Distance Control of Heavy-Duty Vehicle Platooning

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Abstract

Vehicle platooning is a modern technique which enables us to have a group of vehicles driving in a cooperative way. More specifically, in vehicle platooning, a number of vehicles uses a common control system and inter-vehicle communications to cooperatively drive close to each other and in a high speed. The most important consequence of this way of driving is a decrease in aerodynamics drags for the follower vehicles which in turn, leads to considerable reduction in fuel consumption. However, driving at a short inter-vehicle distance in a high speed leads to several big challenges.

In this master thesis, we try to address one of the challenges by designing a longitudinal dynamics controller to keep the distances between vehicles. To simplify the work, it assumes that the vehicles are moving on a flat road. First, the design of P, PD, and linear quadratic tracking (LQT) controller is carried out by considering the dynamics, speed, and acceleration of the vehicles as well as their relative distances. Next, to assess the effectiveness of the designed controllers, numerical simulations are done. Our numerical simulations demonstrate that the LQT and PD controllers have similar performances and show a similar response time to the distance tuning command. Further discussions will be provided to justify the similar performances between PD and LQT controllers.

Finally, to see the sensitivity of the controllers to noise and delay in the estimations, another set of numerical simulations are done. The sensitivity analysis shows that noise and delay in the acceleration estimations have a bigger impact on the overall performance of the controllers. More importantly, when the noise and delay in the estimated accelerations pass a certain threshold, the controller becomes instable implying a sharp requirement on the accuracy and delay in the estimate of the acceleration in a platooning system.
Sammanfattning:


Avslutningsvis görs ytterligare simuleringsar för att illustrera regulatorernas känslighet för brus och fördröjning i insignalerna. Analyser visar att brus och fördröjning i insignaler, specifikt i inkommande accelerationssignal har större påverkan på regulatorernas övergripande prestanda och om brus och fördröjning i signalen överskrider mer än ett tröskelvärde, resulterar att slutna systemet blir instabilt vilket i sin tur sätter ett skarpere krav på noggrannhet och fördröjning i insignaler till kontrollsystemet.
1 Introduction

1.1 Platooning

Air quality today is a hot topic of debate and concern in many European countries. Even though the air pollution reports have shown slow improvements during the recent years, air pollution and CO\textsubscript{2} emissions are still the biggest environmental hazard, and they cause around 400 000 early deaths in 41 European countries every year [1].

Transportation has a big share in creating the air pollution. Particularly, the transport sector accounts for a third of all final energy consumptions and 29% of the total CO\textsubscript{2} emissions in Europe. Moreover, road transport is the dominant mode of transportation, and road-freight transportation specifically stands for 11% of the total CO\textsubscript{2} emissions [2]. This means that there is a big room for reduction in the amount of CO\textsubscript{2} emitted by trucks used for road freights.

These facts, consequently, have created big challenges for vehicle manufactures and have led them to establish or participate in many different research areas, such as vehicle platooning, in order to find solutions for the environmental issues. One of the research areas which attracted a lot of attention in this regard is to develop and implement truck platooning. Truck platooning is a technique in which a group of vehicles, which are equipped with a common control system and operate in a cooperative manner, can drive closely and safely together in high speed [3].

Vehicle platooning has some major impacts like reduced fuel consumption, better traffic flow, and more traffic safety [4-7]. The most important advantage of platooning is the reduction in the fuel consumption. In fact, since the group of vehicles are driving in a short inter-vehicle distance, the aerodynamics drag for the follower vehicles decreases which leads to that the group of followers consumes less fuel [8]. In particular, a reference distance of 10 m between a group of two vehicles having a speed of 80 km/h can lead to reduction of 40 % and 4 % in aerodynamics drag for the second and first vehicle in the platoon, respectively [9]. Moreover, the automatic control of the longitudinal dynamics of the vehicles, which is implemented in platooning, has the potential of reduction in rear-end collisions [10].

In a modern platoon system, vehicles continuously receive actual states of other vehicles in the platoon using vehicle to vehicle communication and also traffic information such as actual speed limits and traffic light status via infrastructural communication. The available information makes the driving better planned which results in a better traffic flow and less fuel consumption. Driving at a close inter-vehicle in high speed contributes also to reduction of fuel consumption thanks to improved aerodynamic effectiveness for the vehicles followed by the platoon leader. Platooning improves safety as human involvement in driving will reduce. For instance, vehicles will brake in time when needed, and accidents will possibly be reduced [11].

