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This is the accepted version of a paper published in . This paper has been peer-reviewed but does not include the final publisher proof-corrections or journal pagination.

Citation for the original published paper (version of record):

Lian, B., Wang, X V., Wang, L. [Year unknown!]

Static and dynamic optimization of a pose adjusting mechanism considering parameter changes during construction

[Journal name unknown!]

Access to the published version may require subscription.

N.B. When citing this work, cite the original published paper.

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Static and dynamic optimization of a pose adjusting mechanism considering parameter changes during construction

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Abstract

Having potentially high stiffness and good dynamic response, a parallel pose adjusting mechanism was proposed for being an attachment to a big serial robot of a macro-micro robotic system. This paper addresses its design optimization problem mainly concerning arrangements of design variables and objectives. Parameter changes during construction are added to the design variables in order to prevent the negative effects to the physical prototype. These parameter changes are interpreted as parameter uncertainty and modeled by probabilistic theory. For the objectives, both static and dynamic performances are simultaneously optimized by Pareto-based method. The involved performance indices are instantaneous energy based stiffness index, first natural frequency and execution mass. The optimization procedure is implemented as: (1) carrying out performance modeling and defining performance indices, (2) reformulating statistical objectives and probabilistic constraints considering parameter uncertainty, (3) conducting Pareto-based optimization with the aid of response surface method (RSM) and particle swarm optimization (PSO), (4) selecting optimal solution by searching for cooperative equilibrium point (CEP). By addressing parameter uncertainty and the best compromise among multiple objectives, the presented optimization procedure provides more reliable optimal parameters that would not be affected by minor parameter changes during construction, and less biased optimum between static and dynamic performances comparing with the conventional optimization methods. The proposed optimization method can also be applied to the other similar mechanisms.

Key Words: parallel mechanism; design optimization; parameter uncertainty; Pareto-based method

1. Introduction

In the assembling of large components in aviation and aerospace, a macro-micro robotic system is usually applied. A large serial robot is employed as transporting unit allowing the components to travel long distance while docking equipment is used to adjust the pose of components for assembling. Three types of docking equipment are mainly applied so far, i.e. pogo positioning platform, multi-suction platform and automated guided vehicle [1]. From topology point of view, these docking equipment are all constructed by serial structures. The serial mechanisms have one motion/force transmission route from the fixed base to the end-effector. It can add up motions of joints hence enlarge the workspace of end-effector, enabling good carrying or adjusting

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capability of the docking equipment. However, the serial mechanisms are easy to accumulate deformations thus reducing rigidity of the docking equipment [2-6]. Since small rigidity is not beneficial for carrying large and heavy aircraft parts, large numbers of serial structures are applied to build the pose adjusting platform in the current docking equipment. Besides compensating the shortages of serial mechanisms by applying multiple units, another alternative is to build the positioning platform by parallel mechanisms (PMs) [7-10].

Having several kinematic chains to support the same end-effector, PMs have the advantages in terms of high stiffness, large load-weight ratio and good dynamic response [11-13]. Correspondingly, they suffer from smaller workspace and more complex control system comparing with their serial counterparts. To find out solutions for pose adjusting mechanism with parallel architecture, the first problem is topology synthesis that invents PMs to achieve large orientation workspace. In this regard, Sun [14, 15] proposed a topology synthesis method that is generic and recognized as a milestone in the topology synthesis of PMs. In his proposed method, derivative mapping between the finite and instantaneous screws are built, from which the full cycle finite motion pattern of limbs specified to moving platform can be computed in an analytical manner. Large orientation workspace can be achieved by the PMs with variable rotational axes, which are difficult for the conventional synthesis approaches but can be addressed by the screw based method [16]. The expected motions of the pose adjusting mechanism are analyzed, based on which the arrangement of limbs and joints are algebraically calculated. Finally, a parallel pose adjusting mechanism named as PaQuad was proposed [17-18]. Instead of the common single moving platform, articulated travelling plate (ATP) is introduced to add extra rotation about the normal direction of moving platform. Contributed by both parallel limbs and the ATP, the PaQuad PM has three rotational and one translational capability. The potentially large orientation workspace and high stiffness of PaQuad PM make it a promising candidate for being applied as docking equipment or pose adjusting mechanism in assembling large components.

After constructing the topological structure, the next problem for the development of the pose adjusting mechanism is the design optimization of PaQuad PM. Design optimization is the process of enhancing concerned performance indices by adjusting structural parameters [19-20]. For the optimization of PMs, performance indices are usually regarded as objectives and structural parameters are the design variables. Constrained conditions are interpreted from the design requirements. With the available objectives, design variables and constrained conditions, the optimal parameters can be searched by the advanced optimization algorithms [21-24]. In such conventional design optimization process, the optimal parameters are directly applied to build the physical prototype of the PM. This indicates that the parameter changes in the construction are not considered in the optimization. As is known, geometric errors are inevitably generated during the construction. These unavoidable geometric errors would lead to minor changes of the optimal parameters. If the performances are sensitive to the parameter changes, the actual performances of the physical prototype will largely deviate from the expected optimal performances. What's worse, the actual parameter might lead to the constrained conditions violating design requirement if the constrained conditions are also sensitive to the parameter changes. In this case, the optimal parameters from design optimization turn out to be unfeasible. In order to prevent such disaster, the possible minor changes of parameters in the construction will be incorporated to the optimization in this paper. Since the minor changes are unknown in the design stage, they are described as parameter uncertainty. The method of addressing parameter uncertainty and discussing their effects to the optimization is one of the contributions of this paper.

Besides the consideration of parameter uncertainty, the arrangement of objectives is another problem in the design optimization of PaQuad PM. Since load carrying capability and assembling motion stability are required, both static and dynamic performances of the PaQuad PM are considered. Several performance indices would be included, resulting in a multi-objective optimization problem. In the PM's research community, three methods are usually adopted for the arrangement of multiple performance indices, i.e. weighting factor, constraint transformation and Pareto-based methods. The first two methods involve converting multi-objective into single objective optimization problem. For instance, Miller [25] adopted weighting factors defined by importance of manipulability and space utilization. Then he carried out the single objective optimization of linear Delta PM. Concerning the determinant and condition number of stiffness matrix, Shin [26] assigned weighting factors whose sum was 1 to the objectives and implemented optimization of a 2-DoF redundantly actuated PM. In the optimization of Gough-Stewart PM, Lou [27] selected feasible workspace as the objective and regarded manipulability, stiffness and dexterity as constraints. Mainly focusing on the energy consumption of a planar PM, Kucuk [28] put the rest required performances (kinematics and dynamics) to the constraints and implemented the optimization. Either through distributing weighting factors to different performance indices or transferring less important indices to constraints, the searching of optimal parameters becomes easier since only single objective is involved. However, subjective operations are performed in these processes, such as determination of weighting factors by designer and importance ranking among performance indices. This subjective information would impose great influence to the optimizations and lead to different optimal results [20, 30].

Alternatively, Pareto-based method simultaneously optimizes several objectives. The optimal result is a cluster of solutions (named as Pareto front) that show the trade-off among objectives. Instead of defining relations before optimization like weighting factor and constraints transformation methods, the relations among multiple performance indices are decided after optimization by choosing optimal parameter set from the Pareto front. In this way, unbiased candidates for optimal results can be obtained without interference of subjective preferences. This method is highly welcome for PM optimizations involving multiple performance indices where there is no previous knowledge or requirements on their relations. For instance, Kelaiaia [31] carried out the Pareto-based optimization of Delta PM considering workspace, stiffness, dexterity and accuracy. Genetic algorithm (GA) was adopted and the final optimum was selected from Pareto front. Stiffness, dexterity and manipulability were all applied as objectives in the multi-objective optimization of a 5-DoF parallel machine tool by Zhang [19]. Pareto-based evolutionary algorithm was developed and optimal solution was chosen by a proposed indices.

