Reducing cognitive load on the working memory by signaling primed colors

Can color improve the learning of mathematics?

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Reducering av kognitiv belastning på arbetsminnet genom färgkodning och priming
Kan färger underlätta inlärningen av matematik?

Abstrakt
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ABSTRACT

Cognitive load theory explains the hardship many young learners faces when trying to learn mathematics [2]. The human brain has a limited capacity when trying to process information [9]. The working memory can become overloaded when trying to process too much information, and as a consequence the learner tends to give up. Designers of flight simulators describes this capacity as the pilots brainbudget [9]. By presenting only relevant information the workload for the users can be minimized. This paper aims to study the effect of color for the purpose reducing cognitive load, by answering the following question: Can the use of color improve the learning of mathematics? An experiment was conducted in Stockholm rural area involving 147 students, aged 12-16. Four colors were carefully selected to enhance the learning material and three methods were developed for the purpose of priming colors. In order to measure the effectiveness of color-coded learning material, a digital test tool was designed. The tool measured various variables such as time, correct answer, attempted tries etc. This allowed a comparison between color coded and grey scaled learning material. A control group did the math test in black&white and two experimental groups did the same test with color-coded material, however one of the experimental groups was primed prior to the experiment. After the test, the participants conducted a small survey and participated in interviews discussing their perception of the color enhanced instructions. Results showed that both signaling, and priming colors have major impact on the student’s ability to solve mathematical tasks. This indicate that color can be used to facilitate the learning process of mathematics, both in regard to accuracy and reduction of time.

CCS Concepts

- Human-centered computing → Information visualization
- Human-centered computing → Visualization techniques.

Keywords

Knowledge Visualization; Cognitive Load Theory; Priming; Signaling; Color; Mathematics; Learning

1. INTRODUCTION

The difficulties in certain areas of mathematics are well documented and occur across cultures, curricula and methodologies [2]. Studies have shown that many children and young students have difficulties in learning math, because they feel that mathematics is very strenuous and complex. Failure to do well in mathematics leads to a negative attitude towards the subject [2]. If the learning material proves to be too difficult for the recipient, the learner tends to give up. Academic confidence is almost exclusively connected to previous success [2]. As a consequence, many young students have developed a negative attitude towards mathematics and their attitude tend to deteriorate with age as it becomes more and more demanding, resulting in many of them trying to avoid it for the rest of their lives [2].

Consider this problem: a young learner facing a system of coordinates for the first time (see figure 1). The learner has to decode the entire system in order to understand the information being presented. There are four aspects the young learner needs to understand, the symbols (the x and y axis, grids, coordinates, origin of coordinates etc.), the procedure (how to determine the x and y coordinates), the concepts (the meaning of the system) and finally the applications (how to use a system of coordinates themselves) [2][18]. Trying to learn all aspects simultaneously unnecessarily puts much pressure on the learner which in turn may lead to failure to understand. Visualizing the procedures with the help of colors can simplify these processes (see figure 2), making it easier for the learner to process larger amount of information. [13] Color can be used to bolster search [13][20], identification and recognition of information [1][11][13][20], and help the learner remember things better [1][20]. If the use of colors proves to have a positive impact on learning, then the results could be of interest to producers of educational material, teachers and students. This paper intends to study the effects of color used to enhance learning material and more specifically aims to answer the question: Can color be used to improve the learning of mathematics?
1.1 Background

Transferring knowledge has been a universal challenge for generations. Books have been written, institutions have been created and people have dedicated their entire lives for the sole purpose of transferring knowledge. There are mainly two kinds of methods to transfer knowledge: the one-to-one transfer of knowledge between a teacher and recipient, and the act of converting knowledge into knowledge artifacts, such as documents, images and videos that are consumed by the learner.

The knowledge base of a person is made up of different types of knowledge [6][22]. Problem solving is a fundamental part of mathematics, and requires different types of knowledge such as conceptual knowledge (the learning of facts, such as formulas), situational knowledge (knowledge that is used in a specific situation), strategic knowledge (knowing how and when to use specific knowledge) and procedural knowledge (knowledge that is acquired by continuous performance of a task) [22].

Different types of knowledge have various degrees of complexity when being transferred to a knowledge artifact [5]. Converting knowledge to artifacts has its challenges and depends for the most part on the type of knowledge being converted. A major challenge is if the possessor of the information has inadequate ability to transfer knowledge to the recipients, causing a deterioration in the efficiency and effectiveness of the information being transferred. If the knowledge holder is not able to specify correctly and communicate precisely what they want to transfer, then the knowledge transferred might be open to misinterpretation and distortion in knowledge transfer occurs. [6]

The effort required by a learner can be derived from various variables. The source material can have different degrees of complexity and is determined by several factors such as presentation, complexity and the preconditions of the learner, be it ability, motivation, or prior knowledge. The person trying to acquire the information will experience different levels of difficulty based on the degree of complexity of the source material. [5][6][22]

Different types of tasks demand different things from the brain [8][11]. Unlike recall tasks, creative problem solving requires a coherent integrated understanding of the chain of cause and effect [13]. To memorize and then to apply a formula requires a deeper understanding of how each component in the formula is connected to the problem thus requires a greater effort from the learner. The difficulty of a mathematical formula is increased by every component that is added to the problem. The complexity of a problem is thus increased by the number of steps required to solve it [18]. A complex problem is defined in this paper as a problem that requires several steps to solve, where some of these steps is previously unknown to the learner. Mathematics is abstract and logical in nature and many concepts in mathematics cannot be explained easily in terms of physical representations or related to everyday life. Mathematics is a type of knowledge that is procedural, similar to language and is obtained through experience [6]. As previously described, learning of mathematics requires understanding of several different processes [2][12]. The symbolic representation of the problem, procedures for solving a problem, concepts for understanding the problem and finally the applications of the problem. Cognitive psychologist has identified the working memory having vital role when trying to learn mathematics [4][10]. When the working memory is overloaded, the learner tends to depend on the memorization of procedures, making understanding a casualty [2].

2. THEORY

2.1 Cognitive Load Theory

This study adopts the theoretical lens of Cognitive Load Theory, a dominating theory within the field of cognitive psychology, to aid in understanding the complications of learning mathematics and how to ease the process of learning.

