Reliability-based analysis of embankment dams

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Master Thesis, 2019
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Abstract

Embankment dams are widely used as water-retaining structures through the world. Two principal structural failure modes of embankment dams are internal erosion and slope instability.

The objective of this thesis is to investigate the feasibility of applying probabilistic methods in embankment dam assessment. Compared to deterministic methods, probabilistic methods considers the variability in material properties and the results are more assessable. Meanwhile, the derived sensitivity factors of the variables can be used to find the most influential one.

Through a literature review, it is found that the difficulties in defining a limit state function for the full process of internal erosion for embankment dams hold back the application of reliability-based methods. However, reliability-based methods have been recognized as suitable for slope stability assessment.

In this thesis, both deterministic and probabilistic calculations have been performed for a homogenous embankment dam. The deterministic calculation is carried out in Slope/W to first find out the most critical slip surface. The failure probability of this slip surface is estimated by both Monte Carlo simulations in Slope/W and FORM analysis in Comrel. Then, the 34 most critical slip surfaces from Slope/W are chosen to calculate the system reliability with simple bounds and applied integration.

The results from the deterministic and probabilistic calculations showed that the studied dam has a higher risk of failure than allowed. In the system reliability calculation, simple bounds gives a wide range, especially the upper bounds. It was found that classification of different slip surfaces into different groups with respect to geometry and material was useful to simplify the calculations. Not much improvement in accuracy of the system probability of failure is found when increasing the
numbers (from 1 to 5) of the most critical slip surfaces from each group. Even the combination of 1 slip surface from each group only gives an error of 6.5%. Categorizing slip surfaces before performing a system reliability analysis is a good way for simplification of the calculations. Due to this, the criteria used for categorization of the slip surfaces are of interest. In this work, it was found that the slip geometry in combination with the materials included in the slip surface constitute a possible way forward to do this.

**Keywords**

Embankment dams, reliability-based analysis, slope stability analysis, internal erosion, dam safety
Sammanfattning

Fyllningsdammar används ofta som dämmande konstruktioner runt om i världen. Två potentiella brottmoder för fyllningsdammar är inre erosion och släntinstabilitet.

Syftet med detta examensarbete är att undersöka möjligheten att tillämpa probabilistiska metoder vid utvärdering av fyllningsdammars säkerhet för dessa brottmoder. Jämfört med deterministiska metoder betraktar probabilistiska metoder även variationen i materialegenskaper. Erhållna känslighetsfaktorer för ingående variabler kan även användas för att hitta de som har störst inverkan på det studerade problemet.

I litteraturstudien framkom att det är svårt att definiera en gränssfunktion som beskriver inre erosion för fyllnadsdammar, vilket begränsar dess användbarhet med probabilistiska metoder. För analys av släntstabilitet är dock probabilistiska metoder användbara.


Resultaten från de deterministiska och probabilistiska beräkningarna visade att den studerade dammen hade en oacceptabel brottsannolikhet. I beräkningarna av gränserna för systemets brottsannolikhet visade resultaten att brottsannolikheten kan variera stort beroende på

...
korrelationen mellan glidytorna. Genom att klassificera glidytorna i olika grupper med hänsyn till geometri och ingående material kan beräkningarna för systemets brottsannolikhet förenklas. Resultaten visade att noggrannheten i beräkningarna för brottsannolikheten av systemet inte förbättrades i någon större utsträckning när antalet valda glidytor (från 1 till 5) från varje grupp av likartade glidytor inkluderades i beräkningen. Kombinationen av 1 glidyta från varje grupp gav endast ett fel på 6,5% jämfört med om alla 34 glidytorna inkluderades. Kategorisering av glidytor av samma karaktär i olika grupper rekommenderas därför att genomföras innan systemets tillförlitlighet analyseras. I detta arbete framkom att glidytan geometri i kombination med ingående material i glidytan utgör ett lämpligt kriterium för att göra denna indelning.

**Nyckelord**

fyllningsdammar, tillförlitlighetsbaserad analys, analys av släntstabilitet, inre erosion, dammsäkerhet
Preface

This thesis is submitted as a degree project for Master of Engineering in Department of Civil and Architectural Engineering in KTH Royal Institute of Technology, Stockholm, Sweden. ÅF Energy, Stockholm, Sweden, financially supports the presented work.

Stockholm, May 2018
Xiaochen Liu
Acknowledgements

First of all, I would like to express my sincere gratitude to my supervisors: Marie Westberg Wilde and Fredrik Johansson for their continuous assistance and encouragement. The thesis work would be impossible without their valuable guidance and suggestions.

Many thanks to ÅF Energy, Hydropower not only for the financial support, also for providing the chance to finish my thesis in ÅF Hydropower, Solna. The groups there have treated me as a member and I will join them soon in September.

Special thanks to Natasha Marxmeier at ÅF, Hydropower, Solna, for her selfless help in embankment dam selection. Andreas Rick had my gratitude for helping me with Geostudio calculation.

Further, I would like to thank Chaoran Fu, my good friend and colleague, who had continuously helped me in both thesis and professional career over the past months.

Last but not least, I want to express my endless love to my family. My great parents, Weidong and Suiping, supported my study in Sweden and have always viewed me as their pride. Also thanks to my partner, Liyan, for her accompanying and sharing my happiness and sorrow.
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1. Introduction

1.1. Background

Building of hydropower in Sweden started at the beginning of 20th century and Hydropower dam construction boomed between the 1950s and 1980s to meet the increasing demand of industrial and residential electricity (Norstedt, 2013). By now, approximately 190 Swedish hydropower dams are internationally classified as large dams (height over 15 m), a majority of them are embankment dams (Norstedt, 2013).

Embankment dams or fill-type dams are generally defined as dams constructed of earth and rock materials (Narita, 2000). They are usually categorized by construction materials as rock-fill dams and earth-fill dams. Besides, embankment dams can also be classified according to their inner configuration, some examples are homogenous embankment dams and embankment dam with a core consisting of low-permeable material.

The statistics for large embankment failures and accidents up to 1986 from Foster et al. (2000) has shown that approximately half of the total failures are caused by structural modes of failure (i.e. piping, slope instability, earthquake etc.) and the rest are related to overtopping. Piping has caused the largest part (46.1%) of failures followed by sliding (Foster et al., 2000). An inventory by Nilsson (1995) indicated that 68 out of 84 Swedish large earth and rock-fill dams has underwent aging issues. The degree of deterioration varies, with ‘Surface erosion in rip-rap’ and ‘Sinkholes in the dam crest or at the shoulder of the dams’ being the two most common ones (Nilsson, 1995).

The safety of dams should always be given highest priority due to severe consequences of failure. In Sweden, the environmental Code and the Civil Protection Act, which is the central part of dam safety regularity
framework, has required dam operators to assess the risk involved in the operation from environmental and safety perspectives (Norstedt, 2013).

The traditional method of evaluating the risk of slope instability is the deterministic method, where representative values of parameters are used to estimate a safety factor. However, this method neglects the variability of materials and when the determined safety factor is applied for conditions with great uncertainties, this factor does not serve as a good representation for the safety of the dam (Zeping Xu, 2017). In addition, the safety factor is determined for the most critical slip surface. However, a number of correlated slip surfaces exists and these should be treated as a system.

Internal erosion is generally assessed by comparing criteria related to filter, transition zones and drainage. The concept of numerical modeling for internal erosion has been developed in recent years, with FEM, FDM and BEM applied for analysis of such problems (Mattsson et al., 2008). These methods could provide detailed information on the safety of embankment dams and furtherly draw conclusion about the initiation of the internal erosion process (Mattsson et al., 2008). However, these deterministic methods all have more or less discarded the uncertainties in input data.

Reliability-based analysis has drawn increasing attention worldwide in the last decade, and constitutes a promising solution to such problems. In reliability-based analysis, parameter uncertainties can be accounted for (Zio, 2013). The limit state function, which contains a sequence of uncertain input variables, is used to calculate the probability of failure (Zio, 2013).

As a quantitative method, reliability-based analysis has made great progress within the embankment dam slope instability assessment. However, the application for internal erosion assessment in embankment dams still need exploration due to great uncertainties from soil properties, dam types, core & filling materials, local climates etc. The increasing demand for a Swedish probabilistic based guideline together with the safety consideration for aging Swedish embankment dams push forward the need for theory development.
1.2. Objectives

This thesis project aims to provide a state-of-the art about the progress and knowledge of reliability based methods for slope stability and internal erosion of embankment dams.

The aim is also to study the influence on the reliability for slope stability if the analysis considers all possible slip surfaces as a system compared to if only the most critical slip surface is studied according to a deterministic approach.

1.3. Outline

Chapter 2 gives an overview about embankment dams including the categorization, failure modes (internal erosion and slope instability) and their mechanisms. A literature review on deterministic and probabilistic analysis is also presented.

In chapter 3, deterministic and probabilistic methods are used to analyze the slope stability of an embankment dam. Outputs from the deterministic calculations are used in the probabilistic analysis to obtain the reliability index and sensitivity factors. 34 of the most critical slip surfaces are chosen and calculated by simple bounds and integration methods to obtain the system reliability. The five most critical surfaces from each group are then combined to investigate the amount needed for a good estimation of the system reliability.

Chapter 4 presents the results after applying the methodologies in chapter 3.

Chapter 5 discusses the critical slip surface from Slope/W in the deterministic calculation, the relationship between the safety factor and failure probability. The performance of simple bounds and the number of slip surfaces needed for a good estimation of the system reliability is also commented. The strategy for slip surfaces grading is discussed in the last part.

In chapter 6, conclusions are given.
1.4. Limitations

In the calculations, a minimum slip surface depth is set to 1 meter in Slope/W, so it only calculates safety factors of slip surfaces that have a depth equal or larger than 1 meter. However, the large slips surfaces usually initiate from some smaller slip surfaces. This setting skips the development of slip surfaces and consider the condition with larger slip surfaces which are hazardous to the structure.

In addition, spatial variability of the fill material in the dam is neglected. The modified embankment dam is a homogeneous dam, but even homogeneous materials have property variances according to their location.

There is also a limitation in the translation of seepage information from Seep/W to Matlab. Piezometric information from Seep/W is a series of points but they are fitted into a rational function in Matlab. This approximation is acceptable but not perfect.

Only the most critical slip surfaces are considered in the system reliability analyses. Slip surfaces with high safety factors and low failure probabilities of failures are not considered in the system reliability analyses. However, they could slightly affect the system reliability, especially when they are correlated with the studied critical slip surfaces.
2. Literature review

2.1. Embankment dams

Embankment dams are generally constructed with natural materials taken close to dam site (soil or rock). Embankment dams is the most popular dam type in the world regarding the number of both completion and construction. The adaptation to critical geometrical condition has allowed embankment dams’ construction on sites where concrete dams are not appropriate, e.g. alluvial deposit and pervious foundations (Narita, 2000). Transportation budget is usually lower than for concrete dams due to the usage of excavated material or adjacent natural materials.

General categorization of embankment dams is based on the material they are composed of. The two most common types are earth-fill or rock-fill embankment dams, according to the inner configuration. A rock-fill dam with a centrally located core is shown in Figure 1.

The central part of an embankment dam is the core, which consists of materials with low permeability, and is essential to prevent water from flowing through the dam (Enegren, 2015). The filter located at both sides of the core retain core fines from being transported with seepage water. Support fill that covers core and filter keep the stability and shield the internal structure form external forces.
2.2. Embankment failure modes and mechanisms

In ICOLD (2015), the word ‘failure’ is defined as ‘Collapse or movement of part of a dam or its foundation, so that the dam cannot retain water. In general, a failure results in the release of large quantities of water, imposing risks on the people or property downstream’.

Foster et al., (2000) made a worldwide survey on failures of large embankment dams up to 1986 with the exception for dams constructed in Japan per-1930 and China, as Figure 2 shows. It is easy to conclude that overtopping (48.4%) and piping (46.1%) are the main failure modes of embankment dams, followed by slides (5.5%). However, overtopping is categorized as hydraulic failure because it generally results from inaccurate hydrological data and inadequate discharge capacity or malfunction of spillway gates rather than due to decisive defects in the embankment dams. Therefore, this thesis focus on the other two main causes of embankment dam failure: internal erosion and sliding failure in the embankment.
2.2.1. Internal erosion

2.2.1.1. Overview about the process of internal erosion

Internal erosion initiates when the unfavorable coincidence of material susceptibility, critical hydraulic load and critical stress condition occur (ICOLD, 2013), as illustrated in the Venn diagram in Figure 3 (Garner and Fannin 2010). The description of these conditions are (ICOLD, 2013):

“**Material susceptibility**: the potential for soil to experience loss of a portion of its finer fraction, as a consequence primarily of grain size and also shape of the grain size distribution curve;

**Critical stress condition**: the inability to resist internal erosion due to the magnitude of effective stress, with recognition that stress varies spatially and/or temporally within the body of the dam; and,

**Critical hydraulic load**: the hydraulic energy required to invoke a mechanism of internal erosion, by means of seepage flow through the dam.”
The three unfavorable coincidences can be concretized and four mechanisms for initiation of internal erosion are described in ICOLD (2017), they are: concentrated leak erosion, backward erosion, contact erosion and suffusion erosion.

The internal erosion process generally follows four phases (ICOLD, 2017, P. 39):
1. Initiation of erosion,
2. Continuation of erosion,
3. Progression to form a pipe or occasionally cause surface instability (sloughing),
4. Initiation of breach
Four general failure pathways were indicated by ICOLD (2017) for failure and incidents caused by internal erosion of embankment dams:

1. Through the embankment, as backward erosion and concentrated leak as illustrated in Figure 4;
2. Internal erosion associated with through-penetrating structures, such as conduits associated with outlet works, spillways walls or adjoining a concrete gravity structure supporting the embankment;
3. Through the foundation, as illustrated in Figure 5;
4. Internal erosion of the embankment into or at the foundation, including (a) seepage through the embankment eroding material into the foundation, or (b) seepage in the foundation at the embankment contact eroding the embankment material, as illustrated in Figure 6.

