Comparison of Euler-Euler Approach and Euler–Lagrange Approach to Model Gas Injection in a Ladle

Yu Liu,* Mikael Ersson, Heping Liu, Pär Jönsson, and Yong Gan

The gas injection in a ladle using a porous plug is simulated using both the Euler-Euler and Euler-Lagrange approaches. The effects of various forces, bubble sizes, and bubble injection frequencies on the flow pattern are modeled. For predicting axial velocity and turbulent kinetic energy, the Euler-Lagrange approach fits better than Euler-Euler approach with the measured data. In the Euler-Euler approach, differences in axial velocities and turbulent kinetic energies for various bubble sizes mainly appears in the plume zone. In the Euler-Lagrange approach, different bubble sizes with the same injection frequency have a small impact on the turbulence dissipation. Furthermore, the turbulent dispersion from the gas phase to the liquid phase has an important effect on the plume structure and spout eye formation. For both modeling, the smaller the bubble diameter is, the larger the axial velocity and turbulent kinetic dissipation are in the central zone. For the bubble coalescence and breakup, according to the comparison of two modeling approaches, the Euler-Lagrange approach is more accurate in predicting the flow pattern for gas injection with a porous plug in the ladle.

1. Introduction

Inert gas bubbling is commonly used in metallurgical processes. Gas injection can increase the homogenization of temperature and alloy elements. In addition, rising bubbles can also enhance the removal of inclusions and impurity elements such as H and N during ladle, tundish and continuous casting processes.^[1–4] Herein, in the industrial ladle process, there are mainly three types of plugs, namely, the hybrid plug, slot plug and porous plug to inject argon bubbles. Different types of plugs generate various kinds of bubble distributions. The porous plug is widely used because the bubble number is easily controlled and stays almost constant at the low flow rate.^[5–7]

In order to study the industrial ladle system, several experiments were carried out with porous plugs to study gas injection. Anagbo and Brimacombe^[5] used the porous plug to

inject discrete bubbles and studied the coalescence regime in the plume. Johansen et al.^[6] studied the distribution of the radial and axial mean and turbulent velocities in the plume zone with the gas bubbling by porous plug. Joo and Guthrie^[8] analyzed the effect of the porous plug location on the mixing mechanisms with single and dual plug injection setups. Guo and Irons^[9,10] set up a NaOH–CO₂ experimental system to simulate the diffusion-controlled decarburization process in the liquid steel. Mazumdar and Guthrie^[11] investigated energy dissipation phenomena with and without overlying second-phase liquid at a low gas flow rate and a specific energy input rate. Recently, a water-bean oil-nitrogen system was set up by Liu et al.^[12] to study the mixing time and open eye formation during gas injection.

In terms of the mathematical modeling to study the phenomena in the gas-stirred ladle, such as the mixing time and open eye formation, there are commonly two advanced approaches, namely, the Euler-Euler approach and the Euler-Lagrange approach. For the Euler-Euler approach, the injection of continuous gas flow in the plume is treated, and the bubble size is set up as unique size.^[13,14] However, the bubbles interact in the plume, such as breakup, aggregation, and coalescence.^[15] Therefore, the size and shape of bubbles vary along the axial line from the gas inlet.^[15,16] Li et al.^[17] combined the Eulerian Multiphase model and Population Balance Model to calculate the effect of the bubble coalescence and breakup on the size of the open eye, mixing time, and wall shear stress. In a recent work by Karouni et al.,^[18] the same method was employed to study the hydrogen degassing in a vacuum arc ladle. In the Euler-Lagrange approach, the flow pattern of the liquid phase was solved by using Eulerian coordinates. With the consideration of bubble particles' interactions in the Lagrange coordinate, the forces of individual bubbles were taken into consideration in the formulation of the liquid phase. In Alexiadis et al. model,^[19] a used-defined population balance model was used in combination with a Discrete Phase Model to predict the statistical bubble distribution in a quasi-steady state flow pattern. Aoki et al.^[20] developed an Euler-Lagrange model with the commercial package ANSYS Fluent[®] to study the bubble morphology affecting the probability of inclusion attachments. Zhang et al.^[21,22] used a similar method to study the flow pattern in the continuous mold casting process coupled with the user-defined code to study the interaction of bubbles. A Volume of

Y. Liu, Prof. H. P. Liu, Prof. Y. Gan
Central Iron and Steel Research Institute
100081 Beijing, P. R. China
E-mail: yuliu2@kth.se

Y. Liu, Dr. M. Ersson, Prof. P. Jönsson
Unit of Processes
Department of Materials Science and Engineering
KTH-Royal Institute of Technology
Brinellvägen 23, SE-100 44 Stockholm, Sweden

DOI: 10.1002/srin.201800494

Fluid Model (VOF model) to track the interface of the melt and a Discrete Phase Model (DPM) to calculate the trajectory of every individual bubble particle was used by Cloete et al.^[23] to improve the Euler-Lagrange approach. The same method was also employed by Li et al.^[24] for studying alloy dispersions. In addition, Liu et al.^[25] used a similar model to study the effect of plug configurations on the open eye formation and the mixing conditions. Li et al.^[26,27] also used the Euler-Lagrange approach coupled with the LES model to study the unsteady state of the open eye.

In the Euler-Euler and Euler-Lagrange approaches, the corresponding forces added to interact with the main momentum equation are compared in **Table 1a**. Based on equations in Table 1a, the drag, virtual mass, and lift forces are generated by the relative velocity, relative acceleration, and shear force between the gas and liquid. The turbulent dispersion force accounts for the turbulent diffusion and momentum transfer at

Table 1. Comparison (a) and additional different forces used in the Euler-Euler approach (b) and in the Euler-Lagrange approach (c) in recent works.