Cognitive Load Theory describes the interaction between the working memory and long-term memory and has a significant role in the learning process [4][7]. Cognitive load refers to the demands put on the working memory during an ongoing task. The working memory has a limited capacity and is the part of the memory that handle, processes and manipulates temporary information [7][14][16]. Too much strain put on the working memory can lead to a cognitive overload which in turn can result in mental exhaustion, fatigue and reduction of concentration and attention. Consequently, the understanding of the subject matter is likely to be compromised. The limited capacity of the working memory allows humans to only store and process a few novel combinations of elements or chunks at any given time [11]. By altering the capacity and duration limits, the longtime memory can aid the working memory in order to permit complex processing [11]. In contrast to working memory, the longtime memory allows for vast quantities of information to be stored [7][11]. The theory is that information stored in the long-time memory can greatly improve the capacity of the working memory [7][11].

2.2 Reducing the cognitive load

To address the challenges put on the working memory in a specific complex situation, certain strategies have been developed to encode and acquire information from the longtime memory in order to minimize the cognitive load. Scientists designing flight simulators describes the mental capacity of the pilot’s brain with the term brain budget, indicating the brain having a limited amount of resources that can be allocated for certain tasks [9]. Minimizing the mental workload for fighter pilots is a huge priority, allowing them to take a more correct and efficient decision, a decision that could be a matter of life and death.

2.2.1 Signaling

Signaling is a technique within the field of information visualization where important visual elements is made to stand out from the rest [11][13][14]. The use of signals to highlight information have proven to have a positive effect on learning [11][13]. Signaling allows the learner to effectively sort information and guide the attention to relevant elements with the help of color, patterns and textures among other things [13]. Signaling helps the working memory to memorize, understand and remember information better [1][11]. To create a greater impact the signal should vastly stand out and distinguish itself
from the surrounding information [13]. The use of signals can also be made to group information with common elements.

2.2.2 Priming
The acquisition of information can be processed consciously or unconsciously [3][19][20]. Priming is a technique that enables unconscious processing of information [17][19][20][21]. When information is processed unconsciously, the load on the working memory decreases [7]. Unconscious retrieval of information can reduce the mental workload put on the learner. The use of signals can facilitate the connection to previously learned knowledge from the long-term memory in order to support the processing of information in the working memory. Reducing the effort created by conscious retrieval of information results in better performance [18]. Furthermore, by associating colors with specific information, colors can be used for priming [15][20][21].

2.3 Colors
Colors have certain distinctive features that make them particularly useful in knowledge visualization. Colors can be used to signal [1][13][23], create relationships between different elements [8][23] and for priming of information [17]. There are various techniques for signalizing using colors. Using different saturation of colors allows attention to be guided gradually [23]. The eye first draws attention to the strongly saturated colors and then looks at the less saturated colors [23]. Colors are also a good way to segment information [11][13], allowing the association of elements with colors and then to connect similar elements with similar colors. Priming can be done by color coding information to a specific color. This allows an unconscious retrieval of information from the long-term memory to the short term memory. Additionally, priming can be done by associating different colors to different methods. Using colors to associate methods such addition and subtraction, the learner can acquire instructions without the need for text and thus prepare the brain for upcoming processes. Colors have a property that makes it easy to group information, making connections between elements [8][21]. The brain always tries to find patterns and shortcuts to more easily understand the outside world. The use of color to create connections between different traits of mathematical problems allows the learner to faster identify the nature of the problem [11][13][20].

This study will use colors for signaling and for priming, to try to ease the process of learning mathematics by reducing the cognitive load it puts on the working memory.

2.4 Research question
2.4.1 Research questions
The question this paper aims to answer is: Can color be used to improve the learning of mathematics? To answer this, the following sub questions will be investigated:

1. Can color enhanced instructional material improve the ability to solve a mathematical problem?
2. Can color enhanced instructional material reduce the time needed to solve a mathematical problem?
3. Can color be used to increase the understanding of a mathematical problem?
4. Can priming of certain colors be used to further improve the learning of mathematics?

2.4.2 Hypothesis
Our hypothesis is that by using colors to enhance learning material, the load induced on the brain can be reduced, thus making it easier for students to understand complex information. The hypothesis is based on theories about cognitive load which implies the search of information creates a load on the working memory. Our theory is that by using colors to identify patterns in mathematical problems, the cognitive load can be decreased thus reducing the load on the working memory which in turn can be used to better understand the mathematical problem in question.

2.5 Delimitations
This paper studies colors used to visually enhance learning material in the field of mathematics. Other enhancement methods to signal information such as size, thickness, position is not being investigated in this study. Consequently, this paper cannot answer if color as a visual enhancement is the most effective way to improve learning but only examines if colors can be used to improve the acquisition of mathematical knowledge.

The participants of the study exclude people with any sort of color blindness due to color having a central feature in this study.

3. METHOD
3.1 Design of the study
3.1.1 Participants
The experimental study was carried out at an upper secondary school in Stockholm County. The participants were students in grades 7 - 9. The target group was chosen based on the fact that all upper secondary students in Sweden follow the same curriculum. This made it easy to know the students’ mathematical skills, which facilitated the design of the study. Another factor that influenced the decision to focus on a young target group was the fact that attitudes toward mathematics have been shown to be of vital importance in this age group [2]. A total of 147 students with aged 12-16 participated in this study. Ethical considerations needed to be made due to the participants’ young age, and therefore a decision was made to not measure the participants’ working memory.

3.1.2 Participants
The main purpose of this paper was to find out if the use of color to teach mathematics can facilitate young learners’ understanding of said subject. To do this, the following things needed to be determined:

- Selection of colors - A set of colors to be used for signaling and priming.
- Designing methods to incorporate chosen colors into effective instructions.
- Creation of problems/tasks that the participants would attempt to solve.
- And finally, a way to measure understanding of mathematical instructions.