Figure 4: Conceptual model for development of piping in embankment
(a) Backward erosion (b) Concentrated leak (Foster M. A., 1999).
2.2.1.2. Concentrated leak erosion

Concentrated leak erosion usually happens through cracks and pre-existing defects. The pre-existing defects can be caused by tensile cracks or hydraulic fractures due to differential settlement in the embankment during construction or operation, adjacent to conduits and wall (Bonelli, 2013). Influence from animal and plants can be another reason for pre-existing defects.

Given a crack in a dam, levee or foundation, the initiation of erosion depends on the geotechnical and hydraulic conditions within the crack and the resulting hydraulic forces imposed on the sides of the crack (Bonelli, 2013). Figure 7 and Figure 4(b) show concentrated leak erosion through an open path. Figure 8 illustrates the events leading to a piping initiated by concentrated leak or suffusion.
2.2.1.3. Backward erosion

Backward erosion can be categorized as backward erosion piping and global backward erosion. Backward erosion piping starts when seepage exits through a free unfiltered surface on the downstream side of the embankment with detachment of soil particles as a result (USACE, 2015). This erosion gradually make the way towards the upstream side of the embankment or its foundation until the formation of a continuous pipe by detached particles transportation (USACE, 2015). The piping in a levee foundation initiated by backward erosion is shown in Figure 9.
Similar processes can also happen in embankment dams. The process of development to the breach can be seen in Figure 4(a).

Global backward erosion refers to the nearly vertical pipe in the core of an embankment (Bonelli, 2013) as Figure 10 shows. This is caused by the general movement of the overlying soil when a collapse happens shortly after the primary formation of backward erosion pipe.
2.2.1.4. Contact erosion

Contact erosion is the selective erosion in fine soils from parallel flow in the coarse soil when coarse and fine soils are in contact with each other. Figure 11 and Figure 12 indicates the process of contact erosion initiation.

Figure 11: Possible location of contact erosion initiation (Beguin et al., 2009)
   a) Homogeneous dam with layered fill due to segregation during construction and a coarse foundation soil
   b) Zoned dam with potential for contact erosion at high reservoir levels above the core and for erosion into coarse layers in the foundation (Beguin et al., 2009)

Figure 12: Illustration of the contact erosion process (Sharp, o.a., 2013)
2.2.1.5. Suffusion

Suffusion erosion is defined by ICOLD (2017) as a form of internal erosion which involves selective erosion of finer particles from the matrix of coarser particles, in such a manner that the finer particles are removed through the voids between the larger particles by seepage flow, leaving behind a soil skeleton formed by the coarse particles. A failure path diagram of piping failure initiated by suffusion or concentrated leak is shown in Figure 8.

According to Bonelli (2013) a geometric criterion, a stress criterion and a hydraulic criterion are necessary for the occurrence of suffusion. An explanation of each criterion is given below (Bonelli 2013):

**Geometric criterion**: The size of the finer soil particles must be smaller than the size of constrictions between the coarser particles, which form the basic skeleton of the soil.

**Stress criterion**: The amount of finer soil particles must be enough to fill the voids of the basic skeleton formed by the coarser particles. If there are more than enough finer soil particles for void filling, the coarse particles will be “floating” in the matrix of fine soil particles, instead of forming the basic soil skeleton.

**Hydraulic criterion**: The velocity of flow through the soil matrix must impose a high enough stress to overcome the stresses imposed on the particles by the surrounding soil and to move the finer soil particles through the constrictions between the larger soil particles.

2.2.2. Slope instability

It is essential to understand the fundamental requirement for stability of slopes: “the shear strength of the soil must be greater than the shear stress required for equilibrium” (Duncan et al., 2014). Therefore, the most fundamental cause of slope instability is concluded to occur when the shear strength of the soil is less than the shear stress required for equilibrium (Duncan et al., 2014). The condition can be reached by two means (Duncan et al., 2014):

1. Through the decrease of shear strength of the soil
2. Through the increase of shear stress
As shown in Figure 2, most of slides occurs in downstream the direction. According to Fell et al. (2005), around 1/5000 embankment dams failed due to slope instability, however about 1/200 embankment dams experienced instability accidents. This indicates that modern guidelines with acceptable safety factors are reasonably conservative, and the excessive deformations when slopes are marginally stable has allowed for in time remedial actions before it has developed into failures.

The shape of failure surfaces of slopes varies depending on the homogeneity of the filling materials (Huang, 1983). The most common shape of a failure surface is cylindrical and it forms if the material is homogeneous because a circle has the least surface area per unit mass (Huang, 1983). In case of an infinite slope (small ratio between depth and length), the critical failure surface will be a plane parallel to the slope (Huang, 1983). The combination of plane and cylindrical surfaces or some other shapes may also exist (Huang, 1983). Slope failure surfaces with different shapes are shown in Figure 13.

![Figure 13: Failure surface shapes (Huang, 1983)](image)

For earth dams, it is significant to consider slope instability in both the upstream and the downstream direction. For an upstream slope, the most critical stages are at the end of construction and during rapid drawdown of the reservoir level (Craig, 2013). The critical stages for a downstream slope are at the end of construction and during steady seepage (Craig, 2013). In addition, pore pressure is a dominant factor for the slope stability, and it influences all stages of an earth dam. Detailed explanations and comparisons are available in Craig, 2013.
2.3. Assessment of embankment dam failure

2.3.1. Internal erosion

2.3.1.1. Particle size based assessment of filter

Filter provides the capability of controlling internal erosion through five functions (ICOLD, 2017):

1. **Retention**. The filter limits the transportation of soil particles.
2. **Self-filtration or stability**. The filter is internally stable and will be subject to finer particles eroding.
3. **No cohesion** so the filter will not hold a crack
4. **Drainage**. The filter is sufficiently permeable to allow flowing of water.
5. **Strength**. The filter transfers stresses within the dam without being crushed or becoming finer.

With these functions, the filter is vital to the continuation or interruption if erosion processes. Therefore, despite the cause of internal erosion are often difficult to identify but it is commonly judged associated with the shortcoming in or lack of downstream filters and drainage (RIDAS, 2011).

In Sweden, RIDAS as the Swedish dam safety guideline, adapted suggestions on gradation and dimension of filter material in ICOLD, see Table 1 and Table 2 (RIDAS, 2011, as cited in Enegren, 2015). The common approach is to check characteristics of the dam (based on test pits & grading curves of the dam materials) to the recommendations below in Table 1 and Table 2.
Table 1: Filter criteria for different base materials (RIDAS, 2011, as cited in Enegren, 2015)

<table>
<thead>
<tr>
<th>Base material with fine-grain soil &lt; 30%</th>
<th>Base material with fine-grain soil 30-80%</th>
<th>Reason for demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{15}/d_{15}&lt;40$</td>
<td>$D_{15}&lt;0.7mm$</td>
<td>Prevent transportation of finer materials</td>
</tr>
<tr>
<td>$D_{15}/d_{85}&lt;4$</td>
<td>$D_{15}&lt;0.7mm$</td>
<td>Prevent transportation of finer materials and ensuring internal stability</td>
</tr>
<tr>
<td>$4&lt;D_{15}/d_{15}$ and $D_{15}&gt;0.1mm$</td>
<td>$4&lt;D_{15}/d_{15}$ and $D_{15}&gt;0.1mm$</td>
<td>Ensuring a satisfying hydraulic conductivity of the filter</td>
</tr>
<tr>
<td>$D_{50}/d_{50}&lt;25$</td>
<td>$D_{50}/d_{50}&lt;25$</td>
<td>Can be used to allow for a larger $D_{50}$</td>
</tr>
<tr>
<td>$D_{max}&lt;60mm$</td>
<td>$D_{max}&lt;60mm$</td>
<td>Applies between filter/central core and filter/filter</td>
</tr>
<tr>
<td>$D_{60}/D_{10}&lt;=6$</td>
<td>-</td>
<td>To avoid separation of stones, applies to coarser filter (If $D_{60}&lt;20mm$ this demand is usually not needed)</td>
</tr>
</tbody>
</table>

Table 2: Maximum stone size of filter to avoid stone separation (RIDAS, 2011, as cited in Enegren, 2015)

<table>
<thead>
<tr>
<th>Minimum $D_{10}$ for filter material, mm</th>
<th>Maximum $D_{90}$ for filter material ,mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.5</td>
<td>20</td>
</tr>
<tr>
<td>0.5-1.0</td>
<td>25</td>
</tr>
<tr>
<td>1.0-2.0</td>
<td>30</td>
</tr>
<tr>
<td>2.0-5.0</td>
<td>40</td>
</tr>
<tr>
<td>5.0-10.0</td>
<td>50</td>
</tr>
<tr>
<td>10.0-50.0</td>
<td>60</td>
</tr>
</tbody>
</table>

Rönnqvist (2015) proposed a unified-plot approach tool where the filter coarseness was combined with the stability index $(H/F)_{min}$ of the filter gradation curve. Rönnqvist (2015) constructed a database consisting of 80 embankment dams, where he plotted the stability index and filter coarseness from each dam. Figure 14 shows the unified-plot showing the relation between stability index $(H/F)_{min}$ and dam-specific ratio for base
soil retention $D_{15\text{max}}/D_{15\text{NE}}$. By this method, dams with potential unstable gradation (i.e. $(H/F)_{\text{min}}<1$) may be identified.

Figure 14: Dimensionless unified-plot of the relation between stability index of the filter gradation and dam-specific ratios for base soil retention: $D_{15\text{max}}/D_{15\text{NE}}$ (Rönnqvist et al., 2014)

Comparing to RIDAS or any other filter gradation specifications, unified-plot approach provides the guidance why filter would permit erosion but it needs implementation in assessing filters that are located into the category in between no-erosion and excessive-erosion boundary (Rönnqvist, 2015).

2.3.1.2. Numerical modeling

Numerical modeling is widely applied in geotechnical engineering. However, Fell et al. (2005) indicated its drawback in application due to the strict requirement of extensive technical knowledge for engineers.

The finite element method (FEM), finite difference method (FDM) and boundary element method (BEM) have undergone great development within the years. However, modeling the general manner of
an internal erosion process is still invalid through the calculated stress and deformation (Mattsson et al., 2008).

It is theoretically possible to analyze the initiation of internal erosion process based on the stress, strains, deformation and pore pressure information from these methods (Mattsson et al., 2008). Some studies applying FEM by e.g. Sherard (1986), Ng and Small (1999), Zhang and Du (1997) and Sharif et al., (2001) concentrated on one or two factors of initiation, e.g. the combined results of a critical hydraulic load and a critical stress condition. Nevertheless, one or two factors is still inadequate to cover all initiations of internal erosion.

Another alternative for modeling internal erosion in embankment dams is to start from the fluid mechanic point of view. By viewing the internal process as fluid flow through porous media, Bear (1972) outlined numerous ways for modeling. However, the difficulties exist in dealing with the anisotropic permeabilities and naturally formed free surfaces (Mattsson et al., 2008). This could possibly be modified by solving the momentum equation for two phases and dealing with the motion of surface water by using an advection equation (Mattsson et al., 2008). Wörman and Xu (2001) studied the internal erosion in heterogeneous stratified soils by applying spectral analysis and Laplace transforms. This study illustrated the feasibility of applying this model to analyze engineering problems in embankment dams with defective filter layers (Wörman and Xu, 2001). The potential of using stochastic analysis in risk analyses of extreme events was also highlighted.

Once internal erosion is established, the following process can be viewed as a type of localization, therefore micromechanical models can be used to describe the constitutive behaviors (Mattsson et al., 2008). However, the great difficulties of this method mentioned by Mattsson et al. (2008), lie in describing the strain localization phenomenon.

2.3.1.3. Surveillance

Surveillance and monitoring are important tools in the qualitatively assessment of internal erosion in embankment dams. It is concluded by Fell et al., (2005), from statistics of failure and accidents, that “many
accidents would have become failures if they had not been detected by monitoring and surveillance and some action taken”. The surveillance can usually be executed from aspects of: seepage flow, surface and internal displacement, pore pressure etc. Geophysical methods, a complement to standard detection methods, have given the theoretically potential of detecting internal erosion at an early stage. However, more research and development are necessary for confirmation of its reliability (Mattsson et al., 2008).

2.3.2. Slope instability

The commonly used deterministic methods for slope stability analysis are Limit equilibrium analysis (LEA) and numerical methods. They compute a factor of safety with different methodologies but share the same definition of FOS (Factor of safety) and they both apply equations of static equilibrium (Duncan et al., 2014).

Fell et al., (2005) illustrated the importance of formulating the problem correctly before deciding to use limit equilibrium or numerical method. Particularly to figure out if undrained strengths (total stress analysis) or drained strengths (effective stress analysis) is more critical. If one is unsure about it, both conditions should be checked and the results adapted from the lowest strength is suggested (Fell et al., 2005).

The limit equilibrium method was found by Mouyeaux et al., (2015) to be more effective than the numerical method in analysis of some simple cases. They compared the two methods of different complexity by applying them to the estimation of failure probability of an embankment dam. Similar results were obtained from both methods but the benefit of the numerical method (FEM in this study) cannot be highlighted in that simple case (Mouyeaux et al., 2015).

2.3.2.1. Limit equilibrium method

Limit equilibrium analysis with clear physic theory is more widely used for slope stability analysis than the finite element method in engineering (Cao et al., 2013). It assumes no effects from localized strain weakening
or progressive failure, which leads to a uniform safety factor along the whole failure surface (Fell et al., 2005).

Some limit equilibrium approaches and their characteristics are shown in Figure 15. The differences of these approaches result from various assumptions and inter-slice boundary conditions (Abramson, 2002). Most of them are based on the method of slice (Liang et al., 1999), where the potential sliding mass is divided into several vertical slices and the critical failure surface is defined as the surface with the minimum safety factor.