Inspired by the current methods on the arrangement of performance indices, the Pareto-based method is chosen for the optimization of PaQuad PM since both stiffness and dynamic performances are assumed to be equally important. However, choosing optimal parameters from Pareto front is usually left to the designers because all the Pareto solutions are optimum from the mathematical point of view, which allows designers make their choice according to the design requirements [31]. For the optimization of the PaQuad PM whose design requirements are not specially defined, a further analysis on the objectives of the Pareto solutions are necessary. A searching principle for the best compromise among multiple performances will be proposed in the following sections, which is another contribution of this paper.

Having reviewed state of the art, this paper mainly addresses the design optimization problem of PaQuad PM for large component assembling in aviation and aerospace. Parameter uncertainty and Pareto-based method are

the two main concerns [and contributions](#). The outline of the paper is as follow. Section 2 carries out the formulation of static and dynamic performance indices, i.e stiffness, elastic dynamic and mass indices. With the consideration of parameter uncertainty, Section 3 reformulates the mapping models between design variables and performance indices, design variables and constrained conditions. Pareto-based multi-objective optimization is implemented in Section 4, where particle swarm optimization (PSO) is chosen as [the](#) algorithm. The discussion on the optimal result is shown in Section 5, as well as the effects of parameter uncertainty level to the optimization. Conclusions are drawn in Section 6.

2. Performance indices

2.1 A parallel pose adjusting mechanism

The proposed parallel pose adjusting mechanism, PaQuad PM, is as shown in Fig. 1a. It consists of a fixed base, four identical PRS limbs and an ATP. Herein, P, R and S denote actuated prismatic joint, revolute joint and spherical joint, respectively. The ATP is composed of in-part 1, in-part 2 and out-part. P joint is achieved by screw-nut and rail-slider. The screws and rails are fixed to the base whereas the nut and slider move along. The 1st and the 3rd PRS limbs connect to the in-part 1 by S joints. The in-part 1 links to out-part through helical (H) joint. Similarly, the 2nd and 4th PRS limbs are fasten to in-part 2 by S joints. An R joint is added to connect in-part 2 and out-part that is regarded as the moving platform. Due to the supports of PRS limbs, out-part is capable of two rotations and one translation. The relative translation between in-part 1 and in-part 2 makes the screw of H joint rotate, which adds the extra rotation of out-part. Therefore, [the studied PM has four degree-of-freedom](#), i.e. one translation and three rotations. The detail mobility analysis is referred to [17].

Fig. 1 The PaQuad PM. (a) Virtual prototype, (b) schematic diagram

In order to describe motions of PaQuad PM, some denotations and coordinate frames are defined as shown in Fig. 1b. The plane of fixed base is formed by connecting points A_i ($i=1,2,\dots,4$), which are symmetrically distributed on the circle with center point O and radius a . Point B_i , C_i and D_i ($i=1,2,\dots,4$) denote centers of P joint, R joint and S joint, [respectively](#). The lengths of in-part 1 and in-part 2 are both $2b$ and the initial distance between them is e . The moving distance of P joint and the length of bar are represented by q_i and l . A

fixed reference frame $O-xyz$ is assigned to point O , of which the x -axis is collinear with OB_2 and the z -axis is vertical to the fixed base. Similarly, a moving reference frame $O'-uvw$ is attached to the point O' of the out-part. Its w -axis points to the same direction as H joint, and the u -axis is parallel to D_4D_2 at home position. Meanwhile, an instantaneous frame $O'-x'y'z'$ is fixed to the point O' and their axes are parallel to those of frame $O-xyz$.

Both static and dynamic performances are concerned in the design optimization of PaQuad PM to minimize deformation and vibration during operation. Hence, mass, stiffness and natural frequency of the PaQuad PM are chosen as objectives. Based on the parametric modeling methods proposed by Sun [14], the mass, stiffness and dynamic performances of PaQuad PM is firstly studied. Owing to the breakthrough in revealing mappings between finite and instantaneous screws, the topological and parametric models of PMs are unified in the same mathematical framework. A stiffness model built by n -DoF virtual spring is then applied [32]. The complete deformations, including couplings between linear and angular deformations of PaQuad PM are expected to be obtained. A modular dynamic modeling method that improves computational efficiency is also adopted [33-36]. On the basis of the comprehensive stiffness and dynamic models, the performance indices, which will be adopted as the objectives in optimization, can be defined.

2.2 Stiffness performance indices

The stiffness modeling of the PaQuad PM has been investigated in [37]. The modeling process is divided into two phases: the formulation of compliance matrix contributed by PRS limbs (C_L) and the formulation of compliance matrix contributed by ATP (C_P). The compliance matrix of the whole PM can be computed by the addition of the C_L and C_P .

Drawing on screw theory, the procedure for computation of C_L is summarized as: (1) obtain compliance matrix of limb (C_L) in joint space by superposition of the compliance matrices of components, (2) apply Jacobian matrix to transfer C_L into stiffness matrix of all the limbs (K_L) in operated space, (3) compute the inverse of K_L and derive C_L .

Compliance matrix of the i th ($i=1,2,\dots,4$) PRS limb in the frame $O'-x'y'z'$ can be formulated as

$$\bar{C}_{L,i} = T_{pi} \bar{C}_{pi} T_{pi}^T + T_{bi} \bar{C}_{bi} T_{bi}^T + T_{si} \bar{C}_{si} T_{si}^T \quad (1)$$

where $\bar{C}_{pi}$, $\bar{C}_{bi}$, $\bar{C}_{si}$ are the compliance matrices of P joint component, R joint component and S joint component in each component reference frames. T_{pi} , T_{bi} , T_{si} are the transmission matrices from each component to point O' in the frame $O'-x'y'z'$.

$$T_p = \begin{bmatrix} R_p^{-1} & R_p^{-1} \cdot [-l_p \times] \\ \mathbf{0} & R_p^{-1} \end{bmatrix}, T_b = \begin{bmatrix} R_b^{-1} & R_b^{-1} \cdot [-l_b \times] \\ \mathbf{0} & R_b^{-1} \end{bmatrix}, T_s = \begin{bmatrix} R_s^{-1} & R_s^{-1} \cdot [-l_s \times] \\ \mathbf{0} & R_s^{-1} \end{bmatrix}$$

herein, R_p , R_b , R_s are the rotation matrix of the frame $O'-x'y'z'$ with respect to component frames. $[-l_p \times]$, $[-l_b \times]$, $[-l_s \times]$ are skew symmetrical matrices. l_p , l_b , l_s are vectors from center of each component to point O' .

Jacobian matrix of PaQuad PM is expressed as follow.

$$\$_t = J_x^{-1} J_\rho \rho, (J_x^{-1} J_\rho)^T \$_w = f \quad (2)$$

where

$$\mathbf{J}_x = \begin{bmatrix} \mathbf{J}_{xs} \\ \mathbf{J}_{xc} \end{bmatrix}, \mathbf{J}_\rho = \begin{bmatrix} \mathbf{J}_{\rho a} & \mathbf{0}_{4 \times 2} \\ \mathbf{0}_{2 \times 4} & \mathbf{0}_{2 \times 2} \end{bmatrix}, \mathbf{J}_{xs} = \begin{bmatrix} \hat{\mathbf{s}}_{ws,1}^T \\ \hat{\mathbf{s}}_{ws,2}^T \\ \hat{\mathbf{s}}_{ws,3}^T \\ \hat{\mathbf{s}}_{ws,4}^T \end{bmatrix}, \mathbf{J}_{xc} = \begin{bmatrix} \hat{\mathbf{s}}_{wc,1}^T \\ \hat{\mathbf{s}}_{wc,2}^T \end{bmatrix}, \mathbf{J}_{\rho a} = \text{diag}(\hat{\mathbf{s}}_{ws,i}^T \hat{\mathbf{s}}_{ta,i}), \boldsymbol{\rho} = \begin{pmatrix} \rho_{ta} \\ \mathbf{0}_{2 \times 1} \end{pmatrix}.$$

herein $\hat{\mathbf{s}}_{ws,i}$ ($i=1,2,3,4$) is named as single wrench offering by actuation and constrained wrenches of i th PRS limb, $\hat{\mathbf{s}}_{wc,j}$ ($j=1,2$) is called combination wrench since it is combined by constraint wrenches of opposite limbs. The details are referred to [17, 37].