1.1.1 Realization of platooning systems

Due to the ever-increasing traffic density and traffic network complexity, it would be very hard for a truck’s driver to drive at a close inter-vehicle distance and to manage to keep this distance relatively constant while driving at a high speed. On the other hand, the reaction time of a typical driver is usually not optimal to continuously adjust the speed and the acceleration to keep the distance to the other trucks fixed. Thanks to very advanced technologies in electronics, wireless communications, and automatic control, it has been become practical to exploit information from different sensors installed on the trucks and to use control mechanism to implement the platooning in a more efficient and safer way [12].
One of the first trials to implement the platooning is the adaptive cruise control (ACC) [13-15] method. In ACC, information is gathered by radars placed in front of each vehicle that are measuring the relative distance and velocity to the preceding vehicle. This information then will be used to control the dynamics of the vehicles to keep the inter-vehicle distance at a fixed value. Due to the delays in estimation of the relative distances and speeds, this method is not able to provide a small inter-vehicle distance without sacrificing the safety [12]. In fact, since each vehicle only knows the relative distance and speed with respect to the preceding vehicle, any action, like acceleration and deceleration, taken by the leading vehicle is known to other vehicles whenever their preceding vehicle takes the same action. This means that the vehicle in the tail gets aware of the head vehicle’s action with a significant delay which basically limits the inter-vehicle distance to avoid having a collision [16].

However, wireless communication can significantly change our ability to more efficiently use the information available from different sources and sensors. More specifically, vehicle-to-vehicle (V2V) communication can be used to exchange precise information between the vehicles about each vehicle’s position, speed, and acceleration with a precision of centimeters that are obtained from global positioning system (GPS) [17, 18]. This means that the precise information about the maneuvering of the lead vehicle is easily available to the other vehicles with a very low amount of delay. When this information is available instantly, one can more easily automate the longitudinal dynamics of the vehicles with a shorter inter-vehicle distance which, in turn, leads to less fuel consumption [18].

Furthermore, when the communication between vehicles becomes easy, extra parameters about each vehicle, like vehicle mass and braking and actuator capabilities, can be easily sent to all other trucks. Availability of this kind of information leads us to a more advanced platooning method known as cooperative adaptive cruise control [18]. From another stand point, vehicle-to-infrastructure (V2I) communication can allow the vehicles in the platoon receive information about the road, topography of the area, traffic status, and weather conditions. This, further, leads to a more efficient platooning by, for example avoiding, unnecessary braking [3, 19, 20]. Further, the communication to infrastructure can help to improve the traffic flow and smoothness.

Exploiting V2V and V2I communication to control the dynamics of the vehicles in a platoon is getting more and more popular. For instance, in [21, 22], platooning of up to 8 vehicles has been implemented and a number of experiments were conducted. The results were quite promising and showed considerable reduction in fuel consumption.

Using the extra information about road topography, more fuel-efficient platooning methods have been introduced. Particularly, look-ahead vehicle control (LAC) uses road map data to take suitable control action by having road grade preview information. For instance, when a downhill is ahead, the control system can send commands to the engine and gear box to lower the speed and hence reduce an unnecessary brake. Likewise, proper actions can be taken when the trucks will face an uphill. For this category of techniques, one can name [23-25]. Based on LAC policy, there is a commercial system available in the market which uses road map data to more efficiently implement a platooning system [23].

1.2 Background

In order to realize the concept of platooning, a prototype system was designed and developed in a cooperation between KTH Royal Institute of Technology, Stockholm, and Scania CV AB, Södertälje; see Figure 1.1 [26]. The system will not be described in detail in this report, yet a general description will be presented to have a better overview of the role of the current thesis in the project. However, the reader can refer to [26] for a complete description of the prototype system.
In this prototype system, basically every vehicle has a unique vehicle ID, and its platoon ID will be the vehicle ID of the platoon leader. The vehicles in the platoon continuously send their actual states to a sub-system called Estimator. Having the states, Estimator estimates the variables that are necessary for the controllers in the form of a state vector. The state vector contains absolute speed, absolute acceleration of each vehicle and the relative distance to the vehicle ahead. More specifically, we have

\[ \vec{X} = \begin{pmatrix} v_{\text{platoon}} \\ a_{\text{platoon}} \\ p_{\text{platoon}} \end{pmatrix} \]  

(1.1)

Where \( v_{\text{platoon}}, a_{\text{platoon}}, p_{\text{platoon}} \) denote the absolute speed, the absolute acceleration, and the relative distance to the vehicle ahead, respectively, for a given vehicle.

Figure 1.1. The figure schematically depicts the prototype system structure. The Estimator is receiving signals from platoon vehicles and estimates a signal that can be used in controllers.

1.3 Theses objective and delimitations

The objective of this thesis is to determine a control system in order to keep a reference distance between platoon vehicles. Reference signals, that are used in the thesis, are the state vector in (1.1), and analyses will be done in order to verify if using all states, as input to the controller, would improve the controlling results in theory and numerical simulations.

In this work, P, PD, and optimal controllers are designed by using different vehicle models. The controllers will later on be implemented in a simulation environment to test and verify the controller design numerically.
2 Vehicle Modeling

In this work, one linear model will be used to design the controller gains for the LQ system. The input is the speed reference, $v_{ref}$, and the output is the absolute speed and acceleration (the first-order derivative of speed with respect to time). One physical model will also be designed for testing of the controllers. To simplify the procedure, the road condition is assumed to be flat with zero slope, and for all the vehicles in platoon, we will use the same model in the testing environment.

