Modelling and Measurement of Industrial Manipulators for High-Precision Contact Applications

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Abstract

This study was conducted with the objective of predicting the accuracy of robotic manipulator models in contact applications. A number of computational robot models, including lumped-parameters models and flexible-body elastic models were used and refined to evaluate the behavior of a manipulator under static, quasi-static and dynamic loading conditions. The simulations were then compared with experimental measurements conducted in the forms of static, quasi-static and dynamic compliance-evaluation series. Additionally, contact application models were constructed and utilized to simulate and assess the associated accuracy level of contact processes involving robotic manipulators.

Results included comprehensive joint-space maps of the manipulator with respect to stiffness which showed the high dependency of the compliance on the configuration of the robot. Computational estimations of heavy-duty contact applications with robotic manipulators predicted the occurrence of self-excited oscillations and low stiffness of the robot when compared to traditional CNC machines. Recommendations and guidelines are derived based on the results to improve the current state of the art and overcome the challenges associated with these applications in the future.
Sammanfattning

Denna studie genomfördes med målet att förutsäga noggrannheten hos robotmodulatormodeller i kontaktapplikationer. Ett antal beräkningsmässiga enskilda robotmodeller, inklusive klumpparametermodeller och flexibla elastiska modeller, användes för att utvärdera beteendet hos en manipulator under statiska, kvasistatiska och dynamiska belastningsförhållanden. Beräkningarna förfinades och simuleringarna jämfördes sedan med replikerade experimentella mätningar utförda i form av statisk, kvasistatisk och en serie av styvhetstutvärderingar. Dessutom konstruerades kontaktapplikationsmodeller och användes för att simulera och utvärdera den associerade noggrannhetsnivån av kontaktprocesser som involverar robotmanipulatorer.

Acknowledgements

The author would like to thank Nikolas Theissen and Andreas Archenti from KTH Royal Institute of Technology for their unparalleled continuous support of the thesis project and assisting in the measurement series of the COMACH experiments.

Special thanks to Jeroen Derkx and Jens L. Andersson from ABB Robotics for their outstanding mentorship and guidance throughout the project. I would also like to sound my deep gratitude for the support of Björn Lunden, Marie Nord, and other colleagues at ABB Robotics.

All of my achievements during the M.Sc. study would have not been possible without the relentless backing of my family. Thanks for igniting the spark of my interest in research and making me believe in my abilities. I would also like to thank my fiancé, Ava, for assisting me in enduring numerous hardships and supporting me during student life.
To Ava.

This would not have been possible without you.
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Chapter 1

1 Introduction

1.1 Background

The tireless endeavor of humankind towards achieving automation can be dated back to the invention of the first wheel. Man has dreamed, portrayed, and created numerous tools by which his life became easier and smarter; yet the capstone of his efforts in creation has fled his grasp until today: Himself. The conservative mind would refrain from this venture to avoid treading on the realm of gods and elude the eternal torment of Prometheus. However, the scientist lives to learn, and to discover; and henceforth he brings the current age of sentient machines on-par with the giant Talos, called Robots.

During the past decade, the sales of industrial robots has seen a constant growth with an average annual increase of 310,000 units between 2015 to 2017 [1]. Figure 1.1 presents an overview of the number of shipments for industrial robots over the past decade. The continuously improving state-of-the-art and demands for enhanced robotics operational abilities gives rise to the need for more sophisticated models and control systems for such designs. The advantages of these models when included in the development phase include achieving an earlier understanding of the behavior of the system in time and frequency domain and identifying critical design parameters [2]. However, attaining an accurately descriptive model for complex robotic manipulators remains a challenge which several recent researches have approached via a number of methods including, lumped spring-mass

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1 In 1921, the Czech playwright Karel Čapek introduced the world to one of his masterpieces, titled “Rossumovi Univerzální Roboti”, or “Rossum’s Universal Robots” in English. The word Robot was derived from the Czech word Robota, translating roughly to “forced labor”, and was used in his play to portray machines who resembled people but worked diligently.

The accuracy of articulated industrial robots in contact applications is insufficient to comply with most geometric dimensions and tolerances in the automotive and aeronautic industries as Conrad [7] and Siciliano [8] state ±1 mm to be the typical range of positioning errors in robotic manipulators. The Swedish part of the COMACH project was established between KTH Royal Institute of Technology, ABB AB, and Hexagon AB with the objective of improving the current dynamic models and control systems in order to ensure an accuracy better than ±250 µm on the location of features for articulated robots in contact applications such as processing, assembling or disassembling.

### 1.2 Thesis Objective

The objective of this thesis work under COMACH is to simulate and analyze various dynamic models for contact applications using robotic manipulators and compare the results with experimental data achieved in the test facilities of ABB Robotics by utilizing the robot which is depicted in Figure 1.2 to validate the robot models and to be able to predict their behavior for contact applications. The robot has a payload of 60 kg and provides a range of 2.05 m at a weight of 425 kg.
For this purpose, a number of models including rigid-body representations, lumped mass-spring models and reduced-order dynamic models based on FE models will be used to estimate the system behavior. Afterwards, the machining process will be modeled and added to the dynamics of the manipulator to estimate the behavior of the combined system.

![ABB Manipulator](image)

*Figure 1.2: An ABB Manipulator*

The experimental phase includes tests carried out by connecting a loaded-double-ball-bar (LDBB) measurement system to the robotic end-effector and evaluating the static and dynamic response of the system. The final objective of the thesis is to compare and analyze the responses from the simulations with experiments conducted in accordance with ISO9283:1998 [9] to refine the current state of simulations and derive predictions and recommendations for prospective contact applications for the robotic system based on the results.

### 1.3 Thesis Structure & Outline

The thesis outline is based on the approach explained below:

1. Describing the objective of the thesis work and the research approach.
2. Reviewing the existing literature in the fields of modelling of robotic manipulators and assessments of contact applications with robotic manipulators.
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3. Illustrating the methodology of the approach towards the modelling and measurement of the robotic manipulator for contact applications.

4. Presenting the findings and basing conclusions on the observations from simulations and experimental data.

In accordance with the described outline, the thesis manuscript pursues the following structure:

1. Chapter 2 reviews the existing literature and previous works in the field of dynamic modelling of articulated robots. Furthermore, the mathematics and theories behind various dynamic modelling approaches towards industrial robotic manipulators will be discussed in this chapter.

2. Continuing the introduction of mathematical models, Chapter 3 introduces the current study and its approaches towards modelling of the robot in order to calculate the dynamic and static behavior of the robot. A series of experimental tests is performed in order to calculate and compare data achieved from the models with experimental investigations. The section illustrates the experimental setup and presents the details and specifications of the utilized equipment for the test procedure.

3. Results from the models and experiments are presented in Chapter 4 to assess the coherence of model data with experiments. The model will then be refined based on the experimental data and expectations for the machining process will be presented. A discussion regarding the state of the results and further estimations follows the chapter.

4. Chapter 5 concludes the thesis study and extrapolates the prospects for future work in this area.
Chapter 2

2 Literature Review and Theoretical Background

This chapter deals with relevant literature terminologies and theories behind modelling of robotic manipulators. Section 2.1 illustrates the definitions and terminologies regarding the structure and performance of industrial robots and section 1.1.1 describes the theories for kinematic and dynamic modelling of industrial manipulators.

2.1 Definitions and Technical Terms for Serial Articulated Manipulators

Sources such as the ISO 8373 standard [10] are used to define the technical terms for serial manipulators. The book “Robotics: modelling, planning and control” [11] provides the essential theory and terminologies used for modeling and describing the structure of industrial robots. The authors describe the robotic manipulator as a sequence of rigid bodies (links) interconnected via articulations (joints) and characterize a robotic manipulator by the arm, the wrist and the end-effector which provide mobility, dexterity, and performance according to the task at hand, respectively. Furthermore, the following definitions regarding industrial robots can be derived from the book which will be of interest during this study:

Workspace: The portion of the environment which the end-effector of the manipulator can access. The size and shape of the workspace depend on the structure of the robot along with the number of DOFs.

Accuracy: This property can be defined as the closeness of agreement between a measured quantity value and a true quantity value of a measurand [12]. When utilizing robotic manipulators, positioning accuracy can be defined using the same description for the measured tool center point (TCP) position in space.
Literature Review and Theoretical Background

Repeatability: This parameter is defined as measurement precision under a set of repeatability conditions of measurement, where repeatability conditions of measurement are the conditions of measurement, out of a set of conditions that includes the same measurement procedure, same operators, same measuring system, same operating conditions and same location, and replicate measurements on the same or similar objects over a short period of time [12].

Serial & Parallel Kinematic Machines: An serial manipulator contains \( n \) number of joints and \( n+1 \) number of links. This forms the minimum requirement for achieving a feasible manipulator structure. When a higher stiffness or reduction of moving mass is required, more links may be implemented between two joints; resulting in a parallel-link articulation which is also called a closed-loop chain. In other words, a physical loop is formed between two joints of the robot in the second scenario.

End-effector: The attachment to the turning disc of the robot which provides the desired functionality for the manipulator towards achieving a task.

The robotic manipulator used in this study consists of the following components as seen in [13]. The 6 axes of the robot plan out the required number of DOFs for achieving at least one solution to reaching any pose in the workspace.

2.2 Modelling of Robotic Manipulators

While the Denavit-Hartenberg (DH) forward kinematics is a mathematical calculation for the positioning of the robot, it does not consider the dynamics of the system such as the elastic behavior of the components and therefore cannot reach a complete prediction of the behavior of the robot. This section describes the theoretical expressions for the dynamics of robotic manipulators and presents mathematical and numerical approaches for modelling the respective behaviors. The following will introduce the rigid-body, lumped mass and flexible-body modelling methods which will be investigated further in this study.
2.2.1 Rigid Body Modelling

The basis of rigid bodies can be labelled as an assembly consisting of infinitely-stiff interconnected links which can produce motion through arbitrary joint movements. In other words, a rigid body model considers the deflection of the body to be negligible when compared to the gross body motion and the distance between any two arbitrary points on the body remains the same at each interval of motion [14]. This assumption allows simple mathematical models of robotic manipulators to be produced based on rigid-body parameter identification [15]. By utilizing a rigid-body model, the shape and position of the manipulator can be determined at each time instance by considering the joint angles. Therefore, the number of degrees of freedom in this case can be directly calculated as \( n = J \) where \( J \) represents the number of joints along the manipulator.