a)					
Euler-Euler approach			Euler-Lagrange approach		
$F_{\text{drag}} = \frac{3\alpha_g \alpha_l C_D}{4d_g} \vec{u}_g - \vec{u}_l (\vec{u}_g - \vec{u}_l)$			$F_{\text{drag}} = \frac{3C_D \rho}{4\rho_b d_{bi}} \vec{u}_{bi} - \vec{u} (\vec{u} - \vec{u}_{bi})$		
$F_{\text{lift}} = \alpha_g C_l \rho_l (\vec{u}_g - \vec{u}_l) \times (\nabla \times \vec{u}_l)$			$F_{\text{lift}} = C_l \frac{\rho}{\rho_b} (\vec{u} - \vec{u}_{bi}) \times \nabla \times \vec{u}$		
$F_{\text{virtual mass}} = \frac{1}{2} \alpha_g \rho_l (d \frac{\vec{u}_g}{dt} - d \frac{\vec{u}_l}{dt})$			$F_{\text{virtual mass}} = \frac{1}{2} \frac{\rho}{\rho_b} (\vec{u}_{bi} \nabla \vec{u} - \frac{d\vec{u}_{bi}}{dt})$		
$F_{\text{turbulence dispersion}} = K_{gl} \frac{D_{lg}}{\sigma_{gl}} (\frac{\nabla \alpha_g}{\alpha_g} - \frac{\nabla \alpha_l}{\alpha_l})$			Discrete random walk model (Section 2.2)		
			$F_{\text{pressure gradient}} = \frac{\rho}{\rho_b} (\vec{u} \cdot \nabla) \vec{u}$		
			$F_{\text{buoyancy}} = \frac{\rho_b - \rho}{\rho_b} \vec{g}$		
b)					
Euler-Euler approach					
	F_{drag}	F_{lift}	$F_{\text{virtual mass}}$	$F_{\text{turbulence dispersion}}$	
Lou and Zhu ^[14,31–33]	+	–	–		+
Yu et al. ^[34–37]	+	+	–		–
Felice, Bellot, and coworkers ^[38,39]	+	+	+		+
Huang et al. ^[40]	+	+	+		–
Li et al. ^[17]	+	+	+		–
Karouni et al. ^[18,41]	+	+	+		+
Present paper	+	+	+		+
c)					
Euler-Lagrange approach					
	F_{drag}	F_{buoyancy}	$F_{\text{virtual mass}}$	$F_{\text{pressure gradient}}$	F_{lift}
Aoki et al. ^[20]	+	–	+	+	+
Singh et al. ^[30]	+	+	–	–	–
Cloete et al. ^[23]	+	+	–	–	–
Li et al. ^[24]	+	+	+	+	+
Liu et al. ^[25]	+	+	+	+	–
Li et al. ^[26,27]	+	+	+	+	–
Present paper	+	+	+	+	–

the interphases in the dispersed flows. The buoyancy force and the pressure gradient force are generated by the density difference between the gas and liquid.^[28,29] The recent works with various options of forces using both approaches are summarized in Table 1b and c. Moreover, in Zhang's work,^[2] the empirical equations for the bubble diameter in liquid steel were concluded, but in some works these equations were also used for bubble diameters' estimation when using the Euler-Euler^[14] and Euler-Lagrange^[30] approaches of water-air systems. The comparison of bubble sizes derived by these formulas are shown in **Table 2**.

In previous works, the parameters for the Euler-Euler and Euler-Lagrange approaches are approximate and not accurately linked with actual physical modeling results. However, the stirring dynamic given by gas bubbling crucially influences the open eye formation and species transport. Therefore, it is meaningful to clarify the relations of parameters for the momentum transport in the mathematical models to prepare for further analysis. In the present paper, two approaches are compared to study the various forces generated by the gas injection by the porous plug to describe the flow pattern in ladles. The flow pattern affected by the bubble generation and rising pattern are compared with existing experimental data.^[5] For the soft bubbling with a lower flow rate of 200 cm³ s^{−1}, discrete bubbles are generated and bubble interacted very few during rising up, so the Euler-Lagrange approach was preferably chose to describe the flow pattern in this condition. For the intense stirring with a higher flow rate of 120 cm³ s^{−1}, the volume fraction of gas in the plume is much larger, therefore the Euler-Euler approach was better employed to describe the flow pattern in this condition.

2. Mathematical Models

In the present paper, the commercial software ANSYS Fluent 19.1 is used to solve the governing equations. In the Euler-Euler approach, the Eulerian Multiphase Model^[48] is used to calculate two sets of equations for the gas and liquid phases in Eulerian coordinates. The Population Balance Model^[49] is employed to predict the bubble distribution along the height direction under a quasi-steady state. In the Euler-Lagrange approach, the Volume of Fluid Model^[48] is used to track the interface of different phases in the Eulerian coordinate system. The bubble particles are tracked by the Discrete Phase Model^[48] in the Lagrange coordinate system. The O'Rourke's Algorithm^[50] in the Discrete Phase Model is used to determine the bubble collision condition. The breakup of discrete bubbles with the diameter less than 2 mm are not considered.

2.1. Euler-Euler Approach (E-E Approach)

In the Euler-Euler approach, multiple sets of equations of continuity, momentum, turbulent energy and dissipation rate are calculated for each phase. For a better description of plume structure, the effect of forces, such as the virtual mass force, drag force, lift force, and turbulent dissipation force are considered in the momentum equation. Therefore, the following conservation equations are solved:

Table 2. Empirical equations of bubble diameter.

	Gas injection	Equation	Water-air system	Steel-argon system
Davidson, Mori et al. ^[42]	Nozzle	$d_b = \left[13.7 \left(\frac{Q_g^2 d_0}{g} \right) \right]^{0.1445}$		55.4 mm
Sano and Mori ^[14,43]	Nozzle	$d_b = \left[\left(\frac{6\alpha d_0}{\rho_l g} \right)^2 + 0.0248 \left(Q_g^2 d_0 \right)^{0.867} \right]^{1/6}$	28.5 mm	28.5 mm
Davidson and Schtiler ^[25,30,44]	Porous plug	$d_b = 0.35 \left(\frac{Q_g^2}{g} \right)^{0.2}$	15.0 mm	15.0 mm
Anagbo and Brimacombe ^[5]	Porous plug		3–50 mm	
Sahai and Guthrie ^[45]	Nozzle	$d_b \propto \left(\frac{\alpha}{\rho_l} \right)^{0.5}$	5.1 mm	17.5 mm
Sheng and Irons ^[46,47]	Nozzle		10 mm, 15 mm	

Continuity equation:

$$\frac{\partial}{\partial t} (a_q \rho_q) + \nabla \cdot (a_q \rho_q \vec{u}) = 0 \quad (1)$$