2.1 System Identification

System identification here basically means building mathematical models of dynamical systems from a set of observations that we make from real experiments. System identification is a very useful tool in particular cases when it is complicated to model a dynamical system or parts of a system numerically. One example of these complicated system is to model the start friction of a dynamic system.

In this case, the input is a speed reference to the truck and the output is the speed of the truck in $m/s$. The data was logged via CAN communication in the vehicle and saved in a computer. The logged data was analyzed later by the Matlab toolbox System Identification in order to estimate a transfer function for the system. Figure 2.1 is showing one example of the plots recorded by real data from the Scania truck.

![Figure 2.1.](image)

Figure 2.1. The data logs are loaded into the system identification toolbox. The speed reference is used as the input and the vehicle speed is used as the output, and a second-order transfer function is estimated to represent the plot. The dashed line is the data sample from the truck, and the solid line is the step response of the estimated transfer function.
Next step is to represent the system in a mathematical model. State space is very practical to use here since system model has two outputs, namely the vehicle speed and the acceleration. The transfer functions logged in the truck have the general form of:

\[ G(s) = \frac{\beta}{s^2 + \alpha s + \beta} \]  

(2.1)

where \( \alpha, \beta \) are some constants. The output \( Y(s) \) of an open loop system is the input \( U(s) \) times the transfer function of the system \( G(s) \); that is,

\[ Y(s) = G(s) \cdot U(s). \]  

(2.2)

Substituting (2.1) into (2.2) gives:

\[ [s^2 + \alpha s + \beta] \cdot Y(s) = \beta \cdot U(s) \]  

(2.3)

By taking the inverse Laplace transform of (2.3), we get

\[ \ddot{y} + \alpha \dot{y} + \beta y = u \cdot \beta \]  

(2.4)

where \( \dot{y} \) and \( \ddot{y} \) represent the first-order and the second-order derivative of \( y \), respectively. The first state variable \( x_1 \) represents the velocity and the second state variable \( x_2 \) is the time derivative of the first state, i.e. the acceleration. More specifically, we have

\[
\begin{cases}
  x_1 = y \\
  x_2 = \dot{y}
\end{cases} \quad \begin{cases}
  \dot{x}_1 = x_2 \\
  \dot{x}_2 = -\beta x_1 - \alpha x_2 + u \beta
\end{cases}
\]  

(2.5)

The general matrix form of a state space model is:

\[
\begin{align*}
  \dot{x} &= A x + B u \\
  y &= C x + D u
\end{align*}
\]  

(2.6)

Putting (2.5) into (2.6) gives the state space matrices \( A \) and \( B \) as

\[
\begin{pmatrix}
  \dot{x}_1 \\
  \dot{x}_2
\end{pmatrix} = \begin{pmatrix}
  0 & 1 \\
  -\beta & -\alpha
\end{pmatrix} \begin{pmatrix}
  x_1 \\
  x_2
\end{pmatrix} + \begin{pmatrix}
  0 \\
  \beta
\end{pmatrix} u
\]  

(2.7)

The matrix \( D \) is set to zero because there is not any feed forward part in the system, and the matrix \( C \) is set to the identity matrix to have the both speed and the acceleration states as output. That is,

\[
C = \begin{pmatrix}
  1 & 0 \\
  0 & 1
\end{pmatrix}
\]  

(2.8)

### 2.2.2 Piece-Wise Affine Model

The tests on the Scania truck were performed on the following conditions:

- start from zero speed and end up in different reference speeds
- start from non-zero speeds and end up in different final speeds
- speed changes by relatively low-speed references
- speed changes by relatively high-speed references
Test logs indicate that the system does not respond in the same way to different steps because of the limitations in the dynamics and the non-linearity in the cruise controller; see Figure 2.2 for further details. To simulate a cruise controller which covers most of the system, the whole cruise controlling system gets divided into five different conditions using five different poles, defined by the logs, in the transfer function; see Figure 2.3.

![Figure 2.2](image)

Figure 2.2. This figure demonstrates non-linearity in the cruise controller and trucks physical limitations. We have two curves that show two different step responses for 17 and 5 m/s as the speed reference from zero speed. Gear changing has obviously a big impact in the step responses of larger reference speeds. Acceleration is around zero for almost two seconds in every gear change. The delays caused by gear changing have not been taken into account in the PWAM in this work for the sake of simplicity.
2.2 Physical Models

This model is a non-linear model and it will take into account the limitations and non-linearity that a real truck will have in longitudinal direction such as dynamic limitations, air drag, etc. The physical model is useful for testing the controllers and analyzing how these non-linearities and limitations in dynamics will affect testing results.

The longitudinal dynamics of a vehicle is divided into two major parts, the vehicle dynamics and the power train dynamics. The vehicle dynamics are influenced by aerodynamic forces, rolling resistance forces, longitudinal tire forces, and gravitational forces. The power train dynamics are basically the power transmitting from the engine to the wheels.

The gravitational forces will be neglected in this work because the system is designed for a flat road condition.

