Relaying Protocols for Wireless Networks

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Abstract

Motivated by current applications in multihop transmission and ad hoc networks, the classical three-node relay channel consisting of a source-destination pair and a relay has received significant attention. One of the crucial aspects of the relay channel is the design of proper relaying protocols, i.e., how the relay should take part into transmission. The thesis addresses this problem and provides a partial answer to that.

In this thesis, we propose and study two novel relaying protocols. The first one is based on constellation rearrangement (CR) and is suitable for higher-order modulation schemes. With CR, the relay uses a bit-symbol mapping that is different from the one used by the source. We find the optimal bit-symbol mappings for both the source and the relay and the associated optimal detectors, and show that the improvement over conventional relaying with Gray mapping at the source and the relay can amount to a power gain of several dB. This performance improvement comes at no additional power or bandwidth expense, and at virtually no increase in complexity. The second one is a half-duplex decode-and-forward (DF) relaying scheme based on partial repetition (PR) coding at the relay. With PR, if the relay decodes the received message successfully, it re-encodes the message using the same channel code as the one used at the source, but retransmits only a fraction of the codeword. We analyze the proposed scheme and optimize the cooperation level (i.e., the fraction of the message that the relay should transmit). We compare our scheme with conventional repetition in which the relay retransmits the entire decoded message, and with parallel coding, and additionally with dynamic DF. The finite SNR analysis reveals that the proposed partial repetition can provide a gain of several dB over conventional repetition. Surprisingly, the proposed scheme is able to achieve the same performance as that of parallel coding for some relay network configurations, but at a much lower complexity.

Additionally, the thesis treats the problem of resource allocation for collaborative transmit diversity using DF protocols with different type of CSI feedback at the source. One interesting observation that emerges is that the joint power-bandwidth allocation only provides marginal gain over the relaying protocols with optimal bandwidth allocation.

Keywords: The relay channel, constellation rearrangement, decode-and-forward, resource allocation.
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Part I

Introduction
Introduction

The classical relay channel [1, 2] has received renewed attention in recent years due to its potential in wireless applications and ad hoc networks [3, 4]. A new concept, user cooperation [5, 6] or cooperative diversity [7, 8], has been introduced as a means to enhance the reliability of communication by an additional transmission of the intended message through another node. The node which collaborates with the main transmitter (user) is called relay. Thereby, when the relay cooperates a network consisting of the source-destination pair and the relay will be configured. An immediate question which emerges from this setup is how the relay should utilize its resources (i.e., power and bandwidth) to enhance the reliability of communication. The main theme of this thesis is to partially answer this question. In what follows we first describe some potential applications of relaying. We then introduce a formal definition of the classical three-nodes relay channel and some well known bounds on the capacity of this channel. Finally, we provide a summary of the contributions in the thesis.

1 Potential Applications of Relaying

Consider a wireless network with some active nodes including a transmitter-receiver pair. Due to the broadcast nature of wireless transmission, some nodes in the network might overhear the transmitted signal. If the direct communication of a transmitter-receiver pair fails (for example, due to the channel variations), those nodes that have a copy of the transmitted signal can help to reestablish the communication between the intended transmitter-receiver pair. The nodes that cooperate in the transmission are called relays. Generally speaking, when the relays cooperate (i.e., take part in the transmission in order to help the transmitter-receiver pair), multiple paths for conveying the transmitter’s message will be available. In other words, a virtual antenna array has been created. Multiple paths in wireless communication often translates into diversity gains. From this setup one can envision various applications of having the relays. In the following we mention some of these applications.
1.1 Relay-Based Cellular Network

One immediate application is the deployment of fixed relays in cellular networks [9]. In the uplink (i.e., mobile to base station) transmission, practical constraints on the size of the handset and the battery life-time preclude the use of Multiple-Input Multiple-Output (MIMO) antenna techniques. MIMO systems are well known to offer multiplexing gain (i.e., an increase in data rate) and diversity gain (i.e., an increase in the slope at which the symbol-error-probability decreases with the average SNR) [10]. Thus one way to obtain the benefits of MIMO links is to use distributed MIMO by the deployment of the relays. Additionally, relay-based networks are more robust to shadow fading (i.e., attenuation of the transmitted signal due to large obstacles) since the placement of relays is not as restricted as that of collocated antennas in the conventional MIMO systems.