An obstacle that comes with the subject of this study is bias, since any result using traditional HCI methods to evaluate might affect the credibility of this study. To address this issue an experimental study using mixed methods was designed. This allowed an additional interpretation of the acquired data, by means of triangulation. Priority was given to the data collected using a digital test-tool (see section 3.2) and any additional data collection was used to primarily enhance the data collected during the
The experimental study consisted of three phases; preliminary study phase, controlled experiment and data analysis. The following methods were used to acquire data:

1. A digital math test using the CEM-Test tool.
2. Observations during the experiment.
3. A national standardized diagnostic math test, UIM-Test, for grade 8 students only.
4. Digital surveys.
5. Group interviews.
6. Follow up questions.

### 3.1.3 Preliminary Study

The purpose of the preliminary study was to set up the controlled experiment. Using an iterative design approach, usability testing was conducted for the purpose of selecting colors, developing priming techniques, construct mathematical questions to perform during the experiment and create an apparatus that could measure their effects. This was followed by a pilot experiment. Three methods were chosen for the purpose of priming (see section 3.1.2). Six different mathematical problems were created to be tested on during the experiment. One week prior to the experiments, the participants in grade 8 took part of a diagnostic test in order to map their mathematical knowledge. The preliminary study phase concluded with a pilot experiment of the CEM-test tool.

### 3.1.4 Priming

In an attempt to test our hypothesis that priming of colors can influence the ability to solve a mathematical problem, methods of priming needed to be developed and then implemented on a group of students. Four colors were selected for the purpose of priming. Green to perform addition, pink to perform subtraction, Yellow to highlight important information, and blue to use for additional purposes. Following methods were used to prime a select group of students.

1. Unconscious training during 3 separate occasions by their teacher during regular math using the different colors when showing worked examples on the whiteboard.
2. A method to solve equations using highlighter pens with the color green and pink was demonstrated for the participants.
3. Two weeks before the experiment, their regular math test was enhanced using the selected colors.

### 3.2 CEM-Test tool

A digital test tool, Color Enhanced Math-Test, was designed for the purpose of collecting data. The tool presented a mathematical task and measured the participants performance. The following things could be measured with the CEM-Test tool:

1) **Total time spent on a task:** Combination three different values:
   a) Completion time - If the participant managed to finish the task; a completion time were registered.
   b) Attempt time - If the participant made an unsuccessful attempt; an attempt time were registered, and the participant was given a new chance.
   c) Skip time - If the subject decided to skip the task; a skip time was recorded.

2) **Number of attempts:** Two types of value were recorded:
   a) Attempts: Number of total amount of attempts
   b) Failed attempt: Each failed attempt was registered.

### 3.3 Controlled experiment

#### 3.3.1 Overview

To measure the effectiveness of color enhanced mathematical learning material a controlled experiment was designed. The digital tests using the CEM-Test tool were conducted in one class at a time for a total of eight occasions.

Participants were divided into three different groups; black&white (BW), color (C), and prime (P) (see figure 5).

The CEM-Test tool presented a total of 12 questions, one at a time, grouped into six different types of questions with an “a” and “b” task. The “a” task was presented with instructions on how to solve the problem while the “b” tasks were presented without any help. For group color and group prime, any examples presented along with task “a” were color-coded (see figure 4). For group black&white no colors were used. The purpose of a “b” task was to investigate if the user, after having solved a question with the help of colors, faced any difficulty solving a question without any kind of aid. All tasks are shown in Appendix A.
The experiments carried out could be viewed as two separate but overlapping experiments (see figure 5).

Experiment A, examined color-coded learning material compared to black and white, where group black&white was the control group and groups color and prime were the experimental groups.

Experiment B, examined the difference in results between group prime and group color, where group color was the control group and group prime the experimental group. Both of the groups did the same CEM-test, the difference being that group prime was primed prior to the test (see section 3.2.2)

As described in section 3.2.2 three different types of effects were measured; total time spent on task, number of attempts, and degree of success. The CEM-test having 12 tasks, the experiment yielded 12x3 output effects generating a total 36 dependent variables for each participant.

### 3.3.3 Other variables
An overview of other possible variables is listed below:

1. Basic understanding of mathematics
2. Amount of color used for each task
3. Colors chosen for each task
4. Difficulty level of each task
5. Method used for priming

### 3.4 Data analysis

#### 3.4.1 Models for data analysis
Data from the CEM-Test was observed by looking at completion rate, degree of success, and overall success rate. Completion rate being a nominal variable indicating if the participant managed to answer the question correctly or not. Degree of success, also a nominal data indicates if the participant managed to answer the question at first attempt or not. Overall success rate shows an ordinal variable indicating the overall success rate.

To test if there was a statistical difference between each group, a Cochran–Mantel–Haenszel– test was performed. A requirement for this was that there could not be an inherent difference between groups. In order to perform the test a subgroup of groups black&white and color was measured by removing 7th grade from both groups. Results from the standardized national test showed that performance results were distributed evenly between groups in grade 8 allowing to perform a fisher's exact test. However, group color had a slightly lower average test-score due to a few students with extremely high-test scores in both groups allowed to further analyze students in grade 8.

In order to transform data sets allowing a fair comparison of time oriented data, a statistical transformational model was designed for the purpose of this study. The model factors in completion rate with a combination of quantile transformation in order to transform data between groups and create homogeneity of variances. The transformation method allows for a two-way analysis of variance between groups followed by post hoc test in an attempt to measure if there is a statistical significant difference of time needed to complete a task. The effectiveness of learning instructions was measured factoring in the ability to solve a question using only one attempt combined with time needed to solve the problem.

## 4. RESULTS

### 4.1 Overview
The purpose of the study is to investigate whether color aided learning material can be used to improve learning of mathematical tasks. Furthermore, the study investigates whether priming of specific colors has any influence on the learning of mathematics. The data collected from the CEM-Test is used to make comparisons between the control and experiment groups. Results from each controlled experiment are presented sequentially showing success rate followed by task completion time. Later the results from the survey are presented, followed by the group interviews. For a representation of the experiments and group compositions see figure 3.
4.2 Experiment A

4.2.1 Success rate

Results measuring the ability to complete a task with the help of color enhanced material indicate a greatly improved rate of success in solving a mathematical problem.

