Managing uncertainties in geotechnical parameters: From the perspective of Eurocode 7

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Abstract

Geotechnical engineering is strongly associated with large uncertainties. Geotechnical site investigations are made only at discrete points and most of a soil volume is never tested. A major issue is therefore how to cost effectively reduce geotechnical uncertainties with respect to structural performance. Managing the geotechnical uncertainties is thus an important aspect of the design process. Guidance on this subject is given in the European design code for geotechnical design, Eurocode 7 (EN 1997), which advocates the use of the partial-factor method, with the added possibility to use the observational method if the uncertainties are large and difficult to assess.

This thesis aims to highlight, develop and improve methods to assess the quality and value of geotechnical site investigations through reliability-based design. The thesis also discusses the limitations of the deterministic partial-factor method, according to its EN 1997 definition, and how to better harmonise this design methodology with the risk-based approach of reliability-based design. The main research contributions are: (1) a presented case study showing the importance of and potential gains with a robust framework for statistical evaluation of geotechnical parameters, (2) the discussion on the limitations of the partial-factor method in EN 1997, and (3) the discussion on how to harmonise the EN 1997 definition of the partial-factor method with the risk-based approach of reliability-based design.

Keywords: Soil and rock engineering, the extended multivariate approach, reliability-based design, Eurocode 7, EN 1997, the partial-factor method, structural safety.

Sammanfattning

Geoteknik förknippas med ett stort antal osäkerheter. Geotekniska undersökningar utförs i diskreta punkter och den absolut största delen av en jordvolym testas aldrig. En viktig fråga inom geotekniken är därför hur man kan minimera osäkerheterna i relation till kostnaderna för undersökningarna och konstruktionens säkerhet. En annan fråga som uppstår när en konstruktion ska dimensioneras är hur osäkerheterna bör hanteras i dimensioneringsprocessen. För detta ges vägledning i de europeiska dimensioneringsprinciperna för geotekniska konstruktioner, Eurokod 7 (EN 1997), som förespråkar partialkoefficientmetoden. Om osäkerheterna är stora och svåra att uppskatta kan observationsmetoden användas som ett komplement.

Syftet med denna avhandling är att belysa, utveckla och förbättra metoder för bygg- och anläggningsbranschen att uppskatta kvalité och värde av geotekniska undersökningar. Ett bayesianskt tillvägagångssätt (multivariabel analys) har använts för att utvärdera osäkerheterna relaterade till de geotekniska undersökningarna. Därtill presenterar avhandlingen en diskussion om begränsningarna i partial-koefficientmetoden, enligt definitionen i EN 1997, och hur denna metod kan utvecklas för att bättre harmonisera med sannolikhetsbaserad dimensionering. De huvudsakliga forskningsbidragen är: (1) den presenterade fallstudien som visar vikten, och den potentiella nyttan, av ett robust ramverk för statistisk utvärdering av geotekniska parametrar som grund för sannolikhetsbaserad dimensionering, (2) diskussionen om begränsningarna med partialkoefficientmetoden enligt definitionen i EN 1997, och (3) diskussionen om hur partialkoefficient metoden kan utvecklas för att bättre harmonisera med sannolikhetsbaserad dimensionering.

Nyckelord: Jord- och bergmekanik, multivariabel analys, sannolikhetsbaserad dimensionering, Eurokod 7, EN 1997, partialkoefficientmetoden.

Preface

The work presented in this thesis was performed at the Division of Soil and Rock Mechanics, KTH Royal Institute of Technology, Sweden. Main supervisor was Professor Stefan Larsson. To Stefan, your support in both personal and professional matters has been invaluable. Assistant supervisors were Dr Johan Spross and Dr Rasmus Müller. I am truly grateful for your support and guidance.

I would also like to express my gratitude to all my colleagues at the Division of Soil and Rock Mechanics for valuable discussions and for making my time at the department such a pleasant stay. Finally, to my wife Frida and my two boys Bertil and Axel, thank you for your endless support; words are powerless to express my gratitude to you.

Borlänge, April 2019
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List of appended papers

This doctoral thesis contains the following scientific articles:

Paper A

Prästings, A., Müller, R., and Larsson, S. 2014. The Observational Method Applied to a High Embankment Founded on Sulphide Clay. Engineering Geology, 181: 112–123, doi: 10.1016/j.enggeo.2014.07.003.

I made the case study and wrote the paper based on work initiated by Müller. Müller and Larsson both contributed to the paper with valuable comments.

Paper B

Lingwanda, M.I., Prästings, A., Larsson, S. and Nyaoro, D. 2017. Comparison of Geotechnical Uncertainties Linked to Different Soil Characterization Methods. Geomechanics and Geoengineering: An International Journal, 12(2): 1–15, doi: 10.1080/17486025.2016.1184761.

Mwajuma Ibrahim Lingwanda performed the analysis and wrote the majority of the paper. I contributed to the paper with writing, comments on the performed analysis and article structure. Larsson and Nyaoro contributed to the paper with valuable comments.

Paper C

Hov, S., Prästings, A., Persson, E. and Larsson, S. 2019. On Empirical Correlations for Normalized Shear Strength from Fall Cone and Direct Simple Sear Tests in Soft Swedish Clays. Submitted to Geotechnical and Geological Engineering.

Hov and I wrote this paper together on the initiative by Hov. I performed the analysis. Persson collected the data within his MSc thesis and Larsson contributed to the paper with valuable comments.

Paper D

Prästings, A., Spross, J., Müller, R., Larsson, S., Bjureland, W. and Johansson, F. 2017. Implementing the Extended Multivariate Approach in Design with Partial Factors for a Retaining Wall in Clay. ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, 3(4): 04017015, doi: 10.1061/AJRUA6.0000918.

I made the case study and wrote the paper. Spross assisted in writing the paper. Müller, Larsson, Bjureland and Johansson contributed to the paper with valuable comments.

Paper E

Prästings, A., Müller, R. and Larsson, S. 2017. Optimizing Geotechnical Site-Investigations. In Proceedings of the 19th International Conference on Soil Mechanics and Geotechnical Engineering, Seoul, South Korea, pp. 639–642.

I made the study and wrote the paper. Müller and Larsson contributed to the paper with valuable comments.

Paper F

Prästings, A., Spross, J. and Larsson, S. 2019. Characteristic Values of Geotechnical Parameters in Eurocode 7. Proceedings of the Institution of Civil Engineers – Geotechnical Engineering, 172(4): 301-311, doi: 10.1680 /jgeen.18.00057.

I made the study and wrote the paper. Spross and Larsson contributed to the paper with valuable comments and discussions.

Other journal papers

As part of my research project, I have contributed to the following journal and conference papers (not part of this thesis).

Prästings, A., Larsson, S. and Müller, R. 2016. Multivariate Approach in Reliability-Based Design of a Sheet Pile Wall. Transportation Geotechnics, 7: 1–12, doi: 10.1016/j.trgeo.2016.03.001.

Bjureland, W., Spross, J., Johansson, F., Prästings, A. and Larsson, S. 2017. Reliability Aspects of Rock Tunnel Design with the Observational Method. International Journal of Rock Mechanics and Mining Sciences, 98(7): 102-110. doi: 10.1016/j.ijrmms.2017.07.004.

Bjureland, W., Spross, J., Johansson, F., Prästings, A. & Larsson, S. 2017. Challenges in Applying Fixed Partial Factors to Rock Engineering Design. In Geo-Risk 2017: Reliability-Based Design and Code Development (Huang, J., Fenton, G.A., Zhang, L. & Griffiths, D.V. (eds)). American Society of Civil Engineers, Reston, VA, USA, GSP 283, pp. 384–393, doi: 10.1061/9780784480700.037.

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1. Introduction

> "In thinking about sources of uncertainty in engineering geology, one is left with the fact that uncertainty is inevitable. One attempts to reduce it as much as possible, but it must ultimately be faced. It is a well-recognised part of life for the engineer. The question is not whether to deal with uncertainty but how?"
>
> (Einstein & Baecher 1982)

1.1. Background

Geotechnical engineering mainly deals with geometries and materials provided by nature. The natural conditions are basically unknown to the geotechnical engineer and must be inferred from geotechnical site investigations, which include observations and measurements performed at discrete locations in soil and rock. In the interpretation of limited site investigations, uncertainties arise because of the difficulties in describing the variable state of nature. Consequently, uncertainties are present in the spatial representation of the geological conditions and in the estimation of geotechnical parameters from a limited number of measurements. Geotechnical engineering therefore differs from structural and mechanical engineering in which the main building materials are man-made, such as steel, concrete, etc., and the geometries and variability of the materials are known.

The uncertainties in geotechnical engineering are largely inductive (Baecher & Christian 2003); based on the given evidence (i.e. observations and measurements) the conclusions (i.e. inferred natural conditions) are probable. In addition to the uncertainty that is related to inherent variability in rock and soil, the unavoidable difference between probable and true – but unknown – conditions represent uncertainty due to lack of knowledge.

A key challenge in geotechnical engineering is to manage these uncertainties in geotechnical design. Historically, a methodology referred to as the observational method has been used to manage large uncertainties in geotechnical design and the risk of undesirable events occurring during construction. Sprung from ideas by Karl Terzaghi (early 20th century) and later formulated by Peck (1969), the observational method offers a rational procedure for managing uncertainties as the practitioner can embrace a form of active design or, as stated by Terzaghi, a learn-as-you-go approach. To account for uncertain geotechnical parameters and other unknown circumstances related to the design, probable conditions and the most unfavourable deviation from these conditions are assessed and supplemented by predefined action plans, should these unfavourable deviations ever occur. Today, the observational method is acknowledged as one of four possible limit state verification methods in the European design codes for geotechnical structures, EN 1997 (CEN 2004). However, as opposed to the simpler learn-as-you-go approach, the occurrence of probable conditions, and unfavourable deviations from these conditions, must be estimated using probability theory in EN 1997. The current definition in EN 1997 is discussed in Spross & Larsson (2014) and Spross & Johansson (2017) present a framework for determining under what circumstances the observational method is favourable.

The field of structural reliability was first introduced in the 1950s and techniques to solve different reliability problems have later been introduced by Cornell (1969), Hasofer & Lind (1974), and Rosenblueth (1975; 1981). Their work made it possible to analytically solve limit-state equations, while accounting for uncertain parameters through their probabilistic descriptions. This development of reliability-based design allowed unacceptable performance (e.g. unfavourable movements or collapse) of a construction to be expressed in terms of probability, based on the uncertainty in the design input. The current approach to ensure acceptable performance of geotechnical structures in the presence of uncertainty (i.e. managing project risks) is to apply partial factors on the parameters in any geotechnical limit state. In EN 1997, this approach is referred to as the partial-factor method and the objective is to calibrate a set of resistance or partial factors to achieve a specified reliability level for a wide range of structures (Olsson & Stille 1984; Thoft-Christensen & Baker 1982). Following EN 1997 and design approach 3 (CEN 2004), the partial factors are applied to geotechnical parameters and to actions (e.g. external loads) to verify that no limit state is violated. That is, that the technical requirements regarding a construction are fulfilled and the probability of collapse or unacceptable performance is sufficiently small. If the uncertainties that are related to the estimation of geotechnical parameters are small (i.e. an extensive number of high-quality measurements have been performed), one may argue that this should produce a relative decrement of the partial factor, and vice versa. In current design practice (i.e. EN 1997), however, the partial factor is fixed, which limits the possibility to account for varying uncertainties in geotechnical design. There is a forthcoming revision of EN 1997 which feeds the discussion in the geotechnical community concerning the current definition of the partial-factor method (Bjureland et al. 2017; Orr 2017; Phoon 2017a; Phoon & Ching 2013; Schneider & Schneider 2013; Wang 2017).

Significant research on the description of uncertainties in geotechnical parameters has been presented by Vanmarke (1977), Lacasse & Nadim (1996) and Phoon & Kulhawy (1999a; 1999b), to mention just a few. Notably, high-quality site investigations are costly and consequently, a trade-off arises between the cost of reducing the uncertainty by performing additional site investigations and the increased structural requirements that are needed to achieve the specified reliability level. An incentive thus exists to combine knowledge gained from site investigations with the prior knowledge of experts and of existing empirical correlations, which may reduce the uncertainty and decrease the structural requirements. For this reason, Bayesian approaches have been adopted in numerous studies related to geotechnical engineering, most recently by Ching et al. (2010), Cao & Wang (2014a; 2014b), Müller et al. (2014; 2016), Cao et al. (2016), Wang et al. (2016), Wang & Zhao (2017), Yang et al. (2017) and Zhang et al. (2018). With a Bayesian approach in geotechnical design, it is possible to reduce the uncertainty due to lack of knowledge through a systematic updating procedure of different sources of information.

1.2. Aim of thesis

The aim of this thesis is to highlight, develop and improve methods to assess the quality and value of geotechnical site investigations.

The aim was formulated in the context of making statistical evaluations of geotechnical parameters and reliability-based design readily accessible to the industry. This naturally includes a discussion on how to make the definition of the EN 1997 partial-factor method to better harmonise with the risk-based approach of reliability-based design.

1.3. Outline of thesis

The thesis consists of a summarising essay accompanied by six appended research papers (A-F). The essay provides a summary of the subjects covered in the research papers, with the objective of putting the papers into a wider context.

Five main chapters (2-6) cover the following subjects: sources of uncertainty in geotechnical design, reliability theory in geotechnical design, geotechnical design in the Eurocodes, Bayesian statistics, and the observational method. This is followed by chapter 7, which gives a summary of the appended papers:

- Paper A is a published, peer-reviewed journal paper. The paper discusses the applicability of the observational method for design and construction of an embankment on soft clay, based on a case study of an embankment project in northern Sweden.
- Paper B is a published, peer-reviewed journal paper. The paper provides a comparison of uncertainties linked to different soil characterisation methods for assessment of the oedometer modulus.
- Paper C is under review in a journal paper. The paper evaluates the performance of common empirical transformation models for the undrained shear strength.
- Paper D is a published, peer reviewed journal paper. The paper discusses how the extended multivariate approach can be implemented in the design framework of EN 1997, based on a case study of a project in Gothenburg, Sweden.
- Paper E is a published, peer-reviewed conference paper. The paper provides an example on how to incorporate the extended multivariate approach in the design framework of EN 1997 and

how the methodology can be of assistance in the planning of site investigations.

- Paper F is a published, peer reviewed journal paper. The paper discusses the suitability of the current definition of the characteristic value of geotechnical parameters (the 5% fractile) and the future progress of the design framework of EN 1997.

The major findings of the papers are presented in chapter 8 and concluding remarks can be found in chapter 9. The concluding remarks cover the most prominent topics of this thesis.

1.4. Main research contributions

The main research contributions are: (1) a presented case study showing the importance and potential gains with a robust framework for statistical evaluation of geotechnical parameters, (2) the discussion on the limitations of the partial-factor method in EN 1997, and (3) the discussion on how to harmonise the EN 1997 definition of the partial-factor method with the risk-based approach of reliability-based design.

1.5. Limitations

The thesis is focused on reliability-based design and the semi-probabilistic approach of the partial-factor method. In general, the thesis assumes a Bayesian interpretation of probability and estimates the reliability of geotechnical structures through the extended multivariate approach. Furthermore, the discussion on characteristic values of geotechnical parameters and on the current design framework of EN 1997 is given in the context of design approach 3 (DA3), which is the approach used in Sweden.

There are uncertainties that cannot be readily accounted for in the probability estimate, which are typically related to human errors. Notably, the Swedish national annex to EN 1997 (IEG 2008b) states that "measurement values that are not representative of the sought-after property shall be removed before estimating the characteristic value". The work performed in this thesis endeavours to achieve an objective evaluation of soil parameters. However, as emphasised in the statement in the Swedish national annex, engineering judgement is needed and has been used to eliminate questionable measurements from the presented studies.

2. Sources of uncertainty in geotechnical design

2.1. Total uncertainty in soil parameters

Uncertainties related to the estimation of soil parameters are commonly divided into two categories: aleatory and epistemic (Lacasse & Nadim 1996; Baecher & Christian 2003; Ang & Tang 2007). Aleatory uncertainty refers to random processes that are unpredictable and occur by chance. One example is the uncertainty due to the inherent variability of soil that arises from unpredictable geological processes. The latter category, epistemic uncertainty, is referred to as knowledge uncertainty or uncertainties of the mind and typically originate from lack of data and lack of understanding of our physical models that are used to describe real world phenomena.

