



The relationship between property transaction prices, turnover rates and buyers' and sellers' reservation price distributions

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Abstract:

This paper analyzes the relationship between movements in property transaction prices and movements in the underlying reservation price distributions of buyers and sellers and how these movements are linked to time varying turnover rate. A main conclusion in previous research is that transaction prices lag changes in buyers' reservation price distribution and that an index tracking transaction prices is less volatile than an index tracking buyer reserves. We show that our less restrictive model of search and price formation reverses the volatility result in previous papers in realistic scenarios, i.e., transaction prices may be more volatile than underlying buyer reserves. We model transaction prices and turnover rates as functions of the moments of buyers' and sellers' reservation price distributions, the search intensity and the average bargaining power among buyers and sellers respectively. We derive the probability density function of transaction prices as a function of these parameters and hence a Maximum-likelihood estimator of the parameters, which serves as a new method of estimating indexes tracking movements in reservation price distributions from transaction data. We perform simulations where we show that the Maximum-likelihood estimator works as intended.

Keywords: Price formation, Transaction price index, Index tracking reservation price distributions, Turnover rates, House price volatility

JEL Codes: C21, C51, D30, R39

1. Introduction

Pro-cyclical liquidity is a well-known phenomenon in property markets, i.e., turnover rates normally increase in up-turns and decrease in down-turns. Since shifts in the reservation price distributions of buyers and seller are partially absorbed by these cyclical changes in the turnover rate, observed transaction prices do not reveal complete information about shifts in buyer (demand) and seller (supply) reserves (Fisher et al., 2003; Goetzmann and Peng, 2006). For example, sticky prices in combination with a dried up liquidity is commonly observed in the initial phase of a down-turn in the property market. A common belief among researches and real estate professionals is that this stickiness is due to demand reacting faster than supply to adverse market conditions, see Clayton et al. (2008) for an analysis of possible reasons for this behavior. That is, buyers initially lower their reservation prices more than sellers, which reduces the turnover rate but to a lesser extent transaction prices since predominantly buyers with higher than average reservation price will match. In a booming property market the opposite may be the case, i.e., the buyers' reservation prices moves faster upwards than the sellers', which increases the turnover rate and initially dampens movements in transaction prices. Consequently, movements in the underlying demand and supply may differ significantly from movements in observed transaction prices over the property cycle. These observations are important in that most research on property markets and their influence on the economy as a whole predominantly use indexes based on transaction prices as the benchmark for measurement of changes in property market conditions. Furthermore, transaction prices are regularly used to infer property returns as well as the value of the stock of properties¹.

A number of papers have brought to attention the discrepancy between changes in observed property prices and changes in the underlying demand and supply and how these variables are related to changes in turnover rates and time on market. Fisher et al. (2003), Fisher et al. (2007) and Goetzman and Peng (2006) model property prices in a search market and develop different methods to estimate indexes tracking, separately, buyers' and sellers' reservation prices². Krainer (2001) uses a search-theoretical model to explain pro-cyclical transaction volumes in the property market. In his model sellers price their homes to sell quickly when prices are high (a "hot" market) in order to reduce the risk of having to sell when buyer valuations are low. When valuations are low (a "cold" market) the opportunity cost of waiting is low with the implication that sellers do not price their homes at levels that would yield the same liquidity as in a hot market. In a similarly spirited paper, Novy-Marx (2009) stresses the importance of a feedback mechanism in the explanation of pro-cyclical liquidity. For example, a positive demand shock increases the ratio of buyers to sellers which improves sellers bargaining position. This makes sellers transact more quickly, which in turn decreases the stock of active sellers relative to buyers which amplifies the initial shock. Clayton et al. (2008) empirically evaluate competing explanations for pro-cyclical liquidity in property markets and finds evidence for an appraisal smoothing/rational updating based explanation as well as the opportunity cost based explanation suggested by Krainer (2001) and Novy-Marx (2009).

¹ For example, according to the accounting standards of the International Accounting Standards Board (IASB), the IAS 40 fair value model for investment properties implies the following: "Gains or losses arising from changes in the fair value of investment property must be included in net profit or loss for the period in which it arises. [IAS 40.35]". Furthermore, "Fair value should reflect the actual market state and circumstances as of the balance sheet date. [IAS 40.38] The best evidence of fair value is normally given by current prices on an active market for similar property....", <http://www.iasplus.com/en/standards/ias/ias40>.