($q = l$ is the equation for liquid phase, $q = g$ is the equation for gas phase)

Momentum equation:

$$\begin{aligned} \frac{\partial}{\partial t} (a_q \rho_q \vec{u}_q) + \nabla \cdot (a_q \rho_q \vec{u}_q \vec{u}_q) \\ = -a_q \nabla p + \nabla \cdot (a_q (\mu + \mu_t) (\nabla \vec{u}_q + \nabla \vec{u}_q^T)) + a_q \rho_q \vec{g} \\ + \vec{F}_{\text{drag},q} + \vec{F}_{\text{TD},q} + \vec{F}_{\text{vm},q} + \vec{F}_{\text{lift},q} \end{aligned} \quad (2)$$

Population Balance Model

Transport equation:

$$\frac{\partial}{\partial t} (n(V, t)) + \nabla \cdot (\vec{u}_i n(V, t)) = B_{ag,i} - D_{ag,i} + B_{br,i} - D_{br,i} \quad (3)$$

$$B_{ag,i} = \frac{1}{2} \int_0^V \Omega_{ag}(V - V', V') n(V - V', t) n(V', t) dV'$$

$$D_{ag,i} = \int_0^\infty \Omega_{ag}(V, V') n(V, t) n(V', t) dV' \quad (4)$$

$$B_{br,i} = \int_{\Omega_V} p \Omega_{br}(V', t) dV', D_{br,i} = g(V) n(V, t) \quad (5)$$

The Luo Aggregation Kernel^[51] is used to calculate the collisions of bubbles with volumes V_i and V_j :

$$\Omega_{ag}(V_i, V_j) = \omega_{ag}(V_i, V_j) P_{ag}(V_i, V_j) \quad (6)$$

where ω_{ag} is the frequency of collision and P_{ag} is the probability that the collision results in coalescence.

$$\omega_{ag}(V_i, V_j) = \frac{\pi}{4} (d_j + d_i)^2 \bar{u}_{ij} \quad (7)$$

where $\bar{u}_{ij}$ is the characteristic velocity of collision of two bubbles with diameters d_i and d_j .

$$\bar{u}_{ij} = (\bar{u}_i^2 + \bar{u}_j^2)^{1/2} \quad (8)$$

where

$$\bar{u}_i = 1.43 (\varepsilon d_i)^{1/3} \quad (9)$$

The expression for the probability of aggregation is:

$$P_{ag}(V_i, V_j) = \exp \left\{ -c_1 \frac{[0.75(1 + x_{ij}^2)(1 + x_{ij}^3)]^{1/2}}{(\frac{\rho_2}{\rho_1} + 0.5)^{1/2} (1 + x_{ij})^3} We_{ij}^{1/2} \right\} \quad (10)$$

where c_1 is a constant of order unity, $x_{ij} = d_i/d_j$, ρ_1 , and ρ_2 are the densities of the primary and secondary phases, respectively, and the Weber number is defined as:

$$We_{ij} = \frac{\rho_l d_i (\bar{u}_{ij})^2}{\sigma} \quad (11)$$

The Luo Breakage Kernel^[52] is used to calculate the breakup of bubbles with volumes V_i and V_j :

$$\begin{aligned} \Omega_{br} &= g(V') \beta(V|V') \\ &= 0.9238 a \left(\frac{\varepsilon}{d^2} \right)^{1/3} \int_{\xi_{\min}}^1 \frac{(1 + \xi)^2}{\xi^n} \exp(-b \xi^m) d\xi \end{aligned} \quad (12)$$

$\xi = \lambda/d$ is the dimensionless eddy size.

2.2. Euler-Lagrange Approach (E-L Approach)

In the Euler-Lagrange approach, a set of equations for continuity, momentum, turbulent energy and dissipation rate are calculated. The conservation equations are shown as follows:

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \quad (13)$$

Momentum equation:

$$\begin{aligned} \frac{\partial}{\partial t}(\rho \vec{u}) + \nabla \cdot (\rho \vec{u} \vec{u}) = & -\nabla p + \nabla \\ & \cdot \left[(\mu + \mu_t) (\nabla \vec{u} + \nabla \vec{u}^T) \right] + \rho \vec{g} \\ & + F_s + F_{bi} \end{aligned} \quad (14)$$

Volume of Fluid Model (VOF Model)

In the VOF model, the equation of the cell's volume fraction is:

$$\frac{\partial a_q}{\partial t} + \vec{u} \cdot \nabla a_q = 0 \quad (15)$$

$$\sum a_q = 1 \quad (16)$$

The density and viscosity of mixed fluid in each cell are as follows:

$$\rho = a_g \rho_g + (1 - a_g) \rho_l \quad (17)$$

$$\mu = a_g \mu_g + (1 - a_g) \mu_l \quad (18)$$

(q is the fluid phase. e.g., liquid and gas)

Continuum Surface Force Model (CSF Model)

$$F_s = \sigma \frac{\rho \kappa \nabla a_l}{0.5(\rho_l + \rho_g)} \quad (19)$$

where $\kappa = \nabla \cdot \hat{n}$, $\hat{n} = \frac{\vec{n}}{|\vec{n}|}$, $\vec{n} = \nabla a_g$, κ is curvature (m^{-2}).

Discrete Phase Model (DPM Model)

In the discrete phase model, the forces, such as the virtual mass, buoyancy, drag, lift, and pressure gradient forces, are added on each bubble particle optionally. The momentum source term is added to the continuous phase momentum by summing up the local contributions from each bubble in the continuous phase flow field:

$$\frac{d\vec{u}_{bi}}{dt} = \frac{\vec{u} - \vec{u}_{bi}}{\tau_r} + \frac{\vec{g}(\rho_b - \rho)}{\rho_b} + \vec{F}_{other} \quad (20)$$

$$\tau_r = \frac{\rho_b d_{bi}^2}{18\mu} \frac{24}{C_D Re} \quad (21)$$

$$F_{bi} = \sum \left(\vec{F}_{drag} + \vec{F}_{buoyancy} + \vec{F}_{other} \right) \dot{m}_b \Delta t \quad (22)$$

C_D is drag coefficient, $\vec{F}_{other}$ is the additional force, Δt is the time step, $\dot{m}_b$ is the mass flow rate of bubbles through each control volume.