The geometrical properties of a robotic manipulator need to be derived based on experimental testing or computational approximation. For rigid-body models, these parameters include the inertia, mass, centers of mass and gravity, and dimensions of the links. In the case of this study, the industrial robot can be considered as a manipulator with six DOFs as depicted previously in Fig. 2.1. Forward kinematics can be used as in equation \( \text{equation} \) to calculate the positioning of the rigid model through the workspace in this case.
Kinematic Calibration

Kinematic calibration of a robot is defined as the process by which the pure kinematic accuracy of an industrial robot is increased [16]. The Denavit-Hartenberg (DH) notation is commonly used in calculating the kinematics of open-loop serial chains. This method provides an explicit, minimal expression towards modelling the kinematics of manipulators by knowing their structural parameters such as joint angles, joint offset, link lengths and link twist. However, this method also disregards the mechanical behavior of the robot (i.e. the deflections of the robot links due to finite stiffness). When coupled with kinematic calibration, the DH parameters can be modified to calculate the positioning of a point on the robot in a more accurate fashion. This procedure is often carried out by the robot manufacturer as an additional service to providing increased accuracy. The DH transformation will ultimately yield the transformation $T_{n}^{0}$ (from joint 0 to joint $n$) which can be described according to equation:

$$T_{n}^{0} = \prod_{i=1}^{n} A_{i+1}^{i} = \begin{bmatrix} R_{3 \times 3} & T_{3 \times 1} \\ 0_{3 \times 3} & 1_{1 \times 1} \end{bmatrix}$$

where $n$ is the number of joints of the manipulator, $A$ is the local transform from joint $i$ to joint $i+1$, and $R$ and $T$ are the rotational and translation components of the transformation, respectively. As a result, an arbitrary point $q_{0} = [x \ y \ z \ 1]^{T}$ can be transformed along the manipulator to $q_{n}$ by applying the transform $T_{n}^{0}$ according to equation:

$$T_{n}^{0} q_{0} = q_{n}$$

2.2.2 Lumped-Parameter Models

In engineering, lumped-parameter approaches are utilized when a distributed parameter of the body is modeled as a property of a single point on the respective structure. When modelling robotic manipulators, the lumped-parameter representations focus on concentrating the stiffness of the manipulators at the joints or at points along the manipulator when dealing with soft robotics, as in Giri and Walker’s work [17]. Hereby, this leads to reduced complexity and computation time while providing a simple mean towards taking the dynamic behavior of the structure into account. When
considering lumped-parameter models, the accuracy of the models increase as the number of flexible points across the body increases which in term contributes to the complexity of the produced result. Therefore, the researcher should consider this trade-off beforehand of producing the lumped-parameter representation as mentioned in Sun and He’s study [3] and the work of Klimchik [18] which assesses the impact of identification accuracy and ill formations of the stiffness matrix on the predictions of behavior for the manipulator. Followed by this procedure, the parameter identification for lumped models is normally carried out by iterative experimental testing and data analysis in order to reach a more accurate simulation.

In the case of this study, a lumped-parameter model is produced following the work of Zimmermann [13] by considering the elasticity of the manipulator to be lumped in the joints. The parameter identification process was carried out by previous works at ABB Robotics and from supplier datasheets which resulted in placement of 3D spring-damper pair models at the first 3 joints of the robot followed by spring-damper pair modelling of the gears at the latter joints as depicted in Figure 2.2. The dynamic response of the resulting model (E-Model) will be assessed with experiments in the following chapters.

![Figure 2.2: E-Model Visualization of The robot [13]](image-url)
2.2.3 **Elastic Body Models**

In principle, real bodies can rarely be considered fully rigid without neglecting the local deformations. When designing high-accuracy machines, even the smallest disturbances must be taken into account to reach the desired accuracy [19]. In order to achieve this objective, a geometry can be divided into small constituting elements, the so-called FE approach is portrayed in Figure 2.3 [20].

![Figure 2.3: FE representation of a 2D body](image)

The FE approach facilitates the piecewise linearization of partial differential equations which govern the properties of the flexible body. The Rayleigh-Ritz method can be generalized with FE models to satisfy the boundary conditions and calculate the deformations. The deformation of a single element can then be described according to Figure 2.4 as in equation (2.3) [21]:

$$d_i = \phi_i u_i + \phi_{i+1} u_{i+1}$$

(2.3)
Equation of Motion

The generalized equation of motion for a moving body can be described based on Lagrange’s equation as seen in equation (2.4) [21]:

$$\frac{d}{dt} \left( \frac{\partial KE}{\partial \dot{\xi}_i} \right) - \frac{\partial KE}{\partial \xi_i} + \frac{\partial PE}{\partial \dot{\xi}_i} = \tau_i$$

(2.4)

Where $\xi$ and $\tau$ represent the coordinate and force elements of the robot. Consequently, the kinetic energy of a link $i$ can be defined as in equation (2.5):

$$(KE)_i = \frac{1}{2} \int_{\gamma_i} \mu \dot{r}_i^2 \, dv$$

(2.5)

where $\mu$, $r$, and $v$ denote the density of the link, coordinates, and volume respectively. The potential energy of a link consists of the gravitational and strain energy components, denoted as $(PE)_g$ and $(PE)_s$, respectively. Equations (2.6) and (2.7) describe these two components for a flexible link $i$.

$$(PE)_g = \int_{\gamma_i} \mu r \int_{\gamma_i} r \, dv$$

(2.7)

Consequently, the potential and kinetic energy of the robotic manipulator can be fully defined as the sum of the respective components as stated in equations (2.8) and (2.9):

$$(KE)_L = \sum_{i=1}^{n} KE_i$$

(2.8)

$$(PE)_L = \sum_{i=1}^{n} (PE)_g + (PE)_s$$

(2.9)

where $n=6$ denotes the number of links. This approach allows the subsequent calculation of the equation of motion for complex elastic bodies based on the FE approach.

2.3 Modal Analysis of Robotic Manipulators

Modal analysis [22] is a field which deals with stability of mechanical structures. Due to the general application purpose of articulated manipulators, the stability of their structure under variant load
profiles requires investigation before industrial deployment. However, the complexity of these machines often makes the modal analysis a gruesome task, and therefore a significant number of researches have been conducted with the goal of simplifying and proposing novel effective and computationally feasible approaches for robotic manipulators. In a review by Dwivedy and Eberhard in 2006 [19], 433 papers were studied which focused on modelling and control of the respective dynamic behavior. They concluded that more research is required to fully model the dynamics introduced by the flexibility of the manipulator and internal eigenmodes. The following will introduce the dynamic sub-structuring and component synthesis methods to simplify the calculation of dynamic motions for flexible bodies.

2.3.1 Dynamic Sub-structuring & Component Mode Synthesis (CMS) Methods

The dynamic sub-structuring method is often incorporated when the motion of a whole flexible body needs to be estimated by the motion of the constituting components. This procedure results in simplifying the motion of the flexible body by reducing the number of DOFs as Craig and Kurdila explain in their work [23]. The respective reduction of DOFs is carried out by FE-reduction methods, also known as component synthesis (CMS) methods. It should be noted that computation time is one of the most challenging factors when accurate dynamic simulations are needed. A more complex model would often result in higher computation time which increases the associated costs of the work. Several researches were conducted to reduce the computational time by simplifying complex models and equations for computational studies. Guyan [5] presented one of the most widely-used CMS method in which the dynamics of the system are reduced by classifying the degrees of freedom (DOF) for a system in the two subclasses of master and slave DOFs. The procedure results in a lower number of external DOFs which reduce the computational time and therefore transform the original system into a reduced representation. In a similar work by Craig and Bampton [6], an approach for reducing the number of DOFs for a whole system was introduced based on the modal analysis of consistent substructure representations of the original model. In their words: “Provision is made, through a Rayleigh-Ritz procedure, for reducing the total number of degrees of freedom of a structure while retaining accurate description of its dynamic behavior.”
By implementing such an approach, the dynamic motion of a flexible body can be expressed as a simplified system of equation consisting of masses, springs and dampers. Equation describes the motion of a system of bodies; where $M$, $C$, $K$, and $F$ are the mass, damping, stiffness and force matrices while $\xi$ represents the degrees of freedom.

$$M \ddot{\xi} + C \dot{\xi} + K \xi = F \tag{2.10}$$

The damping matrix $C$ is often neglected in modelling dynamics of rigid bodies due to its low influence in the equation of motion as well as the difficulty of parameter estimation for this property. Therefore, equation (2.11) can be conveniently achieved from equation (2.10) to describe the motion of a substructure in a multi-body model [23].

$$M \ddot{\xi} + K \xi = r + f \tag{2.11}$$

The component $F$ has been substituted by the subcomponents $r$ and $f$ which represent the internal and external forces respectively. It should be noted that the mass and stiffness matrices are time-invariant and rely on the geometrical properties of the flexible body. One of the approaches towards modelling these parameters is to calculate the values numerically using FE approaches coupled with singular value decomposition as in Hardeman et al.’s [24] work. In order to achieve the respective decomposition, the complex structure of the body is divided into the constituting substructures for modal analysis as seen in Figure 2.5 [22]. The substructures are then connected through FE interface nodes, which allow a complete solution for the motion of the whole body. Previous studies have shown that selecting more nodes along the interface increases calculation accuracy at the cost of increased computation time [13].
By implementing the dynamic sub-structuring approach, the following definition in equation (2.12) can be described for robotic manipulators [24]:

\[ x = T^{(x)}(e^{(m)}) \]  

(2.12)

where \( x \) describes the nodal coordinates and \( e \) represents the joint angles. The transfer \( T \) is therefore a dynamic function which can be achieved in a numerical or analytic fashion. Accordingly, the nodal velocities and accelerations can be defined as in equations (2.13) and (2.14):

\[ \dot{x} = DT^{(x)} \dot{e}^{(m)} \]  

(2.13)

\[ \ddot{x} = DT^{(x)} \dot{e}^{(m)} + (D^{2}T^{(x)}) \ddot{e}^{(m)} \]  

(2.14)

where the geometric transfer function \( T^{(x)} \) and its derivatives are estimated in an iterative process during FE calculations.

In order to implement a CMS method, the following coordinate transformation can be implemented on equation (2.11):
\[ \zeta = R\bar{\zeta} \quad (2.15) \]

where \( R \) represents the Ritz vectors that are chosen by the CMS method. By incorporating such transformation, equation (2.11) can be rewritten as:

\[ \bar{M}_{el} \ddot{\bar{\zeta}} + \bar{K}_{el} \bar{\zeta} = \bar{r} + \bar{f} \quad (2.16) \]

where \( \bar{M}_{el} = R^T M R \), \( \bar{K}_{el} = R^T K R \), \( \bar{f} = R^T f \) and \( \bar{r} = R^T r \) represent the reduced properties of the dynamic system.

