3D Printing a Maxwell Fish Eye Lens With Periodic Structures

Valentine Lin and Tarek Sayed Hamad

Abstract—With the rise of high frequency communication systems such as 5G, new types of antennas has to be developed in order to meet the new requirements. In recent years, lens antennas made of periodic structures has been shown to have desirable performance when increasing operational frequency without increasing the size of the antennas. One way of manufacturing the lenses for the antennas are with 3D printers loaded with dielectrics with specified permittivity. This project group studied the process of designing and manufacturing a flat Maxwell fish eye lens at 5 GHz with a bandwidth of 3.5 GHz to 6 GHz. The resulting design is a lens based on a periodic configuration of cuboid unit cells made from dielectrics which consisted of a hole. By choosing the ratio of dielectric and holes in the unit cells, each part of the lens could be tuned to achieve a specific effective refractive index required for realising the Maxwell fish eye lens.

Index Terms—Maxwell fish eye lens, dielectric lens antenna, metamaterial, periodic structure, 3D printed lenses.

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I. Introduction

In the world of wireless communication systems, lens antennas are gaining popularity since communication systems are developing towards higher operational frequency with technologies such as 5G [1]. Since higher frequency signals require greater directivity, lenses can be utilised to focus the signals in a desired direction as shown by [2]. In the case of 5G, many highly directional transmitters has to be placed in an area to provide full coverage. There need to be line of sight between the sender and receiver because the high frequency signals gets blocked by most objects. One way of solving this problem is by designing integrated lens antennas that can provide several targeted beams from a single transmitter as shown by [3].

Traditional lens antennas has typically been constructed with a homogeneous material by combining different dielectric materials as the bandwidth for dielectric lenses can be very wide while the losses are low [4]. The traditional method has been very expensive and complex in order to manufacture a high performance lens. As an alternative method, 3D printing has been studied as a method of producing periodic structures for use in lens antennas. Studies have shown that these periodic structures can be utilised to design a structure with the same desired properties for a lower manufacturing cost [5]. Periodic structures could also be designed to perform functions that natural materials cannot, such as giving a structure a negative refractive index. In the case of optics, periodic structures could be used to circumvent the physical limitations of traditional optics [6]. For example cloaking an object with a layer of periodic structures could render object invisible for a narrow frequency band if the effective refractive index of the periodic structure is negative [7].

The goal of this project is therefore to produce a prototype of a lens antenna, more specifically a Maxwell fish-eye (MFE) lens. The MFE lens is a type of lens antenna that can be utilised for wireless communication.

A MFE lens is has a refractive index that varies over the radius, these lenses are generally called graded index (GRIN) lenses. At the lens’s centre, the refractive index is largest and gradually decreases to a minimum at the circumference of the lens [8]. The refractive index has a direct correlation to the permittivity of the material [9]. Previous studies have shown that a MFE lens can provide perfect imaging as long as the location of the source is known and within a certain frequency band [4]. Furthermore, with the rise of 3D printers as a cheap and reliable way of manufacturing, previous studies has also shown that a GRIN lens can be created by using a 3D printer with different materials [10]. If holes are introduced in the lens, the transition between materials can be much smoother and the effective permittivity can be tailored. This can be built by a 3D printer in a single process without any complicated machinery and with only a few different materials.

II. Technical Background

In order to manufacture and realise a MFE lens, some theory needs to be established. Therefore, knowledge about lenses, electromagnetic waves, periodic structures, dispersion diagrams and S-parameters are essential and will be presented in this section.

A. Lenses

Lenses are most popular at optic frequencies. They transmit and refracts light in a way that disperses or focuses light to a certain point, a so-called focal point. An image can take form if light is focused on a focal point, which depends on the lens’ geometry and the material’s refractive index [9]. Optical lenses are usually made out of transparent materials such as glass or plastic. However, they can also be made out of non-transparent materials in order to transmit, disperse and refract other electromagnetic waves, not necessarily light.

B. Maxwell Fish Eye Lens

A Maxwell Fish-eye lens is a type of lens antenna with a gradient refractive index. More specifically, it has a refractive index profile that is spherically symmetric, which gives it a certain property.
A source in any excited point of the circumference will converge exactly at the opposite side of the circumference (its focal point), as demonstrated in Fig. 1. Due to the lens’ varying refractive index within the lens itself, the optical path is affected and bends in a way which gives the MFE lens this certain property. Theoretically, a perfect image can be produced on the other side \[4\].