According to Huang (1983), using Mohr-Coulomb theory to calculate soil shear strength, the safety factor is expressed as the ratio between resisting moment and driving moment according to the center of failure arc:

\[
F = \frac{\sum_{i=1}^{n}(\bar{c}b_i \sec \theta_i + \bar{N}_i \tan \bar{\theta})}{\sum_{i=1}^{n}[w_i \sin \theta_i]} \tag{1}
\]

where \(n\) is the number of slices. \(\bar{c}\) is the effective cohesion of the soil, \(\bar{\theta}\) is the effective friction angle, \(a_i\) is the distance from the center of the circle to the centroid of the slice (Liang et al., 1999). A schematic diagram is given as Figure 16.
### Method | Characteristics
---|---
Slope Stability Charts (Janbu, 1968; Duncan et al., 1987) | Accurate enough for some purposes for initial estimates. Faster than detailed computer analyses
Ordinary Method of Slices (Fellenius, 1927) | Only for circular slip surfaces
Satisfies moment equilibrium
Does not satisfy horizontal or vertical force equilibrium
Underestimates the factor of safety in most cases
Bishop’s Modified Method (Bishop, 1955) | Only for circular slip surfaces
Satisfies moment equilibrium
Satisfies vertical force equilibrium
Does not satisfy horizontal force equilibrium
Force Equilibrium Methods (e.g. Lowe and Karafiath, 1960, and U.S. Corps of Engineers, 1970) | Any shape of slip surfaces
Do not satisfy moment equilibrium
Satisfies both vertical and horizontal force equilibrium
Janbu’s Generalised Procedure of Slices (Janbu, 1968) | Any shape of slip surfaces
Satisfies all conditions of equilibrium
Permits side force locations to be varied
More frequent numerical problems than some other methods
Morgenstern and Price’s Method (Morgenstern and Price, 1965) | Any shape of slip surfaces
Satisfies all conditions of equilibrium
Permits side force orientations to be varied
Spencer’s Method (Spencer, 1967) | Any shape of slip surfaces
Satisfies all conditions of equilibrium
Side force are assumed to be parallel

Figure 15: Characteristics of limit equilibrium methods for slope stability analysis (adapted from Duncan 1992; after Duncan and Wright, 1980)

![Figure 15](image)

Figure 16: Schematic diagram over the derivation of the safety factor for a circular slip surface

The Fellenius ordinary method of slices (1936) is one of the earliest methods for slope stability analysis (Liang et al., 1999). By neglecting interactions between slices (normal and shear forces from adjacent slices), the forces considered in Fellenius method becomes as illustrated in Figure 17.
Then $\bar{N}_i$ can be expressed as:

$$\bar{N}_i = w_i \cos \theta_i - u_i l_i$$  \hspace{1cm} (2)

where $l_i$ is slice base length. By substituting equation 2 into equation 1, the safety factor could be expressed as:

$$F = \frac{\sum_{i=1}^{n}[\bar{c} l_i + (w_i \cos \theta_i - u_i l_i) \tan \bar{\theta}]}{\sum_{i=1}^{n}(w_i \sin \theta_i)}$$  \hspace{1cm} (3)

The effect of seepage in slope stability analysis is important for embankment dams. There are different opinions how to regard this factor; the seepage force can be viewed as an external force if the soil skeleton is considered as a free body. But it is an internal force when viewing soil skeleton together with water as a free body (Briaud, 2013).
2.3.2.2. Numerical method

Despite that the numerical method is not as widely applied as the limit equilibrium method, it is more versatile in detailed analysis with respect to deformations and stresses in embankment dams (Chugh, 2007). The theory to calculate the safety factor either includes the gravity increase method (GIM) or strength reduction method (SRM), and usually employs the finite element method as the basic analysis tool (Cao et al., 2013). Both methods have the salient feature of derivation of the critical failure surface and a safety factor without the presumption for the shape and location of the failure surface. (Li et al., 2009), (Rocscience, 2004).

The critical failure surface in the gravity increase method (GIM) is obtained by increasing the gravity gradually but keeping material properties constant (Li et al., 2009). RFPA-GIM, the developed numerical model for slope analysis implemented into the Realistic Failure Process Analysis (RPFA) code and finite element programming gave results which highly corresponded with experimental results, and predictions made by FEM strength reduction method and conventional limit equilibrium analysis (Li et al., 2009). More studies of slope analysis with GIM methods are: Wei-Ya and Wu (2007) and Ya-ming et al., (2006).

The mechanism in the strength reduction method (SRM) is to decrease the shear material strength until reaching the state of boundary equilibrium, which gives a convergence of the numerical solution (Rakić et al., 2013). The factor of safety in that case is the maximum value of the strength reduction factor satisfying the stability condition (Rakić et al., 2013). The conducted analysis on Prvonek dam by Rakić et al., (2013) considered material properties by analysis of material samples with Hoek-Brown and Mohr-Coulomb constitutive models applied. Besides the use of constant values for inhomogeneous materials properties, there remain some discordances between simulation and experiment results, which comes from creep in the dam material (Rakić et al., 2013).
2.4. Embankment dams in Sweden

Moraine was a typical material used for core construction during the period 1950s to 1980s when most of the Swedish embankment dams were constructed. It is well-graded with permeability varying between $10^{-9}$ to $10^{-6}$ m/s and fine content ranging from 25 to 40% (Nilsson, 2007). Figure 18 gives the grain size distribution for a typical core and filter. The maximum grain size of filter materials was not restricted during this period and requirement for material compaction was set as weight ratio >95 % modified proctor (Nilsson, 2007). Water content was required to range from -2% and +2% in relation to the modified optimum water content (Nilsson, 2007). Comparing with present guidelines, material compaction was replaced by void content with the maximum air void content prescribed to 7 or 10 % and water content was adjusted to the range from 0% to +3% over the optimum water content (Nilsson, 2007).

Figure 18: Typical particle size for moraine used as impervious core material and downstream filter material (Nilsson, 2007)

Nilsson (2007) documented embankment dams reported to have sinkholes related to the year of completion, illustrated in Figure 19 and Figure 20. He found that nearly half of the embankment dams completed during 1970-1974 were reported with sinkholes, which was considered to occur because of poor design and/or construction of that time.

The inventory of downstream filter grain size distributions presented by Norstedt et al., (1997) has indicated that a coarser D15 size than present filter specification was used in most documented embankment
dams from that time. D15 size according to present guidelines for moraine with large portion of fines, which is the most common case, is maximum 0.7 mm (Nilsson, 2007). Figure 21 gives the distribution of filter grain size with respect to year of completion.

![Figure 19: Comparison of number of embankment dams with reported sinkholes with total number with respect to year of completion (Nilsson, 2007)](image1)

![Figure 20: Number of embankment dams with reported sinkholes in percent to total number with respect to year of completion (Nilsson, 2007)](image2)
2.5. Reliability-based assessment methods

Reliability-based theory deals with the uncertainties in load and resistance by replacing representative deterministic values of variables with uncertain values, which correspond more with real engineering conditions. Reliability is expressed by CUR (2006) as a probability of proper functioning and the reliability index $\beta$ is now commonly used to represent reliability.

The margin between safety and risk is usually separated by a limit state function, where the reliability is the probability that the limit state is not exceeded (CUR Publicatie, 2006). The limit state function is usually expressed as:

$$ G = R - S $$ (4)

where $R$ is the resistance to failure and $S$ is the load that could potentially lead to failure.

Zero value of $G$ ($R=S$) indicates the limit state, positive value gives 'non-failure' and negative value gives 'failure' (Westberg, 2010). Figure 22 illustrates a failure surface in 2D given by a limit state function.
The probability of failure is:

\[ P_f = P(G \leq 0) = P(S \geq R) \] (5)

Therefore, the reliability value \( P(G > 0) \) is:

\[ P(G > 0) = 1 - P_f \] (6)

The values of \( R \) and \( S \) are usually of great uncertainties, e.g. physical/mechanical uncertainty, statistical uncertainty and model uncertainty (Westberg, 2010). Therefore, in reliability-based analysis, \( R \) and \( S \) are usually modeled as random variables rather than given certain values (CUR Publicatie, 2006).

Methods for derivation of reliability are generally divided into three levels (Westberg, 2010) and (CUR Publicatie, 2006):

1. Level III: attempt to get the best estimate of the probability of failure by considering the probabilistic description of all/most resistance and load variables,
2. Level II: describe variable distribution by applying simplified representation or failure domain idealization, gives an approximation of the probability of failure,
3. Level I: a semi-probabilistic method and gives no probability of failure. It is also called particle factor method in design.

Figure 22: Failure surface in 2D (Westberg, 2010)
2.5.1. Level III methods

2.5.1.1. Fundamental solution

In case there are only two independent variables $R$ and $S$ in a limit state function, the probability of failure can be written in a convolution integral:

$$ p_f = P(R - S \leq 0) = \int_{-\infty}^{\infty} F_R(x) f_S(x) \, dx $$  \hspace{1cm} (7)

Where $F_R(x)$ is the probability that the actual resistance $R \leq x$ or the probability that the actual resistance $R$ of the structural system is smaller than some value $x$ (Westberg, 2010). $f_S(x)$ represents the probability density function of load $S$ and its integral over interval $dx$ ($x \ll S \ll x + \Delta x$) represents the probability of occurrence that $S=x$. The function gives the total failure probability, which correspond with formula (5). Figure 23 below gives a good reference for the solution.

![Figure 23: Basic R-S problem: $f_R()f_S()$ representation (Melchers and Beck, 2017)](image)

Equation (7) can be analytically integrated into equation (8) when the limit state function is linear and the variables are independent normally distributed.

$$ P_f = P[G(x) \leq 0] = \phi\left(\frac{0 - \mu_G}{\sigma_G}\right) = \phi(-\beta) $$ \hspace{1cm} (8)

Where $\mu_G$ and $\sigma_G$ are the mean value and standard deviation of the limit state function $G$, they can be calculated as follow:
\[
\mu_G = \mu_R - \mu_S \quad (9)
\]
\[
\sigma_G^2 = \sigma_R^2 + \sigma_S^2 \quad (10)
\]

\(\beta\) is the reliability index, where a low \(\beta\) value indicates high probability of failure. It can be interpreted to be the number of standard deviation \(\sigma_c\) by which \(\mu_G\) exceeds zero, as Figure 24 shows:

![Figure 24: Normal distribution of safety margin \(G = R - S\) (Melchers and Beck, 2017)](image)

It is worth mention that the reliability index is not invariant, which means expression

\[G = R - S\]

and

\[G = \frac{R}{S} - 1\]

does not yield the same result despite that they share the same concept (Westberg, 2010).

2.5.1.2. Numerical integration

In general the dependence between variables should be considered, where equation (7) does not suit. In that case, numerical integration can be applied.
As is mentioned, the load and resistance are functions of various variables, the reliability function can also be written as:

\[ G = g(x_1, x_2, \ldots, x_n) \]  \hspace{1cm} (11)

The probability of failure can be written in integral:

\[ P_f = \int \int \int \ldots f_{x_1,x_2,\ldots,x_n}(x_1, x_2, \ldots, x_n) \, dx_1 \, dx_s \, \ldots \, dx_n \]  \hspace{1cm} (12)

It can be written as:

\[ P_f = \int_{Z<0} \int \ldots \int_{Z<0} f_{X_1}(x_1) \ldots f_{X_n}(x_n) \, dx_1 \, dx_s \, \ldots \, dx_n \]  \hspace{1cm} (13)

Various numerical integration methods are available and it is efficient to apply the numerical methods for integration of \( n \)-dimensional hypercubes considering the integration limited by 0 to 1 (CUR Publicatie, 2006). Detailed description on a simple numerical integration method according to Riemann-orsicedure is give in CUR Publicatie, (2006).

2.5.1.3. Monte Carlo method

The Monte Carlo method artificially carry out a great number of experiments by using randomly selected values for all variables according to their probability density functions. The probability of failure is approximated as:

\[ P_f \approx \frac{n(G(x_i) \leq 0)}{N} \]  \hspace{1cm} (14)

which gives the ratio between the number of trials that \( G(x_i) \leq 0 \) (failure) and the number of total trials \( N \).
Large number of simulations are necessary for an accurate estimation of $P_f$ and some simplification methods can be used to reduce the number of simulations.

2.5.2. Level II methods

Various methods are available in Level II for specific conditions e.g. linear/ non-linear; normally distributed/ non-normally distributed variables; dependent/ independent variables. The main differences lie in the idealization of the limit state function and the approximation of variables.

2.5.2.1. First order reliability method

As is discussed in chapter 2.5.1.1, the problem with that approach is that the reliability index relies on the choice of linearization point and is not invariant. Hasofer and Lind (1974) addressed this problem by developing a geometric interpretation for the reliability index. According to their theory, all variables have zero mean value and unit standard deviation in standard normal space (Westberg, 2010). The expression of the safety index is expressed as:

$$\beta = \min \left( \sum_{i=1}^{n} y_i^2 \right)^{1/2}$$  \hspace{1cm} (15)

where $y_i$ represent the coordinates of any point on the limit state surface in standard normal space. The point with the shortest distance to origin is called ‘design point’ and is denoted by $y^*$, it represents the highest probability density for the failure domain (Westberg, 2010). The distance between the origin and the failure surface, which is also the distance between the design point and the origin, is the reliability index $\beta$.

As shown in Figure 25, directions cosines $\alpha_i$ are introduced as the direction features of $y^*$, which gives the coordinate of $y^*$:

$$y^* = -\alpha_i \beta$$  \hspace{1cm} (16)

$$\left(\alpha_i^2 + \cdots \alpha_n^2\right) = 1$$  \hspace{1cm} (17)
\( \alpha_i \) represents the sensitivity of the standardized limit state function to changes in \( y_i \). A low value of \( \alpha_i \) indicates a small influence on the \( p_i \) for changes in \( y_i \) and the parameter can rather be treated as a deterministic value rather than a random variable (Melchers & Beck, 2017).