2.2.1 Longitudinal Vehicle Dynamics
Consider a vehicle driving in a flat road (see Figure 2.5), a balance equation considering the influencing forces will be:

$$m\ddot{x} = F_x - F_{aero} - R_x$$  \hspace{1cm} (2.9)

where:

- $F_x$ is the Longitudinal force at the rear tires,
- $F_{aero}$ is the equivalent longitudinal aerodynamic drag force, and
- $R_x$ is the force due to rolling resistance at the front and rear tires.

### 2.2.2 Driveline Dynamics

The longitudinal tire forces $F_x$ are basically the dynamics to transmit the power from the engine over the powertrain to move the vehicle forward. The driveline is basically divided into four parts. The power created in the engine connects to the transmission via the torque converter which finally passes to the wheels. A picture of a typical power train is shown in Figure 2.6.
To simplify, inertia of the engine and inertia of the wheels are included in the total mass. Clutch and driver shaft are assumed to be stiff and will not be mentioned in the model.

2.2.3 Transmission Dynamics

The transmission dynamics consists of a set of gears. If we let \( R \) be the gear ratio, then the value of \( R \) depends on the operating gear including the final gear reduction in the differential. Generally, \( R \) is less than one, and it increases as the gear shifts upwards.

If torque to the wheels is denoted as \( T_{wheels} \), the aforementioned torque to the transmission is represented as \( T_t \), and if the lose in transmission is shown as \( \eta \), then the relation between torques is:

\[
T_{wheels} = \frac{1}{R} T_t \cdot \eta \tag{2.11}
\]

Moreover, the relation between transmission and speed in wheels is:

\[
\omega_t = \frac{1}{R} \omega_w
\]

2.2.4 Aerodynamic Force

The aerodynamic forces can be described as follow:

\[
F_{aero} = \frac{1}{2} c_d A_d \rho_a v^2 \tag{2.13}
\]

where \( A_d \) is the area of the front of the truck and the \( \rho_a \) is the density of the air. Moreover, \( c_d \) is the aerodynamic drag coefficient, and the \( v = \dot{x} \) is the longitudinal velocity.

2.2.5 Rolling Resistance

The simulation of rolling resistance is often one of the most complicated parts to make. It depends on many conditions such as road conditions, pressure in the tier, type of the tier, etc. Therefore, it is
almost impossible to make a model which will perfectly emulate the real system. A relatively simple dynamic model of the roll resistance is a roll coefficient \( c_r \) times the gravity, namely,

\[
F_{\text{roll}} = c_r mg
\]  
(2.14)

where \( c_r = 0.006 + 0.23 \cdot 10^{-6} \cdot v^2 \) [27].

### 2.2.6 The Complete Vehicle Model

Now, back to the dynamic equation (2.9), we have

\[
m \ddot{x} = F_x - F_{\text{aero}} - R_x,
\]

By substituting equation (2.10) into (2.11), \( F_x \) will be:

\[
F_x = T_t \cdot \eta \cdot R_{\text{wheel}}^{-1}
\]  
(2.15)

Further, by applying (2.13), (2.14) and (2.15) into (2.9), a longitudinal dynamic of a vehicle is achieved as

\[
m \ddot{x} = (T_t \cdot \eta \cdot R_{\text{wheel}}^{-1}) - \frac{1}{2} c_d A_d \rho a \dot{x}^2 - (C_r \cdot mg)
\]  
(2.16)

After dividing (2.16) by the mass, an expression of the acceleration will be achieved as [28]:

\[
\ddot{x} = \frac{1}{m} (T_t \cdot \eta \cdot R_{\text{wheel}}^{-1}) - \frac{1}{m} \left( \frac{1}{2} c_d A_d \rho a \dot{x}^2 \right) - (C_r \cdot g)
\]  
(2.17)
3 Distance Control

3.1 Introduction

In order to keep a reference distances $d_o$ between vehicles in the platoon, the distance control is designed. The inputs to controller are the platoon vector including the states of the vehicle in platoon (the vector in 2.2) and including states of the ego vehicle.

In this chapter, four types of controllers are introduced and described. The controllers will later be tested in a simulation environment to be compared to each other.

3.2 Proportional Controller

This proportional controller’s structure is divided into two parts. A feed-forward part which is the speed of the vehicle ahead $V_{lead}$, and an error part which is proportional to the difference in vehicle positions minus the desired control target $d_o$. More specifically,

$$V_{ref} = V_{lead} + K (P_{lead} - P_{ego} - d_o) \quad (3.2)$$

To avoid oscillations, a dead zone $d_d$ is added to the feedback part of the controller (schematic of the implementation can be found in Figure 3.1). The dead zone $d_d$ is implemented to avoid oscillating behavior since control error is almost never equal to zero in this case, i.e. $V_{ref}$ will always be changing.