The refractive index of the lens is greatest at the centre of the lens, and gradually decreases to its lowest value at the circumference. This can be seen in Fig. 2, where the refractive index has a value of \(n = 2\) in the middle, and \(n = 1\) at the circumference. The refractive index of a MFE lens can be described by equation (1).

\[
n = \sqrt{\epsilon_r} = \frac{2n_0}{1 + (\frac{r}{a})^2}
\]

Where \(n\) is the refractive index, \(n_0\) is the refractive index at the border of the lens, \(r\) is the distance from the centre and \(a\) is the radius of the lens. In normal cases, \(n_0 = 1\) since the lens is surrounded by air and there needs to be a smooth transition between the uttermost layer of the lens and the surrounding medium, in this case air; that has a refractive index of \(n = 1\). If the discrepancy between the refractive index of two mediums are too big, it would cause reflections on the surface between the mediums.

\[\text{C. Periodic Structures}\]

Periodic structure are structures that are made up out of a repeated pattern of unit cells, in one, two or three dimensions. A great example of this type of structure are honeycombs, which is made up of hexagonal wax cells and has a repeated pattern of these cells periodically in two directions.

A periodic structure can have similar properties to a material with a specific permittivity, depending on the unit cells. These unit cells are designed to achieve a certain effective permittivity which is needed to realise a lens. A surface made out of a periodic pattern of unit cells, that produces unusual reflection patterns or properties, is called a metasurface [11]. With periodic structures, a GRIN lens is possible, even if it is flat [12].

\[\text{D. Dispersion Diagrams}\]

A dispersion diagram represents a wave’s propagation characteristics through a structure. To understand these diagrams, some theory needs to be established first.

1) Propagation Constant: The propagation constant describes how the wave’s amplitude and phase varies along the propagation direction. In a lossy medium, the propagation constant is complex. It is described with equation (2).

\[
\gamma = \alpha + j\beta
\]

Here \(\alpha\) is the attenuation constant and \(\beta\) is the phase constant [12]. In lossless mediums, \(\alpha = 0\) and \(\beta\) can be calculated using equation (3).

\[
\beta(\omega) = \omega\sqrt{\epsilon\mu}
\]

2) Effective Refractive Index: Dispersion diagrams describes the relation between the phase constant \(\beta\) and angular frequency \(\omega\) of an electromagnetic wave travelling through a structure [12]. The slope of the curve in a dispersion diagram at one frequency is the group velocity, which can be calculated using equation (4).

\[
v_g = \frac{d\omega}{d\beta}
\]
When the slope changes, it means that the effective refractive index of the structure has changed. The effective refractive index can be calculated with equation (5).

$$n_{eff} = \frac{c}{v_g}$$  \hspace{1cm} (5)

By analysing the dispersion diagrams of infinitely repeating unit cells, the effective refractive index can be determined. The relation between the phase constant of the wave in vacuum and in a unit cell provides the effective refractive index for a specific frequency, see equation (6).

$$n_{eff}(\omega) = \frac{\beta_{\text{unit\_cell}}(\omega)}{\beta_{\text{vacuum}}(\omega)}$$  \hspace{1cm} (6)

E. Waveguides

A rectangular waveguide made from a perfect electric conductor (PEC) is designed for feeding the lens. PEC is chosen since it reflects electromagnetic waves fully. A rectangular waveguide can propagate TM and TE modes which are different field distributions. In a rectangular waveguide, with the width $W$ and height $h$, and $W > h$, the dominant mode is called $TE_{10}$. This is due to the fact that this mode has the lowest attenuation of all TE-modes in a rectangular waveguide. These types of rectangular waveguides has a cutoff frequency, which indicates what frequencies of electromagnetic waves can propagate through it. The cutoff frequency can be calculated using equation (7), for cases where $W > h$ [13].

$$f_{c10} = \frac{1}{2W \sqrt{\mu \epsilon}}$$  \hspace{1cm} (7)

This equation also gives some design tips for a waveguide. For example, to make sure that electromagnetic waves of 5 GHz can propagate through the waveguide, a cutoff frequency should be lower than that.

F. S-parameters

The performance of the antenna can be measured numerically by analysing the S-parameters. The S-parameters represent the gain of the signal between two ports and are denoted as $S_{ij}$ where $j$ is the source port and $i$ is the receiving port. If $i$ and $j$ are the same port, then the corresponding S-parameter represents the gain of the reflected electromagnetic waves (reflection coefficient) [14]. If $i$ and $j$ are not the same port, then $S_{ij}$ corresponds to the power transfer between port $i$ and port $j$. Worth noting is the fact that S-parameters can be displayed in either a linear or decibel scale.

