (h_{rk}^* h_{rl} - t h_{ek}^* h_{el}) q_l^* \right\}, \quad (\text{C.7.27})$$

$$= \arg \max_{q_l \in \mathbb{C}} (|h_{rl}|^2 - t |h_{el}|^2) |q_l|^2 + 2 \left| \sum_{\substack{k \in \mathcal{I} \\ k \neq l}} q_k (h_{rk}^* h_{rl} - t h_{ek}^* h_{el}) \right| |q_l| \cos(\alpha - \varphi_l), \quad (\text{C.7.28})$$

$$\stackrel{(\varphi_l = \alpha)}{=} \arg \max_{|q_l| \leq \sqrt{\hat{P}_l}} (|h_{rl}|^2 - t |h_{el}|^2) |q_l|^2 + 2 \left| \sum_{\substack{k \in \mathcal{I} \\ k \neq l}} q_k (h_{rk}^* h_{rl} - t h_{ek}^* h_{el}) \right| |q_l|. \quad (\text{C.7.29})$$

where α and φ_l are phases of $\sum_{\substack{k \in \mathcal{I} \\ k \neq l}} q_k (h_{rk}^* h_{rl} - t h_{ek}^* h_{el})$ and q_l . Since (C.7.29) is a quadratic function, a simple curve discussion reveals that the optimum in the interval $[0, \sqrt{\hat{P}_l}]$ is as follows: (i) If $|h_{rl}|^2 \geq t |h_{el}|^2$ then $|q_l^*| = \sqrt{\hat{P}_l}$, (ii) If $|h_{rl}|^2 < t |h_{el}|^2$ then

$$|q_l^*| = \min \left\{ \sqrt{\hat{P}_l}, \frac{\left| \sum_{\substack{k \in \mathcal{I} \\ k \neq l}} q_k (h_{rk}^* h_{rl} - t h_{ek}^* h_{el}) \right| \cos(\alpha)}{|h_{rl}|^2 - t |h_{el}|^2} \right\}.$$

This proves Theorem C.10. \square

Transmit Beamforming for Single-user Large-Scale MISO Systems with Sub-connected Architecture and Power Constraints

Phuong Le Cao, Tobias J. Oechtering and Mikael Skoglund

Revised version of refereed article published in
IEEE Communication Letter, vol. 22, no. 10, pp. 2096 - 2099, Oct 2018.

©2018 IEEE

Layout has been revised for thesis consistency

Transmit Beamforming for Single-user Large-Scale MISO Systems with Sub-connected Architecture and Power Constraints

Phuong Le Cao, Tobias J. Oechtering and Mikael Skoglund

Abstract

This letter considers optimal transmit beamforming for a sub-connected large-scale MISO system with per-antenna power constraints. The system is configured such that each RF chain serves a group of antennas. Two different methods are considered, fully digital beamforming and hybrid beamforming. For the hybrid scheme, necessary and sufficient conditions to design the optimal digital and analog precoders are provided. It is shown that, in the optimum, the optimal phase shift at each antenna has to match the phase of the channel coefficient and the phase of the digital precoder. In addition, an iterative algorithm is provided to find the optimal power allocation. We study the case where the power constraint on the RF chain is smaller than the sum of the corresponding per-antenna power constraints. Then, the optimal power is allocated based on two properties: Each RF chain uses full power and if the optimal power allocation of the unconstrained problem violates a per-antenna power constraint then it is optimal to allocate the maximal power for that antenna. Finally, numerical examples are provided that illustrate the theoretical results.

D.1 Introduction

In recent years, large-scale multiple-input multiple-output (massive MIMO) wireless communication has received much attention due to its envisioned application in 5G wireless systems. Massive MIMO is to use a very large number of antennas to enhance the spectral efficiency significantly [Mar10, LLS⁺14] and therewith also compensate for the spectral efficiency loss due to the use of higher frequencies (mmWave), which allows hardware systems to reduce the antennas' size and therewith the radiated energy [AALH14, SY16, GDH⁺16, LXD14]. However, massive MIMO and mmWave configurations might cause high hardware cost and large power consumption when each antenna is equipped with a separate RF chain. This problem can be mitigated by using the hybrid analog-digital precoding strategy [AALH14, GDH⁺16, SY16].

For the hybrid precoding, we distinguish between two configurations of hardware, namely *fully-connected* and *sub-connected* large scale antenna systems [SY16, GBKS15, ZHL⁺17, LXD14, LWY⁺17]. In the fully-connected architecture [SY16, GBKS15, ZHL⁺17], each antenna is connected to all RF chains through analog

phase shifters and adders, i.e., each analog precoder output is a combination of all RF signals. One of the biggest drawbacks of this architecture is the requirement of a large number of RF adders and phase shifters which results in both high hardware cost and power consumption. In contrast, a sub-connected architecture has a reduced complexity, where a subset of transmit antennas is connected to one RF chain only. Since this sub-connected architecture requires no adder and less phase shifters, it is less expensive to implement than the fully-connected one but results in less freedom for signalling. Previous studies of transmit strategies for sub-connected architectures have been done in [LXD14, LWY⁺17]. However, these works assume a sum power constraint only. Since each RF chain has a physical limitation, it is reasonable to impose a power constraint on each RF chain. Furthermore, since each RF chain serves more than one antenna, we additionally consider to use power dividers to split the output powers between the antennas. Since it will be optimal to use the maximal power per RF chain and since the splitting ratio of the power divider has a physical limitation as well, we also include a power constraint on each antenna to limit the energy per antenna.

Previous studies of transmit strategies for sub-connected architectures have been done in [LXD14, LWY⁺17]. However, these works assume a sum power constraint only. Since the RF chain has a physical limitation, it is reasonable to impose a power constraint on each RF chain. Furthermore, since each RF chain serves more than one antenna, power dividers are used to split the output powers to its connected antennas. Since it will be optimal to use the maximal power per RF chain and since the splitting ratio of the power divider has a physical limitation, it is reasonable to apply a power constraint on each antenna to limit the energy per-antenna.

Previous works studied optimal transmit strategies for the MISO channel with per-antenna power constraints [Vu11a, MDT14] and joint sum and per-antenna power constraints [COSS16]. However, the problem has so far not been studied for sub-connected architectures. In this letter, we focus on studying the optimal transmit strategy for a single-user large-scale MISO system with sub-connected architecture, per RF chain and per-antenna power constraints. Necessary and sufficient conditions to design the optimal digital and analog precoders for the hybrid beamforming are provided. The hybrid beamforming scheme is considered when the number of RF chains is strictly smaller than the number of antennas.

Notation

We use bold lower-case letters for vectors, bold capital letters for matrices. The superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$ stand for transpose, conjugate, and conjugate transpose. We use $\text{diag}\{\cdot\}$ and $\text{BlockDiag}\{\cdot\}$ for diagonal and block diagonal matrices. \mathbb{N} , \mathbb{R}_+ , and \mathbb{C} are the sets of non-negative integers, non-negative real, and complex numbers.

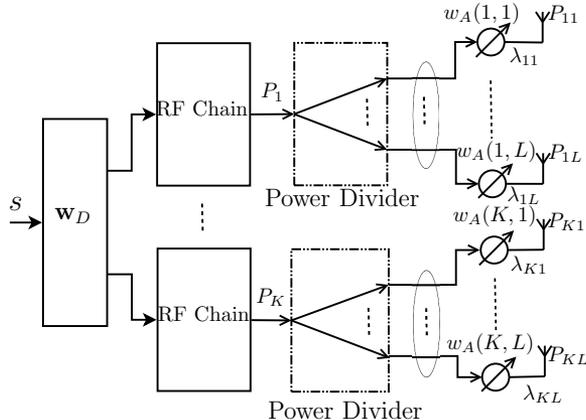


Figure D.1: Transmitter architecture for large-scale antenna system with sub-connected architecture and power constraints.

D.2 System Model

We consider a sub-connected architecture of a single-user large-scale MISO system as depicted in Figure D.1. The transmitter is equipped with K RF chains and M antennas such that each RF chain is connected to a group of L antennas, i.e., $M = KL$. RF chains are indexed by $k \in \mathcal{K} = \{1, \dots, K\}$ and antennas connecting to each RF chain are indexed by $l \in \mathcal{L} = \{1, \dots, L\}$. The transmit data $\mathbf{s} \in \mathbb{C}^{N \times 1}$, where N is number of data stream and $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_N$, is precoded by applying a baseband processing (digital precoder) $\mathbf{W}_D \in \mathbb{C}^{K \times N}$ followed by adjustable power dividers and analog phase shifters. In the hardware setup in Figure D.1, adjustable power dividers as in [AA16] are used to distribute the power from the RF chains to the corresponding transmit antennas. To control the power allocation for each antenna, a block diagonal matrix $\mathbf{\Lambda} \in \mathbb{R}_+^{M \times K}$ is introduced. It is defined as

$$\mathbf{\Lambda} = \text{BlockDiag}\{\mathbf{\Lambda}_1, \dots, \mathbf{\Lambda}_K\} \quad (\text{D.2.1})$$

with $\mathbf{\Lambda}_k = [\lambda_{k1}, \dots, \lambda_{kL}]^T \in \mathbb{R}_+^{L \times 1}$ and $\lambda_{kl} \geq 0, \forall k, l$. Since λ_{kl}^2 denotes the power fraction transmitted from the l -th antenna, $\sum_{l=1}^L \lambda_{kl}^2 = 1$.

Additionally, a diagonal matrix describing analog phase shifters (analog precoder) $\mathbf{W}_A \in \mathbb{C}^{M \times M}$ is used to adjust the phase for each individual antenna. The analog precoder can be written as

$$\mathbf{W}_A = \text{diag}\{\mathbf{w}_A(1), \dots, \mathbf{w}_A(K)\} \quad (\text{D.2.2})$$

with complex phase shift diagonal matrix $\mathbf{w}_A(k) = \text{diag}\{w_A(k, 1), \dots, w_A(k, L)\} \in \mathbb{C}^{L \times L}$ and $|w_A(k, l)|^2 = 1, \forall k, l$.

The channel coefficient vector denoted as $\mathbf{h} = [\mathbf{h}_1^T, \dots, \mathbf{h}_K^T]^T \in \mathbb{C}^{M \times 1}$ with $\mathbf{h}_k = [h_{k1}, \dots, h_{kL}]^T$, $k \in \mathcal{K}$, is known at both transmitter and receiver. Without loss of generality, we assume that $|h_{kl}| > 0$, $\forall k \in \mathcal{K}, \forall l \in \mathcal{L}$. Then, the received signal can be written as

$$y = \mathbf{h}^H \mathbf{W}_A \mathbf{\Lambda} \mathbf{W}_D \mathbf{s} + z, \quad (\text{D.2.3})$$

where $z \sim \mathcal{CN}(0, 1)$ is additive white Gaussian noise.

We consider individual power constraints at each transmit antenna \tilde{P}_{kl} and power constraints at each RF chain \hat{P}_k , $\forall k \in \mathcal{K}, \forall l \in \mathcal{L}$. If $\hat{P}_k \geq \sum_{l=1}^L \tilde{P}_{kl}$, $\forall k \in \mathcal{K}$, then we face the *per-antenna power constraints only* problem. If $\hat{P}_k \leq \sum_{l=1}^L \tilde{P}_{kl}$ for a certain $k \in \mathcal{K}$, i.e., the transmit power on the k -th RF chain is more restricted than the total transmit power on antennas connecting to the k -th RF chain, we face the optimization problem where both sum and per-antenna power constraints can be active. In this work, we focus on the latter case only. Solutions to the other problems follow straight forwardly from this solution.