² Two earlier papers in this strand of literature (Gatzlaff and Haurin, 1997 and 1998) estimate models in the same spirit. In effect, they estimate characteristics of the offer price distributions. But they do not discuss the deeper implications of introducing buyer and seller reserves.

Our paper is most similar to Fisher et al. (2003) and Goetzmann and Peng (2006) in that it focuses on modeling the buyer and seller reservation price distributions as well as the transaction price distribution and the relationship between these distributions and the liquidity in the property market. Both Fisher et al. (2003) and Goetzmann and Peng (2006) makes the point that unless buyers' and sellers' reservation price distributions move in lockstep, movements in equilibrium price (the mean of observed transaction prices) do not track the spread between buyer and seller reserves and their movements over time³. Differential movements of these reserves are instead partially reflected in time varying liquidity.

The observation that movements of transaction prices and the underlying reservation price distributions may differ significantly from each other questions the relevance of using the standard definition of market value, i.e. the expected selling price of a property⁴, as the sole metric for construction of property price indexes regardless of the intended use of the index. In terms of index construction, market value defined as expected selling price is captured by the mean of observed (quality adjusted) transaction prices. Goetzmann and Peng (2006) conclude that observed transaction prices are biased measures of market value and suggest that a plausible definition of market value would be the pair of the means of the buyers' and sellers' reservation price distributions since they track both market demand and market supply simultaneously. They, however, choose to define market value as the mean of the buyers' reservation price distribution alone since this equals the mean of the valuations of all market participants. Similarly, Fisher et al. (2003) differentiate between an index tracking observed transaction prices (a standard transaction price index), which they denote a variable liquidity index, and an index tracking the mean of the buyers' reservation price distribution, which they denote a constant liquidity index⁵. We will adopt the Goetzmann and Peng (2006) definition of market value as the mean of the buyers' reservation price distribution in the remainder of the paper.

We develop a new method to estimate price indexes tracking, separately, the means of buyers' and sellers' reservation price distributions in a search market for heterogeneous goods, and in which the surplus of a transaction is split between the buyer and seller according to a (market wide) bargaining power parameter. We further model a market in which properties are sold through auctions or in an auction like environment. Our method is similar to Goetzmann and Peng (2006) in that only data concerning transacted properties are needed for the estimation of the model. In contrast, Fisher et al. (2003) base their model on a Heckman (1979) approach and hence require data about both sold and unsold properties. The latter approach has the benefit of using more data to estimate the means of the reservation price distributions compared to methods only using data from transacted properties. An important downside, however, is that one seldom has relevant data on the whole population of properties.

³ Both Fisher et al. (2003) and Goetzman and Peng (2006) assume that all moments of the reservation price distributions except the means, as well as all other variables affecting price formation such as e.g. search intensity and negotiation power are constant over time. As we show in this paper, if this does not hold, the equilibrium price will not track changes in buyers' and sellers' reservation price distributions even if their means move in lockstep.

⁴ There exist a number of slightly different versions of this definition with respect to the requisites that define an "arm's-length" transaction, but the common denominator is that market value is derived from observed transaction prices.

⁵ It is not clear from Fisher et al. (2003) exactly why an index tracking the mean of the buyers reservation price distribution would be a constant liquidity index, but they argue that "Asset owners must sell to buyers; hence, it is the buyers who determine the prices that are required to maintain a constant ease of selling....". Our interpretation is that they, similar to Goetzmann and Peng (2006), implicitly assume that all moments of the reservation price distributions except the means, as well as other variables affecting price formation such as search intensity and negotiation power are constant over time.