III. DESIGN

With the theory established, one can design a MFE lens together with a feeding structure. First, the unit cells will be designed. Secondly, they will be used to design the entire lens. Lastly, the feeding will be designed.
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Figure 4. Dispersion diagram of unit cell \( (\varepsilon_r = 3) \) with different sizes of holes. Each line that is dashed corresponds to a unit cell which has a hole with dimension \( \ell \). Other dimensions of the unit cells are fixed and can be seen in table II.

Table II
 DIMENSIONS OF THE HOLES.

<table>
<thead>
<tr>
<th>The hole’s side</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ell_1 )</td>
<td>1.4 mm</td>
</tr>
<tr>
<td>( \ell_2 )</td>
<td>2 mm</td>
</tr>
<tr>
<td>( \ell_3 )</td>
<td>2.6 mm</td>
</tr>
<tr>
<td>( \ell_4 )</td>
<td>3.2 mm</td>
</tr>
<tr>
<td>( \ell_5 )</td>
<td>3.8 mm</td>
</tr>
</tbody>
</table>

B. Unit Cells Results

Each simulation of the unit cells resulted in a dispersion diagram. The dispersion diagrams were further analysed in order to acquire the effective refractive index for each unit cell and its hole, using equation (6). The dispersion diagrams were compiled to one plot which can be seen in Fig. 4 for unit cells with \( \varepsilon_r = 3 \) and in Fig. 5 for unit cells with \( \varepsilon_r = 4 \). This in turn gives the effective refractive index which can be plotted as a function of the hole’s side, as seen in Fig. 6. Since only 5 dispersion diagrams were analysed, the rest of the data to create a smooth graph were generated with spline interpolation using MATLAB.

C. Design of Lens

Since a 3D printer was going to be used to manufacture the lens, a flat MFE lens was designed since it takes less time to 3D print than a three dimensional one.

The final lens consisted of 34 unit cells in diameter, resulting in a diameter of 27.2 cm. The thickness of the lens was 5.4 mm. With simulations, the lens was simulated with different thicknesses, resulting in not so different results. The lens’ thickness was therefore chosen depending on how long it would take to print, since a smaller lens takes less time than a thicker lens. The thickness also depended upon if the lens would be able to hold itself up, hence you cannot have a lens that is too thin. The unit cells in the inner part of the lens was constructed with a material with \( \varepsilon_r = 4.4 \) and the outer part of the lens was constructed with a material with \( \varepsilon_r = 3 \). This can be seen in Fig. 7 where the pink part corresponds to a material with \( \varepsilon_r = 4.4 \) and the green part corresponds to \( \varepsilon_r = 3 \). The reason for why these materials were used, was because those materials were available for us to use in the lab. Since the unit cells are not infinitesimal the dimensions of each unit cell where chosen based on the refractive index the centre of the cell should have according to 1. Note that the lens was covered by PEC parallel plates over and under it.

D. Design of Feeding

Waveguides needed to be designed to be able to test the performance of the lens. A basic cuboid waveguide made
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Figure 7. Top view of the simulated Maxwell fish eye lens. Pink represents a material with $\epsilon_r = 4.4$ and green represents a material with $\epsilon_r = 3$. Its diameter is 27.2 cm and the thickness is 5 mm.

Figure 8. Model of feeding structure, top view. Its dimensions can be seen in table III.

The structure consisted of the MFE lens and eight waveguides symmetrically placed around the lens. All the waveguides where placed 8 mm from the circumference of the lens, since that is the same length as a unit cell. A plot of the field distribution over the lens can be seen in Fig. 9, together with its structure. The S-parameters were also determined and can be seen in Fig. 10.

V. TEST SETUP

A. The 3D Printed Lens

The prototype lens was successfully manufactured using two different materials with different relative permittivities, $\epsilon_r = 3$ and $\epsilon_r = 4.4$ respectively. The diameter of the lens was 27.2 mm and the thickness was 5.4 mm. As can be seen in Fig. 11, the lens is consisting of two parts and the hole varies with the radius.

B. Lens with 8 Waveguides

The S-parameters were measured one at a time with a passive 50 $\Omega$ resistor loaded in each port that was not tested to simulate the load of a coaxial cable. The bottom was covered with a copper plate and the top was covered with a copper net. A copper barrier was also added around the lens. The setup for measuring the lens can be found in Fig. 12.