In geotechnical engineering, epistemic uncertainty is typically associated with measurement errors, statistical uncertainty, and transformation uncertainty. Collectively, these sources of uncertainty originate from the process of characterising soil by performing field and laboratory investigations. Measurement errors inherent from equipment, operator and random testing effects (Jaksa et al. 1997), and statistical uncertainty arises because soil parameters are estimated from a limited set of data (Lacasse & Nadim 1996). Transformation uncertainty is introduced when measurements are transformed to the sought-after parameter from field and laboratory measurements of a property by site-specific or empirical transformation models (Phoon & Kulhawy 1999b).

The mean value of a soil parameter evaluated from any field or laboratory investigation technique, D (e.g. cone penetration test (CPT) or constant rate of strain (CRS) oedometer tests), may be modelled as a product of the mean value evaluated from the field or laboratory measurements, $\bar{\xi}_{\mathrm{m}}$, a transformation factor, C, and a random error factor, $\varepsilon_{\mathrm{tot}}$:

$$\bar{\xi}_{\mathrm{d}} \big| \mathrm{D} = \bar{\xi}_{\mathrm{m}} C \varepsilon_{\mathrm{tot}} \,, \qquad [2.1]$$

where the subscript in $\bar{\xi}_{\mathrm{d}}$ indicates that this is a transformed value of a parameter intended for geotechnical design (e.g. undrained shear strength, s_{u}, or effective internal friction angle, ϕ'). The factor $\varepsilon_{\mathrm{tot}}$ represents the error related to the aleatory and epistemic uncertainties and is commonly modelled as the product of its individual components:

$$\varepsilon_{\mathrm{tot}} = \varepsilon_{\mathrm{in}}\varepsilon_{\mathrm{st}}\varepsilon_{\mathrm{me}}\varepsilon_{\mathrm{tr}} \, , \qquad [2.2]$$

where $\varepsilon_{\mathrm{in}}$ is the error in the estimation of the inherent variability of the soil parameter, $\varepsilon_{\mathrm{st}}$ is the statistical error in the estimation of $\bar{\xi}_{\mathrm{m}}$, $\varepsilon_{\mathrm{me}}$ is the measurement error included in $\bar{\xi}_{\mathrm{m}}$, and $\varepsilon_{\mathrm{tr}}$ is the error in the transformation factor C (i.e. the error given from the uncertainty in the transformation model for the relationship $\xi_{\mathrm{m}} - \xi_{\mathrm{d}}$). Each individual error component ε_i has a mean value μ_{ε_i} and a standard deviation σ_{ε_i}. If the error components are assumed to be uncorrelated, the total uncertainty in $\bar{\xi}_{\mathrm{d}}|\mathrm{D}$ may be expressed in terms of its coefficient of variation (applicable only if the coefficient of variation is < 0.3 (Melchers and Beck 2018)):

$$\mathrm{COV}^2_{\bar{\xi}_{\mathrm{d}}|\mathrm{D}} \approx \mathrm{COV}^2_{\mathrm{in},\bar{\mathrm{D}}} + \mathrm{COV}^2_{\mathrm{st},\bar{\mathrm{D}}} + \mathrm{COV}^2_{\mathrm{me},\bar{\mathrm{D}}} + \mathrm{COV}^2_{\mathrm{tr},\bar{\mathrm{D}}} + \vartheta \, , \qquad [2.3]$$

where, in the subscripts, $\bar{\mathrm{D}}$ indicates that the individual COVs represent the uncertainty in the estimate of the mean value $\bar{\xi}_{\mathrm{d}}$, and ϑ represents a statistical model error, which is ignored in most studies (Müller 2013).

The following sections introduce the uncertainties accounted for in Equation 2.3.

2.1.1. Inherent variability

The inherent variability is characterised by the natural variation of soil from one point to another and arises due to variations in geological, environmental, chemical and physical factors influencing the soil or sedimentation process (Mitchell & Soga 2005). These phenomena cause the soil to vary spatially in both the vertical and the horizontal direction. The spatial variation is possible to model as the sum of a trend, a fluctuating component and the fluctuation scale (Vanmarcke 2010). A schematic representation is presented in Figure 2.1 (Phoon and Kulhawy 1999a), in which the in-situ soil property, ξ, is modelled as a function of

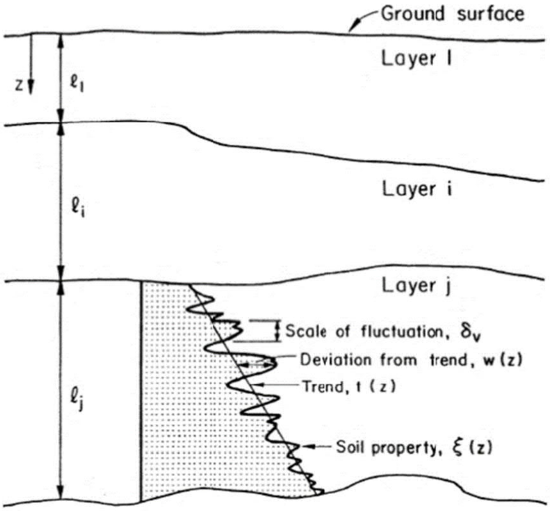

Figure 2.1. Representation of inherent variability of a soil property (Phoon & Kulhawy 1999a. © 1999 Canadian Science Publishing or its licensors. Reproduced with permission.)

depth, z, and decomposed into a smoothly varying trend, $t(z)$, and a fluctuation component, $w(z)$.

The $t(z)$ represents the average value over some volume of soil and is normally evaluated deterministically with regression techniques (Baecher & Christian 2003; Ang & Tang 2007). For a linear trend function (see Figure 2.1), $t(z)$ can be described by:

$$t(z) = a + bz \,, \quad [2.4]$$

where a and b are the regression coefficients, denoting the intercept and slope of the straight line. (It should be noted that soil properties do not always exhibit a linear trend and do not always increase with depth. Examples of other trend functions may be found in Ang & Tang (2007).)

The fluctuating component $w(z)$ in Figure 2.1 represents the inherent variability and is typically modelled as a homogeneous random function (Vanmarcke 2010), which is characterised by a constant mean value and variance (also referred to as stationary). To satisfy the condition of homogeneity in the case of a deterministic trend, $\xi(z)$ must be detrended, which is accomplished by subtracting $t(z)$ from $\xi(z)$ (i.e. evaluating the fluctuation $w(z)$). The mean value and variance are then equal to:

$$\mu_w = \frac{\sum_{i=1}^{n}[w(z_i)]}{n}, \qquad [2.5]$$

and

$$\sigma_w^2 = \frac{\sum_{i=1}^{n}[w(z_i)]^2}{n-1}, \qquad [2.6]$$

where $w(z_i)$ is the fluctuation at depth z_i and n is the number of data points. Furthermore, the coefficient of variation for w is obtained by normalising the standard deviation, σ_w, with respect to the mean value of the trend function, $\bar{t}$:

$$\mathrm{COV}_w = \frac{\sigma_w}{\bar{t}}. \qquad [2.7]$$

The derivation of Equations 2.5 and 2.6 presents a schematic representation of the uncertainty associated with the inherent variability. The σ_w^2 and COV_w thus do not consider the uncertainties that originate from evaluating soil parameters from field and laboratory investigations. In fact, evaluating the inherent variability includes the matter of dealing with measurement artefacts. As such, the inherent variability is estimated from the sample variance, which is calculated based on n number of measurements $\xi_\mathrm{m}(z_i)$ by:

$$\sigma_{\xi_\mathrm{m}}^2 = \frac{\sum_{i=1}^{n}[\xi_\mathrm{m}(z_i) - \bar{\xi}_\mathrm{m}(z)]^2}{n-1}, \qquad [2.8]$$

and relates to the coefficient of variation by:

$$\mathrm{COV}_{\xi_\mathrm{m}} = \frac{\sigma_{\xi_\mathrm{m}}}{\overline{\xi_\mathrm{m}(z)}}, \qquad [2.9]$$

where $\bar{\xi}_\mathrm{m}(z)$ and $\overline{\xi_\mathrm{m}(z)}$ represent the trend function and the mean value of the trend, respectively. As suggested, the calculated $\sigma_{\xi_\mathrm{m}}^2$ and $\mathrm{COV}_{\xi_\mathrm{m}}$ may be imperfect owing to the impact of measurement errors. Consequently, the sample variability evaluated from a quantity of D, $\mathrm{COV}_{\xi_\mathrm{m}|\mathrm{D}}$, consists of both inherent variability and measurement error according to (Orchant et al 1988):

$$\mathrm{COV}^2_{\xi_\mathrm{m}|\mathrm{D}} = \mathrm{COV}^2_{\mathrm{in,D}} + \mathrm{COV}^2_{\mathrm{me,D}}\,, \qquad [2.10]$$

where the inherent variability is estimated by subtracting the measurement error:

$$\mathrm{COV}^2_{\mathrm{in,D}} = \mathrm{COV}^2_{\xi_\mathrm{m}|\mathrm{D}} - \mathrm{COV}^2_{\mathrm{me,D}}\,. \qquad [2.11]$$

The representation of the inherent variability through Equations 2.5 and 2.6 assumes that the soil is statistically homogeneous. This implies that the geological conditions over which the field and laboratory measurements are taken are reasonably uniform. It is therefore vital that only data judged to be coming from the same geological deposit be compared when estimating the characteristics of the inherent variability (Müller 2013).

2.1.2. Measurement error

Measurement errors are inevitably introduced in the execution of field and laboratory investigations, and typically arise from equipment, procedure, operator, and random testing effects (Lumb 1966). Equipment, procedure and operator effects are categorised as systematic (i.e. that they consistently overestimate or underestimate the in-situ soil property and cause a bias in the measurements). For instance, these errors may arise from poorly calibrated equipment and lack of standardised testing procedures, since different operators may perform the same measurement in different ways. Furthermore, one of the major sources of systematic measurement error is sample disturbance, which usually comes from handling of soil samples carelessly, and may cause a reduction of the evaluated mean (Baecher & Christian 2003). The latter, random testing effects, can neither be attributed directly to equipment, operator or procedural effects nor to inherent variability, but are characterised by a scatter, or noise, in the measurements (Jaksa et al. 1997).

Orchant et al. (1988) suggested that the uncertainty related to measurement error may be divided into the three aforementioned sources:

$$\mathrm{COV}^2_{\mathrm{me,D}} = \mathrm{COV}^2_{\mathrm{me,equipment}} + \mathrm{COV}^2_{\mathrm{me,operator}} + \mathrm{COV}^2_{\mathrm{me,random}}\,, \qquad [2.12]$$

where $\mathrm{COV}_{\mathrm{me,equipment}}$ represents the uncertainty related to imperfect equipment, $\mathrm{COV}_{\mathrm{me,operator}}$ represents the uncertainty related to various procedure and operator effects, and $\mathrm{COV}_{\mathrm{me,random}}$ represents the uncertainty related to random measurement errors. Note that these errors are characterised as both systematic and random. However, only random errors are possible to acknowledge as part of the variability off a trend (Jaksa et al. 1997). The uncertainty related to systematic measurement error is therefore typically excluded in $\mathrm{COV}_{\mathrm{me,D}}$. For instance, Orchant et al. (1988) mention that the systematic measurement error is likely to be minimal when good equipment and procedural controls are maintained. By threating the measurement error as a random variable, the $\mathrm{COV}_{\mathrm{me,D}}$ can be reduced by averaging over some volume of soil by the number of measurements, n:

$$\mathrm{COV}^2_{\mathrm{me},\bar{\mathrm{D}}} = \mathrm{COV}^2_{\mathrm{me,D}} \frac{1}{n}, \qquad [2.13]$$

while a potential systematic measurement error would propagate unchanged through a reliability analysis (Whitman 1984).

Separating the random measurement error from inherent variability is complicated (Jaksa et al. 1997), simply because they are both random variables about the trend. A procedure to evaluate random measurement error was presented by Baecher (1983), which utilises the fact that soil properties are spatially correlated, while the random measurement errors are not. Hence, the soil properties at points in close proximity are usually more equal than at points separated by a greater distance. Spatial correlation is commonly described by the soil's autocorrelation structure, which will be discussed further in section 2.2 on random field theory and variance reduction.

During this thesis work, it was found that the measurement errors reported in the literature (e.g. Phoon & Kulhawy 1999a) were sometimes larger than the variability in typical Swedish clays, as evaluated from the sample variance (Equation 2.8). Obviously, this creates a problem when calculating the uncertainty related to inherent variability from Equation 2.11 and the procedure by Baecher (1983) was therefore found to be more stringent, while conducting the case study presented in Paper D. Furthermore, the reported values of measurement error are seldom fully specified (in terms of both random and systematic measurement errors) and should therefore be used with caution.

2.1.3. Statistical uncertainty in the estimation of the trend

The trend function $\bar{\xi}_{\mathrm{m}}(z)$ and its variance $\sigma^2_{\xi_{\mathrm{m}}}$ (Equation 2.8) are point estimates, and hence, because they are estimates, they will probably differ from the actual values. However, they are commonly treated as actual values, which implies that these estimates are determined from an infinite set of measurements, and thus any statistical uncertainty is ignored (Ching et al. 2016a). Soil parameters are typically inferred from a limited number of measurements taken at discrete points in the soil, of which the measurements represent random samples of the variability. This implies that the estimated trend and variance are not fixed if additional measurements are added to a previously performed site investigation (El-Ramly et al. 2002). Hence, the estimated trend function is conditional on a sample of n number of measurements $\{\xi_{\mathrm{m},1}, \xi_{\mathrm{m},2}, \cdots, \xi_{\mathrm{m},n}\}$:

$$\bar{\xi}_{\mathrm{m}}(z) = \hat{a} + \hat{b}z\,, \qquad [2.14]$$

where $\hat{a}$ and $\hat{b}$ are the estimated regression coefficients, typically evaluated based on the least square errors (Ang & Tang 2007). In case of a linear trend, Tang (1980) proposes an unbiased estimate of the variance:

$$\sigma^2_{\xi_{\mathrm{m}}} = \frac{\sum_{i=1}^{n}[\xi_{\mathrm{m}}(z_i) - \bar{\xi}_{\mathrm{m}}(z)]^2 - \hat{b}^2\sum_{i=1}^{n}(z_i - \bar{z})^2}{n-2}\,, \qquad [2.15]$$

where $\bar{z}$ can be interpreted as the centre of gravity of the measurements $\{\xi_{\mathrm{m},1}, \xi_{\mathrm{m},2}, \cdots, \xi_{\mathrm{m},n}\}$.

The statistical uncertainty is the epistemic uncertainty in the assessment of the trend function $\bar{\xi}_{\mathrm{m}}(z)$ and scatter $\sigma^2_{\xi_{\mathrm{m}}}$ about the trend due to limited data. The $\sigma^2_{\xi_{\mathrm{m}}}$ can be expressed in terms of its coefficient of variation (i.e. $\mathrm{COV}_{\xi_{\mathrm{m}}}$) and may be used to calculate $\mathrm{COV}_{\mathrm{in,D}}$ in accordance with Equation 2.11. The effect of statistical uncertainty in the assessment of $\mathrm{COV}_{\mathrm{in,D}}$ may then be evaluated according to (Müller 2013):

$$\mathrm{COV}^2_{\mathrm{st,\bar{D}}} = \mathrm{COV}^2_{\mathrm{in,D}}\psi(n)\,, \qquad [2.16]$$

where $\psi(n)$ is a factor representing the uncertainty in the assessment of the regression coefficients $\hat{a}$ and $\hat{b}$ (i.e. the intersect and slope of the regression line), and $\sigma^2_{\xi_{\mathrm{m}}}$. In the case of no deterministic trend, $\psi(n)$ is equal to $1/n$. For a linear trend, the $\psi(n,z)$ is evaluated according to (Tang 1980):

$$\psi(n,z) = \frac{n-1}{n-3}\left[\frac{1}{n}\left\{1 + \frac{n}{n-1}\frac{(z-\bar{z})^2}{\sigma_z^2}\right\}\right], \qquad [2.17]$$

where σ_z^2 is the sample variance about the centre of gravity for the measurements $\{\xi_{\mathrm{m},1}, \xi_{\mathrm{m},2}, \dots, \xi_{\mathrm{m},n}\}$. The term $(n-1)/(n-3)$ represents the contribution of the uncertainty in the estimate of the basic scatter. Furthermore, the term enclosed in square brackets represents the uncertainty related to the degree of extrapolation of the trend line in the direction of depth, as measured by $(z-\bar{z})^2/\sigma_z^2$ (Zhang et al. 2004; Ang & Tang 2007). Note that for Equation 2.16 to be valid, the sample variance must be determined through Equation 2.15 and the values must be treated as normally distributed, which may require suitable transformation.