We are interested in finding the optimal precoding matrices \mathbf{W}_A , \mathbf{W}_D and the optimal power allocation matrix $\mathbf{\Lambda}$ that achieve the capacity of the point-to-point MISO channel (D.2.3). This is the standard problem of finding the optimal covariance matrix of the zero mean Gaussian distributed input, but here with a certain covariance matrix structure $\mathbf{W}_A \mathbf{\Lambda} \mathbf{W}_D \mathbf{W}_D^H \mathbf{\Lambda} \mathbf{W}_A^H$ reflecting the hardware design. Thus, the optimization problem is given as follows

$$\max_{\mathbf{W}_A, \mathbf{W}_D, \mathbf{\Lambda}} \log(1 + \mathbf{h}^H \mathbf{W}_A \mathbf{\Lambda} \mathbf{W}_D \mathbf{W}_D^H \mathbf{\Lambda} \mathbf{W}_A^H \mathbf{h}) \quad (\text{D.2.4})$$

$$\text{s. t.} \quad \forall k, l : \mathbf{e}_{kl}^T \mathbf{W}_A \mathbf{\Lambda} \mathbf{W}_D \mathbf{W}_D^H \mathbf{\Lambda} \mathbf{W}_A^H \mathbf{e}_{kl} \leq \tilde{P}_{kl}, \quad (\text{D.2.4a})$$

$$\forall k : \sum_{l=1}^L \mathbf{e}_{kl}^T \mathbf{W}_A \mathbf{\Lambda} \mathbf{W}_D \mathbf{W}_D^H \mathbf{\Lambda} \mathbf{W}_A^H \mathbf{e}_{kl} = P_k \leq \hat{P}_k, \quad (\text{D.2.4b})$$

$$\forall k, l : |w_A(k, l)|^2 = 1, \quad (\text{D.2.4c})$$

where (D.2.4a), (D.2.4b), and (D.2.4c) are the per-antenna, RF chain, and phase shifter constraints. $\mathbf{e}_{kl} \in \mathbb{R}^{KL \times 1}$ is a Cartesian unit vector with a one at $((k-1)L + l)$ -th position and zeros elsewhere. Since $\log(1 + \mathbf{h}^H \mathbf{W}_A \mathbf{\Lambda} \mathbf{W}_D \mathbf{W}_D^H \mathbf{\Lambda} \mathbf{W}_A^H \mathbf{h})$ is an increasing function in $\mathbf{h}^H \mathbf{W}_A \mathbf{\Lambda} \mathbf{W}_D \mathbf{W}_D^H \mathbf{\Lambda} \mathbf{W}_A^H \mathbf{h}$, we can equivalently focus on the optimization problem to find an optimal $\mathbf{W}_A \mathbf{\Lambda} \mathbf{W}_D$ for the objective function $|\mathbf{h}^H \mathbf{W}_A \mathbf{\Lambda} \mathbf{W}_D|^2$ instead of (D.2.4).

Note that (D.2.4) can be formed as a convex optimization problem by merging \mathbf{W}_A and $\mathbf{\Lambda}$ together.

D.3 Transmit beamforming design

If $L = 1$, then we have fully digital precoding where every antenna has its own RF chain. In this case $\check{P}_i = \min\{\tilde{P}_i, \hat{P}_i\}$, $\forall i \in \{1, \dots, K = M\}$ gives the per-antenna

power constraint. Then, the optimization problem reduces to

$$\begin{aligned} \max_{\mathbf{\Lambda}, \mathbf{W}_D} \quad & \mathbf{h}^H \mathbf{\Lambda} \mathbf{W}_D \mathbf{W}_D^H \mathbf{\Lambda} \mathbf{h}, \\ \text{s. t.} \quad & \mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i \leq \check{P}_i, \quad \forall i \in \{1, \dots, M\}. \end{aligned} \quad (\text{D.3.1})$$

Following [Vu11a], the optimal solution of the optimization problem (D.3.1) is rank one for $L = 1$, i.e., $\mathbf{W}_D = \mathbf{w}_D$ with elements $w_i = \sqrt{\check{P}_i} \frac{h_i^*}{|h_i|}$, $\forall i \in \{1, \dots, M\}$ and $\mathbf{\Lambda} = \mathbf{I}_M$.

In the following, we study the hybrid beamforming for the case where the number of RF chains is strictly smaller than the number of antennas, i.e., $K < M$ and $L > 1$. The digital precoder \mathbf{W}_D is designed under the assumption that an analog precoder \mathbf{W}_A and a power allocation matrix $\mathbf{\Lambda}$ are given. For a given analog precoding \mathbf{W}_A and a power allocation matrix $\mathbf{\Lambda}$, an equivalent channel \mathbf{g} can be formulated as $\mathbf{g} = \mathbf{\Lambda} \mathbf{W}_A^H \mathbf{h}$. Then the convex optimization problem to find the digital precoder can be written as

$$\begin{aligned} \max_{\mathbf{w}_D} \quad & \mathbf{g}^H \mathbf{W}_D \mathbf{W}_D^H \mathbf{g} \\ \text{s. t.} \quad & (\text{D.2.4a}), (\text{D.2.4b}). \end{aligned} \quad (\text{D.3.2})$$

Following Proposition A.2, we can conclude that the optimal digital precoder \mathbf{W}_D also has rank one, i.e., beamforming is optimal. Therefore, it is sufficient to consider a digital precoder that can be denoted as $\mathbf{W}_D = \mathbf{w}_D \in \mathbb{C}^{K \times 1}$. Moreover, it means that it is optimal to have only one data stream, i.e., $N = 1$, which we will assume in the following.

In hybrid precoding, the analog precoder controls the phase for each antenna. Since it is sufficient to consider a digital precoder of rank one, the phase of the digital precoder can be merged with the analog precoder or simply chosen to be equal to zero. Because of this we assume, without loss of generality, that $\mathbf{w}_D \in \mathbb{R}_+^{K \times 1}$ in the following.

Next, we will derive the optimal analog precoder, the characterization of the amplitude of the optimal digital precoder and optimal power allocation.

D.3.1 Analog precoder

By assuming that a digital precoder $\mathbf{w}_D \in \mathbb{R}_+^{K \times 1}$ and the power allocation $\mathbf{\Lambda}$ are given, we can obtain a necessary condition for the optimal analog precoder \mathbf{W}_A^* by solving the following optimization problem

$$\begin{aligned} \max_{\mathbf{W}_A} \quad & \mathbf{h}^H \mathbf{W}_A \mathbf{\Lambda} \mathbf{w}_D \mathbf{w}_D^H \mathbf{\Lambda} \mathbf{W}_A^H \mathbf{h} \\ \text{s. t.} \quad & (\text{D.2.4a}), (\text{D.2.4c}). \end{aligned} \quad (\text{D.3.3})$$

Proposition D.1. *Let $e^{i\angle h_{kl}} = \frac{h_{kl}}{|h_{kl}|} \quad \forall k, l$. Then the optimal analog precoder has elements given as*

$$w_A^*(k, l) = e^{-i\angle h_{kl}}, \quad \forall k \in \mathcal{K}, \forall l \in \mathcal{L}. \quad (\text{D.3.4})$$

Proof. Given a power allocation $\mathbf{\Lambda}$ and a digital precoder \mathbf{w}_D , then we have

$$\begin{aligned}
& \max_{|w_A(k,l)|^2=1 \forall k,l} |\mathbf{h}^H \mathbf{W}_A \mathbf{\Lambda} \mathbf{w}_D|^2 \\
&= \max_{|w_A(k,l)|^2=1 \forall k,l} \left| \sum_{k=1}^K \sum_{l=1}^L |h_{kl}| e^{i\angle h_{kl}} w_A(k,l) \lambda_{kl} w_D(k) \right|^2 \\
&\leq \left(\sum_{k=1}^K \sum_{l=1}^L |h_{kl}| \lambda_{kl} \sqrt{P_k} \right)^2. \tag{D.3.5}
\end{aligned}$$

The upper bound (D.3.5) is achieved if $w_A(k,l) = e^{-i\angle h_{kl}}$, $\forall k \in \mathcal{K}, \forall l \in \mathcal{L}$. This proves Proposition D.1. \square

Proposition D.1 shows that it is optimal to match the phase at each antenna to the channel coefficient. Therefore, it is sufficient to design the optimal analog precoder by aligning phases of phase shifters to the channel coefficient such that the signal coherently adds up at the receiver.

In the next part, we derive the amplitude of the optimal digital precoder coefficient and the optimal power allocation matrix $\mathbf{\Lambda}$ under the assumption $\hat{P}_k \leq \sum_{l=1}^L \tilde{P}_{kl}$, $\forall k$.

D.3.2 Power allocation

The following proposition shows that it is optimal for the digital precoder to transmit with maximum power on all RF chains.

Proposition D.2. *For the case where $\hat{P}_k \leq \sum_{l=1}^L \tilde{P}_{kl}$ for all k , the optimal solution of (D.3.2) allocates full power on all RF chains, i.e., $\sum_{l=1}^L P_{kl}^* = \hat{P}_k$, $\forall k$.*

Proof. Let $\mathbf{q} = \mathbf{W}_A \mathbf{\Lambda} \mathbf{w}_D$, $\mathcal{Q} := \{\mathbf{q} : \mathbf{e}_{kl}^T \mathbf{q} \mathbf{q}^H \mathbf{e}_{kl} \leq \tilde{P}_{kl}, \sum_{l=1}^L \mathbf{e}_{kl}^T \mathbf{q} \mathbf{q}^H \mathbf{e}_{kl} \leq \hat{P}_k \forall k, l\}$. Suppose there exists an *optimal* \mathbf{q}^* such that there exists a $\bar{k} \in \mathcal{K}$ for which $\mathbf{e}_{\bar{k}l}^T \mathbf{q}^* \mathbf{q}^{*H} \mathbf{e}_{\bar{k}l} = P_{\bar{k}l}^*$ and $\sum_{l=1}^L P_{\bar{k}l}^* < \hat{P}_{\bar{k}}$, then the maximum value of (D.3.2) can be calculated as

$$\begin{aligned}
f^* &= \max_{\mathbf{q} \in \mathcal{Q}} |\mathbf{h}^H \mathbf{q}|^2 = \max_{\mathbf{q} \in \mathcal{Q}} \left| \sum_{\substack{k=1, \\ k \neq \bar{k}}}^K \sum_{l=1}^L h_{kl} q_{kl} + \sum_{l=1}^L h_{\bar{k}l} q_{\bar{k}l} \right|^2 \\
&= \left(\sum_{\substack{k=1, \\ k \neq \bar{k}}}^K \sum_{l=1}^L |h_{kl}| \sqrt{P_{kl}^*} + \sum_{l=1}^L |h_{\bar{k}l}| \sqrt{P_{\bar{k}l}^*} \right)^2 = \left(\sum_{\substack{k=1, \\ k \neq \bar{k}}}^K f_k^* + f_{\bar{k}}^* \right)^2. \tag{D.3.6}
\end{aligned}$$

Since $\sum_{l=1}^L P_{kl}^* = P_k^* < \hat{P}_k \leq \sum_{l=1}^L \tilde{P}_{kl}$, there exists a j and a P_{kj} with $P_{kj}^* < P_{kj} \leq \tilde{P}_{kj}$ and $\hat{P}_k - P_{kj} \geq \sum_{\substack{l=1 \\ l \neq j}}^L P_{kl}^*$, so that $f_k^l = \sum_{\substack{l=1 \\ l \neq j}}^L |h_{kl}| \sqrt{P_{kl}^*} + |h_{kj}| \sqrt{P_{kj}} > f_k^*$. It follows that $f' = \left(\sum_{\substack{k=1, \\ k \neq \bar{k}}}^K f_k^* + f_k^l \right)^2 > \left(\sum_{\substack{k=1, \\ k \neq \bar{k}}}^K f_k^* + f_k^* \right)^2 = f^*$. This contradicts with the optimality of (D.3.6). This implies that the optimal solution of (D.3.2) must meet all RF chain power constraints with equality, i.e., $\sum_{l=1}^L P_{kl}^* = P_k^* = \hat{P}_k \forall k$. \square

Proposition D.2 implies that it is sufficient for the optimization to consider only transmit strategies which allocate full power on all RF chains, i.e., the RF chain power constraints are always active. Accordingly, $w_D(k) = \sqrt{\hat{P}_k}$ is optimal.