Collision and Discrete Coalescence Model

The O'Rourke's algorithm^[50] is used to calculate the bubble collision. The mean expected number of collisions is calculated as:

$$\bar{n} = \frac{n_2 \pi (r_1 + r_2)^2 v_{rel} \Delta t}{V_{cell}} \quad (23)$$

The probability distribution of collision follows a Poisson distribution,

$$P(n) = e^{-\bar{n}} \frac{\bar{n}^n}{n!} \quad (24)$$

The critical offset is calculated as

$$b_{crit} = (r_1 + r_2) \sqrt{\min\left(0.1, \frac{2.4f}{We}\right)} \quad (25)$$

where f is defined as $(r_1/r_2)^3 - 2.4*(r_1/r_2)^2 + 2.7*(r_1/r_2)$, and Weber number is defined as $\frac{\rho u_{rel}^2 \bar{d}_b}{\sigma}$, u_{rel} is the relative velocity between two parcels and $\bar{d}_b$ is mean diameter of two parcels. The actual value of collision b , is $(r_1 + r_2) \sqrt{Y}$, Y is a random number between 0 and 1.

If $b < b_{crit}$, the result of collision is coalescence. Based on the conservation of momentum and kinetic energy, the mass of the new bubble is $m = m_1 + m_2$, and the velocity of new bubble is $v = \frac{m_1 v_1 + m_2 v_2}{m}$. Otherwise, the result of the collision is the grazing collision, the bubble mass is unchanged and the velocity updates to $v'_i = \frac{m_i v_i + m_j v_j}{m_i + m_j} + \frac{m_j (v_i - v_j)}{m_i + m_j} \left(\frac{b - b_{crit}}{r_1 + r_2 - b_{crit}} \right)$.^[53]

Standard Parcel Release Method

In this method, a single bubble parcel is injected per injection stream per time step. The number of bubble particles in the parcel is determined as follows:

$$NB = \dot{m}_s \frac{\Delta t}{m_b} \quad (26)$$

where NB is the number of bubble particles in a bubble parcel, $\dot{m}_s$ is the mass flow rate of the bubble stream, Δt is the time step, and m_b is the particle mass.

For the Euler-Lagrange approach in this paper, there is only one bubble particle in each parcel, which means that each parcel represents a bubble.

Discrete Random Walk Model

In the Euler-Lagrange approach, the discrete random walk model is used to describe the interaction between bubble particles and turbulence eddies in the fluid phase. The velocity fluctuation of eddies are treated as isotropic, as follows:

$$u_i = \bar{u}_i + u' = \bar{u}_i + \zeta \sqrt{2k/3} \quad (27)$$

where ζ is a normally distributed random number greater than zero and less than 1.

The time scale can influence the lifetime of eddies to transfer momentum in the plume zone indirectly. So the time scale has an influence on the characteristics of the plume, such as the

velocity and turbulent dispersion distribution in the plume and the plume width.

2.3. Turbulence Model

In the Euler-Euler approach and Euler-Lagrange approach, the realizable k - ϵ model, which contains two equations of turbulence kinetic energy k and its dissipation rate ϵ , is used to solve the turbulence properties in the viscous flow.

3. Boundary Conditions and Solution Methods

The mesh and boundary conditions in the Euler-Euler and Euler-Lagrange approaches are shown as **Figure 1**. The parameters used in the calculation are listed in **Table 3a**. The comparison of the solvers and solution methods is illustrated in **Table 3b** and **c**. In the Euler-Euler approach, the detailed simulation condition was applied as follows: the outlet condition was set up as degassing, meaning that dispersed gas bubbles were allowed to escape and the continuous liquid phase considered the boundary as a free-slip wall and could not leave the domain; no slip boundary condition was used at the wall; the inlet velocity boundary condition was used to give a gas injection with a constant velocity; the pressure-based solver was chosen to calculate the equations in the transient mode; the multiphase formulation was solved implicitly with a Coupled scheme. In the Euler-Lagrange approach, a pressure-outlet condition was employed at the outlet boundary; no slip boundary condition was also used at the wall but inlet boundary condition was used to inject bubble parcels in the DPM model; the pressure-based solver was chosen to solve the multiphase formulations explicitly with a Pressure-Implicit with Splitting of Operators (PISO) scheme.

4. Results and Discussion

4.1. Mesh Sensitivity

Because the discretization error has been influenced by the calculation grid size,^[54,55] the mesh sensitivity should be firstly checked with different grids when using the Euler-Euler and Euler-Lagrange approaches, respectively. In the Euler-Euler approach, the domains with grid numbers of 81 120, 187 360, and 358 600 are used to construct the calculation systems. The comparisons of velocity magnitude and turbulent kinetic energy at $z = 200$ mm are shown in **Figure 2a** and **b**. The difference of calculation results with the grid of 187 360 and 358 600 are within 5%, and the difference ratios are 1.9% and 4.96% for the velocity magnitude and turbulent kinetic energy on the axial line. However, the results with the grids of 81 120 and 358 600 have a large difference ratio of 7.50% and 48.76% for the velocity magnitude and turbulent kinetic energy on the axial line. Therefore, the mesh with the grid number of 187 360 is used to compare the effects of different parameters on the calculation results using when the Euler-Euler approach. The same comparison is also performed for the Euler-Lagrange approach. Here, the grid numbers of 135 465, 277 200, and 556 875 are used to construct the calculation systems, respectively. The comparisons of velocity magnitude and turbulent kinetic energy at $z = 200$ mm are shown in **Figure 2c** and **d**. The domain with the grid number of 277 200 is used to compare the effects of different parameters on the calculation results in the Euler-Lagrange approach.

4.2. Effect of Different Bubble Sizes on the Flow Pattern in the Euler-Euler Approach

Under the gas flow rate of $1200 \text{ cm}^3 \text{ s}^{-1}$, the axial bubble diameter is between 22 mm and 50 mm.^[5] According to the

Figure 1. Computational mesh and boundary condition. a) Euler-Euler approach, b) Euler-Lagrange approach.