It is possible to reduce the potential bias in the estimation of $\bar{\xi}_{\mathrm{m}}(z)$ by increasing n. Figure 2.2 illustrates the contribution of statistical uncertainty (± one standard deviation) in the estimation of inherent variability of s_{u} from direct simple shear tests, DSS. In Figure 2.2(a) the contribution was estimated from 16 measurements, evenly distributed in the soil layer (between 6 and 30 m of depth). In Figure 2.2(b) the contribution was estimated from 9 measurements which are more concentrated to the middle of the soil layer (between 10 and 25 m of depth) compared to (a). Figure 2.2(b) clearly illustrates the increased statistical uncertainty related to the number of measurements and the degree of extrapolation of the trend line in relation to the location of the measurements.

The presented methodology for estimating the contribution of statistical uncertainty was adopted in Paper D and Paper E. Relatively few studies of the statistical uncertainty are available in the literature. Recently, however, Ching et al. (2016b) presented a paper on the statistical characterisation of random field parameters using frequentist and Bayesian approaches. Ching et al. (2016a) also discuss the impact of statistical uncertainty on reliability calculations.

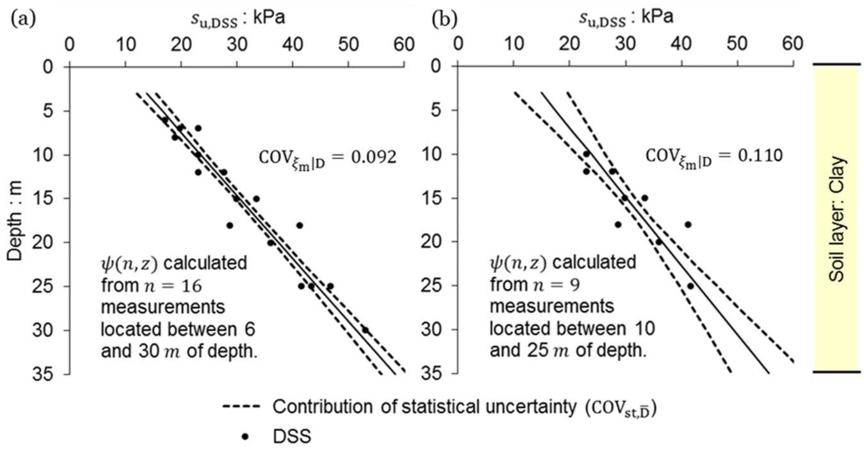

Figure 2.2. Example on the contribution from statistical uncertainty in the assessment of inherent variability of s_u from measurements of DSS; (a) 16 measurements located between 6 and 30 m of depth; (b) 9 measurement located between 10 and 25 m of depth.

2.1.4. Transformation uncertainty

Most field and laboratory investigation techniques register a representative value ξ_m (e.g. the resistance of penetrating the soil with a cone-shaped steel rod, moment of rotating a field vane in the soil, etc.) of the sought-after parameter ξ_d. To determine the sought-after parameter, ξ_m is therefore related to ξ_d via a transformation model, which is represented by the transformation factor C in Equation 2.1. Historically, the transformation factor has been evaluated empirically through back-calculation of embankment and slope failures, which allows the correlation between the analytically derived value of a parameter at the ultimate limit state, and the representative value measured before failure to be studied. Another possibility is to study the pair-wise correlation between the representative value and the parameter, measured directly from laboratory tests. A schematic example of a transformation model is presented in Figure 2.3 (Phoon & Kulhawy 1999b). The transformation model is normally evaluated with regression techniques, and the scatter of data points about the trend represents the error ε_{tr} in the model. The uncertainty is illustrated by the histogram in Figure 2.3, which has a zero mean and variance $\sigma^2_{\varepsilon_{tr}}$ (denoted s_ε in Figure 2.3), and in terms of coefficient of variation, $COV_{tr,\bar{D}}$.

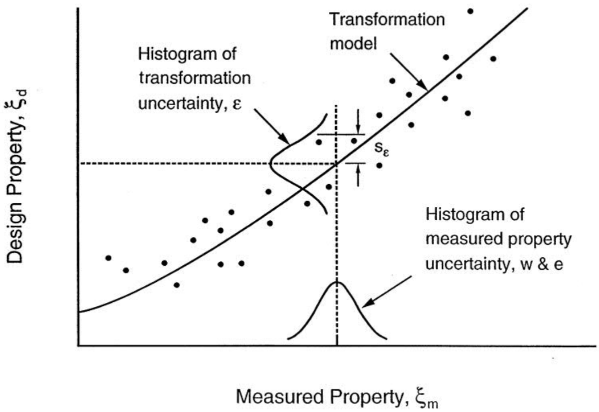

Figure 2.3. Schematic example of a probabilistically evaluated transformation model (Phoon & Kulhawy 1999b. © 1999 Canadian Science Publishing or its licensors. Reproduced with permission.)

The transformation model may systematically overestimate or underestimate the evaluated ξ_{d} and the uncertainty in the transformation is therefore categorised as a systematic uncertainty and thus causes a potential bias in the transformed measurements.

Paper C examines the performance of typical transformation models for fall cone tests (FC)/field vane tests (FV) and for laboratory measurements of CRS oedometer tests. In Paper D and Paper E, the undrained shear strength was evaluated from field measurements of CPT and laboratory measurements of CRS oedometer tests. Empirical relationships were used in the case studies, as suggested by the Swedish Transport Administration (2011 and 2013). The transformation factor can be described by a function of consistency limits, the stress state/history, or both, according to:

$C_{\mathrm{CPT}} = f(w_{\mathrm{L}}, \sigma'_v, OCR)$,	(Larsson & Mulabdic 1991; Demers & Leroueil 2002)	[2.18]
$C_{\mathrm{FV}} = f(w_{\mathrm{L}})$,	(Larsson 1980; Larsson et al. 1987; Larsson et al. 2007)	[2.19]
$C_{\mathrm{FC}} = f(w_{\mathrm{L}})$,	(Larsson 1980; Larsson et al. 1987; Larsson et al. 2007)	[2.20]
$C_{\mathrm{CRS}} = f\left(w_{\mathrm{L}}, \sigma'_p, \sigma'_v, OCR\right)$,	(Mesri 1975; Larsson 1980; Jamiolkowski et al. 1985; Larsson et al. 2007)	[2.21]

where w_{L} is the liquid limit, σ'_{v} is the effective overburden pressure, σ'_{p} is the preconsolidation stress and OCR is the over-consolidation ratio ($\sigma'_{\mathrm{p}}/\sigma'_{\mathrm{v}}$).

Although several theoretical methods exist for establishing transformation factors, Westerberg et al. (2015) mention that additional studies of empirical transformation models are still needed for practical purposes (i.e. in cases when the geotechnical site investigation does not allow site-specific transformation factors to be evaluated).

Based on a large database consisting of FV, DSS and CRS measurements, D'Ignazio et al. (2016) show that there is a weak correlation between the s_{u} determined from FV and the w_{L}. For commonly used transformation models, the potential bias in the prediction of s_{u} may be significant, potentially leading to a relatively large value of $\mathrm{COV}_{\mathrm{tr},\bar{\mathrm{D}}}$. The results in D'Ignazio et al. (2016) were confirmed in Paper C, where a large database of FC and DSS measurements was used to analyse the effectiveness of the transformation models C_{FC} and C_{CRS} in the prediction of s_{u}.

Improving the empirical transformation models reduces the transformation uncertainty. However, an alternative way forward is to adopt Bayesian analysis to cross-validate different sources of information. A Bayesian analysis referred to as the extended multivariate approach (EMA) (Müller et al. 2014 and 2016) was used in Paper D and Paper E. The EMA reduces the transformation uncertainty by cross-validation of different site-specific and empirical transformation models (Equations 2.18-2.21). The concept of Bayesian statistics is introduced in chapter 5.

2.2. Random field theory and variance reduction

2.2.1. Random field theory

As previously stated, soil properties vary spatially in both the vertical and the horizontal direction. Figure 2.1 presents an example of a random field, in which the inherent variability is modelled as the sum of a trend t and a fluctuating component w. However, the complete description of the characteristics of the inherent variability requires the evaluation of the scale of fluctuation from the soil's autocorrelation structure. It should be noted that the variability in the residuals of the trend is estimated by modelling the soil as a homogeneous random field (chapter 2.1.1), i.e. the mean and variance have the same value within the deposit, and the residuals are assumed to be uncorrelated. The vertical and horizontal variability may on the other hand exhibit local areas with somewhat higher and somewhat lower values of the property, and consequently the assumption of uncorrelated residuals tends to be poor for geological data (DeGroot & Baecher 1993).

To demonstrate the vertical case, Figure 2.4 presents a regression analysis on q_c data of CPT, in which local areas of higher and lower values of q_c are illustrated. Note that this implies that the in-situ property at points in close proximity are more alike than at points separated by a greater distance, and the local areas of higher and lower values of q_c thus constitute a spatial correlation structure. The correlation structure describes the phenomenon of inherent variability by investigating the fluctuation of a property (due to sedimentation of soil under various conditions) which is conditional on the observation scale. The vertical case is analogous to the horizontal case. The horizontal fluctuations are

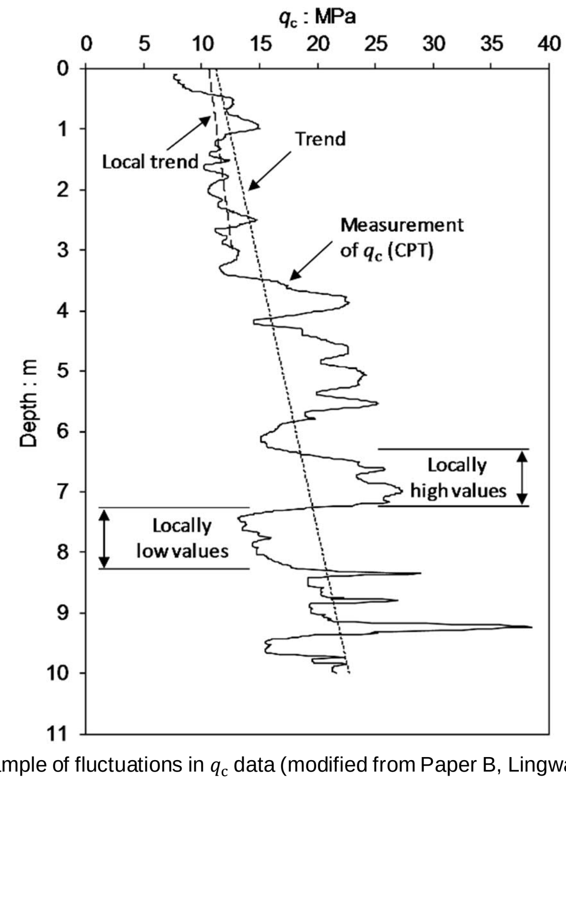

Figure 2.4. Example of fluctuations in q_c data (modified from Paper B, Lingwanda et al. 2017)

however preferably evaluated in relation to the vertical fluctuations (at certain depths). The scale of the horizontal fluctuations would in general be larger, which can be explained by the horizontally isotropic and vertically anisotropic nature of soil (Vanmarcke 1977).

2.2.2. Autocorrelation and scale of fluctuation

The maximum distance at which properties are assumed to be spatially correlated is referred to as the scale of fluctuation and is typically estimated from the autocorrelation structure. The degree of spatial correlation for any soil property evaluated from measurements $\{\xi_{\mathrm{m},1}, \xi_{\mathrm{m},2}, \dots, \xi_{\mathrm{m},n}\}$ at different locations $\{c_1, c_2, \dots, c_n\}$ can be estimated by an autocorrelation function of the separation distance, r (Baecher & Christian 2003):

$$R(r) = \frac{1}{(n-r)\sigma_{\xi_\mathrm{m}}^2} \sum_{i=1}^{n-r} \left[\{\xi_\mathrm{m}(\mathrm{c}_i) - \bar{\xi}_\mathrm{m}\}\{\xi_\mathrm{m}(\mathrm{c}_{i+r}) - \bar{\xi}_\mathrm{m}\}\right]. \qquad [2.22]$$

The r is also referred to as lag distance and defined in an interval (lag) for which $(n-r)$ is the number of data pairs within r. The measurements $\{\xi_{\mathrm{m},1}, \xi_{\mathrm{m},2}, \dots, \xi_{\mathrm{m},n}\}$ may be separated from each other in the horizontal and the vertical direction. For the vertical case, $\xi_\mathrm{m}(c_i)$ represents measurements at different depths z_i and the sample mean $\bar{\xi}_\mathrm{m}$ would then be equal to the trend function $\bar{\xi}_\mathrm{m}(z)$. When two points in space (e.g. two measurements located along the trend of q_c data in Figure 2.4) are close or at zero separation, $R(0)$ would be close to 1 or exactly 1. When the separation distance increases (i.e. r > 0), the autocorrelation will approach zero at a distance where no correlation exists.

An example of a horizontal autocorrelation structure (from Paper B) is presented in Figure 2.5. In the example, the autocorrelation function was calculated for q_c data at a depth from 3 to 4 m. The calculated $R(r)$ is represented by the dots in Figure 2.5 to which a simple triangular autocorrelation model was fitted. The triangular model, together with other commonly used autocorrelation models are presented in Baecher & Christian (2003) and Vanmarcke (2010). The scale of fluctuation (in the example denoted by θ_h to indicate horizontal distance) can then be estimated from the model of $R(r)$ by (Baecher & Christian 2003):

$$\theta = 2\int_0^\infty R(r)\, dr\,. \qquad [2.23]$$

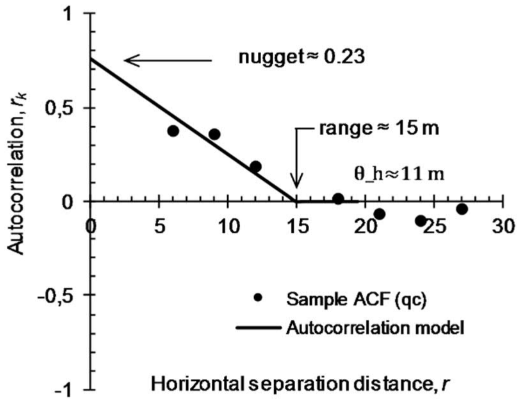

Figure 2.5. Example of an horizontal autocorrelation structure based on q_c data (Paper B, Lingwanda et al. 2017. © 2017 Taylor & Francis. Reused with permission)

2.2.3. Estimating the random measurement error

Estimating the autocorrelation function from a sample of measurements (Figures 2.4 and 2.5) implies that measurement error is present also in the autocorrelation function. Baecher (1983) therefore suggested that $R(r)$ could be expressed according to:

$$R(r) = R_{\text{in}}(r) + R_{\text{me}}(r)\,, \qquad [2.24]$$

where $R_{\text{in}}(r)$ and $R_{\text{me}}(r)$ are the autocorrelation functions of the inherent variability and random measurement error. Owing to the assumed independence in random measurement error between two measurements (i.e. $e(c_i)$ and $e(c_{i+r})$), Baecher (1983) argued that the error be revealed by a spike in the autocorrelation function at $r = 0$, and zero elsewhere. By extrapolating the autocorrelation function to the origin ($r = 0$), an estimate of the fraction of data scatter (typically referred to as the nugget) is obtained. The procedure is exemplified in Figure 2.5, in which the fraction or nugget (assumingly caused by random measurement error) is evaluated to 0.23 for the CPT. The procedure is used to estimate the random measurement error from measurements of CPT in Paper D and Paper E.

2.2.4. Variance reduction

In the example of q_c measurements in Figure 2.4, there are local areas of somewhat higher and somewhat lower values of q_c towards the depth. Typically, in the design of small to medium-sized structures, a specific limit state may be governed by the characteristics that are representative of a specific depth of a soil deposit and averaging of soil properties over the whole deposit depth may therefore be unsuitable. For example, the bearing capacity of a shallow foundation with a small area, A, of 1 m^2, is governed by a near-surface failure and the value of a representative design property should be the local average of the potential failure domain. Ignoring the horizontal extension of the small foundation, a representative local average of q_c between 0 and 3 m of depth in Figure 2.4 is less than the average calculated for the whole deposit depth. Using the global average instead of local averages may thus result in considerable over- or underestimation of a soil property.