Next, the optimal power allocation $\mathbf{\Lambda}^* \in \mathbb{R}_+^{M \times K}$ is designed under the assumption that the optimal digital and analog precoders are given, i.e., $w_D^*(k) = \sqrt{\hat{P}_k} \forall k$ and $|w_A^*(k, l)|^2 = 1$. Let $P_{kl} = |\lambda_{kl} w_A^*(k, l) w_D^*(k)|^2 = \hat{P}_k \lambda_{kl}^2 \forall k, l$. Then we have $\lambda_{kl} = \sqrt{\frac{P_{kl}}{\hat{P}_k}} \forall k, l$, or equivalently

$$\mathbf{\Lambda}_k = \frac{1}{\sqrt{\hat{P}_k}} \left[\sqrt{P_{k1}}, \dots, \sqrt{P_{kL}} \right]^T \quad \forall k. \quad (\text{D.3.7})$$

From the proof of Proposition D.2 we can easily see that the power allocation for one RF chain is independent from the allocation at all other RF chains. Thus, the problem reduces to the problem to find the optimal power allocation for one RF chain. For a given $k \in \mathcal{K}$, the optimal allocated powers $P_{kl}^*, \forall l$ can be obtained by solving the following problem

$$\begin{aligned} & \max_{P_{kl}, \forall l} && \sum_{l=1}^L |h_{kl}| \sqrt{P_{kl}} && (\text{D.3.8}) \\ & \text{s. t.} && \forall l : P_{kl} \leq \tilde{P}_{kl}, \sum_{l=1}^L P_{kl} \leq \hat{P}_k. \end{aligned}$$

This problem is exactly the same as the optimization problem to find the optimal transmit strategy for MISO channels with joint sum and per-antenna power constraints in Paper A. Therefore, the solution of (D.3.8) can be approached by utilizing the solutions of the sum power constraint only and per-antenna power constraints only problems as done in Paper A. In accordance to that we need the optimal power allocation on a group of antennas connecting to one RF chain $k \in \mathcal{K}$ without per-antenna power constraints, which is given by a waterfilling solution [TV05]:

$$P_{kl}^{WF} = \left(\frac{1}{\omega_k} - \frac{1}{|h_{kl}|^2} \right)^+, \forall l \quad (\text{D.3.9})$$

where ω_k satisfies $\sum_{l=1}^L \left(\frac{1}{\omega_k} - \frac{1}{|h_{kl}|^2} \right)^+ = \hat{P}_k$, for any $k \in \mathcal{K}$.

The optimal powers P_{kl}^{WF} , however, may violate the per-antenna power constraints \tilde{P}_{kl} for some k, l . In this case, it is optimal to set those equal to the per-antenna power constraints \tilde{P}_{kl} . The remaining power allocations can then be obtained by solving a reduced optimization problem with a smaller total RF chain power, i.e., $\hat{P}_k - \sum_{l \in \{l \in \mathcal{L} : P_{kl}^{WF} \geq \tilde{P}_{kl}\}} \tilde{P}_{kl}$. The justification of this approach is in Theorem A.5 and reformulated for the considered problem here in the following corollary.

Corollary D.3. *For a given $k \in \mathcal{K}$, let $\mathcal{P}_k := \{l \in \mathcal{L} : P_{kl}^{WF} \geq \tilde{P}_{kl}\}$ and $\mathcal{P} = \bigcup_{k=1}^K \mathcal{P}_k$. If $\mathcal{P} = \emptyset$ then $P_{kl}^* = P_{kl}^{WF} \forall l$, else $P_{kl}^* = \tilde{P}_{kl} \forall l \in \mathcal{P}_k$, and the remaining optimal powers can be computed by solving the reduced optimization problem*

$$\begin{aligned} & \max_{P_{kl} \forall l \in \mathcal{P}_k^c} && \sum_{l \in \mathcal{P}_k^c} |h_{kl}| \sqrt{P_{kl}} && \text{(D.3.10)} \\ & \text{s. t.} && \forall l \in \mathcal{P}_k^c : P_{kl} \leq \tilde{P}_{kl}, \sum_{l \in \mathcal{P}_k^c} P_{kl} \leq \hat{P}_k - \sum_{l \in \mathcal{P}_k} \tilde{P}_{kl}, \end{aligned}$$

where $\mathcal{P}_k^c = \mathcal{L} \setminus \mathcal{P}_k$.

If the waterfilling solution of the reduced optimization problem again violates a per-antenna power constraint, then Corollary D.3 has to be applied again until the waterfilling solution of the reduced optimization problem does not violate any per-antenna power constraints.

We have summarized the approach above to compute the optimal power allocation matrix $\mathbf{\Lambda}^*$ in Algorithm D.3.1.

D.4 Numerical results

In this section, we consider a large-scale MISO system with different transmit antenna configurations. We assume that channels are constant over i.i.d blocks and there is i.i.d between antennas. The results therefore are directly relevant for systems with CSIT and time constant channels. We first evaluate the transmission rate for the case that the number of RF chains and the number of antennas are the same, i.e., fully digital beamforming is used. The system is considered with settings of 16 and 128 pairs of RF chains and transmit antennas respectively. In these settings, the per-antenna power constraints and the RF chain power constraints are the same. Next, we investigate a hybrid beamforming scheme that is configured with $M = 128$ transmit antennas and $K = 16$ RF chains. Each RF chain is designed to serve a group of $L = 8$ antennas. The per-antenna power constraint on each antenna is $\tilde{P}_{kl} = 3$. Curves in Figure D.2 are plotted by gradually increasing \hat{P}_k from $\hat{P}_k = 1$ to $\hat{P}_k = 40$.

We can see from the figure that for the hybrid beamforming, if a RF chain power constraint is more restrictive than the sum of all individual powers of the group of

Algorithm D.3.1: Optimal power allocation matrix

```

1 Compute optimal power allocation  $P_{kl}^{WF}$  using (D.3.9)
2 Denote  $\mathcal{P}_k := \{l \in \mathcal{L} : P_{kl}^{WF} \geq \hat{P}_{kl}\} \forall k$ 
3 Denote  $\mathcal{P} := \bigcup_{k=1}^K \mathcal{P}_k$ 
4 if  $\mathcal{P} = \emptyset$  then
5   |  $P_{kl}^* \leftarrow P_{kl}^{WF} \forall k, l$ 
6   | Go to 16
7 else
8   | for  $k \in \mathcal{K}$  do
9     |   | for  $l \in \mathcal{P}_k$  do
10      |   |   |  $P_{kl}^* \leftarrow \tilde{P}_{kl}$ 
11      |   |   | end for
12      |   |   |  $\mathcal{L} \leftarrow \mathcal{L} \setminus \mathcal{P}_k, \hat{P}_k \leftarrow \hat{P}_k - \sum_{l \in \mathcal{P}_k} \tilde{P}_{kl}$ 
13      |   |   | end for
14   | end if
15 Return to 1.
16 Form  $\Lambda_k^*$  (as in (D.3.7)) and  $\Lambda^*$  with optimal power  $P_{kl}^*$ .
```

antennas connected to that RF chain, i.e., $\hat{P}_k \leq \sum_{l=1}^8 \tilde{P}_{kl} = 24$ (operating point A), then it is optimal to transmit with the maximal per RF chain power \hat{P}_k . After this value the RF power constraint is never active and it is optimal to transmit with the maximal individual power $\tilde{P}_{kl} = 3$ on all antennas.

Next, we compare operating point A of the hybrid beamforming scheme with operating points B and C of fully-digital beamforming schemes that both allocate the same total transmit power. We observe that: (i) By using the same number of RF chains while increasing the number of antennas, we can obtain a significantly higher transmission rate. (ii) With a smaller number of RF chains and the same number of antennas, we can achieve the same transmission rate as the one with fully digital beamforming.

D.5 Conclusions

In this letter, we provide necessary and sufficient conditions to design the optimal beamforming strategies for a single-user large-scale MISO system with a sub-connected architecture, RF chain and per-antenna power constraints. For the digital precoder, the phase can be picked equals zero since the optimal phase shift can be included in the analog precoder. Also, it turns out that it is necessary and sufficient for the optimality to consider rank one precoder only, i.e., beamforming, which allocates maximal RF chain power. Further, we provided an algorithm that allows to compute the optimal power allocation in closed-form. The numerical results illus-

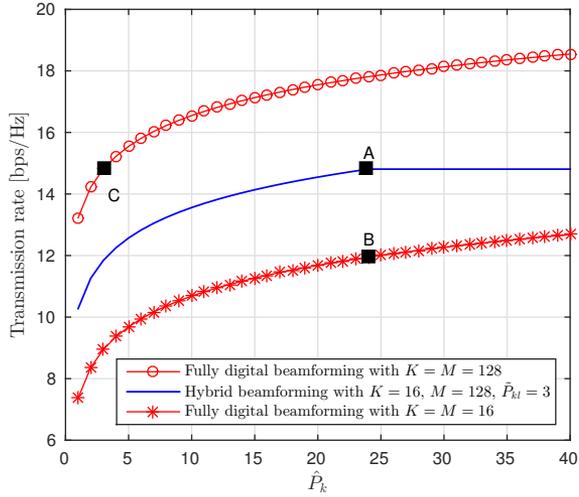


Figure D.2: Transmission rate of the large-scale antenna system with different RF chains and antennas configurations.

trate that, compared to fully digital beamforming, a lower cost hybrid setup with a lower number of RF chains and the same number of antennas can achieve almost the same transmission rate.

Precoding Design for Massive MIMO Systems with Sub-connected Architecture and Per-antenna Power Constraints

Phuong Le Cao, Tobias J. Oechtering and Mikael Skoglund

Revised version of refereed paper published in
The 22nd International ITG Workshop on Smart Antennas (WSA), Mar. 2018.

©2018 VDE

Layout has been revised for thesis consistency

Precoding Design for Massive MIMO Systems with Sub-connected Architecture and Per-antenna Power Constraints

Phuong Le Cao, Tobias J. Oechtering and Mikael Skoglund

Abstract

This paper provides the necessary conditions to design precoding matrices for massive MIMO systems with a sub-connected architecture, RF power constraints and per-antenna power constraints. The system is configured such that each RF chain serves a group of antennas. The necessary condition to design the digital precoder is established based on a generalized water-filling and joint sum and per-antenna optimal power allocation solution, while the analog precoder is based on a per-antenna power allocation solution only. We study the analytically most interesting case where the power constraint on the RF chain is smaller than the sum of the corresponding per-antenna power constraints. For this, the optimal power is allocated based on two properties: Each RF chain uses full power and if the optimal power allocation of the unconstrained problem violates a per-antenna power constraint then it is optimal to allocate the maximal power for that antenna.

E.1 Introduction

In recent years, large-scale multiple-input multiple-output (massive MIMO) for wireless communications has received much attention due to its envisioned application in 5G wireless systems. The motivation of massive MIMO is to use a very large number of antennas to enhance the spectral efficiency significantly [Mar10, LLS⁺14, ML13]. This becomes necessary and possible since at higher frequency (mmWave) the antennas' sizes reduce and therewith also the radiated energy per-antenna [LVGPRH16, SY16].