The virtual work equation of limbs is formulated as

$$\mathbf{s}_w^T \mathbf{s}_t = \mathbf{f}_i^T \mathbf{A}_i \quad (3)$$

where $\mathbf{f}_i$, $\mathbf{A}_i$ are internal forces and deformations in i th PRS limb.

According to Eq. (2) and Hooke's law, the following equations are obtained.

$$\mathbf{s}_{ws,i} = \bar{\mathbf{C}}_{L,i}^{-1} \mathbf{s}_{ts,i}, \mathbf{s}_{wc,j} = (\bar{\mathbf{C}}_{L,j}^{-1} + \bar{\mathbf{C}}_{L,j+2}^{-1}) \mathbf{s}_{tc,i} \quad (4)$$

Assume that n -DoF ($n \leq 6$) virtual spring denotes the deformation of components. The twist at point O' is the superposition of the twist caused by the virtual springs in i th PRS limb ($\mathbf{s}_{t,i}^s$) and the twists of passive joints in the i th PRS limb ($\mathbf{s}_{t,i}^p$). Eq. (4) can be rewritten as

$$\mathbf{J}_x \mathbf{s}_t = \mathbf{J}_x \mathbf{s}_{t,i}^s = \mathbf{A}, \mathbf{J}_x \mathbf{s}_{t,i}^p = \mathbf{0} \quad (5)$$

Substituting Eq. (4) into Eq. (5) yields

$$\mathbf{K}_{Ls,i} = \mathbf{s}_{ws,i} (\mathbf{s}_{ws,i}^T \bar{\mathbf{C}}_{L,i} \mathbf{s}_{ws,i})^{-1} \mathbf{s}_{ws,i}^T, i=1,2,\dots,4 \quad (6)$$

$$\mathbf{K}_{Lc,j} = \mathbf{s}_{wc,j} \left[\mathbf{s}_{wc,j}^T (\bar{\mathbf{C}}_{L,j}^{-1} + \bar{\mathbf{C}}_{L,j+1}^{-1})^{-1} \mathbf{s}_{wc,j} \right]^{-1} \mathbf{s}_{wc,j}^T, j=1,2 \quad (7)$$

Therefore, $\mathbf{C}_L$ is finally computed as

$$\mathbf{C}_L = \left(\sum_{i=1}^4 \mathbf{K}_{Ls,i} + \sum_{j=1}^2 \mathbf{K}_{Lc,j} \right)^{-1} \quad (8)$$

The compliance matrix of ATP $\mathbf{C}_p$ is contributed by the two routes from the two in-parts to the reference point O' . The routes are (1) from in-part 1 through H joint to center screw finally out-part, (2) in-part 2 plus in-part 1 with C joint through R joint to out-part.

By superposition principle, compliance matrix of both routes in the moving reference frame $O'-uvw$ can be calculated as

$$\bar{\mathbf{C}}_{r,1} = \mathbf{T}_{I1,1} \mathbf{C}_{I1,1} \mathbf{T}_{I1,1}^T + \mathbf{T}_{cs} \mathbf{C}_{cs} \mathbf{T}_{cs}^T, \bar{\mathbf{C}}_{r,2} = \mathbf{T}_{I1,2} \mathbf{C}_{I1,2} \mathbf{T}_{I1,2}^T + \mathbf{T}_{I2} \mathbf{C}_{I2} \mathbf{T}_{I2}^T + \mathbf{C}_{be} \quad (9)$$

where $\mathbf{T}_{I1,1} = \begin{bmatrix} \mathbf{R}_{r1}^{-1} & \mathbf{R}_{r1}^{-1} \cdot [-\mathbf{l}_{I1,1} \times] \\ \mathbf{0} & \mathbf{R}_{r1}^{-1} \end{bmatrix}$, $\mathbf{T}_{cs} = \begin{bmatrix} \mathbf{R}_{r1}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{r1}^{-1} \end{bmatrix}$, $\mathbf{R}_{r1} = \text{Rot}(w_{a1}, \gamma)$. $\mathbf{l}_{I1,1}$ denotes the vector of center

screw from in-part 1 to out-part, $\mathbf{C}_{I1,1}$, $\mathbf{C}_{cs}$ represent the compliance matrices of in-part 1, center screw in

component reference frames. $T_{I1,2} = T_{I1,1}$, $T_{I2} = \begin{bmatrix} R_{r2}^{-1} & R_{r2}^{-1} \cdot [-I_{I2} \times] \\ \mathbf{0} & R_{r2}^{-1} \end{bmatrix}$, $R_{I1,2} = \text{Rot}(w_{a1}, \gamma_1) \cdot [-I_{I2} \times]$ is the skew symmetrical matrix relating to the vector I_{I2} from the output point of in-part 2 to point O' . $C_{I1,2}$, C_{I2} , C_{be} are compliance matrix of in-part 1, in-part 2 and R joint between in-part 1 and out-part.

Hence, compliance matrix of ATP C_P is formulated as

$$C_P = T_R \left[\left(\bar{C}_{r,1}^{-1} + \bar{C}_{r,2}^{-1} \right)^{-1} + \bar{C}_{out} \right] T_R^T \quad (10)$$

where $T_R = \begin{bmatrix} R & \mathbf{0} \\ \mathbf{0} & R \end{bmatrix}$, $\bar{C}_{out}$ is compliance matrix of out-part in moving reference frame $O' - uvw$.

With both C_L and C_P available at hand, stiffness model of PaQuad PM is expressed as

$$C = C_L + C_P, \quad K = C^{-1} \quad (11)$$

where C , K are compliance and stiffness matrix of PaQuad PM.

Based on the parametric stiffness model of PaQuad PM, instantaneous energy based stiffness indices proposed in our previous work [37] is adopted. Instantaneous energy is the virtual work of instantaneous payload and deformations. According to the Hooke's law, PM with higher stiffness would generate smaller deformations when the same external payload is exerted. Therefore, smaller instantaneous energy indicates better stiffness performances of PM. Mainly drawing on screw theory, instantaneous energy is defined as

$$E = \$^T_w \$_d = \sum_{j_l=1}^3 | \$_{wl,j_l}^T \$_{dl,j_l} | + \sum_{j_a=1}^3 | \$_{wa,j_a}^T \$_{da,j_a} |, \quad j_l = j_a = 1, 2, 3 \quad (12)$$

where $\$_{wl,j_l}$ ($\$_{wa,j_a}$) is instantaneous pure force (moment), $\$_{dl,j_l}$ ($\$_{da,j_a}$) is instantaneous angular (linear) deformations. Instantaneous energy can be expressed as Eq. (12) because force does not do work on angular deformation, so as to moment and linear deformation.