1.2 Wireless Sensor Networks

Wireless sensors networks [11] are composed of many nodes that are densely deployed in the environment. The main purpose of such networks usually is detection or estimation of a physical phenomenon. The sensors are supposed to work with low-power and have a sort of intelligence to establish the communication and to maintain the network. The collaboration among nodes is of crucial importance in such networks to achieve

- reliable estimation and detection; and
- reasonably long life-time of the network.

In other words, some nodes in the network should take the role of being relays to maintain reliable interconnections between the nodes and to save energy.

1.3 Relay-Assisted Cognitive Radios

Cognitive Radio [12] is motivated by the apparent geographical and temporal under-utilization of the spectrum. The idea is to sense a spectrum hole and try to utilize it in such a way not to harm the primary user (i.e., licensed user). Clearly, one of main challenges of the cognitive radio is to sense the spectrum. This detection problem is intimately related to the cooperative communication in which the cognitive users help one another by relaying relevant information to sense the spectrum [13].

2 The Classical Relay Channel

The Discrete Memoryless Relay Channel was introduced by van der Meulen in 1971 [1]. Figure 1 shows a block diagram of the relay channel. The channel is defined using two finite input sets $\mathcal{X}$ and $\mathcal{X}_r$, two finite output sets $\mathcal{Y}$ and $\mathcal{Y}_r$. 
and a pmf $p(y, y_r|x, x_r)$. Therefore, in Figure 1 we have $X^n \in \mathcal{X}^n$, $Y^n \in \mathcal{Y}^n$, $X^n_r \in \mathcal{X}^n_r$ and $Y^n_r \in \mathcal{Y}^n_r$. Moreover, the channel is assumed to be memoryless:

$$p(y_i, y_{r,i}|x_i, x_{r,i}) = p(y_i, y_{r,i}|x_i, x_{r,i})$$

That is, the received signals at the relay and the destination at time instant $i$ only depend on the transmitted signals from the source and the relay at time instant $i$ (i.e., $x_i, x_{r,i}$).

The goal of the communication channel is to reliably convey the message $W$ uniformly drawn from the set $\{1, 2, \cdots, 2^{nR}\}$ to the destination in $n$ channel uses. To accomplish this task, one needs to design three main components:

- an encoder to map the message $W$ to $X^n$;
- a set of relay functions: $\{f_i\}_{i=1}^n$ such that

$$x_{r,i} = f_i(y_{r,1}, y_{r,2}, \ldots, y_{r,i-1})$$

and

- a decoder to map the received signal $Y^n$ to an estimate of the transmitted message $\hat{W}$.

The capacity of the channel is defined as the supremum of all rates $R$ for which it is possible to achieve $\Pr\{\hat{W} \neq W\} \to 0$ as $n \to \infty$ [14].

### 3 Bounds on Capacity

#### 3.1 Upper bound

We next present an upper bound on the capacity. This upper bound is known as max-flow min-cut or cut-set bound [2] and it consists of two terms:

1. $X^i \triangleq [X_1 X_2 \cdots X_i]$.
2. Here we assume causal relaying, that is the transmitted signal at instant $i$ only depends on the received signals up to time instant $i - 1$. 

Figure 1: Block diagram of the relay channel
**Introduction**

- Broadcast bound: \( C \leq \sup_{p(x,x_r)} I(X; Y, Y_r | X_r) \)
- Multiple access bound: \( C \leq \sup_{p(x,x_r)} I(X, X_r; Y) \)

Figure 2 schematically illustrates these two bounds. One can easily see the analogy between the broadcast bound and the receive diversity in which the relay plays the role of another receiver. On the contrary in the multiple access bound, the relay acts as another transmitter by conveying a fresh codeword of the intended message and the relay channel hence mimics the behavior of the transmit diversity. Combining these two bounds we obtain

\[
C \leq \sup_{p(x,x_r)} \min \{ I(X, X_r; Y), I(X; Y, Y_r | X_r) \}.
\]

(1)

where the supremum is taken over all joint pmfs \( p(x, x_r) \).

### 3.2 Lower bounds

We next provide an overview of some well known lower bounds on the capacity.