The massive increase of antennas leads to new technological challenges from a transceiver hardware perspective. We can distinguish between two configurations of large-scale antenna systems namely fully-connected and sub-connected [SY16, GBKS15, GDH⁺16, LWY⁺17]. In the fully-connected architecture, each antenna is connected to all RF chains through analog phase shifters and adders, i.e., each analog precoder output is a combination of all RF signals. One of the biggest drawbacks of this architecture is the requirement of a large number of RF adders and phase shifters, which results in both high hardware costs and power consumption. Different from the fully-connected architecture, a sub-connected architecture has a reduced complexity, where each RF chain is connected to a subset of transmit antennas only. Since this sub-connected architecture requires no adder and fewer

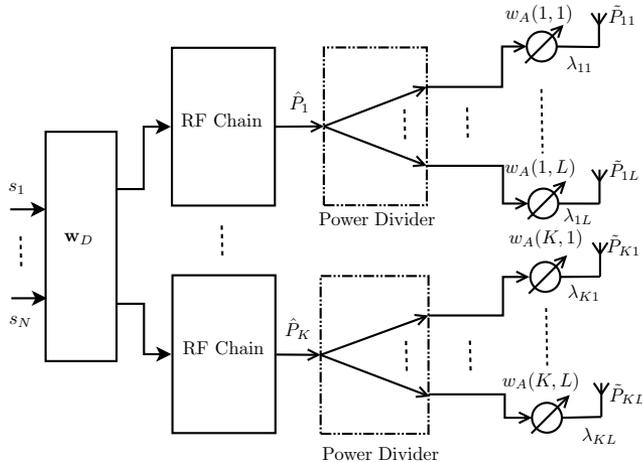


Figure E.1: Sub-connected architecture for Massive MIMO with RF chain and per-antenna power constraints.

phase shifters, it is less expensive to implement than the fully-connected one but results in less freedom for the signalling. Previous studies of transmit strategies for the sub-connected architecture have been done in [LXD14, LWY⁺17]. However, these works assume a sum power constraint only. Since the RF chain impose physical limitations on the transmitter, it is reasonable to impose a power constraint on each RF chain and possibly also on each antenna to limit the average power. Previous works studied optimal transmit strategies for MISO and MIMO channels with per-antenna power constraints [Vu11a, Vu11b, YL07, MDT14, SSB08, COS16, Tun14, BH06, CTJLa07] and joint sum and per-antenna power constraints [COSS16, Loy16, CO17b]. However, the problem has so far not been studied for sub-connected architectures. In this paper, we focus on studying the analog and digital precoders for a massive MIMO system with a sub-connected architecture, RF power constraints and per-antenna power constraints. The single-user large-scale MISO system with a sub-connected architecture, RF power constraints and per-antenna power constraints has been consider in [COS18a].

Notation

We use bold lower-case letters for vectors, bold capital letters for matrices. The superscripts \$(\cdot)^T\$, \$(\cdot)^*\$ and \$(\cdot)^H\$ stand for transpose, conjugate, and conjugate transpose. We use \$\text{diag}\{\cdot\}\$ and \$\text{BlockDiag}\{\cdot\}\$ for diagonal and block diagonal matrices. \$\mathbb{N}\$, \$\mathbb{R}_+\$, and \$\mathbb{C}\$ are the sets of non-negative integers, non-negative real, and complex numbers.

E.2 System model

We consider a sub-connected architecture of a massive MIMO system as depicted in Figure E.1. The transmitter is equipped with K RF chains and M_t transmit antennas such that each RF chain is connected to a group of L antennas, i.e., $M_t = KL$. The receiver is equipped with M_r antennas. RF chains are indexed by $k \in \mathcal{K} = \{1, \dots, K\}$ and antennas connecting to each RF chain are indexed by $l \in \mathcal{L} = \{1, \dots, L\}$. The transmit data $\mathbf{s} \in \mathbb{C}^{N \times 1}$, where N is number of data stream and $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_N$, is precoded by applying baseband processing (digital precoder) $\mathbf{W}_D \in \mathbb{C}^{K \times N}$ followed by a power allocation matrix $\mathbf{\Lambda} \in \mathbb{C}^{KL \times KL}$ and a phase array (analog precoder) $\mathbf{W}_A \in \mathbb{C}^{KL \times KL}$. Due to the architecture, the analog precoder is described by a block diagonal matrix $\mathbf{W}_A = \text{BlockDiag}\{\mathbf{w}_A(1), \dots, \mathbf{w}_A(K)\}$, $\mathbf{w}_A(k) = \text{diag}\{w_A(k, 1), \dots, w_A(k, L)\}$ with complex phase shift constraints $|w_A(k, l)|^2 = 1, \forall k, l$. $\mathbf{\Lambda}$ is a block diagonal matrix to adjust the power allocation and is defined as

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{\Lambda}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{\Lambda}_K \end{bmatrix} \in \mathbb{C}^{KL \times KL}$$

with $\mathbf{\Lambda}_k = [\lambda_{k1}, \dots, \lambda_{kL}]^T \in \mathbb{C}^{L \times 1}, \forall k, l$.

The channel coefficient matrix is denoted as $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ and is known at both transmitter and receiver. Then, the received signal can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{W}_A\tilde{\mathbf{W}}_D\mathbf{s} + \mathbf{z}, \quad (\text{E.2.1})$$

where $\tilde{\mathbf{W}}_D = \mathbf{\Lambda}\mathbf{W}_D \in \mathbb{C}^{M_t \times N}$ is the digital precoder matrix and $\mathbf{z} \sim \mathcal{CN}(0, \mathbf{I})$ is additive white Gaussian noise.

We consider individual power constraints at each transmit antenna \tilde{P}_{kl} and at each RF chain $\hat{P}_k, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}$. If $\hat{P}_k \geq \sum_{l=1}^L \tilde{P}_{kl}, \forall k \in \mathcal{K}$, then we face the per-antenna power constraints only problem. If $\hat{P}_k \leq \sum_{l=1}^L \tilde{P}_{kl}, \forall k \in \mathcal{K}$, i.e., the transmit power on a certain RF chain is more restricted than the total transmit power on antennas connecting to that RF chain, we face the optimization problem where both sum and per-antenna power constraints are active. In this work, we focus on the later case only. Solutions to the other one follow straight forwardly. We are interested in finding the optimal precoding matrices \mathbf{W}_A and $\tilde{\mathbf{W}}_D$ that achieve the capacity of the point-to-point MIMO channel (E.2.1). This is the standard problem of finding the optimal covariance matrix of the zero mean Gaussian distributed input but here with covariance matrix structure $\mathbf{W}_A\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\mathbf{W}_A^H$ reflecting the hardware design. Thus, the optimization problem is given as follows

$$\max_{\tilde{\mathbf{W}}_D, \mathbf{W}_A} f(\mathbf{W}_A, \tilde{\mathbf{W}}_D) \quad (\text{E.2.2})$$

$$\text{s. t.} \quad \mathbf{e}_{kl}^T \mathbf{W}_A \tilde{\mathbf{W}}_D \tilde{\mathbf{W}}_D^H \mathbf{W}_A^H \mathbf{e}_{kl} \leq \tilde{P}_{kl}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}, \quad (2a)$$

$$\sum_{l=1}^L \mathbf{e}_{kl}^T \mathbf{W}_A \tilde{\mathbf{W}}_D \tilde{\mathbf{W}}_D^H \mathbf{W}_A^H \mathbf{e}_{kl} \leq \hat{P}_k, \forall k \in \mathcal{K}, \quad (2b)$$

$$|w_A(k, l)|^2 = 1, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}, \quad (2c)$$

where $f(\mathbf{W}_A, \tilde{\mathbf{W}}_D) = \log |\mathbf{I} + \mathbf{H} \mathbf{W}_A \tilde{\mathbf{W}}_D \tilde{\mathbf{W}}_D^H \mathbf{W}_A^H \mathbf{H}^H|$. (2a), (2b), and (2c) are the per-antenna, RF chain, and phase shifter constraints. \mathbf{e}_{kl} is the Cartesian unit vector with elements at $((k-1)L+l)$ -th position equal to 1 and the rest is 0.

E.3 Precoding Design

If $L = 1$, then we have fully digital precoding where every antenna has its own RF chain. This case has been studied in [Pi12] and [Vu11b]. In this paper we focus on the hybrid precoding for the case where the number of RF chains is strictly smaller than the number of transmit antennas only, i.e., $K < M_t$. We first provide the necessary condition in designing the analog precoder. After that the digital precoder and the power allocation are considered.

E.3.1 Analog precoder

In this part, we provide the necessary condition to design the optimal \mathbf{W}_A assuming that the optimal $\tilde{\mathbf{W}}_D^*$ is given. This gives a necessary condition for the optimal design. Under this assumption, power constraints (2a) and (2b) are already satisfied since \mathbf{W}_A contains phase shifts only. Then the optimization problem to find \mathbf{W}_A can be formed as

$$\max_{\mathbf{W}_A} f(\mathbf{W}_A, \tilde{\mathbf{W}}_D^*), \text{ s. t. (2c)}. \quad (\text{E.3.1})$$

Due to the hardware design, the MIMO precoding matrix can be constructed as $\mathbf{V}^H = \mathbf{W}_A \tilde{\mathbf{W}}_D^*$. By letting $\mathbf{V} = [\mathbf{v}(1), \dots, \mathbf{v}(M_t)]$ where $\mathbf{v}(m) = [v_{m1}, \dots, v_{mN}]^T$ with v_{mn} , $m \in \{1, \dots, M_t\}$, $n \in \{1, \dots, N\}$, is the precoding coefficient for the i -th antenna and the j -th data stream, and $\tilde{\mathbf{W}}_D^* = [\tilde{\mathbf{w}}_D^*(1), \dots, \tilde{\mathbf{w}}_D^*(M_t)]^T$ with $\tilde{\mathbf{w}}_D^*(m) = [\tilde{w}_D^*(m1), \dots, \tilde{w}_D^*(mN)]^T$, we can express the MIMO precoding matrix as follows.

$$\begin{bmatrix} \mathbf{v}^H(1) \\ \vdots \\ \mathbf{v}^H(M_t) \end{bmatrix} = \begin{bmatrix} w_A(1,1) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & w_A(K,L) \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{w}}_D^{*T}(1) \\ \vdots \\ \tilde{\mathbf{w}}_D^{*T}(M_t) \end{bmatrix}. \quad (\text{E.3.2})$$

This implies that

$$\begin{bmatrix} v_{m1}^* \\ \vdots \\ v_{mN}^* \end{bmatrix} = \begin{bmatrix} w_A(k, l) \tilde{w}_D^{**}(m1) \\ \vdots \\ w_A(k, l) \tilde{w}_D^{**}(mN) \end{bmatrix}, \quad (\text{E.3.3})$$

$\forall k \in \mathcal{K}, \forall l \in \mathcal{L}$, and $m = (k-1)L + l$.

As a result, if we let γ_{mn} and θ_{mn} be optimal phases of v_{mn}^* and $\tilde{w}_D^*(mn)$, $\forall n \in \{1, \dots, N\}$, then at the optimum, the elements of the optimal analog precoder have to satisfy the following conditions

$$w_A^*(k, l) = e^{i(\theta_{m1} - \gamma_{m1})} = \dots = e^{i(\theta_{mN} - \gamma_{mN})}, \quad (\text{E.3.4})$$

for all $m \in \{1, \dots, M_t\}$.

Since we assume that the optimal $\tilde{\mathbf{W}}_D^*$ is given, $\tilde{w}_D^*(mn)$ is known. The remaining problem is to find the optimal v_{mn}^* and its equivalent $\mathbf{v}^*(m)$.

Let P_m^* denotes the optimal power allocated for antenna $m \in \{1, \dots, M_t\}$, then the optimal value of \mathbf{V} can be obtained by solving the following problem

$$\begin{aligned} \mathbf{V}^* &= \arg \max_{\mathbf{V}} \log |\mathbf{I} + \mathbf{V}\mathbf{F}\mathbf{V}^H|, & (\text{E.3.5}) \\ \text{s. t. } & \|\mathbf{v}(m)\|^2 \leq P_m^* \quad \forall m = 1, \dots, M_t, \end{aligned}$$

where $\mathbf{F} = \mathbf{H}^H \mathbf{H}$. Note that (E.3.5) is a MIMO channel with per-antenna power constraints only problem in which the per-antenna power constraints $P_m^*, \forall m = 1, \dots, M_t$ are equal to the optimal power allocation given by $\tilde{\mathbf{W}}_D^*$. [Vu11b] showed that there always exists an optimal solution for (E.3.5) that allocates full power on all antennas. However there is no closed-form solution for (E.3.5). The optimal value of \mathbf{V} can be obtained by using techniques in [Pi12] which will be discussed in more detail in the following.