The CMS methods often operate based on dividing the DOFs of the system into the master and slave subcategories. This procedure allows the internal dynamics of the system to be derived based on the external DOFs, thus the recommendations for CMS are to define the master and slave DOFs as the external and internal behaviors. By utilizing such approach, the system defined in equation (2.16) can be redefined as:

\[
\begin{bmatrix}
M_{mm} & M_{ms} \\
M_{sm} & M_{ss}
\end{bmatrix}
\begin{bmatrix}
\ddot{\zeta}_m \\
\ddot{\zeta}_s
\end{bmatrix}
+
\begin{bmatrix}
K_{mm} & K_{ms} \\
K_{sm} & K_{ss}
\end{bmatrix}
\begin{bmatrix}
\zeta_m \\
\zeta_s
\end{bmatrix}
=
\begin{bmatrix}
f_m \\
0
\end{bmatrix}
+
\begin{bmatrix}
r_m \\
r_s
\end{bmatrix}
\quad (2.17)
\]

where \( m \) and \( s \) represent the master and slave DOF notations. In this study, a Guyan reduction method [5] is implemented to achieve the reduced-order robot model. The derivation of the slave DOFs based on the masters in this approach can be achieved by assuming \( f_s = r_s = 0 \), which yields:

\[ y_s = -K_{ss}^{-1}(M_{sm}\ddot{y}_m + M_{ss}\ddot{y}_s K_{sm} y_m) \quad (2.18) \]

While the Guyan method is one of the most widely-accepted CMS methods, the following should be taken into consideration when implementing such an approach [23]:

- While the static solution of the Guyan-reduced model is as accurate as the full dynamic model, the accuracy weaves off as the excitation frequency of the system is increased. The accuracy of the system in this case depends on the number and selection of the master nodes (representing the FE DOFs). It should be noted that an increased number of master nodes increases accuracy and calculation time for the dynamic model.
The Guyan reduction may lead to a more expensive eigensolution as it destroys the sparseness of the mass and spring matrices despite reducing the number of DOFs [13]. The resulting reduced-order flexible-body model can be implemented to achieve an affordable, yet satisfactory solution to complex dynamics of multi-bodies such as industrial robotic arms as is one of the goals of this study.

2.4 **Contact Applications Involving Robotic Manipulators**

This section describes the theoretical background and past approaches towards machining with industrial robotic manipulators. Furthermore, the sources of errors in such operations will be identified based on literature research and a list of methods will be prescribed towards mitigating the adverse effects of the respective sources.

2.4.1 **A Review on General Machining Behavior and Related Definitions**

Machining is perhaps the most widely-used contact application with need of automation in the industry. Machining processes can be defined as any sort of process where a product with a desired final shape and size is realized by a controlled material-removal contact mechanism. One of the common forms of machining is created by a relative motion between a rotary workpiece and stationary tool, and vice versa, as seen in Figure 2.6.

![Figure 2.6: Front view of facing while turning [25]](image)

As machining processes rely on causing material removal, they result in high contact forces acting on the work piece and the milling machine based on the type of material. These forces can introduce various static and dynamic responses from the involved systems which affect the accuracy and
precision of the machining process and inherently the final quality of the product. One of the recognized sources of error in these processes is chatter [26], which can be defined as the self-induced oscillatory behavior of the system during a machining process. Quintana and Ciurana [26] mention the following as the negative effects of chatter on product quality:

- Poor surface quality.
- Unacceptable inaccuracy.
- Excessive noise.
- Disproportionate tool wear.
- Machine tool damage.
- Reduced material removal rate (MRR).
- Increased costs in terms of production time.
- Waste of materials.
- Waste of energy.
- Environmental impact in terms of materials and energy.
- Costs of recycling, reprocessing or dumping non-valid final parts to recycling points.

Chatter can be classified into the following categories based on the source of the dynamic excitation and nature of oscillations:

1. *Frictional chatter*: The slip-stick force between the tool and the workpiece can cause oscillations which are commonly called the "dry friction" response amongst machinists. Lubricating the system can minimize this effect and create stability.

2. *Regenerative chatter*: The most common and researched source of chatter for the past sixty years [26], this type of chatter is caused by the interaction of the approaching tooth and the wavy surface created on the workpiece by the returning tool as depicted in Figure 2.7.

3. *Mode-coupling chatter*: This type of chatter is caused by the coupling of two vibration modes during operation as portrayed in Figure 2.8 [27]. The system vibrates at the resonance frequency in this case, making mode-coupling chatter hard to distinguish from external force excitations of the system.
4. *Thermomechanical chatter*: Induced by the thermal expansion and contraction of the tool or workpiece during machining processes, these effects often become more apparent when extreme operation parameters such as depth of cut are considered.

![Figure 2.7: Regenerative chatter caused by the interaction between the teeth of the tool and the workpiece [26]](image)

*Figure 2.7: Regenerative chatter caused by the interaction between the teeth of the tool and the workpiece [26]*

![Figure 2.8: Diagram of 2D mode coupling chatter [27]](image)

*Figure 2.8: Diagram of 2D mode coupling chatter [27]*
2.4.2 Previous Studies on Contact Applications with Robotic Manipulators

There have been recent attempts in modelling robot manipulators utilized for contact applications. In a work by Garnier et al. [28], a theoretical model was introduced based on the kinematics of a robotic manipulator and the dynamic parameters of a drilling process. Although the model was then simulated in MATLAB, it was not validated with experimental data to support the conclusions. In another similar work by Mariappan and Veerabathiran [29], the kinematics and dynamics of a robotic manipulator were modeled for a drilling application and simulated in SimScape and MATLAB. The study also suffered from not considering the experimental aspect in the scope of the work. Yin et al. [30] evaluated the performance of a robotic manipulator for a drilling application through theoretical Cartesian compliance modelling and experiments. The results demonstrated the need for further studies and modelling to improve the accuracy of contact applications with robotic manipulators and reduce chatter. In a study by Devlieg [31], a robotic manipulator was combined with CNC control to achieve precise drilling operations with errors in the order of ±0.25mm. The results showed the possibility of implementation of machining mechanisms on manipulators with a closed-loop control. These studies support the objectives of the COMACH project.

While previous studies suggest that such operations will be more cost- and time-efficient when performed in an elaborate procedure with an industrial manipulator [32], almost no level of industrial development with such combination has seen the light of day. Pandremonos et al. [33] take a critical vantage point towards machining with robotic manipulators in their paper “Machining With Robots: A Critical Review” to depict the underlying reasons for the complexity of development of such operations. They point out that while industrial manipulators are superior to conventional CNC machines due to their extended flexibility and operational range from a conceptual point of view, they suffer from accuracy issues, vibrations (chatter) and complex programming and structural design.

In a series of works by ABB Corporate Research [34], the complexity of combined robotic machining operations were thoroughly studied by constructing a combined robot-spindle mechanism as illustrated in Figure 2.9. The authors observed severe low-frequency chatter (≈10Hz) as depicted in Figure 2.10 which led to their assumption for the dominance of mode-coupling chatter in the respective experimental setup. They then derived theoretical stability criteria for assessing the
occurrence of mode-coupling chatter in machining processes with industrial robots to explain the instability of the experiments.

In another experimental work by Cen and Melkote [35] who utilized a similar combined system with a KUKA KR210, low-frequency oscillations were observed due to the resonance of the system. In a work by Bu et al., chatter was observed when drilling with a KUKA KR500-2 robot as seen in Figure 2.11.

![Image of IRB6400 robot](image.png)

*Figure 2.9: The IRB6400 robot used by ABB Corporate Research for the machining process [36]*

![Graph showing cutting forces](graph.png)

*Figure 2.10: Chatter occurrence during a machining process with IRB6400 [36]*
As seen in the reviewed literature, the quality of the resulting product when using a combined robot-spindle system is inferior to conventional CNC machines. Therefore, one would need to model and analyze the underlying mechanical behaviors of the system to pinpoint the sources of errors and develop methods for reducing their influence in real-life behavior. The following section will introduce the theoretical models developed to model the respective combined system.

Figure 2.11: Results of machining from Bu et al.'s work [37]
2.4.3 **Identification of Sources of Errors**

This section presents a summary of the identified sources of error for robotic contact applications based on previous literature and findings. Furthermore, approaches and solutions will be put forward for each source to provide an organized structure towards reducing the errors associated with such processes.

The following are the major sources of error in robotic contact applications based on the reviewed literature and theoretical models:

- **Links & Joints Compliance**: The compliance of links and joints depends on the configuration of the robot due to the nonlinear behavior of the torsional stiffness of the gearbox and drive shafts actuating the joints [38] and the change in the dynamic properties of the robot such as the effective mass and stiffness matrices. Moreover, the inertial and external loads, coupled with gravitational effects can increase the loading on joints. In the words of Schneider [39]: “Link and joint compliance, causing the deflection of the links and finally the TCP, contribute up to 8-10% of the position and orientation errors of the TCP”. While the value may not be certain in this case, the effect is well-known and can be evaluated by simulations and experimental assessments.

- **Backlash & Hysteresis**: These properties can be traced back to the imperfections and deviations of a developed gearbox from an ideal model. Backlash, or commonly called “Play” is caused by the looseness of the gears during the mating process; specially when a preload is not present which allows a range of free motion near the unloaded operational condition. While this error is considerable and is more significant than geometric tolerances [40], it can be reduced by calibrating the robot. Hysteresis is a property which is commonly referred to as the “Memory” of the gearbox, which conveys the nonlinear closed-loop behavior of the gear when loaded in a range as seen in Figure 2.12 [41]. While these effects will remain present after calibration, they can be observed in laboratory experiments and added to the gearbox models of the manipulator.

- **Chatter**: Chatter can be defined as the negative impact of self-excited vibrations on the system during contact applications which involve dynamic loads as discussed in previous
sections [26]. The resulting effects of chatter include loss of accuracy and poor surface quality which are critical to operational success. As a result, this phenomenon needs to be predicted based on observations and sophisticated models involving the robot and contact application which will be a subject of discussion in the further chapters.

- **Cell Environment**: Environmental disturbances can induce unforeseen effects during the machining process. An ideal cell provides an extremely stiff mounting for the robot (such as a concrete block) which is securely mounted on the floor of the basement level. Moreover, the operation cell must be thermally isolated for sensitive applications such as contact operations to avoid unforeseen thermal effects during performance or data analysis [39].

- **Geometric Tolerances and Wear**: The physical tolerances in the manipulator arise from the two main sources of production tolerances (including manufacturing and machining) and wear due to extensive use. Even the smallest error in the joint angles or link tolerances can lead to a high error at the TCP position due to the length of the links. However, these errors can be reduced or compensated for by kinematic calibration which then modifies the DH parameters of the robot [33, 39].

- **Thermo-mechanical Effects**: These effects are caused by the energy dissipation from the material and the tool at the contact point and heavily rely on the setup, including the feed rate, depth of cut and the rotational velocity of the spindle. While these errors are persistent across any experiment; they are not in the scope of the current work due to the focus of the study on more dominant errors [33, 39].

