Modelling of turbulent gas-particle flow

by

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Abstract

An Eulerian-Eulerian model for dilute gas-particle turbulent flows is developed for engineering applications. The aim is to understand the effect of particles on turbulent flows. The model is implemented in a finite element code which is used to perform numerical simulations. The feedback from the particles on the turbulence and the mean flow of the gas in a vertical channel flow is studied. In particular, the influence of the particle response time and particle volume fraction on the preferential concentration of the particles near the walls, caused by the turbophoretic effect is explored. The study shows that the particle feedback decreases the accumulation of particles on the walls. It is also found that even a low particle volume fraction can have a significant impact on the turbulence and the mean flow of the gas. A model for the particle fluctuating velocity in turbulent gas-particle flow is derived using a set of stochastic differential equations. Particle-particle collisions were taken into account. The model shows that the particle fluctuating velocity increases with increasing particle-particle collisions and that increasing particle response times decrease the fluctuating velocity.

Descriptors: turbulent gas-particle flows, modelling, turbophoresis, two-way coupling

Preface

This thesis studies turbulent gas-particle flows. In the first part a short review of the basic concepts and methods is presented. The second part consists of the following papers:

Paper 1. Strömgren T., Brethouwer G., Amberg G. and Johansson A. V., 2008
"Modelling of turbulent gas-particle flows with focus on two-way coupling effects on turbophoresis", Submitted to AIChE Journal,

Paper 2. Strömgren T., Brethouwer G., Amberg G. and Johansson A. V., 2008
"Modelling of particle fluctuations in turbulence by stochastic processes", Technical report, Linné Flow Centre, Dept. of Mechanics, KTH, Stockholm Sweden

Division of work between authors

The research project was initiated by Gustav Amberg (GA) and Arne Johansson (AJ) who also acted as supervisors. GA, AJ, Geert Brethouwer (GB) and Tobias Strömgren (TS) have continuously discussed the progress of the project during the course of the work.

Paper 1

The code development and calculations was done by TS with feedback from GA. The paper was written by TS with inputs from GB, GA and AJ.

Paper 2

The derivation was done by TS with input from GA. The paper was written by TS with input from GB, GA and AJ.

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Part I

Introduction

CHAPTER 1

Introduction

Turbulent gas-particle flows are found in a wide variety of areas in nature and industry. The dispersion of atmospheric aerosols, volcanic ash eruptions and dust storms are examples of gas-particle flows in nature. Engineering applications are for example spray drying, separation of particles in cyclones, circulating fluidized beds and pneumatic transport of metal powders.

The nature of single-phase turbulent flows is complex and difficult to predict due to the non-linearity and the wide range of length- and time scales. A turbulent gas flow with dispersed particles not only inherits all of the difficulties from single-phase turbulent flows but also the complexity of the interaction between the dispersed and the gas phase. The particles are transported by the mean flow and dispersed by the turbulence. On the other hand the flow will be affected by the particles due to exerted drag and by particle-particle collisions for increasing particle concentrations. The presence of particles can significantly enhance or reduce the turbulent intensity. The overall behaviour of a turbulent gas-particle laden flow depends on many parameters, for example volume fraction of particles, particle size and density and wall-roughness if walls are present. In this study rather high Reynolds numbers will be studied which can lead to complicated phenomena such as preferential concentration of particles in regions with low turbulent intensity.

To be able to predict and understand turbulent gas-particle flows engineering models are of great importance. Many industrial applications can be developed using such models. However, turbulent gas-particle flows are difficult to model due to their complexity and their sensitivity to particle properties and volume fractions.

The present thesis is part of a long term project, with the aim to develop turbulence models for dilute particle-laden flows. In the present work the effect of particle feedback on the gas-phase turbulence and mean flow is investigated. It is also studied how the preferential concentration of particles in wall bounded flows is affected by particle response time and particle volume fraction. The influence of particle-particle collisions on the ratio of turbulent and particle kinetic energy is investigated by use of stochastic differential equations.

CHAPTER 2

Gas-particle flow

2.1. Forces on a single particle

To be able to describe a flow with dispersed particles and the effect of the particles on the flow the forces acting on a single particle in a flow field need to be examined. A particle could for example be dust, flyashes or metal powder. Hereafter the gas and particle phase will also be referred to as the continuous and dispersed phase, respectively.