![Direction cosines](image)

Figure 25: Direction cosines \( \alpha_i \) of design point for a 2D limit state function (Westberg, 2010)

The equation of the tangent hyperplane equation \( g_L(y) \) to the transformed failure surface \( g(y) \) through the design point is therefore given by:

\[
g_L(y) = \beta + \sum_{i=1}^{n} \alpha_i y_i
\]

(18)

If the limit state function is differential, the sensitivity factor can be calculated according to Thoft – Cristensen and Baker (2012) as:

\[
\alpha_i = \frac{-\frac{\partial g}{\partial Y_i}}{\sqrt{\sum_{i=1}^{n} \left( \frac{\partial g}{\partial Y_i} \right)^2}}
\]

(19)

where \( n \) is the number of basic variables in limit state function, \( \frac{\partial g}{\partial Y_i} \) is the partial derivative of the transformed limit state function \( g(Y) \) for the normalized random variable \( Y_i \) (Westberg, 2010).

2.5.2.2. Hasofer-Lind transformation

The special condition discussed in chapter 2.5.1.1 is seldom valid in practice (Baecher and Christian, 2005) and the convolution integral
cannot be solved analytically (Westberg, 2010). In order to get independent standard normal variables, the following transformation function can be applied on independent normal variables:

\[ y_i = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}} \] (20)

where \( X_i \) are independent normal variables and \( Y_i \) are standard normal variables, \( \mu_{x_i} \) and \( \sigma_{x_i} \) are mean values and standard deviations of variables \( X_i \). Accordingly, the limit state function is transferred from \( G(x) = 0 \) to \( g(y) = 0 \). A schematic picture of transformation in 2D-space is shown as Figure 26.

![Figure 26: Schematic picture of the transformation to standard normal space in 2D space.](image)

2.5.3. System reliability

The reliability of a single failure mode or single element is discussed in previous sections, but failures are commonly caused by the resultant of defection of several elements or occurrences of several failure modes.

An ideal series system fails if any of its component fails, as shown in Figure 27 (a), therefore,

\[ P_f = P(\cup E_i) = P(\cup \{G_i(x) \leq 0\}) = P(\{\min G_i(x) \leq 0\}) \] (21)

where \( E_i \) is the event of failure of component \( i \) (or in failure mode \( i \)).
An ideal parallel system fails if all its component fail, as shown in Figure 27 (b), therefore,

\[ P_f = P(\cap E_i) = P(\cap \{G_i(x) \leq 0\}) = P(\{\max G_i(x) \leq 0\}) \] (22)

where \( E_i \) is the event of failure of component \( i \) (or in failure mode \( i \)) (Hohenbichler and Rackwitz, 1982).

The difficulties in assessing system reliability lies in the determination of joint probability, which depends on the correlation \( \rho_{ij} \) between event \( E_i \) and \( E_j \) (Westberg, 2010).

A schematic table about the effect from correlation on system reliability is shown in Figure 28.
Assume that \( n \) failure modes (or \( n \) elements) are identified and their limit state functions \( G_i(x_1, \ldots, x_k, \ldots, x_m) \) and safety indices \( \beta_i \) are derived. According to Eq 18, the transformed limit state function in standard normal space is:

\[
g_n = \beta_n + \alpha_{n1}y_1 + \alpha_{n2}y_2 + \cdots + \alpha_{nm}y_m
\]

where \( \alpha_{ik} \) is the sensitivity factor for each stochastic variables \( k \) of failure mode \( i \). And variables \( y_k \) are standard normal variables.

Correlations between different failure modes \( i \) and \( j \) can be calculated by:

\[
\rho_{ij} = \sum_{k=1}^{n} \alpha_{ik} \alpha_{jk}
\]

2.5.3.1. Simple bounds

Simple bounds for a series system are:

\[
\max_{i=1}^{n} P(E_i) \ll P_f \ll 1 - \prod_{i=1}^{n} (1 - P(E_i))
\]
Narrower bounds for a series system with \( n \) elements derived by Ditlevsen (CUR Publicatie, 2006) can be calculated as:

\[
P(E_i) + \sum_{i=2}^{n} \max \left[ \left( P(E_i) - \sum_{j=1}^{i-1} P(E_i \cap E_j) \right), 0 \right] \ll P_f
\]

\[
\leq \sum_{i=1}^{n} \left( P(E_i) - \max_{j<i} P(E_i \cap E_j) \right)
\]

Combined probability of event \( E_i \) and \( E_j \) in lower bound is

\[
P(E_i \cap E_j) = \Phi(-\beta_i^*) \Phi(-\beta_j) + \Phi(-\beta_i) \Phi(-\beta_j^*)
\]

and that in upper bounds:

\[
P(E_i \cap E_j) = \Phi(-\beta_i^*) \Phi(-\beta_j)
\]

where

\[
\beta_i^* = \frac{\beta_i - \rho_{ij} \beta_i}{\sqrt{1 - \rho_{ij}^2}}
\]

Simple bounds for a parallel system are:

\[
\prod_{i=1}^{n} P(E_i) \leq P_f \ll \max_{i=1}^{n}(P(E_i))
\]

for \( \rho_{ij} > 0 \);

Narrower bounds for a parallel system with \( n \) elements derived by Ditlevsen (CUR Publicatie, 2006) can be calculated as:

\[
\prod_{i=1}^{n} P(E_i) \leq P_f \ll \max_{i=1}^{n}(P(E_i \cap E_j))
\]

for \( \rho_{ij} > 0 \).
\( P(E_i \cap E_j) \) here can be either calculated by Eq. 28 for upper bounds or by integration as described below.

2.5.3.2. Integration

Despite that it is easy to estimate the system reliability through simple bounds, the range is wide with low use in practice (Westberg, 2010). Therefore, methods of integration can be applied to help estimate a more accurate result.

The joint probability of two components, \( P(E_i \cap E_j) \) can be expressed in a bivariate normal distribution function (Nikolaidis et al., 2004) as:

\[
\Phi_2(E_i, E_j; \rho_{ij}) = \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} \varphi_2(t_i, t_j; \rho_{ij}) dt_i dt_j
\]  

(32)

where the bivariate normal density function with zero mean \( \varphi_2 \) is given by

\[
\varphi_2(t_i, t_j; \rho_{ij}) = \frac{1}{2\pi\sqrt{1-\rho_{ij}^2}} \exp\left(-\frac{1}{2(1-\rho_{ij}^2)}(t_i^2 + t_j^2 - 2\rho_{ij}t_it_j)\right)
\]  

(33)

The approximated reliability index \( \beta_{sys} \) for a series system with two components can be calculated as:

\[
\beta_{sys} = -\Phi^{-1}(P_f) \approx -\Phi^{-1}(1 - \Phi_2(\beta_i, \beta_j; \rho_{ij}))
\]  

(34)

And for a parallel system with two components:

\[
\beta_{sys} = -\Phi^{-1}(P_f) \approx -\Phi^{-1}(\Phi_2(-\beta_i, -\beta_j; \rho_{ij}))
\]  

(35)

The reliability for a system with \( n \) elements can be obtained by repeating the combing procedure of two elements, or it can be calculated by a multivariate standard normal distribution of \( n \)-dimensions (\( \Phi_n \)) (Westberg, 2010).
The system sensitivity factor is also updated and it can be calculated form the following procedure (Westberg, 2010):

1. Calculate $\beta_i$ and associated $\alpha_{ik}$ and $\beta_j$ and associated $\alpha_{jk}$ for the original limit state function. The transformed limit state function in normal standard space can be expressed as:

   \[
   g_i = \beta_i - \alpha_{i1}y_1 - \cdots - \alpha_{im}y_m
   \]
   \[
   g_j = \beta_j - \alpha_{j1}y_1 - \cdots - \alpha_{jm}y_m
   \]

2. Calculate $\beta_{sys}$ by integration or Monte Carlo simulation and the system limit state function is:

   \[
   g_{ij} = \beta_{sys} - \alpha_1y_1 - \cdots - \alpha_my_m
   \]

3. Increase the mean value of variable $k$ in the $g_i$ and $g_k$ equation by a value $\varepsilon$, giving

   \[
   g_i' = \beta_i - \alpha_{i1}y_1 - \cdots - \alpha_{ik}(y_k + \varepsilon_k) - \cdots - \alpha_{im}y_m
   \]
   \[
   g_j' = \beta_j - \alpha_{j1}y_1 - \cdots - \alpha_{jk}(y_k + \varepsilon_k) - \cdots - \alpha_{jm}y_m
   \]

   and calculate the new $\beta_{sys}'$.

4. Approximate values of $\alpha_k$ can be obtained from

   \[
   \alpha_k = \frac{\delta \beta}{\delta \varepsilon_k} = \frac{\beta_{sys}'}{\varepsilon_k}
   \]

2.6. Reliability-based assessment of embankment dam failure

2.6.1. Internal erosion

The failure of an embankment dam due to internal erosion are generally divided into four phases: initiation, continuation, progression and breach. There are plenty of variables in each phase and the bound between
adjacent phases are vague. Therefore, the main problem of applying reliability-based theory in assessing internal erosion problems in embankment dams, according to Redaelli (2014), lies in the difficulties of defining a performance function for reliability analysis.

Due to the reason above, the probability of an internal erosion process is more difficult to quantify with a completely probabilistic approach. The quantification has been circumvent e.g. by expert elicitation, subjective approach and probabilistic models for part of the internal erosion problem. Some probabilistic studies are also performed on govern parameters in certain phase of internal erosion.

In quantitative risk analysis, it is common to analyse potential failure modes. USBR (2011) described potential failure modes from initiation, through a step-by-step progression. The annual load probability is typically included at the beginning of the tree. The probability for each subsequent event of an event tree can then be estimated. Where the probability of an event to occur is objectively assessed, but often subjective probabilities are necessary and expert elicitation is used.

The reliability rating system combines traditional way of reliability analysis and subjective probabilities. The innovation part is the elicitation of subjective probabilities to incorporate epistemic uncertainties caused by lack of information and important geotechnical aspects that are neglected (Redaelli, 2014). The reliability rating system was applied for assessing the probability of breaching resulting from both under-piping (Redaelli, 2009) and through-piping (Redaelli, 2014). This method gives a way to quantify the impact of epistemic uncertainty.

Similar to other methods that elicit probability from subjective judgement or expert judgement, the reliability rating system is also combined with the event tree method. As Figure 29 shows, the occurrence of failure is decomposed in a sequence of simpler events (Redaelli, 2014).

The event tree is set previously and probability of occurrence is filled in for each event. The probability of some or all events which are absent by other means can be assessed by expert judgement (Redaelli, 2014). Reliability rating system is not a full reliability analysis method and a
rigorously structured procedure must be followed with the consumption of tremendous time and manpower.

![Event tree for breaching by piping through the earthfill of flood embankments](image)

**Figure 29:** Event tree for breaching by piping through the earthfill of flood embankments (Redaelli, 2009)

Dotted branches connect events that are not mutually exclusive; WL = water level, PP = pore pressure

Andreini et al., (2016) developed a probabilistic model for the two governing parameters of the erosion phenomena (critical shear stress $\tau_c$ and coefficient of erosion $C_e$). The critical shear stress was defined as the tangential stress threshold at which the erosion mechanism initiates and the coefficient of erosion is the constant that express the magnitude of the rate of erosion (Andreini et al., 2016). Probabilistic models were developed by combing empirical models and the results from hole erosion test (Wan and Fell, 2002, 2004a,b; Bonelli et al., 2013). Developed models were then applied in the estimation of conditional probability of initiation of concentration leak erosion with no detection, intervention, and repair for a typical earth dam. With contributions from both cohesive and noncohesive components considered, the models provided the potential of predicting the influence of physical and geotechnical properties of soils on the erodibility behavior in water-retaining structures (Andreini et al., 2016).

Lee and Lee (2017) evaluated the probability of backward erosion and suffusion in an alluvial foundation soil based on the comparison of distribution feature (mean and variance) of erodible particle size and erosion conduit size. The final result gave a reliability index against internal erosion as shown in Figure 30 (Lee and Lee, 2017). The placement of a seepage stability berm near dam toe was considered to be
more cost effective than installation of a slurry wall or sheet piles underneath the dam core in prevention of backward erosion (Lee and Lee, 2017).

![Figure 30: Probability of failure and reliability index](image)

(a) Probability distribution of random values of $Q$ and $R$
(b) Distribution of limit state function values

However, starting from the point of governed material properties for internal erosion can only give the probability of initiation but could hardly describe the following processes.

The Netherlands has developed some specifications to assess probability of backward erosion based on probability. A probabilistic calculation software named ‘Hydra-Ring’ is under development.

The WBI 2017 Code Calibration associated probabilistic (First order reliability method) with semi-probabilistic method (partial factor method) by equating the design values of the different stochastic variables to their design point value. The design point values was obtained from FORM-analysis and design point values for a structure are calculated as follow:

$$S_d = S_{rep} \cdot \gamma_S$$  \hspace{1cm} (42)
$$R_d = R_{rep}/\gamma_R$$  \hspace{1cm} (43)

where $S_d$ and $R_d$ are design values for the load and resistance, $S_{rep}$ and $R_{rep}$ are representative value of the load and resistance, and $\gamma_S$ and $\gamma_R$ are particle safety factors for load and resistance.
As for backward erosion assessment, failure is considered to arise from contribution of all sub-failure mechanisms (uplift, heave and piping) (Jongejan, 2017). The limit state function is given as:

\[
Z = \max\{Z_{up}, Z_{he}, Z_{pip}\}
\]

(44)

Where \(Z_{up}, Z_{he}\) and \(Z_{pip}\) are limit state functions for uplift, heave and piping and can be expressed as:

**Uplift:**

\[
Z_u = m_u \Delta \phi_{c,u} - (\phi_{exit} - h_{exit})
\]

(45)

**Heave:**

\[
Z_h = i_{c,h} - \frac{\phi_{exit} - h_{exit}}{D_{cover}}
\]

(46)

The limit state function for piping is based on the Sellmeijer 2011-model (Sellmeijer et al., 2011):

\[
Z_p = m_p H_c - (h - h_{exit} - r_c D_{cover})
\]

(47)

where \(m_u\) is the model factor uplift, \(\Delta \phi_{c,u}\) the critical head difference for uplift, \(\phi_{exit}\) the piezometric head at the exit point, \(h_{exit}\) the phreatic level at the exit point, \(i_{c,h}\) the critical heave gradient, \(D_{cover}\) the total thickness of the cover layer, \(m_p\) a model factor, \(H_c\) the critical head difference over the structure between entry and exit point, \(h\) the actual water level and \(r_c\) a reduction factor.