Group color (C) had a higher success rate across all tasks for grades 7-9 with a completion rate of 77% (240/312) compared to group black&white (B), having a 67% rate of success (238/354). Further breakdown of each task indicates that the rate of success is higher across all tasks in favor of group color.

| Table 1 – Rate of success for each task grade 7-9 |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| N              | Task 1          | Task 2          | Task 3          | Task 4          | Task 5          | Task 6          |
| Black          | 59              | 78%             | 83%             | 73%             | 54%             | 68%             | 47%             |
| Color          | 52              | 90%             | 92%             | 79%             | 63%             | 79%             | 58%             |

Supporting this evidence is the completion rate from group prime having a completion rate of 81% (179/222) relatively to group black&white grade 8-9 having a completion rate of 61% (140/228). Further breakdown of each task can be seen in Table 2.

| Table 2 – Rate of success for each task grade 8-9 |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| N              | 1A              | 2A             | 3A             | 4A             | 5A             | 6A             |
| Group B        | 38              | 66%            | 76%            | 68%            | 50%            | 63%            | 45%            |
| Group P        | 37              | 97%            | 97%            | 81%            | 68%            | 76%            | 65%            |

Using a Cochran Mantel Haenszel - test can confirm a statistical significant difference in favor of group color compared to group black&white on task 1 (P = 0.0058), task 2 (P = 0.035) and task 4 (P=0.0207)(see table 3). Comparing group black&white and group prime further confirms a statistically significant difference to complete task 1(P = 0.004) and task 2 (P = 0.0088)(see table 4).

<table>
<thead>
<tr>
<th>Table 3 - CMH-test group BW v group C</th>
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</thead>
<tbody>
<tr>
<td>Task</td>
</tr>
<tr>
<td>DF</td>
</tr>
<tr>
<td>Chi-Square</td>
</tr>
<tr>
<td>P-Value</td>
</tr>
</tbody>
</table>

Table 4 - CMH -Test group BW v group P

| Task | 1A | 2A | 3A | 4A | 5A | 6A |
| DF   | 1  | 1  | 1  | 1  | 1  | 1  |
| Chi-Square | 13 | 6.0 | 1.8 | 8.3 | 1.8 | 8.2 |
| P-Value | 0.0004 | 0.0088 | 0.18 | 0.07 | 0.18 | 0.074 |

The UIM-Test conducted on subjects in grade 8 prior to the experiment showed an average test score of 29/55 for participants in group color and 35/55 for both groups prime and black&white. This indicated a very similar level of prior mathematical knowledge between groups prime and black&white while group color has a slight disadvantage. Furthermore, the test score was evenly distributed within both groups prime and black&white, which allowed for a Fisher’s exact test.

Results comparing the groups in grade 8 only show the ability to complete a task was 64% for group black&white grade 8, compared to group color with a 69% rate of success and group P with an 86% rate of success. A further breakdown of each task can be seen in Table 5.

| Table 5 - Rate of success for each task grade 8 |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Group | N | 1a | 2a | 3a | 4a | 5a | 6a | Total |
| BW   | 22 | 77% | 77% | 68% | 50% | 68% | 45% | 64% |
| C    | 16 | 81% | 88% | 69% | 56% | 65% | 50% | 69% |
| P    | 19 | 95% | 95% | 95% | 79% | 79% | 74% | 86% |

Fisher’s exact test conducted between group prime and group black&white (see table 6) showed a statistically significant difference in completing at first attempt in task 1 (P=0.0113), task 2 (P=0.0383), task 4 (P=0.008), and task 5 (P=0.0204 ) in favor of group prime. No test between group color and group black&white was conducted due to inherent differences between groups as indicated from the UIM - test scores.

<table>
<thead>
<tr>
<th>Table 6 - Fisher’s Exact test results</th>
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</thead>
<tbody>
<tr>
<td>Task</td>
</tr>
<tr>
<td>Task 1</td>
</tr>
<tr>
<td>Task 2</td>
</tr>
<tr>
<td>Task 3</td>
</tr>
<tr>
<td>Task 4</td>
</tr>
<tr>
<td>Task 5</td>
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<tr>
<td>Task 6</td>
</tr>
</tbody>
</table>

The matrix (see Figure 4) illustrates a breakdown of level of success in each group. Each row indicates the attempts made by every single participant in grade 8, each column illustrates a task. Green indicates success on the first try, blue indicates success after several tries and red indicates failure. The matrix illustrates the huge difference in failed tasks between the groups.

The total amount of subjects that managed to finish all tasks is on the first try shows a great advantage for group prime (see table 7).

<table>
<thead>
<tr>
<th>Table 7 – Subjects who completed all tasks on first try</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
</tr>
<tr>
<td>BW</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>P</td>
</tr>
</tbody>
</table>

4.2.2 Task completion time

Results indicate that time needed to solve a mathematical problem can be considerably shortened with the aid of color enhanced learning material. Completion time across all task indicate a significant advantage towards the use of color.

Results from group color (C) compared to group black&white (BW) shows an overall of 34% less time spent for group color. Breakdown of completion time for each task is shown in Table 8.

There was a statistically significant difference between groups as determined by two-way ANOVA. Post hoc comparisons using the Dunnett’s test indicated that the mean score for group color was significantly different in task 1, 2, 4 and 5 as presented in table 9. Together these results suggest that color enhanced learning material has a statistically significant effect in reducing the time needed to complete a mathematical task.

Supporting this evidence, task completion time across all task indicate a significant advantage in favor of group prime (P) against group black&white (BW), witch an exception for task 3.
Comparisons of mean values show that a total advantage of 35% for group prime (table 8). Dunnett’s test indicates a significant difference in task 1, 2 and 4. Further, the results from the survey indicate that 89% of the participants felt the use of color increased their understanding of mathematics, and that the perceived difficulty of the test was highest for group black&white.

Further breakdown of success rate of each task is shown in table 10.

Results using nominal data measuring success at first attempt indicate that priming of colors can vastly increases the ability to understand how to properly solve a mathematical question. Results indicate the ability to complete a task at first attempt was 45% for group color (C) grade 8 compared to group prime (P)
grade 8 with 75% rate of success at first attempt. Further illustration of each task is shown in figure 6.

4.3.2 Task completion time
Results indicate that time needed to solve a mathematical problem can be considerably shortened with the aid of color enhanced learning material. Completion time across all task indicate a significant advantage towards the use of color.

