Bidding models for bond market auctions

KRISTOFER ENGMAN
Bidding models for bond market auctions

KRISTOFER ENGMAN
Abstract

In this study, we explore models for optimal bidding in auctions on the bond market using data gathered from the Bloomberg Fixed Income Trading platform and MIFID II reporting. We define models that aim to fulfill two purposes. The first is to hit the best competitor price, such that a dealer can win the trade with the lowest possible margin. This model should also take into account the phenomenon of the Winner’s Curse, which states that the winner of a common value auction tends to be the bidder who overestimated the value. We want to avoid this since setting a too aggressive bid could be unprofitable even when the dealer wins. The second aim is to define a model that estimates a quote that allows the dealer to win a certain target ratio of trades. We define three novel models for these purposes that are based on the best competitor prices for each trade, modeled by a Skew Exponential Power distribution. Further, we define a proxy for the Winner’s Curse, represented by the distance of the estimated price from a reference price for the trade calculated by Bloomberg which is available when the request for quote (RFQ) arrives. Relevant covariates for the trades are also included in the models to increase the specificity for each trade. The novel models are compared to a linear regression and a random forest regression method using the same covariates.

When trying to hit the best competitor price, the regression models have approximately equal performance to the expected price method defined in the study. However, when incorporating the Winner’s Curse proxy, our Winner’s Curse adjusted models are able to reduce the effect of the Winner’s Curse as we define it, which the regression methods cannot. The results of the models for hitting a target ratio show that the actual hit ratio falls within an interval of 5% of the desired target ratio when running the model on the test data. The inclusion of covariates in the models does not impact the results as much as expected, but still provide improvements with respect to some measures. In summary, the novel methods show promise as a first step towards building algorithmic trading for bonds, but more research is needed and should incorporate more of the growing data set of RFQs and MIFID II recorded transaction prices.
Budgivningsmodeller för auktioner på obligationsmarknaden

Sammanfattning


När målet är att träffa det bästa konkurrentpriset ger regressionsmodellerna ungefär samma resultat som expected price-modellen som definieras i denna studie. När man däremot integrerar effekten av Winner’s Curse med den definierade indirekta variablen kan vår Winner’s Curse-justerade modell minska effekten av Winner’s Curse, vilket regressionsmetoderna inte kan. Resultaten av modellerna som ämnar vinna en förbestämd andel av transaktionerna visar att den faktiska andelen transaktioner som man vinner faller inom ett intervall på 5% kring den önskade andelen när modellen körs på testdata. Att inkludera kovariat i modellerna påverkar inte resultaten till den grad som uppskattades, men ger mindre förbättringar med avseende på vissa måttal. Sammanfattningsvis visar de nya metoderna potential som ett första steg mot att bygga algoritmisk handel för obligationer, men mer forskning behövs och bör utnyttja mer av den växande datamängden av RFQs och MIFID II-rapporterade transaktionspriser.
Acknowledgements

A big thank you to my mentors Morten Karlsmark and Victor Shcherbakov, and the rest of the Fixed Income Quant team: Jonas Nilsson, Stefan Sandberg and Fredrik Jäfvert. It has been both educational and a lot of fun to work with you, and your support has been invaluable. Also, a thanks to some additional people that have been very helpful during the process: Jacob Hallmer, David Rydberg, Patrik Karlsson, Hanna Hultin, Ibrahim Senyuz, Claes Cramer, and many others at SEB. I would finally like to express my gratitude for the opportunity to write my thesis at SEB.

Lastly, I want to thank my supervisor at the Royal Institute of Technology, Pierre Nyquist, for the advice and support during and before the start of the study.
# Contents

1 Introduction  

2 Theory  
   2.1 Dealer markets  
   2.2 Dealer markets as an inventory optimization problem  
   2.3 Pricing of fixed-coupon bonds  
   2.4 Multi-Dealer-to-Customer platforms  
   2.5 MIFID II  
   2.6 Skew Exponential Power (SEP) distribution  
   2.7 Noncentral t-distribution  
   2.8 Optimization methods  
   2.9 Model evaluation methods  
   2.10 Regression Trees  

3 Methodology  
   3.1 Data  
   3.2 Modelling  
   3.3 Testing  

4 Results  
   4.1 Comparison of parametric distributions  
   4.2 Hitting the cover price  
   4.3 Hitting the target ratio  
   4.4 Feature importance of covariates  

5 Conclusions  

6 References  

7 Appendix
1 Introduction

The recent years of heavy regulatory pressure have caused a need for rapid digitization of trading operations within the financial industry. Due to regulations such as MIFID II, which requires trade information to be reported shortly after execution, securities trading has been forced to become more digitized. As a result, electronic trading has in the last century engulfed the traditional trading practices, and have irrevocably changed the landscape for trading. Previously, trading was exclusively done through dealer-customer trades, often through so-called voice trading [1]. This market was defined by dealers, persons who hold an account of securities such that they can sell to or buy from their customers and provide market making, and customers such as asset managers, pension funds and corporations. Today, most trades are done electronically through platforms such as Bloomberg, and the distinction between dealer and customer has become less clear. However, the rate of electronification is largely dependent on the liquidity of the security that is traded. While equity and foreign exchange trading quickly have become almost fully electronified, less liquid securities such as some fixed income instruments are lagging behind [1]. To facilitate this process, one important step is to investigate and define models for bidding in competitive bond market auctions, which is the purpose of this report.

Fixed income instruments are products that provide a stream of cash flows through periodic payments along with an eventual return of a notional at maturity. The risk associated with these instruments is primarily related to the credit risk of the issuer and interest rate movements. In the case of the issuer defaulting, the periodic payments will be lost, and the notional may be as well depending on the seniority of the instrument. Most of these instruments are therefore rated to assess the inherent credit risk. At the same time, interest rate movements will affect the relative value of the security compared to other securities, which in turn affects its price. One of the most traded fixed income instruments is the bond.

The basic concept of the bond is simple. Say a company is in need of more funds to invest in a new technology to expand its operations. The company can then decide to issue a bond, or several bonds, to secure these funds. The bond represents a loan to the company, the bond issuer, from the buyer of the bond, the bondholder. In return, the bond issuer will pay interest in the form of coupon payments to the bondholder. At some time in the future, at the maturity of the bond, the loan, the notional, is paid back to the bondholder. While the above description captures the essence of the bond, there are many nuances to bonds that make them more complex.

Some examples of bond issuers are corporations, governments, municipalities, and banks. Depending on the issuer, the credit risk and the terms of the bond contract may vary greatly. The maturity of the bond is the date when the bond contract ends, and by extension the time-to-maturity, the time during which the bond contract lasts, is a key feature of any bond. This will determine the number of coupon payments that will be
made, and also the period under which the contract is at risk. The coupon rate of the bond decides the amount that is paid in the periodic payments of the bond, and the notional is the amount that is "lent" by the bondholder to the issuer, which the issuer will pay back to the bondholder at maturity. However, the terms of repaying the notional can be different depending on the type of bond.

The main difference between bonds and stocks is their heterogeneity and liquidity. While stocks only reference one company, a bond for the same company (or other type of issuer) may have different maturities, coupons, notionals and contractual terms. Evidently, this plethora of similar bonds causes each single bond to be less liquid than stocks, with a few exceptions such as US treasury bonds and other benchmark bonds. This is one of the reasons why electronification of bond trading has lagged behind many other asset types.

In the fixed income sphere, trading was traditionally divided into two segments; the dealer-customer segment where dealers trade with their client, and the inter-dealer segment, where dealers trade between one another. In the dealer-customer segment, customers contacted a dealer when they wanted to purchase or sell a security and were offered a quote, so the market was quote-driven, meaning that prices were only revealed to potential customers when asked for. Recently, fixed income trading has become increasingly order-driven, where indicative prices for some sample notionals are streamed continuously by dealers, and trades are often executed through so-called Multi-Dealer-to-Customer (MD2C) platforms such as Bloomberg Fixed Income Trading (Bloomberg FIT), Tradeweb or MarketAxess. On these platforms, customers can post a request for quote (RFQ) to several dealers simultaneously, and thus starting an e-auction for the security in question. The contacted dealers may be presented with composite prices for the relevant security provided by the platform and some other information about the bond and the RFQ, but are otherwise blind to what the other dealers will quote.

The study of MD2C platforms for trading is a recent phenomenon since the platforms themselves have not existed for long. However, this type of trading platform can be likened to common value auctions, which have been extensively studied. Common value auctions are auctions where the value of the item on sale is the same to all bidders, but each bidder has a different guess of the true value. Since pricing of financial instruments is relatively standardized on the market, we can assume that the price of the bond to all bidders will be the same, but the value estimate of the instrument depends on the dealers’ expectations of the future return of the instrument. Thus, when averaging over all bidders the value estimate should be unbiased, which would correspond to a common value auction scenario.

A recurring phenomenon present in common value auctions is the Winner’s Curse. This was first observed by Capen et al. in their analysis of auctions of parcels of land for oil drilling. Given the difficulty of estimating the amount of oil in the parcel of land, the valuations made by experts from the companies participating in the auction will differ quite a lot. However, if we assume that the valuations are unbiased with common
mean, then Capen et al. mean that the winner of the auction tends to be the company whose expert estimated the highest value of the parcel of land. In a later paper, Thaler summarizes the phenomenon and describes that the winner of a common value auction will suffer from one of two consequences [3]

1. The winning bid exceeds the value of the parcel of land, i.e. the company loses money.
2. The value of the parcel of land is less than the expert’s estimate, i.e. the company will be disappointed even if they make a profit.

In either case, the company will be at a loss, so the winner can be said to be "cursed". Capen et al. find three general rules of bidding in auctions under competition [2]

1. The less information one has compared to the competitors, the lower one should bid.
2. The more uncertain one is about the value estimate, the lower one should bid.
3. The more bidders (above three), the lower one should bid.

The first two rules are quite evident, but the third may be less obvious. In this case, Capen et al. mean that "The more serious bidders we have, the further from the true value we expect the top bidder to be." [2] In other words, as the number of competitors increase, it is increasingly unprofitable to attempt to win the auction. This phenomenon was also found by Kagel and Levin, who analyzed auctions before the publication and widespread adoption of the Winner’s Curse phenomenon in auction bidding theory. In their study, they find that auctions with a large number of bidders (6 to 7 bidders) have more aggressive bidding than auctions with fewer bidders (3 to 4 bidders) [4]. To provide good pricing in the e-auctions on MD2C platforms, it would be of interest to incorporate the effect of the Winner’s Curse in bidding models to reduce the risk of suffering from its consequences.

Since we are considering common value auctions of trading financial instruments, it is also important to understand the process of trading and how it is modeled. There have been several analyses of the processes of trading and how to do it optimally, but these have primarily taken on the economic perspective of supply and demand in the form of an inventory problem common in market microstructure modelling. Ho and Stoll are well known for their research on this subject. In one of their studies, they derive models for the optimal bid and ask prices that optimize the dealer’s expected utility dependent on his current position, with stochastic demand and return on stock [5]. In a later paper, they analyze the behaviour of competing dealers on the market, again through an inventory problem formulation and derive reservation bid and ask prices of dealers [6]. These papers provide a useful theoretical framework for market microstructure modelling, that is still used in papers published today.

Previous studies have also been performed in the context of electronic trading in a limit order book (LOB), where limit orders are aggregated. Oomen models the properties
of execution in an aggregator, and also observe the Winner’s Curse in the case where many liquidity providers (LP) are competing in the LOB [7]. Similarly, Avellaneda and Stoikov presented an inventory-based strategy for submitting bid and ask quotes in a LOB where transactions arrive as a Poisson stochastic process [8].