![Figure 3.1. Schematic representation of the implementation of the proportional controller.](image-url)
3.3 PD Controller

Adding a derivative part in the P controller in (3.2) makes the controller faster and helps the vehicle to react much quicker in high acceleration changes. More precisely, we have

\[ V_{ref} = V_{lead} + K_i (P_{lead} - P_{ego} - d_0) + K_D (V_{lead} - V_{ego}) \]  \hspace{1cm} (3.3)

3.3.1 Implementation

The implementation schedule is depicted in Figure 3.2. Using large parameters causes oscillating behavior when approaching the desire distance and using small parameters makes the system slow. In this sense, a strategy has been used by dividing the controller operation in two different parts and two different parameters is used for each section.

In the first section, when the distance is getting higher than \( d_{limit} \), the parameters \( K_{i1} \) and \( K_{d1} \) will be used, and it will change to \( K_{i2} \) and \( K_{d2} \) when approaching \( d_{limit} \).

![Figure 3.2. PD controller implementation.](image)
3.4 Optimal Controllers

Optimal control theory is about to determine the control signal that will cause a process to satisfy the physical constrains and at the same time minimize (or maximize) some performance criterion [29].

3.4.1 Linear Quadratic Control

Linear Quadratic control problem is an optimal control problem description when the dynamical system is described as linear and the costs are a quadratic function of states and inputs.

A dynamical system in state space function is generally described as:

\[ \dot{x} = Ax + Bu \]

The standard cost function in a finite horizon time invariant system, defined on: \( x \in \mathbb{R}^n, u \in \mathbb{R}^m \) is described by:

\[ J(t_0) = \frac{1}{2} x^T(T)Q(T)x(T) + \frac{1}{2} \int_{t_0}^{T} (x^TQ(t)x + u^TR(t)u)dt. \]  

(3.4)

and the feedback control law which will minimize the value of the cost is:

\[ u(t) = -Kx(t) \]  

(3.5)

where \( K \) is given by:

\[ K = R^{-1}B^TP(t) \]  

(3.6)

and \( P \) is given by solving the Riccati equation:

\[ A^TP(t) + P(t)A - P(t)BR^{-1}B^TP(t) + Q = -\dot{P}(t) \]  

(3.7)

in the time interval between \([t_0, T]\).

This type of LQ problem which is in a limit time bound requires information ahead of current time. A simpler case is to set up the system non-time bounded in an infinite-horizon time. This is made possible by letting the final time tends to infinity, i.e. \( T \to \infty \).

The problem is to find the solution of the stationary case by minimizing the infinite-horizon cost function:

\[ J(t_0) = \int_{t_0}^{\infty} (x^TQ(t)x + u^TR(t)u)dt \]  

(3.8)

The solution is to solve \( P \) out of the equation (4), considering the system to be in steady state and letting \( T \to \infty \) so do we have the \( P_\infty = 0 \). This means that we have

\[ A^TP + PA - PBR^{-1}B^TP + Q = 0 \]

And \( K \) is given by (3) and we put it in (2) to get the feedback gain [30].
3.4.2 Reference Tracking Problem

The target is to minimize the differences between the leading vehicle references and the ego vehicle states, namely,

\[ e_1 = r - H_1 x_1. \]  \hfill (3.12)

where

\[ x_1 = x_{ego} = \begin{pmatrix} v_{ego} \\ a_{ego} \\ p_{ego} \end{pmatrix} \]  \hfill (3.9)

is the state vector of the ego vehicle, \( A_1, B_1 \) and \( C_1 \) are the system matrices taken from the linear model, and \( z_1 \) are the states that will be minimized. The description of state space model of the system is:

\[
\begin{cases}
    \dot{x}_1 = A_1 x_1 + B_1 u_1 \\
    y_1 = C_1 x_1 \\
    z_1 = H_1 x_1
\end{cases}
\]  \hfill (3.10)

and the reference vector from the leading vehicle is:

\[ r = \begin{pmatrix} v_{lead} \\ a_{lead} \\ p_{lead} \end{pmatrix}. \]  \hfill (3.11)

To control the distance, an integral action has to be implemented as a new state. This new state is the integral of the deviation of speeds of the vehicles:

\[ x_2 = \int_0^t e_1(\tau) d\tau. \]

The state vector of the control system is now:

\[ X = \begin{pmatrix} \dot{x}_1 \\ x_2 \end{pmatrix} \]

and the extended system matrices will be:

\[
\begin{cases}
    \dot{X} = A X + B u + G r \\
    y = C X
\end{cases}
\]  \hfill (3.13)

which is:

\[ \dot{X} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} A_1 & 0 \\ -H_1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} B_1 \\ 0 \end{pmatrix} u + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r. \]

The error vector contains now:
\[ e = \left( r - H_1 x_1 \right) x_2 = M r + H x, \quad (3.14) \]

where:
\[ M = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } H = \begin{pmatrix} -H_1 \\ 0 \\ 1 \end{pmatrix}. \]

### 3.5 Linear Quadratic Tracking Control

The target is now to minimize the cost function \( J \) of the error vector (3.14) from the system (3.13):
\[
J = \frac{1}{2} \int_0^\infty \left( e^T Q e + u^T R u \right) dt,
\]
where:
\[ Q = Q^T \geq 0, R = R^T > 0. \]

The solution is to calculate \( P \) by solving the Riccati equation. By substituting, the cost function can be rewritten as:
\[
J = \frac{1}{2} \int_0^\infty \left( x^T H^T Q H x + 2 r^T M^T Q H x + r^T M^T Q M r + u^T R u \right) dt.
\]

Now, the Riccati equation can be solved by standard methodology:
\[
\dot{P} = -PA - A^T P - H^T Q H + P B R^{-1} B^T P \quad (3.15)
\]
\[
\dot{g} = (P B R^{-1} B^T - A^T) g - (H^T Q M + P G) r \quad (3.16)
\]

Equation (3.15) is a standard Riccati equation, and equation (3.16) is an auxiliary vector equation that defines the feed forward gain.