Instead solving (E.3.5) for \mathbf{V} with all $\mathbf{v}(m)$ at once, we derive a necessary condition for each $\mathbf{v}(m)$, $m \in \{1, \dots, M_t\}$, separately. In order to extract the contribution of $\mathbf{v}(m)$ of the m -th antenna to the objective function of (E.3.5), we can reorder \mathbf{F} as

$$\mathbf{F}_m = \begin{bmatrix} f_m & \mathbf{f}_m^H \\ \mathbf{f}_m & \tilde{\mathbf{F}}_m \end{bmatrix}, \quad (\text{E.3.6})$$

where $\tilde{\mathbf{F}}_m \in \mathbb{C}^{(M_t-1) \times (M_t-1)}$ obtained by removing m -th row and columns from \mathbf{F} , \mathbf{f}_m is the m -th column of \mathbf{F} without diagonal item f_m . Then, following [Pi12], the objective function of (E.3.5) can be written as

$$\begin{aligned} & \log |\mathbf{I} + \mathbf{V}\mathbf{F}\mathbf{V}^H| \\ &= \log \left| \mathbf{I} + [\mathbf{v}(m) \quad \tilde{\mathbf{V}}_m] \begin{bmatrix} f_m & \mathbf{f}_m^H \\ \mathbf{f}_m & \tilde{\mathbf{F}}_m \end{bmatrix} \begin{bmatrix} \mathbf{v}(m)^H \\ \tilde{\mathbf{V}}_m^H \end{bmatrix} \right| \end{aligned}$$

$$\begin{aligned}
 &= \log \left| \mathbf{I} + \tilde{\mathbf{V}}_m \tilde{\mathbf{F}}_m \tilde{\mathbf{V}}_m^H + f_m (\mathbf{v}(m) \mathbf{v}^H(m) \right. \\
 &\quad \left. + \mathbf{v}(m) \mathbf{w}_m^H + \mathbf{w}_m \mathbf{v}^H(m)) \right| \\
 &= \log \left| \mathbf{I} + \tilde{\mathbf{V}}_m \tilde{\mathbf{F}}_m \tilde{\mathbf{V}}_m^H - f_m \mathbf{w}_m \mathbf{w}_m^H + f_m \mathbf{w}_m \mathbf{w}_m^H \right. \\
 &\quad \left. + f_m (\mathbf{v}(m) \mathbf{v}^H(m) + \mathbf{v}(m) \mathbf{w}_m^H + \mathbf{w}_m \mathbf{v}^H(m)) \right| \\
 &= \log \left| \mathbf{D}_m + f_m (\mathbf{v}(m) + \mathbf{w}_m) (\mathbf{v}(m) + \mathbf{w}_m)^H - f_m \mathbf{w}_m \mathbf{w}_m^H \right| \\
 &= \log |\mathbf{D}_m| + \log \left(1 + f_m (\mathbf{v}(m) + \mathbf{w}_m)^H \mathbf{D}_m^{-1} (\mathbf{v}(m) + \mathbf{w}_m) \right. \\
 &\quad \left. - f_m \mathbf{w}_m^H \mathbf{D}_m^{-1} \mathbf{w}_m \right), \tag{E.3.7}
 \end{aligned}$$

with $\mathbf{D}_m = \mathbf{I} + \tilde{\mathbf{V}}_m \tilde{\mathbf{F}}_m \tilde{\mathbf{V}}_m^H$, $\mathbf{w}_m = \frac{1}{f_m} \tilde{\mathbf{V}}_m \mathbf{f}_m$, and $\tilde{\mathbf{V}}_m$ is the sub-matrix of \mathbf{V} with $\mathbf{v}(m)$ removed.

Since $\log(\cdot)$ is a monotonically increasing function, and \mathbf{D}_m , \mathbf{w}_m , and f_m are independent of $\mathbf{v}(m)$ for a fixed matrix $\tilde{\mathbf{V}}_m$, we can reformulate the optimization problem (E.3.5) into

$$\tilde{\mathbf{v}}^*(m) = \arg \max_{\tilde{\mathbf{v}}(m): \|\tilde{\mathbf{v}}(m)\|^2 \leq P_m^*} (\tilde{\mathbf{v}}(m) + \tilde{\mathbf{w}}_m)^H \Sigma_m (\tilde{\mathbf{v}}(m) + \tilde{\mathbf{w}}_m), \tag{E.3.8}$$

where \mathbf{Z}_m and Σ_m are left singular and diagonal matrices obtained from the singular value decomposition of \mathbf{D}_m , i.e., $\mathbf{D}_m = \mathbf{Z}_m \Sigma_m \mathbf{Z}_m^H$, as well as $\tilde{\mathbf{v}}(m) = \mathbf{Z}_m^H \mathbf{v}(m)$ and $\tilde{\mathbf{w}}_m = \mathbf{Z}_m^H \mathbf{w}_m$.

Following [Pi12], the objective function is maximized when the phases of complex numbers \tilde{v}_{mn} and \tilde{w}_{mn} , $\forall n \in \{1, \dots, N\}$ are the same. Let a_{mn} and b_{mn} be the amplitudes of \tilde{v}_{mn} and \tilde{w}_{mn} . Then (E.3.8) can be simplified to

$$\mathbf{a}_m^* = \arg \max_{\mathbf{a}_m} \sum_{n=1}^{N^*} \frac{(a_{mn} + b_{mn})^2}{\sigma_{mn}}, \text{ s. t. } \sum_{n=1}^{N^*} a_{mn}^2 \leq P_m^*, \tag{E.3.9}$$

where σ_{mn} , $n \in \{1, \dots, N^*\}$ are the diagonal elements of Σ_m , $\mathbf{a}_m = [a_{m1}, \dots, a_{mN}]$, $\mathbf{b}_m = [b_{m1}, \dots, b_{mN}]$, and $N^* = \text{rank}(\tilde{\mathbf{W}}_D) \leq N$. Following [Pi12], we know that the contours of the objective function in (E.3.9) are N^* -dimension ellipsoids centered at $(-b_{1n}, \dots, -b_{M_n})$, and the per-antenna power allocation must satisfy all the per-antenna power constraints with equality. Therefore, the optimal solution of (E.3.9) has to satisfy the following condition

$$(\rho_m^* \Sigma_m - \mathbf{I}) \mathbf{a}_m^* = \mathbf{b}_m, \tag{E.3.10}$$

where $\rho_m^* > 0$ is the Lagrange multiplier chosen such that $\mathbf{a}_m \mathbf{a}_m^H = \sum_{n=1}^{N^*} a_{mn}^2 = P_m^*$. The value of ρ_m^* can be found by solving the following equation derived from (E.3.10)

$$\sum_{n=1}^{N^*} \frac{b_{mn}^2}{(\rho_m^* \sigma_{mn} - 1)^2} = P_m^* \tag{E.3.11}$$

using, e.g., Newton's method [BV09] (see Appendix E.6.1).

From (E.3.10) and the fact that \tilde{v}_{mn}^* and \tilde{w}_{mn} are in-phase for all $n \in \{1, \dots, N\}$, we can obtain the solution of (E.3.8) as

$$\tilde{\mathbf{v}}^*(m) = (\rho_m^* \boldsymbol{\Sigma}_m - \mathbf{I})^{-1} \tilde{\mathbf{w}}_m. \quad (\text{E.3.12})$$

Thus in the optimum, it is necessary that

$$\mathbf{v}^*(m) = \frac{1}{f_m} \mathbf{Z}_m (\rho_m^* \boldsymbol{\Sigma}_m - \mathbf{I})^{-1} \mathbf{Z}_m^H \tilde{\mathbf{V}}_m^* \mathbf{f}_m, \quad (\text{E.3.13})$$

with $\tilde{\mathbf{V}}_m^*$ is the optimal sub-matrix of \mathbf{V} . This necessary condition can be used in a person-by-person optimality algorithm to find $\mathbf{v}^*(m)$, $\forall m \in \{1, \dots, M_t\}$. From (E.3.13) and the assumption that the optimal digital precoder is given, the necessary condition for the optimal analog precoder is provided.

E.3.2 Digital precoder

Next, we derive the conditions for the design of the digital precoder $\tilde{\mathbf{W}}_D$ with a given optimal \mathbf{W}_A^* above. Note that in the hybrid precoding design, the analog precoder contains phases only, while the power adjustment is performed by the digital precoder. The convex optimization problem to find the digital precoder $\tilde{\mathbf{W}}_D$ can be formed as

$$\begin{aligned} \max_{\tilde{\mathbf{W}}_D} \quad & f(\mathbf{W}_A^*, \tilde{\mathbf{W}}_D) & (\text{E.3.14}) \\ \text{s. t.} \quad & \forall k, l : \mathbf{e}_{kl}^T \mathbf{W}_A^* \tilde{\mathbf{W}}_D \tilde{\mathbf{W}}_D^H \mathbf{W}_A^* \mathbf{e}_{kl} \leq \tilde{P}_{kl}, \\ & \forall k : \sum_{l=1}^L \mathbf{e}_{kl}^T \mathbf{W}_A^* \tilde{\mathbf{W}}_D \tilde{\mathbf{W}}_D^H \mathbf{W}_A^* \mathbf{e}_{kl} \leq \hat{P}_k. \end{aligned}$$

Proposition E.1. *With $\hat{P}_k \leq \sum_{l=1}^L \tilde{P}_{kl}, \forall k$, there exist always an optimal solution of (E.3.14) which allocates full power on each RF chain.*

Proof. Suppose that there exists no optimal transmit strategy with full power allocation on each RF chain. Let \mathbf{Q}^* denotes the optimal solution of (E.3.14), then there exists at least one k , $1 \leq k \leq K$ such that $\sum_{l=1}^L \mathbf{e}_{kl}^T \mathbf{Q}^* \mathbf{e}_{kl} = \sum_{l=1}^L P_{kl}^* = P_k^* < \hat{P}_k$. Since for those k , $\sum_{l=1}^L P_{kl}^* = P_k^* < \hat{P}_k \leq \sum_{l=1}^L \tilde{P}_{kl}$ and $P_{kl}^* \leq \tilde{P}_{kl}$, there exists a positive semi-definite Hermitian matrix $\mathbf{Q} \succcurlyeq \mathbf{Q}^*$ such that by increasing the per-antenna power allocation from \mathbf{Q}^* , full power on each RF chain is allocated in \mathbf{Q} . Let R^* denotes the maximum transmission rate $R^* = f(\mathbf{Q}^*)$, then we have $f(\mathbf{Q}) \geq R^* = f(\mathbf{Q}^*)$ since $f(\mathbf{Q})$ is matrix-monotone in \mathbf{Q} [JB07]. This contradicts with the assumption that there does not always exist an optimal solution which allocates full power on each RF chain. \square

Accordingly, it is sufficient for the optimization to consider only transmit strategies which allocate full power on all RF chains, i.e., the RF chain power constraints are always active. Let $\tilde{\mathbf{H}} = \mathbf{H}\mathbf{W}_A^*$, then (E.3.14) can be equivalently expressed as

$$\begin{aligned} \max_{\tilde{\mathbf{W}}_D} \quad & \log |\mathbf{I} + \tilde{\mathbf{H}}\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\tilde{\mathbf{H}}^H| & \text{(E.3.15)} \\ \text{s. t.} \quad & \forall k, l : \mathbf{e}_{kl}^T \mathbf{W}_A^* \tilde{\mathbf{W}}_D \tilde{\mathbf{W}}_D^H \mathbf{W}_A^{*H} \mathbf{e}_{kl} \leq \tilde{P}_{kl}, \\ & \forall k : \sum_{l=1}^L \mathbf{e}_{kl}^T \mathbf{W}_A^* \tilde{\mathbf{W}}_D \tilde{\mathbf{W}}_D^H \mathbf{W}_A^{*H} \mathbf{e}_{kl} \leq \hat{P}_k. \end{aligned}$$

The optimal solution of (E.3.15) subject to the RF chain power constraints only is given by the following generalized water-filling solution from [XFZP15] and Paper B.