The limit state function is time invariant in the Sellmeijer model. This process-based model was developed and tested for uniform homogeneous and horizontal soil layers (Jongejan, 2017).

### 2.6.2. Slope instability

The development of probabilistic theory has provided another approach for assessing embankment dam slope stability. Despite that the method has underwent rapid development in the last decades, the application of probabilistic theory within this field is still a subject of debate. Fell et al. (2000) & Fell et al. (2005) argued against the application due to the low incidence of failure of embankment dams from slope instability and that

Consenters believe probabilistic methods in the analysis of embankment slope instability can work as a rational tool to account for uncertainties in e.g. soil properties, modeling errors etc. (Cao et al., 2017). In addition to the results, reliability index can be used in conjunction of cost analysis (Liang et al., 1999). Zeping Xu (2017) pointed out that reliability analysis constitute a good complement for the traditional deterministic method and it will be more significant in embankment dam slope stability assessment with the accumulation of experience and statistics. Christian et al (1994) presented convincing conclusions on its application based on the analysis of the James Bay embankment dam project.

The most effective and widely-used application on probabilistic assessment of slope instability in embankment dams is the first-order second moment (FOSM) method (Wu and Kraft, 1970; Cornell, 1971; Alonso, 1976; Tang et al. 1976; Vanmarcke, 1977, as cited in Christian et al., 1994), together with the Monte Carlo simulation method (MCSM). Four recognized slope stability analysis methods are used: ordinary method of slices, simplified Bishop’s method, simplified Janbu’s method and Spencer’s method (Malkawi et al., 2000).

The advantages of applying reliability-based theory is recognized for considering uncertainties of random variables in the calculation (Zeping Xu, 2017; Liang et al., 1999). The potential of conjugating the probabilistic analysis with cost-benefit analyses is also described by Liang et al., (1999). However, the influence of epistemic uncertainties, the complexity in computation of the reliability index, the accuracy of collected data and applied statistical method are the issues to be addressed (Zeping Xu, 2017).

It was observed by Liang et al., 1999, Chen and Chang (2011) and Wu et al., (1999) that the slip surface with the minimum safety is not necessarily the slip surface with maximum failure probability.
Nevertheless, it could still be used as an initial trial for the search of slip surfaces with minimum safety factor or maximum failure probability (Liang et al., 1999).

An interesting comparison was carried out by Sivakumar Babu and Srivastava (2010) between results from the combination of response surface methodology (RSM) & first order reliability method (FORM) and the combination of MCSM & limit equilibrium method. The influences from distribution method, correlation coefficient, variance of reservoir full level and seismic acceleration were highlighted.

In Netherlands code WBI 2017, slope stability analysis is based on Uplift-Van model, which is an extension of Bishop’s model according to local conditions in the Netherlands. A brief introduction is given in 2.6.1. It differs from previously used specifications as the shear strength is modelled on the basis of Critical State Soil Mechanics (CSSM) (Schofield and Wroth, 1968) instead of using the Mohr-Coulomb model. CSSM uses the Stress History and the Normalized Soil Engineering Properties (SHANSEP) approach (Ladd and Foot, 1974). Spatial variabilities are dealt with using random fields models (Vanmarcke, 2011; Vanmarcke, 1985; Vanmarcke, 1977) and modern numerical analyses, e.g.Random Finite-Element Method (RFEM) (Griffiths and Fenton, 2004) and Nonlinear Deformation Analysis (NDAs) (Boulanger and Montgomery, 2016). Material properties over certain segments in limit equilibrium stability analysis are estimated based on the average values (Jongejan, 2017). Results from test cases showed that reliability index and safety factor were insensitive to outer water level, which suggested a potential update in reliability estimations based on performance observations (Jongejan, 2017).
3. Methodology

In order to obtain the second aim of this thesis, a homogeneous embankment dam is used as a calculation example. The calculation scheme is shown in Figure 31. The calculations consists of three different parts: in the first part a deterministic analyses is performed, in the second part a probabilistic analysis is performed based on the most critical slip surface in the deterministic calculation and in the third part several slip surfaces correlated with each others are considered as a system and the joint probability of failure for the whole system is calculated.

In the first part, a deterministic slope stability analysis based on the Fellenius ordinary method is carried out in Slope/W, where the seepage condition is simulated in Seep/W for a normal reservoir level.

Slope/W also provides the ability to perform Monte Carlo simulations, therefore a reliability-based analysis using randomly generated parameters of the materials (friction angle and density) is done in the second part.

Figure 31: Flow chart over the calculation procedure.
A deterministic slope stability analysis is also set up with Fellenius ordinary method in Matlab with seepage data from Seep/W applied to fit into a piezometric line. Then the results from limit equilibrium calculations are used to define a limit state function in Comrel. The reliability analysis using FORM in Comrel gives the reliability index and sensitivity factors for the variables.

Slope failure of the embankment dam is considered as a series system and each potential slip surface as a failure mode. Slip surfaces with low safety factor or reliability index are chosen and integrated to study system reliability, which is done in the third part. Both methods of integration and simple bounds will be applied to get failure probability of this series system. Besides, different numbers of slip surfaces are chosen to be integrated in order to find out the amount of slip surfaces needed to get a reliable estimation of the failure probability.

3.1. Dam description

3.1.1. Geometry

The dam geometry is based on a real homogeneous dam with no inner filters. This simple structure is chosen because of the convenience in further calculating the seepage condition and decrease the difficulty in the application of the Fellenius ordinary method. There are no transition zones between different materials, thus only a limited number of material properties should be considered.

The embankment dam is 3.74 meters in height with normal reservoir water level (DG) at 3.04 meters. Upstream and downstream slopes are 6.5 and 14.1 meters long with 30 and 15 degree angles separately. Embankment and foundation both consist of silty sand. The analyzed section is shown in Figure 32.
3.1.2. Material assumption

The main material used for the embankment dam is silty sand with different degrees of compaction. Material properties for deterministic calculations in GeoStudio are shown in Table 3.

<table>
<thead>
<tr>
<th>Material</th>
<th>Unit weight (kN/m³)</th>
<th>Friction angle (°)</th>
<th>Hydraulic conductivity (K(m/s))</th>
<th>Volumetric water content (VWC(%))</th>
<th>Calculation model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Embankment (Silty sand)</td>
<td>20</td>
<td>32</td>
<td>5*10⁻⁶</td>
<td>40</td>
<td>Mohr Coulomb Saturated/ Unsaturated</td>
</tr>
<tr>
<td>Foundation (Silty sand</td>
<td>20</td>
<td>35</td>
<td>1*10⁻⁸</td>
<td>50</td>
<td>Mohr Coulomb Saturated/ Unsaturated</td>
</tr>
<tr>
<td>permanently compressed)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Test pit experiment data from SWECO gave material grain sizes data, as shown in Table A.1, where mean values of grain sizes are used for the calculation. Locations of these drilling tests are shown in Figure A.1. With these material properties, VWC (Volumetric Water Content) and hydraulic conductivity of the dam body and foundation materials according to matric suction are shown in Figure 33 and Figure 34. Soil matric suction is commonly referred to as the negative pore-water pressure. This parameter is introduced because the change of soil shear
stress may vary with different rates in condition that negative and positive pore-water pressures change (GEO-SLOPE International, 2017).

Figure 33: Left: Dam body VWC (Volumetric Water Content) function; Right: Foundation VWC (Volumetric Water Content) function

Figure 34: Left: Dam body hydraulic conductivity function; Right: Foundation hydraulic conductivity function.

In probabilistic calculations, friction and unit weight of foundation material and dam body material are assumed to be normally distributed, which is also recognized as the most usual distribution in geotechnical
reliability analysis (Rackwitz et al., 2002). Rackwitz et al., (2002) also suggested the following material properties for silty sand: 18-20 kN/m$^3$ for dry unit weight, 19.5-20.5 kN/m$^3$ for saturated unit weight and 0.45-0.6 for the internal friction ($\tan(\varphi)$). The standard deviation are 5-10% of the unit weight and 10-20% of the drained internal friction. Therefore, the material properties were assumed according to Table 4.

Table 4: Material properties input random variables in the probabilistic calculation.

<table>
<thead>
<tr>
<th>Material property</th>
<th>Distribution method</th>
<th>Value</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Embankment</td>
<td>Unit weight(kN/m$^3$)</td>
<td>20</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Friction angle(°)</td>
<td>32</td>
<td>32</td>
<td>2</td>
</tr>
<tr>
<td>Foundation</td>
<td>Unit weight(kN/m$^3$)</td>
<td>20</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Friction angle(°)</td>
<td>35</td>
<td>35</td>
<td>2</td>
</tr>
</tbody>
</table>

### 3.2. Deterministic calculation

#### 3.2.1. Slope/W calculation

Fellenius ordinary slice method is applied for the deterministic calculation in the software GeoStudio, where seepage condition through the embankment is simulated in SEEP/W before doing further slope instability analysis in SLOPE/W.

Usually, dangerous slip surfaces develop from surfaces with larger safety factors but at smaller scale. However, the process of slip surface development is neglected in SLOPE/W by setting a minimum slip surface depth. Considering the dimension of dam, it is assumed that only slip surfaces with depth larger than 1 meter can potentially cause hazard structural failure. It also means that the program will filter out any slip surface depth less than 1 meter.

By using ‘Grid and Radius’ option, a 30*30 grid and 30 control lines are defined to search for the most critical slip surface. Each point in the grid above the embankment dam geometry represents a potential center
of a slip surface and it gives the radii by finding a tangency point from the control lines.

3.2.2. Matlab approach

In order to carry out the probabilistic analyses of the embankment dam, Matlab is used to build the limit state function based on the critical slip surface from SLOPE/W. The Matlab code is presented in Appendix B.

3.2.2.1. Piezometric line

In order to obtain the best fitting input parameters in the Matlab calculation based on the SLOPE/W data, the input piezometric line data is derived from SLOPE/W. For the convenience of calculation, these data are simulated as functions using the curve-fitting tool in Matlab. The simulated result gives a rational function for the piezometric line in the main body according to the following equation:

\[ f(x) = \frac{(p_1 x^2 + p_2 x + p_3)}{(x^3 + q_1 x^2 + q_2 x + q_3)} \]  

(48)

With coefficients:

\[ p_1 = 250; p_2 = -1.237 \times 10^4; p_3 = 1.53e \times 10^5; \]
\[ q_1 = 8.723; q_2 = -2564; q_3 = 4.504e + 04 \]

3.2.2.2. Adaption of Fellenius express

Neglecting seismic and cohesive force (c equals to 0 for silty sand), Eq 2 can then be expressed as:

\[ F = \frac{\sum_{i=1}^{n} [(w_i \cos \theta_i - u_i l_i) \tan \bar{\theta}] \sqrt{\sum_{i=1}^{n} [w_i \sin \theta_i]}}{\sum_{i=1}^{n} [w_i \sin \theta_i]} \]  

(49)
Replacing pore water force expression $u_i l_i$ with $\gamma_w Z_e l_i$, where $\gamma_w$ represents unit weight of water, $l_i$ is the length of each slice base and $Z_e$ is

$$Z_e = Z_w * (\cos \varphi)^2$$  \hspace{1cm} (50)$$

where $Z_w$ is the vertical distance of the center point of each slice below the piezometric line. $\varphi$ is the angle between pore water head at the middle of the slice and the vertical line, and it is approximated to be the angle between the slice piezometric line and the horizontal line. Since the slice width is small enough, the piezometric line is approximated to be straight and this angle is marked as $\varphi$ in Figure 35. Therefore, the expression is updated as follow:

$$F = \frac{\sum_{i=1}^{n} [(w_i \cos \theta_i - \gamma_w Z_e l_i) \tan \bar{\theta}]}{\sum_{i=1}^{n} [w_i \sin \theta_i]}$$  \hspace{1cm} (51)$$

where soil weight for each slice $w_i$ can be expressed as $A_i \gamma_i$, where $A_i$ is soil area and $\gamma_i$ is the soil unit weight. The expression can then be expressed as:

$$F = \frac{\sum_{i=1}^{n} [(A_i \gamma_i \cos \theta_i - \gamma_w Z_e l_i) \tan \bar{\theta}]}{\sum_{i=1}^{n} [A_i \gamma_i \sin \theta_i]}$$  \hspace{1cm} (52)$$
Using the soil properties in embankment for the whole slip surface and used term $\Delta_{bas-dam}$ as a correction for this simplification. Only slices that exist in both embankment and foundation need to be corrected and the number of these slices is denoted as $m$. Then the expression becomes:

$$F = \frac{\sum_{i=1}^{n}[A_i \gamma_{dam} \cos \theta_i - \gamma_w Z_{el} l_i] \tan \bar{\phi}_{dam} + \sum_{i=1}^{m} A_i \Delta_{bas-dam}}{\sum_{i=1}^{n}[A_i \gamma_{dam} \sin \theta_i]}$$  \hspace{1cm} (53)$$

where $\gamma_{dam}$ in the denominator denotes soil unit weights in both embankment ($\gamma_{dam}$) and foundation ($\gamma_{bas}$) as they are the same in values.