If we consider matrices \( A, B, C, Q \) as constants and verify that \((A, B)\) is stabilizable and \((A, \sqrt{Q})\) is detectable, there will exist one unique steady state solution \((\dot{P} = 0)\).

The same solution for \( g \) (in steady state) will be:
\[ g_{ss} = (P_{ss} B R^{-1} B^T - A^T)^{-1} (H^T Q M + P_{ss} G) r_{ss}. \]

The control law is described as:
\[ u = -K_x X - K_r r, \]
where:
\[
K_x = R^{-1} B^T P_{ss} \quad (3.17)
\]
\[
K_r = R^{-1} B^T (P_{ss} B R^{-1} B^T - A^T)^{-1} (H^T Q M + P_{ss} G). \quad (3.18)
\]

Finally, the closed loop of the system is described as:
\[ \dot{x} = (A - B K_x) x + (G - B K_r) r. \]
3.5.1 Implementation

A schematic of the system is drawn in Figure 3.3.

Figure 3.3. Implementation of the LQT controller.
3.6 Linear Quadratic Trajectory Planner

In this section, acceleration signal from the leading vehicle will be added as an input to the controllers in order to improve controlling results.

If (3.9) is the state vector and the system (3.10) is the system matrices of the ego vehicle and if the leading vehicle’s dynamics are described as:

\[
\dot{\mathbf{z}} = F \mathbf{z}, \quad (F, \text{observable})
\]

where \( \mathbf{z} \) includes:

\[
\mathbf{z} = \mathbf{x}_{\text{lead}} = \begin{pmatrix} \mathbf{v}_{\text{lead}} \\ \mathbf{a}_{\text{lead}} \\ \mathbf{d}_{\text{lead}} \end{pmatrix}
\]

the augmented system is then:

\[
\hat{\mathbf{x}} = \hat{\mathbf{A}} \mathbf{x} + \hat{\mathbf{B}} \mathbf{u}
\]  \( (3.19) \)

where \( \hat{\mathbf{x}}, \hat{\mathbf{A}}, \hat{\mathbf{B}} \) and \( \hat{\mathbf{Q}} \) are defined as:

\[
\hat{\mathbf{x}} = \begin{pmatrix} \mathbf{x}_{\text{ego}} \\ \mathbf{x}_{\text{lead}} \end{pmatrix}, \quad \hat{\mathbf{A}} = \begin{pmatrix} \hat{A}_1 \\ 0 \end{pmatrix}, \quad \hat{\mathbf{B}} = \begin{pmatrix} \hat{B}_1 \\ 0 \end{pmatrix}, \quad \hat{\mathbf{Q}} = \begin{pmatrix} Q & -Q \\ -Q & Q \end{pmatrix}.
\]

The cost function to be minimized can be described as:

\[
J = \int_{0}^{\infty} \hat{\mathbf{x}}^T \mathbf{Q} \hat{\mathbf{x}} + \mathbf{u}^T \mathbf{R} \mathbf{u}
\]

and the control law from a basic LQR is generally:

\[
u(t) = -K \hat{\mathbf{x}}(t); \quad (3.20)
\]

where:

\[
K = \mathbf{R}^{-1} \hat{\mathbf{B}}^T \hat{\mathbf{P}}(t)
\]  \( (3.21) \)

and \( \hat{\mathbf{P}}(t) \) is the solution of the Riccati differential equation:

\[
\begin{cases}
-\dot{\hat{\mathbf{P}}}(t) = \hat{\mathbf{A}}^T \hat{\mathbf{P}}(t) + \hat{\mathbf{P}}(t) \hat{\mathbf{A}} - \hat{\mathbf{P}}(t) \hat{\mathbf{B}}^T \mathbf{R}^{-1} \hat{\mathbf{B}} \hat{\mathbf{P}}(t) + \hat{\mathbf{Q}} \\
\hat{\mathbf{P}}(T) = 0
\end{cases}
\]  \( (3.22) \)
The problem that arises now is that the system (3.19) is not controllable and therefore the Riccati equation (3.22) cannot be solved with standard methodology. What we can utilizes here is that not every part of $\tilde{P}(t)$ will influence the calculation of feedback gain $K$. A solution will, therefore, be to partition $\tilde{P}(t)$ and divide the calculation in two parts [31].