By applying eigenvector and eigenvalue of the compliance matrix C , the local stiffness index can be defined as follow.

$$\eta = \left| \sum_{i=1}^6 \left(\left(\sum_{j_l=1}^3 \rho_{l,j_l} \delta_{l,i,j_l} + \sum_{j_a=1}^3 \rho_{a,j_a} \delta_{a,i,j_a} \right)^2 \lambda_{c,i} \right) \right| \quad (13)$$

where η is the overall stiffness index of PM at a given point within workspace. ρ_{l,j_l} and ρ_{a,j_a} represent the magnitudes of $\$_{wl,j_l}$ and $\$_{wa,j_a}$. δ_{l,i,j_l} and δ_{a,i,j_a} denote the linear coefficients from eigenvector of compliance matrix C , whose computation process is shown in [37]. $\lambda_{c,i}$ is the eigenvalue of compliance matrix C .

The global stiffness index of PM is finally formulated by evaluating stiffness performance η all over the workspace as

$$\kappa_\eta = \frac{\int_V \eta dV}{V}, \quad \sigma_\eta = \frac{\sqrt{\int_V (\eta - \bar{\eta})^2 dV}}{V} \quad (14)$$

where V describes the volume of the workspace, $\bar{\eta}$ represents the mean value of η over the workspace. It was pointed out that the unit for instantaneous energy is always Joule no matter linear stiffness, angular stiffness or their combined effects are evaluated. The formulated stiffness indices are unit invariant.

2.3 Elastic dynamic performance indices

Being applied as pose adjusting mechanism, PaQuad PM *tends to carry* the large component to move slowly and precisely. In this case, the coupling effects of rigid body motion and elastic deformation can be ignored. Kineto-elastic dynamic (KED) method is adopted to build the elastic dynamic model of the PaQuad PM. The modeling process is divided into four levels, i.e. element, part, substructure and the whole mechanism. First of all, kinematic differentiation equation of element is formulated. Herein, Euler-Bernoulli beam is adopted as the basis element, whose differential motion equation is referred to [38]. Then, kinematic differentiation equation of part is assembled from the elements. According to the connections on the ajoint nodes, elastic dynamic models of substructures are assembled from parts. Finally, elastic dynamic model of the PaQuad PM is formulated by substructures considering the deformation compatibility.

Fig. 2 Substructures, parts and elements of PaQuad PM

According to the physical structure of the PaQuad PM, three substructures are divided as shown in Fig. 2. Translation and bar substructures are from the $\underline{P}$ RS limb. The former is to realize the translations of $\underline{P}$ joint while the latter connects to the R and S joint. ATP is categorized as a separate substructure because of the internal H and R joints and the special linkages between parts.

The translation substructure is constructed through screw pair and guide-slider which are represented by two and three beam elements. By assigning element coordinates, kinematic differentiation equation of elements are formulated, on which basis the one for translation substructure is assembled as

$$\mathbf{M}_{ip} \ddot{\mathbf{U}}_{ip} + \mathbf{K}_{ip} \mathbf{U}_{ip} = \mathbf{F}_{ip} + \mathbf{Q}_{ip}, \quad i=1, \dots, 4 \quad (15)$$

where $\mathbf{U}_{ip}$ is generalized coordinates of translation substructure.

$$\mathbf{U}_{ip} = [\mathbf{U}_{iN7} \cdots \mathbf{U}_{iN12} \quad \mathbf{U}_{iN16} \quad \mathbf{U}_{iN19} \cdots \mathbf{U}_{iN24} \quad \mathbf{U}_{iN28} \quad \mathbf{U}_{iN31} \cdots \mathbf{U}_{iN42}]^T_{26 \times 1}$$

$$\mathbf{M}_{ip} = \sum_{j=1}^2 \mathbf{B}_{ipj}^T \mathbf{M}_{ipj} \mathbf{B}_{ipj}, \quad \mathbf{K}_{ip} = \sum_{j=1}^2 \mathbf{B}_{ipj}^T \mathbf{K}_{ipj} \mathbf{B}_{ipj}, \quad \mathbf{F}_{ip} = \sum_{j=1}^2 \mathbf{B}_{ipj}^T \mathbf{F}_{ipj}, \quad \mathbf{Q}_{ip} = \sum_{j=1}^2 \mathbf{B}_{ipj}^T \mathbf{Q}_{ipj}.$$

herein $\mathbf{M}_{ipj}$, $\mathbf{K}_{ipj}$, $\mathbf{F}_{ipj}$ and $\mathbf{Q}_{ipj}$ are the mass matrix, stiffness matrix, external force vector and internal force vector of screw pair or guide-slider. $\mathbf{B}_{ipj}$ is the transformation matrix considering the boundary conditions and connections between parts. The screw pair is fixed to the motor at node $iN1$ and connects to the base with ball bearing at node $iN3$. Helical connection is adopted between screw pair and guide-slider. Hence,

$$[U_{iN8} \ U_{iN9} \ U_{iN11} \ U_{iN12}]^T = [U_{iN26} \ U_{iN27} \ U_{iN29} \ U_{iN30}]^T, \ U_{iN25} = U_{iN7} + \frac{P_h}{2\pi}(U_{iN28} - U_{iN10}).$$

The bar substructure is denoted by ten elements with nine nodes. With the kinematic differentiation equations of each element available at hand, differential motion equation of the bar is expressed as

$$\mathbf{M}_{ip}\ddot{\mathbf{U}}_{ip} + \mathbf{K}_{ip}\mathbf{U}_{ip} = \mathbf{F}_{ip} + \mathbf{Q}_{ip}, \ i = 5, \dots, 8 \quad (16)$$

where $\mathbf{M}_{ip}$, $\mathbf{K}_{ip}$, $\mathbf{F}_{ip}$ and $\mathbf{Q}_{ip}$ are the mass matrix, stiffness matrix, external force vector and internal force vector of the bar structure. They are assembled by kinematic differentiation equations of the ten elements.

The ATP substructure consists of in-part 1, in-part 2 and out-part. In-part 1 and in-part 2 connect to out-part by H joint and R joint, respectively. Between the two in-parts, cylindrical structure is added to ensure the stability of motion transformations. Four beam elements are assigned to represent the in-parts and two beam elements are applied to describe elastic deformations of the screw in H joint. The cylindrical structure and out-part are regarded as concentrated masses. Hence, there are eight elements in the ATP. From the differential motion equations of each element, the kinematic differentiation equation of ATP is formulated as

$$\mathbf{M}_{9p}\ddot{\mathbf{U}}_{9p} + \mathbf{K}_{9p}\mathbf{U}_{9p} = \mathbf{F}_{9p} + \mathbf{Q}_{9p} \quad (17)$$

where $\mathbf{U}_{9p}$ is a 40×1 vector representing generalized coordinates. $\mathbf{M}_{9p}$, $\mathbf{K}_{9p}$, $\mathbf{F}_{9p}$ and $\mathbf{Q}_{9p}$ are the mass matrix, stiffness matrix, external force vector and internal force vector of ATP. They are computed by the eight elements considering the relations of connecting nodes. These relations include: (1) fixed connection between screw of H joint and out-part, (2) relative rotation between in-part 2 and out part, (3) cylindrical connection between cylindrical connections and (4) helical connection between screw of H joint and in-part 1.

Based on the kinematic differentiation equations of substructures, the elastic dynamic model of the PaQuad PM can be assembled as

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F} \quad (18)$$

where $\mathbf{M}$, $\mathbf{K}$, $\mathbf{F}$ are the mass matrix, stiffness matrix and external forces of PaQuad PM.

$$\mathbf{M} = \sum_{i=1}^9 \mathbf{D}_i^T \mathbf{M}_{ip} \mathbf{D}_i, \ \mathbf{F} = \sum_{i=1}^9 \mathbf{D}_i^T \mathbf{F}_{ip} + \sum_{i=1}^4 \mathbf{A}_i^T \mathbf{Q}_{is}, \ \mathbf{K} = \sum_{i=1}^9 \mathbf{D}_i^T \mathbf{M}_{ip} \mathbf{D}_i + \sum_{i=1}^4 \mathbf{A}_i^T \mathbf{K}_{is} \mathbf{A}_i,$$

herein $\mathbf{D}_i$ and $\mathbf{A}_i$ are formulated concerning the connections between substructures. Translational substructure is connected to bas substructure by R joint, indicating that the rotations about the R joint are different but the rest are the same. The bar substructure is linked to the in-parts of ATP by S joint. It means that the three rotations at the connecting nodes are the same. The detail is referred to [39].