**Block Markov Encoding**

With block Markov encoding [2] (also known as decode-and-forward (DF) [15–18]), the transmission occurs in \( B \) consecutive blocks. The relay transmits a fresh codeword by decoding the previous block received at the relay. This scheme relies on the decoding at the relay and hence its performance is limited by the quality of the source-relay link. Using DF, the following rate is achievable [2]

\[
R = \sup_{p(x,x_r)} \min \{ I(X, X_r; Y), I(X; Y_r | X_r) \}
\]

(2)

where

- \( I(X, X_r; Y) \) is the multiple access bound on the capacity; and
- \( I(X; Y_r | X_r) \) reflects the capacity of the source-relay link.
One can readily deduce that DF operates close to the capacity upper bound when the quality of the source-relay link is "reasonably" good.

DF relaying achieves the capacity when the relay channel is physically degraded, that is

\[ p(y, y_r|x, x_r) = p(y|x, x_r)p(y_r|x_r) \]

or equivalently \( X \rightarrow (Y_r, X_r) \rightarrow Y \) form a Markov chain. This, loosely speaking, means that the relay receives a better copy of the transmitted message than the destination. Note that by this definition the the broadcast bound simplifies to \( I(X; Y_r | X_r) \) and the rate obtained by DF hence coincides with the cut-set bound.

**Side Information Encoding**

DF relaying requires that the relay decodes the received signal and it therefore results in a performance degradation when the source-relay link is not strong enough. In a situation when decoding is not possible, the relay can transmit an estimate of the received signal \( (\hat{Y}_r) \) and the destination can perform decoding with the side information \( Y \). One can show that the rate [2]

\[ R = \sup_{p(x)p(x_r)p(y_r | x_r, y)} I(X; Y, \hat{Y}_r | X_r) \]

subject to \( I(Y_r; \hat{Y}_r | X_r, Y) < I(X_r; Y) \) \( (3) \)

is achievable. If the relay-destination link supports high data rate transmission (i.e., \( I(X_r; Y) \) is very large), the destination can recover \( Y_r \) with low distortion and the rate obtained by this scheme hence approaches the broadcast bound on the capacity. This scheme is also known as compress-and-forward (CF) [19].

**4 Frequency-Division Gaussian Relay Channel**

Figure 3 illustrates a Gaussian relay channel with orthogonal receive components [20]. This model is motivated by practical limitations in the radio hardware where the relay cannot transmit and receive simultaneously. One solution is that the relay receives and transmits in different frequency bands. This makes the source-relay and the relay-destination links orthogonal to each other. The received signal at the relay is given by

\[ y_r = ax + z_r \] \( (4) \)

where \( a \) is the channel gain between the source and the relay, and \( z_r \sim \mathcal{N}(0, 1) \) is the additive white Gaussian noise with unit variance. The received signals at destination are given by

\[ y_1 = x + z_1 \]
\[ y_2 = bx_r + z_2 \]
where $b$ is the channel gain between the relay and the destination, $z_r \sim \mathcal{N}(0,1)$ and $z_2 \sim \mathcal{N}(0,1)$. We assume that $z_r$, $z_1$, and $z_2$ are mutually independent. The source and the relay are operating under average power constraints. That is $\mathbb{E}[X^2] \leq P_s$ and $\mathbb{E}[X_r^2] \leq P_r$.

### 4.1 Upper bound on the capacity

Using the cut-set bound given by (5), the following upper bound on the capacity can be obtained \[20\]

$$C \leq \min \left\{ \frac{1}{2} \log(1 + (a^2 + 1)P_s), \frac{1}{2} \log(1 + P_s) + \frac{1}{2} \log(1 + b^2P_r) \right\}. \tag{5}$$

which is calculated by choosing independent Gaussian alphabets at the source and at the relay (i.e., $X \sim \mathcal{N}(0,P_s)$ and $X_r \sim \mathcal{N}(0,P_r)$).