$\tilde{P}(t)$ is generally described as:

$$\tilde{P}(t) = \begin{bmatrix} P(t)P_{12}(t) \\ P_{12}^T(t)P_{22}(t) \end{bmatrix};$$

and (3.20) can be rewritten as:

$$u(t) = K_x(t)\hat{x}_{ego} + K_r(t)\hat{x}_{lead}$$

where:

$$\begin{cases} 
K_x = -R^{-1}B^TP(t) \\
K_r = -R^{-1}B^TP_{12}(t).
\end{cases}$$

As it seems, the only parts in (3.23) which are used in calculation (3.24) are $P$ and $P_{12}$. Substituting $P$ and $P_{12}$ into expression (3.22), we get

$$\begin{cases} 
-\dot{P} = -PA - A^TP - PBR^{-1}B^TP + Q \\
-\dot{P}_{12} = P_{12}F + A^TP_{12} - PBR^{-1}B^TP_{12} - Q
\end{cases}$$

(3.26) can now be solved by the standard methodology. A schematic for the system is depicted in Figure 3.4.

Figure 3.4. Implementation of the LQR trajectory planner.
4 Analyses

4.1 Introduction

In this chapter, a simulation of the described controllers will be demonstrated. The controllers are implemented on represented physical model. The purpose of the simulations is to see how vehicles in a platoon are being affected by the disturbances caused by speed changes of leading vehicles and signal disturbances. The scenario is that vehicles start from speed $0 \text{ m/s}$ by an initial distance of 20 meter. The leading vehicle will start to drive off at time zero and other vehicles will follow and try to keep 20 meters distance. The platoon leader will start to accelerate to speed $15 \text{ m/s}$ at time zero and at time 80 seconds decelerates to speed $7.5 \text{ m/s}$ and finally speeds up to $10 \text{ m/s}$.
4.2 P and PD Controller

In the first simulation, P and PI controllers are implemented; see Figure 4.1.

Figure 4.2. This figure shows P controller in the left and PD controller to the right. The top plot shows speeds of three vehicles in a simulated platoon including the leading vehicle. The second and third plot are the distances between leading vehicle and vehicle 2 and distances between the second and the last (third) vehicle. The fourth plot shows the control signals. No difference in distances is observable between the third and second vehicle in plots by the Proportional controller. It is due to the low fluency of the error to the control signal. As one can observe in the fourth plot to the left, the control signals to vehicle two and three are identical while leader reference speed changes from 15 to 7.5 m/s. The static error is eliminated also for the third vehicle in platoon using PD controller.
4.3 LQ Controllers

To design LQT controllers, parameters from linearized identified model was used in order to decide system parameters $A_1$, $B_1$ and $C_1$. In the following analyzes parameters for the speed changing from 10 to 15 m/s has been used from plots of Scania truck.

Figure 4.13 shows plot of LQT controller contra LQT trajectory planner. The scenario is that leading vehicle accelerates from zero speed to 15m/s, at time 80s it decelerates to 7.5 m/s and speeds goes up to 10 m/s at time 120 seconds. The initial and also the reference distance is set to 20 meters.

![Figure 4.13](image-url)

Figure 4.13. The LQT controller to the left and LQT trajectory planner (with contribution of acceleration signal from the leading truck) to the right in platoon of three vehicles. The figure on the top shows the speed trace and the following second and third figures from top are showing distance controllers of following trucks. The last figure is showing control signals, i.e. reference speed sent to the cruise controller. It is obvious that the acceleration signal helps to have quicker response from the following vehicles. It seems that the control signals to following vehicles gets higher than reference speed of the leading vehicle for a short time to compensate the deviation in control error.
4.3.1 Comparison of LQT and LQT Trajectory

The structure of these two controllers is basically the same. The main difference is the added part in the trajectory planner solution, i.e. the contribution from the acceleration signal of the leading vehicle.

Identical weights of the matrix Q elements will give the same feedback gains in both solutions, except in LQT, when feedback gain for the acceleration of the leading vehicle, i.e. the second element of $K_{r(LQT)}$, is always equal to zero. Accordingly, if we remove the acceleration signal from both solutions, the results are expected to be as the same, provided that identical Q parameters are used in both solutions.

The reason can be shown by following the calculation. The matrix $P$ is identical in both solutions, regarding to equations (3.15) and (3.22), which gives the same $K_x$ gain in both cases. The first elements are also identical in $K_r$ gains in both solutions.

$K_r$ gains for the LQT and LQT trajectory are given by:

$$K_{r(trajectory)} = R^{-1}B^T[P_{12}(t)]$$

(4.4) and:

$$K_{r(LQT)} = R^{-1}B^T[P_{ss}BR^{-1}B^T - A^T]^{-1}(H^TQM + P_{ss}G).$$

(4.5)

Obviously, the difference is in the last multiple terms in the solutions $P_{12}(t)$ is a full rank matrix, which gives all the elements in $K_{r(trajectory)}$ (4.4) to be different from zero. The result of the gridded term in (4.5) is a one-dimensional matrix with only the first column different from zero and the first column is also identical to the first column in $P_{12}(t)$. Consequently, the first elements in both $K_r$ vector gains will be equal to each other.
5 Disturbance analyses

In theory, as it was shown in Figure 4.13, the acceleration signal from leading vehicle would help to achieve a quicker respond of the LQT distance controllers provided that the signal meets certain quality criteria. In this section, some analyzes will be done to examine impact of noise and delays in acceleration signal to control results.