Table 1 summarizes the presented main sources of errors and proposed approaches towards solutions for assessing or reducing the inherent negative impact on contact applications.
### Table 1: List of Errors and Proposed Solutions for Machining Processes

<table>
<thead>
<tr>
<th>Error Source/Type</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Links &amp; Joints Compliance</td>
<td>Developing Accurate Predictive Models</td>
</tr>
<tr>
<td>Backlash &amp; Hysteresis</td>
<td>Modelling &amp; Experimental Testing</td>
</tr>
<tr>
<td>Chatter</td>
<td>Experimental Testing &amp; Model Development</td>
</tr>
<tr>
<td>Cell Environment</td>
<td>Cell Calibration</td>
</tr>
<tr>
<td>Geometric Tolerances and Wear</td>
<td>Kinematic Calibration</td>
</tr>
<tr>
<td>Thermo-Mechanical Effects</td>
<td>Isolating the Operational Environment</td>
</tr>
</tbody>
</table>

*Figure 2.12: Hysteresis response modelling and observations of a robotic manipulator [41]*
2.4.4 **Theoretical Modelling**

The robotic machining process can be modeled by assessing the behavior of the combined system based on the following two main subsystems:

1. The behavior and response of the individual robotic manipulator
2. The influences of the machining process on the combined system

The dynamics and mechanics of the robotic manipulator have been studied previously as discussed before [13, 22]. The machining process response can be defined based on the observations of Pan et al. [36] and a derivation from Stone [25] which yields:

\[
m_1 \frac{d^2 x_1(t)}{dt^2} + c_1 \frac{dx_1(t)}{dt} + k_1 x_1(t) = -Rb(x_1(t) \cos \alpha + x_2(t) \sin \alpha) \cos (\beta - \alpha),
\]

\[
m_2 \frac{d^2 x_2(t)}{dt^2} + c_2 \frac{dx_2(t)}{dt} + k_2 x_2(t) = Rb(x_1(t) \cos \alpha + x_2(t) \sin \alpha) \sin (\beta - \alpha),
\]

where equations (2.19) and (2.20) represent the oscillations of the tool in the two main modal directions as seen in Figure 2.8. \( b \) is the contact length and \( R \) is the constant cutting force coefficient. By following this depiction, the depth of cut \((d)\) can be simply modeled as:

\[
d = x_1(t) \cos \alpha + x_2(t) \sin \alpha
\]

which models the mode-coupling chatter observations of Pan et al. [36] in a theoretical fashion. The range of motion increases exponentially when chatter occurs and therefore the depth of cut would change in an oscillatory motion which leads to the chatter marks as observed in Figure 2.13.

*Figure 2.13: Chatter observations in Pan et al.’s experiments [36]*
Chapter 3

3 Methodology

This chapter focuses on presenting the simulation and experimental setup and approach towards analysis of the results in the current study. For this purpose, the chapter consists of the following sections:

1. Dynamic Response Analysis of Robotic Manipulators
2. Static Response Analysis of Robotic Manipulators
3. Development of Contact Application Models in Dymola
4. Experimental Setup & Testing

It should be noted that the major part of mechanical simulations of this work were performed in Dymola (Dynamic Modeling Laboratory [42]) which provides the basis for assessing complex integrated mechanical bodies such as robotic manipulators. Post-processing of data is performed in MATLAB [43] to visualize the desired simulation and experimental results. The following will present the methodology of analyzing the response behavior of the robot.

3.1 Dynamic Response Analysis of The Robot

Industrial robots have been traditionally associated with low-frequency resonance behavior which ultimately became a pressing disadvantage when dealing with dynamic excitations [32]. Henceforth, assessment of the first resonance frequency for the robot was of crucial importance for stability analysis of contact applications. Previous studies observed a very low-frequency (~10 Hz) oscillatory behavior for a manipulator during contact applications [36].

The manipulator was analyzed by linearizing the system as a state-space system according to equation (3.1):
\[ \dot{X} = AX + Bu \]
\[ Y = CX + Du \]  
(3.1)

where \( X \), \( Y \), and \( u \) depict the assumed system states (DOFs), outputs, and inputs, respectively. \( A \), \( B \), \( C \), and \( D \) are constant matrices that are calculated through linearization of the states in Dymola. This procedure allows for the calculation of the static gain as well as the dynamic properties of the robot such as the eigen frequencies at a given robot pose. The following section will describe the method used to calculate the eigen frequencies of the manipulator.

### 3.1.1 Evaluation of The Eigen frequencies & Dynamic Gain Calculation

Eigen frequencies of the robotic manipulators were of high importance in this study due to the presence of dynamic excitations in a number of contact applications. In order to visualize the dynamic behavior of the robot according to the models, Bode plots of the linearized system at a given pose were produced. These plots provide an accurate depiction of the estimated behavior based on transfer functions, where a single transfer function can be defined as \( TF = \frac{\text{output}}{\text{input}} \). By achieving the desired linearized matrices \( Y \) and \( X \), one can achieve a full system of transfer functions according to equation (3.2):

\[ SYS = TF_{\text{output-input}} \]  
(3.2)

Therefore, the full dynamic response of the system can be estimated by a bode plot with the size of \( TF \). As a result, the eigen frequencies can be observed in the bode plot by drastically increased values on the output side. It should be noted that while his approach is only valid for the calculated pose of the robot, the first eigen frequency of robotic manipulators is commonly known to occur at low frequencies.

### 3.2 Static Response Analysis of Robotic Manipulators

This section delves into the utilization of the Dymola models for calculating the static-response of the manipulator. One of the main properties of concern when evaluating the performance and precision of robotic manipulators is TCP stiffness. This parameter can be derived based on Hooke’s law of elasticity by depicting the TCP as a 6DOF spring with elastic properties based on the robotic manipulator. As a result, equations (3.3) and (3.4) can be stated for the displacement and orientation
of the TCP under a certain wrench, neglecting the coupling terms which are the non-diagonal terms in the stiffness matrix:

\[
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} =
\begin{bmatrix}
K_{xx} & K_{xy} & K_{xz} \\
K_{yx} & K_{yy} & K_{yz} \\
K_{zx} & K_{zy} & K_{zz}
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix}
\]

(3.3)

\[
\begin{bmatrix}
T_x \\
T_y \\
T_z
\end{bmatrix} =
\begin{bmatrix}
K_{rx} & K_{ry} & K_{rz} \\
K_{rx} & K_{ry} & K_{rz} \\
K_{rx} & K_{ry} & K_{rz}
\end{bmatrix}
\begin{bmatrix}
\Delta \theta_x \\
\Delta \theta_y \\
\Delta \theta_z
\end{bmatrix}
\]

(3.4)

Under small deformations, the Hooke’s law stays accurate for describing the static behavior of the manipulator, as the deflections are assumed to be linearly elastic. As the identification of the rotational stiffness matrix is not trivial and not in the scope of the current work, the latter parts will mainly focus on the displacement stiffness matrix. The stiffness matrix can then be defined as the following in equation (3.5):

\[
K =
\begin{bmatrix}
K_{xx} & K_{xy} & K_{xz} \\
K_{yx} & K_{yy} & K_{yz} \\
K_{zx} & K_{zy} & K_{zz}
\end{bmatrix}
\]

(3.5)

where the diagonal terms are commonly referred to as the “Axial Stiffness” values and the non-diagonal terms are termed the “Coupling Stiffness” values [44]. Hence, the compliance matrix can be calculated according to equation (3.6):

\[
C = K^{-1} =
\begin{bmatrix}
C_{11} & C_{21} & C_{31} \\
C_{21} & C_{22} & C_{23} \\
C_{31} & C_{32} & C_{33}
\end{bmatrix}
\]

(3.6)

The following section will elaborate on the concepts of static gains and axial stiffnesses for robotic manipulators and their method of implementation.

### 3.2.1 Static Gain & Axial TCP Compliance Evaluation

The concept of static gains was born with transfer functions where the static gain of a transfer function can be defined as in equation (3.7):

\[
G = TF_{(s \rightarrow 0)}
\]

(3.7)
which conveys the meaning of the system gain in the presence of a unit static input. By assuming the displacements in the axial directions as outputs, the components of the static gain can be derived as:

\[ G_y^x = \frac{y}{x} \]  

(3.8)

It is common practice in structural mechanics to neglect the coupling terms of the stiffness matrix and introduce a completely diagonal matrix for stiffness as in equation (3.9):

\[ K = \begin{bmatrix} K_{xx} & 0 & 0 \\ 0 & K_{yy} & 0 \\ 0 & 0 & K_{zz} \end{bmatrix} \]  

(3.9)

This allows a simpler one-to-one transformation between the deflection and stiffness values. By assuming a linearized system to be present, the components of the compliance matrix in equation (3.10) are equal to the static gains when a unit force in each direction is exerted on the system. This allows the calculation of the stiffness matrix based on a linearized state-space representation of the model which yields:

\[ C = K^{-1} = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} G_{Fx}^{Ax} & 0 & 0 \\ 0 & G_{Fy}^{Ay} & 0 \\ 0 & 0 & G_{Fz}^{Az} \end{bmatrix} \]  

(3.10)

This assumption leads to equation (3.11) for the compliance matrix:

\[ C = K^{-1} = \begin{bmatrix} C_{11} & 0 & 0 \\ 0 & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} = \begin{bmatrix} K_{xx}^{-1} & 0 & 0 \\ 0 & K_{yy}^{-1} & 0 \\ 0 & 0 & K_{zz}^{-1} \end{bmatrix} \]  

(3.11)

In this case, the compliance matrix values \( C_{11}, C_{22}, \text{ and } C_{33} \) are equal to the static gain components of the linearized system from force to displacement. However, more complex structures such as robotic manipulators behave in a nonlinear fashion and therefore do not support neglection of the coupling terms. This assumption will be analyzed further in the results chapter.

### 3.2.2 Mapping of the 3D Joint-Space of The Robot With Respect to Axial TCP Stiffness

The key role of axial TCP stiffness has so far been deduced from the theory. The linearization approach allows automated computation of the system properties in desired poses and
configurations. By defining the configuration of the manipulator by inputting the six motor positions (defined by the encoder readings) or the joint angles (defined by the angle of mechanical joints), it is possible to linearize the resulting system at different points in the joint-space to map the stiffness values to their respective joint values. Figure 3.1 describes the process flow from the desired robot configurations to generation of axial stiffness data using MATLAB and Dymola. The following sections will describe the automation process between MATLAB and Dymola for data transfer in more detail.

![Diagram](image)

*Figure 3.1: Architecture of work flow for joint-space stiffness calculations*

**Interfacing MATLAB and Dymola**

Dymola is based on the Modelica coding language which allows external scripts to call the functions with input arguments. Dymola scripts can then be produced by MATLAB for calculations to be carried out according to the inputs as visualized in the diagram in Figure 3.1. The following functions were used in Dymola for producing the results:
• *DymolaCommands.SimulatorAPI.linearizeModel:* Linearize the Dymola model into a state-space representation according to equation (3.1). The outputs are produced in the form of the $A$, $B$, $C$, and $D$ matrices.

• *DymolaCommands.SimulatorAPI.simulateExtendedModel:* This command allows the user to simulate the model by specifying the start and end time; as well as recording the desired outputs into an array. This command also takes in the required input parameter values which in this study were defined as the joint angles of the robot.

• *writeMatrix:* This function allows the user to write existing variables in the workspace to a file for data transfer between software. In this case, the axial deflections at the TCP were recorded with respect to the input force values.