Task completion time for task 1 show 2 indicate an advantage towards group Prime. However, task 3 and 5 show an disadvantage for group Prime. Table 11 illustrates further breakdown in mean value.

<table>
<thead>
<tr>
<th>Tasks</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>93</td>
<td>69</td>
<td>192</td>
<td>246</td>
<td>70</td>
<td>202</td>
<td>762</td>
</tr>
<tr>
<td>P</td>
<td>97</td>
<td>46</td>
<td>144</td>
<td>230</td>
<td>104</td>
<td>207</td>
<td>778</td>
</tr>
</tbody>
</table>

A two-way between subjects ANOVA was conducted to compare the effect of primed color enhancement on task completion time for each task (1a - 6a) (see table 12). Results indicate that there was a was a statistically significant difference between groups as determined by two-way ANOVA on task 1 and task 4.

Post hoc comparisons using the Dunnett’s test indicated that the mean score for group prime was significantly different each task those tasks. Priming of colors resulted in a statistically significant advantage on task 1, and disadvantage on task 5.

A common factor for task 3, task 5 and task 6 was the inclusion of an additional third color, yellow.

Later follow-up interviews (see section 4.6) explained that some students had misunderstood how yellow was supposed to be perceived and spent a considerable amount of time in order to understand its use.

| Group interview – color | C | P |  |  |  |  |  |  |  |
|-------------------------|--|--|--|--|--|--|--|---|
| Group interview – prime |  |  |  |  |  |  |  |  |

They then discussed the idea of having color-coded instructions in their learning material and math books. This was an idea all of the subjects considered would be very helpful. “In the math books, in the parts where they explain what to do, it would be great [to use color-coding]” one student said. “That would make things a lot simpler” added another.

4.4.2 Group interview – prime
The interview participants from group prime shared the same opinion about the color-coding as group color. All of the subjects agreed the colors had been very helpful. Further, all of them agreed it had been helpful knowing the meaning of the colors in advance. “The colors were very practical, even when I didn’t grasp the written instructions that well I could still understand because you could see the color-patterns”, said one of the students from group prime. The group conveyed that the colors would have been helpful even if they had not known their meaning, but that it “simplified things even more”. Some also made the argument that they would have figured out the meaning of the colors regardless, since green and red are generally connected positives and negatives. One of the students brought up the association with green and red lights at pedestrian crossings.

All of the subjects in the group had a very positive attitude towards color-coding and expressed that it had helped them alot. “You saw what you were supposed to do by looking at the colors, so you didn’t have to think as much,” said one student from group prime. “The best thing for me was that I didn’t need to read the instructions, I looked at the colors to see a pattern and understood what to do” said another.

4.4.3 Group interview – Control group
Group black白宫t started with discussing the difficulty of the test. “There was a lot of text, so first I thought to myself, ‘Oh, no!’ but it wasn’t that bad once you read it”, said one of the student from group black白宫t. The group thought the difficulty of the test varied, some problems were hard, and some were easier. Two of them specifically mentioned problem number 4 as the hardest one.

The group was shown what the test had looked like with color-enhanced instructions, I looked at the colors to see a pattern and understood what to do” said another.

4.4 Interviews
Three group interviews were conducted in smaller groups of 3-5 students to discuss their thoughts about the conducted test and further thoughts about color-coding. The dialogue started with an open question about their general thoughts about the CEM-test.

4.4.1 Group interview - color
From group color, all of the participants in the group interview agreed the colors enchanted material had been very helpful. “The colors helped because it made it easier to see patterns, they made it less confusing” said one student from group color. “...And you could see the connections easier” added another. The group was asked if they thought they could have completed the test without the help of colors. Students expressed that: “It would’ve taken a much longer [amount of time]”. The general consensus however was that they would have completed the test nonetheless.
group black&white, as it would make instructions easier to understand.

5. DISCUSSION

5.1 Summary of findings
The question we set out to answer in this study (presented in section 2.4.1) was if color could be used to improve the learning of mathematics.

Results measuring the ability to complete a task with the help of color enhanced material show a greatly improved rate of success in solving a mathematical problem. The study’s results also suggest that color enhanced learning material can considerably reduce the time needed to complete a mathematical task. Results indicate that priming of colors can increase the ability to understand how to solve a mathematical problem.

Above findings shows that color can be used to improve the learning of mathematics. Our study supports our hypothesis (see section 2.4.2) surrounding the cognitive load theory and how color-coding and priming could reduce the load on the working memory, however this study cannot confirm any correlation.

5.2 Analysis of findings
The effective use of signals to highlight patterns, by grouping elements in a visual scene with a common property [13], in task 1 and 2 provides an explanation to the higher success rate for group color and prime. This supports the hypothesis that colors make it easier to understand mathematical problems. The color green facilitated in the decision making indicating what information is important to process and what information is redundant – helping the learner to focus only on the relevant information.

Furthermore, group prime required substantial less amount of time to solve task 1, comparatively to both group black and group color, with an average of 70% (40 seconds) less time than group black, and 40% less time than group color. This indicated that with the additional effects of priming, the subjects were helped in solving task 1. The color green was infused with instruction about which procedures to perform, in this case adding all green numbers. However in task 2, the difference between group prime and color was reduced. This can be related to an occurrence of natural priming, having recently solved a problem with similar use of the color resulting in an extended effect. Understanding how the color can be used reduced the effort needed to finish the task. An unintentional mistake in the design of task 3 further supports this explanation. The color yellow was primed with the intention to aid the learner focus on important aspects with regards to text and color. However as reported in the interviews, the usage of yellow was confusing for the subjects.

The original intent by the designers were to prime yellow to mean multiply and blue to divide, however the testing of colors during the preliminary study phase indicated a difficulty trying to convey multiple hidden meanings in various colors without compromising the integrity of the experiment. The colors green and pink were the colors that conveyed a proper meaning while yellow became confusing for the subjects. This is revealed in the results from task 3 where we see a significantly higher time required to solve for group prime. The same thing cannot be observed in group color, having a consistent faster completion time compared to group black.

The introduction of a 3rd color in task 3 and 5 provides an additional explanation to the increased time needed to finish tasks 3 and 5. This gives the subjects an additional task to perform – to understand the meaning of the color yellow inducing an information overload for the working memory. The use of signaling to convey important information loses its effect when too much information is presented.