Our paper contributes in several ways to previous studies on property price index construction and the estimation of indexes tracking buyers' and sellers' reservation price distributions. Firstly, while previous papers recognize that property markets are search markets, their model specifications implicitly or explicitly imply a very specific search market in which sellers always meet exactly one buyer during an index period. We model a market in which sellers meet a random number of buyers and where the search intensity, defined by a market wide search intensity parameter, is estimated by the model together with the bargaining power parameter⁶. Secondly, previous models require that buyers' and sellers' reservation price distributions are identical except for their first moments. We apply no restrictions on the reservation price distributions. For example, and importantly, their variances may differ. Thirdly, in contrast to previous models we explicitly derive, in a very general form, the probability density function of observed transaction prices. This allows a straight-forward Maximum-likelihood estimation of all model parameters and makes the model very flexible with respect to distributional assumptions. Basically any "well-behaved" reservation price distributions, as well as a distribution describing search intensity, can easily be inserted in the model. Fourthly, unlike earlier models the empirical turnover rate is not a necessary input for our Maximum-likelihood estimator. This is because we derive the probability density function of observed transaction prices and may thus base the estimation solely on observed prices. This is an advantage since it is not always easy to correctly measure the turnover rate⁷. Finally, the explicit derivation of the probability density function of observed transaction prices as a function of the parameters of the model provides a simple analytical framework for analysis of how changes in observed property prices and market liquidity are related to changes in underlying demand and supply.

To summarize our contributions, we derive a model of search and price formation in the property market that is in important ways richer than models used in previous studies on property price index construction and the estimation of indexes tracking buyers' and sellers' reservation price distributions. In previous papers changes in observed property prices and turnover rate are solely driven by a time varying distance between the means of buyers' and sellers' reservation price distributions. Furthermore, sellers always meet exactly one buyer during an index period. These assumptions impose strong restrictions on the price formation in property markets which are contradicted by empirical observations.

We apply our theoretical model and the Maximum-likelihood estimator in simulations where we show the importance of enriching the model as discussed above. For example, a main conclusion in both Fisher et al. (2003) and Goetzmann and Peng (2006) is that a standard transaction price index is less volatile than the mean of the underlying buyer reservation price distribution. We show that this conclusion mainly follows from the strong restrictions their models impose on the search process and the price formation in property markets. In our richer model setting, a standard transaction price index may realistically be more volatile than the underlying reservation price distributions.

⁶ In our model the exact determinants of how the surplus is divided is unimportant. We are only interested in estimating a bargaining power parameter (all buyers are assumed to have the same bargaining power) in each index period. See e.g. Quan and Quigly (1991) or Novy-Marx (2009) for a discussion on bargaining power and the division of the surplus of a transaction.

⁷ It is a fairly straightforward exercise to include the turnover rate in the estimation if deemed advantageous in some situations. Inclusion may then be done in a way that, unlike previous papers, does not assume that observed turnover exactly equals the theoretically expected one.

2. Model

In this section we model the transaction price distribution for properties in a simple search market. We consider two different selling mechanisms. First we model the case in which a property for sale is visited sequentially by potential buyers and where the property, after a negotiation, is sold to the first visitor with a reservation price higher than the seller's. Thereafter we model the case in which sales are executed through auctions or in an auction like environment⁸.

We derive an expression for the probability density function of observed property prices and show how this function is related to the probability distribution of error terms in a hedonic model of property prices. We furthermore present a Maximum-likelihood estimator of the reservation price distribution of buyers and sellers respectively, as well as of a search intensity parameter and, in the case of sequential search, a bargaining power parameter.⁹

Search market with sequential arrival

Assume a search market in which a seller of a property is visited by M buyers during the time period $[t-1, t]$, where $0 \leq M \leq N$. The property is sold to visitor M , the first of the sequentially arriving buyers with a reservation price higher than the seller's. Hence, if none of the N visitors has a reservation price that is higher than the seller's, the property remains unsold during the time period. The variable N is a Poisson distributed random variable with mean λ and represents the maximum number of buyers that visit a seller during the time period. The reservation price s of a seller and the reservation price b of a buyer visiting the seller are realizations of i.i.d. continuous random variables denoted by S and B respectively.

⁸ While private negotiation is the dominant sales method in most markets, sales through auctions or in auction-like environments are common in several countries, e.g. in Australia, New Zealand, Ireland, and the Nordic countries. In Sweden, virtually all sales of residential real estate in larger cities are executed through auctions, either online or conducted by the broker using text message and phone calls, see Hungria-Gunnelin (2014). As pointed out by Han and Strange (2014), the fraction of houses sold in a bidding war, i.e. in an auction-like environment, has increased considerably in the U.S. during the last decade and while the bust of the housing market has decreased this fraction, it has not come down to historical levels.

⁹ It should be noted that by buyers and sellers we mean not only the buyers and sellers that actually take part in a transaction but also potential buyers and sellers.