In the 1-dimensional case, the local average of some measurements $\{\xi_{m,1}, \xi_{m,2}, \dots, \xi_{m,n}\}$, located within the averaging length, L, is defined by:

$$\bar{\xi}_{m,L} = \frac{1}{L}\int_L \xi_m(\mathbf{c})\, d\mathbf{c}\,, \qquad [2.25]$$

where $\mathbf{c} = \{c_1, c_2, \dots, c_n\}$ is a vector representing the locations of the measurements. The averaging length L can be considered to be the length within the soil volume that is affected by a potential failure influence zone. As the influence zone's size increases, the probability of over- or underestimating the soil property, owing to the influence of locally higher or lower values, decreases. Naturally, the bearing capacity of a small foundation is more likely to be influenced by a local weak spot in the soil than that of a structure whose failure influence zone averages over a larger volume of soil that contains both locally weak and strong zones. The $\bar{\xi}_{m,L}$ calculated for a structure with a large influence zone will thus be close to the global average and the effect of uncertainty due to inherent variability will decrease in relation to a variance reduction factor, $\Gamma^2(L)$. The relationship may be expressed in terms of coefficient of variation:

$$\mathrm{COV}^2_{in,\bar{D}} = \Gamma^2(L)\mathrm{COV}^2_{in,D}\,, \qquad [2.26]$$

where $\mathrm{COV}_{\mathrm{in},\bar{\mathrm{D}}}$ represents the reduced uncertainty due to inherent variability because of spatial averaging. When the size of a failure influence zone is small, the variance reduction factor will approach its upper bound $\Gamma^2(L) = 1$. When the size of the failure zone increases, the spatial average will approach the global average and eventually $\Gamma^2(L) = 0$.

Vanmarke (1977) proposed that the variance reduction could be approximated in 3 dimensions based on the vertical and horizontal scale of fluctuation (θ_{v} and θ_{h}). As such, a potential failure influence zone spans a vertical distance, L_{v}, and the horizontal lengths, $L_{\mathrm{h},x}$ and $L_{\mathrm{h},y}$, in the coordinate directions of x and y. For the simple case when $L_{\mathrm{h},x} = L_{\mathrm{h},y} = L_{\mathrm{h}}$ (i.e. the area, A, of the failure influence zone in the horizontal plane is square), and if the correlation between points on the 2-dimensional horizontal plane is symmetric (i.e. the correlation shows no preferable direction), the variance reduction factor becomes:

$$\Gamma_{3\mathrm{D}}^2\left(L_{\mathrm{v,h}}\right) = \Gamma_{1\mathrm{D}}^2(L_{\mathrm{v}})\Gamma_{2\mathrm{D}}^2(A)\,, \qquad [2.27]$$

where

$$\Gamma_{1\mathrm{D}}^2(L_{\mathrm{v}}) = \begin{cases} 1 & L_{\mathrm{v}} \leq \theta_{\mathrm{v}} \\ \dfrac{\theta_{\mathrm{v}}}{L_{\mathrm{v}}} & L_{\mathrm{v}} \geq \theta_{\mathrm{v}} \end{cases}, \qquad [2.28]$$

and

$$\Gamma_{2\mathrm{D}}^2(A) = \begin{cases} 1 & L_{\mathrm{h}} \leq \theta_{\mathrm{h}} \\ \dfrac{\theta_{\mathrm{h}}^2}{A} & L_{\mathrm{h}} \geq \theta_{\mathrm{h}} \end{cases}. \qquad [2.29]$$

The $\Gamma_{1\mathrm{D}}^2(L_{\mathrm{v}})$ is the variance reduction factor in 1 dimension due to spatial averaging in the vertical direction, and $\Gamma_{2\mathrm{D}}^2(A)$ is the variance reduction factor in the 2-dimensional horizontal plane. These expressions represent a simple model of the variance function. Other models may be found in Vanmarcke (2010).

3. Reliability theory in geotechnical design

3.1. Definition of structural reliability

In general terms, structure reliability relates to the performance of a structure (i.e. the structure's ability to fulfil its design purpose for some specified time), and in mathematical terms the probability that the specified limit states for a structure will be violated during a specified reference period (ultimate and serviceability limit states) (Thoft-Christensen & Baker 1982). The general term may be difficult to embrace in practical engineering as there is no strict design constraint related to the performance of a structure. However, the general definition introduces a question that must be answered before specifying the relevant limit states (i.e. what defines structural performance?). As noted by Mašín (2015) and Spross (2016), the probability of exceeding a specified limit state is typically referred to as the probability of failure, but this may be misleading because the behaviour at limit state attainment may not be the ultimate and sudden collapse. In this thesis, the terms failure and probability of failure will therefore be used to address unsatisfactory performance, and the reader should be aware that the term failure may concern both ultimate and serviceability limit states.

It should be noticed that the terms probability and reliability are used interchangeably, as in the statements above. The term probability is typically referred to in combination with failure (i.e. probability of failure), and is the probability that an undesired event will occur that deteriorates the performance of a structure. The term reliability is basically the opposite to probability of failure, commonly defined as the probability of satisfactory performance. These terms are given mathematical meaning in the following sections.

3.2. Factor of safety to probability of failure

In geotechnical practice, safety is typically addressed by analysing the relationship between the load, S, and resistance, R. This relationship is referred to as the limit state function in deterministic design and introduces a safety margin, SM, or a factor of safety, FS, which are defined as:

$$SM = R - S \,, \qquad [3.1]$$

and

$$FS = \frac{R}{S} \,. \qquad [3.2]$$

Traditionally, geotechnical engineers are more familiar with the FS and, theoretically, safety is attained at an FS of 1.0. However, due to the apparent uncertainties in the estimated values of S and R, it is unlikely that satisfactory performance of a construction can be assured over the specified reference period. Consequently, the basic principle is to apply an FS that is larger than 1.0 that corresponds to an acceptable safety. Quantifying the acceptable safety, in terms of the SM or FS is largely a societal and political issue, for which the actual decision is often delegated to the code writers.

Traditionally, the required SM or FS should account for five major types of sources of failure (Doorn & Hansson 2011):

1) "higher loads than those foreseen,
2) worse properties of the material than foreseen,
3) imperfect theory of the failure mechanism in question,
4) possibly unknown failure mechanisms, and
5) human error in design."

Factors 3 to 5 are eventualities that are difficult to characterise in probabilistic terms (Doorn & Hansson 2011), while factors 1 and 2 can be characterised as variabilities (i.e. representing the uncertainty) of S and R, respectively. However, a deterministic SM or FS does not take into account the uncertainties in the estimated values of S and R. Instead, determining the magnitude of the required SM or FS involves a great deal of experience of typical variabilities and occasional failures that may occur due to one or a combination of the five failure sources listed above. It is to be noted the same value of the SM or FS is often used in many different design scenarios, with no regard to the variable uncertainty (Duncan 2000). This may be unfortunate as large uncertainties may require an SM or FS that exceeds the norm, or vice versa. The performance of the construction may thus be either over- or underestimated.

To account for the uncertainties in S and R, a probabilistic approach to geotechnical design may provide a rational framework for addressing variable uncertainties and estimating project risks (i.e. defining risk by the probability of failure multiplied by the consequences of failure). The larger the uncertainties the greater the incentive to evaluate its effect on the results (Nadim 2003). Over the last few decades, attempts have been made to replace deterministic design with reliability-based design (Doorn & Hansson 2011). Treating both S and R as random variables, the probability that the load will exceed the resistance can be expressed as:

$$p_{\mathrm{F}} = P[G(R,S)] = P(R - S \leq 0)\,, \qquad [3.3]$$

where $G(R,S)$ represents the limit state function. (In reliability-based design, $G(R,S)$ is commonly referred to as the performance function of the limit state.) Figure 3.1 illustrates probability distributions of S and R and the corresponding distributions of the limit states (in terms of SM and FS). Failure may occur within the area where the distributions overlap in Figure 3.1(a) and to calculate p_{F}, each possible value of either S or R within the overlapping area must be checked against limit state violation. The p_{F} is then the probability that FS<1 or SM<0, illustrated by the hatched area in Figure 3.1(b) and (c), respectively. The lines that separate the failure region from the safe region are commonly denoted the failure surface of the limit state function in higher dimensions.

In addition to the probability of failure, a common measure of safety is the reliability index, β (graphical interpretation in Figure 3.1). In the simple case when SM and FS are normally distributed, β is calculated by normalising the respective mean values with their standard deviation (Cornell 1969):

$$\beta = \frac{E[SM]}{\sigma_{\mathrm{SM}}}\,, \qquad [3.4]$$

and

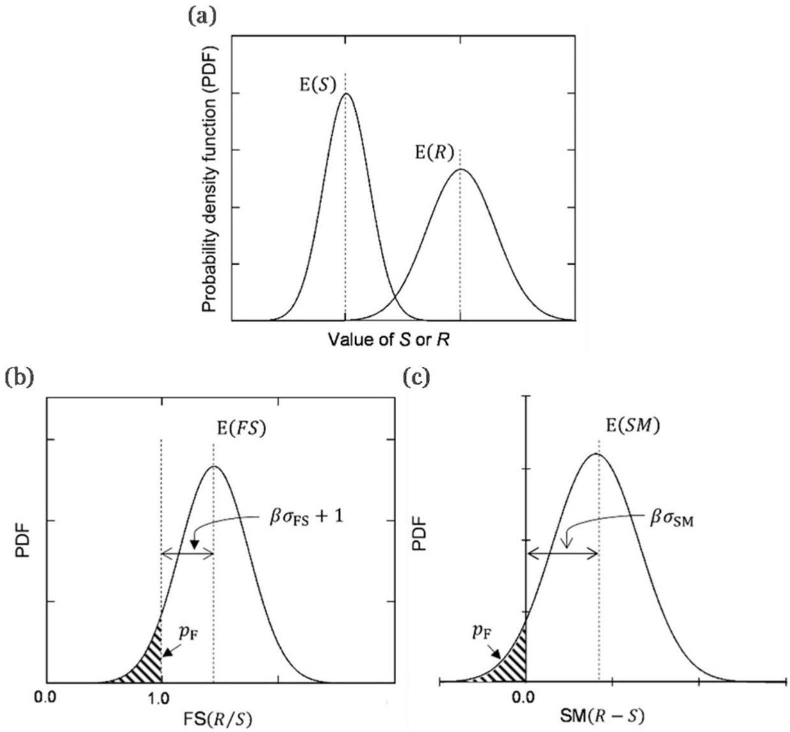

Figure 3.1. Probability distributions for (a) load (*S*) and resistance (*R*); (b) factor of safety (*FS*); (c) safety margin (*SM*).

$$\beta = \frac{E[FS] - 1}{\sigma_{\text{FS}}}. \qquad [3.5]$$

The β then relates to the probability of failure by

$$p_{\text{F}} = \Phi(-\beta)\,, \qquad [3.6]$$

where Φ is the standard normal distribution function. The complement to p_{F} is commonly referred to as the structural reliability $(1 - p_{\text{F}})$.

In general, the limit state function consists of multiple random variables $\boldsymbol{X} = \{X_1, X_2, \dots, X_n\}$ (in the following denoted basic variables). (X_i may represent the design value of a soil parameter ξ_{d} (e.g. the undrained shear strength, s_{u}, or effective internal friction angle, ϕ')). The basic variables may be related to the load (S) or resistance (R), and hence the limit state function $G(\boldsymbol{X})$ may be divided into $G_S(\boldsymbol{X})$ and $G_R(\boldsymbol{X})$ respectively. For several basic variables, a generalisation of Equation 3.3 becomes a problem of integration over the failure domain $G(\boldsymbol{X}) \leq 0$ (Melchers & Beck 2018):

$$p_{\mathrm{F}} = P[G(\boldsymbol{X}) \leq 0] = \int \dots \int_{G(\boldsymbol{X}) \leq 0} f_{\mathbf{X}}(\boldsymbol{x})\, dx\ , \qquad [3.7]$$

where $f_{\mathbf{X}}(\boldsymbol{x})$ is the joint probability distribution for the n-dimensional vector $\boldsymbol{X}$ and $\boldsymbol{x}$ contains the values $\{x_1, x_2, \dots, x_n\}$ of $\boldsymbol{X}$. In the general case, the limit state function $G(\boldsymbol{X})$ may not be linear (e.g. in the simple form of $G(\boldsymbol{X}) = X_1 - X_2$), and the basic variables may not be normally distributed. As such, Equation 3.7 may be difficult to integrate analytically. Several approximative methods are described in the literature (Baecher & Christian 2003; Melchers & Beck 2018; Fenton & Griffiths 2008), a few of which are presented in the following section.

3.3. Reliability methods

In the preceding chapter, limit-state design was discussed based on consideration to probabilistic interference, hence "Factor of safety to probability of failure". The degree of probabilistic interference in limit state verification is commonly divided into three reliability levels (Melchers & Beck 2018):

- Reliability level I accounts for uncertainty by adding partial factors or load and resistance factors to characteristic values of the basic variables. Level I design is commonly associated with the semi-probabilistic approach of the partial-factor method.
- Reliability level II accounts for uncertainty through the mean value, standard deviation, and correlation coefficients of the basic variables. (The basic variables are assumed normally distributed.) Level II design is commonly associated with first-order reliability methods.
- Reliability level III accounts for uncertainty by considering the joint probability distribution of all basic variables. An example is the fully probabilistic analysis of Monte Carlo simulations.

The partial-factors method is introduced in chapter 4, Geotechnical Design in the Eurocodes. An introduction to the most commonly used reliability models of level II and III is given in the following sections.

3.3.1. First-order second-moment reliability method (FOSM)

The FOSM reliability method circumvents the integration of Equation 3.7 and instead provides an alternative way of including the variability of the basic variables into the estimation of p_{F} through β (Equations 3.4-3.6). The FOSM reliability method approximates the mean value and variance of $G(\boldsymbol{X})$ by using the first-order terms of a Taylor's expansion. The Taylor's expansion linearises the failure surface $G(\boldsymbol{X})$ = 0 in relation to the mean values of the basic variables $G(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n)$ (Baecher & Christian 2003):

$$G(\boldsymbol{X}) \approx G(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n) + \sum_{i=1}^{n} (x_i - \bar{x}_i) \frac{\partial G}{\partial x_i} . \quad [3.8]$$

When the variables are uncorrelated the first-order approximation to the mean and variance (second-moment) of the limit state function are given by integration (Baecher & Christian 2003):

$$E[G(\boldsymbol{X})] = G(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n) , \quad [3.9]$$

and

$$\sigma_{\mathrm{G}}^2 \approx \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial G}{\partial x_i} \frac{\partial G}{\partial x_j} \rho_{X_i X_j} \sigma_{X_i} \sigma_{X_j} , \quad [3.10]$$

where $\rho_{X_i X_j}$ is the correlation coefficient between variables X_i and X_j, and $\partial G / \partial x_i$ is the partial derivative, usually taken at the means $\{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n\}$. Equation 3.9 and 3.10 are then used to calculate β through Equation 3.4. The partial derivative can be difficult to calculate directly and an approximation can be found by numerical differentiation by increasing and decreasing a variable by small increments (Christian 2004). However, caution should be taken when using the approximate procedure and the increments should be as small as possible until rounding error affects the result (Baecher & Christian 2003).

One advantage of the FOSM approach is that each parameter's sensitivity to the limit state function can easily be assessed by (Baecher & Christian 2003)

$$\alpha_i = \frac{\partial G / \partial x_i}{\sqrt{\sum_{i=1}^{n} (\partial G / \partial x_i)^2}} . \quad [3.11]$$

The sensitivity α_i of the basic variables to the limit state function may be highly useful in the process of planning for additional field and laboratory investigations and helps answer questions about which variables require the most attention in design (Castillo et al. 2004).

An extension of the FOSM reliability method is the Rosenblueth's point estimate method (Rosenblueth 1975). Details on the point estimate method are considered to be beyond the scope of this thesis and may be found for example in Baecher & Christian (2003) and Fenton & Griffiths (2008).

3.3.2. First-order reliability method (FORM)

Evidently, the FOSM reliability method would only give the exact probability of failure for a linear limit state function with normally distributed variables. To create an invariant format of the reliability problem (i.e. where the calculated reliability is independent of the formulation of the limit state) a development from the FOSM reliability method was presented by Hasofer & Lind (1974), and is commonly known as the FORM or Hasofer-Lind methodology.