The particles are assumed to be spherical. The equation of motion of a small rigid spherical particle in an unsteady flow for small particle Reynolds numbers ($Re_p << 1$) is described by the well known Basset-Boussinesq-Oseen equation. Here $Re_p = \frac{D_p|u_p - u_g|}{\nu_g}$ where D_p is the particle diameter, u_p is the particle velocity, u_g is the gas velocity and ν_g is the kinematic viscosity of the gas phase. An expression for this equation has been derived by Maxey & Riley (1983). Here it is presented without including the effects of a non-uniform flow

$$\begin{aligned}\frac{du_p}{dt} =& \frac{\rho_g}{\rho_p}\frac{Du_g}{Dt} - \frac{1}{2}\frac{\rho_g}{\rho_p}\frac{d}{dt}(u_p - u_g) - \frac{1}{\tau_p}(u_p - u_g) \\ &+ \frac{18}{4}\frac{\mu_g}{D_p\rho_p}\int_{-\infty}^{t}\frac{\frac{d}{d\tau}(u_p - u_g)}{(\pi\nu_g(t-\tau))^{\frac{1}{2}}}d\tau + (1 - \frac{\rho_g}{\rho_p})g \qquad (2.1)\end{aligned}$$

where ρ_g and ρ_p are the density of the gas and particle phase, respectively, μ_g is the dynamic viscosity of the gas phase and $\tau_\mathrm{p} = \frac{\rho_\mathrm{p} \mathrm{D}_\mathrm{p}^2}{18\mu_\mathrm{g}}$ is the particle response time, which is the time it takes for a particle to adjust itself to the fluid velocity. The first term on the right hand side (R.H.S) is the fluid acceleration term. When a particle accelerates or decelerates it also accelerates or decelerates the surrounding fluid. This results in a force on the particle which is represented by the second term on the R.H.S, sometimes called the added mass term. The third term on the R.H.S. is the Stokes drag, a force due to viscous drag and pressure forces on the particle, without the correction given by Faxén (Crowe *et al.* 1998). This force acts in the opposite direction to the added mass force. If $Re \sim O(1)$ or larger Stokes drag is not a good approximation and the term must be multiplied with a drag factor. Schiller & Naumann (1933) presented a drag factor f valid for particle Reynolds numbers up to 1000

$$f = 1 + 0.15 Re_p^{0.687}. \qquad (2.2)$$

There are also corrections for non-spherical particles but only spherical particles are considered here. When a particle is accelerating or decelerating a force called the history or Basset force is caused by the lagging of the boundary layer development on the particle. This term is represented by the fourth term on the R.H.S. For larger particle Reynolds numbers there are no satisfactory models for this contribution. The last term on the R.H.S. is the body force due to gravity. Other body forces could be coulomb forces or termophoretic forces but are neglected here.

The fluid acceleration and added mass terms are only of importance when $\rho_p/\rho_g \simeq 1$, that is for liquid-solid flows. The history and added mass forces can only make a contribution if the gas- or particle flow is accelerating or decelerating (unsteady flow). For large density ratio between the particle and gas phase, which is the case considered here, the dominating forces are the drag and the gravitational forces.

Particles moving in a shear flow experience a lift force due to the non-uniform pressure distribution around the particle. This can be of importance in wall bounded flows (Young and Leeming 1997). Saffman (1965) and Saffman (1968) derived an expression for the lift force which is only valid for certain conditions such as low Reynolds numbers. In a wall boundary layer this lift force is towards the wall if the particles are leading the flow and in the opposite direction if the particles lag the flow. Correction functions for higher Reynolds numbers have been derived but are not satisfactory (Sommerfeld *et al.* 2007).

A rotating particle in a flow may experience a lift force called Magnus force, due to the deformation of the flow field around the particle that causes a pressure difference between the two sides of the particle. Since particles can acquire high angular velocity after wall collisions this effect can be of importance in the near wall region (Sommerfeld 2003).

In the present study only the drag- and gravitational forces are taken into account. The lift forces could have a contribution near the walls in shear flows but are neglected for the moment. In this study it is thus assumed that the equation describing the motion of particles in a gas flow for Re_p up to 1000 is approximated by

$$\frac{du_p}{dt} = -(1 + 0.15Re_p^{0.687})\frac{1}{\tau_p}(u_p - u_g) + (1 - \frac{\rho_g}{\rho_p})g \tag{2.3}$$

2.2. Coupling between the phases

For small particles and small volume fractions of particles the continuous phase is unaffected by the presence of particles. This is called one-way coupling because no momentum is transferred from the particles to the continuous phase. If the particle volume fraction increases so that particle mass fraction is significant the momentum transfer from the dispersed- to the continuous phase will be large enough to affect the turbulent flow field. This is called two-way coupling. If the particle volume fraction increases even more the collisions between particles will produce significant stresses in the particle phase and the distance between the particles decreases. This is called four-way coupling. For large volume fractions the particle motion is controlled by particle-particle collisions, but this is outside the scope of this thesis. Here the focus will be on relatively dilute flows where the particle motion is mainly governed by hydrodynamic forces. The different regimes and the corresponding particle volume fractions, ϕ, are illustrated in figure 2.1.