Table 3. Physical parameters (a), dimensions of ladle (b), and boundary conditions (c) used in the simulation.

a)				
$\rho_{\text{water}} [\text{kg m}^{-3}]$	$\rho_{\text{air}} [\text{kg m}^{-3}]$	$\mu_{\text{water}} [\text{kg (m s)}^{-1} \text{m}^{-1}]$	$\mu_{\text{air}} [\text{kg (m s)}^{-1} \text{m}^{-1}]$	$\sigma [\text{N m}^{-1}]$
998.2	1.225	1.003×10^{-3}	1.7894×10^{-5}	0.0728
b)				
	Euler-Euler approach (Eulerian multiphase model)		Euler-Lagrange approach (VOF + DPM)	
Diameter of ladle	500 mm		500 mm	
Height of ladle	400 mm		550 mm (400 mm steel filled height)	
Diameter of porous plug	60 mm		60 mm	
Wall	No-slip		No-slip	
Outflow	Degassing		Pressure-outlet	
Solver type	Pressure-based, transient		Pressure-based, transient	
Formulation	Implicit		Implicit	
Pressure-velocity coupling scheme	Coupled		PISO	
Convergence criteria	10^{-4}		10^{-4}	
Time step size	$1 \times 10^{-4} \text{ s}$		$1 \times 10^{-4} \text{ s}$	

comparison in Table 1a, the bubble size mainly influences the drag force to interact with the momentum calculation in each control volume. In the Euler-Euler approach, the bubble sizes of 5, 20, 35, and 50 mm are compared.

In **Figure 3**, the water axial velocity, radial velocity, and turbulent kinetic energy at different heights with different bubble sizes are compared. According to the comparison of the water axial velocity with experiment data^[5] in Figure 3a, the

Figure 2. The velocity magnitude and turbulent kinetic energy at $z = 200 \text{ mm}$ (half in the axial direction) in the Euler-Euler approach and Euler-Lagrange approach with different grids. a) Velocity magnitude in the Euler-Euler approach, b) Turbulent kinetic energy in the Euler-Euler approach, c) Velocity magnitude in the Euler-Lagrange approach, and d) Turbulent kinetic energy in the Euler-Lagrange approach.

prediction of axial velocity is 83–165% larger than the experimental data around the plume zone. However, the plume width is 30–48% smaller than the measured value. In Figure 3b, it is clear that the Euler-Euler approach predicts the rotational flow at the central height in the ladle less accurately than when using Euler-Lagrange approach. In terms of the effect of different bubble sizes on the flow pattern, the smaller bubble size predicts the narrower plume width, larger axial velocity and larger turbulent kinetic energy. However, the radial velocity is not affected to a large extent. Moreover, the differences between axial velocity and turbulent kinetic energy values for various bubble sizes mainly appear at the plume zone, shown as Figure 3a and c. Therefore, if the inclusion removal and open eye formation are calculated using the Euler-Euler approach, the effect of bubble size in the plume zone can be neglected outside the plume zone but not for central plume zone. In previous studies, Sahai and Guthrie,^[45] Stapurewicz and Themelis^[56] argued that the flow recirculation and mixing condition were independent if the gas injection was done by nozzle, tuyere, or plug. So no matter the bubble distribution is, the total injection dynamic has no difference at the same flow rate. However, the bubble size has an obvious impact on the velocity pattern and turbulence distribution when using the Euler-Euler approach.

4.3. Effect of Different Bubble Sizes and Bubble Generation Frequency on the Flow Pattern in Euler-Lagrange Approach

Based on the Standard Parcel Release Method, the relation among the gas flow rate, bubble diameter, bubble injection frequency, and bubble number per unit injection time are calculated as follows:

$$Q = \frac{4}{3} \pi \left(\frac{d_b}{2} \right)^3 * f * N_b \quad (28)$$

where Q is the total flow rate (m^3/s), d_b is the bubble diameter (m), f is the bubble injection frequency (s^{-1}), N_b is the bubble number per unit injection time (stream number). In the actual experiment with gas injection at a flow rate of $200 \text{ cm}^3 \text{ s}^{-1}$, the bubble frequency is about 104 s^{-1} at a position 30 mm above the porous plug. In the present paper, the bubble frequency is chosen as 100 s^{-1} in order to facilitate a comparison with various bubble injection conditions, shown as Table 4. The relationship among the bubble injection frequency, bubble diameter and bubble number per unit injection time are linked and various bubble generation patterns are shown in Figure 4. The predicted axial velocity is compared with experiment data^[5] in Figure 5a and c. For the axial velocity, the comparison shows that the simulation result is consistent with experiment data among different injection patterns. It is shown that the bubble injection pattern does not have critical influence of the velocity distribution at the same flow rate, which is consistent with the experimental result in previous studies.^[45,56] The turbulent kinetic energy with different bubble generation pattern is compared at the central height and at the gas–liquid interface. In Figure 5b, it is shown that different injection frequencies with the same bubble size do not have an obvious influence on the turbulence dissipation, because the bubble injection is

Figure 3. a) The axial velocity, b) radial velocity, and c) turbulent kinetic energy at different heights in the Euler-Euler approach with different bubble sizes.

symmetrical. In Figure 5d, it is shown that different bubble sizes with the same injection frequency have a small impact on the turbulence dissipation. The smaller the bubble diameter is, the larger the axial velocity and turbulent kinetic dissipation are in the central zone.

Table 4. Parameters of gas injection in the water experiment.^[5]

Diameter of porous plug [mm]	Pore size [mm]	Flow rate [cm ³ s ⁻¹]	Group	Bubble frequency [f] [s ⁻¹]	Bubble diameter [d _b] [mm]	Stream number [N _b]
60	1 mm	200	A	100	2	480
			B	200	2	240
			C	1000	2	48
			D	100	1	3840
			E	100	4	60

4.4. Effect of Turbulent Dispersion in the Euler-Lagrange Approach

To describe the turbulent dispersion in the Euler-Lagrange approach, the Discrete Random Walk Model is used to calculate the kinetic energy dissipation of eddies. The characteristic lifetime of the eddy is defined as a constant $\tau_e = 2T_L$, and T_L is the fluid Lagrangian integral time which may be expressed as follows:

$$T_L = C_L \frac{k}{\varepsilon} \quad (28)$$

where C_L is a constant to be determined as it is not well known. For the standard k - ε model and its variants, the default value of C_L is 0.15.