Results show a significantly higher rate of success for group prime, even more so when looking at the subjects ability to solve tasks on the first try. The hardest task to complete, and the one which required most time to complete, was task 4. This task is also where priming helped the most, indicating that priming had a higher impact on more difficult problems. Paying attention to the fact that a higher amount of weaker students managed to finish the task further strengthens the theory regarding mitigating the load on the working memory being especially important when the subject lacks prior understanding of the problem [18].

The theory of increased mental effort resulting in an overload of the working memory indicates that the extended time needed for group priming to complete task 5 and 6 correlates with their rate of success in finishing task 4. Follow-up questions with subjects from group prime, confirms that the students who made a larger effort to finish task 4 took a slight mental break to finish the upcoming tasks, a problem many subjects from groups color and black did not have as many skipped task 4.

All students participating in the group interviews (see section 4.4) perceived the color-coded material made it easier to understand and solve the problems. The results of the survey analysis also tell us that a huge majority (89%) of the students felt that the colors had helped them. The surveys also tell us that the perceived difficulty of the test was higher for group black&white compared to the experimental groups (color and prime).

Analyzing the difference between task “a” and “b” tasks, don’t indicate any specific difference correlation between groups, if participants managed to complete task “a”, they usually tend to complete task “b”.

5.3 Method criticism

5.3.1 Inherit design flaws in the CEM-Test
The time measured by the CEM-test was not usable data on its own since, unless the student had completed all problems, the total time could be very skewed. A student skipping half of the problems would (almost) always get a faster time than someone who spent time to solve each problem. Looking at the time difference one problem at a time, only counting those who solved the problem, could solve this issue, however we cannot dismiss the problems that were skipped from analysis. Looking at the data from the CEM-test it was obvious that many of the students skipped several problems without trying, even though the test was voluntary. In most of these cases the student had put in effort during the first few tasks, often succeeding, but then choosing to skip the rest of the problems. Some of the data was very obvious surrounding this, however it is not entirely easy to determine if the student had put in any effort or not when the skip was not made in the first few seconds. We chose to dismiss the unsolved problems if the student had spent less than 30 seconds on said problem, a completely arbitrary line drawn by us, and we cannot disclose with certainty that our way of handing this data is completely fair.

5.3.2 Experimental design model
An inherent problem with a between study experimental model, is that it does not map out if the participant would fail to complete the corresponding learning material in the other group. The prior mathematical knowledge between groups could have major influence on final completion rate. Thus, careful consideration is
needed to be done in order to filter and make selections of the source material prior to analysis.

5.3.3 Priming and the usage of colors
It could be argued that students from group color, who were not supposed to know the meaning of the colors, might have understood some of them anyway, due to the selection of color green and pink. These color might have prior associations for the participants in group color, as which might diminish the difference in results between group color and group prime less significant. This speculation is supported by the group interviews where students from both group color and group black could figure out some of the colors meaning quickly, and where students themselves suggested the meaning of the colors used for addition and subtraction could universally be associated with positives and negatives. Out of the four colors used, the students seemed to be able to figure out when and why three of the colors were used, green, pink and blue, however, yellow they could not, some even expressed confusion around the color. After examining our usage of the color yellow, we do believe it was used in a confusing matter in at least two of the three problems it was used, and not used the way the primed students expected it to be used, causing some confusion. The three problems where yellow was used were also the three problems which gave negative results for group prime compared to group color (see table 10). We believe this caused some skewed results, and therefore argue our study somewhat diminishes the positive effect of priming. This is however not something we can validate and has to be seen as speculation from our part.

5.4 Future research
Both the results of this study and the group interviews indicate that a poor usage of color in color-coding material can infuse confusion for the learner, therefore further research should be made into the correct usage of colors in color-coding for the purpose of learning.

Although the results of this study show that priming can add to the positive effect of color-coding, the method used can be criticized. Therefore, it is possible that the true effect of priming is substantially different from its estimated effect in this study, and further research should be made on the area.

5.5 Conclusion
Using colors in learning material to aid the young learners ability to understand and solve mathematical problems has proven by this study to be very effective on students aged 12-16. The study shows with high confidence that color enchanted instructional material can improve the ability to solve a mathematical problem, reduce the time needed to solve a mathematical problem, and aid in understanding the mathematical problem. The study also shows with moderate confidence that priming of certain colors can be used to further improve the learning of mathematics. We can also conclude that the use of color can have a positive effect on the young learner’s perception of the learning material. Teachers in mathematics and producers of educational material should look to implement color-coding into instructional mathematics, however further studies on correct color usage is needed to ensure a good effect.

6. ACKNOWLEDGMENTS
Our thanks to all the participants of this study, to Hazim Deirmenc for helping us develop the CEM-test tool and finally Shaheen Waliullah for your help during the preliminary study phase.

7. REFERENCES


Appendix A
Tasks in CEM-test

Task 1a - colored

För att räkna ut svaret i detta mönster adderar du ihop värdet på varannan sifra och du börjar med det första siffran!

Exempel

\[ 4 \quad 6 \quad 6 \quad 7 \quad 3 \quad 3 \quad 2 \quad 0 \quad 1 \quad 4 \quad 4 \quad 6 = \square \]

Uppgift 1a)
Beräkna värdet i rutan

\[ 4 \quad 6 \quad 6 \quad 7 \quad 3 \quad 3 \quad 2 \quad 0 \quad 1 \quad 4 \quad 4 \quad 6 = \square \]

Task 1b

Uppgift 1b)
Beräkna värdet i rutan

\[ 3 \quad 3 \quad 2 \quad 5 \quad 4 \quad 0 \quad 6 \quad 7 \quad 1 \quad 2 \quad 0 = \square \]

Task 2a - colored

För att räkna ut svaret i detta mönster adderar du ihop värdet på alla uthålliga siffror och ignorera de jämna talen!