The first step in the Hasofer-Lind methodology is to transform the basic variables into standardised normal distributions with zero mean and unit variance. The joint probability distribution $f_{\mathbf{X}}(\boldsymbol{x})$ of the basic variables is then represented by $f_{\mathbf{U}}(\boldsymbol{u})$ in the standard normal space, and the limit state function by $G(\boldsymbol{U})$. The optimal path between the zero mean of $f_{\mathbf{U}}(\boldsymbol{u})$ and a failure surface represented by $G(\boldsymbol{U})$ = 0 can then be estimated through a minimisation procedure (Melchers & Beck 2018):

$$\beta = \min_{G(\boldsymbol{U})=0} \sqrt{\sum_{i=1}^{n} u_i^2} \,, \qquad [3.12]$$

where u_i represents coordinates on the failure surface $G(\boldsymbol{U})$ = 0. A graphical representation of β is presented in Figure 3.2 for the simple case of a linear limit state. The point on the failure surface that is closest to the origin in Figure 3.2(b) is referred to as the design point or checking point, u^*. The coordinates of the design point in the standard normal space is defined by:

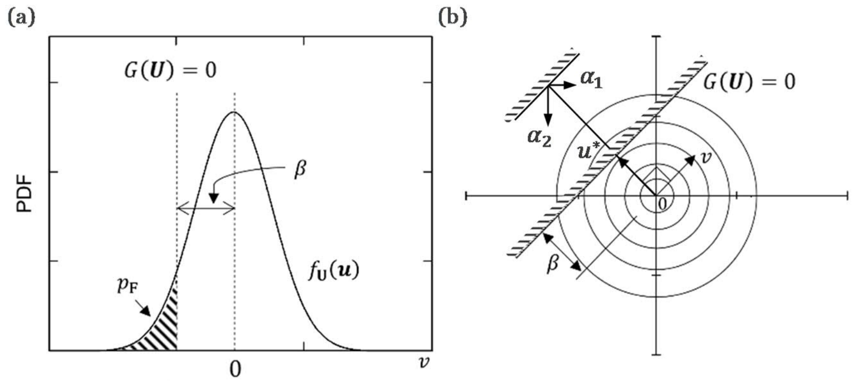

Figure 3.2. Graphical representation of the reliability index, (a) joint probability distribution $f_{\mathrm{U}}(\boldsymbol{u})$ of the linear limit state function in the space of the standardised normal variables; (b) contours of the joint probability distribution $f_{\mathrm{U}}(\boldsymbol{u})$ in the 2-dimensional standard normal space.

$$u^* = -\alpha_i \beta \, , \tag{3.13}$$

where α_i is typically defined as an outward normal to the design point (Melchers & Beck 2018).

Equation 3.12 is, in its most basic form, only valid for uncorrelated variables. When the basic variables are correlated, one has to adopt a transformation procedure that considers the correlations, such as the Cholesky approach (see e.g. Baecher & Christian 2003) or alternatively adopt Low and Tang's (1997) approach, which addresses the correlated variables directly through their covariance matrix, $\mathbf{C}$, without the aid of transformation. In their formulation of the reliability index, Low and Tang use the vector notation $\boldsymbol{X}$ of the basic variables to express the corresponding values, $\boldsymbol{x}^*$, that satisfy the failure criterion $G(\boldsymbol{X}) = 0$, and $\bar{\boldsymbol{x}}$ to represent the corresponding mean values $\{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n\}$ by:

$$\beta = \min_{G(\boldsymbol{X})=0} \sqrt{(\boldsymbol{x}^* - \bar{\boldsymbol{x}})^T \mathbf{C}^{-1} (\boldsymbol{x}^* - \bar{\boldsymbol{x}})} \, . \tag{3.14}$$

3.3.3. Monte Carlo simulation

An alternative approach for dealing with a non-linear limit state and basic variables that deviate from the normal distribution is the Monte Carlo simulation technique, which allows the integration of Equation 3.7 to be solved by numerical approximation. The procedure includes random sampling of the basic variables $\{X_1, X_2, \dots, X_n\}$, given their type of distribution. Each random variable X_i is thus sampled randomly to generate a set of sample values $\boldsymbol{x} = \{x_1, x_2, \dots, x_n\}$, which is then inserted into the limit state function to check for possible limit state violation (i.e. if $G(\boldsymbol{x}) \leq 0$). For N number of randomly chosen vectors $\boldsymbol{x}$ of x_i values, the p_F is estimated as the average value of the amount of simulations leading to failure, n_f (Melchers & Beck 2018):

$$p_\mathrm{F} = \frac{n_f(G(x_i) \leq 0)}{N}. \quad [3.15]$$

The uncertainty in the calculation may be evaluated in relation to the number of samplings as:

$$\mathrm{COV}p_\mathrm{F} = \sqrt{\frac{1 - p_\mathrm{F}}{p_\mathrm{F} N}}. \quad [3.16]$$

One of the greatest advantages of Monte Carlo simulation is that Equation 3.7 can be calculated without simplifications regarding the shape of the joint probability distribution $f_\mathbf{X}(\boldsymbol{x})$.

4. Geotechnical design in the Eurocodes

4.1. Design methodology

In EN 1990 (CEN 2002), the basic design criterion implies that a structure shall have adequate structural resistance, serviceability and durability. To ensure that the acceptable performance of a structure is met, it must be verified that no relevant limit state is violated (i.e. no states beyond which the structure no longer fulfils the design criteria are allowed) (Orr & Breysse 2008). According to EN 1997 (CEN 2004) these limit states shall be verified by one or a combination of approaches:

- “design by calculation with analytical, semi-empirical or numerical models,
- adoption of prescriptive measures,
- experimental models and load test, and/or
- an observational method.”

The last approach, “an observational method” is favourable when the prediction of geotechnical behaviour is difficult due to large uncertainties (CEN 2004). In such cases, it may not be possible to verify acceptable behaviour solely with calculation models. Adopting the observational method thus allows a design to be reviewed and potentially altered during construction based on estimates of the structural behaviour and predetermined actions, should the behaviour deviate from the estimated (Peck 1969). The observational method is further discussed in chapter 6.

In design by calculations, the partial-factor method is normative for verification of ultimate and serviceability limit states. The concept of partial-factor design was first introduced in the 1950s by Brinch Hansen (1956). Since then, versions of the partial-factor method have been implemented in various design codes (e.g. the load and resistance factor design (LRFD), which is frequently used in North American design codes (Fenton et al. 2016)). The development of Eurocode 7 started in the 1980s and the first draft of this limit state design code was published in 1987

(Orr 2006). In this thesis, an introduction to the partial-factor method is given from the perspective of design according to EN 1997.

4.2. The partial-factor method

4.2.1. Basic design principles

Verification of limit states in the Eurocodes includes studying the relationship between the design value of effect of actions, E_{d}, and the corresponding resistance, R_{d}, by:

$$E_{\mathrm{d}} \leq R_{\mathrm{d}} \,, \qquad [4.1]$$

where E_{d} may be represented by for example bending moment, shear force and bearing pressure, and R_{d} by for example bending capacity, shear resistance and bearing capacity (Bond 2013). To calculate the design values, partial factors for effects, actions, resistance and geotechnical parameters (γ_{E}, γ_{F}, γ_{R} and γ_{M}) are introduced in a deterministic manner, where the partial factors for structural and ground limit states (STR and GEO) are applied either on the effects of representative actions and resistance (CEN 2004):

$$\gamma_{\mathrm{E}} E\left\{F_{\mathrm{rep}}; \frac{x_{\mathrm{k}}}{\gamma_{\mathrm{M}}}; a_{\mathrm{d}}\right\} \leq \frac{R\{\gamma_{\mathrm{F}} F_{\mathrm{rep}}; x_{\mathrm{k}}; a_{\mathrm{d}}\}}{\gamma_{\mathrm{R}}} \,, \qquad [4.2]$$

or on the representative actions and geotechnical parameters themselves:

$$E\left\{\gamma_{\mathrm{F}} F_{\mathrm{rep}}; \frac{x_{\mathrm{k}}}{\gamma_{\mathrm{M}}}; a_{\mathrm{d}}\right\} \leq R\left\{\gamma_{\mathrm{F}} F_{\mathrm{rep}}; \frac{x_{\mathrm{k}}}{\gamma_{\mathrm{M}}}; a_{\mathrm{d}}\right\} , \qquad [4.3]$$

where F_{rep} is the representative actions, x_{k} is the characteristic value of some geotechnical parameter, X (with point values x), and a_{d} is the design value of geometrical data.

Each Eurocode member country is entitled to choose the particular partial factors (and their values), which are published in national annexes to EN 1997. The introduction of partial factors is divided into three different design approaches (DA1-3), from which the member countries may choose:

- DA1 requires separate verifications using two different combinations of partial factors that are applied to actions and to geotechnical parameters (except for piles and anchors for which partial factors are applied to resistance instead, in one of the two combinations).

- DA2 applies partial factors simultaneously to actions or to the effect of actions, and to resistance.
- DA3 applies partial factors simultaneously to actions or to the effect of actions, and to geotechnical parameters.

4.2.2. Partial factors for geotechnical parameters

In DA3, the main issue of the partial-factor method is to calculate design values of geotechnical parameters, $x_{\mathrm{d},i}$, from their respective characteristic values, $x_{\mathrm{k},i}$:

$$x_{\mathrm{d},i} = \frac{x_{\mathrm{k},i}}{\gamma_{\mathrm{M},i}}, \quad [4.4]$$

where $\gamma_{\mathrm{M},i}$ is accounting for uncertainties in geotechnical parameters and structural resistance models (CEN 2004). The partial factors and the uncertainties that they cover are described in EN 1990 (CEN 2002; Figure C3). The relationship between the individual partial factors is given therein as $\gamma_{\mathrm{M},i} = \gamma_{\mathrm{Rd},i}\gamma_{\mathrm{m},i}$, where $\gamma_{\mathrm{Rd},i}$ is the partial factor for uncertainty in the structural resistance model and $\gamma_{\mathrm{m},i}$ is the partial factor for the material or product property.

Further, the characteristic value is defined in EN 1997 as a “cautious estimate of the value affecting the occurrence of the limit state” (CEN 2004) and may be chosen as a subjectively estimated cautious value or, if statistical methods are used, as the 5% fractile of the representative value. Fundamentally, the selection of the characteristic value shall reflect the aleatory uncertainty due to inherent variability and epistemic uncertainty due to characterisation uncertainties (Schneider & Schneider 2013).

The partial factors in the Eurocodes are fixed but may be calibrated by the Eurocode member countries to suit specific geological conditions and to conform to existing national design codes. According to EN 1990, partial factors should be calibrated such that the reliability levels for representative structures are as close as possible to the target reliability level (represented by β_{T} or p_{FT}) (CEN 2002). Calibration of national partial factors for geotechnical parameters should be performed by either fully probabilistic methods (Level III) or first order reliability methods (FORM) (Level II) (CEN 2002; Figure C1). Based on the FORM (level II), the partial factors can be derived from the general expression (Melchers & Beck 2018):

$$\gamma_{\mathrm{m},i} = \frac{x_{\mathrm{k},i}}{x_{\mathrm{d},i}} = \frac{x_{\mathrm{k},i}}{F_{\mathrm{X}_i}^{-1}[\Phi(u_i^*)]}, \qquad [4.5]$$

where the design value $x_{\mathrm{d},i}$ is calculated through back-transformation of the FORM design point u_i^* (i.e. $F_{\mathrm{X}_i}^{-1}[\Phi(u_i^*)]$). The back-transformation results in the following expressions, for normally distributed variables (CEN 2002; Table C3):

$$\gamma_{\mathrm{m},i} = \frac{x_{\mathrm{k},i}}{\bar{x}_i(1 - \alpha_i \beta_{\mathrm{T}} \mathrm{COV}_i)}, \qquad [4.6]$$

and for log-normally distributed variables:

$$\gamma_{\mathrm{m},i} = \frac{x_{\mathrm{k},i}}{\bar{x}_i e^{-\alpha_i \beta_{\mathrm{T}} \mathrm{COV}_i}}. \qquad [4.7]$$

The design value is then related to the mean values $\bar{x}_i$, the sensitivity factors α_i, the target reliability index β_{T} and the coefficient of variation COV_i. Values for the effect of actions E and resistance R may typically be chosen as $\alpha_E = -0.7$ and $\alpha_R = 0.8$, when the condition of $0.16 < \alpha_E/\alpha_R < 7.6$ is satisfied (CEN 2002; Annex C). Furthermore, in calibration of partial factors, COV_i represents the variability in the material or product property that the code developer regards as appropriate to provide the required level of safety for the typical design situation.

It can be inferred from the discussion in Paper D and Paper F that the partial-factor method in the Eurocodes may not produce designs that are more consistent with the target reliability than the method of factor of safety (i.e. the deterministic Equations 3.1 and 3.2). This is partly because the partial factors that are calibrated by the Eurocode member countries do not consider a variable COV_i. Figure 4.1 illustrates the relationship between $\gamma_{\mathrm{m},i}$ and COV_i for normally distributed variables (assuming $x_{\mathrm{k},i} = \bar{x}_i$), which clearly shows that a small change in COV_i renders a significant change in $\gamma_{\mathrm{m},i}$. According to Phoon & Ching (2013), a solution to this problem may be that a range of partial factors be calibrated for different values of COV_i to suit different design scenarios. Phoon & Kulhawy (2008) also suggest typical ranges of soil parameter variability for reliability calibration.

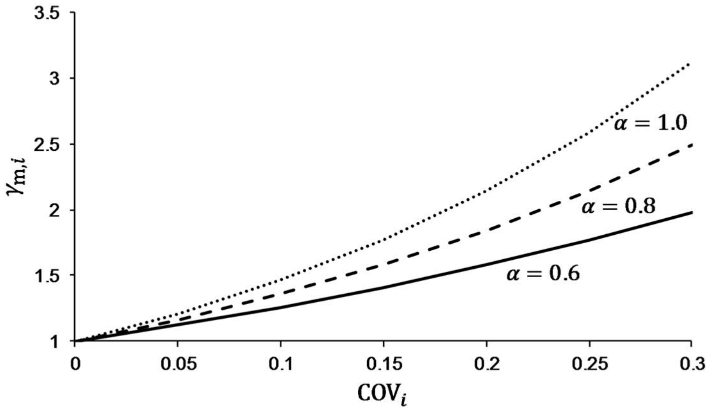

Figure 4.1. Relationship between $\gamma_{m,i}$ and COV_i for normally distributed variables.

4.2.3. Selection of characteristic values

4.2.3.1. According to the Eurocodes

Because the Eurocodes stipulates fixed partial factors, variable uncertainty in geotechnical parameters is managed in the selection of the characteristic value. The cautious estimate or its statistical equivalent the 5% fractile should account for the following factors (CEN 2004; Clause 2.4.5.2(4)P):

- "geological and other background information,
- the inherent variability of the material properties,
- the extent of field and laboratory investigations,
- the type and number of samples,
- the extent of the ground zone governing the behaviour of the geotechnical structure at the limit state being considered, and
- the ability of the geotechnical structure to transfer loads from weak to strong zones in the ground."

In addition, Clause 2.4.5.2(1) augments that the selection of characteristic values shall be complemented by well-established experience. According to Orr (2017) the requirement in 2.4.5.2(1) indicates that the selection of characteristic values should not be based purely on statistical analysis but should involve the engineer's subjective judgements complemented by experience.

When a potential failure may be regarded as local (i.e. the influence zone is significantly smaller than the scale of fluctuation), the characteristic value shall be selected as the 5% fractile of the PDF representing the variability of X. Alternatively, in the case of a non-local failure the characteristic value shall be selected with a 95% confidence in $\bar{x}$ for the potential failure influence zone (N.B: this definition is also consistent with the 5% fractile of $\bar{x}$). A graphical interpretation of the characteristic values is given in Figure 4.2. Using statistical methods, the $x_{\mathrm{k},i}$ for stationary data (i.e. the data do not exhibit a trend) may be calculated according to (Frank et al. 2004):

$$x_{\mathrm{k},i}^{\{stationary\}} = \bar{x}\left(1 - k_n \, \mathrm{COV}_X\right), \qquad [4.8]$$

where the factor k_n is calculated from the Student's t-factor and depends on the type of failure (local or non-local), the number of performed measurements and the statistical level of confidence required for the assessed characteristic value. Additionally, COV_X represents the variability of X.