A recent study published by Fermanian et al. presents a modelling framework for dealer behaviour on a MD2C platform on the corporate bond market [9]. They describe likelihood functions for different outcomes of the auctions, and use RFQ data from Bloomberg FIT to fit the trade data to distributions that they deem suitable given the data set and required financial assumptions using maximum-likelihood. They model the dealer quotes with a Skew Exponential Power (SEP) distribution, and the customer’s reservation price, the ’worst’ price the customer is ready to accept, with a Gaussian distribution. The models are then extended such that covariates containing more information, such as the credit rating of the bond and notional, are included in the model and also introduce a model that incorporates the probability of a dealer participating in the RFQ when they are requested. The authors furthermore define distributions to estimate hit ratios and the ’best’ price that will be quoted among the competitors in the trade. The resulting models are examined with respect to the number of dealers requested for the trade to see how the dealer behaviour is affected, and they find that it indeed has an effect.

In this study, we will examine a data set of RFQs from the MD2C platform Bloomberg FIT combined with post-trade transparency reporting data from MIFID II. This combination allows us to see the best prices competitors have posted both in the case where the inspected dealer has won the RFQ, and in the case where the inspected dealer has lost the RFQ. With this data, we will define models that aim to fulfill two goals. The first aim is to hit the cover price of a trade, which would allow one to win the RFQ with the smallest possible margin. We would also like to define a pricing model for hitting the cover price that at the same time takes into account the effect of the Winner’s Curse. The model should reduce the risk of suffering from the phenomenon while still retaining sufficient accuracy in hitting the cover price. The second aim is to define a model that allows the user to decide a target ratio of RFQs that they want to win, and then achieves a similar hit ratio on a set of trades. It would also be interesting to incorporate the Winner’s Curse effect in this model such that the dealer can protect itself from overpricing.

We will firstly fit a parametric distribution to the best prices posted out of all competitors in each RFQ, where the data is partitioned with respect to the requested number of dealers and the side of the trade. This represents the density of the best price that will be sent out of all competitors in an RFQ, and can be used as a base for creating optimal bidding models. To account for the Winner’s Curse, a proxy will be defined using a reference price available to the bond trader when an RFQ is received. Using the best competitor price density, we can define the probability of winning given that a certain quote is sent. We can use the probability of winning and the Winner’s Curse proxy to minimize the risk of being afflicted by the Winner’s Curse phenomenon in a trade using an optimization methodology. By defining a target ratio of trades the
dealer wants to win, we can use a similar optimization set up to estimate the quote that corresponds to winning a certain target ratio of RFQs. Furthermore, additional information included in the RFQ can be utilized as covariates to increase the accuracy of the resulting models.

Since MD2C platforms are a relatively new phenomenon, there have only been a handful of studies using this data. However, there has yet to be any study to combine this data set with trade prices yielded from the MIFID II regulatory reporting. Combining these data sets should provide a further insight into how competitors set their bids on an MD2C platform.

The rest of the study will be structured as follows. Section 2 will describe some preliminaries to establish the basis for the modelling, followed by the methodology used to set up the models in Section 3. Finally, the results will be presented in Section 4 and conclusions in Section 5.
2 Theory

We will start by establishing some preliminaries for the concepts used in the modelling methodology. Firstly, we will look at some economic theory, presenting how bonds are priced along with a description of how dealer markets work and can be described mathematically. Thereafter, the main mathematical backbone for the methodology will be presented, including some topics of statistics and optimization.

2.1 Dealer markets

Before the widespread electronification of trading, the main trading venue was the trading floor, where dealers performed trades face-to-face with one another. These floor traders have largely been replaced by electronic limit order markets, where limit orders in the form of "Buy 200 shares at 25 SEK/share" are consolidated in limit order books (LOB). The LOB matches the first buyer and seller that have matching limit orders on each respective side. One key prerequisite for this system is that the traded security has sufficient liquidity, such that limit orders are matched within a reasonable time frame. This is not a problem for asset classes such as equity and foreign exchange, but some fixed income instruments are too illiquid to trade this way. For fixed income securities, dealer markets are more common, in which a trader acts as an intermediary for a customer on the market.

On the traditional dealer market, the customer calls the dealer and requests a quote (price) of a security. This is known as voice trading. The dealer responds with their bid and ask prices, and the customer may choose to sell at the bid price or buy at the ask price. The price in the middle of the ask and the bid price is called the mid price of the security. For a large or important trade, the customer may contact several dealers to get the best price possible. In this type of interaction, the dealer will have an inventory of securities that can be traded. Managing their inventory is important to be able to provide the best service to their customers and to reduce the risk of losing money from being forced to buy or sell to balance the current inventory level. Therefore, inter-dealer trading is also important, where a dealer contacts another dealer to replenish or diminish their current position in a security.

One large drawback for the customer in quote-driven markets is their inherent low transparency, since dealers only provide their quotes in response to a customer request and rarely post their prices publicly. Voice trading is still common for fixed income today, however it is increasingly common for the customer to instead start an e-auction for the trade on a Multi-Dealer-to-Customer (MD2C) platform. These will be discussed in Section 2.4.
2.2 Dealer markets as an inventory optimization problem

To create a better understanding of how dealer markets work, we will look at a classic modeling setup where the market is modeled as an inventory optimization problem. The branch of finance that studies this type of topic is called market microstructure, a term that can be traced back to a paper by Garman in 1976 [10].

In his book Empirical Market Microstructure, Hasbrouck presents the Roll model of bid, ask and transaction prices, first described by Roll [11, 12]. The model is defined as follows. Let the efficient price $u_t$ be a zero-drift random walk, defined as

$$u_t = u_{t-1} + \epsilon_t, \quad \epsilon_t \sim \text{IID}(0, \sigma^2),$$

where $\sigma \in \mathbb{R}^+$. Given a cost of trade $c$ and a margin $m$, we can define the bid and ask prices $b_t$ and $a_t$, and the bid-ask spread $\delta_t$ as

$$b_t = u_t - (c + m), \quad a_t = u_t + (c + m), \quad \delta_t(a_t, b_t) = a_t - b_t = 2(c + m).$$

More generally, we can define the trade price as

$$p_t = u_t + q_t(c + m),$$

where $q_t$ is a sign function for the trade direction defined as

$$q_t = \begin{cases} 
1, & \text{if a customer is buying;} \\
-1, & \text{if a customer is selling.} 
\end{cases}$$

With this model in mind, Hasbrouck considers the following setup. Buyers and sellers arrive according to Poisson or Exponential stochastic processes, and transact a single quantity. We define the arrival intensities of buyers and sellers as functions of the price $\lambda_{\text{Buy}}(p_t)$ and $\lambda_{\text{Sell}}(p_t)$, respectively. $\lambda_{\text{Sell}}(p_t)$ increases as $p_t$ increases, and $\lambda_{\text{Buy}}(p_t)$ decreases as $p_t$ increases. Without loss of generality, we can assume that the cost of trade $c$ is zero for the remainder of the thesis.

If the dealer quotes the ask price for buyers and the bid price for sellers, the dealer will make a profit corresponding to the spread on each unit turned over. Assuming that $\lambda_{\text{Buy}}(a_t) = \lambda_{\text{Sell}}(b_t)$, i.e. that supply and demand balance on average, then the average profit $\Pi(a_t, b_t)$ per unit time is

$$\Pi(a_t, b_t) = (a_t - b_t)\lambda_{\text{Buy}}(a_t) = (a_t - b_t)\lambda_{\text{Sell}}(b_t) = \delta(a_t, b_t)\lambda^{\text{Optimal}}.$$
The arrival intensity of buyers and sellers given a certain price. The shaded area represents the profit $\Pi(a_t, b_t)$ given that the dealer sells at the ask price $a_t$ and buys at the bid price $b_t$.

See Figure 1 for a visualization of the relationship presented in (5). From the definition of the arrival intensities, we see that increasing the spread increases the profit per trade, but will at the same time decrease the arrival intensity of customers. Therefore, it is important for the dealer to set a price level which delivers sustainable profits. For example, by offering the same price to buyers and sellers, $P_{Eq}$, the dealer makes no profit.

To accommodate for asynchronous buying and selling, the dealer needs to maintain buffer inventory of the security and cash. The key constraint is that inventory levels cannot drop below a certain threshold, e.g. zero or such that you avoid being too long or short in a certain security. With $\lambda^{Buy}(a_t) = \lambda^{Sell}(b_t)$, the holdings of stock follow a zero-drift random walk, and the holdings of cash follow a positive drift random walk (due to profits from stock turnover). Inherently from the zero-drift random walk, the dealer will eventually run out of inventory with probability one. To circumvent this, dealers need to position bid and ask prices to create an imbalance in buy and sell orders to push their inventory levels to a preferred level. When the dealer’s inventory approaches their upper boundary, the dealer needs to set their bid quotes to decrease arrival rates of sellers (and force it to zero at the boundary) and vice versa for the lower boundary. As a result, bid and ask prices are monotone decreasing functions of the current inventory level.
2.3 Pricing of fixed-coupon bonds

On the MD2C platform, the dealers stream current indicative prices of bonds with some standard notionals. The basic premise of calculating these prices is relatively straightforward. The price of a bond is equal to the aggregate value of its discounted cash flows. Say we have a bond with a notional of $N$, and a coupon rate of $c$ paid $\alpha$ times a year (i.e. an annual coupon of $cN = C$), and $M$ whole coupon payments left until maturity. Further, let the discount rate be $r$. Then the price of the bond is

$$
P = \sum_{i=1}^{M} \frac{C/\alpha}{(1 + r/\alpha)^i} + \frac{N}{(1 + r/\alpha)^M}$$

$$
= \frac{C}{r} \left(1 - \frac{1}{(1 + r/\alpha)^M}\right) + \frac{N}{(1 + r/\alpha)^M}.
$$

The discount rate $r$ is not available from the market, but represents how the person pricing the bond views the time value of money. However, given that the price of the bond is available, we can solve for the discount rate corresponding to this price by using for example the Newton-Raphson method. The resulting discount rate then represents the bond yield. The bond yield and the bond price are interchangeable ways to view the price of a bond.

When pricing between coupon dates, we have to take accrued interest into account. This represents the interest that accrued from the time of the last coupon payment until the settlement date of the bond trade. The accrued interest is defined as

$$
I = C \cdot \frac{\text{Days since last coupon payment}}{\text{Days in the current coupon period}},
$$

with $C$ defined as in (6). The pricing of the bond between coupon payments is similar to (6), but includes an adjustment of the discount factor with the accrued factor $w$ of the current coupon period defined by some day-count convention. The price is then defined

$$
P = \sum_{i=1}^{M} \frac{C/\alpha}{(1 + r/\alpha)^{i-w}} + \frac{N}{(1 + r/\alpha)^{M-w}}

= (1 + r/\alpha)^w \left[ \frac{C}{r} \left(1 - \frac{1}{(1 + r/\alpha)^M}\right) + \frac{N}{(1 + r/\alpha)^M} \right].
$$

Often, it is more interesting to look at the bond price without the effect of the accrued interest, since this will change every day. This price is called the clean price, while the
price defined in (8), which includes the accrued interest, is called the dirty price. As such, we define the clean price as

\[ P_{\text{clean}} = P_{\text{dirty}} - I, \]  

(9)

where \( I \) is the accrued interest as defined in (7).

In Figure 2, an example is shown of the dirty and clean prices over time of four different bonds in a scenario where we have a constant yield of 5%. When the coupon is larger than the yield, the bond is said to be a premium bond. If the coupon is equal to the yield, it is a par bond and if the coupon is larger than the yield, it is a discount bond. In Figure 2, the top line is a premium bond, the second is a par bond, and the bottom two are discount bonds.

