Control and Analysis of Pulse-Modulated Systems

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Abstract

The thesis consists of an introduction and four appended papers. In the introduction we give an overview of pulse-modulated systems and provide a few examples of such systems. Furthermore, we introduce the so-called dynamic phasor model which is used as a basis for analysis in two of the appended papers. We also introduce the harmonic transfer function and finally we provide a summary of the appended papers.

The first paper considers stability analysis of a class of pulse-width modulated systems based on a discrete time model. The systems considered typically have periodic solutions. Stability of a periodic solution is equivalent to stability of a fixed point of a discrete time model of the system dynamics.

Conditions for global and local exponential stability of the discrete time model are derived using quadratic and piecewise quadratic Lyapunov functions. A griding procedure is used to develop a systematic method to search for the Lyapunov functions.

The second paper considers the dynamic phasor model as a tool for stability analysis of a general class of pulse-modulated systems. The analysis covers both linear time periodic systems and systems where the pulse modulation is controlled by feedback. The dynamic phasor model provides an $\mathbf{L}_2$-equivalent description of the system dynamics in terms of an infinite dimensional dynamic system. The infinite dimensional phasor system is approximated via a skew truncation. The truncated system is used to derive a systematic method to compute time periodic quadratic Lyapunov functions.

The third paper considers the dynamic phasor model as a tool for harmonic analysis of a class of pulse-width modulated systems. The analysis covers both linear time periodic systems and non-periodic systems where the switching is controlled by feedback. As in the second paper of the thesis, we represent the switching system using the $\mathbf{L}_2$-equivalent infinite dimensional system provided by the phasor model. It is shown that there is a connection between the dynamic phasor model and the harmonic transfer function of a linear time periodic system and this connection is used to extend the notion of harmonic transfer function to describe periodic solutions of non-periodic systems. The infinite dimensional phasor system is approximated via a square truncation. We assume that the response of the truncated system to a periodic disturbance is also periodic and we consider the corresponding harmonic balance equations. An approximate solution of these equations is stated in terms of a harmonic transfer function which is analogous to the harmonic transfer function of a linear time periodic system. The aforementioned assumption is proved to hold for small disturbances by proving the existence of a solution to a fixed point equation. The proof implies that for small disturbances, the approximation is good.
Finally, the fourth paper considers control synthesis for switched mode DC-DC converters. The synthesis is based on a sampled data model of the system dynamics. The sampled data model gives an exact description of the converter state at the switching instances, but also includes a lifted signal which represents the inter-sampling behavior. Within the sampled data framework we consider H-infinity control design to achieve robustness to disturbances and load variations. The suggested controller is applied to two benchmark examples; a step-down and a step-up converter. Performance is verified in both simulations and in experiments.

**Keywords**
Pulse-width modulation, Periodic systems, Stability analysis, Harmonic analysis, Lyapunov methods, Dynamic phasors, Harmonic transfer function, Switched mode power converters, Sampled data modeling, H-infinity synthesis