On the one hand, the bubble behavior in the primary region is mainly related to the inlet momentum. On the other hand, the bubble behavior is mainly controlled by the rising distance in the buoyancy region.^[57,58] The velocity and turbulence dissipation of bubbles have a major impact on the stirring pattern in the ladle as well as on the spout eye formation at the gas-liquid interface.^[11,59] In the Euler-Lagrange approach, the turbulent dispersion between bubble particles and turbulent eddies has a large influence on the plume structure, as shown in **Figure 6a–d**. Based on the experimental data,^[5] the plume width is chosen as the boundary of the plume with gas fractions equal to zero in the gas fraction profile. For the mathematical modeling, the plume width is measured as the boundary with the ratio including total gas particles larger than 99%. In the actual experiment, the plume waggles because of the bubbles' random rising. However,

Figure 4. Regime of discrete bubbles formed when injecting gas through porous plugs. Experiment is reproduced with permission.^[5] 1990, Springer US.

Figure 5. The velocity magnitude and turbulent kinetic energy at different heights with different bubble injection frequencies and bubble sizes. a) Velocity magnitude with different injection frequencies, b) Turbulent kinetic energy with different injection frequencies, c) Velocity magnitude with different sizes, and d) Turbulent kinetic energy with different sizes.

in the mathematical modeling, the realizable $k-\varepsilon$ model is an average method. Although the Discrete Random Walk Model is added, the plume swing is underestimated to a certain extent. In Figure 6e, with the constant C_L increasing, the plume becomes wider. In the model, the default value of C_L is 0.15 in which the plume width predicted by modeling is less than the experimental result. In Figure 6f, when the constant C_L increases, the spout width increases while the spout height decreases. According to the plume structure in Figure 6a, in the primary zone, the plume width decreases along the axial direction from the plug exit because of counteraction by the liquid stirring; in the transition zone, the bubble velocity increases and reaches the terminal velocity. At the same time, the plume width increases because of the bubble turbulent energy dispersion to the surrounding liquid. In the buoyancy zone, the bubble rises up vertically and the plume width stays fairly constant. When the constant C_L is 0.15, the modeling

result gives a similar trend that the plume width increases more gradually along the axial direction from the plug exit.

4.5. Bubble Coalescence and Breakup

4.5.1. Transition Theorem between the DPM Model and the VOF Model

For the bubble cluster generated by the porous plug, the bubble size is mainly determined by the pore size. The initial bubble size can be assumed to be constant at 2 mm. With the flow rate increasing, the bubble injection frequency rises, and the possibility of bubble coalescence also rises, so the discrete bubbling regime gradually changes to a transition coalescence regime, then jetting bubbling. For the discrete bubbling regime,

Figure 6. Influence of the time scale constant on the plume structure. a) Experiment, b) $C_L = 0$, c) $C_L = 0.05$, d) $C_L = 0.15$, e) plume width, and f) spout size. Experiment is reproduced with permission.^[5] 1990, Springer US.

Figure 7. Transition mechanism between VOF model and DPM model.

Figure 8. Three different bubbling regimes for porous plug and their bubble distribution along the vertical distance from bottom. a) and d) $200\text{ cm}^3\text{ s}^{-1}$, b) and e) $600\text{ cm}^3\text{ s}^{-1}$, c) and f) $1200\text{ cm}^3\text{ s}^{-1}$, g) Bubble distribution along the vertical distance from bottom.

Figure 9. Bubble distribution along the vertical distance from bottom. a) Previous work, b) Present work.

the Euler-Lagrange approach is well-suited to use because the volume fraction of gas in the plume zone is low. For the jetting regime, the Euler-Euler approach is more suitable and the population balance model can be coupled to account for the bubble size distribution.^[17] The Euler-Lagrange can also be improved to describe the transition coalescence regime and jetting regime, and the population balance model is user-defined and combined with the Euler-Lagrange approach.^[19] But the gas volume fraction in the domain cannot be properly described. In the present paper, the small bubble can be tracked in the Lagrange coordinate using DPM model. Meanwhile, the large bubble can be transferred to be tracked in the Eulerian coordinates using a VOF model. The bubble droplet size can be statistically analyzed using a user-defined function code. The gas phase can be tracked by the transition method from DPM to VOF using a user-defined code,^[60,61] and the vice versa of transition method from VOF to DPM is calculated by using a new model in the ANSYS Fluent package.^[48,62,63] as shown in Figure 7.

The modeling of the bubble coalescence and breakup are compared with experimental data^[5] in Figure 8a–f. For the discrete bubbling with $200 \text{ cm}^3 \text{ s}^{-1}$, a small part of discrete particles collide in the central plume zone. Only a few small size bubbles between 3 and 4 mm are tracked by the VOF model. With the gas flow rate increasing to $600 \text{ cm}^3 \text{ s}^{-1}$, more discrete particles collide and large bubbles with 20–35 mm diameters appear. However, there are still a large number of discrete bubble particles outside the plume zone. When the gas flow rate increases to $1200 \text{ cm}^3 \text{ s}^{-1}$, most of the bubble particles disappear and are directly combined into large bubbles of 23–42 mm at the injection zone of porous plug. Five figures in the calculation result are post-processed to get the average value of the bubble diameter. The bubble distributions along the vertical position from the bottom with various gas flow rates are also compared with experimental data in Figure 8g. For the bubble size distribution along the vertical direction, the bubble diameter decreases as the distance from the bottom increases.

4.5.2. Bubble Breakup and Coalescence in the Euler-Euler Approach Coupled with the Population Balance Model

The bubble distributions along the vertical direction in two previous models^[18,19] are compared with experimental data^[5] in Figure 9a. It is shown that the Discrete Phase Model is better

than the Eulerian Multiphase Model in predicting the bubble distribution when considering bubble collisions, coalescences, and breakups. The governing equations to describe bubble behaviors in the Population Balance Model are given in Section 2.2.4.