This is only one example of the many complexities when pricing bonds. The reader is referred to [15, 16] for more information on this subject.

2.4 Multi-Dealer-to-Customer platforms

As previously mentioned, a common way to buy bonds is through a Multi-Dealer-to-Customer (MD2C) platform. An MD2C platform is a platform where a customer can interact with multiple dealers simultaneously when performing a trade. The process of using the MD2C platform when trading bonds is the following
1. A client connects to the MD2C platform and can see indicative prices streamed by dealers, presented with a reference size. If the client is interested in a trade, they can start sending an RFQ.

2. The client selects dealers, this can be up to 15 dealers for Bloomberg FIT, and sends them an RFQ for the bond with the desired notional and side (buy/sell).

3. The requested dealers receive the RFQ from the client and can answer with a price. Dealers can see which client has requested a quote, the number of dealers that was requested, and perhaps some composite prices of the best streamed prices for the bond calculated by the provider of the MD2C platform. In Bloomberg FIT, the composite prices are called the CBBT (Composite Bloomberg Trader) bid/ask/mid price. There is also a reference price which represents the price that Bloomberg values the trade to.

4. The client receives the prices from the dealers as they are sent, and may deal at any time with the dealer that currently has the best price. They can also decide to not trade altogether.

5. When the auction has ended, all dealers are informed of their result, and if there was a trade. The dealers are given information based on the outcome of the trade. If they won ("Done"), they get to see the five next best prices. They will know their placement if they came in second ("Covered") or if they were tied with the winning competitor but did not win ("Tied Traded Away"). Otherwise they will know they came third or worse ("Traded Away"). If they did not post a price before the auction was closed, they will not get any information about the other dealers’ prices.

For later notation, we will note that the second best bidding price in the RFQ is called the cover price. The optimal winning bid in an RFQ should be just above or below this price depending on the side to minimize the money "left on the table" in the auction when winning. In addition, the terminology 'better' price used in the subsequent sections will refer to prices that increase the probability of winning the trade.

Evidently, the MD2C platform provides vastly more information than the voice trading of dealer markets. However, due to the confidentiality of the data that the MD2C platform provides, there have been few studies that utilize it. One problem with using this data to study competitor bidding prices is that the dealer only knows the competitors’ prices when the dealer themselves has won, which may introduce bias. However, due to the introduction of MIFID II regulations, it is now possible to get the winning competitor price of an RFQ even when you do not win.

### 2.5 MIFID II

MIFID II (Markets in Financial Instruments Directive II), that came into force in the beginning of January 2018, is one of the recent regulations that has driven the digitization
of the financial industry. The main contributor to this is its requirement for regulatory reporting and trade transparency [17].

MIFID II states that trade information such as the trade price and the trade size should be disclosed publicly within a certain time frame depending on the size of the trade. The goal is that trades should be reported immediately as they are executed, but due to technical limitations there is a grace period for reporting it. For a standard trade, this time frame is 15 minutes, but for some trades larger than €100,000 or trades that may have a significant effect on market liquidity, the reporting may be deferred for a significant period of time. In some cases the reporting of the trade may even be deferred indefinitely. The grace period for standard trades is slated to be reduced to five minutes in 2020, but the rules for deferral of trades is expected to remain [18].

The requirement to post the transaction data shortly after trade execution has presented large challenges for banks. However, it can also be seen as an opportunity if one can leverage the data reported through MIFID II as we aim to do in this report.

2.6 Skew Exponential Power (SEP) distribution

The data we will be looking at in this study consists of observations of bond prices where the mid price of the trade has been subtracted, which will show how much margin dealers put on their trades. Since the bids rarely cross the mid price, the data should be skewed and could also show kurtosis from dealers posting quotes that they know will not win. This is the same set up as in the study of Fermanian et al., and will be described further in Section 3.2 [9].

To be able to handle data with this type of widely varying appearances, flexible parametric distributions that can handle a continuous variation from normality to non-normality with skew and kurtosis is of high interest. The Skew Exponential Power (SEP) distribution is one of these distributions, and also has an analytical expression for its log-likelihood function. The distribution was defined by Azzalini in 1986 and has the following density [19]

$$f_{SEP}(x; \mu, \sigma, \lambda, \alpha) = \frac{2\Phi(w)}{\sigma c} \exp(-|z|^\alpha / \alpha),$$

(10)

where $\mu > -\infty$, $\sigma > 0$, $\lambda < \infty$, $\alpha > 0$, $z = (x - \mu) / \sigma$, $w = \text{sign}(z)|z|^{\alpha/2}\lambda(2/\alpha)^{1/2}$, $c = 2\alpha^{1/\alpha}\Gamma(1/\alpha)$, $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution, and $\Gamma(\cdot)$ is the gamma function.

The SEP distribution has three special cases. The distribution reduces to the Exponential Power distribution when $\lambda = 0$, the Skew Normal distribution when $\alpha = 2$, and the Gaussian distribution when $(\lambda, \alpha) = (0, 2)$. 
The Exponential Power (EP) distribution was studied extensively by for example Box and has the following density [20]

\[ f_{EP}(x; \mu, \sigma, \alpha) = (\sigma c)^{-1} \exp(-|z|^\alpha / \alpha), \]  

where \( \mu \in (-\infty, \infty) \), \( \sigma > 0 \), \( \alpha > 0 \), \( z = (x - \mu) / \sigma \), and \( c = 2^{\alpha/\alpha - 1} \Gamma(1/\alpha) \). The Gaussian distribution is achieved with \( \alpha = 2 \).

The Skew Normal (SN) distribution was introduced by Azzalini in 1985 and has density defined as [21]

\[ f_{SN}(x; \mu, \sigma, \lambda) = \frac{2}{\sigma} \Phi(\lambda z) \phi(z), \]  

where \( \mu > -\infty \), \( \sigma > 0 \), \( \lambda < \infty \), \( z = (x - \mu) / \sigma \), and \( \phi(\cdot) \) is the density function of the standard normal distribution. The Gaussian distribution is achieved with \( \lambda = 0 \).

We can simplify the density of the SEP distribution using (11) to get

\[ f_{SEP}(x; \mu, \sigma, \lambda, \alpha) = 2 \Phi(w) f_{EP}(x; \mu, \sigma, \alpha), \]  

where \( \mu > -\infty \), \( \sigma > 0 \), \( \lambda < \infty \), \( \alpha > 0 \), \( z = (x - \mu) / \sigma \), and \( w = \text{sign}(z)|z|^\alpha/\alpha \).

The SEP distribution has the following log-likelihood function

\[ l(\theta; x) = -(1/\alpha - 1) \ln \alpha - \ln \Gamma(1/\alpha) - \ln \sigma + \ln \Phi(w) - |z|^\alpha/\alpha, \]  

where \( \theta = (\alpha, \lambda, \mu, \sigma) \), \( x \) is the observation, \( \Gamma(\cdot) \) is the gamma function, and the remaining variables are defined as in equation (13). Diciccio and Monti note that the likelihood function may attain its maximum at the boundary of the parameter space when the number of observations is lower than 100 [22]. For example, the \( \lambda \) parameter was sometimes found to have a monotonically increasing profile likelihood, and similar problems were present for the \( \alpha \) parameter. Diciccio and Monti found that this behaviour was rare in cases where the number of samples was above 100. In this study, the maximum likelihood estimations of the SEP distribution always has more observations than this threshold to ensure stability of the parameter estimation.

The SEP density with some example parameters is presented in Figure 3, Panel a). As we can see, the distribution is flexible and accommodates various levels of skew and kurtosis, which is relevant when looking at data that rarely crosses a certain threshold. This distribution was used by Fermanian et al. in their study [9].
2.7 Noncentral t-distribution

The noncentral t-distribution (NCT) is similar to the SEP distribution in its ability to accommodate both skew and kurtosis, but can be considered to be a more established distribution. Therefore, it will be investigated as an alternative to the SEP distribution in this study.

The NCT distribution is a generalized variant of the Student’s t-distribution and was first described by Fisher in 1931 [23]. $X$ is a noncentral t-distributed variable with $k > 0$ degrees of freedom and noncentrality parameter $c$ if

$$X = \frac{Y + c}{\sqrt{V/k}},$$

(15)

where $Y$ is a standard normal random variable, $V$ is an $\chi^2$ random variable with $k$ degrees of freedom and $c$ is a real-valued noncentrality parameter. The distribution reduces to the standard Student’s t-distribution when $c = 0$. The original distribution (15) can be adjusted as a location-scale distribution with location $\tau$ and shape $\nu$ as follows

$$X = \frac{(Y - \tau)/\nu + c}{\nu\sqrt{V/k}}.$$  

(16)

The density of the NCT distribution without location and scale parameters is defined as follows

$$f_{\text{NCT}}(x; k, c) = \frac{k^\frac{k}{2} \exp\left(-\frac{kx^2}{2(x^2+k)}\right)}{\sqrt{\pi} \Gamma\left(\frac{k}{2}\right) \frac{2\frac{x}{2}^\frac{k}{2}}{k} \frac{1}{2} \frac{x^2+k}{\sqrt{x^2+k}}} \int_0^\infty y^k \exp\left(-\frac{1}{2} \left(y - \frac{cx}{\sqrt{x^2+k}}\right)^2\right) dy,$$  

(17)

where $\Gamma(\cdot)$ is the gamma function and the remaining variables are defined as in (15). Unfortunately, there is no closed-form expression for the log-likelihood of this distribution similar to (14). However, since the noncentral t-distribution is implemented in Python’s SciPy stats module via the function nct, we can use this to estimate the parameters.

The NCT density with some example parameters is presented in Figure 3, Panel b). The noncentral t-distribution is similar to the SEP distribution in most cases, but does not have the same steep dip in the density around a value that the SEP distribution can have.
2.8 Optimization methods

To incorporate the effect of the Winner’s Curse in our models and to perform the maximum likelihood estimation of the parameters for the density of the prices, we will utilize optimization methods. We use two methods, the COBYLA method and the Nelder–Mead method.

The COBYLA (Constrained Optimization by Linear Approximation) method is a derivative-free numerical optimization algorithm that was first described by Powell in 1994 [24]. The COBYLA method iterates an approximation of the complete optimization problem as linear programming problems. Each iteration solves the approximate problem, creating a candidate for the optimal solution. The objective function value in the candidate point is evaluated and as the solution converges, the step size is reduced. The algorithm stops when a certain tolerance threshold is reached.

The Nelder–Mead method is another numerical optimization method that was first described by Nelder and Mead in 1965 [25]. The method uses the concept of simplex to solve the optimization problem, and is thus also not reliant on knowing the derivative of the objective function. For an optimization problem of \( n \) dimensions, the algorithm has \( n + 1 \) test points arranged as a simplex, a polytope. The objective function is evaluated in each of the test points, and the centroid point of all points except point \( n + 1 \) is calculated. If the objective function value in a point extended in the direction of the centroid is the best thus far, the new point will replace the worst test point. If the new test point is not better, the whole simplex is shrunk towards the current best point and the algorithm is restarted. The algorithm is similarly stopped when a certain tolerance threshold is reached.

Figure 3: Examples of densities of the SEP and NCT distributions. Only positive values of \( \lambda \) and \( c \) are presented; if the signs of the parameters were reversed, the density would be reflected about the origin. The continuous lines are the (approximately for NCT) Gaussian cases.
The reader is referred to \[24, 25\] for more information about these optimization methods.