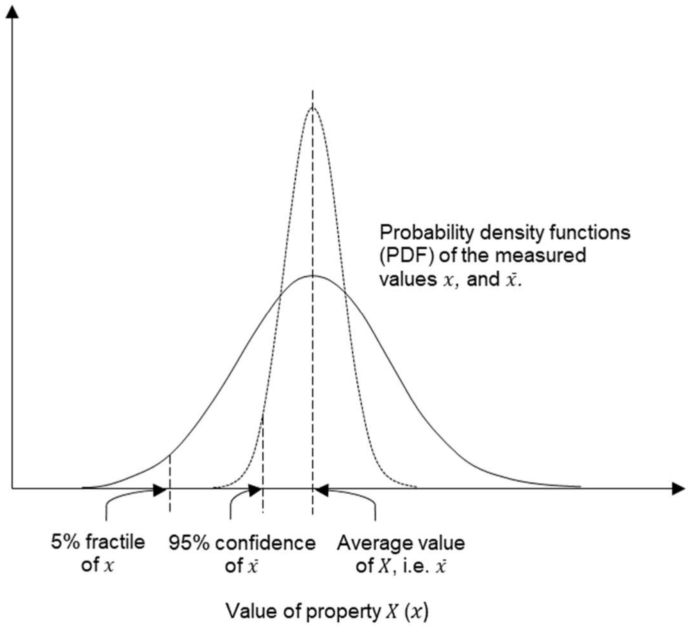

Figure 4.2. Graphical representation of the characteristic value for local and non-local failure.

If the data are non-stationary (i.e. the data exhibit a trend), Frank et al. (2004) recommend calculating the 5% fractile for the local failure according to:

$$x_{\mathrm{k},i}^{\{non-stationary\}}(X|z) = [\bar{x} + b(z-\bar{z})] - t_{n-2}^{0.95}\sigma_{X|z}\ , \qquad [4.9]$$

and for the non-local failure:

$$x_{\mathrm{k},i}^{\{non-stationary\}}(\bar{x}|z) = [\bar{x} + b(z-\bar{z})] - t_{n-2}^{0.95}\sigma_{\bar{x}|z}\ , \qquad [4.10]$$

where b is the slope of the regression line, $\bar{z}$ can be interpreted as the centre of gravity of the measurements $\{x_1, x_2 \dots\ x_n\}$, $t_{n-2}^{0.95}$ is the t-factor of student's distribution and $\sigma_{X|z}$ and $\sigma_{\bar{x}|z}$ are the standard deviation of the variability in X and $\bar{x}$, respectively. For the local failure, $\sigma_{X|z}$ is calculated from the variance according to:

$$\sigma_{X|z} = \sqrt{\sigma_X^2 + \left(\underbrace{\frac{1}{n} + \frac{(z-\bar{z})}{\sum_{i=1}^{n}(z_i-\bar{z})^2}}_{\text{Leverage}}\right)\sigma_X^2}\ , \qquad [4.11]$$

and for the non-local case:

$$\sigma_{\bar{x}|z} = \sqrt{\left(\underbrace{\frac{1}{n} + \frac{(z-\bar{z})}{\sum_{i=1}^{n}(z_i-\bar{z})^2}}_{\text{Leverage}}\right)\sigma_X^2}\ , \qquad [4.12]$$

where the contents within the brackets is referred to as the leverage about the centre of gravity of the measurements $\{x_1, x_2 \dots\ x_n\}$ (Wakefield 2013). The methodology for non-stationary data is discussed in Paper F, which presents a review of the uncertainties accounted for in Equations 4.9 and 4.10 through $\sigma_{X|z}$ and $\sigma_{\bar{x}|z}$.

Paper D and Paper F discuss whether the 5% fractile accounts properly for the factors listed in EN 1997, Clause 2.4.5.2(4)P. It should be noted that the two design situations (local and non-local failure) are equivalent to the extremes regarding the contribution of inherent variability in a design. For a local failure, when the failure influence zone is small compared to the scale of fluctuation, the reduction factor Γ^2 (introduced in chapter 2.2.3) will approach its upper bound $\Gamma^2 = 1$. As the size of the potential failure influence zone increases, the spatial average will instead approach the global average and $\Gamma^2 = 0$. When calculating the characteristic values according to Equations 4.9–4.12, no variance reduction is therefore performed for the local failure but complete variance reduction for the non-local failure. For the typical geotechnical problem, the variance reduction factor would be between 0 and 1. Variance reduction is not specifically covered in the Eurocodes; however, EN 1997 does not preclude the use of variance reduction and there are suggestions (Schneider & Schneider 2013) that do incorporate the variance reduction factor in the calculation of characteristic values.

4.2.3.2. The Swedish interpretation of the Eurocodes

National annexes to EN 1997 are published by the various national standard bodies. The Swedish National Annex (SNA) to EN 1997 (IEG 2008a) advocates the use of DA3 in most geotechnical structures, except for pile design, where DA2 is used. The SNA also defines the characteristic value as the product of a mean value and a conversion factor, η_i, rather than a 5% fractile, and calculates the design value according to:

$$x_{\mathrm{d},i}^{\{\mathrm{SNA}\}} = \frac{x_{\mathrm{k},i}^{\{\mathrm{SNA}\}}}{\gamma_{\mathrm{M},i}} = \frac{x_{\mathrm{chosen}}\eta}{\gamma_{\mathrm{M},i}}, \qquad [4.13]$$

where x_{chosen} is selected by the geotechnical engineer as either a subjectively estimated mean value or as $\bar{x}$, and η_i is chosen from a set of listed values in the national annex (IEG 2008a). Compared to the cautiously selected value (CEN 2004) the selection of x_{chosen} is not intended to account for uncertainties in geotechnical parameters. The uncertainty is instead managed via the selection of η, which is basically constructed to account for the uncertainties and factors listed in EN 1997 (clause 2.4.5.2(4)P) on x_{chosen}. The evaluation of η is based on 8 sub-factors:

$$\eta = \eta_1\eta_2\eta_3\eta_4\eta_5\eta_6\eta_7\eta_8\,, \qquad [4.14]$$

where the sub-factors are presented in Table 4.1. Each individual η_i is chosen by the geotechnical engineer from a set of listed values in the national annex (IEG 2008a). An example of this procedure is given in Table 4.2.

Table 4.1. Definitions of conversion factors in the SNA to EN 1997 (IEG 2008a)

η_i	Definition
η_1	The inherent variability of the material properties
η_2	Number of independent measurement points
η_3	Uncertainty related to the assessment of soil properties
η_4	The location of measurement points in relation to the structure
η_5	The extent of the ground zone governing the behaviour of the geotechnical structure at the limit state being considered
η_6	The ability of the geotechnical structure to transfer loads from weak to strong zones in the ground
η_7	Type of failure mechanism (i.e. ductile or brittle failure)
η_8	The sensitivity of the material design property on the limit state

Table 4.2. Example from the evaluation of η_3 in the SNA to EN 1997 (IEG 2008b)

Influencing factors	η_3
One method of cone penetration test/field vane/fall cone has been used	0.90
Two to three methods have been used – large variability in the results[A]	0.95
Two to three methods have been used – small variability in the results	1.00
Two to three methods have been used – small variability in the results and empirical relationships[B] confirm the results	1.05
Direct simple shear tests or triaxial tests confirm the results from other investigations and empirical relations.	1.10

[A] Obviously incorrect results are to be rejected.
[B] Refers to empirical relationships between the undrained shear strength and the pre-consolidation stress evaluated from constant-rate-of-strain oedometer tests.

It is suggested in IEG (2008b) that the η may be calibrated statistically in relation to $\gamma_{\mathrm{M},i}$ for normally distributed variables:

$$\eta = \frac{\gamma_{\mathrm{M},i}}{1 - e^{\alpha_i \beta_\mathrm{T} \mathrm{COV}_i}}, \tag{4.15}$$

and for log-normally distributed variables:

$$\eta = \frac{\gamma_{\mathrm{M},i}}{e^{\alpha_i \beta_\mathrm{T} \mathrm{COV}_i}}. \tag{4.16}$$

Inserting Equation 4.16 in Equation 4.13 (for the example of log-normally distributed variables) then gives:

$$x_{\mathrm{d},i}^{\{\mathrm{SNA}\}} = \frac{x_{\mathrm{k},i}^{\{\mathrm{SNA}\}}}{\gamma_{\mathrm{M},i}} = \frac{x_{\mathrm{chosen}}\eta}{\gamma_{\mathrm{M},i}} = \frac{x_{\mathrm{chosen}}}{e^{\alpha_i \beta_\mathrm{T} \mathrm{COV}_i}}. \tag{4.17}$$

In Paper D, x_{chosen} is selected as the mean value $\bar{x}$ in Equation 4.17, and thus, presents the adjustable partial factor through η:

$$\frac{\gamma_{\mathrm{M},i}}{\eta} = e^{\alpha_i \beta_\mathrm{T} \mathrm{COV}_i}. \tag{4.18}$$

The example given in Paper D implements the extended multivariate approach in the evaluation of $\bar{s}_\mathrm{u}$, and the corresponding total uncertainty $\mathrm{COV}_{\bar{\xi}_\mathrm{d}}$ from multiple types of field and laboratory investigation techniques to calibrate $\gamma_{\mathrm{M},i}/\eta$.

4.3. Acceptable probability of limit exceedance

Common to all reliability levels (I-III) is the basis for dealing with acceptable risk (previously defined in chapter 3.2) (Olsson & Stille 1984). In level I design, an acceptable risk is achieved by applying in-code defined partial factors (e.g. through $\gamma_{\mathrm{M},i}$ in the Eurocodes) which may be calibrated in relation to reliability levels II and III (CEN 2002). It is to be noted that the calibration of partial factors in relation to the FORM design point provides a connection between $\gamma_{\mathrm{M},i}$ and the level of risk through β_T and COV_i (Equations 4.6 and 4.7), where the acceptable probability of limit exceedance is determined through β_T.

The target values (i.e. β_{T} and p_{FT}) that are introduced in the design code should be calibrated by back-calculation from existing structures to yield a safety level considered acceptable by society. Johansson et al. (2016) note that a discrepancy commonly occurs between the back-calculated β_{T} and the β_{T} given in the code. If the back-calculated β deviates from target values given in the code it might indicate that the safety of the analysed structure is either insufficient or too high, or that the calculation model and its parameters do not incorporate the most relevant aspects. The Eurocodes define three consequence classes (CC1 to CC3) (Table 4.3). These consequence classes are in turn related to the reliability classes RC1 to RC3, which define suitable values of β_{T} and p_{FT} (Table 4.4).

Table 4.3. Definition of consequence classes cited from the Eurocodes (CEN 2002; Table B1)

Consequence class	Description	Example
CC1	Low consequence for loss of human life, and economic, social or environmental consequences small or negligible	Agricultural buildings where people do not normally enter (e.g. storage buildings), greenhouses
CC2	Medium consequence for loss of human life, economic, social or environmental consequences considerable	Residential and office buildings, public buildings where consequences of failure are medium (e.g. an office building)
CC3	High consequence for loss of human life, or economic, social or environmental consequences very great	Grandstands, public buildings where consequences of failure are high (e.g. a concert hall)

Table 4.4. Acceptable levels of safety according to the Eurocodes (CEN 2002; Table B2)

Reliability class	β_{T} A/ B	p_{FT} A/ B
RC1	4.20/3.3	$1.33 \times 10^{-5}/0.50 \times 10^{-3}$
RC2	4.70/3.8	$1.30 \times 10^{-6}/7.23 \times 10^{-5}$
RC3	5.20/4.3	$1.00 \times 10^{-7}/8.54 \times 10^{-6}$

A 1 year reference period
B 50 years reference period

Quantifying the acceptable risk in terms of β_{T} is a highly debated question (e.g. Fenton et al. 2016 and Fenton et al. 2015), and is according to Spross (2016) a question that needs more research. As noted by Phoon (2017a), the specified β_{T} in the Eurocodes may be suitable for structural engineering, but not necessarily for geotechnical engineering. However, the Eurocodes offer no classification of β_{T} based on structural branch. In the geotechnical community, the β_{T} specified in the Eurocodes has long been considered to be too high for geotechnical purposes, see for example Al-Naqshabandy & Larsson (2013) and U.S. Army Corps of Engineers (1997).

As previously stated, EN 1997 does not allow for any variation in the partial factor. (The recommendations in EN 1997 for national partial factors on geotechnical parameters are presented in Table 4.5.) Thus, to achieve the required reliability of geotechnical designs EN 1997 recommends that the design work should be assigned to one of three geotechnical categories (GC 1, GC 2 or GC 3) (CEN 2004) to account for different levels of complexity in the geotechnical design. According to Orr (2013) the geotechnical categories provide a framework for managing different levels of risk in geotechnical design and require that greater attention is given to the quality of the geotechnical design work.

Table 4.5. Partial factors on geotechnical parameters suggested in EN 1997 (CEN 2004; Table A.4)

Geotechnical parameter	$\gamma_{M,i}$	
Angle of shearing resistance	$\gamma_{M,\varphi'}$	1.25[A]
Effective cohesion	$\gamma_{M,c'}$	1.25
Undrained shear strength	γ_{M,s_u}	1.4
Unconfined strength	γ_{M,q_u}	1.4
Weight density	$\gamma_{M,\gamma}$	1.0

[A] Factor is applied to $\tan \varphi'$

5. Bayesian statistics

5.1. The Bayesian vs. the frequentist approach

Many authors use the simple example of tossing a coin to distinguish the Bayesian approach from the frequentist approach, as in Glickman & van Dyk (2007). A frequentist's approach for determining the probability of a coin toss landing heads would only consider long series of trials (even though it would be intuitive to predict a probability of ½). The Bayesian approach allows prior judgement to be made before tossing the coin and will therefore take the intuitive prediction of the probability into account. As such, a significant difference between the two approaches lies in the definition of the probability (Baecher & Christian 2003; Vick 2002). That is, the Bayesian approach treats the probability estimate of the coin toss as a degree-of-belief, and hence includes prior judgement, or belief, in the prediction. The frequentist approach to the probability estimate relies on the observed frequency of the outcome of the coin toss, with no reference to prior judgement.

Chapter 3.2 discusses how probability distributions must be assigned to the basic variables $\boldsymbol{X}$ to calculate p_{F} (Equation 3.7). This includes assigning probability distributions to the soil parameters evaluated from field and laboratory investigations. However, as long series of trials seldom exist, the assessment of probability distributions may be a cumbersome task (Orr & Breysse 2008; Forrest & Orr 2010). It is therefore important to combine the results from measurements with prior knowledge of the site conditions, which may be drawn from correlations to sites with similar geotechnical conditions or subjective assessments made by the local expertise (Cao et al. 2016). In geotechnical engineering, the Bayesian definition of the probability as a degree of belief is thus highly preferable to the frequentist approach as it allows prior information, based on for example subjective judgement to be systematically included in the reliability analysis prior to performing field and laboratory investigations.

When field and laboratory measurements are available, the prior knowledge may be updated and thus may combine the prior information based on subjective judgement and measurements into a single expression of the probability. The following sections discuss Bayesian methodologies in geotechnical engineering. In addition, Paper F presents a discussion on the suitability of using the 5% fractile to account for varying geotechnical uncertainties in the partial-factor method from the perspective of a Bayesian point of view, as opposed to the frequentist point of view.