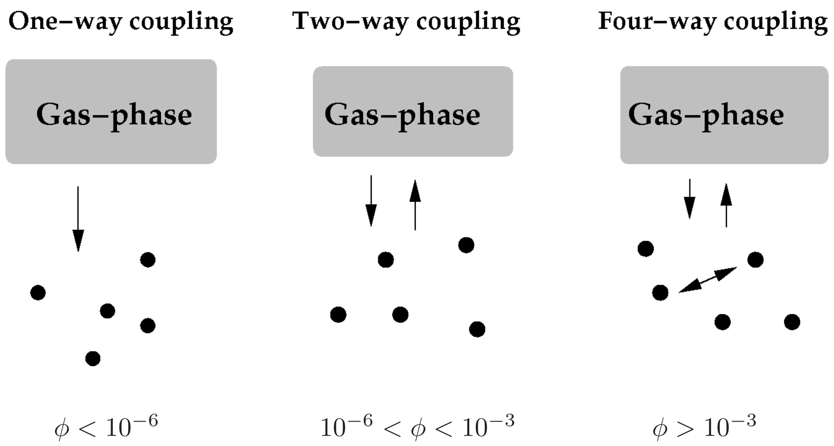

FIGURE 2.1. *Different regimes of interaction between particles and the continuous phase with corresponding particle volume fractions, ϕ.*

Particle-laden flows in the one-way coupling regime are relatively well understood. However, in the two-way coupling regime the interaction between the phases have a highly nonlinear nature. How the gas-phase turbulence is affected by the particle-phase depends on particle volume fraction, particle size and flow configuration.

2.3. Turbulence

Flows are in general turbulent when the Reynolds number of the flow is sufficiently high. The turbulence in single-phase (unladen) flow is disordered, consists of a wide range of time- and length scales and is unpredictable. The understanding of the physics of turbulence in unladen flows is still incomplete despite all the research that has been done the last century.

2.4. Effect of turbulence on particles

Particle inertia affects the diffusion of particles in turbulence. The parameter establishing the degree of particle dispersion is the Stokes number St which is the ratio between the particle response time to the flow time scale, τ_g, i.e. $St = \frac{\tau_p}{\tau_g}$. Particles with $St << 1$ respond instantaneously even to the smallest eddies of the flow and as a result the turbulent particle diffusion is the same as for the fluid. For $St >> 1$ particles are not affected by the turbulence they will retain the memory of their previous velocity and their turbulent diffusion is thus approximately zero. Particles with $St \sim 1$ will filter high frequency turbulence fluctuations and will be centrifuged to the peripheries of the turbulent structures. In this regime the turbulent particle diffusion will be larger than the turbulent diffusion of the fluid because particles have smaller velocity fluctuations but larger autocorrelation time than the flow (Reeks 1977). The memory of a particle to its previous velocity increases with increasing Stokes number (inertia) as can be seen in figure 2.2 (Squires & Eaton 1991).

Experiments and numerical simulations have highlighted that particles with $St \sim 1$ tend to have a convective drift in non-homogeneous turbulent flows as they move from regions with high gas turbulence intensity to regions with low gas turbulence intensity. This is because particles are "thrown" out from regions of high turbulence intensity to regions of lower turbulence intensity where there are no eddies with enough energy to disperse the particles back. This leads to a mean migration of particles counter to the fluctuating velocity gradient, which is often referred to as the effect of turbophoresis. In a channel- or pipe-flow particles with St ~ 1 accumulate in the viscous wall-layer close to the wall and possibly deposit on the walls (Fessler *et al.* 1994). This non-zero wall-normal mean particle velocity is counteracted by diffusion (Haarlem *et al.* 1998).

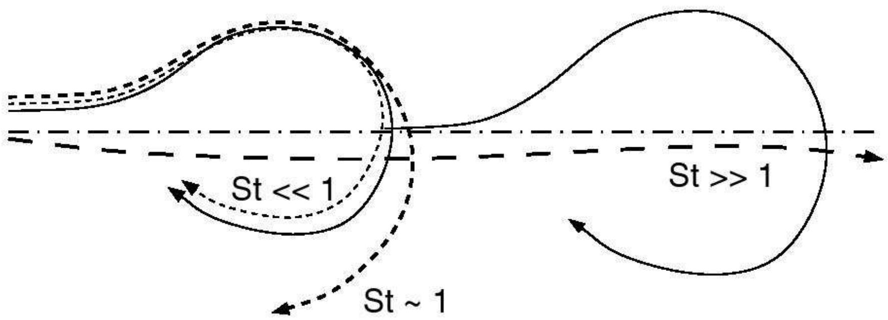

FIGURE 2.2. *Effects of Stokes number on particle dispersion, from Crowe* et al. *(1998).*

A model for particle transport to the wall and deposition was presented by Friedlander & Johnstone (1957). The distance where the drift towards the wall starts depends on the particle inertia and the intensity of the turbulent fluctuation velocity. Reeks (1983) described dispersion of discrete particles in a turbulent shear flow. An addition to the normal diffusion transport was found to be proportional to the gradient of the local turbulence velocity correlation in a direction that transports particles from high to low turbulence intensities and was referred to the as effect of turbophoresis.