In the present work, the Eulerian Multiphase Model coupled with Population Balance Model are also used to predict the bubble size distribution. The calculation systems in Figure 1a and b are used for the condition without top gas (non-free surface) and with top gas (free surface), respectively. In the calculation modeling without top gas, the degassing condition is applied on the gas outlet boundary for a flat non-free surface, so the bubble size distribution is controlled by the model itself. Nevertheless, in the modeling of the gas leaving the melt, the bubble diameter can be adjusted manually. The bubble diameter along vertical distance from the bottom in two calculation systems are compared in Figure 9b. It is shown that the calculation system using gas backflow as an outlet boundary condition is better than the system using degassing as an the outlet boundary condition. However, for the calculation, the drawback of this model is that a decreasing rate of the bubble diameter along the vertical distance from the bottom is different from the actual condition: in the experimental data, the bubble diameter decreases sharply at the primary zone and the decreasing rate of the bubble diameter along the vertical distance from the bottom decreases gradually. However, in the calculation, the bubble diameter also decreases to a lower extent at the primary zone and the bubble diameter decreases sharply at the surface zone.

5. Conclusions

The paper presents the flow pattern in the ladle using the Euler-Euler approach and Euler-Lagrange approaches. The effects of various forces, bubble sizes and bubble generation frequencies on the flow pattern have been discussed. In the Euler-Euler approach, differences of axial velocity and turbulent kinetic energy values with various bubble sizes mainly appear at the plume zone. In the Euler-Lagrange approach, the simulation result is consistent with the

experiment data among different injection patterns with different bubble injection frequencies and bubble sizes. Furthermore, different injection frequencies with the same bubble size do not have an obvious influence on the turbulence dissipation, otherwise, different injection frequencies with the same bubble size has a small influence on the turbulence dissipation. When the turbulent dispersion increases, the plume width and the spout eye width increases while the spout eye height decreases. For both modeling approaches, the smaller the bubble diameter is, the larger the axial velocity and turbulent kinetic dissipation are in the central zone. For the bubble coalescence and breakup, the Euler-Euler approach and Euler-Lagrange approach are compared. In the Euler-Lagrange approach, the improved model for the bubble distribution including a transition between the DPM model and the VOF model agrees well with the experimental result; in the Euler-Euler approach, the Eulerian Multiphase Model coupled with Population Balance Model has a drawback of decreasing rate of the bubble diameter along the vertical distance from the bottom. Regarding the comparison of the two approaches, the Euler-Lagrange approach is more accurate in predicting the flow pattern for gas injection with a porous plug in the ladle.

Acknowledgement

Yu Liu extended his sincere appreciate to the CSC (China Scholarship Council) for financial support of his PhD study in KTH-Royal Institute of Technology, Sweden.

Conflict of Interest

The authors declare no conflict of interest.

Nomenclature

d_b	Bubble diameter (m)
d	Inner nozzle diameter (m)
$\rho, \rho_l, \rho_g, \rho_b$	Mixture density; liquid density; gas density; density of bubble particles (kg m^{-3})
α_l, α_g	Liquid volume fraction; gas volume fraction
$\vec{u}, \vec{u}_q, \vec{u}_{rel}, u_{bi}$	Velocity component of mixture fluid; velocity component of liquid phase; relative velocity between gas and liquid; bubble particle velocity; plume velocity (m s^{-1})
g	Acceleration of gravity (m s^{-2})
p	Total pressure (N m^{-2})
μ, μ_t	mixture viscosity; turbulent viscosity ($\text{kg m}^{-1} \text{s}^{-1}$)
F_s	Surface tension force (N m^{-3})
σ	Surface tension coefficient (N m^{-1})
κ	Curvature (m^{-2})
$B_{ag,i}, B_{br,i}$	Birth term due to aggregation; birth term due to breakage
$D_{ag,i}, D_{br,i}$	Death term due to aggregation; death term due to breakage
F_{Di}	Drag force (N)
Q_{bi}	Bubble injection mass flow rate (kg s^{-1})

Δt	Time step (s)
V	Volume of the bubble (m^3)
τ_r	Bubble relaxation time

Keywords

bubble coalescence and breakup, ladle, mathematical modeling, porous plug

Received: September 19, 2018

Revised: December 13, 2018

Published online: February 27, 2019

- [1] D. Mazumdar, R. I. L. Guthrie, *ISIJ Int.* **1995**, 35, 1.
- [2] L. F. Zhang, S. Taniguchi, *Int. Mater. Rev.* **2000**, 45, 59.
- [3] D. Sichen, *Steel Res. Int.* **2012**, 83, 825.
- [4] G. Irons, A. Senguttuvan, K. Krishnapisharody, *ISIJ Int.* **2015**, 55, 1.
- [5] P. E. Anagbo, J. K. Brimacombe, *Metall. Trans. B* **1990**, 21, 637.
- [6] S. T. Johansen, D. G. C. Robertson, K. Woje, T. A. Engh, *Metall. Trans. B* **1988**, 19, 745.
- [7] B. Trummer, W. Fellner, A. Viertauer, L. Kneis, G. Hackl, *Bulletin* **2016**, 1, 35.
- [8] S. Joo, R. I. L. Guthrie, *Metall. Trans. B* **1992**, 23, 765.
- [9] D. Guo, G. A. Irons, *Metall. Mater. Trans. B* **2000**, 31, 1447.
- [10] D. Guo, G. A. Irons, *Metall. Mater. Trans. B* **2000**, 31, 1457.
- [11] D. Mazumdar, R. I. L. Guthrie, *Metall. Mater. Trans. B* **2010**, 41B, 976.
- [12] Z. Q. Liu, L. M. Li, B. K. Li, *ISIJ Int.* **2017**, 57.
- [13] O. J. Illegbusi, J. Szekely, *ISIJ Int.* **1990**, 30, 731.
- [14] W. T. Lou, M. Y. Zhu, *Metall. Mater. Trans. B* **2013**, 44B, 1251.
- [15] G. C. Wang, H. C. Zhou, Q. R. Tian, X. G. Ai, L. F. Zhang, *ISIJ Int.* **2017**, 57, 805.
- [16] Y. G. Xu, M. Ersson, P. G. Jönsson, *ISIJ Int.* **2016**, 56, 1982.
- [17] L. M. Li, Z. Q. Liu, B. K. Li, H. Matsuura, F. Tsukishashi, *ISIJ Int.* **2015**, 55, 1337.
- [18] F. Karouni, B. P. Wynne, J. Talamantes-Silva, S. Phillips, *Steel Res. Int.* **2018**, 89, 1700550.
- [19] A. Alexiadis, P. Gardin, J. F. Domgini, *Metall. Mater. Trans. B* **2004**, 35, 949.
- [20] J. Aoki, L. F. Zhang, B. G. Thomas, *3rd Int. Congress on Science & Technology of Steelmaking*, Association for Iron & Steel Technology, Warrendale **2005**.
- [21] T. Zhang, Z. G. Luo, C. L. Liu, H. Zhou, Z. S. Zou, *Powder Technol.* **2015**, 273, 154.
- [22] T. Zhang, Z. G. Luo, H. Zhou, B. Ni, Z. S. Zou, *ISIJ Int.* **2016**, 56, 116.
- [23] S. W. P. Cloete, J. J. Eksteen, S. M. Bradshaw, *Prog. Comput. Fluid Dyn.* **2009**, 9, 345.
- [24] Y. L. Li, L. F. Zhang, Y. Ren, *Proc. of the Extraction and Processing Division Symposium on Pyrometallurgy in Honor of David G. C. Robertson*, San Diego, February **2014**.
- [25] H. P. Liu, Z. Y. Qi, M. G. Xu, *Steel Res. Int.* **2011**, 82, 440.
- [26] L. M. Li, Z. Q. Liu, M. X. Cao, B. K. Li, *JOM* **2015**, 67, 1459.
- [27] L. M. Li, B. K. Li, Z. Q. Liu, *ISIJ Int.* **2017**, 57, 1.
- [28] Z. Q. Liu, F. S. Qi, B. K. Li, M. F. Jiang, *Metall. Mater. Trans. B* **2015**, 46B, 933.
- [29] Z. Q. Liu, B. K. Li, *Metall. Mater. Trans. B* **2017**, 48B, 1833.
- [30] R. Singh, D. Mazumdar, A. K. Ray, *ISIJ Int.* **2008**, 48, 1033.
- [31] W. T. Lou, M. Y. Zhu, *Metall. Mater. Trans. B* **2013**, 44B, 762.
- [32] W. T. Lou, M. Y. Zhu, *ISIJ Int.* **2014**, 54, 9.
- [33] W. T. Lou, M. Y. Zhu, *Metall. Mater. Trans. B* **2014**, 45B, 1706.
- [34] S. Yu, S. Louhenkilpi, *Metall. Mater. Trans. B* **2013**, 44B, 459.