It should be noted that robot configurations were defined as arrays containing the joint angle values in MATLAB. Then the `sprintf` MATLAB function was used to print the desired script into a file. The Dymola data were produced in the form of .m MATLAB readable files and were imported into MATLAB as arrays of data. The joint-space of the joints 2 and 3 was discretized with respect to the mechanical limits and the calculated deflection values were input into Dymola. This allows a mapping of the joint-space for joints 2 and 3 to be produced based on Dymola simulations in MATLAB. The visualization of the joint-space data was performed by creating a surface of the stiffness components with respect to the motor angles. The components of the stiffness matrix were then calculated according to equation (3.5) which will be discussed further in the next section.
3.2.3 **Evaluation of The TCP Stiffness Matrix for Robotic Manipulators**

The stiffness and compliance matrices are of primary concern when modelling the mechanics of robotic manipulators. This is due to their influence on the dynamic and static behaviors of the system, including TCP accuracy and precision, and resonance frequency. TCP stiffness can be defined by taking the applied forces and deflections at the TCP into account to model the accuracy of the robot under various loads.

In order to calculate the TCP stiffness matrix with accordance to equations $(3.3)$ and $(3.5)$, the experiments can be designed with three arbitrary load direction and magnitudes that are non-linear multiples of one and another to produce a valid system of equations for computing the stiffness components. This leads to the following description in equation $(3.12)$:

\[
\begin{bmatrix}
F_x \\
F_y \\
F_z \\
F_x \\
F_y \\
F_z \\
F_x \\
F_y \\
F_z \\
F_x \\
F_y \\
F_z
\end{bmatrix} =
\begin{bmatrix}
K_{xx} & K_{xy} & K_{xz} \\
K_{yx} & K_{yy} & K_{yz} \\
K_{zx} & K_{zy} & K_{zz} \\
K_{xx} & K_{xy} & K_{xz} \\
K_{yx} & K_{yy} & K_{yz} \\
K_{zx} & K_{zy} & K_{zz} \\
K_{xx} & K_{xy} & K_{xz} \\
K_{yx} & K_{yy} & K_{yz} \\
K_{zx} & K_{zy} & K_{zz}
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z \\
\Delta x \\
\Delta y \\
\Delta z \\
\Delta x \\
\Delta y \\
\Delta z \\
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix}
\]

which can be shortly described as in equation $(3.13)$:

\[
F_{\text{exp}} = K\Delta x_{\text{exp}}
\]

This system of equations can then be solved using a simple computational software such as MATLAB for achieving the stiffness components. It should be noted that the resulting stiffness matrix is a linearized description of the system behavior and therefore depends on the orientation and position of the TCP. This method is applicable to simulations and real-life experiments as it only depends on the knowledge of exerted force values and deflections.
It should be noted that while the evaluation of TCP stiffness with this method is convenient, the calculated values are only applicable to the joint-space values used for the initial setup. Producing a detailed mapping of the joint-space can lead to a satisfactory estimation of the robot behavior in the workspace as discussed in the previous section. The following section will focus on modelling contact applications utilizing a robotic manipulator to provide a deeper understanding of the combined operation behavior.

### 3.3 Development of Contact Application Models in Dymola

Contact applications can be modeled based on the results of the literature survey and previous experiments involving robotic manipulators. In this study, the interaction of the robot TCP with the workpiece is modeled according to Figure 2.8 with the goal of modelling the chatter behavior of the robot and assessing the dynamic behavior of the TCP in these applications. The following models are created to provide a low-level and high-level understanding of the architecture of interactions between the tool and the workpiece in chatter behavior:

- **Combined 1-DOF model:** This model was made according to equations (2.19) and (2.20), with the oscillations being divided into two respective DOFs. This model allows the understanding of low-level interactions and provides more insight into the nature of self-excited vibrations in contact applications.

- **2-DOF model:** This model was constructed with respect to Figure 2.8 as an abstract model to visualize and provide a higher-level understanding of mode-coupling chatter. The movement of the TCP and interactions of the two modes can be seen in the oscillation plane in this model.

The objective of this study was to measure and quantify the influencing properties on system behavior. As noted by previous authors [36], the lower stiffness value of the manipulator when compared to CNC machine is of concern in contact applications and stability criteria should be derived based on modelling and simulations. Therefore, the produced models were used to evaluate and estimate the stability of the robot based in contact applications. Figure 3.2 and Figure 3.3 show the 1D and 2D contact models in Dymola respectively.
While modelling often provides accurate estimations and can lead to an earlier evaluation during the design phase, verification and validation of the models are as significant as the models themselves. Therefore, several experiments were designed and set up to provide data for the verification of the
respective models. The following section describes the involved setup equipment and sets of experimental evaluation for this purpose.

3.4 Experimental Setup & Testing

The experiments were designed and carried out in the ABB laboratories with the objective of providing reference data for the models. This phase was divided into the following sets of experiments which measure different robot properties:

- Static measurement series of the manipulator: These experiments were designed to evaluate the TCP stiffness and deflection values under various magnitudes and directions of static forces.
- Quasi-static measurement series of the manipulator: These investigations were planned to evaluate the variations in TCP properties while following a pre-defined path in a quasi-static manner [45].
- Dynamic measurement series of the manipulator: These experiments were carried out to confirm the dynamic behavior of the robot against oscillatory excitations.

As experimental measurement series often yield a high amount of raw data, data analysis was an important aspect in this study as the quality of the final results relied directly on the analysis process. MATLAB (R2019a) was employed for the purpose of raw data analyses and to also stimulate a smart procedure for running the Dymola models. The following subsections will describe the mentioned experimental procedures and utilized setup in more detail to provide a comprehensive overview of the measurement and approach strategy.

*Quasi-static behavior can be defined as motion at speeds where the acting frequency of inertial forces can be assumed to be zero or lower than 1 Hz.
3.4.1 Static Measurement Series of The Manipulator

One of the key properties for assessing the behavior of the robot in contact applications is static accuracy. This term can be defined as the accuracy of the robot after when none of the terms in equation (2.10) are time-dependent. In real life, this conveys the meaning of no mechanical movement in the involved components [46]. The measurement series were conducted with the goal of assessing TCP accuracy and stiffness under different static loading conditions. To achieve this, the robot was tested in three configurations which aimed in various points in the workspace with respect to Figure 3.4, Figure 3.5 and Figure 3.6. Configurations 1 and 2 were chosen near the boundaries of the workspace as extremities and configuration 3 was selected as a typical operational configuration. Table 2 presents the joint values for the selected configurations in the workspace.

Table 2: Joint angles for measurement configurations
### Table 1: Experimental Setup & Testing Configuration

<table>
<thead>
<tr>
<th>Config. No.</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
<th>$\theta_5$</th>
<th>$\theta_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>52</td>
<td>-19</td>
<td>12</td>
<td>-15</td>
<td>-11</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>66</td>
<td>0</td>
<td>-66</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>32</td>
<td>20</td>
<td>101</td>
<td>-98</td>
<td>-35</td>
</tr>
</tbody>
</table>

In order to construct the experimental setup, a loaded double ball bar (LDBB) system [47] was used as the external loading mechanism as seen in Figure 3.7. The LDBB is comprised of a pressured air inlet and a moving piston which allows external loads of 100-800 N to be applied at the connection point. The adapter for LDBB was attached to mounting plates to secure a stiff connection to the ground. The mounting plates were constructed as stacks of a steel alloy to validate the stiffness of the setup. This modular development facilitated the quick change and construction of secure mountings for the test configurations which can be seen in Figure 3.8. The cell temperature was recorded to guarantee thermal stability and the robot underwent a startup cycle of 2 hours prior to testing. The turning disc of the robot was equipped with a custom-designed adapter for attachment to the LDBB as seen in Figure 3.9.
Figure 3.7: Experimental setup for the static measurement series

Figure 3.8: Stackable mounting plates used for creating a secure ground connection
In order to calculate the deflections and changes in coordinates, a Leica Absolute Tracker AT960 system was employed in conjunction with a SMR mounted on the robot as seen in Figure 3.10. The base uncertainty level of this tracker is 15 µm for the given experimental setup [48], which is coherent with the Golden rule of measurement regarding test uncertainty ratio (TUR 10:1) [49]. The robot was tested in the “Motors ON” state which releases the mechanical brakes in the joints as in a real operation. The TCP deflections were then calculated based on the measurements of the tracker and transferred to MATLAB for data processing. The joint values of the robot were recorded at each configuration for replication of the experiments in simulation via the Dymola models. This procedure ultimately allows the results from measurements and simulations to be compared and to provide reference data for improving the robot models in Dymola based on static accuracy.
Table 3 tabulates the testing conditions for the performed static measurements. It should be noted that the experiments were conducted at constant values of external force ranging from zero up to 800N to cover the payload range of the robot.

Table 3: Overview of testing conditions for static measurements

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Direction of Measurement (In Base Coordinate System)</th>
<th>External Force Values (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X, Y, Z</td>
<td>0 – 800</td>
</tr>
<tr>
<td>2</td>
<td>X, Y, Z</td>
<td>0 – 800</td>
</tr>
<tr>
<td>3</td>
<td>Y</td>
<td>0 – 800</td>
</tr>
</tbody>
</table>
3.4.2 Quasi-Static Measurement Series of The Manipulator

These series of experiments were designed based on the traditional double ball bar testing of machining equipment [50] with the goal of evaluating the compliance of the TCP when loaded along a circular trajectory. This allows the path accuracy of the robot to be evaluated during operation and therefore assess the coherence of the models with reality. Figure 3.11 shows the experimental setup for these series which is similar to the previous articulation. The circle trajectory was chosen along the X-Y plane in the global coordinate system and the LDBB was used as the mechanism for external loading. The trajectory was created in ABB RobotStudio and input to the controller as a RAPID code which can be seen in Figure 3.12 and Appendix B.

The Leica laser tracker was used to log the position of the robot during the experiments. The recorded positions were then translated back to joint positions using inverse kinematics with a DH approach. The resulting joint positions were recorded and transferred to MATLAB for visualization and data analysis. It should be noted that these sets of experiments were also conducted at loads between zero and 800 N for assessing the path accuracy of the robot along a planar trajectory. In order to replicate the same experiments in Dymola, the movement along the designed path was logged using the controller signals in ABB TuneMaster. The resulting motor positions were put in the Dymola models to recreate the trajectory using the simulations and perform a comparison between real life and simulation data.
Figure 3.11: Setup for the quasi-static measurement series

Figure 3.12: RobotStudio model for quasi-static testing
3.4.3 **Dynamic Measurement Series of The Manipulator**

These series of measurements were based on assessing the eigen frequencies of the robot to predict chatter occurrence and behavior in contact applications. Previous research suggested that the first eigen frequency of the robot is placed in the low-frequency bandwidth (~10Hz) which causes extreme oscillatory behavior when dynamic excitation is exerted on the manipulator [36]. The experimental setup is similar to Figure 3.7 for these series. However, in this case, the LDBB was also equipped with two piezo actuators which were able to provide a dynamic excitation force to the system. The experiments were then repeated by replacing the LDBB mechanism with an LMS shaker as seen in Figure 3.13 for comparison between the produced results.