The natural frequency of the PaQuad PM can be further computed by Eq. (18) as

$$\det(-\omega^2 \mathbf{M} + \mathbf{K}) = 0 \quad (19)$$

where ω_f ($f = 1, 2, \dots, n$) is the f th natural frequencies.

Therefore, the elastic dynamic indices of PaQuad PM over the workspace are formulated as

$$\kappa_\omega = \frac{\int_V \omega_0 dV}{V}, \ \sigma_\omega = \frac{\sqrt{\int_V (\omega_0 - \bar{\omega}_0)^2 dV}}{V} \quad (20)$$

2.4 Mass indices

The mass of execution part of the PaQuad PM comes from the limbs and ATP. The limb mass is formulated as

$$M_{\text{limb}} = M_T + M_B + M_{sj} \quad (21)$$

where M_T , M_B and M_{sj} are the mass of translation substructure, bar substructure and S joint. And

$$M_T = \rho \left(\frac{\pi d_{sc}^2}{4} l_{sc} + \sum_{i=1}^2 l_{sdi} b_{sdi} h_{sdi} \right), M_B = \rho l_b b_b h_b, M_{sj} = M_{sj1} + M_{sj2} + \rho l_{sj} \frac{\pi d_{sj}^2}{4}.$$

herein ρ denotes the density of the 45# steel. d_{sc} , l_{sc} are the diameter and length of screw in translation substructure, l_{sdi} , b_{sdi} and h_{sdi} are the length, width and height of elements of slider. l_b , b_b and h_b are the length, width and height of bar. M_{sj1} , M_{sj2} are the mass of first and second components of S joint, whereas d_{sj} and l_{sj} are the diameter and length of the third component.

The mass of ATP is formulated as

$$M_{ATP} = M_{in1} + M_{in2} + M_{out} + M_{cy} + M_H + M_R \quad (22)$$

where M_{in1} , M_{in2} , M_{out} , M_{cy} , M_H and M_R are the mass of the in-part 1, the in-par2, the out part, the cylindrical structure, H joint and R joint.

$$M_{in1} = 2\rho l_{ip1} b_{ip1} h_{ip1}, M_{in2} = 2\rho l_{ip2} b_{ip2} h_{ip2}, M_H = \rho \frac{\pi d_{hj}^2}{4} l_{hj}.$$

herein, l_{ip1} , b_{ip1} , h_{ip1} (l_{ip2} , b_{ip2} , h_{ip2}) are the length, width and height of in-part 1 (in-part 2). d_{hj} , l_{hj} are the diameter and length of central screw in H joint. The rest masses are assessed by SolidWorks software.

The mass of the PaQuad PM can be expressed as

$$M = M_{limb} + M_{ATP} \quad (23)$$

3. Statistical objectives and probabilistic constraints

Substantial parameters are involved in the parametric models of the performance indices shown in Eq. (14), Eq. (20) and Eq. (23). The large numbers of parameters increase the difficulty of design optimization hence parameter sensitivity is carried out for selecting dominant parameters as design variables in our previous work [40]. Response surface model (RSM) method is firstly adopted to build the explicit mapping model between performance indices and all parameters. On this basis, reliability sensitivity indices are proposed to evaluate the effects of parameters to performance indices. The selected design variables of PaQuad PM are listed in Table I.

Table I Dsign variables of the PaQuad PM (units: m)

Parameter		Parameter	
a	radius of fixed base	b_b	Bar width
b	radius of moving platform	d_{sj}	diameter of S joint
l_{sd2}	vertical length of slider	l_{ip2}	length of in-part 2
h_{sd2}	vertical height of slider	h_{ip2}	height of in-part 2
b_{sd2}	vertical width of slider	b_{ip2}	width of in-part 2
l_b	bar length	d_{hj}	H joint diameter

Conventionally, the design variables are optimized with the aid of optimization algorithm. Then the optimal parameters are directly applied to build the physical prototype. During machining and assembling of parts, geometric errors are generated and accuracy improvement is implemented before putting the physical prototype into practical application. In such development procedure, however, performance changes resulted from the

parameter changes caused by geometric errors are not evaluated. In other words, parameter deviations in the construction are not considered in the optimal design.

Parameter deviation refers to the differences between optimal and actual parameters. They are unavoidable in the manufacturing of PMs. If the performance indices are sensitive to these parametric changes, the actual performances of physical prototype would largely deviate from the expected values, resulting in non-optimal performances. The situation would be worse when the constrained conditions are easily affected by the parameter changes. In this case, the actual parameters might result in violating the constrained conditions although the corresponding optimal parameters from optimization satisfy the constraints. The physical prototype couldn't be used since it cannot meet the engineering requirements. In order to prevent such disaster, parameter deviation during construction (mainly the geometric errors) would be concerned in the design optimization. For this purpose, modeling of parameter deviation and reformulation of objectives/constraints are addressed in this section.

Parameter deviations cannot be directly added to the design variables since they are unknown in the design phase. For instance, parameter deviation of the radius of the fixed base a is resulted from the geometric error whose value is controlled by machining and assembling tolerance Δa . We can predict that the actual a is within the range $[a - \Delta a, a + \Delta a]$, but we don't know the exact value. Due to this feature, parameter deviation is called parameter uncertainty in the presented study.

Mathematically, parameter uncertainty can be expressed by probabilistic simulation method. Taking a as an example, N random values are generated by specifying mean value $\bar{a}$ and standard deviation σ_a . Mean value $\bar{a}$ is given by the nominal value while standard deviation is assigned by the tolerance. If the sample size N is large enough, there is a big possibility that the minor change of a during construction is captured.

In the conventional optimization, design variables are changed by optimizer to search for the optimal parameters. Considering parameter uncertainty, the values of design variables visited by optimizer are changed into mean value of the N samples. The standard deviation of the sample is determined by probabilistic distribution of machining errors which are assumed to be subjected to normal distribution. Therefore, by applying these random samples of parameters, design variables become

$$\mathbf{X} = \{\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_j, \dots, \mathbf{X}_N\}, \quad j = 1, 2, \dots, N \quad (24)$$

$$\mathbf{X}_j = \{a_j, b_j, l_{sd2,j}, h_{sd2,j}, b_{sd2,j}, l_{b,j}, b_{b,j}, d_{sj,j}, l_{ip2,j}, h_{ip2,j}, b_{ip2,j}, d_{hj,j}\} \quad (25)$$

where $\mathbf{X}$ is the model of design variables with parameter uncertainty. $\mathbf{X}_j$ denotes the j th set of random design variables.