Many studies of turbophoresis have been conducted through experiments, DNS and modelling, for example Fessler *et al.* (1994), Liu & Agarwal (1974), Young & Hanratty (1991) Li *et al.* (2001), Vance *et al.* (2006), Botto *et al.* (2005), Vreman (2007), Young and Leeming (1997), Cerbelli *et al.* (2001) and Slater *et al.* (2003). However, most studies of turbophoresis are about flows with very small particle volume fractions where the effects of particle feedback on the gas-phase can be neglected. Li *et al.* (2001) studied the effects of turbophoresis for larger particle volume fractions where the effects of two-way coupling must be taken into account. Most models used to study turbophoresis assume perfectly absorbing walls, i.e. particles are removed when they reach the wall (particle deposition). Cerbelli *et al.* (2001) assumed that particles are elastically reflected at the wall. In real world collisions are not totally elastic nor are all particles deposited at the wall so a more realistic boundary condition should lie somewhere between these two.

The "crossing trajectories effect" is when the particles have a mean relative velocity with respect to the gas flow (Reeks (1977), Squires & Eaton (1991)). Gravity, for example can lead to particles crossing the turbulent vortices relatively fast. The consequence is that the vortices have only a short time to affect the particles leading to reduced turbulent particle dispersion (Wells & Stock 1983).

2.5. Turbulence and how it is affected by the particle-phase

2.5.1. *How particles affect the turbulence*

Depending on mass-loading, particle size, particle density and turbulence intensity, particle-laden flows can show turbulence intensities that are significantly reduced or enhanced compared to an unladen case. Experiments have shown that small particles tend to attenuate the turbulence while larger particles augment the turbulence (Tsuji *et al.* (1984), Kulick *et al.* (1994)). Kulick *et al.* (1994) showed through experiments that the gas-turbulence intensity is significantly reduced, up to 80 % for small particle mass loadings (~ 0.1), and that the attenuation is stronger for increasing distance from the wall, Stokes number and particle mass loading. Gore & Crowe (1991) analysed available data of pipe-flows and showed that the ratio between particle diameter to turbulent length scale is the most appropriate parameter to correlate the increase or decrease of turbulent intensity due to particles. However, the magnitude of turbulence attenuation or enhancement does not depend in an obvious way on any simple set of parameters.

There are two basic physical mechanisms for the turbulence attenuation in wall-bounded flows (Vreman 2007). The first is that the mean slip velocity between the gas and dispersed phase is non-uniform and large close to the wall. This is due to particle slip at the wall and leads to momentum transfer in the direction of the wall. The second mechanism is that the particle-fluid interaction leads to an extra term in the turbulent kinetic energy equation that will decrease turbulent kinetic energy. However, the presence of large particles can augment the gas-turbulence due to turbulence produced in the wakes behind the particles. Hadinoto *et al.* (2005) showed in experiments that above a certain Reynolds number the gas phase turbulence intensity increases with increasing Reynolds number, see also the experiments by Hwang & Eaton (2006).

2.5.2. *How is the mean-profile affected by particles?*

Nearly all experiments of wall-bounded particle laden flows have found that the fluid-velocity profiles are slightly flattened by the particles (Tsuji *et al.* (1984), Kulick *et al.* (1994)). An explanation of this is that particles keep their high streamwise velocity as they migrate from the core of the pipe to the wall because of their high inertia. When the particle inertia increases, the particles will retain more and more of their streamwise bulk velocity on their way to the boundary layer (Johansen 1991). Consequently, the particles have a higher velocity than the gas near the wall and thus increase the gas-velocity there through the drag force (Vreman 2007). As the particle volume fraction increases the effect on the mean flow increases.

2.5.3. *Particle-particle collisions and wall roughness*

As particle volume fraction increases particle-particle collisions can have a significant influence on the particle behaviour and the particle phase dynamics. Particle-particle collisions occur due to relative motion between particles that can be caused by several mechanisms, for example Brownian motion of particles, laminar or turbulent fluid shear or particle inertia in turbulent flows (Sommerfeld 2000). The importance of particle-particle collisions for rather low volume fractions ($\sim 10^{-4}$) has been highlighted by Vance *et al.* (2006) and Li *et al.* (2001). In wall-bounded flows wall roughness can have a strong effect on the particle fluctuating motion and affect the particle-wall collisions (Sommerfeld 1992).

CHAPTER 3

Numerical simulation of turbulent particle-laden flow

Numerical simulation of the continuous phase in turbulent particle-laden flows is often divided into three categories: Direct Numerical Simulation (DNS), Large Eddy Simulation (LES) and closure models. Extensive reviews on modelling and simulation of turbulent dispersed two-phase flows can be found in Elghobashi (1994), Mashayek & Pandya (2003) and Loth (2000).