![Image](image.png)

*Figure 3.13: The LMS shaker used for the dynamic measurement series*

Table 4 lists the overview of the dynamic test conditions and setup for each configuration. The robot was statically preloaded to secure the LDBB connection and provide torque on the joints. Also, the nonlinear behavior of the gearboxes would be examined in this manner as the robot is estimated to behave differently under various static loads.
Table 4: Overview of the tests for the dynamic measurement series

<table>
<thead>
<tr>
<th>Test. No.</th>
<th>Config. No.</th>
<th>$F_{x,\text{stat}}/F_{x,\text{dynm}}$</th>
<th>$F_{y,\text{stat}}/F_{y,\text{dynm}}$</th>
<th>$F_{z,\text{stat}}/F_{z,\text{dynm}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>100/20</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>500/20</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>500/20</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>500/20</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>100/20</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>500/20</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>500/20</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>0</td>
<td>500/20</td>
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<tr>
<td>9</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>100/20</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>500/20</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>500/20</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>0</td>
<td>500/20</td>
<td>0</td>
</tr>
</tbody>
</table>
Chapter 4

4 Results & Discussion

This chapter presents the results of the simulations and experiments and follows up by sparking a discussion based on the respective remarks. The chapter is divided into the topics below to make it easier to follow for the reader:

1. Static analysis of the manipulator
2. Quasi-static analysis of the manipulator
3. Dynamic analysis of the manipulator
4. Prediction results for contact behavior of the manipulator

The rest of the chapter will focus on technical scrutiny of the results according to the layout. The objective of this chapter is to provide insight into the underlying mechanisms and interactions for contact applications with robotic manipulators through the final discussion.

4.1 Static Analysis of The Manipulator

This section concentrates on presenting the results of the static measurement series and comparing them with the replicated simulation results in Dymola. This is to assess the TCP stiffness of the manipulator at a given point in the workspace through experimentation and simulation in order to lay a comprehensive comparison structure between the respective two methods of evaluation.
4.1.1 Static TCP Deflection Evaluation

Figure 4.1 presents a sample depiction of the measurement data for the static experiments. It is observed that the deflection trend is nonlinear at lower force values and deviates to a linear behavior as the force increases. Furthermore, the effect of the mechanical couplings can be seen as deformations are observed in all three principal axes under uni-axial loads. This provides evidence that a simplified assumption of a diagonal stiffness matrix leads to high calculation errors for robotic manipulators. Robotic manipulators are traditionally considered as serial articulated connections; however, elements such as a coupled wrist can introduce nonlinearities to the elastic behavior of the manipulator. The average measured deflection is marked with a red line to provide more clarity.

![Figure 4.1: Measurements of static deflection for configuration 1 with uni-axial loading in Y direction](image)

Figure 4.2 presents a sample figure of the deflection measurements from Dymola simulations and laboratory experiments. The full list of figures can be found in Appendix C. The highest measured deflection was more than 6 mm when the robot was loaded in the Y direction in configuration 1. This is in-line with the expectations as the Y direction is assumed to be the least stiff axis for the robot. While the magnitude of error between simulations and experiments varies based on the configuration.
and external force, the relative error in most cases is less than 25%. However, as the absolute error is 0.2 mm, introduction of accurate contact applications would not be advisable based on sole simulations. The average calculated deflection values from experiments and simulations are drawn as blue and red lines respectively. The errors for the simulations are calculated as:

\[
\text{Error}_{\text{abs}} = |\text{Average Measured Deflection} - \text{Average Simulated Deflection}|
\]

(4.1)

\[
\text{Error}_{\text{rel}} = \frac{|\text{Average Measured Deflection} - \text{Average Simulated Deflection}|}{\text{Average Measured Deflection}}
\]

(4.2)

Table 5 presents the magnitudes of absolute errors for the static measurement series. The maximum absolute error value present in the experiments was 0.79 mm. It should be noted that the relative error value is highly dependent on the absolute value of the measurement and can be misleading especially when the measurement has a significantly small value. With respect to the objectives of the COMACH project, the measured levels of accuracy are not satisfactory for accurate contact applications. Another subject of note when observing the static deflections is the sudden rise in deflection magnitude which occurs around exerted force values of 400N. This was suspected to be due to the gravitational effects on the manipulator specially when the lower arm was extended downwards. In other words, the hanging lower arm was lifted by the external force, causing the gears on the joints to switch direction and therefore visualizing the looseness of the gears, or the so-called backlash. This idea was tested by recreating the circumstances in Dymola and checking the motor torque values which confirmed the assumption of motors switching rotation direction. The motor torques for motors 2 and 3 are visualized in Figure 4.3 for configuration 2 as the load is exerted in the Z+ direction.
Table 5: Absolute error values for the static measurement series

<table>
<thead>
<tr>
<th>Configuration No.</th>
<th>Direction of External Force</th>
<th>Absolute Error (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>X</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>Y</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td>0.63</td>
</tr>
<tr>
<td>3</td>
<td>Y</td>
<td>0.21</td>
</tr>
</tbody>
</table>
Figure 4.2: Sample Measurements and Experimental Results for Static Deflection
As can be seen, motors 2 and 3 are switching directions between the +200 N and +500 N force values which correspond to the occurrence of backlash in the gears and a sudden increase in the interpreted deflection values. The following section will focus on the interpretation of the stiffness values from measurements and simulation.

4.1.2 Static TCP Stiffness Evaluation

The TCP stiffness is perhaps one of the most critical mechanical properties when modelling the behavior of a robotic manipulator. This parameter can in most cases define the compliance of the manipulator with respect to the desired objective. As a result, the evaluation of this property has been a main subject in this study. This section will describe the evaluated data and undergone procedure from simulation and experiments.

The stiffness matrix was first evaluated by the simplified assumption of non-existing coupling values to provide a comprehensive comparison between the two stiffness matrices presented in equations (3.5) and (3.9). Figure 4.4 shows a sample of the calculated axial stiffness values from the experiments with respect to equation (3.9). The full list of figures is available in Appendix C. The first subject of note when observing this evaluation is the nonlinear behavior of the axial stiffness values.
While these evaluations often do not follow a specific trend, they agree on the stiffening of the TCP in the loaded direction with an increase in load values. The average axial stiffness values are tabulated and visualized in the bar graph of Figure 4.5.

*Figure 4.4: Measurement of axial stiffness values with the assumption of a diagonal stiffness matrix*
Figure 4.5: Comparison of average axial stiffness values between simulations and experiments
It can be seen that the incoherence between the measurements and simulation are of a high degree in this case. The lowest average error between the measurements and simulations was calculated when the robot was loaded in the Y+ direction. The evaluations of axial stiffness in the X and Z directions with the assumption of a diagonal matrix are therefore not accurate. As a result, the validity of the simplified assumption of a diagonal stiffness matrix is challenged in this case. Even more, the stiffness values do not follow a specific trend and high deformations are often caused in the coupling directions. This led to the full evaluation of the stiffness matrix with respect to equation (3.12).

Figure 4.6 plots an extract of the calculated stiffness values by the uni-axial stiffness (diagonal stiffness matrix) and full stiffness matrix assumptions versus simulation data. It can be seen that the evaluated stiffness values from the simulation and experiments match best when the full stiffness matrix is evaluated. The error is significantly reduced when the coupling terms are considered, with this case resulting in the reduction of relative error from 48.9% to 13.9%.

![Figure 4.6: TCP stiffness evaluation through various methods for the simulations and experiments for configuration 1 when loaded in X+](image)

While these results prove useful whence evaluating the stiffness at a given point in space, they fall short of deriving a mapping of stiffness over the workspace to provide more insight into the selection of the working configuration of the robot before the experimental phase. As a result, in an endeavor
to address the respective shortcoming, the joint-space of the robot was mapped for the operational ranges of joints 2 and 3 which constitute a significant portion of the stiffness of the robot. This will be elaborated on in the next section.

4.1.3 **Evaluating the TCP Stiffness Over the Joint-Space of The Manipulator**

The TCP stiffness of the manipulator was evaluated over the joint space of joints 2 and 3 with the goal of creating a comprehensive reusable map for future users who wish to have a reference for stiffness values over the workspace. This was achieved through extensive Dymola simulations of the manipulator and linearizing the resulting system with respect to the achieved axial deformations and unit force inputs. Figure 4.7, Figure 4.8, and Figure 4.9 portray the mapping of axial stiffness over the mechanical joint-space of joints 2 and 3 in the X, Y and Z principal axes respectively. Likewise, Figure 4.10, Figure 4.11, and Figure 4.12 present the 2D view of the same visualization with a color-bar for improved readability inside the thesis script.

![Figure 4.7: 3D view of joint-space stiffness evaluation in the global X direction](image-url)
Figure 4.8: 3D view of joint-space stiffness evaluation in the global Y direction

Figure 4.9: 3D view of joint-space stiffness evaluation in the global Z direction
Figure 4.10: 2D view of joint-space stiffness evaluation in the global X direction

Figure 4.11: 2D view of joint-space stiffness evaluation in the global Y direction
As measured by the joint-space evaluation, the highest stiffness in the global X direction occurs when joints 2 and 3 are at 100 and -90 degrees respectively. This configuration is visualized in Figure 4.13. The stiffness in the global Y direction is shaped like a saddle, which conveys the meanings of minimizing the bending moment caused around the base of the robot by having the TCP close to the base. A configuration which is based around this idea is portrayed in Figure 4.14.
Figure 4.14: A sample configuration with high stiffness in the global Y direction

Figure 4.15 visualizes the robot in a stiff Z configuration according to Figure 4.12. The sensible conveyance of this result is to have the mass of the manipulator acting against the direction of the exerted force.

Figure 4.15: The stiffest configuration in the global Z direction
A result of note from the joint-space stiffness evaluation is the weakness of the manipulator in the global Y direction in almost all of the configurations. This is mainly due to the acting bending moment on the base of the robot when the robot is loaded. While the stiffness of the robot in the X and Z axes can reach up to values in the order of $10^7$, the stiffness in Y can only extend up to the order of $10^5$, thus making it weaker than the other axes by two orders of magnitude.

The following section will elaborate on the findings of the quasi-static measurements and compare the results with simulation data.

### 4.2 Quasi-Static Analysis of The Manipulator

The quasi-static measurements were conducted with the goal of visualizing the path accuracy of the robot under a loaded configuration. The robot was designed to follow a path as described in Figure 3.12. The TCP positions were then evaluated via three different methods:

- Signals from the virtual controller (VC): The VC provides real-time signals of the TCP position by evaluating the kinematics of the robot using the nominal DH parameters and sensed motor positions.
- Dymola simulations including gravity: Dymola simulations were performed for the same trajectory with the inclusion of the effect of gravity on the robot.
- Experimental measurements: In-lab measurements of the TCP positions using the Leica laser tracker whence following the designed trajectory.