Exempel

\[ 8 \quad 1 \quad 4 \quad 8 \quad 3 \quad 4 \quad 1 \quad 6 \quad 4 \quad 0 \quad 2 \quad 5 \quad 5 \quad 2 \quad 7 \quad 6 \quad 3 \quad 4 = 25 \]

Uppgift 2a)
Beräkna värdet i rutan

\[ 3 \quad 1 \quad 6 \quad 8 \quad 2 \quad 0 \quad 5 \quad 4 \quad 1 \quad 8 \quad 3 \quad 0 \quad 1 \quad 5 \quad 6 \quad 8 \quad 0 \quad 6 = \square \]

Task 2b

Uppgift 2b)
Beräkna värdet i rutan

\[ 3 \quad 5 \quad 6 \quad 4 \quad 2 \quad 4 \quad 5 \quad 4 \quad 1 \quad 2 \quad 1 \quad 0 \quad 5 \quad 3 \quad 0 \quad 8 \quad 1 \quad 6 = \square \]

Task 1a - black&white

För att räkna ut svaret i detta mönster adderar du ihop värdet på varannan sifra och du börjar med det första siffran!

Exempel

\[ 1 \quad 4 \quad 2 \quad 9 \quad 3 \quad 5 \quad 0 \quad 2 \quad 2 \quad 6 \quad 3 \quad 4 = 11 \]

Uppgift 1a)
Beräkna värdet i rutan

\[ 4 \quad 6 \quad 6 \quad 7 \quad 3 \quad 3 \quad 2 \quad 0 \quad 1 \quad 4 \quad 4 \quad 6 = \square \]

Task 2a - black&white

För att räkna ut svaret i detta mönster adderar du ihop värdet på alla uthålliga siffror och ignorera de jämna talen!

Exempel

\[ 8 \quad 1 \quad 4 \quad 8 \quad 3 \quad 4 \quad 1 \quad 6 \quad 4 \quad 0 \quad 2 \quad 5 \quad 5 \quad 2 \quad 7 \quad 6 \quad 3 \quad 4 = 25 \]

Uppgift 2a)
Beräkna värdet i rutan

\[ 3 \quad 1 \quad 6 \quad 8 \quad 2 \quad 0 \quad 5 \quad 4 \quad 1 \quad 8 \quad 3 \quad 0 \quad 1 \quad 5 \quad 6 \quad 8 \quad 0 \quad 6 = \square \]
**Task 3a - colored**

1) Mönstret är en talföljd där man adderar de första två talen och subtraherar därefter det tredje talen.

2) Det fjärde talet är svaret från steg 1!

Mönstret återupptas igen från nästkommande tal.

**Uppgift 3a)**

Vad ska stå istället för frågetecknet?

\[ 9 \quad 3 \quad 4 \quad 8 \quad 5 \quad 8 \quad 4 \quad 9 \quad 5 \quad 1 \quad 3 \quad 4 \quad 3 \quad 1 \quad ? \]

---

**Task 3b**

**Uppgift 3b)**

Vad ska stå istället för frågetecknet?

\[ 6 \quad 3 \quad 3 \quad 6 \quad 5 \quad 7 \quad 4 \quad 8 \quad 5 \quad 2 \quad 3 \quad 4 \quad 7 \quad 3 \quad 1 \quad ? \]

---

**Task 4a - colored**

1) Ta första sifran i raden ovanför strecket och addera med alla tre tal under under strecket och sätt in summan i första bomen under strecket.

2) Ta andra sifran i raden ovanför strecket och subtrahera med alla tre tal under den och sätt in differensen i andra bomen under strecket.

3) Ta tredje talet i raden ovanför strecket och addera med alla tre tal under den och sätt in summan i tredje bomen under strecket.

4) Ta sista talet i raden ovanför strecket och subtrahera med alla tre tal under den och sätt in differensen i sista bomen under strecket.

5) Du ska nu addera öppa första och tredje talet under strecket och sedan subtrahera det med andra och fjärde talet under strecket och rikna ut svarnet!

**Exempel**

\[
\begin{array}{c}
2918 \\
1430 \\
2242 \\
4104 \\
\hline
9282 = 13
\end{array}
\]

**Uppgift 4a)**

\[
\begin{array}{c}
2918 \\
1729 \\
\hline
2 \quad 9 \quad 1 \quad 8 \\
0 \quad 3 \quad 1 \quad 7 \\
4 \quad 2 \quad 2 \quad 1 \\
4 \quad 1 \quad 0 \quad 4 \\
\hline
9 \quad 2 \quad 8 \quad 2 = 13
\end{array}
\]

---

**Task 4b**

**Uppgift 4b)**

Beräkna värdet i den stora rutan

\[
\begin{array}{c}
2 \quad 7 \quad 5 \quad 8 \\
1 \quad 0 \quad 0 \quad 4 \\
5 \quad 4 \quad 1 \quad 0 \\
1 \quad 1 \quad 2 \quad 1 \\
\hline
\end{array}
\]

= [Blank]

---

**Task 3a - black&white**

1) Mönstret är en talföljd där man adderar de första två talen och subtraherar därefter det tredje talen.

2) Det fjärde talet är svaret från steg 1!

Mönstret återupptas igen från nästkommande tal.

**Uppgift 3a)**

Vad ska stå istället för frågetecknet?

\[ 9 \quad 3 \quad 4 \quad 8 \quad 5 \quad 8 \quad 4 \quad 9 \quad 5 \quad 1 \quad 3 \quad 4 \quad 3 \quad 1 \quad ? \]

---

**Task 4a - black&white**

1) Ta första sifran i raden ovanför strecket och addera med alla tre tal under under strecket och sätt in summan i första bomen under strecket.

2) Ta andra sifran i raden ovanför strecket och subtrahera med alla tre tal under den och sätt in differensen i andra bomen under strecket.

3) Ta tredje talet i raden ovanför strecket och addera med alla tre tal under den och sätt in summan i tredje bomen under strecket.

4) Ta sista talet i raden ovanför strecket och subtrahera med alla tre tal under den och sätt in differensen i sista bomen under strecket.

5) Du ska nu addera öppa första och tredje talet under strecket och sedan subtrahera det med andra och fjärde talet under strecket och rikna ut svarnet!