3.1. Direct numerical simulations

In DNS the governing equations are solved without any models. All scales are resolved and all details of the flow are captured. However, DNS is computationally very expensive and is only feasible for simple geometries and rather low Reynolds numbers. Therefore, it is not suitable for any engineering applications, but DNS is very useful for getting a basic physical understanding of dispersed turbulent two-phase flows and can support the development of closure models. The particle phase is treated in a Lagrangian context, i.e. the particle phase is represented by a number of particles whose trajectories x_{pi} are computed by integrating:

$$\frac{dx_{pi}}{dt} = U_{pi} \tag{3.1}$$

and equation (2.3) simultaneously. If the particle size is smaller than the smallest length scales of the flow, particles can be considered as 'point' particles, i.e. the flow around each particle is not resolved. Most DNS-studies do not take into account the feedback from the particles on the flow (one-way coupling). However, Vreman (2007) and Li *et al.* (2001) have taken into account the feedback from the particles on the flow (two-way coupling). Examples of DNS studies are Haarlem *et al.* (1998), Rouson & Eaton (2001), Narayanan *et al.* (2003) and Botto *et al.* (2005).

3.2. Large eddy simulations

In LES the large scales are explicitly resolved, while the smaller scales are unresolved and accounted for by closure models. Thus LES can be used for higher Reynolds numbers and more complex geometries compared to DNS. In LES of turbulent particulate flows, the particle phase is treated in a Lagrangian way. Examples of LES in the literature are Vance *et al.* (2006), Yamamototo *et al.* (2001) and Wang & Squires (1996).

3.3. Closure models

In closure models the governing equations are averaged and modelled before they can be solved. This method is the least computationally demanding and can be used for engineering problems. However, unclosed terms will appear that need to be modelled. Two approaches can be distinguished. The particle phase can either be considered as a continua similar to the continuous phase or represented by discrete particles. These are the Eulerian-Eulerian approach and the Eulerian-Lagrangian approach, respectively. In the latter approach the number of particles is limited because each particle is tracked, but more physics of the particles can be modelled such as particle rotation and wakes behind particles. Eulerian-Lagrangian models for turbulent dispersed two-phase flows have been developed by for example Sokolichin & Eigenberger (1997) and Böttner & Sommerfeld (2002).

In the Eulerian-Eulerian approach both phases are considered as a continuum and obey conservation equations of mass and momentum. The variables are an average over a control volume that is much larger than the particle size and much smaller than a characteristic length scale of the fluid. For the particle phase this requires that the particle concentration is large enough to ensure that the ensemble averaging is statistically meaningful. The instantaneous mass and momentum equations read

$$\frac{\partial}{\partial t}(\phi_k\rho_k) + \frac{\partial}{\partial x_j}(\phi_k\rho_k u_{kj}) = 0 \tag{3.2}$$

$$\frac{\partial}{\partial t}(\phi_k\rho_k u_{ki}) + \frac{\partial}{\partial x_j}(\phi_k\rho_k u_{ki}u_{kj}) = \frac{\partial}{\partial x_j}(\phi_k\tau_{kij}) + \phi_k(-1)^k\frac{f_i}{\tau_p} + \phi_k\rho_k g_i \tag{3.3}$$

where the index k=1 represents the continuous phase, k=2 represents the particle phase, ρ is the density, u is the instantaneous velocity, τ_{ij} is the stress tensor, $\mathrm{g_i}$ is the gravitational acceleration and f_i is the drag term. For the particle-phase the stress tensor in dilute flows only takes into account the pressure gradient of the continuous flow. For denser flows ($\phi_p > 10^{-3}$) particle-particle collisions and the influence of surrounding particles on the relative velocity of an individual particle must be taken into account. Here we consider a dilute flow giving the following definition of the stress tensor:

$$\tau_{kij} = -p + \nu_k\rho_k(\frac{\partial u_{ki}}{\partial x_j} + \frac{\partial u_{kj}}{\partial x_i}). \tag{3.4}$$

where p is the pressure and ν is the kinematic viscosity. The global continuity implies that

$$\phi_1 + \phi_2 = 1 \tag{3.5}$$

For a complete derivation of the equations governing two-phase flow see for example Anderson & Jackson (1967), Ishii (1975), Enwald *et al.* (1996) and Jackson (1997). For turbulent gas-particle flows there are two commonly used averaging methods: Reynolds averaging (non-weighted) and a volume fraction weighted averaging ($\sim$ Favre averaging) defined as

$$F_i = \frac{< f_i \phi >}{< \phi >} \tag{3.6}$$

where $< \cdot >$ indicates time- or ensamble-averaging. The variables can be decomposed according to Reynolds decomposition into a mean and a fluctuating term, i.e. $u_i = U_i + u'_i$ where u_i is the instantaneous value, U_i is the mean value and u'_i is the fluctuating part. In the same way ϕ is decomposed into a mean Φ and a fluctuation ϕ', i.e. $\phi = \Phi + \phi'$. Using the volume fraction averaging method (3.2) and (3.3) become