### 2.9 Model evaluation methods

One of the metrics used to evaluate the models defined in this report is the Root Mean Squared Error (RMSE). This is a measure of how far a model’s estimated values are from observed true values, and thus represent a way to compare different models’ predictive ability. It is defined as

\[
\text{RMSE} = \sqrt{\frac{\sum_{k=1}^{N}(y^*_k - y_k)^2}{N}},
\]

(18)

where \( y^*_k \) is the estimated value, \( y_k \) is the true value and \( N \) is the number of observations.

The lower the RMSE is, the better the estimation. A RMSE of zero represents a perfect match between the estimated values and the observed true values.

To make sure that the distribution we use for the prices is adequate, the Kolmogorov–Smirnov test will be used. It is a commonly used statistical test that evaluates if a set of data follows a certain distribution. The test was first described by Kolmogorov in 1933 \[26\], and then refined by Smirnov in 1944 \[27\]. The Kolmogorov–Smirnov test compares the difference between the empirical distribution function of the data to the cumulative distribution function of a candidate distribution. The largest absolute distance between these is named the Kolmogorov–Smirnov statistic \( D_n \), defined as

\[
D_n = \sup_x |F_n(x) - F(x)|,
\]

(19)

where \( F_n(x) \) is the empirical distribution function of the observed data and \( F(x) \) is the candidate distribution function. The test is constructed as follows

\[
\sqrt{n}D_n > K_\alpha,
\]

(20)

where \( K_\alpha \) is the value such that \( P(K \leq K_\alpha) = 1 - \alpha \) for a chosen significance level \( \alpha \), and \( K \) is Kolmogorov distributed. The null hypothesis is that the observed data is distributed as the candidate distribution, and thus a higher p-value indicates that it is more likely that the observed data follows the candidate distribution.
2.10 Regression Trees

As a comparison for the models defined in this report, regression trees will be used. This is a popular method of estimating an unknown value given a set of observations. The methodology is based on classification of data into a tree of hierarchies based on importances of different splits of the data. Each split of the data is based on the split purity, which measures how much a split contributes to the correct classification of the data. Some measures of this are the RSS and the Gini index, which are described in [28]. To decrease the risk of overfitting the data and improve prediction, the aggregated result of several regression trees can be considered. This is called a random forest regression.

One benefit of using regression trees is that they give a measure of the chosen covariates’ importance for the estimation, called feature importance. The variables with the highest feature importance are the variables that contribute to the highest split purity of the regression tree or random forest. The features importances are normalized such that the sum of all feature importances equals one.

Since regression trees will not be the focus of the study, the reader is referred to [28] for more information on the mathematics behind this methodology.
3 Methodology

This section will describe the methodology used to obtain the results of the study. The data used in the study will be explained along with a description of how the novel bidding models are constructed. Finally, we will describe how the models are tested and compared.

3.1 Data

The primary data set for the analysis consists of RFQs from the MD2C platform Bloomberg FIT that were received between mid 2017 and the first quarter of 2019. The RFQs are for a large number of different bonds, so to ensure sufficient quality of the models, a subset of more liquid AAA-rated Swedish bonds is used. These include government bonds, index-linked bonds, mortgage bonds and credit bonds. The modelling approach is, however, applicable for any type of bond. The RFQ data set contains, among others, the following columns of interest for the analysis:

- The status of the trade (Done, Covered, Traded Away, etc.);
- The trade date;
- The notional of the trade;
- The ISIN of the bond;
- The maturity date;
- The inspected dealer’s posted quote;
- Cover quotes 1 through 5 (cover quote 1 is what we call the cover price);
- The number of competing dealers;
- The composite Bloomberg trader mid, ask and bid quotes;
- The composite Bloomberg trader reference price.

Some of the above columns contained prices represented as both the bond price or the bond yield, so a conversion algorithm is used to get all the prices in terms of bond yields using the fundamental theory presented in Section 2.3. In this study, the bond yield is used as the representation of the bond price for all subsequently presented analysis.

To train the models, observations of RFQs resulting in trades from mid 2017 to the beginning of 2019 are used. From these, we use the best price of the competitors when the inspected dealer was the winning dealer, and other trades where we know the winning price of the competitor through MiFID II reporting. Note that the MiFID II prices are from cases where the inspected dealer did not win the auction, which gives us information about the competitors’ prices in cases that would not be possible by solely using data.
from the MD2C platform. The quotes included in the training data set represent the best competitor quotes posted for each trade that we have information on.

The test data consists of observations of RFQs resulting in trades from the beginning of 2019 to the end of Q1 2019. It contains observations from trades where the inspected dealer was accepted and tied traded away. Using MIFID II data, a number of trades are added with other trade statuses. In addition, trades where the dealer was tied traded away from the same period as the training data set is added to the test data set, as these are not used in the training data.

In addition, we also use a data set of approximately 6500 trades where the inspected dealer lost the trade and were traded away or covered, where we do not have the MIFID II reported winning price. These are taken from the same time period as the training data, but were not included in the training nor the test data set. This data set is also used to test the models, but it is not referred to as the test data set in the report.

All the data sets are filtered to only include trades where one to four dealers along with the inspected dealer were requested. This is done since the case where the inspected dealer has zero competing dealers means we have no competitor price to model after, and because approximately 93% of the trades are in this subset. After filtering, the test data is approximately 10% of the size of the training data. The training and test data is described further in Tables 1 and 2 respectively. For more information about the full training and test data set before filtering along with the specification of their trade status distribution, see the Appendix.

### Table 1: Training data set specifications after filtering.

<table>
<thead>
<tr>
<th>Dealers</th>
<th>Buy RFQs</th>
<th>Sell RFQs</th>
<th>Total</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>222</td>
<td>146</td>
<td>368</td>
<td>4.1%</td>
</tr>
<tr>
<td>2</td>
<td>759</td>
<td>676</td>
<td>1435</td>
<td>16.0%</td>
</tr>
<tr>
<td>3</td>
<td>623</td>
<td>698</td>
<td>1321</td>
<td>14.7%</td>
</tr>
<tr>
<td>4</td>
<td>2979</td>
<td>2863</td>
<td>5842</td>
<td>65.2%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>4583</strong></td>
<td><strong>4383</strong></td>
<td><strong>8966</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>

### Table 2: Test data set specifications after filtering.

<table>
<thead>
<tr>
<th>Dealers</th>
<th>Buy RFQs</th>
<th>Sell RFQs</th>
<th>Total</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19</td>
<td>16</td>
<td>35</td>
<td>3.8%</td>
</tr>
<tr>
<td>2</td>
<td>74</td>
<td>84</td>
<td>158</td>
<td>17.2%</td>
</tr>
<tr>
<td>3</td>
<td>86</td>
<td>108</td>
<td>194</td>
<td>21.1%</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
<td>231</td>
<td>531</td>
<td>57.8%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>477</strong></td>
<td><strong>439</strong></td>
<td><strong>918</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>
3.2 Modelling

Similar to Fermanian et al., we start by transforming the bond yields of the RFQs to reduced quotes $\delta$ \cite{9}. For each trade $i$, the reduced quote $\delta_i$ corresponding to the bond yield $P_i$ is defined as

$$\delta_i = \frac{P_i - \text{CBBT}_i^{\text{Mid}}}{\Delta_i^{\text{CBBT}}}, \quad (21)$$

where $\text{CBBT}_i^{\text{Mid}}$ is the Composite Bloomberg Trader mid yield, and $\Delta_i^{\text{CBBT}}$ is half the CBBT spread, i.e. $\Delta_i^{\text{CBBT}} = (\text{CBBT}_i^{\text{Bid}} - \text{CBBT}_i^{\text{Ask}})/2$.

The reduced quotes allow us to model the distance of the quotes to the mid price, which will most likely be affected by the number of dealers present in the auction as was shown by Fermanian et al. in their study \cite{9}. Since the spread of a security indicates how liquid it is, dividing by this factor helps to normalize the data.

The reduced quotes $\delta$ are then fit using maximum likelihood to the SEP distribution using equation (14). We optimize the maximum likelihood estimation using the COBYLA method. We create a SEP model of the reduced quotes for each number of requested dealers $n$ and each side of the trade. These models are created by separating the reduced quotes into partitions based on the trades’ number of requested dealers and the side, and fitting the SEP distribution to each subset. Since the training data comprises the best competitor prices for each trade, the fitted SEP densities represent the probability density of the best price of all competitors that will be posted as a quote for a trade. Here we make the assumption that the bids of the competing dealers have the same underlying distribution. In our data set, almost all of the trades were answered by all requested dealers, so we do not take into account the effect of the probability of a requested dealer not answering the RFQ.

The subsequently presented models all use the fitted SEP densities as a base for finding the best quote to post as a bid for each trade. The information about the number of requested dealers and the side of the trade is available when the RFQ is received by the dealer, so we can choose the appropriate model for each trade using this information. To estimate the quote for a trade, each model produces their optimal reduced quote, which is then converted to the corresponding bond yield. For the trade $i$, the transformation is

$$P_i^* = \delta_i^* \cdot \Delta_i^{\text{CBBT}} + \text{CBBT}_i^{\text{Mid}}, \quad (22)$$

where $\delta_i^*$ is the reduced quote estimated by the model and $P_i^*$ the corresponding bond yield. The remaining variables are defined as in (21). If we assume that the estimated yield should correspond to the cover price of the RFQ, we can define the optimal win price as the cover price adjusted with a 0.1 basis point (1 basis point = 0.01%) margin.
to make sure that the dealer wins, but with a minimal cost. This margin was decided from expert knowledge about the bonds included in the data. The choice of a suitable margin depends on for example the liquidity of the bond that is traded.

One way to define the best quote to post using the density of the best competitor prices is the quote corresponding to the expected value of this density. This will provide the best pricing in most cases, since the expected value will fall close to the largest mass of the probability density of the reduced quotes. If we let $g(\delta; n, \cdot)$ denote the best competitor price density when the trade has $n$ requested dealers, and where $\cdot$ can correspond to either the buy or the sell case, the expected value is defined

$$\delta^* = \int_{-\infty}^{\infty} g(\delta'; n, \cdot) d\delta',$$

where $\delta^*$ is the expected value and thus the estimated best reduced quote to bid for this model. This pricing methodology is henceforth called the Expected Price method.

With the same notation as in (23), it is easy to define the probability of winning on the sell side with the reduced quote $\delta$ when there are $n$ competing dealers as

$$P_{\text{win}}(\delta; n, \text{sell}) = \mathbb{P}(\delta_{\text{cmp}} < \delta) = \int_{-\infty}^{\delta} g(\delta'; n, \text{sell}) d\delta' = G(\delta; n, \text{sell}),$$

and on the buy side as

$$P_{\text{win}}(\delta; n, \text{buy}) = \mathbb{P}(\delta_{\text{cmp}} > \delta) = 1 - \mathbb{P}(\delta_{\text{cmp}} < \delta) = 1 - \int_{-\infty}^{\delta} g(\delta'; n, \text{buy}) d\delta' = G(\delta; n, \text{buy}),$$

which corresponds to the probability that the dealer’s reduced quote $\delta$ is better than the best reduced quote of all competitors, $\delta_{\text{cmp}}$. Using (24) and (26), we can adjust the chosen reduced quote such that the dealer’s pricing is not too generous or vice versa. Henceforth, the probability of winning a trade with $n$ requested dealers using the reduced quote $\delta$ is denoted $G(\delta; n, \cdot)$, where $\cdot$ again can correspond to either the buy or the sell case.