### 4.2 Achievable rate of DF

Since the transmitted signals from the source and the relay do not interfere at the destination, we modify the achievable rate obtained by DF given in (2) as

$$R_{DF} = \max \left\{ \sup_{p(x)} I(X;Y_1), \sup_{p(x,x_r)} \min \{I(X,X_r;Y_1,Y_2), I(X;Y_r|X_r)\} \right\}. \tag{6}$$
It turns out the optimal choice of \( p(x, x_r) \) to maximize (6) is \( p(x)p(x_r) \) where \( X \sim N(0, P_s) \) and \( X_r \sim N(0, P_r) \). This therefore yields [20]

\[
R = \max \left\{ \frac{1}{2} \log(1 + P_s), \min \left\{ \frac{1}{2} \log(1 + a^2 P_s), \frac{1}{2} \log(1 + P_s) + \frac{1}{2} \log(1 + b^2 P_r) \right\} \right\}. \tag{7}
\]

Therefore, DF achieves the capacity if \( a \geq \sqrt{1 + b^2(P_r + \frac{P_r}{P_s})} \).

### 4.3 Achievable rate of CF

To evaluate the achievable rate using CF, one needs to find the optimal choice of \( p(x)p(x_r)p(\hat{y}_r|x_r, y) \) which is a difficult task. However, the achievable rate by employing Gaussian codebook at the source and the relay in conjunction with Gaussian quantization can be calculated in closed form. That is \( X \sim N(0, P_s), X_r \sim N(0, P_r), Y_r = Y + N_q \) where \( X, X_r, \) and \( N_q \) are mutually independent and \( N_q \) are chosen to meet the constraint \( I(Y_r; \hat{Y}_r|X_r, Y) < I(X_r; Y) \). This yields the following achievable rate [20]

\[
R_{CF} = \frac{1}{2} \log \left( 1 + P_s + \frac{a^2 b^2 P_r (P_s + 1)}{1 + (a^2) P_s + b^2 P_r (1 + P_s)} \right). \tag{8}
\]

From (8), it is clear that \( R_{CF} \) is always greater than the capacity of the direct transmission. Note that with CF the relay needs to know the quality of side information, that is the source-destination channel gain. Finally, we note that

\[
\lim_{a \to \infty} R_{CF} = \frac{1}{2} \log \left( 1 + P_s + b^2 P_r (P_s + 1) \right)
\]

\[
\lim_{b \to \infty} R_{CF} = \frac{1}{2} \log \left( 1 + P_s + a^2 P_s \right).
\]

In another words, CF asymptotically achieves the multiple access and broadcast bounds as \( a \to \infty \) and \( b \to \infty \) respectively.

### 4.4 Achievable rate of linear relaying

With linear relaying [20–23], the relay’s output is a linear combination of the received signals at the relay. One of the simplest forms of linear relaying is instantaneous linear relaying in which the relay transmits an amplified version of the received signal, that is

\[
x_r = \sqrt{\frac{P_r}{\mathbb{E}[y_r^2]}} y_r. \tag{9}
\]
This scheme is also known as amplify-and-forward (AF). The achievable rate can be calculated as

\[ R_{AF} = \frac{1}{2} \log \left( 1 + P_s + \frac{a^2 b^2 P_s P_r}{1 + a^2 P_s + b^2 P_r} \right). \]

(10)

From (10), it is seen that \( R_{AF} \) is always greater than the capacity of the direct link and it achieves the broadcast bound as \( b \to \infty \). By comparing (8) and (10), one can see that \( R_{CF} > R_{AF} \).

5 Slow-Fading Relay Channel

We next consider the relay channel when the channel gains (for example \( a \) and \( b \) in Figure 3) are random variables but they stay constant during the transmission of one block. That is the channels are quasi-static. For this type of channels the classical Shannon capacity [24] might be zero. For example, this is so when the channel gains are Rayleigh distributed. Thus other measures such as outage probability are usually used [25]. The outage probability shows how often the destination can successfully decode a packet transmitted with a fixed rate \( \beta \) bits per channel use and it is defined as

\[ P_{\text{out}} = \Pr \{ R < \beta \} \]

(11)

where \( R \) is the achievable rate obtained by a given scheme.

6 Resource Allocation for Time-Division Relay Channels

When the relay and the source are restricted to use the same frequency bands, due to the practical limitations, the relay cannot transmit and receive simultaneously. As a remedy, many researchers have assumed a time-division half-duplex mode in which the reception and the transmission occur in non-overlapping timeslots [15, 19, 26, 27]. Note that this is slightly different from the frequency-division relay channel discussed in Section 4. The relay therefore uses a fraction of the available timeslot for the listening and the remaining part for the transmission. Hence, one can optimize the length of the timeslot\(^4\) used for listening and for transmission as well as the conventional power allocation. The works [28–30] consider variable rates transmission protocols for the slowly fading relay channel. The work of [29, 30] formulate criteria for joint power-bandwidth allocation. The former investigates outage probability while the latter studies delay-limited

\(^3\)There are however some cases such as the full-duplex non-orthogonal relay channel where AF can improve on CF with Gaussian quantization [20].