Figure 4.16 pictures a 3D view of the results of the three methods of evaluation. By a simple observation, one can deduce that the original circle path and direct TCP signals from the controller are in-line with each other. The difference in the Z value between the Dymola simulations and VC signals is due to the gravitational effects on the flexible manipulator. The Z variation between the Dymola models and the measurements is thought to be due to the difference between the nominal and actual DH parameters of the robot which is rooted in the wear and geometric tolerances and the difference between the actual and observed base coordinate system origin. Figure 4.17 presents the top-view of the same trajectories. It is evident that the circle lies well within the tolerances of all
measurement methods in the XY plane. The different scales on the three principal axes should be taken into consideration whence viewing the figure.

Figure 4.16: 3D view of the results of the three methods of evaluation for the unloaded QSC measurements

Figure 4.17: Top-view of the three different evaluation methods for the unloaded QSC measurements
The loaded series of experiments were conducted at constant force levels of 0-800 N over the course of the trajectory. Figure 4.18 and Figure 4.19 show the 3D and top-view of the measurement data for the TCP coordinates during the experiments.

![Figure 4.18: 3D view of the TCP position during the experimental QSC measurement series](image)

![Figure 4.19: 2D view of the TCP position during the experimental QSC measurement series](image)
By comparing Figure 4.12 and Figure 4.18 one can deduce the reasoning for the Z variations along the trajectory. The TCP is stiffer when closer to the robot’s base according to the joint-space data which can also be seen in the measurements whom depict a lower deflection in the Z direction when the TCP is closer to the origin of the system. Figure 4.20 provides a magnified view of the TCP position during the experiments. It can be seen that the XY planar deflection increases with force as expected.

![Figure 4.20: Magnified view of the TCP position during the experimental QSC measurement series](image)

The experiments were replicated in Dymola to provide a one-to-one assessment between real-world experiments and simulation scenarios. Figure 4.21 portrays the 3D view of the simulation results based on the experimental exerted force. The lower stiffness of the TCP in the regions further away from the base coordinate system can be clearly seen in the depiction. Another notable fact is the tilt of the unloaded condition due to the effect of gravity which is not considered in the direct VC signals. This is due to the acting weight of the hanging lower arm in this configuration which can cause backlash as discussed previously. Figure 4.22 and Figure 4.23 show the increasing deflection of the TCP under increasing exerted external force.
Figure 4.21: 3D view of the TCP position during the simulations for the QSC measurement series

Figure 4.22: 2D view of the TCP position during the simulations for the QSC measurement series
Figure 4.23: Magnified view of the TCP position during the simulations for the QSC measurement series

Figure 4.24 and Figure 4.25 picture the results from the experiments and simulations for a visual comparison. The difference between the initial kinematics influences the deflected visualizations; however, whilst looking at the top-view, one can deduce the conformity of the experiments and simulations.

Figure 4.24: 3D view of measurement results from simulation and experiments
In order to provide a comprehensive visualization of the deflection values from the simulations and experiments, the relative deflections for each case were calculated and plotted. Figure 4.26 draws a sample presentation of the results in this case.

Figure 4.25: Magnified 2D view of measurement results from simulation and experiments

Figure 4.26: Sample quasi-static deflection measurement from experiments and simulations
Whence viewing the deflection values of the quasi-static measurement series, one can conclude that the accuracy of the manipulator heavily suffers under high external loadings which often occur in heavy-duty contact applications. Another subject of note is the level of error involved in the predictions of the simulations. While the errors are small for cases where high loadings are not present, they are still not within the tolerance goals of the COMACH project (±250µm). Moreover, it should be noted that the deflections along the Y axis are significantly higher than those of X and Z. This is due to the lower axial stiffness of the robot along the Y axis as displayed previously.

The next section will proceed with the analysis of the system response for the manipulator from a dynamic point of view.

4.3 Dynamic Analysis of The Manipulator

The objective of this section is to analyze the manipulator as a dynamic system whose response is dependent on a dynamic excitation force based on equation (2.10). The system was considered as a linearized system around a point in space in order to proceed with the dynamic response calculation. Three models were used as the basis for reducing the system into the linear format in order to visualize and compare the differences between the respective models:

- The E-model: The linear E-model with lumped parameters at the joints of the robot.
- The flexible model: The flexible-body model achieved by a Guyan reduction of the FE representation of the manipulator.
- The flexible model with coupled wrist: The flexible-body model enhanced with a coupled wrist representation.

Figure 4.27 and Figure 4.28 present the bode responses for the robot at a sample configuration using the respective models. The most significant result of the dynamic analyses is the value of the first eigen frequency of the robot which corresponds to just over 10Hz. This confirms the observations of previous researchers who experienced the same range of self-excitation of the manipulator at this frequency [36].
Figure 4.27: Sample full bode plot of the robot when subjected to axial excitations
Furthermore, one can notice the differences between the various models using the bode plots. The following can be deduced based on the dynamic response of the manipulator:

- It is clear that the static-gain and behavior of the models are in-line with each other. Deviation in the predicted behavior arises as the frequency increases.
- It is evident that the E-model fails to predict any dynamic behavior after the 100Hz bandwidth. This is expected as the lumped parameters are designed for static behavior and fail to include the elasticity and flexibility of the full body.
- The flexible-body model predicts dynamic behavior up to 1kHz and agrees with the E-model in the lower bandwidth range. However, as this model is more computationally expensive, the use of it should be limited to sophisticated dynamic applications in the upper bandwidth range.
- The inclusion of the coupled wrist to the flexible body model has added extra dynamic behaviors which can be seen up to 1kHz.
- As the Guyan reduction method [5] was used for the analyses, the cutoff frequency of 100Hz for the Guyan reduction affects the gain of the estimated response. This is in-line with previous estimations of the Guyan behavior which predicted the respective model to drop
accuracy in the upper bandwidth range [23]. Nevertheless, the predicted behavior in the upper bandwidth range is still more feasible than the results of the E-model.

The following section will proceed to derive predicted estimations for the contact behavior of the manipulator based on the previous simulation and experimental observations.

4.4 Prediction results for the contact behavior of robotic manipulators

This section focuses on deriving the estimated behavior of a manipulator for heavy-duty contact applications involving dynamic excitations. Previous researches suggested the presence of mode-coupling chatter during experiments with a robotic manipulator [36]. Therefore, two models were constructed according to Figure 3.2 and Figure 3.3 to recreate the experimental procedure. The simulations were performed with the goal of comparing the robotic manipulator to a CNC machine which is significantly stiffer.

Figure 4.29 portrays the simulated time-responses of the robotic manipulator and a CNC machine whence disturbed with an initial input. While the CNC machine proceeds to reduce the disturbance and converge to a steady state, the robotic manipulator undergoes a self-excitation behavior which leads to higher magnitudes of oscillation.

![Figure 4.29: Time response of a robotic manipulator and CNC machine during a contact application](image)
This in turn causes the manipulator to reach the mechanical limits and continue its oscillations within that range. This can also be visualized as the movement of the attached tool in both cases which is portrayed in Figure 4.30 and Figure 4.31.

Figure 4.30: Dynamic tool positioning of a CNC machine after a disturbance

Figure 4.31: Dynamic tool positioning of a robotic manipulator after a disturbance
The sheer magnitude of the TCP movement along the X and Y directions in the two cases presents a plausible case on the more suitable behavior of the CNC machine. It should also be noted that the robotic manipulator follows an oval shape once excited which corresponds to the oscillation plane and stiffness values in the principal axes. As a result, the simulations confirm previous laboratory observations regarding the occurrence of self-excited oscillations in the form of mode-coupling chatter.
Chapter 5

5 Conclusion & Future Work

This study was conducted with the objective of modelling and analyzing the behavior of a robotic manipulator for contact applications. Based on the results of the laboratory and simulation testing, the following can be concluded:

- The static compliance results indicate that while the models often produce a relatively satisfactory representation of reality, they cannot be solely relied on for accurate applications due to the presence of nonlinear behavior in the gears and flexibility of the robotic manipulators body. Also, the difference between nominal and actual kinematics from Dymola and the experiments can be influential here.

- The static deflection values demonstrated the high deformation of the robot at the TCP when associated with loads correspondent to heavy-duty operations such as milling. Therefore, such operations should be performed with care with robotic manipulators by increasing axial stiffness at the TCP.

- The discretization of the joint-space of the robot for mapping stiffness measures the Y direction (orthogonal axis to the lower arm) to be the least stiff axis of loading for almost all of the configurations of the robot. Comprehensive reusable maps are produced for future users so that the stiffness of a configuration can be estimated before operation or computational evaluation.

- The results of the quasi-static measurements illustrate the lower stiffness of the robot along the Y axis. The coherence between the simulated and experimental deflections is of a good degree; however, it does not lie within the goals of the COMACH project. Another subject of note is the discrepancy between the actual and nominal robot kinematic models, which can cause errors in future estimations.
• The dynamic measurements illustrate the low first eigen frequency of the robot to be at \( \sim 10 \text{Hz} \). This is in-line with previous in-lab measurements [36] and shows the weakness of the robot in applications involving high-magnitude excitation forces.

• Predictive estimations for the behavior of the robotic manipulator have been derived for contact applications. These estimations forecast the behavior of the manipulator to be significantly worse than the CNC machine due to the occurrence of the self-excitation behavior. The TCP of the manipulator is in this case expected to oscillate which would damage the workpiece and cause operation failure.

Due to the mentioned concerns the work strongly recommends the selection of a rather stiff configuration with accordance to the provided joint-space maps for contact applications. Moreover, the process should be designed in such a way that the excitation force does not induce a high component along the global Y axis due to the lower stiffness value. While these measures do not guarantee stability, they can serve as baseline recommendations for a more favorable initial setup.

The following describes a set of preliminary guidelines for further work and studies to be conducted in order to improve the current state of the art:

• Identifying the robot parameters based on experimental procedures in the laboratory to improve and refine the Dymola models.

• Adding the nonlinear gear properties such as backlash and hysteresis into the Dymola manipulator models.

• Constructing a process model based on the contact application to consider the process parameters such as feed rate and depth of cut. These would allow more accurate depictions of the contact behavior to be produced based on the combined robotic application model.

• The research approach would be to iterate the design of the robot, especially in the second and third joints, to achieve a higher stiffness and stiffness-to-mass ratio.

By implementing the recommended refinements, a more stable high-duty contact operation with robotic manipulators would possibly become within reach.
References


[34] !!! INVALID CITATION !!! [28-36].


**Appendix A: Dymola Documentation**

This section presents the modelling structures of the manipulator in Dymola. While the robot is constructed of numerous parts with various sizes in real-life, a rather simplified model can be reached in Dymola based on the DH parameters and mechanical parameter identification of the robot. The following introduce the Dymola blocks and individual components of the robot, followed by a depiction of the full models.

### A.1 Inputs and Outputs

Dymola uses triangular and circular colored blocks from the Modelica library to depict the inputs and outputs of a system and subsystems. Figure A.1 shows the basic Dymola input/output blocks which were used to create the robot models. It should be noted that in this case, real values were used inside the models and Boolean values were used as checks for the success of the runs to verify the validity of simulations.