VI. RESULTS

The result of the measurements, found in Fig. 13, show that the lens has a S7.1 varying between 0.18 and 0.6 in the frequencies between 3 GHz and 6 GHz. The S1.1 of the

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Magnitude [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width W</td>
<td>50</td>
</tr>
<tr>
<td>Length L</td>
<td>80</td>
</tr>
<tr>
<td>Height H</td>
<td>5.6</td>
</tr>
<tr>
<td>Thickness t</td>
<td>0.3</td>
</tr>
<tr>
<td>Gap height gh</td>
<td>5</td>
</tr>
<tr>
<td>Focal point fp</td>
<td>25</td>
</tr>
<tr>
<td>Focal height fh</td>
<td>5</td>
</tr>
</tbody>
</table>

Table III: Dimensions waveguides.
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Figure 10. S-parameters (linear scale) for lens with 8 waveguides. S1.1 is equal to the reflection coefficient, and S7.1 is equal to the power transfer to the target port.

Figure 11. 3D printed MFE lens prototype. The lens’ diameter was 27.2 cm and the thickness was 5.4 mm. It consisted of two dielectric materials with $\varepsilon_r = 3$ and $\varepsilon_r = 4.4$ respectively.

The prototype is lowest between 5.1 GHz and 5.6 GHz. At the same frequencies, the S7.1 is the largest, which means we have the greater power transfer between those frequencies.

VII. DISCUSSION

In order to minimise reflections along the edges of the lens, the effective refractive index along the circumference had to be close to $n = 1$ which proved to be quite difficult to achieve without making the walls of the unit cells too thin to manufacture. The resulting unit cells at the edge had a wall thickness of 0.4 mm which made them very fragile. But since the operational frequency of the lens is relatively low, at 5 GHz, small imperfections does not affect the performance of the lens. A solution to this problem would be to use a dielectric with a lower permittivity for the outermost layer which would result in thicker unit cells at the circumference.

It is worth to mention sources of errors or things that contribute to the differences between the simulations and real world results. First of all, a net of copper was used as the top layer of the construction instead of a copper plate used for the bottom layer. By using a copper net, it can act as a Faraday cage which blocks the waves from going out of the lens construction. The net was installed since it gives people the ability to see the actual lens and the construction. With a copper plate, this would not have been possible. However, the copper net introduces some irregular airgaps between the actual lens and the net itself, which affects the performance. By installing a copper plate which lies along the lens’ upper surface instead, this problem could be fixed which would improve the performance. The net introduced unwanted reflections together with the waveguide ports.

Secondly, the entire construction around the lens was hand-made out of copper plates. Although a template was used to construct the waveguides, it must be stated that their dimensions and geometry is not as perfect as the waveguides in the simulations. The same goes for the rest of the entire construction. This problem could be used by improved methods of constructing metallic structures, such using machines and/or
letting a professional metallic worker construct the parts.

Thirdly, the inner part of the lens needed to be rasped a little bit in order for it to fit inside the outer part of the lens. When the inner part was later installed, it introduced a slight bending of the entire lens. Therefore, the geometry of the lens is likewise not as perfect as in the simulations.

VIII. CONCLUSIONS

A Maxwell fish eye lens was realised. It was based on periodic structures and was 3D printed using dielectric materials with \( \epsilon_r = 3 \) and \( \epsilon_r = 4.4 \). Also, a feeding structure for the lens was realised. The result was a prototype lens that performed similarly to the simulations. The lens performed well even if the waveguides were not pointed towards each other. It had acceptable power transfer and low reflections, especially if the waveguides were not pointed towards each other. It was realised. The lens performed well even if the waveguides were not pointed towards each other. The result was a prototype lens that performed similarly to the simulations. The lens performed well even if the waveguides were not pointed towards each other. It had acceptable power transfer and low reflections, especially between 5 GHz and 5.5 GHz. The results also show that the power transfer to the other ports is higher than in the measurements compared to the simulations. This means that the lens does not focus the electromagnetic waves as well as it does in the simulations and some of the power is scattered into other ports.

Differences came mainly from unwanted reflections from both the copper net and waveguide ports, losses in the materials which were not taken into account in the simulations and lastly the geometry of every hand-made component which were not as perfect as in the simulations. These things can be improved in the future to realise even better lenses.

The final prototype of the lens and its construction worked, as can be seen in the results. The VNA used to measure the S-parameters was calibrated for a frequency band of 3 GHz-6 GHz, hence why there is no measured results for other frequencies.

The power transfer in the measurement were significantly lower compared to the simulations and the transfer to the other ports also appears to be higher.

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REFERENCES