Affected by the N samples of design variables, there would be N performance indices and constrained conditions at each design point. For instance, $a_0 \in (a^L, a^U)$ is assigned by the optimizer during the optimization. Herein, a^L and a^U are the lower and upper bounds for radius of fixed base given by design requirements. Replace a_0 with $\mathbf{a}_0 = \{a_{0,1} \quad \dots \quad a_{0,k} \quad \dots \quad a_{0,N}\}$ and the rest design variables are addressed in the same manner. performance indices in Eq. (14), Eq. (20) and Eq. (23) become

$$\boldsymbol{\kappa}_{\eta 0} = \{\kappa_{\eta 0,1} \quad \kappa_{\eta 0,2} \quad \dots \quad \kappa_{\eta 0,k} \quad \dots \quad \kappa_{\eta 0,N}\}, \quad \boldsymbol{\sigma}_{\eta 0} = \{\sigma_{\eta 0,1} \quad \sigma_{\eta 0,2} \quad \dots \quad \sigma_{\eta 0,k} \quad \dots \quad \sigma_{\eta 0,N}\} \quad (26)$$

$$\boldsymbol{\kappa}_{\omega 0} = \{\kappa_{\omega 0,1} \quad \kappa_{\omega 0,2} \quad \dots \quad \kappa_{\omega 0,k} \quad \dots \quad \kappa_{\omega 0,N}\}, \quad \boldsymbol{\sigma}_{\omega 0} = \{\sigma_{\omega 0,1} \quad \sigma_{\omega 0,2} \quad \dots \quad \sigma_{\omega 0,k} \quad \dots \quad \sigma_{\omega 0,N}\} \quad (27)$$

$$\mathbf{M}_0 = \{M_{0,1} \quad M_{0,1} \quad \cdots \quad M_{0,k} \quad \cdots \quad M_{0,N}\} \quad (28)$$

The statistical features of these indices are evaluated to address the effects of parameter uncertainty as follows.

$$\kappa_0(\kappa_{\eta 0}) = \frac{\sum_{k=1}^N \kappa_{\eta 0,k}}{N}, \quad \sigma_0(\kappa_{\eta 0}) = \frac{\sum_{k=1}^N (\kappa_{\eta 0,k} - \kappa_0(\kappa_{\eta 0}))^2}{N} \quad (29)$$

$$\kappa_0(\kappa_{\omega 0}) = \frac{\sum_{k=1}^N \kappa_{\omega 0,k}}{N}, \quad \sigma_0(\kappa_{\omega 0}) = \frac{\sum_{k=1}^N (\kappa_{\omega 0,k} - \kappa_0(\kappa_{\omega 0}))^2}{N} \quad (30)$$

$$\kappa_0(\mathbf{M}_0) = \frac{\sum_{k=1}^N M_{0,k}}{N}, \quad \sigma_0(\mathbf{M}_0) = \frac{\sum_{k=1}^N (M_{0,k} - \kappa_0(\mathbf{M}_0))^2}{N} \quad (31)$$

At each design point during optimization, possible changes of performances would be evaluated by the mean value and standard variations of performance indices under parameter uncertainty. Those who with low mean value and high standard variations would not be chosen by optimizer. It indicates that the set of design variables are discarded if parameter uncertainty has great influence on the performance indices. The higher mean values and lower standard variations, the more reliable and [more](#) stable of the future performances of physical prototype.

Constrained conditions are modified considering parameter uncertainty in a similar manner. The conventional constrained condition $g(\mathbf{x}_0) \leq 0$ becomes

$$P(g(\mathbf{X}_0) \leq 0) \geq P_R \quad (32)$$

where $P(g(\mathbf{X}_0) \leq 0)$ is the probability of N sets of design variables satisfy $g(\mathbf{x}_0) \leq 0$, $P_R \in (0,1)$ is a predefined value given by designer.

Monte Carlo Simulation is [adopted](#) for the probabilistic computation as follows

$$P(g(\mathbf{X}_0) < 0) = \frac{1}{N} \sum_{i=1}^N I(\mathbf{X}_0) \quad (33)$$

where $I(\mathbf{X}_0)$ is an indicator function.

$$I(\mathbf{X}_0) = \begin{cases} 1 & g(\mathbf{X}_0) \leq 0 \\ 0 & g(\mathbf{X}_0) > 0 \end{cases} \quad (34)$$

Assume that $N = 10,000$, $P_R = 0.95$. Eq. (33) means that the design point $\mathbf{x}_0$ would have little influence on the constrained condition if [more than](#) 95% of the 10,000 sets of random design variables satisfy $g(\mathbf{X}_0) \leq 0$.

In this way, performance indices and constrained conditions are reformulated considering the parameter uncertainty. They are names as statistical objectives and probabilistic constraints.

4. Pareto-based optimization

When directly applying statistical objectives and probabilistic constraints to the optimization, the [calculation](#) is time consuming and sometimes difficult to be performed if we want a large sample size for better estimation of parameter uncertainty. To deal with this issue, the RSM model applied in the sensitivity analysis [41] is adopted again to build the explicit mapping model between performance indices and design variables. RSM is the technique of building approximate model through data training. Herein, data training is implemented by design of experiment (DoE) and the RSM models are obtained through least square calculation. Mathematically, the RSM models are expressed as

$$f_{\text{linear}}(\mathbf{x}) = a_0 + \sum_{i=1}^{12} b_i x_i, \quad f_{\text{quadratic}}(\mathbf{x}) = a_0 + \sum_{i=1}^{12} b_i x_i + \sum_{i=1}^{12} c_i x_i^2 + \sum_{i=1}^{12} \sum_{i < j}^{12} d_{ij} x_i x_j, \quad (35)$$

$$f_{\text{cubic}}(\mathbf{x}) = a_0 + \sum_{i=1}^{12} b_i x_i + \sum_{i=1}^{12} c_i x_i^2 + \sum_{i=1}^{12} \sum_{i < j}^{12} d_{ij} x_i x_j + \sum_{i=1}^{12} e_i x_i^3 \quad (36)$$

$$f_{\text{quartic}}(\mathbf{x}) = a_0 + \sum_{i=1}^{12} b_i x_i + \sum_{i=1}^{12} c_i x_i^2 + \sum_{i=1}^{12} \sum_{i < j}^{12} d_{ij} x_i x_j + \sum_{i=1}^{12} e_i x_i^3 + \sum_{i=1}^{12} f_i x_i^4 \quad (37)$$

where $\mathbf{x}$ denote the design variables. a , b_i , c_i , d_{ij} , e_i and f_i are calculated regression coefficients. $x_i x_j$ denotes interactions of any two parameters. x_i^2 , x_i^3 and x_i^4 represent the second, third and fourth order nonlinearity.

Extra data is required to assess the accuracy of the RSM models. Four metrics, relative average absolute error (RAAE), relative maximum absolute error (RMAE), root mean square error (RMSE) and R square (R^2) are applied as

$$\text{RAAE} = \frac{\sum_{q=1}^{m_l} |y_q - \hat{y}_q|}{\sum_{q=1}^{m_l} |y_q - \bar{y}|}, \quad \text{RMAE} = \frac{\max\{|y_q - \hat{y}_1|, \dots, |y_q - \hat{y}_{m_l}|\}}{\sum_{q=1}^{m_l} |y_q - \bar{y}| / m_l}, \quad (38)$$

$$\text{RMSE} = \sqrt{\frac{\sum_{q=1}^{m_l} (y_q - \bar{y})^2}{m_l}}, \quad R^2 = 1 - \frac{\sum_{q=1}^{m_l} (y_q - \hat{y}_q)^2}{\sum_{q=1}^{m_l} (y_q - \bar{y})^2} \quad (39)$$

where y_q denote the exact value at evaluation parameter set q , $\hat{y}_q$ is the calculated value by the surface function, $\bar{y}$ is the mean value of y_q , m_l is the number of additional evaluation parameter sets. In general, a large value of R^2 and small values of RAAE, RMAE, RMSE are preferred [42].

With the aid of Isight software, the RSM models of performance indices are formulated. Note that linear stiffness along each axis is required to be larger than certain values. The RSM models of minimum linear stiffness are also calculated. The accuracy of these RSM models is shown in Table II. The selected models are painted gray in the table.