The soil area in foundation is denoted as $A_{bas}$, which has been included as a part of soil area in slice ($A_i$) in the first term of the numerator in Eq 53 with the soil property of embankment. The correction term $\Delta_{bas-dam}$ subtracts that value and introduces the correct soil properties for a $A_{bas}$:

$$\Delta_{bas-dam} = \cos \theta_i A_{bas} (\gamma_{bas} \tan \bar{\phi}_{bas} - \gamma_{dam} \tan \bar{\phi}_{dam})$$  \hspace{1cm} (54)$$

Substituting the correction into Eq 53, the equation becomes:

$$F = \frac{\gamma_{dam} \tan \bar{\phi}_{dam} \sum_{i=1}^{n}[A_i \cos \theta_i] - \gamma_w \tan \bar{\phi}_{dam} \sum_{i=1}^{n} Z_{el} l_i}{\gamma_{dam} \sum_{i=1}^{n}[A_i \sin \theta_i]} + \frac{(\gamma_{bas} \tan \bar{\phi}_{bas} - \gamma_{dam} \tan \bar{\phi}_{dam}) \sum_{i=1}^{m} \cos \theta_i A_{bas}}{\gamma_{dam} \sum_{i=1}^{n}[A_i \sin \theta_i]}$$  \hspace{1cm} (55)$$

It is important to note that derived safety factors with Fellenius ordinary slice method usually underestimates the safety factor about 5-20% percent compared with methods that are more accurate, e.g. the Bishop’s modified method and Spencer’s method (Craig, 2013). Despite that the results may not be practically applicable regarding the safety factor, this method is considered accurate enough to compare the approaches due to its transparency.
3.3. Probabilistic approach

The probabilistic analyses are also carried out in Slope/W and Comrel for the same critical slip surface as the one obtained in the deterministic approach.

3.3.1. Monte Carlo simulation

Monte Carlo simulations are used to estimate the density and distribution functions of the safety factors in Slope/W. Friction angle and unit weight in both embankment and foundation are simulated 1 million times based on the distributions and parameters given in Table 4.

3.3.2. FORM analysis

A FORM analysis is performed in Comrel with the same parameters as input variables as in the previous Monte Carlo simulation. The Comrel code and inputs are presented in Appendix C. The limit state function is built based on Fellenius ordinary method and applied coefficients of variables (tot1, tot2, tot3, and tot4) are from the results in Matlab. ‘Uwd’ and ‘Uwb’ represent unit weight of the soil in the embankment and foundation respectively and their friction angles are denoted ‘Phid’ and ‘Phib’ respectively.

3.4. Embankment dam system reliability

An embankment dam fails if any of its potential slip surface fails. From this point, slope failure of embankment dam can be considered as a series system and each potential slip surface as a component in that system.

In order to get a good approximation of the failure probability of the whole embankment dam instead of a single critical slip surface, sufficient slip surfaces must be studied. The selection and further division of these slip surfaces are given in Chapter 3.4.1.
The theory of system reliability is applied to get the joint reliability of different slip surfaces. Integration and simple bounds of all chosen slip surfaces are calculated.

Combinations of 1–5 slip surfaces from each group are also studied to investigate the number of slip surfaces necessary for a good estimation of the system reliability. Simple bounds calculation is also performed for comparison.

### 3.4.1. Choice of critical surfaces to analyze

A series of slip surfaces are first selected from SLOPE/W. In order to include sufficient critical slip surfaces, 13 slip surfaces with lowest safety factor value and 24 slip surfaces with highest failure probability (after Monte Carlo simulation) are selected. Slip surface 10068, 10037 and 8952 appear twice, therefore the final database consists of 34 slip surfaces.

The obtained center coordinates and radii of slip surfaces in SLOPE/W are used in the probabilistic calculations for each slip surface in Matlab and Comrel to get sensitivity factors and the reliability index.

To simplify, all slip surfaces are divided into two groups. The first group consists of 19 slip surfaces that exist only or mostly in the embankment and the second groups consists of 15 slip surfaces that exist both in the embankment and in the foundation. Two examples are shown in Figure 36 and Figure 37.
Figure 36: Example of a slip surface that only exists in embankment.

Figure 37: Example of slip surface that exists in both the embankment and in the foundation.
The reason for this classification is not only due to the geometry of the slip surfaces but also due to the influences of materials. For the first group, there is almost no influence from the foundation material as it exists only or mostly in the embankment. Therefore, sensitivity factors of variable ‘Uwb’ and ‘Phib’ is zero. Besides, as given in Table 6, the other two variables ‘Uwd’ and ‘Phid’ barely vary between different slip surfaces and the correlation between different slip surfaces approximately equals to 1 for this group, which indicates the possibility of using simple bounds. It should be noticed that only small parts of the slip surfaces are located in the foundation (low sensitivity factor of ‘Uwb’ and ‘Phib’), e.g. Slip surface 9045, Slip surface 7929 and Slip surface 8983 are categorized in this first group.

3.4.2. Joint probability of the two groups of slip surfaces

3.4.2.1. Joint failure probability in first group

In the first group, the correlation is observed to be close to 1 because they share similar sensitivity factors. For this group, system reliability can be estimated with simple bound theory, as given by Eq 21.

3.4.2.2. Combination of two groups of slip surfaces

The second group is formed by 15 slip surfaces that are not so highly correlated and they cannot be simply assumed to be fully-correlated or non-correlated. If simple bound is applied in this case, the $p_f$ would vary in the range of 0.03 to 0.90, which is too wide to give more precise results.

Therefore, integration is carried out to get the whole system reliability with the Matlab, see Appendix D. The integration begins by combing slip surface 7836 and slip surface 7217, so the reliability from the first group is considered as the first combination. The combined reliability index and sensitivity factors of the first 2 slip surfaces are then integrated with the third one ‘Slip surface 7248’ to get the joint probability of the first three slip surfaces. To obtain the system reliability, additional slip surfaces are continuously integrated to get the new combined reliability.
3.4.2.3. Simple bounds

As a series system, simple bounds results can be easily obtained from Eq 25.

3.4.3. Combination of different numbers of slip surfaces

The 5 most critical slip surfaces from each group are chosen for the investigation of the number of failure modes needed for a good estimation of the system reliability. Both simple bounds and integration calculations are performed.

In the first group, system reliability can still be approximated as that of Slip surface 7836, which has the highest probability of failure. It means that Slip surface 7836 represent system reliability of group 1 no matter how many slip surfaces that are chosen in group 1.

Similar to previous joint probability calculation, system reliability of both groups is calculated by continuously combining the reliability of group 1 (it is the reliability of Slip surface 7836 in this case) with that in group 2. The 1-5 most critical slip surfaces from each group are integrated.

Calculations are also performed for simple bounds of different numbers of slip surfaces.
4. Results

4.1. Deterministic calculations

4.1.1. Slope/W calculation

Pore-water pressure contour lines are shown in Figure 38 and the blue dashed line represents the piezometric line through the embankment.

The color map shown in Figure 39 illustrates the safe and dangerous domain of the centers for the slip surfaces. The change in color from green to blue indicates the increasing safety factor from 1.3 to over 1.75 with a step of 0.05. The red point illustrated where the center of the critical surface is located.

Final result gives a safety factor of 1.375, with the center coordinate of the critical slip surface at xy (22.263, 7.1044). As illustrated by the white curve in Figure 39, the critical slip surface develops in both the dam body and in the embankment.
Figure 39: Slip surface color map. It gives the domain of curve and center.

4.1.2. Deterministic analysis with Matlab

Matlab calculation results are given in Table 5, the sums below are marked as ‘tot1’, ‘tot2’, ‘tot3’ and ‘tot4’ respectively.

Table 5: Results of Matlab calculation.

<table>
<thead>
<tr>
<th></th>
<th>$\sum_{i=1}^{n} A_i \cos \theta_i$</th>
<th>$\sum_{i=1}^{n} \cos \theta_i A_{bas}$</th>
<th>$\sum_{i=1}^{n} Z_{el} l_i$</th>
<th>$\sum_{i=1}^{n} [A_i \sin \theta_i]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.5297</td>
<td>0.7623</td>
<td>4.6048</td>
<td>1.1322</td>
</tr>
</tbody>
</table>

The final safety factor is 1.30 in the Matlab analyses and a schematic figure of the critical slip surface is given in Figure 40.
4.2. Probabilistic calculations

4.2.1. Monte Carlo simulation

The result from the Monte Carlo simulation gave a probability of failure equal to 0.0005, corresponding to a reliability index of 2.87. The density and distribution functions of the safety factor are shown in Figure 41.
Figure 41: Probability density function of safety factors in the Monte Carlo simulation.

Figure 42: Probability distribution function of the safety factor obtained with Monte Carlo simulation.
4.2.2. FORM analysis

The results from Comrel give a reliability index of 2.42 and a probability of failure equal to 0.0077. Sensitivity factors for different variables are shown in Figure 43. Obviously, Uwd (Unit weight of dam body material) has the largest influence on the reliability index with a sensitivity factor of 0.84. This parameter is followed by Uwb (Unit weight of foundation material), for which the sensitivity factor is 0.39. Friction angles of the foundation and the dam body materials have the same effect and both have a sensitivity factor of 0.27.

![Figure 43: Sensitivity factors of variables](image)

4.3. Embankment dam system reliability

4.3.1. Choice of critical surfaces to analyze

Calculation results of selected slip surfaces are shown in Appendix E. In Slope/w, the calculated range of safety factor is from 1.375 to 1.574 and the probability of failure varies between 0.003 to 0.014. The results does not clarify the relationship between the safety factor and the probability of failure as a lower safety factor does not necessarily correspond to a high probability of failure.

Matlab gives a maximum value of the safety factor from equal to 1.5466 and a minimum value of 1.2391.
Analysis in Comrel gives a wider range of the probability of failure compared to the results in Slope/W, which ranges from 0.0017 to 0.0649. The unit weight and friction angle of the soil in embankment, denoted as 'Uwd' and 'Phid', has the highest and lowest sensitivity factors in most slip surfaces that go through the foundation. This rule also applies to slip surfaces which only exist in the embankment, where sensitivity factors for the unit weight and friction angle of the foundation are equal to 0.
4.3.2. Joint probability of the two groups of slip surfaces

4.3.2.1. Joint failure probability for the first group

The first group consists of slip surfaces that are only or mostly located in the embankment. With a correlation approximated to be 1, the combined failure probability of this group is decided by the slip surface with maximum failure probability. As is highlighted in grey in Table 6, it is 0.065 with a reliability index equal to 1.52.

Table 6: Calculation for slip surfaces in group 1, highly correlated slip surfaces

<table>
<thead>
<tr>
<th>Slip surface number</th>
<th>Beta</th>
<th>Pf</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>8952</td>
<td>1,741</td>
<td>0,0408 0,91 0 0,42 0 0</td>
<td></td>
</tr>
<tr>
<td>8921</td>
<td>1,73</td>
<td>0,0418 0,91 0 0,41 0 0</td>
<td></td>
</tr>
<tr>
<td>10068</td>
<td>1,962</td>
<td>0,0249 0,9 0 0,44 0 0</td>
<td></td>
</tr>
<tr>
<td>8890</td>
<td>1,717</td>
<td>0,0430 0,91 0 0,41 0 0</td>
<td></td>
</tr>
<tr>
<td>10037</td>
<td>1,96</td>
<td>0,0250 0,9 0 0,44 0 0</td>
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</tr>
<tr>
<td>10006</td>
<td>2,001</td>
<td>0,0227 0,9 0 0,43 0 0</td>
<td></td>
</tr>
<tr>
<td>9975</td>
<td>1,956</td>
<td>0,0252 0,9 0 0,43 0 0</td>
<td></td>
</tr>
<tr>
<td>8859</td>
<td>1,703</td>
<td>0,0443 0,92 0 0,43 0 0</td>
<td></td>
</tr>
<tr>
<td>7836</td>
<td>1,515</td>
<td>0,0649 0,93 0 0,37 0 0</td>
<td></td>
</tr>
<tr>
<td>9944</td>
<td>1,95</td>
<td>0,0256 0,91 0 0,42 0 0</td>
<td></td>
</tr>
<tr>
<td>9913</td>
<td>1,942</td>
<td>0,0261 0,91 0 0,42 0 0</td>
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<tr>
<td>8828</td>
<td>1,687</td>
<td>0,0458 0,92 0 0,39 0 0</td>
<td></td>
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<tr>
<td>10 099</td>
<td>1,976</td>
<td>0,0241 0,9 0 0,44 0 0</td>
<td></td>
</tr>
<tr>
<td>9 882</td>
<td>1,932</td>
<td>0,0267 0,91 0 0,41 0 0</td>
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<tr>
<td>9 881</td>
<td>1,883</td>
<td>0,0299 0,91 0 0,41 0 0</td>
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<tr>
<td>9 851</td>
<td>1,922</td>
<td>0,0273 0,91 0 0,41 0 0</td>
<td></td>
</tr>
<tr>
<td>9 045</td>
<td>1,886</td>
<td>0,0296 0,91 0,04 0,42 0,03</td>
<td></td>
</tr>
<tr>
<td>7 929</td>
<td>1,679</td>
<td>0,0466 0,92 0,04 0,39 0,03</td>
<td></td>
</tr>
<tr>
<td>8983</td>
<td>1,767</td>
<td>0,0386 0,91 0,01 0,42 0 0</td>
<td></td>
</tr>
</tbody>
</table>
4.3.2.2. Combination of two groups of slip surfaces

The calculation process and the result are presented in Table 7. Slip surface 7836 (from the first group) is combined with all slip surfaces in the second group. The joint probability of the whole database continues to increase to the value 0.077 for each combined slip surface.