5.1 Noise in signals

The first simulation plots the distances of a five-vehicle platoon, shown in Figure 5.1. The figure shows three different plots with three different signal conditions. The blue curve is the acceleration signal from leading vehicle without any noise, the green and red curve are noisy signal with 0.01 and 0.1 bandwidth, respectively. Figure 5.2 shows that a noise bigger than \(0.5 \, \text{m/s}^2\) in acceleration signal makes the closed loop system unstable.

The scenario is as the same as in Section 4. The leader starts to accelerate from zero speed to 15 m/s and in time 80s decelerates to 7.5 m/s and again accelerates to 10m/s in time 120s.

![Graph showing distances between vehicles with noise](image)

Figure 5.1. Platoon with noisy acceleration signal. The noisy acceleration signal seems to mostly affect the fourth vehicle in this case, and the distance goes up to 30 meters before it adjusts to 10 meters.
Figure 5.2. Acceleration signal higher than 0.5 bandwidth makes instability in the system. All following vehicles will give up platoon by following the vehicle number two which decelerates to a complete stop at time 50 seconds.
5.2 Delays in Signals

Another problem that may occur is delays in signals in real time systems. As it is shown in Figure 5.3, delays higher than 0.5 seconds in acceleration signal will have more significant impact in the results.

Figure 5.5. This figure shows distance tuning between leading vehicle and the following vehicle. The scenario is the same as in Figures 5.2 and 5.3, initial distance is 0 and target is 10 m. As it is shown in the figure, negative effect on control results becomes more remarkable on signal delays larger than 0.5 seconds.
6 Conclusion

In this thesis, a longitudinal dynamics controller has been designed to automate the control of a number of vehicles grouped as platoon. For the sake of simplicity, we assumed that the vehicles are moving on a flat road. The controllers aimed to keep a reference distance between the vehicles and are basically P, PD, and optimal controllers. The design and analysis of the controllers are done by taking into account the dynamics of the vehicles and the available information about the vehicles like speed, acceleration, and distance to the other vehicles.

Numerical simulations show that the performance of the LQT and PD controllers are quite similar, and they show a similar response time to the distance tuning command. According to the analyses carried out in this thesis, the main reason for the quite similar performance is that their algorithmic structures are fairly similar. In fact, we showed that the result of longitudinal dynamics control with these two methods is the same, if one uses the same matrix coefficients P, D, and Q. It is, however, much easier to regularize the controlling results for the LQ controller by penalizing the matrix Q to realize a focus on the energy consumption or a focus on the quickness in the controlling of the inter-vehicle distances. This is mainly because that for the PD controller, one needs to simultaneously adjust both coefficient matrices P and Q to regularize the controller to consider the energy consumption or the speed of reaction to distance changes.

To extend our understanding from the dynamics control of a platoon system, in Section 5, we utilized all the available information in the platoon to design a faster controller. Moreover, to analyze the sensitivity of the designed controller to disturbances like noise in the estimations as well as delay in the estimation, a number of numerical simulations has been done. The numerical simulations showed that noise and delay in the acceleration signal has a negative impact on the controlling performance, while other signals have less important impact on the performance of the controller. More importantly, when the noise and delay exceed some certain values, the closed-loop control system becomes instable. This means that among the set of state variables, the acceleration data plays an important role to keep the controller system stable, and one needs to be careful to have accurate and low-delay estimates of the acceleration of the vehicles.

For the future studies, it would be very interesting to design a more complex controller based on the model-predictive control theory and to analyze the performance of this controller for the problem of longitudinal dynamics controller design. A comparison to the performance of P and PD controller may let us understand how using a more complex controller would help to have a higher performance.

Moreover, designing a controller that has less sensitivity to the error and delay in the estimation of the acceleration signal would also be very crucial. A potential study can be started by a more thorough and deeper analysis of the sensitivity of the controller to error and delay, and then one might be able to come up with some modification to the original controller to make it more robust against the sources of error.

Last but not least, as discussed throughout this thesis, platooning can be very effective in reducing fuel consumption, improving the traffic flow in the roads, and decreasing the number of vehicle collisions. This basically means that with the ever-increasing demand for the freight transportation, platooning would be a key solution in the future to limit the fossil fuel consumption and hence green-house emissions. Moreover, a smoother traffic flow in the roads, which is another consequence of having an efficient platooning system, will lead to a reduction in the other vehicle’s fuel consumption, increasing the impact of this technology. Having these facts in mind, investment in designing improved dynamics controller methods will fundamentally help us to have a sustainable economy growth as well as ecosystems in the future, where the fuel consumption and air pollution are the biggest concerns.
Bibliography


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