$$\frac{\partial}{\partial t}(\Phi_k \rho_k) + \frac{\partial}{\partial x_j}(\Phi_k \rho_k U_{kj}) = 0 \tag{3.7}$$

$$\begin{aligned} &\frac{\partial}{\partial t}(\Phi_k \rho_k U_{ki}) + \frac{\partial}{\partial x_j}(\Phi_k \rho_k U_{ki} U_{kj}) = -\frac{\partial}{\partial x_j}(\Phi_k P_k) \\ &+ \frac{\partial}{\partial x_j}(\nu_k \Phi_k (\frac{\partial U_{ki}}{\partial x_j} + \frac{\partial U_{kj}}{\partial x_i})) - \frac{\partial}{\partial x_j}(\rho_k \Phi_k < u'_{ki} u'_{kj} >) \\ &+ \Phi_k (-1)^k \frac{f_i}{\tau_p} + \Phi_k \rho_k g_i \end{aligned} \tag{3.8}$$

The third term on the right hand side (the Reynolds stress) is an unclosed term. To solve this a closure model is needed. The most common approach to close the Reynolds stress is the Boussinesq hypothesis,

$$< u'_{ki} u'_{kj} > = -\nu_{kT} \frac{\partial U_{ki}}{\partial x_j} \tag{3.9}$$

where ν_T is the turbulent viscosity determined by the velocity and length scales of the large energetic eddies of the turbulence. The eddy viscosity of the gas phase can be modelled through use of one-equation or more commonly by two-equation models with equations for both these quantities. Examples of two-equation models are the $K - \omega$ or $K - \epsilon$ models, where K is the turbulent kinetic energy, ω is the inverse time scale of the turbulence and ϵ is the energy dissipation rate. The equation for K_g derived from (3.8) reads

$$\begin{aligned} &\rho_g \frac{\partial K_g}{\partial t} + \rho_g U_{gj} \frac{\partial K_g}{\partial x_j} = 2 \rho_g \nu_{tg} S_{ij} S_{ij} - \rho_g C_\mu \omega K_g \\ &+ \frac{\partial}{\partial x_j}[\rho_g(\nu_g + \frac{\nu_{tg}}{\sigma_K}) \frac{\partial K_g}{\partial x_j}] - (1 + 0.15 Re_r^{0.687}) \frac{\rho_p}{\tau_p} \Phi_p (< u'_{gi} u'_{gi} > - < u'_{pi} u'_{gi} >) \end{aligned} \tag{3.10}$$

where $S_{ij} = \frac{1}{2}(\frac{\partial U_{gi}}{\partial x_j} + \frac{\partial U_{gj}}{\partial x_i})$, $C_\mu = 0.09$ and $\sigma_K = 2$. The last term in (3.10) represents the interaction between the gas phase and the particle phase and will only appear when two-way coupling is taken into account. This implies that the mean kinetic energy of the gas phase is affected by the particle phase.

For turbulent dispersed particle flows usually a simple model is used for the particle turbulent viscosity. An often used model is $\nu_{Tp} = \frac{\nu_{Tg}}{1 + (\frac{\tau_p}{T_L})^2}$ where ν_{Tp} is the turbulent particle viscosity and T_L is the Lagrangian time scale of the fluid (Choi & Chung 1983). This approach is valid for small particles and rather small volume fractions. The mean particle kinetic energy can be modelled as

$$K_p = < u'_{pi} u'_{pi} > = \frac{K_g}{1 + \frac{\tau_p}{T_L}}, \tag{3.11}$$

also used by Tu & Fletcher (1994) and Young and Leeming (1997). For a derivation see Hinze (1959). To improve prediction a similar two-equation model to that used for the gas phase can be used for the particle phase. Wang *et al.* (1997) developed a $K - \epsilon - K_p - \epsilon_p$ model where mean turbulent kinetic energy and energy dissipation transport equations for the particle phase and the gas phase are similar.

A shortcoming of the Boussinesq assumption is the poor description of strongly anisotropic turbulence. Reynolds stress transport models that solve transport equations for the Reynolds stresses are better suited to model strongly anisotropic turbulence. An intermediate type of models is the Explicit Algebraic Reynolds Stress Model (EARSM) that uses an explicit relation between the Reynolds stress anisotropy and the mean strain and rotation rate tensors. This kind of model can capture the physics in strongly anisotropic turbulence by only solving two transport equations for the turbulence quantities (two-equation models) (Wikström *et al.* 2000). Reeks (1993) used a kinetic approach to evaluate the Reynolds stresses and compared it with the Boussinesq assumption and found that his approach was better for anisotropic turbulence.