Table II Accuracy assessment of performance indices and constrained conditions

	Order	κ_η	κ_ω	M	k_{Lx}	k_{Ly}	k_{Lz}
RAAE	linear	0.15155	0.07052	0.02075	0.0789	0.08473	0.06258
	quadratic	0.03118	0.07476	0.00096	0.02467	0.02629	0.04894
	cubic	0.01954	0.09199	0.00092	0.02527	0.03007	0.04022
	quartic	0.02049	0.08047	0.00103	0.02348	0.02191	0.04651
RMAE	linear	0.47332	0.58342	0.05513	0.25277	0.27843	0.2421
	quadratic	0.13967	0.70735	0.0508	0.19097	0.18894	0.2067
	cubic	0.18967	0.66039	0.0426	0.20618	0.20871	0.281
	quartic	0.19441	0.67017	0.0516	0.15002	0.14778	0.36324
RMSE	linear	0.19178	0.11896	0.02521	0.10412	0.10966	0.09061

R ²	quadratic	0.04077	0.11766	0.00137	0.03504	0.03696	0.06384
	cubic	0.03448	0.15552	0.00125	0.03754	0.04089	0.05994
	quartic	0.03097	0.12803	0.00134	0.03178	0.03067	0.07089
	linear	0.5557	0.51231	0.99407	0.87857	0.86354	0.86882
	quadratic	0.96554	0.28159	0.99997	0.98349	0.98134	0.88027
	cubic	0.97127	0.28673	0.99998	0.97993	0.97698	0.89676
	quartic	0.97114	0.24439	0.99997	0.98352	0.98457	0.82969

The workspace of PaQuad PM and the engineering requirements are shown in Table III. Herein, α , β and γ are rotational angles about x , y and z -axis. k_{lx} , k_{ly} and k_{lz} are the minimal required linear stiffness along each axis. Referring to the Pareto-based method, objectives shown in Eq. (29) to Eq. (31) are simultaneously applied. The optimization problem can be formulated as

$$\begin{aligned} & \max \{ \kappa(\kappa_\eta), \kappa(\kappa_\omega), \kappa(\mathbf{M}) \} \& \min \{ \sigma(\kappa_\eta), \sigma(\kappa_\omega), \sigma(\mathbf{M}) \} \\ & \left\{ \begin{aligned} & s.t. \quad P(k_{lx}(\mathbf{X}) - 0.5 \geq 0) \geq P_R, P(k_{ly}(\mathbf{X}) - 0.5 \geq 0) \geq P_R \\ & \quad P(k_{lz}(\mathbf{X}) - 10 \geq 0) \geq P_R \\ & \quad \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U \end{aligned} \right. \end{aligned} \quad (40)$$

where the standard variance of the parameter uncertainty is set as 0.03 considering machining/assembling errors and the sampling size is $N = 10,000$. The acceptable probabilistic level P_R is given as 0.95. $\mathbf{x}^L$ and $\mathbf{x}^U$ denote the lower and upper bounds of design variables that are given in Table I.

Table III Workspace and engineering requirements of PaQuad PM

workspace				Engineering requirement		
α (°)	β (°)	γ (°)	z (mm)	k_{lx} (N/ μ m)	k_{ly} (N/ μ m)	k_{lz} (N/ μ m)
40	40	360	500	0.5	0.5	10

Therefore, at each design point, 10,000 sets of design variables are randomly generated centering on the design point within variance range 0.03. Linear stiffness along each axis under random design variables is calculated to check if they meet the requirements. When 95% of the random variable sets make the linear stiffness satisfy the demands, corresponding design point is assumed to be a candidate that engineering constraints would not be violated under parameter change in application phase. Then mean values and standard variances of stiffness, elastic dynamic and mass under the random samples are evaluated. The optimizer will choose solutions with higher mean values and lower standard variances as Pareto points, which indicates reliable performances that won't be affected by the future parameter changes.

During this process, the optimizer is driven by particle swarm optimization (PSO). It is a population-based stochastic optimization technique inspired by social behavior of bird flocking or fish schooling. PSO has the merits such as fast convergence to global optimum, easy implementation and few parameters to adjust thus widely applied in the optimization of PMs [43-44]. The optimization flow is shown in Fig. 3.

With the aid of Matlab and Isight software, the mulit-objective PSO algorithm is set according to the parameters in Table IV. 100 particles are assigned to run 100 iterations, hence there are $100 \times 100 = 10,000$

design variable sets would be assessed by PSO. At each design point, $N = 10,000$ random design variable sets would be generated by MCS.

Fig. 3 Flow chart of PSO algorithm

Table IV Settings of multi-objective PSO algorithm

Parameter	Setting
Maximum iteration	100
Number of particles	100
Inertia weight	0.729
Global learning coefficient	1.49
Personal learning coefficient	1.49

5. Result and discussion

It is found from the optimal results that the standard variance of stiffness, mass and elastic dynamic performances with parameter uncertainty remain zero. That is to say, the unavoidable parameter changes in the application phase would not impose any influence to the concerned performances. Selection of optimal result relies mainly on the mean values of performance indices.

Pareto front among mean values of performance indices are shown in Fig. 4. The ideal optimum is the Pareto point with maximum first natural frequency κ_{ω} , minimum overall instantaneous energy κ_f and minimum

execution mass M . However, due to the conflicting features among the performance indices, it is impossible for these objectives simultaneously obtaining ideal values with the same parameter set. For instance, increasing structural parameters is beneficial for rigidity and stiffness of PaQuad PM, but it would also increase the mass of the whole mechanism, which is against the requirements on light weight structure. This in return proves the necessity of applying Pareto-based method in the optimization of PaQuad PM.

Fig 4 Pareto front of optimization considering stiffness, mass and first frequency under parameter uncertainty

Pareto front is a cluster of solutions that none of the objectives can be further improved without a decrease of the remaining objectives. It shows the compromise among multiple objectives. Designer can choose any Pareto point as the final optimum according to the application requirements. In our previous work, a cooperative equilibrium searching method [20] is proposed for the case when there is no preference or the objective are assumed to be equally important. The procedure is illustrated as follow.

First of all, the ideal optimum is constructed by the best value of each objective. On the Pareto front (κ_ω , κ_η , M), the maximum first frequency corresponds to the point (167.7699Hz, 0.6428×10^{-6} J, 947.2457kg), the minimum instantaneous energy is the point (85.7289, 0.02700, 806.1609) and the minimum execution mass is the point (25.9202Hz, 3.7997×10^{-6} J, 645.6126kg). The ideal point is formed by selecting the best performance indices as (167.7699Hz, 0.02700×10^{-6} J, 645.6126kg).

Then, all the objectives on Pareto front are made to be dimensionless by

$$\hat{\kappa}_{\omega,f} = \frac{\kappa_{\omega,f} - \bar{\kappa}_\omega}{\tilde{\kappa}_\omega}, \quad \hat{\kappa}_{\eta,f} = \frac{\kappa_{\eta,f} - \bar{\kappa}_\eta}{\tilde{\kappa}_\eta}, \quad \hat{M}_f = \frac{M_f - \bar{M}_f}{\tilde{M}_f}, \quad f = 1, 2, \dots, n_p \quad (41)$$

where $\bar{\kappa}_\omega$, $\bar{\kappa}_\eta$ and $\bar{M}_f$ are the mean value of each objective on the Pareto front, $\tilde{\kappa}_\omega$, $\tilde{\kappa}_\eta$ and $\tilde{M}_f$ are the corresponding standard variance, n_p is the total number of the Pareto front. This step is to eliminate the effects of different units among multiple performance indices.