5.2. Bayesian updating

Updating the prior knowledge of some variable X_i is possible by utilising the Bayes' theorem (Bayes 1763). To increase the knowledge about X_i, a set of observations $\boldsymbol{x} = \{x_1, x_2, \ldots, x_n\}$ is drawn from an experimental and uncertain process. In general terms, the random variables $\boldsymbol{X}$ may thus be described by a conditional probability density function, $f_{\mathbf{X}|\boldsymbol{\theta}}(\boldsymbol{x}|\boldsymbol{\theta})$, where the vector $\boldsymbol{\theta}$ defines the parameters of the probability density function, for example the mean value and standard deviation of a normal distribution. Given some additional information, the parameters $\boldsymbol{\theta}$ and the probability density function $f_{\mathbf{X}|\boldsymbol{\theta}}(\boldsymbol{x}|\boldsymbol{\theta})$ can be updated. The prior knowledge of $\boldsymbol{\theta}$ is represented by the probability density function, $f'_{\boldsymbol{\Theta}}(\boldsymbol{\theta})$, which contains the information about $\boldsymbol{\theta}$, prior to any observations being made. ($\boldsymbol{\Theta}$ represents the random variable whose values represent possible values of $\boldsymbol{\theta}$.) The updated probability density function given $\boldsymbol{x}$, is given by (Melchers & Beck 2018):

$$f''_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) = \frac{P(\boldsymbol{x}|\boldsymbol{\theta}) f'_{\boldsymbol{\Theta}}(\boldsymbol{\theta})}{\int P(\boldsymbol{x}|\boldsymbol{\theta}) f'_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) d\boldsymbol{\theta}}, \qquad [5.1]$$

where $P(\mathbf{x}|\boldsymbol{\theta})$ is the probability of observing $\boldsymbol{x}$ conditional on $\boldsymbol{\theta}$. The $P(\boldsymbol{x}|\boldsymbol{\theta})$ is commonly referred to as the likelihood function of $\boldsymbol{\theta}$ and may also be written as $L(\boldsymbol{\theta}|\boldsymbol{x})$. Simplifying the expression in Equation 5.1 gives:

$$f''_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) = kL(\boldsymbol{\theta}|\boldsymbol{x}) f'_{\boldsymbol{\Theta}}(\boldsymbol{\theta}), \qquad [5.2]$$

where $k = [\int L(\boldsymbol{\theta}|\mathbf{x}) f'_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) d\boldsymbol{\theta}]^{-1}$ is a normalising constant and independent of $\boldsymbol{\theta}$.

Using Bayesian sampling theory in geotechnical engineering, $\boldsymbol{X}$ may typically be represented by the population of some geotechnical parameter, and the observed values $\boldsymbol{x}$ by field and laboratory measurements. In the design of geotechnical structures, the variable of interest is often expressed in the form of a mean value, which is estimated from a set of measurements and hence is itself a random variable. It should be noted that the uncertainty in $\boldsymbol{X}$ then represents both the estimation of the variability of the population and the uncertainty related to the estimation of the mean value. Combining the two renders the predictive distribution of $\boldsymbol{X}$ that incorporates both sources of uncertainty (e.g. Ang & Tang 2007; Melcher & Beck 2018):

$$f_{\mathbf{X}}(\boldsymbol{x}) = \int_{\theta} f_{\mathbf{X}|\mathbf{\Theta}}(\boldsymbol{x}|\boldsymbol{\theta}) f''_{\mathbf{\Theta}}(\boldsymbol{\theta}) d\boldsymbol{\theta}. \qquad [5.3]$$

The predictive distribution can be interpreted as a weighted average of all possible distributions $f_{\mathbf{X}|\mathbf{\Theta}}(\boldsymbol{x}|\boldsymbol{\theta})$. As additional information on $\boldsymbol{\theta}$ becomes available (i.e. as the uncertainty in $\boldsymbol{\theta}$ is reduced), $f_{\mathbf{X}}(\boldsymbol{x})$ will approach the true probability density function of $\boldsymbol{X}$ (Müller 2013).

The use of Bayesian applications in geotechnical engineering has been studied by a number of authors. Zhang et al. (2004) use the concept of Equation 5.3 to reduce the prediction uncertainty from empirical transformation models based on global data sets with regional data sets. Cao & Wang (2013, 2014b) use a Bayesian approach to estimate statistically homogeneous soil layers and select spatial correlation functions for soil properties. Applications of Bayesian updating for characterisation of soil variability when the data is limited have also been studied by Cao & Wang (2014a), Wang & Xu (2015), Cao et al. (2016), Wang et al. (2016), Wang & Zhao (2017) and Yang et al. (2017).

5.3. The extended multivariate approach

5.3.1. The multivariate approach

Uncertainties arise in the prediction of soil parameters from field and laboratory investigations according to Equation 2.3 (recapitulated here for the sake completeness):

$$\mathrm{COV}^2_{\bar{\xi}_{\mathrm{d}}|\mathrm{D}} \approx \mathrm{COV}^2_{\mathrm{in},\bar{\mathrm{D}}} + \mathrm{COV}^2_{\mathrm{st},\bar{\mathrm{D}}} + \mathrm{COV}^2_{\mathrm{me},\bar{\mathrm{D}}} + \mathrm{COV}^2_{\mathrm{tr},\bar{\mathrm{D}}} + \vartheta\,.$$

The effect of inherent variability can be reduced by averaging over the failure domain (Equations 2.27-2.29); the statistical uncertainty and random measurement error can also be reduced by averaging over the performed measurements (i.e. reducing the uncertainties by increasing the amount of measurements (Equations 2.13 and 2.16)). However, the transformation uncertainty (introduced in chapter 2.1.4) propagates unchanged throughout the reliability analysis, and notably may contribute considerably to the uncertainty in the design parameter $\bar{\xi}_{\mathrm{d}}$. As stated previously, the results in Paper B and Paper C indicate that the transformation uncertainty may be considerable in common transformation models. This was also shown recently by D'Ignazio et al. (2016), who studied a multivariate database consisting of index tests and s_{u} evaluated from direct simple shear tests and field vane tests. On this subject, Ching et al. (2010) proposed a simplified Bayesian updating procedure (the multivariate approach) that cross-validates different sources of information on empirical correlations used for the transformation of $\bar{\xi}_{\mathrm{m}}$ to $\bar{\xi}_{\mathrm{d}}$. This procedure reduces the effect of transformation uncertainty on the evaluated $\bar{s}_{\mathrm{u}}$.

5.3.2. The extension of the multivariate approach

The multivariate approach proposed by Ching et al. (2010) targets the transformation uncertainty in their simplified updating procedure and hence ignores the error terms $\varepsilon_{\mathrm{inh}}$, $\varepsilon_{\mathrm{st}}$ and $\varepsilon_{\mathrm{me}}$. Müller et al. (2014) therefore proposed an extension of the procedure that includes the evaluation of the total uncertainty (i.e. accounts for the complete expression of Equation 2.3) prior to adopting the updating procedure. In this procedure, the posterior average value (expected value) and corresponding variance of a parameter $\bar{\xi}_{\mathrm{d}}$ are estimated on the basis of prior information, $\mathrm{D}_{\mathrm{prior}}$, and measurements from multiple types of field and laboratory investigation techniques D_i, and represented by their respective average values $\bar{\xi}_m$. In the following Equations 5.5 and 5.6 (Ang & Tang 2007; Fenton & Griffiths 2008), $y = \bar{\xi}_{\mathrm{d}} | \mathrm{D}_{\mathrm{prior}}$ and

$$\boldsymbol{x} = [\mathrm{D}_1, \mathrm{D}_2 \dots \mathrm{D}_n]^{\mathrm{T}} = \begin{bmatrix} \bar{\xi}_{m_1}(z) \\ \bar{\xi}_{m_2}(z) \\ \vdots \\ \bar{\xi}_{m_n}(z) \end{bmatrix} = \begin{bmatrix} \hat{a}_1 + \hat{b}_1 z \\ \hat{a}_2 + \hat{b}_2 z \\ \vdots \\ \hat{a}_n + \hat{b}_n z \end{bmatrix}. \qquad [5.4]$$

The posterior expected value, $\mathrm{E}[y|\boldsymbol{x}]$, and the corresponding variance, $\mathrm{Var}[y|\boldsymbol{x}]$, both of which are conditional on $\mathrm{D}_{\mathrm{prior}}$, can then be calculated by:

$$\mathrm{E}[y|\boldsymbol{x}] = \mathrm{E}[y] + \mathrm{Cov}(y, \boldsymbol{x})\mathrm{Var}[\boldsymbol{x}]^{-1}(\boldsymbol{x} - \mathrm{E}[\boldsymbol{x}]) \,, \qquad [5.5]$$

$$\text{Var}[y|\boldsymbol{x}] = \text{Var}[y] - \text{Cov}(y,\boldsymbol{x})\text{Var}[\boldsymbol{x}]^{-1}\text{Cov}(y,\boldsymbol{x})^T \,, \qquad [5.6]$$

where E$[y]$ and Var$[y]$ are the prior expected value and variance of y, respectively, Cov$(y,\boldsymbol{x})$ is the covariance vector between y and $\boldsymbol{x}$, Cov$(y,\boldsymbol{x})^{\mathrm{T}}$ is the transposed covariance vector, and E$[\boldsymbol{x}]$ and Var$[\boldsymbol{x}]$ are the expected value and covariance matrix of $\boldsymbol{x}$, respectively. As such, Var$[y]$ represents the total uncertainty in $\bar{\xi}_{\mathrm{d}}\big|\mathrm{D}_{\mathrm{prior}}$ and Var$[\boldsymbol{x}]$ represents the total uncertainty in $\bar{\xi}_{\mathrm{d}}\big|\mathrm{D}$ (evaluated in accordance with Equation 2.3 prior to adopting the simplified updating procedure) (Müller 2013). A detailed step-by-step example on how to solve Equations 5.5 and 5.6 can be found in the appendix to Müller et al. (2014).

The multivariate approach is only valid for normally distributed variables, which is rarely the case for geotechnical parameters. Suitable transformation to a normal distribution must therefore be made prior to performing the calculations. This is the case in Paper D and Paper E, where the posterior $\bar{s}_{\mathrm{u}}''$ and COV$_{\bar{s}_{\mathrm{u}}}''$ are given by back-transformation of E$[y|\boldsymbol{x}]$ and Var$[y|\boldsymbol{x}]$ to lognormally distributed values.

6. The observational method and reliability-based design

R. B. Peck (1969) refers to Terzaghi as the father of the observational method, but Peck (1969) indeed provided the definition that has been used successfully for many years. The focus of the observational method is on prediction and monitoring and Peck provided a set of principles for how to apply the method. Powderham (2002) explains the basic idea: "Essentially, the observational method facilitates design changes during construction and establishes a framework for risk management". Powderham (2002) also provided a somewhat condensed definition of Peck's principles in four points:

- "commence construction with a design providing an acceptable level of risk to all parties;
- maintain or decrease this level of risk;
- progress construction in clearly defined phases; and
- implement appropriate changes progressively and demonstrate acceptable performance through observational feedback."

The observational method is an alternative when design is difficult to carry out in a regular fashion because of a high level of uncertainty. However, as discussed by Nicholson et al. (1999), the approach is far from appropriate in all projects solely because there are substantial uncertainties. For the observational method to work as intended, it must first be possible to change the design during construction and, second, the nature of a potential failure cannot be brittle (i.e. it must, within a reasonable timeframe, be possible to measure and draw conclusions as to whether the construction is approaching failure or not).

The observational method allows a design and its safety to be updated continuously during the construction process. That is, uncertainties in design that are related to the behaviour of a construction can be reduced as new information becomes available during the construction process.

According to Christian (2004), Bayesian statistics are closely related to the observational method. Bayesian updating techniques, like the extended multivariate approach, allow the uncertainties to be updated (re-calculated) as new information is put into context.

In recent years, the industry's interest in the observational method has increased as the design approach has made its way into EN 1997. Therein, the observational method is formalised in 5 paragraphs where P denotes principles that must not be violated (CEN 2004):

(1) "When prediction of geotechnical behaviour is difficult, it can be appropriate to apply the approach known as 'the observational method', in which the design is reviewed during construction.

(2) P The following requirements shall be met before construction is started:

   a) acceptable limits of behaviour shall be established;

   b) the range of possible behaviour shall be assessed and it shall be shown that there is an acceptable probability that the actual behaviour will be within the acceptable limits;

   c) a plan of monitoring shall be devised, which will reveal whether the actual behaviour lies within the acceptable limits. The monitoring shall make this clear at a sufficiently early stage, and with sufficiently short intervals to allow contingency actions to be undertaken successfully;

   d) the response time of the instruments and the procedures for analysing the results shall be sufficiently rapid in relation to the possible evolution of the system;

   e) a plan of contingency actions shall be devised, which may be adopted if the monitoring reveals behaviour outside acceptable limits.

(3) P During construction, the monitoring shall be carried out as planned.

(4) P The results of the monitoring shall be assessed at appropriate stages and the planned contingency actions shall be put into operation if the limits of behaviour are exceeded.

(5) P Monitoring equipment shall either be replaced or extended if it fails to supply reliable data of appropriate type or in sufficient quantity."

Spross (2016) provides an extensive review of the observational method in general, and of the implementation of the approach, according to the definition in EN 1997. Spross concludes that the definition of the observational method in EN 1997 may, in some cases, be too strict, and argues that a less strictly defined observational method may be good enough for many applications.

In Paper A, a case study is presented on the implementation of the observational method in the construction of a high embankment. Based on the study, the importance of implementing a guide on how to use a reliability-based approach to limit state verification in the observational method should be emphasised (i.e. verify that the design fulfils the design requirements with an acceptable risk). On this subject, Spross & Johansson (2017) recently exemplified how to introduce reliability constraints in the observational method. They also proposed a probabilistic methodology to aid in the decision of choosing between the observational method and conventional design by answering the question: When is the observational method favourable?

7. Summary of appended papers

7.1. Paper A

> Prästings, A., Müller, R., and Larsson, S., 2014. The Observational Method Applied to a High Embankment Founded on Sulphide Clay. Engineering Geology, 181: 112–123.

This journal paper presents a case study on how the observational method was adopted in design and construction of a high embankment. The study compares the observational method (as per the defined in EN 1997) to the actual implementation in the project. The construction consisted of a 16-m high embankment of crushed rock-fill, situated on loose sediments of silt, clay and sulphide clay. The observational method was successfully implemented in the project, although it is questionable if the execution fully meet the requirements specified in EN 1997. For example, the Eurocodes require that there is an acceptable probability that the behaviour of the structure will be within the acceptable limits. In the design of the embankment, the acceptable limits of the construction behaviour were deterministically defined in relation to a factor of safety (FS≥1.5) for potential shear failure. Uncertainties in the soil parameters were not evaluated and a probabilistic analysis of the embankment behaviour was not performed. The behaviour of the embankment was instead evaluated with regards to different combinations of probable and unfavourable values on the most sensitive parameters.

7.2. Paper B

> Lingwanda, M.I., Prästings, A., Larsson, S. and Nyaoro, D., 2016. Comparison of Geotechnical Uncertainties Linked to Different Soil Characterization Methods. Geomechanics and Geoengineering: An International Journal, 12(2): 1–15.

This journal paper presents a study that compares the evaluated uncertainties from cone penetration tests, standard penetration tests and the light dynamic probing method (performed jointly and in parallel) with the purpose of determining the soil confined modulus. The modulus was evaluated from oedometer tests and site-specific correlations were evaluated for the field tests. The field and laboratory tests were conducted close to the main campus of the University of Dar es Salaam. The geology at the test site consisted of kaolinic sandstone overlaid by clay-bound sands. The tests were performed in a grid pattern with 6-m intervals and at a total of 36 points. The variability was evaluated for the measurements and transformation uncertainties were evaluated for the respective correlation. The standard penetration test was proven to be related to the highest total uncertainty and the transformation uncertainty had a considerable impact. The oedometer tests were related to the least amount of uncertainty.

7.3. Paper C

Hov, S., Prästings, A., Persson, E. and Larsson, S., 2019. On Empirical Correlations for Normalized Shear Strength from Fall Cone and Direct Simple Shear Tests in Soft Swedish Clays. Submitted to Geotechnical and Geological Engineering.

This journal paper presents a study that investigates the performance of common transformation models for undrained shear strength and tests the recommended correction for fall cone tests. The paper presents a multivariate database consisting of 499 data points from sites which are mainly located in south-eastern Sweden. The database consists of evaluated shear strengths obtained from direct simple shear tests and fall cone tests, including index tests. Common transformation models for the relationship between normalised undrained shear strength and index tests where evaluated using the database. It was found that the normalised shear strength evaluated from direct simple shear tests and fall cone tests conform to Swedish and Norwegian recommendations. The results indicate that the recommended correction for fall cone tests (and field vane tests) may not be applicable to a wide range of liquid limits. Furthermore, there is a poor agreement between the results evaluated from fall cone tests and direct simple shear tests. It is suggested that this poor agreement is related to natural variations and sample disturbance.

7.4. Paper D

Prästings, A., Spross, J., Müller, R., Larsson, S., Bjureland, W. and Johansson, F., 2017. Implementing the Extended Multivariate Approach in Design with Partial Factors for a Retaining Wall in Clay. ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, 3(4): 04017015.