Next, we want to incorporate the Winner’s Curse into the model. We define a proxy of the amount of Winner’s Curse a given reduced quote suffers from using a similar formula to that presented in a study by Laffont in 1997 [29].
where $\delta^{\text{ref}}$ is the reduced quote corresponding to a reference price of the trade, and $q_t$ is a sign function defined as in (1). In this report, we use the CBBT reference price as the value for $\delta^{\text{ref}}$. As described in Section 2.4, the CBBT reference price represents the composite price that Bloomberg suggests for the trade given the current streaming prices from dealers of the bond. We can use this as a proxy for the true value of the trade. As such, we can say that the more one overprices compared to this, the more likely one is to suffer from the Winner’s Curse. Note that this proxy takes on negative values when one gives a better price than the reference price, and will decrease linearly as one chooses a more aggressive quote. The best quote to post that adjusts for the Winner’s Curse proxy can then be defined as the reduced quote solving the following maximization problem

$$
\delta^* = \arg \max_{\delta} \quad q_t(\delta - \delta^{\text{ref}})G(\delta; n, \cdot).
$$

Since the Winner’s Curse proxy decreases as the probability of winning given the chosen reduced quote increases, the optimization problem is convex and has a global optimum. The Winner’s Curse adjusted method should estimate a more conservative quote than the expected price method (23), which reduces the risk of overbidding on the given RFQ. In theory, this should have two effects. The first is to reduce the amount of money 'left on the table' in the auction, i.e. the money lost due to winning the trade with a too large margin from the second best price. The second is to reduce the risk of winning trades that are not profitable. While the expected price method will try to hit the cover price for every trade, this model could set a price that is lower if the cover price would be too far from the reference price, since this would indicate that the cover price is maybe too high with respect to the true value of the bond. This pricing methodology is henceforth called the Winner’s Curse adjusted method.

To have more control of the Winner’s Curse adjusted model, we introduce constraints on (28) as follows

$$
\delta^* = \arg \max_{\delta} \quad q_t(\delta - \delta^{\text{ref}})G(\delta; n, \cdot)
\quad \text{s.t.} \quad \max(p - m, 0) \leq G(\delta; n, \cdot) \leq \min(p + m, 1),
$$

for some chosen probability of winning $p \in [0, 1]$ and target ratio margin $m \in [0, 1]$. The probability $p$ can be chosen as a value depending on the RFQ, such as $p = \frac{1}{n}$, where $n$ is the number of requested dealers for the trade, or so as to win a certain ratio of trades to secure ones market share. The target ratio margin $m$ defines how much we let the Winner’s Curse proxy adjust the desired target ratio to protect from overpricing. Setting it to zero will remove the effect of the Winner’s Curse proxy altogether. This pricing methodology is henceforth called the Constrained Winner’s Curse adjusted method.
An extended approach for the aforementioned novel methods is to add scaled covariates to the model. The hypothesis is that the covariates will incorporate more information when setting the quote of the bond, which should give better prices. We introduce the adjusted reduced quote $\delta_{\text{adj}}$, defined as

$$\delta_{\text{adj}} = \delta + C\beta^T,$$  \hspace{1cm} (30)

where $C$ is a matrix containing one observation of a set of covariates for each observation of the reduced quote $\delta$, and $\beta$ is a vector with scaling parameters for each covariate. Thus, $C\beta^T$ is a vector that contains a scalar adjustment of each reduced quote due to the covariates of each trade. After adjusting all the observations contained in the training set, the adjusted reduced quotes can be fit to the SEP distribution as described in the beginning of this section and the resulting covariate-adjusted densities $g_{\text{adj}}(\delta_{\text{adj}}; n, \cdot)$ can be used in place of the original best competitor price density $g(\delta; n, \cdot)$ in the aforementioned models. The models using the covariate-adjusted densities will estimate the adjusted reduced quote $\delta_{\text{adj}, i}$ for the trade $i$, and similar to (22), we get the corresponding quote for the trade $P^*_i$ as

$$P^*_i = (\delta_{\text{adj}, i}^* - C_i\beta^T) \cdot \Delta_{i,\text{BBT}} + \text{CBBT}_{i,\text{Mid}},$$  \hspace{1cm} (31)

where $C_i$ is a vector containing the observations of the covariates for trade $i$, and $\beta$ is defined as in (30). The vector $\beta$ is estimated by fitting the reduced quotes to a SEP distribution using maximum likelihood, while at the same time incorporating the expression (30) to adjust the reduced quotes. Since every iteration of the maximum likelihood estimation will change $\beta$, and by extension the adjusted reduced quotes, the maximum likelihood estimation of the $\beta$ along with the other SEP parameters is difficult to implement.

To ensure the proper fit of all the parameters of the SEP distribution along with $\beta$, the distribution is fitted in two steps. Firstly, the reduced quotes are fitted using an adjusted maximum likelihood estimation incorporating (30) into the likelihood function (14) using a Nelder–Mead optimization. The resulting estimated $\beta$ vector is used to create a vector of adjusted reduced quotes as in (30), and the resulting adjusted reduced quotes are then fit to the SEP distribution with the original likelihood function (14) using a COBYLA optimization, giving more robust estimations of the original SEP parameters. Thus, the first optimization yields the estimate of $\beta$, and the second yields the estimation of the SEP parameters.

We chose to include the following covariates in our model, which are all available when an RFQ is received by the dealer:

- The notional value of the trade;
- The number of days until the maturity of the bond;
• The number of days of accrued interest as of the trade date;
• The reference CBBT price of the trade;
• The coupon of the bond;
• The CBBT bid-ask spread.

3.3 Testing

Before the analysis, the training data is cleaned by removing the outliers of the reduced quotes that fall outside the 99% and 1% percentiles. It is then divided into subsets depending on the number of requested dealers and the side of the trade. Then, to test if the SEP distribution is a suitable choice for the analysis, we perform an assessment of the fit of the training data to the Gaussian distribution, the NCT distribution, and finally the proposed SEP distribution. Firstly, we plot the fitted probability density function of each candidate distribution against the empirical density of the reduced quotes for each subset of the data. Secondly, we measure the goodness-of-fit of the distributions using a Kolmogorov–Smirnov test. To assess the skew and kurtosis of the data, the empirical skew and kurtosis and accompanying statistical tests are calculated using functions from Python’s stats module. The statistical tests show if the skew and kurtosis are significantly different from zero.

Finally, we compare the performance of the expected price model when using the different distributions as the parametric distribution for the best competitor price density \( g(\delta; n, \cdot) \). We compare the results using the Root Mean Squared Error (RMSE), bias and hit ratio to see which gives the best pricing. The definitions of these measures are presented in Table 3.

The novel methods described in Section 3.2 are tested on the test data described in Table 2. In these tests, we use the SEP distribution as the parametric distribution for \( g(\delta; n, \cdot) \). For each RFQ in the test data, the corresponding SEP model of the best competitor price density \( g(\delta; n, \cdot) \) is chosen and used as the base for the pricing models. The measures used to assess the models is presented in Table 3. Note that the true price \( P_{true} \) corresponds to the actual best competitor price in the trade, which we want the models to hit. The reference price \( P_{ref} \) is, as before, the CBBT reference price calculated by Bloomberg for each trade. The measures are compared between the different models to see how well each model achieves its purpose.

Furthermore, the pricing models are tested using the measures in Table 3 on approximately 6500 observations of trades where the inspected dealer lost the trade, and we do not know the winning price, as described in Section 3.1. When calculating the measures with this data set, they will use the the price that the inspected dealer lost with, \( P_{trAway} \), instead of the true price, \( P_{true} \). The results will give an indication if the chance of winning would have been higher using the novel methods compared to the present pricing. In this case, the target values of the measures presented in Table 3 should instead be
Table 3: Details about the measures used to compare models. The first four measures analyze the distance from the true price, the two thereafter measure the Winner’s Curse distance, and the last two are again based on the distance to the true price.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Target value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>Root Mean Squared Error as described in Section [2,9]</td>
<td>0</td>
</tr>
<tr>
<td>$</td>
<td>P^* - P_{true}</td>
<td>$</td>
</tr>
<tr>
<td>Bias</td>
<td>The average distance of the estimated price from the true price. A winning price has positive bias, and a losing price is negative. (The distance is defined as $P^* - P_{true}$ if sell side, $P_{true} - P^*$ if buy side).</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma_{true}$</td>
<td>The standard deviation of the distance of the estimated price from the true price $P_{true}$. The distances are defined as the ones averaged in the Bias measure.</td>
<td>0</td>
</tr>
<tr>
<td>$</td>
<td>P^* - P_{ref}</td>
<td>$</td>
</tr>
<tr>
<td>$\sigma_{ref}$</td>
<td>The standard deviation of the distance of the estimated price from the reference price $P_{ref}$. The distances are defined as the ones averaged in the Bias measure, except for the usage of $P_{ref}$ instead of $P_{true}$.</td>
<td>0</td>
</tr>
<tr>
<td>HR</td>
<td>Hit ratio. The ratio of trades where the estimated model price wins the trade. A high hit ratio would in most cases indicate that the model is overpricing, and vice versa.</td>
<td>Depends</td>
</tr>
<tr>
<td>HR/</td>
<td>Bias</td>
<td></td>
</tr>
</tbody>
</table>
above zero (except for the Winner’s Curse distance, which should still be low), and the hit ratio should be higher than for the test data. Some measures are excluded for the analysis of the lost trades since they are not as relevant.

The hit ratio measure presented in Table 3 is calculated as if we assume that the models have calculated the cover price. Therefore, when calculating this, a 0.1 basis point margin is added or subtracted depending on the side of the trade to simulate the pricing one would post when using the models. In this case, the hit ratio should be approximately 50% when using the models on the test data. When adjusting for the Winner’s Curse, the hit ratio should be lower since we are purposely lowering the bid to reduce the risk of suffering from the Winner’s Curse. In the traded away and covered data set, the hit ratio should be higher, since it will not represent the ratio of won trades, but instead trades where the estimated quote is better than the quote posted which resulted in a lost trade.

For the constrained Winner’s Curse adjusted method, the calculated hit ratios do not include the 0.1 basis point adjustment. This is because we assume that this model estimates the price that would correspond to having the chosen hit ratio, and not the cover price of the trade as before. Since the constrained Winner’s Curse model does not aim to hit the cover price, but to win a certain ratio of trades, we only look at these non-adjusted hit ratios as a measure of the performance of this model. A set of target ratios, corresponding to \( p \) in (29), are chosen and compared to the resulting actual hit ratios on the test data with a target ratio margin \( m \) of 0% to test its accuracy, and 5% to test how the Winner’s Curse proxy affects the result.

The proposed models are also compared with a linear regression model and a random forest regressor, which represent current standard methods for prediction. These models similarly have the reduced quotes as their input and use the covariates presented in Section 3.2 as independent variables along with dummies indicating if the trade is on the buy or sell side. The random forest regressor parameters are determined by a three-fold cross-validation randomized grid search on the training data set. The grid parameters and chosen parameters are presented in Table 4.

The random forest regressor and linear regression are performed using the functions RandomForestRegressor and LinearRegression from Python’s SkLearn module. The random forest regressor uses the parameters presented in Table 4 for both training and for prediction of the optimal reduced quotes to post. The linear regression model is performed with the default values of the Python function. We test the regression methods similar to to the other methods. First we train the models on the training data, and then we input the covariates for each RFQ in the test data to estimate the best reduced quotes for each trade. The reduced quotes are then converted to the bond yield using (22). The regression models are also evaluated with the measures presented in Table 3, here with the 0.1 basis point adjustment for the hit ratio since they also aim to hit the cover price. To get a more stable result, the random forest regression is calculated six times with random state variable equal to \{0, 1, 2, 3, 4, 5\}, and the average
Table 4: Grid parameters and optimal choice with respect to Mean Squared Error after randomized grid search for random forest regression training.