![Basic Dymola input/output blocks](image)

*Figure A.1: Basic Dymola input/output blocks*

### A.2 Sensors

Similar to real-life scenarios, Dymola incorporates sensor blocks to convert mechanical properties of bodies to readable signals for the user. In this fashion, the user would need to connect the sensor in the proper connection order (serial or parallel) to the desired frames of the body and reference which
would allow extraction of parameters such as translation/rotational acceleration, velocity and displacement. Figure A.2 draws the typical block of a sensor in Dymola.

![Figure A.2 : A sensor block in Dymola](image)

### A.3 Motors

The motors are defined by the ABB Robotics Library as a transfer to the reference value in the mechanical system followed by an inertia property. The reference value can be the torque or the position of the motor. Figure A.3 portrays the motor block when the reference is set to a position signal. Figure A.4 shows the structure of the motor block for the same conditions.

![Figure A.3 : The motor block in Dymola](image)

![Figure A.4: The structure of the motor block](image)
A.4 Gearboxes

The ideal gearbox block in Dymola is present in the Modelica library which allows the conversion of torque and rotation according to the gear ratio. The ideal gearbox was used in the production of the rigid-body model to create the required kinematics for the robot. The elastic gearbox models were produced in the ABB Robotics Library by combining an ideal gear with an inertia and a rotational spring-damper mechanism. Figure A.5 and Figure A.6 show the elastic gearbox block and its internal structure respectively.

![Figure A.5: The elastic gearbox block in Dymola](image)

![Figure A.6: The structure of the elastic gearbox block](image)

A.5 Joints

The joints were modeled as simple revolute joints for the rigid-body model. For the E-Model, the joints of the robot are modeled as connections of revolute joints coupled to spring and dampers to represent the elastic rotational behavior of the robot joints according to Figure A.7. For the flexible model, revolute joints were coupled with their respective flexible body to produce elastic rotation. An example of a 3 DOF revolute joint can be seen in Figure A.7.
A.6 Solid Bodies & Payload

The solid bodies and the payload block in Dymola are modeled as a body with inertial properties such as moment of inertias and mass. A translation frame is added to model the length of the body and therefore the body includes another input in the form of center of mass coordinates. Figure A.8 shows the structure of the body block in Dymola.
A.7 Flexible Bodies

The flexible body models followed a Guyan reduction method [5] for this work based on previous developments for the robotic manipulator models at ABB Robotics [13, 22]. The reduction procedure was based on the single-node FE model and the resulting file (also known as the .SID file) was imported into the Dymola flexible-body element which can be seen in Figure A.9. The block also accepts an optional Wavefront (.OBJ) file which can provide the means towards visualizing the CAD model and producing an animation of the body.

![modalBody](image)

*Figure A.9: The flexible-body block in Dymola*

A.8 Rigid-Body Model of The Manipulator

The rigid-body model was produced by neglecting all elasticity in the manipulator to create a kinematic representation of the robot. This was produced by dividing the body of the manipulator into the following parts as portrayed in and Figure 2.2:

- Base
- Frame
- Lower Arm
- Arm-housing
- Tube-shaft
- Tilt-housing
- Turning Disc

Each substructure was modeled as a rigid connection of a motor, a gearbox, and a joint block which facilitated the kinematic calculations for the manipulator. The resulting body visualized the motion of the manipulator by taking the CAD files as inputs and calculating the kinematics of the robot at a given
configuration. Figure A.10 portrays the rigid-body model in this case where the motor positions are taken in as inputs to the model and the TCP position is designed as the output.

![Figure A.10: Layout of the rigid-body model in Dymola (motor position to TCP position/rotation)](image)

**A.9 E-Model of The Manipulator**

In order to create the E-Model based on a lumped-parameter approach towards the robot, the rigid-body model was transformed to produce an elastic behavior at the joints. Each substructure was modeled as an elastic connection of a motor, a gearbox, and a joint block which facilitated the lumped-parameter approach towards creating the complete manipulator model. As a result, the model provides elastic behavior at the joints whose mechanical properties are calibrated based on previous experiments at ABB Robotics. Therefore, a configuration file (.VCSIM) is created as an input to the E-Model which feeds the experimental data into the model and creates a modular interface for replacing such values with less effort. Figure A.11 pictures the layout of the E-Model for the manipulator in Dymola.
It should be noted that while the specific robot was the subject of study in this work, the E-Model persists in structure for any type of robot due to its modular elements and replaceable configuration file.

![Diagram of the E-Model in Dymola](image)

**Figure A.11: Layout of the E-Model in Dymola (motor position to TCP position/rotation)**

### A.10 Flexible-Body Model of The Manipulator

This model was created through the dynamic sub-structuring approach as portrayed in Figure 2.5. The respective constituting sub-structures were meshed in a FE software and the Guyan reduction method was used on the resulting grid to create the .SID inputs for the Dymola model. The resulting structure was connected in a similar fashion to the previous model to create the complete structure. Figure A.12 depicts the layout of the flexible-body model.

While the flexible-body model has proven to be more accurate in estimations of the dynamic behavior of the robot [13], the model is also significantly more time-consuming which is a critical aspect in engineering calculations. The user should always consider the difference in gained value when selecting the flexible-body model as the E-Model has been proven to behave closely to the flexible-body model while having a much lesser computation time.
This model is an enhanced version of the flexible-body model which takes the mechanical coupling of the wrist into account. The mechanical coupling exists between joints 4, 5 and 6 through the gearboxes which transfers torque and rotation and therefore has been known to produce nonlinear behavior in dynamic movements. As a result, it is added to as a coupled-wrist block to the ABB Robotics Library. Figure A.13 and Figure A.14 show the coupled wrist block and structure in Dymola respectively.
Appendix A: Dymola Documentation

Figure A.13: The coupled wrist block in Dymola

Figure A.14: The coupled wrist structure in Dymola
Appendix B: Robot Studio Code for Quasi-Static Testing

MODULE Module1
  CONST robtarget Target_50:=[[883.561,-287.318,583.49],[0.707106781,0.0,0.707106781,0],[-1,0,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_20:=[[1037.38,-144.255,583.49],[0.707106781,0.0,0.707106781,0],[-1,0,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_60:=[[882.84,22.68,583.24],[0.707106781,0.0,0.707106781,0],[0.0,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_40:=[[882.84,-127.639,738.419],[0.707106781,0.0,0.707106781,0],[-1,0,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_70:=[[1488.34,47.78,588.13],[0.5,0.5,0.5,-0.5],[0.1,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_80:=[[1333.34,202.78,588.13],[0.5,0.5,0.5,-0.5],[0.1,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_90:=[[1178.34,47.78,588.13],[0.5,0.5,0.5,-0.5],[0.1,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_100:=[[1333.34,-107.22,588.13],[0.5,0.5,0.5,-0.5],[0.1,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_110:=[[1488.34,47.78,588.13],[0.5,0.5,0.5,-0.5],[0.1,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_120:=[[1451.767,47.78,688.13],[0.5,0.5,0.5,-0.5],[0.1,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_130:=[[1331.633,166.195,688.13],[0.5,0.5,0.5,-0.5],[0.1,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_140:=[[1214.962,44.367,688.13],[0.5,0.5,0.5,-0.5],[0.1,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_150:=[[1335.047,-70.635,688.13],[0.5,0.5,0.5,-0.5],[0.1,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_160:=[[1451.767,47.78,688.13],[0.5,0.5,0.5,-0.5],[0.1,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_170:=[[1476.256,47.78,684.13],[0.5,0.5,0.5,-0.5],[0.1,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_180:=[[1331.454,190.684,684.13],[0.5,0.5,0.5,-0.5],[0.1,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_190:=[[1190.474,44.007,684.13],[0.5,0.5,0.5,-0.5],[0.1,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_200:=[[1335.226,-95.124,684.13],[0.5,0.5,0.5,-0.5],[0.1,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_210:=[[1476.256,47.78,684.13],[0.5,0.5,0.5,-0.5],[0.1,-1,1],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_220:=[[1451.767,47.78,688.13],[0.0,-70.7106781,0.707106781,0],[0.0,0.0,0],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_230:=[[1331.633410479,-70.634702977,688.13],[0.0,0.07205425,0.99974041,0],[0.0,0.0,0],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_240:=[[1214.962185537,51.19282468,688.13],[0.717223104,0.696843612,0],[0.0,0.0,0],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_250:=[[1335.04589521,166.194702977,688.13],[0.0,0.99974041,-0.007205425,0],[0.0,0.0,0],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST robtarget Target_260:=[[1451.767,47.78,688.13],[0.0,-70.7106781,0.707106781,0],[0.0,0.0,0],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
  CONST jointtarget calib_post:=[[0,0,0,0,0,0],[9E+09,9E+09,9E+09,9E+09,9E+09,9E+09]];
Appendix B: Robot Studio Code for Quasi-Static Testing

Author: SEARHOS
Version: 1.0

***********************************************************
***********************************************************

Procedure main

This is the entry point of your program

***********************************************************

PROC main()
    MoveJ Target_170,v10,fine,tool0\WObj:=wobj0;
    MoveC Target_180,Target_190,v10,fine,tool0\WObj:=wobj0;
    MoveC Target_200,Target_210,v10,fine,tool0\WObj:=wobj0;
ENDPROC

PROC Path_10()
    MoveL Target_50,v50,fine,tool0\WObj:=wobj0;
    MoveC Target_20,Target_60,v50,fine,tool0\WObj:=wobj0;
    MoveC Target_40,Target_50,v50,fine,tool0\WObj:=wobj0;
ENDPROC

PROC Path_20()
    MoveL Target_40,v1000,z100,tool0\WObj:=wobj0;
    MoveL Target_50,v1000,z100,tool0\WObj:=wobj0;
    MoveL Target_60,v1000,z100,tool0\WObj:=wobj0;
ENDPROC

PROC Path_30()
    MoveJ Target_70,v10,fine,tool0\WObj:=wobj0;
    MoveC Target_80,Target_90,v10,fine,tool0\WObj:=wobj0;
    MoveC Target_100,Target_110,v10,fine,tool0\WObj:=wobj0;
ENDPROC

PROC Path_40()
    MoveJ Target_120,v10,tool0\WObj:=wobj0;
    MoveC Target_130,Target_140,v10,tool0\WObj:=wobj0;
    MoveC Target_150,Target_160,v10,tool0\WObj:=wobj0;
ENDPROC

PROC Path_50()
    MoveJ Target_170,v10,tool0\WObj:=wobj0;
    MoveC Target_180,Target_190,v10,tool0\WObj:=wobj0;
    MoveC Target_200,Target_210,v10,tool0\WObj:=wobj0;
ENDPROC

PROC Path_60()
    MoveL Target_220,v1000,z100,tool0\WObj:=wobj0;
    MoveC Conc,Target_230,Target_240,v1000,z100,tool0\WObj:=wobj0;
    MoveC Conc,Target_250,Target_260,v1000,z100,tool0\WObj:=wobj0;
ENDPROC
ENDMODULE