\(^4\)This is also known as bandwidth or dimension allocation as well.
capacity. The work of [31–33] study resource allocation for the ergodic Gaussian relay channel. There is some other related work that only considers optimal power allocation for the relay channel [34–36].

7 Contributions of the Thesis

The thesis investigates different transmission protocols for the slow fading relay channel. The contributions of the thesis can be divided in two major groups:

- the study of uncoded transmission under a modulation constraint, in which we study optimal receivers and mapping optimization at the nodes; and

- the study of coded transmission with an emphasis on DF relaying.

Short summaries of the papers are given in the following:

**Paper A: Improving Collaborative Transmit Diversity by Using Constellation Rearrangement**

Published at the *IEEE Wireless Communications and Networking Conference (WCNC)*, 2007.

We propose an enhancement to cooperative transmit diversity based on uncoded detect-and-forward, by using so-called constellation rearrangement (CR) [37–40] at the relay. With CR, the relay uses a bit-symbol mapping that is different from the one used by the source. We find the optimal bit-symbol mappings for both the source and the relay and the associated optimal detectors, and show that the improvement over conventional relaying with Gray mapping at the source and the relay can amount to a power gain of several dB. This performance improvement comes at no additional power or bandwidth expense, and at virtually no increase in complexity.

**Paper B: Receiver Design for Wireless Relay Channels with Regenerative Relays**

Published at the *IEEE International Conference on Communications (ICC)*, 2007.

We develop a general framework for design of receivers for the wireless relay channel. We derive the optimum detectors for various degrees of channel state information (CSI) at the destination. We consider both the case when the destination has access to full knowledge of the CSI and the case when it only knows the statistics of the channel. High-SNR and low-SNR approximations of the detectors are presented as well.
Paper C: Cooperative Transmission Based on DF Relaying with Partial Repetition Coding


We propose a novel half-duplex decode-and-forward relaying scheme based on partial repetition coding at the relay. In the proposed scheme, if the relay decodes the received message successfully, it re-encodes the message using the same channel code as the one used at the source, but retransmits only a fraction of the codeword. We analyze the proposed scheme and optimize the cooperation level (i.e., the fraction of the message that the relay should transmit). We compare our scheme with conventional repetition in which the relay retransmits the entire decoded message, and with parallel coding, and additionally with dynamic decode-and-forward (DDF). We provide a finite SNR analysis for all the collaborative schemes. The analysis reveals that the proposed partial repetition can provide a gain of several dB over conventional repetition. Surprisingly, the proposed scheme is able to achieve the same performance as that of parallel coding for some relay network configurations, but at a much lower complexity.

Paper D: Analytical Results on Block Length Optimization for Decode-and-Forward Relaying with CSI Feedback

Published at the *IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2007.

We consider block length optimization for collaborative transmit diversity using a decode-and-forward protocol assuming that the source and the relay have access to the magnitudes of all path gains. Moreover, we propose a simple scheme which requires only one bit of channel state information (CSI) feedback. We analyze the outage probabilities of all schemes for both selection combining and maximum ratio combining (MRC) at the destination. Analytical results show that even one bit of CSI feedback can provide a significant gain over conventional (non-adaptive) collaborative schemes.

Paper E: A Spectrally Efficient Transmission Scheme for Half-Duplex DF Relaying

Published at the *IEEE Global Telecommunications Conference (GLOBECOM)*, 2007.

We propose a spectrally efficient transmission scheme for the half-duplex relay channel. In the proposed scheme, the relay combines $N$ detected $r$-dimensional symbols and generates $M$ new $r$-dimensional symbols, where $M < N$, using a linear transformation. The proposed linear transformation preserves the signal energy, and it facilitates decoupled symbol detection at the receiver. We also present an optimized design for the case of complex scalar modulation ($r = 2$) with $N = 2$ and $M = 1$. This design increases the spectral efficiency by 33% compared to conventional decode- or amplify-and-forward relaying.
References