Let B_k , where $k = 1, 2, \dots$, be the reservation price of visitor k . Define the highest reservation price among all visitors as:

$$B^{\max} := \begin{cases} \max\{B_1, B_2, \dots, B_N\}, & N > 0 \\ -\infty, & N = 0. \end{cases} \quad (1)$$

It is convenient to define an indicator variable Z as:

$$Z := \begin{cases} 1, & B^{\max} \geq S \\ 0, & B^{\max} < S. \end{cases} \quad (2)$$

Hence, a property is sold during the time period $[t-1, t]$ if $Z = 1$. Define visitor M as:

$$M := \begin{cases} \min\{k : B_k \geq S\}, & Z = 1 \\ \infty, & Z = 0. \end{cases} \quad (3)$$

Define B^{seq} , the reservation price of the first of the sequentially arriving buyers who fulfill $B \geq S$, as:

$$B^{\text{seq}} := \begin{cases} B_M, & Z = 1 \\ -\infty, & Z = 0 \end{cases} \quad (4)$$

Applying the law of total expectation, the probability that a seller with reservation price s does not match during the time interval $[t-1, t]$ can be expressed as:

$$\begin{aligned} P(Z = 0 | S = s) &= \sum_{n=0}^{\infty} P(Z = 0 | N = n, S = s) \cdot P(N = n | S = s) \\ &= \sum_{n=0}^{\infty} P(Z = 0 | N = n, S = s) \cdot P(N = n) \end{aligned} \quad (5)$$

Due to independence between N and S , the last probability in (5) equals $P(N = n)$. Denote the cumulative distribution functions of buyers' and sellers' reservation prices with F^b and F^s

respectively. Assuming that the reservation price b of a buyer visiting a seller, and the reservation price s of the seller are independent, the probability that the buyer and the seller do not match equals $F^b(s)$. Given that $N \sim Po(\lambda)$, expression (5) equals:

$$\begin{aligned} P(Z=0|S=s) &= \sum_{n=0}^{\infty} (F^b(s))^n e^{-\lambda} \frac{\lambda^n}{n!} \\ &= e^{-\lambda} e^{\lambda F^b(s)} \sum_{n=0}^{\infty} e^{-\lambda F^b(s)} \frac{(\lambda F^b(s))^n}{n!} = e^{-\lambda(1-F^b(s))} \end{aligned} \quad (6)$$

Denote the stock of properties by Q^{Tot} . Let Q denote the number of transacted properties and Q^{rate} denote the turnover rate during the time interval $[t-1, t]$. The expected number of transacted properties can be expressed as:

$$E[Q] = Q^{Tot} P(Z=1) = Q^{Tot} E[Q^{rate}] \quad (7)$$

Initially, for ease of derivation, we assume that all properties are identical. This assumption will later be relaxed. Again applying the law of total expectation, expression (7) becomes:

$$\begin{aligned} E[Q] &= Q^{Tot} P(Z=1) = Q^{Tot} \int_{-\infty}^{\infty} f^s(s) P(Z=1|S=s) ds \\ &= Q^{Tot} \int_{-\infty}^{\infty} f^s(s) (1 - P(Z=0|S=s)) ds = Q^{Tot} \int_{-\infty}^{\infty} f^s(s) \left[1 - e^{-\lambda(1-F^b(s))} \right] ds \end{aligned} \quad (8)$$

where $f^s(s)$ is the probability density function of the sellers' reservation price distribution. All buyers are assumed to have the same bargaining power, denoted by w , and all sellers have the bargaining power $(1-w)$, where $0 \leq w \leq 1$ represents the fraction of the surplus (the difference between the seller's and the buyer's reservation price) that goes to the buyer in a successful match. Hence, a match results in the transaction price TP equal to $ws + (1-w)b$. The expected transaction price conditional on a match, $E[TP|Z=1]$, equals¹⁰:

¹⁰ For the derivation of the model it is convenient to define the unconditional expected transaction price as

$E[TP] = E[TP|Z=1]P(Z=1) + E[TP|Z=0]P(Z=0)$, where $E[TP|Z=0] = 0$ is not of interest in this analysis since we do not observe any transaction price if there is no match.