Table 7: Joint reliability of group 1 with group 2

<table>
<thead>
<tr>
<th>Slip surface number</th>
<th>Beta</th>
<th>Pf</th>
<th>Alfa</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Uwd</td>
<td>Uwb</td>
</tr>
<tr>
<td>7836</td>
<td>1,515</td>
<td>0.0649</td>
<td>0.93</td>
<td>0</td>
</tr>
<tr>
<td>7217</td>
<td>2.421</td>
<td>0.0077</td>
<td>0.84</td>
<td>0.39</td>
</tr>
<tr>
<td>2 surfaces</td>
<td>1,5125</td>
<td>0.0652</td>
<td>0.6167</td>
<td>0.4497</td>
</tr>
<tr>
<td>7248</td>
<td>2.577</td>
<td>0.0050</td>
<td>0.81</td>
<td>0.44</td>
</tr>
<tr>
<td>3 surfaces</td>
<td>1,5125</td>
<td>0.0652</td>
<td>0.5685</td>
<td>0.4905</td>
</tr>
<tr>
<td>6194</td>
<td>2,593</td>
<td>0.0048</td>
<td>0.74</td>
<td>0.55</td>
</tr>
<tr>
<td>4 surfaces</td>
<td>1,5125</td>
<td>0.0652</td>
<td>0.5498</td>
<td>0.5107</td>
</tr>
<tr>
<td>6163</td>
<td>2,456</td>
<td>0.0070</td>
<td>0.78</td>
<td>0.49</td>
</tr>
<tr>
<td>5 surfaces</td>
<td>1,5122</td>
<td>0.0652</td>
<td>0.5515</td>
<td>0.5054</td>
</tr>
<tr>
<td>7279</td>
<td>2.745</td>
<td>0.0030</td>
<td>0.78</td>
<td>0.49</td>
</tr>
<tr>
<td>6 surfaces</td>
<td>1,5122</td>
<td>0.0652</td>
<td>0.5515</td>
<td>0.5046</td>
</tr>
<tr>
<td>6225</td>
<td>2,767</td>
<td>0.0028</td>
<td>0.68</td>
<td>0.6</td>
</tr>
<tr>
<td>7 surfaces</td>
<td>1,5122</td>
<td>0.0652</td>
<td>0.5373</td>
<td>0.5196</td>
</tr>
<tr>
<td>6132</td>
<td>2.34</td>
<td>0.0096</td>
<td>0.82</td>
<td>0.44</td>
</tr>
<tr>
<td>8 surfaces</td>
<td>1,5108</td>
<td>0.0654</td>
<td>0.5541</td>
<td>0.5000</td>
</tr>
<tr>
<td>7310</td>
<td>2.921</td>
<td>0.0017</td>
<td>0.75</td>
<td>0.54</td>
</tr>
<tr>
<td>9 surfaces</td>
<td>1,5108</td>
<td>0.0654</td>
<td>0.5473</td>
<td>0.5107</td>
</tr>
<tr>
<td>5140</td>
<td>2.584</td>
<td>0.0049</td>
<td>0.64</td>
<td>0.64</td>
</tr>
<tr>
<td>10 surfaces</td>
<td>1,5107</td>
<td>0.0654</td>
<td>0.5307</td>
<td>0.5259</td>
</tr>
<tr>
<td>6256</td>
<td>2.9</td>
<td>0.0019</td>
<td>0.65</td>
<td>0.63</td>
</tr>
<tr>
<td>11 surfaces</td>
<td>1,5107</td>
<td>0.0654</td>
<td>0.5299</td>
<td>0.5265</td>
</tr>
<tr>
<td>9014</td>
<td>1,822</td>
<td>0.0342</td>
<td>0.91</td>
<td>0.2</td>
</tr>
</tbody>
</table>
4.3.2.3. Simple bounds

Simple bounds are calculated according to Eq 25. Clearly, the simple bounds are not very informative as it gives a range of the probability of failure between 0.065 - 0.54 corresponding to the reliability indices from -0.09 to 1.52.

Table 8: Simple bounds for the probability of failure of the two groups

<table>
<thead>
<tr>
<th>Group number</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0649</td>
<td>0.4860</td>
</tr>
<tr>
<td>2</td>
<td>0.0342</td>
<td>0.0989</td>
</tr>
<tr>
<td>1 and 2</td>
<td>0.0649</td>
<td>0.5369</td>
</tr>
</tbody>
</table>

4.3.3. Combination of different number of slip surfaces from each group

In the joint probability calculation, the reliability of slip surface 7836 can still be used as a combined reliability of 1 up to five critical slip surfaces in group 1 due to the high correlations in between.
Table 9: Joint probability of 5 most critical slip surfaces in group 1

<table>
<thead>
<tr>
<th>Slip surface number</th>
<th>Beta</th>
<th>Pf</th>
<th>Alfa</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7836</td>
<td>1,515</td>
<td>0,0649</td>
<td>0,93</td>
<td>0,37</td>
</tr>
<tr>
<td>8859</td>
<td>1,703</td>
<td>0,0443</td>
<td>0,92</td>
<td>0,4</td>
</tr>
<tr>
<td>8890</td>
<td>1,717</td>
<td>0,0430</td>
<td>0,91</td>
<td>0,41</td>
</tr>
<tr>
<td>8921</td>
<td>1,73</td>
<td>0,0418</td>
<td>0,91</td>
<td>0,41</td>
</tr>
<tr>
<td>8952</td>
<td>1,741</td>
<td>0,0408</td>
<td>0,91</td>
<td>0,42</td>
</tr>
</tbody>
</table>

Then integration results of slip surface 7836 with the 5 most critical slip surfaces in Group 2 are shown as Table 10, which give a reliability index of 1.45 and a failure probability of 0.073.

Since more critical slip surfaces in group 2 are combined with slip surface 7836 prior to less critical slip surfaces, the step-by-step integration in Table 10 also gives the combination results of 1-4 most critical slip surfaces from each group. For example, derived reliability index equal to 1.458 and a probability of failure equal to 0.0724 are also the combination results of the two most critical slip surfaces from each group (as can be seen on the line marked 3 surfaces).
Table 10: Joint reliability of group 1 with group 2

<table>
<thead>
<tr>
<th>Slip surface number</th>
<th>Beta</th>
<th>Pf Alfa</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Uwd</td>
<td>Uwb</td>
</tr>
<tr>
<td>7836</td>
<td>1,515</td>
<td>0,0649</td>
<td>0,93</td>
</tr>
<tr>
<td>2102</td>
<td>2,3970</td>
<td>0,0083</td>
<td>0,16</td>
</tr>
<tr>
<td>2 surfaces</td>
<td>1,458</td>
<td>0,0723</td>
<td>0,5354</td>
</tr>
<tr>
<td>7217</td>
<td>2,4210</td>
<td>0,0077</td>
<td>0,84</td>
</tr>
<tr>
<td>3 surfaces</td>
<td>1,458</td>
<td>0,0724</td>
<td>0,5542</td>
</tr>
<tr>
<td>6163</td>
<td>2,4560</td>
<td>0,0070</td>
<td>0,78</td>
</tr>
<tr>
<td>4 surfaces</td>
<td>1,4578</td>
<td>0,0724</td>
<td>0,5512</td>
</tr>
<tr>
<td>4116</td>
<td>2,5370</td>
<td>0,0056</td>
<td>0,32</td>
</tr>
<tr>
<td>5 surfaces</td>
<td>1,4531</td>
<td>0,0731</td>
<td>0,4972</td>
</tr>
<tr>
<td>7248</td>
<td>2,5770</td>
<td>0,0050</td>
<td>0,81</td>
</tr>
<tr>
<td>6 surfaces</td>
<td>1,4527</td>
<td>0,0732</td>
<td>0,5440</td>
</tr>
</tbody>
</table>

The simple bounds results are given in Table 11, where the lower and upper limits are illustrated for the combinations of 1-5 slip surfaces.

Table 11: Simple bounds for the probability of failure for different slip surface numbers.

<table>
<thead>
<tr>
<th>Group number</th>
<th>Limit</th>
<th>Simple bounds according to slip surfaces numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Lower</td>
<td>0,0649</td>
</tr>
<tr>
<td></td>
<td>Upper</td>
<td>0,0649</td>
</tr>
<tr>
<td>2</td>
<td>Lower</td>
<td>0,0083</td>
</tr>
<tr>
<td></td>
<td>Upper</td>
<td>0,0083</td>
</tr>
<tr>
<td>1 and 2</td>
<td>Lower</td>
<td>0,0649</td>
</tr>
<tr>
<td></td>
<td>Upper</td>
<td>0,0726</td>
</tr>
</tbody>
</table>
5. Discussion

Probabilistic methods are a promising tool for the assessment of embankment dams but there is still a long way to go. Probabilistic methods give the failure probability, which is more assessable than the results from a deterministic method. Sensitivity factors give information about the influences of different variables. Moreover, the method can be easily incorporated in cost analysis and risk assessment.

The application in embankment dam internal erosion assessment is in its infancy. Systematic reliability of four processes of internal erosion: initiation, continuation, progression, and breach still cannot be quantified in a completely probabilistic way. Some approaches circumvent the problem of epistemic uncertainty by using elicited subjective probabilities with the traditional way of assigning probability. Other probabilistic analyses concentrate on some essential phases e.g. initiation and progression.

Despite that there were some arguments about the application of reliability theory in slope stability assessment, there exists a rather extensive research work performed on this subject. The general outline of this application has been developed but there remain some issues to be addressed, e.g. correlation between different slip surfaces and the influence from spatial variability of the materials.

5.1. Critical slip surface

Material properties are assumed as shown in Table 4 for the deterministic and probabilistic calculation. The result will change according to material properties and different results will be given if larger variabilities of the input parameters (unit weight and friction angle) are assumed. In order
to get a realistic result, field tests and experiments are necessary for a good approximation of input parameters.

Both deterministic and probabilistic methods indicate that the homogeneous dam has a high risk of failure, which is larger than what could be considered acceptable. For the most critical slip surface in Slope/W, results of the deterministic and probabilistic calculation are shown in Table 12. For a real dam, a comparison shall be done with the requirement of load case 2 in Table 13. Calculated safety factors correspond to the probabilities of failures of 0.005 and 0.0077 respectively. These values are quite high compared to the suggested values for major structures from USACE (1997), which are 5 for the reliability index with a corresponding probability of failure equals to 2.87E-07.

Table 12: Result of deterministic and probabilistic calculation for critical slip surface

<table>
<thead>
<tr>
<th>Methods</th>
<th>Deterministic calculation (Fs)</th>
<th>Probabilistic calculation (Pf)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slope/W Matlab</td>
<td>MC simulation in Slope/W FORM in Comrel</td>
</tr>
<tr>
<td>Value</td>
<td>1.38</td>
<td>β=2.86, pf=0.005</td>
</tr>
<tr>
<td></td>
<td>1.30</td>
<td>β=2.42, pf=0.0077</td>
</tr>
</tbody>
</table>

Table 13: Specified stability safety factor for embankment dams at different load cases (RIDAS, 2012)

<table>
<thead>
<tr>
<th>Load case</th>
<th>Description</th>
<th>Safety factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Finished construction of dam before reservoir is filled</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>Normal operation condition with stationary flow through dam body</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>Extreme operation condition with over dam in associated with design flood</td>
<td>1.3</td>
</tr>
<tr>
<td>4</td>
<td>After rapid decrease of water level</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Sensitivity factors can be derived by using the probabilistic method and they indicate the influences on probability of failure from different factors. The result from Comrel shows that unit weight of the embankment constitute the largest contribution to the limit state function.
in all cases. In terms of soil properties, the limit state function is more sensitive to unit weight than friction angle. These results highlight the key points in embankment dam surveillance and stability enhancing measures with respect to slope stability.

5.2. Safety factor and failure probability

Comparing the calculated safety factor and the reliability index, the slip surface with the minimum safety factor is not the slip surface with the highest probability of failure. As Appendix D shows, slip surface 7217 has the minimum safety factor in Slope/W but slip surface 8952 has the highest probability of failure both in Slope/W and Comrel.

Liang et al., (1999), also described this. Liang et al., considered that the slip surface with minimum safety factor can still be used as a close initial trial to search for the slip surfaces with minimum failure probability.

In the present analysis, there is a tendency that the slip surfaces that exist both in the foundation and in the embankment have lower failure probabilities than embankment slip surfaces, even with lower safety factors. A potential explanation would be the differences between material properties – higher friction angle is assumed in the foundation than in the embankment. Therefore, different material combinations could cause different results.

5.3. Simple bounds

As a simple method used to get an approximation of system reliability, simple bounds makes simple assumption concerning the correlation between elements. Full-correlation is assumed in lower bound and non-correlation for upper bounds.

The lower bounds is only decided by the failure probability of the most critical slip surface, which is 0.065 here. This number is not far from the combined failure probability of 34 slip surfaces – 0.0770 in this specific
case. This indicates that the slope stability is dominated by the most critical slip surface.

Non-correlation is assumed for upper bounds, meaning its value will keep increasing as more slip surfaces are studied. This bound increases sharply to an unrealistic value after including additional slip surfaces for several times as shown in Table 8 and Table 11.

Simple bounds are calculated according to Eq 25. Clearly, the simple bounds are not very informative as it gives a range of the probability of failure between 0.065-0.54 corresponding to the reliability indices from -0.09 to 1.52.

In the dam studied in this thesis, the results show that the selected slip surfaces are more on the fully-correlated side and the lower bounds give better estimation than the upper bounds. This conclusion will certainly vary depending on specific cases.

5.4. Required number of critical surfaces to obtain an accurate estimation of system reliability

From the point of accuracy, it is theoretically beneficial to integrate as many slip surfaces as possible. An asymptotic line is expected, which comes infinitely close to the ‘true’ probability of failure and any extra combination would only add small changes to the value. However, the derivation of the reliability and the joint probability for a large amount of slip surfaces is complex and time consuming. One methodology to simplify this process is to integrate several of the most critical slip surfaces.