<table>
<thead>
<tr>
<th>Option</th>
<th>Grid parameters</th>
<th>Chosen parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of trees</td>
<td>{200, 400, ..., 2000}</td>
<td>1000</td>
</tr>
<tr>
<td>Max depth</td>
<td>{1, 6, ..., 100}</td>
<td>11</td>
</tr>
<tr>
<td>Max features</td>
<td>{N_{variables}, \sqrt{N_{variables}}, \log_2(N_{variables})}</td>
<td>\sqrt{N_{variables}}</td>
</tr>
<tr>
<td>Min samples for</td>
<td>{2, 5, 10}</td>
<td>10</td>
</tr>
<tr>
<td>splitting internal node</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min samples required at leaf node</td>
<td>{1, 2, 4}</td>
<td>4</td>
</tr>
<tr>
<td>Bootstrap samples</td>
<td>{Yes, No}</td>
<td>Yes</td>
</tr>
</tbody>
</table>

results of the measures and the feature importances of the covariates is calculated.

In general, note that the estimated quotes and model evaluation measures presented in the results are denoted in terms of bond yields. The bond yields are represented in decimal form, which is in line with how they are shown in Bloomberg. This means that if the bias of a model is -0.0001, then the average estimated quote for that model is losing the trade by 1 basis point.
4 Results

This section will present the results divided into four subsections. The first will present the analysis of the choice of the parametric distribution of the reduced quotes. The second and third will present the results of the models when trying to hit the cover price and a target hit ratio, respectively. Finally, the fourth section will present an analysis of the chosen covariates with respect to their feature importance in the random forest regression.

4.1 Comparison of parametric distributions

Since the parametric distribution model of the reduced quotes represents the backbone of all the pricing models, it is important that a suitable choice is made. The empirical densities of the reduced quote subsets with four dealers on the buy and sell side are presented in Figure 4. As can be seen in Tables 1 and 2, these subsets are the most prevalent in the data, and should therefore affect the final result the most. The fit of these data sets to the Gaussian, NCT, and SEP distributions is presented in Figure 5.

![Empirical density of reduced quotes for buy side.]

![Empirical density of reduced quotes for sell side.]

Figure 4: Empirical density of reduced quotes for the subset with four competing dealers.

When considering the empirical skew and kurtosis presented in Table 5, it shows that the test for presence of skew is highly significant while the test for presence of kurtosis is only significant on a 95% confidence level for the buy side. The skew can be seen from Figure 4, but it is not as extreme as the skew observed in the study by Fermanian et al. [9]. One would expect there to be a more distinct cutoff around zero, since for the buy (sell) case a value above (below) zero means that the quote has crossed the mid price, which should not happen that frequently. Similarly, the empirical densities show significantly less kurtosis than in the same study. Fermanian et al. hypothesized that the kurtosis in their data came from prices where the dealer did not want to win, and therefore posted bad price guaranteed not to win the trade [9]. However, since we only
(a) Fit of Gaussian distribution to reduced quotes on buy side.

(b) Fit of Gaussian distribution to reduced quotes on sell side.

(c) Fit of NCT distribution to reduced quotes on buy side.

(d) Fit of NCT distribution to reduced quotes on sell side.

(e) Fit of SEP distribution to reduced quotes on buy side.

(f) Fit of SEP distribution to reduced quotes on sell side.

Figure 5: The fit of the Gaussian, NCT and SEP distributions to the reduced quotes. The empirical density of the reduced quotes with four competing dealers is presented in blue (bars), and the fitted density in orange (line).
include the best dealer prices in our data set, that effect should not be present in our data.

Table 5: The empirical skew and kurtosis as calculated by the functions Skew and Kurtosis and corresponding tests SkewTest and KurtosisTest in Python’s stats module. A lower p-value indicates that it is more likely that there is skew or kurtosis present.

<table>
<thead>
<tr>
<th></th>
<th>Skew</th>
<th>SkewTest (p-value)</th>
<th>Kurtosis</th>
<th>KurtosisTest (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 dealers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buy</td>
<td>-0.259814</td>
<td>0.00000</td>
<td>0.216704</td>
<td>0.025325</td>
</tr>
<tr>
<td>Sell</td>
<td>0.294391</td>
<td>0.00000</td>
<td>-0.030802</td>
<td>0.786773</td>
</tr>
</tbody>
</table>

One reason for why our data set is not as skewed as that of Fermanian et al. could be that the CBBT mid price is not sufficiently accurate. Since Bloomberg does not give out details of how their CBBT mid price is calculated, it is difficult to determine if this is the case. The model performance could most likely be improved by using the same mid prices that traders use, since these should be more in line with the mid price that is used by market participants.

We can see from Figure 5 that the Gaussian distribution is centered around the mass of the empirical density of the reduced quotes, which should produce good results in most cases. However, especially in cases with lower number of dealers, as the example of three dealers presented in the Appendix, the data is more skewed and then the Gaussian distribution would not be as suitable to use. In addition, since the NCT and SEP distributions both reduce to the Gaussian distribution as special cases, they should still cover the case where the data is in fact Gaussian distribution.

When looking at Figure 5, the NCT and SEP distributions provide a similar fit to the data set. The main difference in fit lies in the mode, which for the SEP is farther from the origin compared to the NCT distribution. The steeper skew in the SEP distribution seems to better capture the skew in the empirical density of the reduced quotes than the NCT manages to do. The difference is slightly more apparent when looking at the subset of data where we have three requested dealers, again presented in the Appendix. Using a better CBBT mid price, the skew would perhaps have been steeper even in the case of four dealers, and then the SEP distribution would most likely have an advantage over the NCT distribution. However, in our case the difference between the fit is small.

We also look at the Kolmogorov–Smirnov test calculated for the subsets with four and three requested dealers. The null hypothesis is that the data follows the candidate distributions. The p-values from the statistical test are presented in Table 6. The best test values are bolded.
Table 6: Kolmogorov–Smirnov test p-values. \( H_0 \): sample comes from the candidate distribution.

<table>
<thead>
<tr>
<th></th>
<th>Gaussian</th>
<th>NCT</th>
<th>SEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 dealers</td>
<td>Buy</td>
<td>0.0</td>
<td>0.001567</td>
</tr>
<tr>
<td></td>
<td>Sell</td>
<td>0.0</td>
<td>0.014663</td>
</tr>
<tr>
<td>3 dealers</td>
<td>Buy</td>
<td>0.001926</td>
<td>0.216274</td>
</tr>
<tr>
<td></td>
<td>Sell</td>
<td>0.004259</td>
<td>0.830005</td>
</tr>
</tbody>
</table>

In this test, the SEP distribution outperforms the others by a wide margin, even though all distributions are rejected by the test on standard significance thresholds. The SEP distribution produces a noticeably better fit for the data where three dealers are requested than when four are requested, which is expected since the data set with three requested dealers has a more significant skew. In both cases, the Gaussian distribution produces a significantly worse fit according to the Kolmogorov–Smirnov test.

The RMSE, the bias and the hit ratio of the expected price method using the three different distributions are presented in Table 7. The best result for each measure is bolded. We see that the best distributions according to these measures is again the SEP distribution, but the difference between the models is negligible. This is not surprising given that all candidate distributions are centered around the bulk of the density mass in the training data, which should correspond quite well to the test data.

Table 7: Results of the expected price method using the parametric distributions Gaussian, NCT and SEP for the reduced quotes.

<table>
<thead>
<tr>
<th></th>
<th>Gaussian</th>
<th>NCT</th>
<th>SEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>0.009437</td>
<td>0.009436</td>
<td>0.009435</td>
</tr>
<tr>
<td>Bias</td>
<td>-0.000445</td>
<td>-0.000443</td>
<td>-0.000431</td>
</tr>
<tr>
<td>HR</td>
<td>0.50</td>
<td>0.50</td>
<td>0.501089</td>
</tr>
<tr>
<td>HR/(</td>
<td>Bias</td>
<td>)</td>
<td>1123.60</td>
</tr>
</tbody>
</table>

The choice of SEP as the underlying parametric distribution for the competitor prices ultimately seems warranted given the above results and its use on similar data in the study by Fermanian et al.

4.2 Hitting the cover price

The first aim of the novel pricing models is to hit the best competitor price for each trade. This would allow putting a margin on the estimated price to win the trade with minimal cost. The best competitor price would in this case be the cover price, so we
Table 8: The measures presented in Table 3 calculated for each model on the test data set. Results are presented for the expected price (EP) method and Winner’s Curse adjusted (WCA) method along with corresponding covariate versions (EPC, WCAC). Regression results from linear regression (LM) and random forest (RF) are also presented.

<table>
<thead>
<tr>
<th></th>
<th>EP</th>
<th>EPC</th>
<th>WCA</th>
<th>WCAC</th>
<th>LM</th>
<th>RF</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>0.009435</td>
<td>0.009359</td>
<td>0.009680</td>
<td>0.009683</td>
<td>0.009413</td>
<td>0.009228</td>
</tr>
<tr>
<td>(P^* - P^{true})</td>
<td>0.006242</td>
<td>0.005995</td>
<td>0.006659</td>
<td>0.006538</td>
<td>0.006209</td>
<td>0.005774</td>
</tr>
<tr>
<td>Bias</td>
<td>-0.000431</td>
<td>-0.000787</td>
<td>-0.00228</td>
<td>-0.002842</td>
<td>-0.000592</td>
<td>-0.001086</td>
</tr>
<tr>
<td>(\sigma^{true})</td>
<td>0.009425</td>
<td>0.009326</td>
<td>0.009407</td>
<td>0.009256</td>
<td>0.009394</td>
<td>0.009178</td>
</tr>
<tr>
<td>(P^* - P^{ref})</td>
<td>0.014914</td>
<td>0.014585</td>
<td>0.013064</td>
<td>0.012502</td>
<td>0.014752</td>
<td>0.014386</td>
</tr>
<tr>
<td>(\sigma^{ref})</td>
<td>0.005081</td>
<td>0.005523</td>
<td>0.004067</td>
<td>0.004160</td>
<td>0.004596</td>
<td>0.006007</td>
</tr>
<tr>
<td>HR</td>
<td>50.1089%</td>
<td>50.3268%</td>
<td>39.1068%</td>
<td>36.0566%</td>
<td>49.7821%</td>
<td>50.4176%</td>
</tr>
<tr>
<td>HR/</td>
<td>Bias</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1162.62</td>
<td>639.476</td>
<td>171.521</td>
<td>126.871</td>
<td>840.914</td>
<td>464.250</td>
<td></td>
</tr>
</tbody>
</table>

This aims to "hitting the cover price". Another aspect is that we want to minimize the Winner’s Curse while still retaining a good hit ratio, and this will also be discussed. The results of the models for hitting the cover price are presented in Table 8. The best value for each measure is bolded.

The expected price (EP) method shows good accuracy given the simplicity of the model. The bias of the model is slightly below zero, which means that the model tends to be slightly conservative when estimating the price. The bias is small, but for our purposes it is larger than desired. By adding the 0.1 basis point adjustment, the bias will still be negative and the average bid would still be below the actual cover price. The negative bias could be due to outliers in the test data, such as trades where the final price was abnormally high due to an error on the part of a dealer. Alternatively, another explanation could be that the training data is biased due to mostly training the models on trades where the inspected dealer won or was tied with the winning competitor. Because of the addition of the MIFID II data, this effect should be offset, but it could still affect the results. The hit ratios of the expected price method is around 50%, which is to be expected since it is taking the expected value of the best competitor price density, which in our case is relatively symmetrical for the largest subset.