Lemma E.2 ([XFZP15] and Paper B). *Let $\mathbf{A} = \tilde{\mathbf{W}}_D \tilde{\mathbf{W}}_D^H$, then the optimal \mathbf{A}^* of (E.3.15) subjects to the RF chain power constraints only is given by*

$$\mathbf{A}^* = \mathbf{D}^{-\frac{1}{2}} [\mathbf{U}]_{:,1:R} [\mathbf{U}]_{:,1:R}^H \mathbf{D}^{-\frac{1}{2}} - [\mathbf{U}]_{:,1:R} \mathbf{\Delta}^{-1} [\mathbf{U}]_{:,1:R}^H \quad \text{(E.3.16)}$$

where diagonal matrix \mathbf{D} is a Lagrange multiplier; diagonal matrix $\mathbf{\Delta}$ and the first $R = \min(M_t, M_r)$ columns of a unitary matrix $[\mathbf{U}]_{:,1:R}$ are obtained from eigenvalue decomposition $\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} = \mathbf{U} \begin{bmatrix} \mathbf{\Delta} & 0 \\ 0 & 0 \end{bmatrix} \mathbf{U}^H$. The diagonal elements of $R \times R$ diagonal matrix $\mathbf{\Delta}$ are positive real values in decreasing order.

The elements of the diagonal matrix \mathbf{D} for the optimal solution of (E.3.15) can be computed as follows. For all $k \in \mathcal{K}$, at high SNR, we have

$$[\mathbf{D}]_{k,l} = \frac{\sum_{l' \in \mathcal{L}} [[\mathbf{U}]_{:,1:R} [\mathbf{U}]_{:,1:R}^H]_{kl'}}{\hat{P}_k + \sum_{l' \in \mathcal{L}} [[\mathbf{U}]_{:,1:R} \mathbf{\Delta}^{-1} [\mathbf{U}]_{:,1:R}^H]_{kl'}} \quad \forall l \in \mathcal{L}. \quad \text{(E.3.17)}$$

However it may happen that the optimal powers allocated under RF chain power constraints only problem may exceed the maximum allocated per-antenna powers. Following Paper A and Paper B, if an antenna has an optimal power allocation that violates the per-antenna power constraint, then it is optimal to allocate the maximal per-antenna power on that antenna. This behaviour is explained in the following lemma

Lemma E.3 (Lemma A.9 and Lemma B.2). *For all $k \in \mathcal{K}$, let $\mathcal{P}_k := \{l \in \mathcal{L} : P_{kl}^{WF} > \tilde{P}_{kl}\}$ where P_{kl}^{WF} are the corresponding diagonal elements of \mathbf{A}^* in (E.3.16). Then, the optimal power can be allocated as*

$$\begin{cases} \forall k, \forall l \in \mathcal{L} : P_{kl}^* = P_{kl}^{WF}, & \text{if } \mathcal{P}_k = \emptyset, \\ \forall k, \forall l \in \mathcal{P}_k : P_{kl}^* = \tilde{P}_{kl}, & \text{otherwise.} \end{cases} \quad \text{(E.3.18)}$$

We observe from Lemma E.3 that if there exists any optimal power which violates the per-antenna power constraint, then full per-antenna power is allocated on that antenna. The remaining optimal power allocation can be found by considering a reduced optimization problem as follows

$$\begin{aligned}
\max_{\tilde{\mathbf{W}}_D} \quad & \log |\mathbf{I} + \tilde{\mathbf{H}} \tilde{\mathbf{W}}_D \tilde{\mathbf{W}}_D^H \tilde{\mathbf{H}}^H| & (\text{E.3.19}) \\
\text{s. t.} \quad & \forall k, \forall l \in \mathcal{P}_k : \mathbf{e}_{kl}^T \mathbf{W}_A^* \tilde{\mathbf{W}}_D \tilde{\mathbf{W}}_D^H \mathbf{W}_A^H \mathbf{e}_{kl} = \tilde{P}_{kl}, \\
& \forall k, \forall l \in \mathcal{L}_k : \mathbf{e}_{kl}^T \mathbf{W}_A^* \tilde{\mathbf{W}}_D \tilde{\mathbf{W}}_D^H \mathbf{W}_A^H \mathbf{e}_{kl} \leq \tilde{P}_{kl}, \\
& \forall k, \forall l \in \mathcal{L}_k : \sum_l \mathbf{e}_{kl}^T \mathbf{W}_A^* \tilde{\mathbf{W}}_D \tilde{\mathbf{W}}_D^H \mathbf{W}_A^H \mathbf{e}_{kl} \leq \hat{P}'_k.
\end{aligned}$$

where $\hat{P}'_k = \hat{P}_k - \sum_{l \in \mathcal{L}_k} \tilde{P}_{kl}$, $\mathcal{L}_k = \mathcal{L} \setminus \mathcal{P}_k$ is a set of indices of antennas connecting RF chain k -th that have not been fixed to full per-antenna powers, and \mathcal{P}_k is defined in Lemma E.3.

The optimal solution of the reduced optimization problem (E.3.19) can be solved by using Lemma E.2 again. However, the diagonal matrix \mathbf{D} corresponding to the water level is different since some optimal powers have been fixed to maximal per-antenna powers following Lemma E.3. The elements of the diagonal matrix \mathbf{D} for the optimal solution of (E.3.19) therefore can be computed as follows. For all $k \in \mathcal{K}$, at high SNR, we have

$$[\mathbf{D}]_{k,l} = \frac{[[\mathbf{U}]_{:,1:R}[\mathbf{U}]_{:,1:R}^H]_{kl}}{\tilde{P}_{kl} + [[\mathbf{U}]_{:,1:R} \mathbf{\Delta}^{-1} [\mathbf{U}]_{:,1:R}^H]_{kl}} \text{ if } l \in \mathcal{P}_k \quad (\text{E.3.20})$$

and

$$[\mathbf{D}]_{k,l} = \frac{\sum_{l' \in \mathcal{L}} [[\mathbf{U}]_{:,1:R}[\mathbf{U}]_{:,1:R}^H]_{kl'}}{\hat{P}'_k + \sum_{l' \in \mathcal{L}} [[\mathbf{U}]_{:,1:R} \mathbf{\Delta}^{-1} [\mathbf{U}]_{:,1:R}^H]_{kl'}} \text{ if } l \in \mathcal{L}_k. \quad (\text{E.3.21})$$

However it may still happen that optimal power powers in the optimal solution of the (E.3.19) may exceed the remaining maximum per-antenna powers. Therefore, an iteration process starting from Lemma E.3 can be used to allocate optimal powers on the remaining antennas.

Note that with a reduced optimization problem, in Lemma E.3, \mathcal{L} will be replaced by \mathcal{L}_k in every iteration. This process stops when all power constraints are satisfied.

Based on analysis above, an iterative algorithm is proposed to compute the optimal value of $\mathbf{W}_D \mathbf{W}_D^H$. To see this, we consider the following sequence of optimization problems

$$\begin{aligned}
\max_{\tilde{\mathbf{W}}_D \in \mathcal{S}(\emptyset)} f(\tilde{\mathbf{W}}_D) &= \max_{\tilde{\mathbf{W}}_D \in \mathcal{S}(\emptyset) \cap \mathcal{R}(1)} f(\mathbf{W}_A^*, \tilde{\mathbf{W}}_D) \\
&\geq \max_{\tilde{\mathbf{W}}_D \in \mathcal{S}(\emptyset) \cap \mathcal{R}(2)} f(\tilde{\mathbf{W}}_D)
\end{aligned}$$

Algorithm E.3.1: Optimal power allocation in digital precoder

```

1 Solve (E.3.15) subject to RF chain power constraints only to find  $P_{kl}^{WF}$  using
  Lemma E.2 and (E.3.17).
2 for  $k = 1 : K$  do
3   Check the optimal power allocation  $P_{kl}^{WF}$  with per-antenna power
   constraints  $\tilde{P}_{kl}$  and form  $\mathcal{P}_k := \{l \in \mathcal{L} : P_{kl}^{WF} > \tilde{P}_{kl}\}$ .
4   while  $\mathcal{P}_k \neq \emptyset$  do
5      $P_{kl}^* \leftarrow \tilde{P}_{kl}, \forall l \in \mathcal{P}_k$ .
6      $\mathcal{L}_k = \mathcal{L} \setminus \mathcal{P}_k$ .
7     Form the reduced optimization problem (E.3.19).
8     Solve (E.3.19) using Lemma E.2, (E.3.20) and (E.3.21).
9      $\mathcal{L} \leftarrow \mathcal{L}_k$ .
10    Return 3.
11  end while
12   $P_{kl}^* \leftarrow P_{kl}^{WF}, \forall l \in \mathcal{L}$ .
13 end for
    
```

$$\dots$$

$$\geq \max_{\tilde{\mathbf{W}}_D \in \mathcal{S}(\emptyset) \cap \mathcal{R}(K(L-1))} f(\tilde{\mathbf{W}}_D) = (\text{E.3.15}). \quad (\text{E.3.22})$$

where $\mathcal{S}(\emptyset) := \{\tilde{\mathbf{W}}_D \in \mathbb{C}^{M_t \times N} : \sum_{l=1}^L \mathbf{e}_{kl}^T \mathbf{W}_A^* \tilde{\mathbf{W}}_D \tilde{\mathbf{W}}_D^H \mathbf{W}_A^{*H} \mathbf{e}_{kl} \leq \hat{P}_k \forall k\}$; $\mathcal{R}(i) = \{\tilde{\mathbf{W}}_D \in \mathbb{C}^{M_t \times N} : \mathbf{e}_{kl}^T \mathbf{W}_A^* \tilde{\mathbf{W}}_D \tilde{\mathbf{W}}_D^H \mathbf{W}_A^{*H} \mathbf{e}_{kl} \leq \tilde{P}_{kl}, \forall k, \forall l \in \mathcal{P}_k(i)\}$ with $\mathcal{P}_k(i)$ is the set of indices of powers which violates the per-antenna power constraints connecting to k -th antenna at the i -th iteration with $\mathcal{R}(1) = \emptyset$.

Every reduced optimization problem in (E.3.22) is convex. The global convergence of the proposed iteration method is guaranteed after solving at most $K(L-1)$ convex optimization problems.

E.4 Numerical Results

In this section, we consider a massive MIMO system with different transmit antenna configurations. We first evaluate the transmission rate for the case that the number of RF chains and the number of antennas are the same, i.e., fully digital beamforming is used. The considered system has either $K = 4$ or $K = 28$ RF chains and $M_t = 28$ antennas with per-antenna power constraints $\tilde{P}_{kl} = 3$. Further, we includes the case with $K = 4$ and $M_t = 4$ with per-antenna power constraints $\tilde{P}_{kl} = 21$. At the receiver, we set the number of received antennas as $M_r = 4$. For the plotted curves in Figure E.2, we have the same transmit power constraint \hat{P}_k on all RF chains that we gradually increase from $\hat{P}_k = 1$ to $\hat{P}_k = 30$.