$$\begin{aligned}
\mathbb{E}[TP|Z=1] &= \int_{-\infty}^{\infty} \mathbb{E}[TP|S=s, Z=1] f^s(s|Z=1) ds \\
&= \int_{-\infty}^{\infty} g(s) \frac{f^s(s) P(Z=1|S=s)}{P(Z=1)} ds = \int_{-\infty}^{\infty} g(s) \frac{f^s(s) \left[1 - e^{-\lambda(1-F^b(s))}\right]}{\mathbb{E}[Q^{rate}]} ds
\end{aligned} \tag{9}$$

where $g(s) = \mathbb{E}[TP|S=s, Z=1]$ and equals:

$$g(s) = ws + (1-w) \mathbb{E}[B^{seq} | Z=1] = ws + (1-w) \mathbb{E}[B | B \geq S] = ws + (1-w) \left(\frac{\int_s^{\infty} u \cdot f^b(u) du}{1 - F^b(s)} \right) \tag{10}$$

where $f^b(b)$ is the probability density function of the buyers' reservation price distribution. Since B_1, B_2, \dots are independent we have that $\mathbb{E}[B^{seq} | Z=1] = \mathbb{E}[B | B \geq S]$ in expression (10).

In order to empirically estimate the parameters of the reservation price distributions, as well as the Poisson parameter λ and the bargaining power parameter w , it is convenient to derive an expression for the probability density function of observed transaction prices as a function of these parameters since the parameters can then be estimated using a straight-forward Maximum-likelihood approach.

Denote with F^{TP} and f^{TP} the cumulative distribution function and the probability density function of observed transaction prices respectively. If the transaction price $TP = y$ and the reservation price of the seller $S = s$, then $B^{seq} = b$ where $y = ws + (1-w)b$ according to the bargaining power assumption. Since the derivation concerns a match we have that $y \geq s$ and that both $F^{TP}(y|S=s, Z=1)$ and $f^{TP}(y|S=s, Z=1)$ equal zero for $y < s$. We now derive f^{TP} from F^{TP} :

$$\begin{aligned}
F^{TP}(y|S=s, Z=1) &= P(TP \leq y|S=s, Z=1) \\
&= P(wS + (1-w)B^{seq} \leq y|S=s, Z=1) \\
&= P(ws + (1-w)B^{seq} \leq y|S=s, Z=1) \\
&= P\left(B^{seq} \leq \frac{y-ws}{1-w} | S=s, Z=1\right) \\
&= F^{B^{seq}}\left(\frac{y-ws}{1-w} | S=s, Z=1\right) \\
&\Rightarrow \\
f^{TP}(y|S=s, Z=1) &= \frac{d}{dy} F^{TP}(y|S=s, Z=1) \\
&= \frac{d}{dy} F^{B^{seq}}\left(\frac{y-ws}{1-w} | S=s, Z=1\right) \\
&= \frac{1}{1-w} f^{B^{seq}}\left(\frac{y-ws}{1-w} | S=s, Z=1\right) \\
&= \frac{1}{1-w} \frac{f^b\left(\frac{y-ws}{1-w}\right)}{1-F^b(s)}
\end{aligned} \tag{11}$$

We further know that

$$f^s(s|Z=1) = \frac{P(Z=1|S=s)}{P(Z=1)} f^s(s) = \frac{\left[1 - e^{-\lambda(1-F^b(s))}\right]}{E[Q^{rate}]} f^s(s) \tag{12}$$

Combining (11) and (12) and integrating we have the following expression for the probability density function of observed transaction prices:

$$\begin{aligned}
f^{TP}(y|Z=1) &= \int_{-\infty}^y f^{TP}(y|S=s, Z=1) f^s(s|Z=1) ds \\
&= \frac{1}{1-w} \int_{-\infty}^y \frac{f^b\left(\frac{y-ws}{1-w}\right)}{1-F^b(s)} \frac{\left[1 - e^{-\lambda(1-F^b(s))}\right]}{E[Q^{rate}]} f^s(s) ds \\
&= \frac{1}{1-w} \int_{-\infty}^y \frac{f^b\left(\frac{y-ws}{1-w}\right) \left[1 - e^{-\lambda(1-F^b(s))}\right] f^s(s)}{\left(\int_s^{\infty} f^b(u) du\right) E[Q^{rate}]} ds
\end{aligned} \tag{13}$$