If (3.2) and (3.3) are Reynolds averaged instead of Favre averaged more unclosed terms will appear. If comparison to experiments are made Favre averaging is to prefer because experimental results are usually Favre averaged. Elghobashi & Abou-Arab (1983) discuss the unclosed terms in Reynolds averaged equations and their modelling.

Examples of models using Eulerian-Eulerian modelling are given by Elghobashi & Abou-Arab (1983), Kataoka (1986), Kataoka & Serizawa (1989), Young and Leeming (1997), Guha (1997), Slater *et al.* (2003) and Shin *et al.* (2003). Most models use a two-equation model for the gas phase turbulence and a simpler model for the particle phase. For small particle volume fractions and small particles this approach can give reasonably good results.

For increasing particle volume fractions when particle-particle collisions must be taken into account kinetic-theory based models can be used, see Zhang & J.M.Reese (2003), Reeks (1991), Lun *et al.* (1984) and Hrenya & Sinclair (1997). In the limit of very large particle concentrations the flow becomes a granular flow. Modelling of particle-particle collisions can either be described in a deterministic or in a stochastic way. In a deterministic model the particles are tracked and when particles collide a new velocity is calculated using Newtons equations of motion (Lun & Liu 1997). In stochastic modelling the probability of particle-particle collisions are calculated from kinetic-theory (Sommerfeld 2003). Modelling of particle-wall collision dynamics and wall roughness is found to have a very large impact on the particle velocity statistics in the flow (Sommerfeld (1992), Konan *et al.* (2006), Vreman (2007)).

CHAPTER 4

Numerical treatment

The system of partial differential equations that describes turbulent gas-particle flow, presented in paper 1, are solved numerically by using finite elements. The system of equations on weak form are specified in a Maple worksheet together with boundary conditions, initial conditions and the method used to solve each equation. The femLego toolbox then generates a finite element code in C/fortran (Amberg *et al.* 1999). Triangular elements were used with piecewise linear base functions for both the velocity and the pressure. The solver is based on a projection method for incompressible Navier-Stokes (Guermond & Quartapelle 1997) where a sequence of decoupled equations for velocity and pressure is solved at each time step.

Each equation leads to a linear system of algebraic equations that must be solved. For dense matrices direct methods are preferred. The unsymmetric multifrontal method (UMFPACK) provides fast and accurate direct solvers (Davis 2004). For sparse matrices iterative methods are often used such as the generalised minimum residual method (GMRES) and the conjugate gradient method (CG). The CG method is applicable to symmetric and positive definite matrices and was used for the pressure equation. GMRES was used for the momentum equations and the turbulence quantities.

In the momentum equations the convective term is written semi-explicitly which allows larger time step. To obtain stability in the convective terms streamline diffusion is used which adds stability without sacrificing accuracy (Hansbo & Szepessy 1990). A stabilisation term is also added to the pressure- and particle volume fraction equation.

The two-way coupling problem can give rise to numerical difficulties. In order to highlight how the two-way coupling is handled in the present work a simplified two-way coupled system with just the advective term on the left hand side and the Stokes coupling on the right hand side is shown below

$$\frac{U^{n+1} - U^n}{dt} = -\frac{1}{\tau}(U^{n+1} - V^n) \tag{4.1}$$

$$\frac{V^{n+1} - V^n}{dt} = \frac{1}{\tau}(U^{n+1} - V^{n+1}) \tag{4.2}$$

where U is the gas velocity, V the particle velocity and n denotes the time step. Here all terms in the Stokes coupling are implicit except for the particle velocity in the gas-phase equation which is explicit. It can be shown with von Neumann stability analysis that this system is stable for all dt.

A channel geometry was constructed with uniform spacing in the streamwise direction and refined spacing was used in the wall-normal direction. In the streamwise direction cyclic boundary conditions were used, i.e. the values in the nodes at the end of the channel were copied to the beginning of the channel.

The inverse time scale of the turbulence ω has the following near wall behaviour

$$\omega = \frac{6\nu}{\beta y^2} \tag{4.3}$$

where ν is the gas kinematic viscosity, β is a constant and y is the wall distance. This causes numerical difficulties in the near-wall region since $\omega \to \infty$ as $y \to 0$. In order to capture the rapid growth of ω in the near wall-region a decomposition was introduced, $\omega = \tilde{\omega} + \omega_w$ (Gullman-Strand *et al.* 2004). An equation is solved for $\tilde{\omega}$ which is zero at the wall. ω_w is chosen according to (4.3).

CHAPTER 5

Summary of Papers

5.1. Paper 1

In paper 1 an Eulerian-Eulerian model was developed for an upward turbulent gas-particle flow in a vertical channel. A $K - \omega$ model was used to describe the gas phase turbulence. The feedback from the particles on the flow was taken into account. The model results were in good agreement with available DNS data. The aim of the study was to investigate the difference between simulations with one-way coupling , i.e. with no feedback of the particles on the flow, and simulations with two-way coupling, i.e. with feedback of the particles on the flow, for different volume fractions and particle diameters. Also the effect of two-way coupling on the preferential concentration of particles near the wall due to the turbophoretic effect was studied.