We observe from the figure that for the hybrid beamforming, the transmission

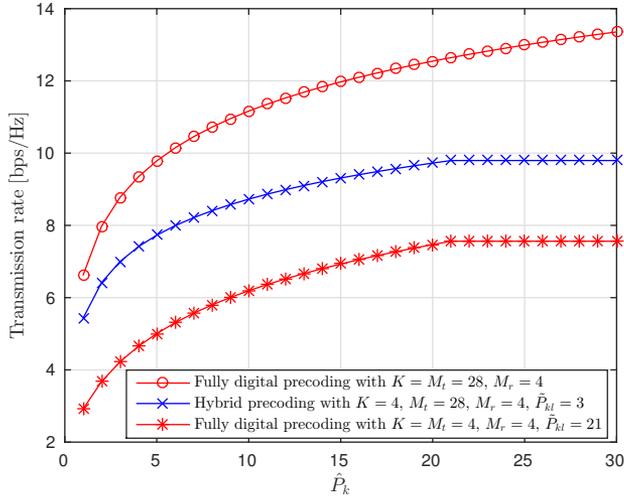


Figure E.2: Transmission rate of the sub-connected architecture massive MIMO system with different RF chain and antenna configurations.

rate increases together with the increasing of the RF chain power if the RF chain power constraint is more restrictive than the sum of all individual powers of a group of antennas connected to that RF chain, i.e., in the example $\hat{P}_k \leq 21$. Beyond that, i.e., $\hat{P}_k > 21$, the transmission rate remains constant with increasing RF chain power constraints since the allocated RF chain powers remain constant due to the per-antenna power constraints. It means the RF power constraint is never active for any $\hat{P}_k > 21$, and it is optimal to transmit with the maximal individual power $\hat{P}_{kl} = 3$ on all antennas.

In comparison to the fully digital precoding, we observe that the sub-connected architecture provides significant performance gains if a hardware with a few RF chains should be used, but performs worse if a fully connected system with many antennas is available. The gap between the fully digital and the hybrid precodings is due to the per-antenna power constraints.

E.5 Conclusions

In this paper, we provide necessary conditions to design the optimal analog and digital precoders for a massive MIMO system with sub-connected architecture. It is shown that precoders can be found by applying similar techniques from the optimization problems of the point-to-point MIMO system with per-antenna power constraints only and with joint sum and per-antenna power constraints. If the sum

of the per-antenna power constraints of antennas is larger than the RF chain power constraint of which they are connecting to, then there exists always an optimal policy that allocates full RF power. The power allocation on antennas follows the allocated behaviour of one with joint sum and per-antenna power constraints. The overall phase is controlled by the analog precoder while the power allocation is adjusted by the digital precoder only.

E.6 Appendix

E.6.1 Newton's method

Let $f(\rho_m)$ be a function of ρ_m ,

$$f(\rho_m) = \sum_{n=1}^{N^*} \frac{b_{mn}^2}{(\rho_m \sigma_{mn} - 1)^2} - P_m^*. \quad (\text{E.6.1})$$

Then, ρ_m^* is a root to the equation $f(\rho_m) = 0$. Following the Newton's method, at the i -th iteration, the value of ρ_m is updated as follows

$$\rho_m(i) = \rho_m(i-1) + \frac{f(\rho_m(i-1))}{f'(\rho_m(i-1))}, \quad (\text{E.6.2})$$

where $f'(\rho_m(i-1))$ is the first derivative of $f(\rho_m)$ with respect to ρ_m at iteration $(i-1)$. The initial value $f(\rho_m(0))$ can be set as $\max_n \frac{1}{\sigma_{mn}} (1 + \sqrt{\frac{b_{mn}^2}{P_m^*}})$. \square

References

- [AA16] E. A. Abbas and A. Abbosh. Adjustable millimeter Wave in-phase Power Divider for 5G using a Three-line Coupler Structure. In *IEEE International Symposium on Antennas and Propagation (APSURSI)*, Jun 2016.
- [AALH14] A. Alkhateeb, O. E. Ayach, G. Leus, and R. W. Heath. Channel Estimation and Hybrid Precoding for Millimeter Wave Cellular Systems. *IEEE Journal of Selected Topics in Signal Processing*, 8(5):831–846, Oct 2014.
- [Ash90] R. B. Ash. *Information Theory*. Dover Publications, Inc., Newyork, 1990.
- [BDP+13] R. Baldemair, E. Dahlman, S. Parkvall, Y. Selen, K. Balachandran, T. Irnich, G. Fodor, and H. Tullberg. Future Wireless Communications. In *IEEE 77th Vehicular Technology Conference (VTC Spring)*, pages 1–5, Jun 2013.
- [BH06] F. Boccardi and H. Huang. Optimum power allocation for the MIMO-BC Zero-forcing precoder with Per-antenna power constraints. In *Conference on Information Systems and Sciences (CISS)*, Mar 2006.
- [BJBO11] E. Bjornson, N. Jalden, M. Bengtsson, and B. Ottersten. Optimality Properties, Distributed Strategies, and Measurement-Based Evaluation of Coordinated Multicell OFDMA Transmission. *IEEE Transactions on Signal Processing*, 59(12):6086–6101, Dec 2011.
- [BV09] S. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, 2009.
- [CHSY07] C.-C. Chong, K. Hamaguchi, P. F. M. Smulders, and S.-K. Yong. Millimeter-Wave Wireless Communication Systems: Theory and Applications. *EURASIP Journal on Wireless Communications and Networking*, (1):072831, 2007.
- [CK78] I. Csiszár and J. Körner. Broadcast Channel with Confidential Messages. *IEEE Transactions on Information Theory*, 24(3):339–348, May 1978.

- [CO17a] P. L. Cao and T. J. Oechtering. Optimal Trade-off Between Transmission Rate and Secrecy Rate in Gaussian MISO Wiretap Channels. In *21th International ITG Workshop on Smart Antennas - WSA*, pages 1–8, Mar 2017.
- [CO17b] P. L. Cao and T. J. Oechtering. Optimal Transmit Strategy for MIMO Channels with Joint Sum and Per-antenna Power Constraints. In *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pages 3569–3573, Mar 2017.
- [CO18] P. L. Cao and T. J. Oechtering. Optimal Transmit Strategies for Gaussian MISO Wiretap Channels. *IEEE Transactions on Information Forensics and Security*, 2018. submitted to.
- [COS16] P. L. Cao, T. J. Oechtering, and M. Skoglund. Optimal transmission with Per-antenna Power Constraints for Multiantenna Bi-directional Broadcast Channel. In *The Ninth IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM)*, Jul 2016.
- [COS18a] P. L. Cao, T. J. Oechtering, and M. Skoglund. Transmit Beamforming for Single-user Large-scale MISO Systems with Sub-connected Architecture and Power Constraints. *IEEE Communications Letters*, 22(10):2096 – 2099, Oct 2018.
- [COS18b] P. L. Cao, T. J. Oechtering, and M. Skoglund. Precoding Design for Massive MIMO Systems with Sub-connected Architecture and Per-antenna Power Constraints. In *The 22nd International ITG Workshop on Smart Antennas (WSA)*, Mar 2018.
- [COSS15] P. L. Cao, T. J. Oechtering, R. F. Schaefer, and M. Skoglund. Optimal Transmission Rate for MISO Channels with Joint Sum and Per-antenna Power Constraints. In *IEEE International Conference on Communications (ICC)*, Jun 2015.
- [COSS16] P. L. Cao, T. J. Oechtering, R. F. Schaefer, and M. Skoglund. Optimal Transmit Strategy for MISO Channels With Joint Sum and Per-Antenna Power Constraints. *IEEE Transactions on Signal Processing*, 64(16):4296–4306, Aug 2016.
- [CT06] T. Cover and J. Thomas. *Elements of Information Theory*. John Wiley and Sons, Inc., 2nd edition, 2006.
- [CTJLa07] M. Codreanu, A. Tölli, M. Juntti, and M. Latva-aho. MIMO downlink weighted sum rate maximization with power constraint per antenna group. In *IEEE VTC Spring*, Apr 2007.
- [EGK12] A. El Gamal and Y.-H. Kim. *Network Information Theory*. Cambridge University Press, 2012.

- [EU12a] E. Ekrem and S. Ulukus. Capacity-Equivocation Region of the Gaussian MIMO Wiretap Channel. *IEEE Transactions on Information Theory*, 58(9):5699–5710, Sep 2012.
- [EU12b] E. Ekrem and S. Ulukus. Capacity Region of Gaussian MIMO Broadcast Channels With Common and Confidential Messages. *IEEE Transactions on Information Theory*, 58(9):5669–5680, Sep 2012.
- [FG98] G. Foschini and M. Gans. On Limits of Wireless Communications in a Fading Environment when Using Multiple Antennas. *Wireless Personal Communications*, 6(3):311–335, Mar 1998.
- [FJL⁺13] D. Feng, C. Jiang, G. Lim, L. J. Cimini, G. Feng, and G. Y. Li. A Survey of Energy-efficient Wireless Communications. *IEEE Communications Surveys Tutorials*, 15(1):167–178, 2013.
- [Fos96] G. J. Foschini. Layered Space-time Architecture for Wireless Communication in a Fading Environment when using Multi-element Antennas. *Bell Labs Technical Journal*, 1(2):41–59, Autumn 1996.
- [GBKS15] H. Ghauch, M. Bengtsson, T. Kim, and M. Skoglund. Subspace Estimation and Decomposition for Hybrid Analog-Digital millimetre-wave MIMO Systems. In *IEEE International Workshop on Signal Processing Advance in Wireless Comms. (SPAWC)*, pages 395–399, Jun 2015.
- [GDH⁺16] X. Gao, L. Dai, S. Han, C. L. I, and R. W. Heath. Energy-Efficient Hybrid Analog and Digital Precoding for MmWave MIMO Systems With Large Antenna Arrays. *IEEE Journal on Selected Areas in Communications*, 34(4):998–1009, Apr 2016.
- [GETL15] X. Gao, O. Edfors, F. Tufvesson, and E. G. Larsson. Massive MIMO in Real Propagation Environments: Do All Antennas Contribute Equally? *IEEE Transactions on Communications*, 63(11):3917–3928, Nov 2015.
- [Gol01] A. Goldsmith. Design Challenges for Future Wireless Systems. Technical report, NAE Symposium on Frontiers of Engineering, 2001.
- [GTER12] X. Gao, F. Tufvesson, O. Edfors, and F. Rusek. Measured propagation characteristics for very-large MIMO at 2.6 GHz. pages 295–299, 2012.
- [HJ13] R. A. Horn and C. R. Johnson. *Matrix Analysis*. Cambridge University Press, 2013.

- [HPC10] H. Huh, H. Papadopoulos, and G. Caire. Multiuser MISO transmitter optimization for intercell interference mitigation. *IEEE Transactions on Signal Processing*, 58(8):4272–4285, Aug 2010.
- [HS10] M. F. Hanif and P. J. Smith. On MIMO cognitive radio with antenna selection. In *IEEE Wireless Communications and Networking Conference*, 2010.
- [HU13] C. Hellings and W. Utschick. Energy efficiency optimization in MIMO broadcast channels with fairness constraints. In *IEEE International Workshop on Signal Processing Advance in Wireless Comms. (SPAWC)*, 2013.
- [JB07] E. A. Jorswieck and H. Boche. *Majorization and Matrix-monotone Functions in Wireless Communications*. Now Publishers Inc., 2007.
- [JMZ⁺14] V. Jungnickel, K. Manolakis, W. Zirwas, B. Panzner, V. Braun, M. Lossow, M. Sternad, R. Apelfrojd, and T. Svensson. The role of small cells, coordinated multipoint, and massive MIMO in 5G. *IEEE Communications Magazine*, 52(5):44–51, May 2014.
- [JPKR11] J. Jose, N. Prasad, M. Khojastepour, and S. Rangarajan. On Robust Weighted-sum Rate Maximization in MIMO Interference Networks. In *IEEE International Conference on Communications (ICC)*, 2011.
- [KW10a] A. Khisti and G. Wornell. Secure Transmission with Multi-antenna II: The MIMOME Wiretap Channel. *IEEE Transactions on Information Theory*, 56(11):5515–5532, 2010.
- [KW10b] A. Khisti and G. W. Wornell. Secure Transmission With Multiple Antennas I: The MISOME Wiretap Channel. *IEEE Transactions on Information Theory*, 56(7):3088–3104, Jul 2010.
- [KYFV07] K. Karakayali, R. Yates, G. Foschini, and R. Valenzuela. Optimum Zero-forcing Beamforming with Per-antenna Power Constraint. In *IEEE International Symposium on Information Theory (ISIT)*, June 2007.
- [LETM14] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta. Massive MIMO for Next Generation Wireless Systems. *IEEE Communications Magazine*, 52(2):186–195, Feb 2014.
- [LHHT⁺13] Q. Li, M. Hong, W. H.-T., W.-K. Ma, Y.-F. Liu, and Z.-Q. Luo. An alternating optimization algorithm for the MIMO secrecy Capacity problem under sum power and Per-antenna power constraints. In *Acoustics, Speech and Signal Processing (ICASSP)*, May 2013.