As expected, the Winner’s Curse adjusted (WCA) method does not improve the estimation of the cover price. The model has slightly worse values for the average absolute distance from the true competitor price, the bias and the RMSE compared to the expected price model. However, using this model, the Winner’s Curse distance is noticeably decreased. The Winner’s Curse distance as defined by the measure defined in equation (27) of the WCA method compared with the EP method is decreased by 12%. When looking at the test data, this corresponds to saving on average 0.011% of the notional of each trade given that you still win the trade with the more conservative quote. Since
the trade size is often large, this could potentially save a lot of money for the dealer. For example, if the notional value for a trade is 10 000 000 SEK, this would correspond to saving 1100 SEK. The effect is also reflected in the hit ratios, that are now around 40% instead of 50%. The model indicates that by "paying" ten percentage points of hit ratio, the risk of overbidding can be decreased. In other words, the model has decreased the risk of suffering from the Winner’s Curse when participating in these auctions.

Similar to the CBBT mid price, the chosen reference value is a CBBT price calculated by Bloomberg, and the performance of the WCA method depends on the accuracy of this parameter. It would be interesting to see how the performance would change if the accuracy of this parameter was increased. Traders should have access to an internal pricing system that gives a reference price, and maybe by using this in place of the CBBT reference price the performance could be improved. One could imagine that the Winner’s Curse adjusted model could in that case outperform the expected price model with respect to hitting the cover price, since the Winner’s Curse proxy should move the price towards a robust reference value through the optimization. This would also give a better protection against overbidding while not decreasing the probability of winning significantly.

The expected price method with covariates (EPC) has similar performance to the model without covariates, albeit having slightly better performance as evidenced by some of the measures. The absolute distance from the cover price is marginally smaller, and so is the Winner’s Curse distance. The bias is slightly more negative, which means the model is even more conservative. The RMSE measure indicates that EPC method is better than the EP method, but the difference is negligible. It is interesting that the expected price method with covariates does not significantly outperform the expected price method without covariates. The results for the Winner’s Curse adjusted method with covariates (WCAC) are similar, with only negligible differences from the model without covariates. Notably, the Winner’s Curse distance and the hit ratio have been decreased further, which like for the EPC method indicates that the model will give more conservative quotes.

The results of the models with covariates may indicate that the methodology for incorporating the effect of the covariates is not optimal in its current state. Some measures show improvement, but others are worse when incorporating covariates. Maybe more covariates, such as which customer the trade is done with, could be incorporated and tested to see if they give better results. However, it is likely that another implementation of covariates in the models altogether is needed to get more value from using the covariates.

The regression models also show good performance when tested on the test data. When comparing with the EP methods, all measures are quite similar, but they outperform the EP method with respect to for example the RMSE and the distance to the true price. The main difference is that the bias is larger for both the linear regression model (LM) and the random forest regressor (RF) compared to the EP method. If the goal is
Table 9: The measures presented in Table 3 calculated for each model on the data set with traded away and covered trades. Results are presented for the expected price (EP) method and Winner’s Curse adjusted (WCA) method along with corresponding covariate versions (EPC, WCAC). Regression results from linear regression (LM) and random forest (RF) are also presented. Note that these results use the quote that the inspected dealer lost with as the "true price" in the first two measures and the hit ratio.

<table>
<thead>
<tr>
<th></th>
<th>EP</th>
<th>EPC</th>
<th>WCA</th>
<th>WCAC</th>
<th>LM</th>
<th>RF</th>
</tr>
</thead>
<tbody>
<tr>
<td>[P^* - P_{\text{trAway}}]</td>
<td>0.009881</td>
<td>0.008975</td>
<td>0.008690</td>
<td>0.007977</td>
<td>0.009601</td>
<td>0.008383</td>
</tr>
<tr>
<td>Bias</td>
<td>0.008184</td>
<td>0.006737</td>
<td>0.006186</td>
<td>0.004895</td>
<td>0.007900</td>
<td>0.006079</td>
</tr>
<tr>
<td>[P^* - P_{\text{ref}}]</td>
<td>0.014717</td>
<td>0.013297</td>
<td>0.012719</td>
<td>\textbf{0.011428}</td>
<td>0.014433</td>
<td>0.012659</td>
</tr>
<tr>
<td>(\sigma_{\text{ref}})</td>
<td>0.004483</td>
<td>0.004901</td>
<td>0.003517</td>
<td>0.003577</td>
<td>0.005064</td>
<td>0.005274</td>
</tr>
<tr>
<td>HR</td>
<td>84.0055%</td>
<td>80.8959%</td>
<td>76.9089%</td>
<td>73.2297%</td>
<td>84.2057%</td>
<td>80.2083%</td>
</tr>
<tr>
<td>HR/</td>
<td>Bias</td>
<td></td>
<td>102.646</td>
<td>120.031</td>
<td>124.302</td>
<td>149.570</td>
</tr>
</tbody>
</table>

To hit the cover price, the ease of use and implementation of these regression methods could make them superior to the EP methods. However, when comparing to the WCA methods, the Winner’s Curse distances of the regression methods are larger, just like for the EP methods. Since you cannot control the effect of the Winner’s Curse in the regression models, the novel methods are superior if the dealer wants to avoid suffering from this phenomenon. In addition, the hit ratio to bias is more than double for the EP method compared to the RF method, indicating that the EP method will on average overprice less to get the same hit ratio compared to the RF method. In conclusion, there are advantages and disadvantages with both the regression methods and the novel methods, and the preferred choice depends on the user’s requirements and goals.

In the case where we do not know the winning price, it would still be interesting to see how the tested methods fare against previously sent bids. If the models estimate a better price than the historical data, then the dealer’s chances of winning the RFQ could have been higher. The results when running the model on historic traded away and covered trades are presented in Table 9. As discussed in Section 3.3, the measures in Table 9 are calculated against the quotes that the inspected dealer lost with and not the true price, and should be interpreted accordingly. Therefore, there are no inherent ‘best’ values for the first two measures in Table 9.

When comparing the pricing of the novel methods compared to the prices set on historic traded away data, we see that both the EP method and WCA method estimate prices that are better than the quote that the inspected dealer lost with in over 70% of the trades. Since we do not have the actual winning price, we cannot see how well the models priced the trade, but we see that by using these methods the probability of winning the RFQ could have been higher. The EP method gives better prices in approximately 80% of the cases, while retaining the Winner’s Curse distance at almost the same value as
on the test data. For the WCA method, the price is closer to the winning price in approximately 75% of the trades, and here the Winner’s Curse distance is again similar to the value calculated for the test data. Since the reference price should be of the same quality here as in the test data, we can make the assumption that the dealer should win approximately the same amount of times as they did in the test data presented in Table 8 if the models have the same Winner’s Curse distance. In that case, the dealer could have won approximately 50% of these trades using the EP method and about 40% using the WCA method.

The models with covariates have a slightly more pronounced effect on the measures calculated on the lost trades compared to the models without covariates. The EPC method has a smaller distance from the traded away quote and the Winner’s Curse distance is significantly decreased. The bias is similarly noticeably smaller. Similarly, the WCAC method gives better values for the estimates in most cases. Notably, the Winner’s Curse distance is decreased even further than the WCA method. The smaller bias of the WCAC method contributes to it having the best hit ratio to bias measure, and the second best is the WCA method. Therefore, these methods should give good pricing results while protecting from overbidding.

As with the test data, the regression models have similar performance to the EP methods. The results of the LM method are especially similar to that of the EP method, the only larger difference being the standard deviation of the Winner’s Curse distance. One noticeable difference is that the Winner’s Curse distance for the random forest regression is smaller than the EP methods and marginally smaller than the WCA method without covariates. In particular, it is much smaller than it was on the test data set. The reason for this could for example be that the traded away data set was favorable to the random forest method. Since the training data and the data where the inspected dealer lost are both from the same time period, the market is in the same state, and thus the pricing could be more similar than the test data, which is primarily in a later time period. This effect is something that the non-linear random forest regression model would capture better than the novel methods. However, while the average absolute distance to the reference price is lower, the standard deviation is almost 50% higher than that of the WCA method, most likely due to the random forest model sometimes giving anomalously high or low prices. The EP and WCA models will inherently protect against these types of outliers, since the EP is based on the expected value, and WCA is penalized by going over the reference price. Therefore, the EP and WCA methods can be considered to be more safe choices for pricing, since they are more stable.

4.3 Hitting the target ratio

The second aim is to set quotes that win a target ratio of trades. These models make use of the framework from the EP and WCA methods, and in particular the definition of the probability of winning defined in (24) and (26). As discussed in Section 3.3, the hit ratios presented for these models do not include the 0.1 basis point adjustment, unlike
Figure 6: The resulting actual hit ratios (HR) when setting the target ratio to $p$ for the constrained Winner’s Curse adjusted method with a target ratio margin $m$, and the hit ratios when covariates are included in the model (HRC).

<table>
<thead>
<tr>
<th>p</th>
<th>HR</th>
<th>HRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.7930%</td>
<td>5.7734%</td>
</tr>
<tr>
<td>0.1</td>
<td>14.8148%</td>
<td>13.9434%</td>
</tr>
<tr>
<td>0.25</td>
<td>27.1242%</td>
<td>25.9259%</td>
</tr>
<tr>
<td>0.5</td>
<td>41.2854%</td>
<td>41.1765%</td>
</tr>
<tr>
<td>0.75</td>
<td>68.8453%</td>
<td>66.2309%</td>
</tr>
<tr>
<td>0.9</td>
<td>86.0566%</td>
<td>83.8780%</td>
</tr>
<tr>
<td>1</td>
<td>94.6623%</td>
<td>93.6819%</td>
</tr>
</tbody>
</table>

(a) The resulting hit ratios when $m = 5\%$.

<table>
<thead>
<tr>
<th>p</th>
<th>HR</th>
<th>HRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>0.1</td>
<td>10.0218%</td>
<td>9.9129%</td>
</tr>
<tr>
<td>0.25</td>
<td>21.8954%</td>
<td>23.2026%</td>
</tr>
<tr>
<td>0.5</td>
<td>46.2963%</td>
<td>44.9891%</td>
</tr>
<tr>
<td>0.75</td>
<td>74.1830%</td>
<td>71.5686%</td>
</tr>
<tr>
<td>0.9</td>
<td>90.3050%</td>
<td>88.4532%</td>
</tr>
<tr>
<td>1</td>
<td>99.7821%</td>
<td>99.7821%</td>
</tr>
</tbody>
</table>

(b) The resulting hit ratios when $m = 0\%$.

the EP and WCA methods. The results of the constrained Winner’s Curse adjusted models with a target ratio margin of 5\% and 0\% are presented in Figure 6, Panel a) and b), respectively.

When setting the target ratio margin $m = 5\%$, we can see the effect of the Winner’s Curse proxy on the results. The model captures the desired hit ratio within the allowed constraints in most cases, and stays within an interval of 10\% in all cases. A noticeable trend is that the model hits the boundary point of the optimization that is closest to the hit ratios resulting from the WCA and WCAC methods shown in Table 8. From this, we can see that the model still takes the Winner’s Curse proxy into account. Of course, the effect is reduced when the allowed optimization space is reduced, but it could still be useful to minimize the risk of setting prices that are too aggressive or conservative by some amount.

When setting the target ratio margin to 0\%, the hit ratio is closer to the desired target ratio and stays within an interval of 5\% of the target ratio in all cases. The increased accuracy is likely because the Winner’s Curse proxy is no longer having an effect; the method is in effect just calculating the inverse of the probability of winning function defined in (24) and (26) for some given target ratio. This could be interpreted as evidence of how well the densities of the best competitor prices are tuned from the training data.

Using covariates in this model does not clearly improve nor decrease its performance. In some cases, the target ratio is much closer using the models with covariates, and in some cases it is much farther away. In general, it seems that incorporating covariates in this model decreases its reliability.