**Exempel**

\[
\begin{array}{c}
2918 \\
1430 \\
2242 \\
4104 \\
\hline
9282 = 13
\end{array}
\]

**Uppgift 4a)**

\[
\begin{array}{c}
2918 \\
1729 \\
\hline
2 \quad 9 \quad 1 \quad 8 \\
0 \quad 3 \quad 1 \quad 7 \\
4 \quad 2 \quad 2 \quad 1 \\
4 \quad 1 \quad 0 \quad 4 \\
\hline
9 \quad 2 \quad 8 \quad 2 = 13
\end{array}
\]

---

**Task 4b**

**Uppgift 4b)**

Beräkna värdet i den stora rutan

\[
\begin{array}{c}
2 \quad 7 \quad 5 \quad 8 \\
1 \quad 0 \quad 0 \quad 4 \\
5 \quad 4 \quad 1 \quad 0 \\
1 \quad 1 \quad 2 \quad 1 \\
\hline
\end{array}
\]

= [Blank]
**Task 5a - colored**

1) Addera (plussa) ihop alla siffror i varje hörn och räkna ut summan.
2) Ta summan från steg 1 och multiplicera med det mittensta talet.
3) Det är svaret!

**Exempel**

\[
\begin{align*}
1 & \quad 4 & \quad 5 & \quad 9 & \quad 3 \\
9 & \quad 8 & \quad 2 & \quad 4 & \quad 6 \\
4 & \quad 1 & \quad 0 & \quad 7 & \quad 5
\end{align*}
\]

\[
\begin{align*}
&\text{Summa:} & 1+3+4+5 &= 13 \\
&\text{Värde:} & 13 \times 2 &= 26
\end{align*}
\]

**Uppgift 5a)**

Beräkna värde i rutten

\[
\begin{align*}
1 & \quad 5 & \quad 4 & \quad 3 & \quad 4 \\
5 & \quad 7 & \quad 3 & \quad 4 & \quad 6 \\
3 & \quad 1 & \quad 7 & \quad 6 & \quad 2
\end{align*}
\]

**Task 5b**

**Uppgift 5b)**

Beräkna värde i rutten

\[
\begin{align*}
3 & \quad 1 & \quad 2 & \quad 7 & \quad 2 \\
4 & \quad 9 & \quad 2 & \quad 3 & \quad 9 \\
1 & \quad 7 & \quad 5 & \quad 3 & \quad 1
\end{align*}
\]

**Task 6a - colored**

1) Ta den första sifran på det första raden ovanför strecket och multiplicera det med summan av de två nedanställda talen
2) Ta svaret från steg 1 och sätt in det i första rutnan under strecket
3) Ta den andra sifran på det första raden ovanför strecket och multiplicera det med summan av de två nedanställda talen
4) Ta svaret från steg 3 och sätt in det i andra rutnan under strecket
5) Ta svaret från steg 3 och subtrahera det med svaret från steg 3 och det är svaret!

**Exempel**

\[
\begin{align*}
2 & \quad 1 \\
6 & \quad 4 \\
1 & \quad 6 & \quad 5 & \quad 11
\end{align*}
\]

\[
\begin{align*}
&\text{Svaret:} & 2 \times (6+2) - 1 \times (5+1) \\
&\text{= 16 - 5} \\
&\text{= 11}
\end{align*}
\]

**Uppgift 6a)**

Beräkna värde i den stora rutnan

\[
\begin{align*}
3 & \quad 2 \\
3 & \quad 1 \\
2 & \quad 3
\end{align*}
\]

**Task 6a - black & white**

1) Ta den första sifran på det första raden ovanför strecket och multiplicera det med summan av de två nedanställda talen
2) Ta svaret från steg 1 och sätt in det i första rutnan under strecket
3) Ta den andra sifran på det första raden ovanför strecket och multiplicera det med summan av de två nedanställda talen
4) Ta svaret från steg 3 och sätt in det i andra rutnan under strecket
5) Ta svaret från steg 3 och subtrahera det med svaret från steg 3 och det är svaret!

**Exempel**

\[
\begin{align*}
2 & \quad 1 \\
6 & \quad 4 \\
1 & \quad 6 & \quad 5 & \quad 11
\end{align*}
\]

\[
\begin{align*}
&\text{Svaret:} & 2 \times (6+2) - 1 \times (5+1) \\
&\text{= 16 - 5} \\
&\text{= 11}
\end{align*}
\]

**Uppgift 6a)**

Beräkna värde i den stora rutnan

\[
\begin{align*}
3 & \quad 2 \\
3 & \quad 1 \\
2 & \quad 3
\end{align*}
\]
Task 6b

Uppgift 6b)
Beräkna värdet i den stora rutan

\[
\begin{array}{c}
5 & 2 \\
3 & 1 \\
2 & 2
\end{array}
\]
APPENDIX B

The design on the final product differs slightly since it was handwritten in Swedish and colored using marking pens. The symbols used for priming on a whiteboard was written with the color green and pink instead of being highlighted using a marker pen due to practical reasons.

Sample test questions for the purpose of priming.

1) **Simplify**
   a) \(20 + (4x + 10) - 2x\)
   b) \(6x - 8x + 10x\)
   c) \(5x - (2x - 10) + (x + 2)\)

2) The circumference of a rectangular field is **30 meters** and the length of the field is **four times longer** than the width.
   a) What is the length and width of the field?
   b) What is the area of the field?

Sample instructions during the preliminary phase (in Swedish).

1)
   Definition 1. Låt \(A\) vara en kvadratisk matris. 2\(\times\)2 Matris. Determinanten av matrisen beräknas på följande sätt:

   \[
   \text{Det}(A) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \times a_{22} - a_{12} \times a_{21}
   \]

2)
   Definition 1. Låt \(A\) vara en kvadratisk matris.

   \[
   \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{vmatrix}
   \]

   Summan av alla diagonalelement i en kvadratisk matris \(A\) kallas matrisens spår och betecknas \(tr(A)\) (från engelskans ”trace”).

   Alltså \(tr(A) = a_{11} + a_{22} + \ldots a_{nn}\)

   **Exempel.** Låt \(A = \begin{vmatrix} 4 & 5 & 3 & 7 \\ 3 & 5 & 8 & 7 \\ 4 & 2 & 2 & 3 \\ 2 & 3 & 0 & 7 \end{vmatrix}\)

   Då är \(tr(A) = 4+5+2+7\)