Next, distances from the Pareto points to the ideal optimum are computed by

$$D_f = \sqrt{(\hat{\kappa}_{\omega,f} - \max(\hat{\kappa}_{\omega,f}))^2 + (\hat{\kappa}_{\eta,f} - \min(\hat{\kappa}_{\eta,f}))^2 + (\hat{M}_f - \min(\hat{M}_f))^2} \quad (42)$$

Finally, cooperative equilibrium point ($\kappa_\omega = 136.6057$ Hz, $\kappa_\eta = 0.6473 \times 10^{-6}$ J, $M = 760.8232$ kg) is defined as the Pareto point with minimum distance $\min(D_f) = 1.4718$. It indicates that this Pareto point is the closet

solution to the ideal optimum and has the best compromise among all the objectives. Therefore, corresponding structural parameters are the optimal parameters of the optimization, which is shown in Table V.

Table V Ranges of structural parameters and the optimal parameters (unit: m)

	Lower bound	Upper bound	CEP		Lower bound	Upper bound	CEP
a	0.2	0.4	0.2	b_b	0.08	0.18	0.08
b	0.2	0.4	0.2	d_{sj}	0.06	0.18	0.18
l_{sd2}	0.08	0.15	0.15	l_{ip2}	0.55	0.8	0.55
h_{sd2}	0.06	0.12	0.06	h_{ip2}	0.04	0.08	0.076
b_{sd2}	0.06	0.15	0.15	b_{ip2}	0.2	0.4	0.2
l_b	0.45	0.65	0.65	d_{hj}	0.02	0.4	0.04

* CEP denotes the cooperative equilibrium point

Fig 5 Distribution of Pareto points with standard variance 0.03 and acceptable probability 0.95

From the optimal parameters, it is found that most of them are driven to the limits. For instance, a , b , h_{sd2} , l_{ip2} , b_{ip2} are exactly the values of lower bounds while l_{sd2} , b_{sd2} , l_b , d_{sj} chose the upper bounds. Only h_{ip2} and d_{hj} are the values within the ranges, but they are also very close to the bounds. In the conventional optimization without consideration of parameter uncertainty, if the design variables close to the boundary are chosen as optimal parameters and applied to build physical prototype, the unavoidable parameter change in application phase would easily make these parameters out of the required ranges. In this case, it is difficult to make sure that the performances are still as expected and the constraints are not violated. In our optimization method, the uncertainty has been considered and the possible changes of the chosen parameters would not impose influence to the performances and the constraints. The physical prototype is reliable under minor parameter changes during construction.

Fig. 5 shows the 2D version of the Pareto front. It is found from the distribution of Pareto points that instantaneous energy κ_η is the dominant objective since most of the Pareto points are located in the area where κ_η is smaller. Probability of linear stiffness along each axis is computed at all the 10,000 design points chosen by PSO. These calculated probabilities are assessed whether to meet requirements. There are only 2753 feasible design points, indicating that the constraints play a key role in optimization since abandoned design points violating constraints are twice more than the feasible points. The values of feasible points are illustrated in Fig. 6. Probabilistic constraints of most feasible points are 1, implying that physical prototype built by these design

points won't be affected by minor parameter changes in application phase. The numbers of points that make the value of probabilistic constraints located between 0.95 to 1 are $P(k_{lx}(\mathbf{X})-10 \geq 0)$, $P(k_{ly}(\mathbf{X})-0.5 \geq 0)$, $P(k_{lz}(\mathbf{X})-0.5 \geq 0)$ in descending order. Probabilistic constraints about linear stiffness along z -axis of several design points are very close to the boundary $P_R = 0.95$. It reveals that the linear stiffness along z -axis is easier to be changed under parameter uncertainty, which needs special attention in the future construction.

Fig 6 Distribution of probabilistic constraints of feasible points

In order to investigate the effects of parameter uncertainty samplings to the optimal results, standard variance are given as 0.01 and 0.06 and the Pareto fronts are shown in Fig. 7 and Fig. 8. When the standard variance is set as 0.01, 0.03 and 0.06, the number of Pareto points is 114, 79 and 58. It implies that options for optimal solutions become less with the increasing of parameter uncertainty level. The distributions of Pareto points for the three cases are similar. Most of them are located in the area where κ_η is smaller. The cooperative equilibrium point $(\kappa_\omega, \kappa_\eta, M)$ for the three cases are $(143.1045\text{Hz}, 0.7273 \times 10^{-6}\text{J}, 755.0992\text{kg})$, $(136.6057\text{Hz}, 0.6473 \times 10^{-6}\text{J}, 760.8232\text{kg})$ and $(110.2216\text{Hz}, 0.7061 \times 10^{-6}\text{J}, 738.1848\text{kg})$. As the parameter uncertainty level goes up, the first frequency κ_ω drops, instantaneous energy κ_η decreases then increases while execution mass M increases and then drops. In general, performances of the PM have to be sacrificed to keep the reliability of optimum if parameter uncertainty level is increasing. Since different parameter uncertainty level would lead to different optimal results, a more accurate estimation on parameter uncertainty is required for more precise optimization in the future.

Fig 7 Distribution of Pareto points with standard variance 0.01 and acceptable probability 0.95

Fig 8 Distribution of Pareto points with standard variance 0.06 and acceptable probability 0.95

6. Conclusion

This paper dealt with the static and dynamic optimization of a parallel pose adjusting mechanism called PaQuad PM being applied as an attachment to a big serial robot of a macro-micro robotic system. The contributions of the proposed optimization method are the arrangements of design variables and objectives. For the design variables, parameter uncertainty resulted from minor parameter changes during construction is considered. For the objectives, Stiffness, elastic dynamic and mass performances are simultaneously optimized by Pareto-based method. Conclusions are drawn as follow:

- (1) Performance indices are firstly formulated based on the performance modeling. Parametric stiffness model of the PaQuad PM is computed by the compliance models of PRS limbs and ATP. Then instantaneous energy based stiffness index is adopted for evaluating overall stiffness performance within predefined workspace. Elastic dynamic model of PaQuad PM is formulated from beam element, parts, substructures and the whole mechanism. First natural frequency is employed as elastic dynamic performance indices. Execution mass of PaQuad PM is defined as the mass index.
- (2) Parameter uncertainty is modeled by generating random samples to the design variables. These random samples are defined by specifying mean value and standard variance of design variables. Based on the design variables with parameter uncertainty, mean values and standard variance of performance indices are formulated and adopted as statistical objectives by MCS. Probabilistic constraints that evaluating numbers of random design variables satisfying engineering requirements are computed. By the statistics theory and probability calculation, parameter uncertainty is considered in the design optimization, making the optimal solution not being affected by minor parameter changes in construction.
- (3) After Pareto front is obtained with the aid of RSM method and PSO, a cooperative equilibrium point is defined as the optimal solution that has the best compromise among multiple objectives. The analysis of optimal parameters and the distribution of probabilistic constraints further confirm the necessity of considering parameter uncertainty in design optimization.

The proposed optimization procedure provides reliable optimal parameters that would not be affected by minor parameter changes in construction, and also unbiased optimal result that denote the best compromise among multiple performances. The optimization method is applicable to any PM. In future work, a more

precise knowledge on the probabilistic distribution of geometric errors would be investigated to address the more accurate estimation of parameter uncertainty of PMs.

Acknowledgements

This research work was supported by the National Natural Science Foundation of China (No. 51875391), the Jing-Jin-Ji District Special foundation for Key Research Program of Tianjin (No. 17JCZDJC40400), the Major program and Special foundation for the New Generation of Intelligent Technology of Tianjin (No. 18ZXZNGX00350), the Key Development and Major Technological Support Program of Tianjin (No. 18YFZCSF00590) and the International Postdoctoral Exchange Fellowship Program (No. 32 Document of OCPC, 2017).

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