- [LHW⁺13] Q. Li, M. Hong, H.-T. Wai, Y.-F. Liu, W.-K. Ma, and Z.-Q. Luo. Transmit Solutions for MIMO Wiretap channels using Alternating Optimization. *IEEE Journal on Selected Areas in Communications*, 31(9):1714–1727, Sep 2013.
- [LLPS13] R. Liu, T. Liu, H. V. Poor, and S. Shamai. New Results on Multiple-Input Multiple-Output Broadcast Channels With Confidential Messages. *IEEE Transactions on Information Theory*, 59(3):1346–1359, Mar 2013.
- [LLS⁺14] L. Lu, G. Y. Li, A. L. Swindlehurst, A. Ashikhmin, and R. Zhang. An Overview of Massive MIMO: Benefits and Challenges. *IEEE Journal of Selected Topics in Signal Processing*, 8(5):742–758, Oct 2014.
- [LML12] S. R. Lee, S. H. Moon, and I. Lee. Downlink Distributed Antenna Systems: Optimal Beamforming Designs and Capacity Behavior. In *IEEE International Conference on Communications (ICC)*, Jun 2012.
- [Lo99] T. K. Y. Lo. Maximum Ratio Transmission. *IEEE Transactions on Communications*, 47(10):1458–1461, Oct 1999.
- [Loy16] S. Loyka. The Capacity of Gaussian MISO Channels under Total and Per-antenna Power Constraints. In *IEEE International Symposium on Information Theory (ISIT)*, pages 325–329, Jul 2016.
- [LP09] R. Liu and H. V. Poor. Secrecy Capacity Region of a Multiple-Antenna Gaussian Broadcast Channel With Confidential Messages. *IEEE Transactions on Information Theory*, 55(3):1235–1249, Mar 2009.
- [LP10] J. Li and A. Petropulu. Optimal Input Covariance for Achieving Secrecy Capacity in Gaussian MIMO Wiretap Channels. In *IEEE International Conference on Acoustics Speech and Signal Processing (ICASSP)*, Mar 2010.
- [LTV06] A. Lozano, A. M. Tulino, and S. Verdú. Optimum Power Allocation for Parallel Gaussian Channels with Arbitrary Input Distribution. *IEEE Transactions on Information Theory*, 51(1):141–154, Jan 2006.
- [LVGPRH16] R. Lopez-Valcarce, N. Gonzalez-Prelcic, C. Rusu, and R. W. Heath. Hybrid Precoders and Combiners for mmWave MIMO Systems with Per-Antenna Power Constraints. In *IEEE Global Communications Conference (GLOBECOM)*, pages 1–6, Dec 2016.
- [LWY⁺17] N. Li, Z. Wei, H. Yang, X. Zhang, and D. Yang. Hybrid Precoding for mmWave Massive MIMO Systems With Partially Connected Structure. *IEEE Access*, 5:15142–15151, 2017.

- [LXD14] L. Liang, W. Xu, and X. Dong. Low-Complexity Hybrid Precoding in Massive Multiuser MIMO Systems. *IEEE Wireless Communications Letters*, 3(6):653–656, Dec 2014.
- [LYCH78] S. Leung-Yan-Cheong and M. E. Hellman. The Gaussian Wiretap Channel. *IEEE Transactions on Information Theory*, 24(4):451–456, Jul 1978.
- [Mar10] T. L. Marzetta. Noncooperative Cellular Wireless with Unlimited Numbers of Base Station Antennas. *IEEE Transactions on Wireless Communications*, 9(11):3590–3600, Nov 2010.
- [Mar15] T. L. Marzetta. Massive MIMO: An Introduction. *Bell Labs Technical Journal*, 20:11–22, 2015.
- [MDT14] M. Maamari, N. Devroye, and D. Tuninetti. The Capacity of the Ergodic MISO Channel with Per-antenna Power Constraint and an Application to the Fading Cognitive Interference Channel. In *IEEE International Symposium on Information Theory (ISIT)*, Jul 2014.
- [ML13] S. K. Mohammed and E. G. Larsson. Per-Antenna Constant Envelope Precoding for Large Multi-User MIMO Systems. *IEEE Transactions on Communications*, 61(3):1059–1071, Mar 2013.
- [MXL13] J. Mao, G. Xie, and Y. Liu. Energy Efficiency Optimization for Cognitive Radio MIMO Broadcast Channels. *IEEE Communications Letters*, 17(2):337–340, Feb 2013.
- [OH08] F. Oggier and B. Hassibi. The Secrecy Capacity of the MIMO Wiretap Channel. In *IEEE International Symposium on Information Theory (ISIT)*, 2008.
- [PCL03] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas. Joint Tx-Rx beamforming design for multicarrier MIMO channels: a unified framework for convex optimization. *IEEE Transactions on Signal Processing*, 51(9):2381–2401, Sep 2003.
- [Pi12] Z. Pi. Optimal MIMO Transmission with Per-antenna Power Constraint. In *Global Communications Conference (GLOBECOM)*, Dec 2012.
- [RAL14] S. M. Razavizadeh, M. Ahn, and I. Lee. Three-Dimensional Beamforming: A new enabling technology for 5G wireless networks. *IEEE Signal Processing Magazine*, 31(6):94–101, Nov 2014.
- [SSB08] S. Shi, M. Schubert, and H. Boche. Per-antenna Power Constrained Rate Optimization for Multiuser MIMO Systems. In *Proc. International ITG Workshop Smart Antennas.*, 2008.

- [SSC12] Q. Shi, E. Song, and G. Chen. Signaling Strategy Optimization for Gaussian MIMO Wiretap Channel. In *IEEE International Conference on Communications (ICC)*, 2012.
- [SY16] F. Sotirios and W. Yu. Hybrid Digital and Analog Beamforming Design for Large-Scale Antenna Arrays. *IEEE Journal of Selected Topics in Signal Processing*, 10(3):501–513, Apr 2016.
- [TCJ07] A. Tölli, M. Codreanu, and M. Juntti. Minimum SINR Maximization for Multiuser MIMO Downlink with per BS Power Constraints. In *IEEE Wireless Communications and Networking Conference (WCNC)*, Mar 2007.
- [TCJ08] A. Tölli, M. Codreanu, and M. Juntti. Linear Multiuser MIMO Transceiver Design with Quality of Service and Per-antenna Power Constraints. *IEEE Transactions on Signal Processing*, 56(7):3049–3055, Jul 2008.
- [Tel99] I. Telatar. Capacity of multi-antenna Gaussian channels. *European Transactions on Telecommunications*, 10(6):585–595, Nov 1999.
- [Tho87] J. A. Thomas. Feedback Can at Most Double Gaussian Multiple Access Channel Capacity. *IEEE Transactions on Information Theory*, 33(5):711–716, Sep 1987.
- [Tun14] D. Tuninetti. On The Capacity of The AWGN MIMO Channel Under Per-Antenna Power Constraints. In *IEEE International Conference on Communications (ICC)*, Jun 2014.
- [TV05] D. Tse and P. Viswanath. *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.
- [VA87] R. G. Vaughan and J. B. Andersen. Antenna Diversity in Mobile Communications. *IEEE Transactions on Vehicular Technology*, 36(4):149–172, Nov 1987.
- [Vu11a] M. Vu. MISO Capacity with Per-Antenna Power Constraint. *IEEE Transactions on Communications*, 59(5):1268–1274, May 2011.
- [Vu11b] M. Vu. MIMO Capacity with Per-Antenna Power Constraint. In *Global Communications Conference (GLOBECOM)*, pages 1–5, Dec 2011.
- [WESS08] A. Wiesel, Y. Eldar, and S. Shamai (Shitz). Zero-forcing Precoding and Generalized Inverses. *IEEE Transactions on Signal Processing*, 56(9):4409–4418, Sep 2008.

- [WKX⁺18] Y. Wu, A. Khisti, C. Xiao, G. Caire, K. Wong, and X. Gao. A Survey of Physical Layer Security Techniques for 5G Wireless Networks and Challenges Ahead. *CoRR*, abs/1801.05227, 2018.
- [WOB⁺08] R. F. Wyrembelski, T. J. Oechtering, I. Bjelakovic, C. Schnurr, and H. Boche. Capacity of Gaussian MIMO Bidirectional Broadcast Channel. In *IEEE International Symposium on Information Theory (ISIT)*, pages 584–588, Jul 2008.
- [WSNW17] V. W. S. Wong, R. Schober, D. Ng, and L.-C. Wang. *Key Technologies for 5G Wireless Systems*. Cambridge University Press, 2017.
- [WSS06] H. Weingarten, Y. Steinberg, and S. Shamai (Shitz). The Capacity Region of the Gaussian Multiple-Input Multiple-Output Broadcast Channel. *IEEE Transactions on Information Theory*, 52(9):3936–3964, Sep 2006.
- [Wyn75] A. Wyner. The Wiretap Channel. *The Bell System Technical Journal*, 54:1355–1387, 1975.
- [XFZP15] C. Xing, Z. Fei, Y. Zhou, and Z. Pan. Matrix-field Water-filling Architecture for MIMO Tranceiver Designs with Mixed Power Constraints. In *2015 IEEE 26th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)*, Sep 2015.
- [XQ13] J. Xu and L. Qiu. Energy Efficiency Optimization for MIMO Broadcast Channels. *IEEE Transactions on Wireless Communications*, 12(2):690–701, Feb 2013.
- [YC04] W. Yu and J. Cioffi. Sum Capacity of Gaussian Vector Broadcast Channels. *IEEE Transactions on Information Theory*, 52(2):361–374, Feb 2004.
- [YC06] S. K. Yong and C.-C. Chong. An Overview of Multigigabit Wireless through Millimeter Wave Technology: Potentials and Technical Challenges. *EURASIP Journal on Wireless Communications and Networking*, 2007(1):078907, Dec 2006.
- [YL07] W. Yu and T. Lan. Transmitter Optimization for the Multi-antenna Downlink with Per-antenna Power Constraint. *IEEE Transactions on Signal Processing*, 55(6):2646–2660, Jun 2007.
- [Zha10] R. Zhang. Cooperative Multi-cell Block Diagonalization per-base-station Power Constraints. *IEEE Journal on Selected Areas in Communications*, 28(9):1435–1445, Dec 2010.

-
- [ZHL⁺17] G. Zhu, K. Huang, V. K. N. Lau, B. Xia, X. Li, and S. Zhang. Hybrid Beamforming via the Kronecker Decomposition for the Millimeter-Wave Massive MIMO Systems. *IEEE J. Sel. Areas in Commun.*, 35(9):2097–2114, Sep 2017.
- [ZLS17] M. Zhou, Y. Liao, and L. Song. *Full-Duplex Wireless Communications for 5G*, pages 299–335. Oct 2017.
- [ZXL09] L. Zhang, Y. Xin, and Y.-C. Liang. Weighted Sum Rate Optimization for Cognitive Radio MIMO Broadcast Channels. *IEEE Transactions on Wireless Communications*, 8(6):2950 – 2959, Jun 2009.