- [35] S. Yu, J. Miettinen, S. Louhenkilpi, *Mater. Sci. Forum* **2013**, 762, 253.
- [36] S. Yu, J. Miettinen, S. Louhenkilpi, *Steel Res. Int.* **2014**, 85, 1393.
- [37] S. Yu, J. Miettinen, L. Shao, S. Louhenkilpi, *Steel Res. Int.* **2015**, 86, 466.
- [38] V. De Felice, I. L. A. Daoud, B. Dussoubs, A. Jardy, J. P. Bellot, *ISIJ Int.* **2012**, 52, 1273.
- [39] J. P. Bellot, V. De Felice, B. Dussoubs, A. Jardy, S. Hans, *Metall. Mater. Trans. B* **2014**, 45B, 13.
- [40] A. Huang, H. Z. Gu, M. J. Zhang, N. Wang, T. Wang, Y. Zou, *Metall. Mater. Trans. B* **2013**, 44B, 744.
- [41] F. Karouni, B. P. Wynne, J. Talamantes-Silva, S. Phillips, *Steel Res. Int.* **2018**, 89, 1700551.
- [42] M. Sano, K. Mori, T. Sato, *Tetsu-to-Hagane* **1977**, 63, 2308.
- [43] M. Sano, K. Mori, *Trans. Jpn Inst. Met.* **1976**, 17, 344.
- [44] S. T. Johansen, F. Boysan, *Metall. Trans. B* **1988**, 19, 755.
- [45] Y. Sahai, R. I. L. Guthrie, *Metall. Trans. B* **1982**, 13, 193.
- [46] Y. Y. Sheng, G. A. Irons, *Metall. Trans. B* **1993**, 24, 695.
- [47] Y. Y. Sheng, G. A. Irons, *Metall. Mater. Trans. B* **1995**, 26, 625.
- [48] ANSYS, ANSYS FLUENT Theory Guide Version 19.1, ANSYS Inc., Canonsburg **2017**.
- [49] ANSYS, ANSYS Population Balance Module Manual Version 19.1, ANSYS Inc., Canonsburg **2017**.
- [50] P. J. O'Rourke, *Doctoral Thesis*, Princeton University, **1981**.
- [51] H. Luo, H. F. Svendsen, *AIChE J.* **1996**, 42, 1225.
- [52] H. Luo, *Doctoral Thesis*, Trondheim University, **1993**.
- [53] W. Gumprich, M. Chrigui, M. Braun, A. Sadiki, *7th Int. Conf. on Multiphase Flow, Tampa, May 30th–June 4th 2010*.
- [54] I. B. Celik, U. Ghia, P. J. Roache, *J. Fluids Eng.* **2008**, 130, 078001.
- [55] Z. Q. Liu, B. K. Li, *Chem. Eng. J.* **2018**, 338.
- [56] T. Stapurewicz, N. J. Themelis, *Can. Metall. Q.* **1987**, 26, 123.
- [57] A. H. Castillejos, J. K. Brimacombe, *Metall. Trans. B* **1987**, 18, 659.
- [58] M. Iguchi, H. Takeuchi, Z. Morita, *ISIJ Int.* **1991**, 31, 246.
- [59] Y. Kishimoto, Y. Y. Sheng, G. A. Irons, J. S. Chang, *ISIJ Int.* **1999**, 39, 113.
- [60] L. M. Li, W. Y. Ding, F. F. Xue, C. Xu, B. K. Li, *JOM* **2018**, 70, 2900.
- [61] M. J. Sun, B. K. Li, L. M. Li, *Metall. Mater. Trans. B* **2018**, 49B, 2821.
- [62] ANSYS, Using the VOF-to-DPM Model Transition for Dispersion of Liquid in Gas, https://ansyshelp.ansys.com/account/secured?returnurl=/Views/Secured/corp/v191/flu_ug/flu_ug_sec_vof_dpm.html, accessed: **2018**.
- [63] ANSYS, VOF to DPM CFD Spray Model, <https://www.ansys.com/blog/fluvent-19-speeds-cfd-spray-simulations>, accessed: **2018**.