This journal paper presents a case study that discusses the limitations of the partial-factor method (as defined in EN 1997). The paper implements the extended multivariate approach in design with partial factors by adopting a simple statistical extension of the Swedish definition of the characteristic value and design value, which use a conversion factor, η, instead of the 5% fractile, to account for varying geotechnical uncertainties. In the case study, the design value is evaluated based on the statistical extension of the Swedish methodology and compared to the methodology in EN 1997. It is concluded that the EN 1997 definition of the partial-factor method does not allow the safety level to be reduced to less than is already incorporated in the partial factor. It is also suggested that the 5% fractile does not adequately account for variable uncertainties in the geotechnical parameters.

7.5. Paper E

Prästings, A., Müller, R. and Larsson, S. 2017. Optimizing Geotechnical Site-investigations. In Proceedings of the 19th International Conference on Soil Mechanics and Geotechnical Engineering, Seoul, South Korea, pp. 639–642.

This conference paper builds on the case study presented in Paper D. The paper proposes a simplified approach of implementing the extended multivariate approach in design with partial factors (as per the definition in the SNA to EN 1997). The paper investigates the potential reduction of uncertainties in the estimation of the undrained shear strength, based on: (1) the soil's inherent variability, (2) the number of measurements, (3) the type of field or laboratory soil test, and (4) the combination of different soil tests. The potential reduction of uncertainties is presented in illustrative figures and it is proposed that these can be of assistance in the planning of field and laboratory investigations.

7.6. Paper F

Prästings, A., Spross, J. and Larsson, S. 2019. Characteristic values of Geotechnical Parameters in Eurocode 7. Proceedings of the Institution of Civil Engineers – Geotechnical Engineering, 172(4): 301-311.

This journal paper presents a discussion on the lack of harmonisation between reliability-based design and the current definition of the partial-factor method in EN 1997. Based on the differences between the frequentist and the Bayesian school of thought, the paper discusses whether the 5% fractile is suitable to use to account for varying geotechnical uncertainties. The paper also discusses, from a Bayesian point of view, whether the curse of the small sample size is a valid argument to dismiss estimation of geotechnical property statistics, and finally how to bridge the gap between reliability-based design and design with partial factors (as per the definition in EN 1997). The paper presents the Swedish definition of the characteristic value and design value, which utilise a conversion factor, η, instead of the 5% fractile, to account for varying geotechnical uncertainties. The extended multivariate approach can be used to calibrate the η factor and the definition may bridge the gap between reliability-based design and design with partial factors.

8. Concluding remarks

8.1. Uncertainties in geotechnical parameters

Geotechnical uncertainties need to be managed stringently throughout the design and construction process. For this purpose, Paper A suggests that the extended multivariate approach (or Bayesian methodologies in general) is suitable to combine with the features of reliability-based design and may also be used in combination with an observational method.

Based on the results in Paper B and Paper C, I suggest that the transformation uncertainty is typically the most significant uncertainty in Equation 2.3, which is supported by the conclusions drawn by D'Ignazio et al. (2016) and Müller et al. (2014). Evidentially, the extended multivariate approach allows the uncertainties in Equation 2.3 to be systematically reduced with additional information. In addition, when several sources of geotechnical information are available, the systematic, and so often large, transformation uncertainty may be reduced through cross-validation of different field and laboratory investigation techniques via a Bayesian procedure. Moreover, the extended multivariate approach determines a weighted mean value based on the uncertainty in each of the investigation techniques used for the project at hand. This means that the most reliable source of information has the highest impact on the evaluated mean value. It should be noted that the SNA to EN 1997 (IEG 2008a) states that the potentially greater trust in high-quality investigations shall be reflected in the selection of the characteristic value, and entail a form of subjective weighing. By implementing the extended multivariate approach, the weighing is less subjective as it is based on the more objective information of $COV_{\bar{\xi}_d|D}$.

By implementing the extended multivariate approach in partial-factor design (Paper D and Paper E), site investigation efforts may be linked to potential design savings through the evaluated posterior uncertainty and the weighted mean value. However, for the design to be pertinent with respect to the societal demands concerning risk and safety given in EN 1997 (as regulated by the target reliability index (β_T)), the uncertainty estimates should reflect the influence on design from soil-inherent variability and the imperfections related to field and laboratory investigations as accurately as possible. According to Forrest & Orr (2010) and Orr & Breysse (2008) the geotechnical property data are often too sparse to generate meaningful statistics, and hence the uncertainties in Equation 2.3 may therefore be difficult to determine with accuracy. Paper F discusses the so-called curse of the small sample size (coined by Phoon 2017b) in geotechnical engineering and suggests that the uncertainties in geotechnical parameters should be estimated even in cases where the geotechnical field and laboratory investigation is limited. Bayesian methodologies may be used to increase accuracy in the statistical inference of small samples of data. Typical values compiled by Phoon & Kulhawy (1999a) and Phoon & Kulhawy (1999b) on inherent variability, measurement error and transformation uncertainty may also be used as input in the calculation of $\mathrm{COV}_{\bar{\xi}_{\mathrm{d}}|\mathrm{D}}$.

However, choosing from a set of typical uncertainty values given in the literature introduces an undesirable amount of subjectivity into the estimation of $\mathrm{COV}_{\bar{\xi}_{\mathrm{d}}|\mathrm{D}}$, which threatens the proclaimed objectivity of the extended multivariate approach. Nevertheless, as suggested in Paper F, in the absence of geotechnical field and laboratory investigations, or when the investigations are limited, the uncertainties may be determined by engineers' best knowledge, following the Bayesian approach to statistics. On this subject, my co-authors and I state in Paper F that "From our Bayesian point of view, we would rather have highly uncertain estimates – but probabilistic estimates – of the geotechnical properties, than no estimates at all.". From this perspective, the design is still relevant if the lack of knowledge is properly reflected in the design output.

8.2. Implementing the extended multivariate approach in design with partial factors

Paper D implements the extended multivariate approach in design with partial factors. A statistical extension of the Swedish definition of the characteristic value and design value (IEG 2008b) – which use the conversion factor η, instead of the 5% fractile – is used to account for varying uncertainties in geotechnical parameters. The extended multivariate approach is then implemented in the Swedish definition through calibration of the conversion factor η. In the study, it is shown that performing qualitative field and laboratory investigations should potentially allow the partial factor $\gamma_{\mathrm{M},i}$ to be lower than the fixed values suggested by the Eurocodes (Table. 4.5). The statistical definition of η (Equations 4.15 and 4.16) implies that risk reduction can be performed either by reducing the uncertainties (i.e. reducing $\mathrm{COV}_{\bar{\xi}_{\mathrm{d}}|\mathrm{D}}$) or by reducing the expected consequences, as they are regulated through the β_{T}.

Paper D also discusses the limitations of the current definition of the partial-factor method in EN 1997 and suggests that the uncertainties in geotechnical parameters are not stringently incorporated into the definition of the design value and characteristic value. The limitations are related to the fact that the partial factor is fixed and to the definition of the characteristic value as a cautious estimate (corresponding to a 5% fractile). In the paper, the characteristic value and design value, calculated by way of the current definition of the partial-factor method in EN 1997 (CEN 2004), are compared to the characteristic value and design value, calculated by way of the definition in the SNA to EN 1997 (IEG 2008b). The comparison illustrates the results of a major difference between the approach in EN 1997 and the approach in the SNA. In EN 1997, the fixed partial factor provides the lower bound of safety that is incorporated in the limit state. In the Swedish definition, however, the conversion factor η (Equation 4.13) can be lower than, higher than or equal to 1, where a value of $\eta > 1.0$ principally allows for a reduction of the safety level incorporated in the limit state through the partial factor. For the stability problem in Paper D, the reduction of uncertainties through the extended multivariate approach thus gives $\eta > 1.0$, and the design value calculated in accordance with the definition in the SNA is therefore higher (closer to the mean value) than the design value calculated in accordance with EN 1997.

Paper D further shows that if enough field and laboratory investigations are performed, the 5% fractile for the evaluated mean value will be roughly the same as the mean value itself. Under these conditions, is it possible to claim that the characteristic value properly accounts for the uncertainties and other factors given in EN 1997, Clause 2.4.5.2(4)P? For example, EN 1997 states that the characteristic value shall account for “the type and number of samples” (CEN 2004). Does this statement imply that the engineer should account for systematic uncertainties such as the transformation uncertainty, or is the transformation uncertainty merely accounted for in the fixed partial factor? (The Eurocodes are not clear about this.)

Schneider and Schneider (2013) suggest that the characteristic value – the 5% fractile – may be calculated from the distribution of $\mathrm{COV}_{\bar{\xi}_{\mathrm{d}}|\mathrm{D}}$ (Equation 2.3) and therefore also include the transformation uncertainty, as opposed to the procedure presented in Frank et al. (2004). In contrast to my opinion, they suggest that the transformation uncertainty can be set to zero in the calculation of the 5% fractile. As previously pointed out, based on the findings in Paper B and Paper C, I suggest that the transformation uncertainty can be significant. In theory, the erroneous management of the transformation uncertainty may lead to unreasonably safety levels if not properly accounted for in the design through a reduction procedure (e.g. through the implementation of the extended multivariate approach in design with partial factors).

8.3. Harmonising the partial-factor method with reliability-based design

According to Paper F, difficulties in implementing reliability methods in geotechnical design may originate from the lack of harmonisation between reliability-based design and the definition of the partial-factor method in EN 1997. It is argued that the Swedish definition (managing variable uncertainties in η) provides a stronger connection to reliability-based design and to the failure probability which is regulated by the target reliability index β_{T}, owing to: 1) the definition is based on the evaluated mean value (rather than the 5% fractile), and 2) by way of the variable η, the lower bound of safety that is provided by the design value is not fixed in relation to the partial factor.

Implementing the extended multivariate approach in design with partial factors, according to the suggestion in Paper D and Paper E, harmonises the partial-factor method with reliability-based design. However, in my opinion it is not realistic to believe that such statistical methods, like the extended multivariate approach, can be implemented into code documents without proper simplifications. Paper E therefore investigates the possibility to produce simple charts to indicate how the amount of measurements and the combination of different field and laboratory investigation techniques affect the uncertainties in $\mathrm{COV}_{\bar{\xi}_{\mathrm{d}}|\mathrm{D}}$ (Equation 2.3) and thereby the conversion factor η (Equation 4.13). Paper E concludes that these charts can produce general estimates of η that may be used for guidance on how to plan for effective site-investigations in clay.

9. Future research and practical implementation

During my years as a PhD-candidate I have realised that the introduction of reliability-based design and statistics in geotechnical engineering is limited by the researchers' attempts to communicate the potential gain in probabilistic methods. Most geotechnical engineers feel satisfied in producing designs that follow the current definition of the partial-factor method in EN 1997. Many of whom do not reflect about the geotechnical uncertainties and potential deviation from the specified target reliability these uncertainties may lead to.

I therefore suggest that future research should focus on the actual implementation of reliability-based design in engineering practise. First, the potential gains from using reliability methods as a complement to the current partial-factor method need to be thoroughly communicated to the industry – from a practical perspective. Second, this may include simplifications to (or realization of) mathematical expressions, as per the example in paper E. However, to fully implement reliability methods in engineering practise I believe that the industry (in close collaboration with the research community) will need to develop commercial software's to facilitate producing probabilistic descriptions of geotechnical parameters.

Furthermore, in recent years, the industry has shown increased interest in different Geotechnical BIM applications in which the visualisation of underground information in combination with a preliminary design may constitute a platform to communicate project risks to clients during the design process. Imagine the opportunity to visualise uncertainties in geotechnical parameters in combination with the effect that these uncertainties may bring to the design. Clients might be more amenable in providing resources for risk reduction in the design stage if the potential gain were easy to communicate.

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Figure 2.5: © 2017 Taylor & Francis. Reproduced with permission. In: Lingwanda, M. I., Prästings, A., Larsson, S., & Nyaoro, D. L. 2016. Comparison of geotechnical uncertainties linked to different soil characterization methods. Geomechanics and Geoengineering, 12(2), 137–151.

Erratum (Paper D)

Prästings, A., Spross, J., Müller, R., Larsson, S., Bjureland, W. and Johansson, F., 2017. Implementing the Extended Multivariate Approach in Design with Partial Factors for a Retaining Wall in Clay. ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, 3(4): 04017015. doi: 10.1061/AJRUA6.0000918.

Errors have been found in the published paper. The errors include corrections to Eqs. (8) and (9). Furthermore, a mistake in the calculation of the uncertainty related to random measurement error, $\mathrm{COV}_{\mathrm{e},D_i}$, requires corrections to Table 4 and to Figs. 7, 9 and 10. The corrections of the calculation results are inconsequential and do not alter the conclusions of the published paper. The corrections have been submitted to ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering.

Erratum (Paper E)

Prästings, A., Müller, R. and Larsson, S. 2017. Optimizing Geotechnical Site-investigations. In Proceedings of the 19th International Conference on Soil Mechanics and Geotechnical Engineering, Seoul, South Korea, pp. 639–642.

Errors have been found in the published paper. The errors include a correction of eq. 12. Furthermore, a mistake in the calculation of the uncertainty related to random measurement error, COV_{me,D_i}, requires corrections to Figs. 2, 4 and 5. The corrections of the calculation results are inconsequential and do not alter the conclusions of the published paper. The corrections are given below.

<u>Corrections to section 3.1 (Fig. 2):</u>
A mistake in the calculation of COV_{me,D_i} was found for which corrections are provided in Fig. 2. The correct equation is $COV^2_{me,D_i} = 0.15COV^2_{D_i}$ (for the CPT) and $COV^2_{me,D_i} = 0.3COV^2_{D_i}$ (for the CRS and DSS). This causes subsequent errors in the estimation of the inherent (spatial) variability COV_{sp,D_i} (Spat.), and the statistical uncertainty COV_{st,D_i} (Stat.).

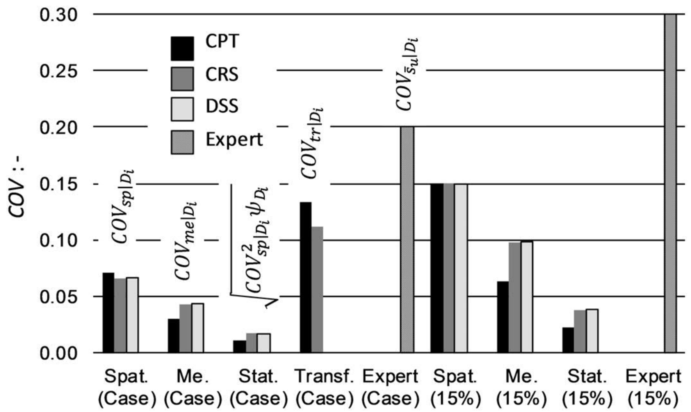

Figure 2. "(Case)" represents uncertainty values obtained in the case project, "(15%)" represents uncertainty values evaluated based on an assumed inherent variability ($COV_{sp|D_i}$) of 15% (see ch. 4).

Correction to section 5.1 (eq. 12):

Equation 12 is incorrectly stated and should be

$$X_d = \frac{\eta}{\gamma_M} \times X_k \ . \tag{12}$$

Subsequent corrections to section 5.2 (Fig. 4):

The correction of the COVs in Fig. 2 requires a subsequent modification to the evaluated conversion factor, η, in Fig. 4.

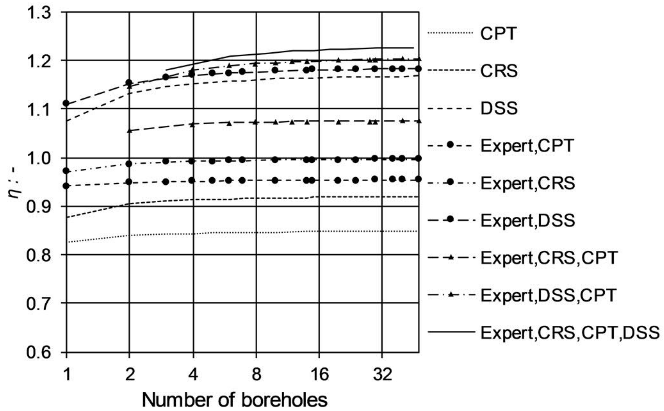

Figure 4. The η in relation to number of boreholes and to different combinations of site-investigation methods performed in the case project.

Subsequent corrections to section 5.3 (Fig. 5):

The correction of the COVs in Fig. 2 requires a subsequent modification to the evaluated conversion factor, η, in Fig. 5. The effect on Fig. 5 is negligible.