The methods for hitting a target ratio could be powerful tools when used for trading,
as one could decide the market share that is desired and let the model find a suitable estimate for the corresponding price. One of the possible reasons for why the performance of this model is good is that the pricing of bonds follows a simple pattern, which is often the case today. Given some reference price, the dealer can choose to add or subtract a margin, often of standardized quantities, depending on the current market situation and the relationship with the client that wants to trade. In this case, this type of model should be able to give good performance.

4.4 Feature importance of covariates

While the covariate implementation for the novel pricing models did not have an as significant impact on the result as was hypothesized, it is still interesting to analyze their importance for hitting the correct price. Since the random forest regression model was good at hitting the best competitor price, this indicates that the covariates’ feature importance may be relatively similar to the true importance of the covariates when pricing. The feature importances of the covariates included in the covariate-adjusted methodologies and in the regression methods as independent variables are presented in Table 10.

Table 10: Feature importances for the covariates, including buy and sell indicators, as calculated by the random forest regressor. The feature importances calculated for the covariates are normalized such that they add up to one.

<table>
<thead>
<tr>
<th>Covariate</th>
<th>Feature importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Days to maturity</td>
<td>0.249111</td>
</tr>
<tr>
<td>CBBT reference price</td>
<td>0.157196</td>
</tr>
<tr>
<td>CBBT spread</td>
<td>0.115514</td>
</tr>
<tr>
<td>Notional</td>
<td>0.115537</td>
</tr>
<tr>
<td>Coupon</td>
<td>0.106980</td>
</tr>
<tr>
<td>Days of accrued interest</td>
<td>0.089954</td>
</tr>
<tr>
<td>Sell indicator</td>
<td>0.085024</td>
</tr>
<tr>
<td>Buy indicator</td>
<td>0.080685</td>
</tr>
</tbody>
</table>

The highest feature importance is given to the number of days until the bond’s maturity. As was described in Section 2.3, the price of a bond is the equal to the aggregated value of its discounted cash flows. Since the days to maturity is a proxy for the number of cash flows remaining for the bond, it is not surprising that it is of importance when setting the price of the bond.

The CBBT reference price is the covariate with the second highest feature importance. This is an indication that the reference price is sufficiently close to the actual execution price of the RFQ to give a good indication of the cover price of the trade. Since the
reference price is available when the RFQ is sent to the dealer, it should be leveraged by dealers as a good guess of the final price to compare with an internally calculated reference price.

The notional and the CBBT spread are given similar feature importance according to the random forest regression. Similarly to the days to maturity, the notional is included in the calculation of the bond price, and should therefore be important to the pricing. However, since many bonds could have the same notional, it is perhaps not as good for partitioning the trades as the number of days until maturity. The CBBT spread indicates the liquidity of the bond, and this should naturally also affect the price of the bond. However, since it is already included in the transformation between yields and reduced quotes as defined in (21), the effect may have been diminished.

The remaining covariates have lower feature importance. The coupon, like the notional, is often the same among many trades, and maybe even more so than the notional since we have a limited number of bonds. This should diminish its importance for each individual trade. The accrued days of interest is maybe less important since the prices we look at are quoted as the clean bond price, and since the days to maturity in some way already incorporates this effect when looking at trades of same bond. Finally, the fact that the buy and sell side indicators are not as important could indicate that the true prices are often crossing the mid, since this would make the side of the trade less important. This could for example be attributed to that the CBBT mid prices used in this report are not sufficiently accurate, or that trading of bonds are often done as a service to the client and not to make a profit, but it is difficult to determine. However, the low feature importance might also be due to that these indicators are not specific to a certain price, which means that they do not significantly contribute to the split purity despite still being important for the overall regression.

Overall, the feature importances for the covariates are in line with expectations from financial theory of bond pricing and trading.
5 Conclusions

This study has defined six novel models for estimating the optimal bidding price in bond market auctions. The expected price (EP) method and the Winner’s Curse adjusted (WCA) method along with the same models with covariates (EPC, WCAC) are models that aim to hit the best competitor price of a trade, to be able to win with the lowest margin. The WCA methods incorporate the effect of the Winner’s Curse in the pricing using a proxy for the Winner’s Curse defined using a reference price available to the trader at the time of the auction. The constrained Winner’s Curse adjusted (CWAC) method along with a version with covariates aims to hit a target ratio of trades using a probability of winning the trade defined using the density of the best competitor prices.

When considering the first aim of the study, the EP and WCA methods show good results. When looking at previously lost trades, the EP method calculates quotes that are better in over 80% of the trades compared to those sent when the dealer lost, which could represent a large improvement over the previous pricing. The results indicate that the EP method could have won 50% of the lost trades, which is a good result. The EP method is also good at hitting the best competitor prices, and given more training data the results could most likely be improved. The WCA method succeeds in its purpose of reducing the risk of suffering from the Winner’s Curse by reducing the distance from a reference price set for each trade. In the test data, the WCA method reduces the Winner’s Curse proxy by approximately 12% compared to the EP method at the same time as reducing the hit ratio of trades by about 10%. This reduction in the Winner’s Curse proxy corresponds to saving on average 0.011% of the notional given that we still win the trade according to the test data set. The regression models also show good results when tested. The random forest regressor is especially good at hitting the cover price, but the estimated prices have higher bias than the novel methods. Since the regression methods cannot penalize the risk of suffering from the Winner’s Curse, the novel methods are especially useful if avoiding the Winner’s Curse is of interest to the dealer.

The aim to win a target ratio of trades chosen by the dealer is fulfilled by the CWAC methods, which show great promise in this respect. The results on the test data shows that when choosing a target ratio margin of 5%, the actual hit ratio stays within an interval of 10% around the target ratio, and when choosing a margin of 0%, the actual hit ratio stays within an interval of 5% of the target ratio. Taking into account the simplicity of the model, these results are promising. Choosing a certain target ratio to win is something that the regression models cannot do, so again the novel pricing method framework is shown to be useful.

The implementation of covariates in the novel models defined in the study did not have an as significant effect on the results as was expected. This could be due to the choice of covariates, but since the regression methods fared well with the same
covariates it seems unlikely. Another reason could be that the implementation does not incorporate the effect of the covariates in an adequate way. As a result, the inclusion of covariates in the implementation used in this report seem less useful, especially given the complexity of fitting the extra scaling parameters for the covariates. When looking at the feature importances of the covariates as calculated by the random forest regressor, the importances seem to be in line with expectations from financial theory.

Some limitations are present in the study. Firstly, the testing set used is relatively small compared to the training set. Over time additional test data could be accumulated to give more precise results. Since the data in the training set to a large extent consisted of the cases where the inspected dealer was accepted as the best price or they had the same price as the winner, there is a risk that a bias was introduced for estimating lower competitor prices. The addition of MIFID II data should offset this to some extent, but the data could still be unbalanced with respect to the status of the trades. With more MIFID II prices, the analysis could have been protected against this bias. Additionally, the choice of the SEP distribution was not a clear choice from the results when comparing the distributions. This could partly be attributed to the CBBT indicative prices from Bloomberg, that are most likely not the most accurate reference values. However, since this was the best reference for the mid, bid, ask and the bond price that could be found. Finally, when participating in an RFQ, the responding dealer may often only post quotes at specific ticks. This was not considered in the study. Incorporating the effect of this limitation for the dealer could be interesting to investigate further.

This study opens many topics that could be the subject of future studies. Firstly, it would be interesting to investigate alternative implementations of covariates in the novel pricing methods. With a better implementation, the additional information of the covariates could give more specificity to the estimation of each trade, which should theoretically give a better pricing. One example could for example be to replace the best competitor price density \( g(\delta; n, \cdot) \) with a conditional density, where the reduced quote is conditioned on the covariates for the specific trade. Since only a subset of the bonds were used in this study, a next step would be to see how the novel models fare with a more diverse set of bonds.

In addition, the implementation of the Winner’s Curse proxy in this case is quite simple and could be made more complex. For example, one could adjust the strength of the Winner’s Curse proxy in the optimization of the WCA method depending on the number of dealers requested and other relevant variables to better incorporate the rules presented by Capen et al. [2]. One could also look further into improving the regression models since they already provided good results. For example, one could test the effect of introducing new covariates in the models, perhaps indicator of specific larger customers, or proxies for market factors such as current price trends.

The novel methods presented in this report show promise as a first step towards creating algorithms for setting optimal bidding prices in bond market auctions. With further research in this area that leverages more data from MD2C platforms and MIFID II
recorded transactions, the lagging electronification of bond trading can get the final push it needs to become fully electronic and perhaps increase the liquidity of the bond market.
6 References


7 Appendix

Table 11: Complete unfiltered training data set specifications.

<table>
<thead>
<tr>
<th>Dealers</th>
<th>Buy RFQs</th>
<th>Sell RFQs</th>
<th>Total</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>284</td>
<td>305</td>
<td>589</td>
<td>3.4%</td>
</tr>
<tr>
<td>1</td>
<td>339</td>
<td>235</td>
<td>574</td>
<td>3.4%</td>
</tr>
<tr>
<td>2</td>
<td>1332</td>
<td>1082</td>
<td>2414</td>
<td>14.1%</td>
</tr>
<tr>
<td>3</td>
<td>1177</td>
<td>1229</td>
<td>2406</td>
<td>14.1%</td>
</tr>
<tr>
<td>4</td>
<td>5693</td>
<td>4752</td>
<td>10445</td>
<td>61.1%</td>
</tr>
<tr>
<td>5</td>
<td>156</td>
<td>136</td>
<td>292</td>
<td>1.7%</td>
</tr>
<tr>
<td>&gt; 5</td>
<td>194</td>
<td>168</td>
<td>362</td>
<td>2.1%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>9175</strong></td>
<td><strong>7907</strong></td>
<td><strong>17086</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>

Table 12: Complete unfiltered test data set specifications.

<table>
<thead>
<tr>
<th>Dealers</th>
<th>Buy RFQs</th>
<th>Sell RFQs</th>
<th>Total</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>23</td>
<td>28</td>
<td>51</td>
<td>2.8%</td>
</tr>
<tr>
<td>1</td>
<td>37</td>
<td>22</td>
<td>59</td>
<td>3.2%</td>
</tr>
<tr>
<td>2</td>
<td>148</td>
<td>167</td>
<td>315</td>
<td>17.3%</td>
</tr>
<tr>
<td>3</td>
<td>147</td>
<td>171</td>
<td>318</td>
<td>17.5%</td>
</tr>
<tr>
<td>4</td>
<td>511</td>
<td>454</td>
<td>965</td>
<td>53.0%</td>
</tr>
<tr>
<td>5</td>
<td>24</td>
<td>28</td>
<td>52</td>
<td>2.9%</td>
</tr>
<tr>
<td>&gt; 5</td>
<td>31</td>
<td>29</td>
<td>60</td>
<td>3.3%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>921</strong></td>
<td><strong>899</strong></td>
<td><strong>1820</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>

Table 13: Trade status distribution of the observations included in the training and test data after filtering.

<table>
<thead>
<tr>
<th>Trade status</th>
<th>Training data</th>
<th>Test data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covered</td>
<td>1490</td>
<td>95</td>
</tr>
<tr>
<td>Done</td>
<td>4898</td>
<td>296</td>
</tr>
<tr>
<td>Not participated</td>
<td>31</td>
<td>2</td>
</tr>
<tr>
<td>Traded away</td>
<td>2547</td>
<td>146</td>
</tr>
<tr>
<td>Tied traded away</td>
<td>0</td>
<td>379</td>
</tr>
</tbody>
</table>
Figure 7: The fit of the Gaussian, NCT and SEP distributions to the reduced quotes of subsets of the data with three competing dealers. The empirical density of the data is presented in blue (bars), and the fitted density in orange (line).