The simulations with the model developed here showed that two-way coupling decreases the concentration of particles in the near-wall region due to a decrease in the mean particle kinetic energy which drives the turbophoresis. Increased particle volume fraction also decreased the preferential concentration of particles in the near wall region. These two effects are shown in figure 5.1. The model simulations also showed that the particle concentration in the near wall region increases for increasing particle response times up to a certain value whereafter it decreases. This is due to a diminishing interaction between turbulence and particles when timescales becomes disparate. Depending on the particle diameter, the mean gas velocity was up to 30% lower in simulations with two-way coupling in comparison with simulations with only one-way coupling for a volume fraction of $2 \cdot 10^{-4}$, see figure 5.2. For increasing particle diameter the effect of particles on the mean flow decreases. Two-way coupling effects are thus substantial even at rather low particle volume fractions, and thus need to be taken into account in engineering models.

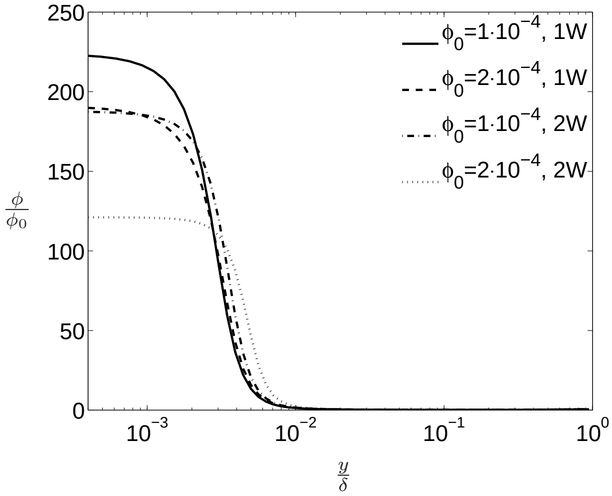

FIGURE 5.1. *Simulated profiles of the particle volume fraction ϕ_p normalised with the initial volume fraction, ϕ_0, for one- and two-way coupling cases and two different initial particle volume fractions. The distance to the wall is scaled with the half-channel width.*

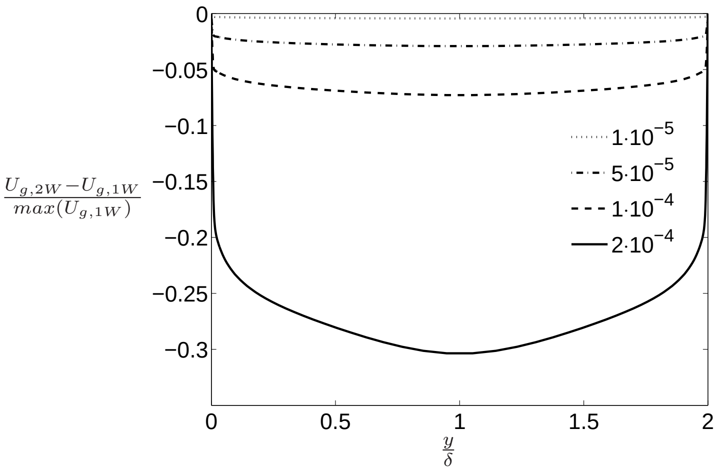

FIGURE 5.2. *Difference between mean gas velocities obtained from simulations with one- and two-way coupling and a particle diameter of 40μm as a function of the distance to the wall y scaled with the half-channel width. The mean gas velocity difference is scaled with the maximum mean gas velocity for one-way coupling.*

5.2. Paper 2

In paper 2 a model for the turbulent kinetic energy of the particle phase is derived based on stochastic differential equations. The model takes into account both the effect of the turbulence on the particle phase and the diffusion due to particle-particle collisions. It is found that particle-particle collisions can significantly increase the particle kinetic energy in turbulence. The study also shows that the particle kinetic energy decreases with increasing particle response times. For quite large particle volume fractions the particle kinetic energy reaches its maximum at particle response times greater than zero.

Outlook

The current model will be used on a backward facing step which is a more complex configuration. Comparison will be made to available experimental data and also to other models.

In practical applications of turbulent gas particle flows the anisotropy of the turbulence and the mean shear can be substantial and have a significant effect on particle fluxes. These have not been taken into account in the present model. Highly anisotropic turbulent flows with strong mean shear and passive scalar fluxes in the flows can be accurately described by Explicit Algebraic Reynolds Stress Models (EARSM). An idea is to apply this modelling approach to turbulent gas-particle flows.

The derivation of mean particle kinetic energy using a stochastic approach will also be developed.

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