In the present study, all slip surfaces are sorted into two groups according to their geometries. One group of the most critical slip surfaces which only appears in the embankment and the other group where the critical slip surfaces appear in both the embankment and the foundation. High correlation are found within each group, but the correlation is comparatively low between these two groups. As shown in Table 14 the joint probability increases from 0.0723 to 0.0732 as more slip surfaces from group two are added to the system.
Table 14: Combined system reliability of different slip surface numbers

<table>
<thead>
<tr>
<th>Group number</th>
<th>System reliability according to the numbers of included slip surfaces</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1 combined with 2</td>
<td>0.0723</td>
</tr>
</tbody>
</table>

Only small changes in joint probability can be seen after integrating 4 and 5 slip surfaces from each group. It can also be expected that if more slip surfaces are integrated, the number would still be around 0.073, which shows that 4 or 5 slips surfaces might be sufficient. The integrated results of a few slips surfaces from each group are actually very close to the integrated result of all 34 most critical slip surfaces, with an error of around 5%. This indicates that the potential to simplify the integration calculation by just investigating a few slip surfaces from groups of slip surfaces of similar type might be sufficient from a system perspective.

### 5.5. Categorization of groups in the system

The comparison above is based on the categorization of slip surfaces into two different groups. The criterion of categorization is slip surface geometry, which is actually the combinations of different materials.

Different material combinations give different ‘groups’ in the studied case. Each group is internally highly correlated but lowly correlated with another. It is easy to get the joint probability for each group and then combine them, e.g. through simple bounds or integration. But it is difficult to find criteria for categorization and then ensure that the calculation has cover all ‘groups’. The result would be too optimistic to represent the real risk if any of these ‘groups’ is neglected. For example, if we just combine several global critical slip surfaces, the result will be the same as that we get for group 1 because the slip surfaces from the second group are not included.

Then the question comes if there are more criteria for categorization other than material combinations. This is quite important for
embankment dams with complex structures, e.g. inner core, multi-layers, erosion protections etc.

In this study, categorization is carried out for 34 selected high-risk slip surfaces. Slip surfaces with low safety factors and/or low reliability indices are studied. However, other slip surfaces with high safety factors or high reliability indices could potentially influence the results. If their failure probabilities are low enough to be neglected or they are highly correlated with the existing two groups, the sorted two groups are good representation of the true slope reliability. Otherwise, other internally-correlated groups need to be figured out or more slip surfaces need to be integrated.
6. Conclusion and suggestions for future research

To analyze internal erosion in a complete probabilistic way is difficult with the present stage of theoretical knowledge. It is more practical to start from the key phases of the erosion process, e.g. initiation and continuation.

The application of probabilistic methods in slope stability analysis has made great progress. The general outline for calculations have been developed, but some questions remains to be solved to make it more applicable and reliable. For example, more accurate material properties are required for a good estimation of the failure probability. The problem of spatial variability also needs to be addressed.

Through this study, it is found that a system approach is necessary for an accurate determination of the probability of failure in slope stability assessment. The probability of one slope failure is the combination of failure probabilities of numerous slip surfaces. Investigating only slip surface with a minimum safety factor or maximum failure probability neglects the influences from other slip surfaces. Despite that their failure probabilities sometimes may be close to the ‘true’ system reliability, these values cannot represent system reliability.

System reliability is influenced by correlations between different components. The extreme example is the lower and upper bounds of a series system, where full-correlation and non-correlation are assumed respectively. Therefore, correlations of slip surfaces must be considered when assessing embankment dam slope stability through a system approach.

It is proved to be possible to get a good estimation of the failure probability by combining several featured slip surfaces. In this study, a 6.5% of error (combined failure probability compared to the failure
probability by combing all 34 slip surfaces) is reached at the first combination of the most critical slip surface from each group.

Categorizing failure slip surfaces is a good way to find out these representative slip surfaces and it is more efficient and reliable than casually integrating several of the most critical slip surfaces. The main difficulties lie in finding out the criteria used for categorization and correlations between different groups. Material combination could be used as one such criterion for group division. However, more complex structures and more material combinations are to be studied.
References


liquefiable deposit. Soil Dynamics and Earthquake Engineering, 91, 222-233.


U.S. Department of the interior, B. o. (1997). *Risk-based analysis in geotechnical engineering for support of planning studies, engineering and design*.


Appendix A: Material properties

Table A.1: Grain size data from pit test

<table>
<thead>
<tr>
<th>Section</th>
<th>Depth</th>
<th>D10 Mean value</th>
<th>D60 Mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 right dam crest</td>
<td>1,300</td>
<td>0,007</td>
<td>0,010</td>
</tr>
<tr>
<td>6 left dam crest</td>
<td>1,400</td>
<td>0,013</td>
<td>0,010</td>
</tr>
<tr>
<td>5 right dam toe</td>
<td>0,900</td>
<td>0,005</td>
<td>0,012</td>
</tr>
<tr>
<td>7 left dam toe</td>
<td>0,600</td>
<td>0,018</td>
<td>0,476</td>
</tr>
</tbody>
</table>

Figure A.1: Locations of drilling tests
Appendix B: Slope analysis Matlab code

clc;
clear all;
clear;

xr=22.717; yr=8.2438; R=8.6106; % values in this line should be change for different slip surfaces
% draw seepage; seepagefit.m is used, this fitting function is from cftool
p1 = 250 ;
p2 = -1.237e+04 ;
p3 = 1.53e+05 ;
q1 = 8.719 ;
q2 = -2564 ;
q3 = 4.503e+04 ;
x1=(15:0.05:25);
y1=Seepagefit(x1,p1,p2,p3,q1,q2,q3);
plot(x1,y1,'b');hold on;

% draw embankment dam profile; profile.m is used
x2=(15:0.05:25);
y2=profile(x2);
plot(x2,y2,'r');hold on;

% draw slip surface; slip.m is used
xs=(15:0.01:25);
ys=slip(xs);
plot(xs,ys,'g'); hold on;
grid on; axis equal;
legend('Piezometric line','Embankment profile','Slip surface');
k=3.74/(9.93-24); d=3.74*24/14.07; %downstream slope and intercept
[xinter1,yinter1]=linecirc(k,d,xr,yr,R);
[xinter2,yinter2]=linecirc(0,0,xr,yr,R);
%find the intersection point of slip surface and profile

len=(xinter2(1)-xinter1(2))/30; %length of each slice, watch out for the solve%
funsoil2=@(x)0-(yr-sqrt(R*R-(x-xr).*(x-xr)))/0; %o-function slip surface to get the depth of slice in foundation

for i=1:30
    %calculate soil area of each slice
    dx(i)=xinter1(2)+(i-1)*len;
dx(i+1)=dx(i)+len;
dxmid(i)=(dx(i)+dx(i+1))/2;
    A(i)=(profile(dxmid(i))-slip(dxmid(i)))*len; %area of soil for each slice
    sinalfa(i)=(xr-dxmid(i))/R; %sin of each slice
    cosalfa(i)=sqrt(1-sinalfa(i)*sinalfa(i)); %cos of each slice

    %calculate water head for each slice
    Hw=Seepagefit(dxmid(i),p1,p2,p3,q1,q2,q3);
    Hs=slip(dxmid(i));
    if Hw>Hs
        umid(i)=Hw-Hs; %use water head at middle of each slice
        w1(i)=Seepagefit(dx(i),p1,p2,p3,q1,q2,q3);
w1(i+1)=Seepagefit(dx(i+1),p1,p2,p3,q1,q2,q3);
cosseep(i)=len/sqrt(len^2+(w1(i)-w1(i+1))^2);
l(i)=umid(i)*cosseep(i)^2;
    end

    yw(i)=slip(dx(i));
yw(i+1)=slip(dx(i+1));

    % to get the curve length of each slice
    D(i)=sqrt(len^2+(yw(i)-yw(i+1))^2);
degree(i)=2*asin(D(i)/2/R);
curveleng(i)=degree(i)*R;
end

%calculate the foundation area in each slice
if integral(funsoil2,dx(i),dx(i+1))>0
    Abas(i)=integral(funsoil2,dx(i),dx(i+1));
else Abas(i)=0;
end
end

%input unit weight and friction angle
tandam=tan(pi*32/180);rdam=20;rwater=9.807;tanbas=tan(pi*35/180);rbas=20;

for i=1:30
    %calculate for different forces
    R1(i)=rdam*tandam*A(i)*cosalfa(i);%resistance force from soil weight
    R2(i)=(tanbas*rbas-tandam*rdam)*Abas(i)*cosalfa(i);%correction for differences in foundation and dam body
    R3(i)=tandam*curveleng(i)*l(i)*rwater;%resistance force from water uplift
    S(i)=rdam*A(i)*sinalfa(i);%driving force from soil weight

    %calculate and sum up for the constants used in comrel
    C1(i)=A(i).*cosalfa(i);tot1=sum(C1);%slice area*cos
    C2(i)=Abas(i).*cosalfa(i);tot2=sum(C2);%slice foundation area *cos
    C3(i)=curveleng(i)*l(i);tot3=sum(C3);%water head*slip surface curve length
    C4(i)=A(i).*sinalfa(i);tot4=sum(C4);%slice area*sin
end

Fs=(tot1*rdam*tandam+tot2*(tanbas*rbas-tandam*rdam)-tandam*rwater*tot3)/(tot4*rdam)
Appendix C: Comrel code and inputs

\[ FLIM(1) = \text{FUNC}(1) + \text{FUNC}(2) + \text{FUNC}(3) + \text{FUNC}(4) \]

\[ \text{DEFFUNC}(1)() = c1 \cdot Uwd \cdot \tan(\text{Phid} \cdot \pi/180) \]

\[ \text{DEFFUNC}(2)() = c2 \cdot (\tan(\text{Phib} \cdot \pi/180) \cdot Uwb - Uwd \cdot \tan(\text{Phid} \cdot \pi/180)) \]

\[ \text{DEFFUNC}(3)() = -Uww \cdot c3 \cdot \tan(\text{Phid} \cdot \pi/180) \]

\[ \text{DEFFUNC}(4)() = -(c4 \cdot Uwd) \]

<table>
<thead>
<tr>
<th>ID...</th>
<th>Comment</th>
<th>Distribution</th>
<th>Value</th>
<th></th>
<th></th>
<th></th>
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<td>0</td>
<td>2</td>
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<tr>
<td>R Uwb</td>
<td>Unit weight of basement</td>
<td>Normal (Gauss)</td>
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<td>0</td>
<td>2</td>
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<tr>
<td>R Phid</td>
<td>Friction angle of dam body</td>
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</table>
Appendix D: Joint probability calculation

%function
[alfa,beta,ra]=system_reliability_bivariate_normal_series(alfa,b1,beralf);

b1=[1.5076;1.822]'; %beta values of original ';
%
alfa=[0.538920982728161 0.517543432458873 0.454092312006519
0.48529696276691;
0.91 0.2 0.42 0.2]'; % original sensitivity factors;

eps=0.1; %increase of my
tole=1e-10; %tolerance of integration
re_calc = 1;
if min(b1)>4
    re_calc = normcdf(-4)/normcdf(-min(b1));
b1 = [-norminv(normcdf(-b1(1))*re_calc) -norminv(normcdf(-b1(2))*re_calc)];
end
calc =1;
ra=alfa(:,1)'*alfa(:,2);
rar = 0;
alfagam = alfa;

if ra>0.99
    rar = ra;

    num = find(b1 ==min(b1));
    beta = b1(num);
    alfa = alfa(:,num)';
calc =0;
else
    \%disp('ra out of bounds (0<ra<0.99). press Ctrl+C')

fi1=[1/(2*\pi*sqrt(1-ra^2))];
fi2=-(1/(2*(1-ra^2)));

\%series system

F = @(t1,t2)1/(2*\pi*sqrt(1-ra^2))*exp(-(1/(2*(1-ra^2)))*(t1.^2+t2.^2-2*ra*t1*t2));
Q = dblquad(F,-10,b1(1),-10,b1(2),tole);
bseries=-norminv(1-Q);
    \% if (1-Q)<0
    \%      tole = 1e-12;
    \%      F = @(t1,t2)1/(2*\pi*sqrt(1-ra^2))*exp(-(1/(2*(1-ra^2)))*(t1.^2+t2.^2-2*ra*t1*t2));
    \%      Q = dblquad(F,-10,b1(1),-10,b1(2),tole);
    \%      bseries=-norminv(1-Q);

    if (1-Q)<0; num = find(b1 ==min(b1(1),b1(2))); bseries = b1(num); alfa =alfa(:,num)';jakob = 1;
        calc =0;
    end

at=0;
le=size(alfa,1);

if calc==1
    for i=1:le;
        var=zeros(le,1);
        var(i)=eps;

        b1n=b1-alfa'*var; \%plus/minus?
        eval(['Q',num2str(i),'] = dblquad(F,-10,b1n(1),-10,b1n(2),tole);']);
        eval(['bser',num2str(i),']=-norminv(1-Q',num2str(i),');']);

        eval(['a',num2str(i),']=(bseries-bser',num2str(i),')/eps;']);
        eval(['at=a',num2str(i),'^2+at;']);
    end
end
alfas=zeros(1,le);
for i=1:le;
    if eval(['a',int2str(i),'~=0'])
        eval(['alfas(',int2str(i),')=(a',int2str(i),'^2/at)^0.5*(a',int2str(i),'/abs(a',int2str(i),'));']);
    else
        alfais(i)=0;
    end
end
bseries;
alfaseries=alfais;
disp('series')
alfa=alfaseries;
end
beta=bseries;

beta = -norminv(normcdf(-beta)/re_calc)
end
### Appendix E: Selected critical slip surface with the reliability indices and sensitivity factors

<table>
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<tr>
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<th>Matlab</th>
<th>Comel</th>
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*Slip surface number marked with green mean these surfaces appeared twice in database.*