Similar to expression (9), the expected transaction price conditional on a match equals¹¹:

$$E[TP|Z=1] = \int_{-\infty}^{\infty} y \cdot f^{TP}(y|Z=1) dy \quad (14)$$

Sales through auctions

Assume that properties are sold through auctions or following a bidding war as discussed in Han and Strange (2014). Assume further that an observed transaction price equals the reservation price of the winning bidder¹², i.e. B^{max} in expression (1). Let N be a Poisson distributed random variable with mean λ , representing the number of bidders in an auction. Denote with $G^{B^{max}}$ and $F^{B^{max}}$ the cumulative distribution functions of B^{max} and observed transaction prices respectively. $G^{B^{max}}$ conditional on n bidders equals:

$$\begin{aligned} G^{B^{max}}(s|N=n) &= P(B^{max} \leq s|N=n) = P(\max\{B_1, B_2, \dots, B_n\} \leq s|N=n) \\ &= P(B_1 \leq s, B_2 \leq s, \dots, B_n \leq s) = \prod_{j=1}^n P(B_j \leq s) = F^b(s)^n \end{aligned} \quad (15)$$

Noting that $B^{max} \geq s$ for observed transactions and using (15), the probability density function of observed transaction prices conditional on $S = s$, $f^{B^{max}}(y|S=s, Z=1)$, can be derived as follows:

¹¹ The expectation in expressions (14) of course coincide with the expectation in expression (9), the difference in derivation being that in (9) the law of total expectation is applied to the expected price conditional on the reservation price of the seller.

¹² A more realistic assumption would be that the sales price ends up somewhere between the winner's and the second highest bidder's reservation price. However, this will not alter the qualitative results of our model.

$$\begin{aligned}
F^{B^{\max}}(y|N=n, S=s, Z=1) &= P(B^{\max} \leq y|N=n, S=s, Z=1) = P(B^{\max} \leq y|N=n, S=s, B^{\max} \geq s) \\
&= \frac{F^{B^{\max}}(y|N=n) - F^{B^{\max}}(s|N=n)}{1 - F^{B^{\max}}(s|N=n)} = \frac{F^b(y)^n - F^b(s)^n}{1 - F^b(s)^n}
\end{aligned}$$

\Rightarrow

$$\begin{aligned}
f^{B^{\max}}(y|N=n, S=s, Z=1) &= \frac{d}{dy} F^{B^{\max}}(y|N=n, S=s, Z=1) \\
&= \frac{d}{dy} \frac{F^b(y)^n - F^b(s)^n}{1 - F^b(s)^n} = nf^b(y) \frac{F^b(y)^{n-1}}{1 - F^b(s)^n}
\end{aligned}$$

\Rightarrow

$$\begin{aligned}
f^{B^{\max}}(y|S=s, Z=1) &= \sum_{n=0}^{\infty} f^{B^{\max}}(y|N=n, S=s, Z=1) \cdot P(N=n|S=s, Z=1) \\
&= \sum_{n=1}^{\infty} nf^b(y) \frac{F^b(y)^{n-1}}{1 - F^b(s)^n} \frac{P(Z=1|N=n, S=s) \cdot P(N=n|S=s)}{P(Z=1|S=s)} \\
&= \sum_{n=1}^{\infty} nf^b(y) \frac{F^b(y)^{n-1}}{1 - F^b(s)^n} \frac{P(Z=1|N=n, S=s) \cdot P(N=n)}{1 - e^{-\lambda(1-F^b(s))}} \\
&= \sum_{n=1}^{\infty} nf^b(y) \frac{F^b(y)^{n-1}}{1 - e^{-\lambda(1-F^b(s))}} \frac{e^{-\lambda} \lambda^n}{n!} = f^b(y) \frac{\lambda e^{-\lambda}}{1 - e^{-\lambda(1-F^b(s))}} \sum_{n=1}^{\infty} \frac{(\lambda F^b(y))^{n-1}}{(n-1)!} \\
&= f^b(y) \frac{\lambda e^{-\lambda(1-F^b(y))}}{1 - e^{-\lambda(1-F^b(s))}}
\end{aligned} \tag{16}$$

Integrating the conditional probability density function in (16) we have the following expression for the probability density function of observed transaction prices¹³:

¹³ Assuming that each auction is visited by only one buyer who bids her reservation price and the reservation price distributions only differ in their first moments, our auction model collapses to the model of Goetzmann and Peng (2006) and the simplified probability density

function of observed transaction prices equals: $f^{B^{\max}}(y|Z=1) = \frac{f^b(y)F^s(y)}{E(Q^{rte})} = \frac{f^b(y)F^s(y)}{\int_{-\infty}^{\infty} f^s(s)(1-F^b(s))ds}$

$$\begin{aligned}
f^{B^{\max}}(y|Z=1) &= \int_{-\infty}^y f^{B^{\max}}(y|S=s, Z=1) f^s(s|Z=1) ds \\
&= \int_{-\infty}^y f^b(y) \frac{\lambda e^{-\lambda(1-F^b(y))}}{1 - e^{-\lambda(1-F^b(s))}} \frac{1 - e^{-\lambda(1-F^b(s))}}{E[Q^{rate}]} f^s(s) ds \\
&= f^b(y) \frac{\lambda e^{-\lambda(1-F^b(y))}}{E[Q^{rate}]} \int_{-\infty}^y f^s(s) ds \\
&= f^b(y) \frac{\lambda e^{-\lambda(1-F^b(y))}}{E[Q^{rate}]} F^s(y)
\end{aligned} \tag{17}$$

The expected transaction price in the auction case equals:

$$E[TP|Z=1] = \int_{-\infty}^{\infty} y \cdot f^{B^{\max}}(y|Z=1) dy \tag{18}$$

3. A method for index estimation and correction of sample selection bias

In order to use expression (13) or (17) to construct an empirically applicable Maximum-likelihood estimator of the parameters of the model we now relax the assumption that properties are identical. Following Goetzmann and Peng (2006), we define the market value of property i at time t , $MV_{i,t}$, as the mean of all potential buyers' private valuations:

$$MV_{i,t} = \bar{b}(m_{i,t}) \tag{19}$$

where $m_{i,t}$ are the attributes of property i at time t . Denote by $\bar{s}(m_{i,t})$ the mean of all sellers' reservation prices for a property with the attributes $m_{i,t}$. The reservation price of the buyer, $b_{i,t}$, in a transaction of property i equals the market value plus an error term, $\varepsilon_{i,t}^b$:

$$b_{i,t} = \bar{b}(m_{i,t}) + \varepsilon_{i,t}^b \tag{20}$$

Similarly, the reservation price of the seller, $s_{i,t}$, can be expressed as:

$$s_{i,t} = \bar{s}(m_{i,t}) + \mu_t + \varepsilon_{i,t}^s = \bar{s}(m_{i,t}) + \varepsilon_{i,t}^s \tag{21}$$

where $\mu_t = \bar{s}(m_{i,t}) - \bar{b}(m_{i,t}) = \bar{s}_t - \bar{b}_t$, i.e. the distance between the means of the reservation price distributions (assumed to be the same for all i) and $\varepsilon_{i,t}^s$ is the error term of the seller. Since we model an observed transaction it must be the case that $b_{i,t} \geq s_{i,t}$, or equivalently $\varepsilon_{i,t}^b \geq \mu_t + \varepsilon_{i,t}^s$. Applying our bargaining power assumption the price of property i can be expressed as:

$$\begin{aligned}
 TP_{i,t} &= w_t \cdot s_{i,t} + (1 - w_t) \cdot b_{i,t} \\
 &= w_t \cdot [\bar{b}(m_{i,t}) + \mu_t + \varepsilon_{i,t}^s] + (1 - w_t) \cdot [\bar{b}(m_{i,t}) + \varepsilon_{i,t}^b] \\
 &= \bar{b}(m_{i,t}) + w_t \cdot \mu_t + w_t \cdot \varepsilon_{i,t}^s + (1 - w_t) \cdot \varepsilon_{i,t}^b \\
 &= \bar{b}(m_{i,t}) + \varepsilon_{i,t}
 \end{aligned} \tag{22}$$

where

$$E[\varepsilon_{i,t}] = w_t \cdot \mu_t + E[w_t \cdot \varepsilon_{i,t}^s + (1 - w_t) \cdot \varepsilon_{i,t}^b \mid \varepsilon_{i,t}^b \geq \mu_t + \varepsilon_{i,t}^s] \tag{23a}$$

In the auction case expression (23a) becomes:

$$E[\varepsilon_{i,t}] = E[\varepsilon_{i,t}^b \mid \varepsilon_{i,t}^b \geq \mu_t + \varepsilon_{i,t}^s] \tag{23b}$$

When relaxing the assumption that properties are identical, differences between properties must be controlled for in the construction of a property price index. Assuming $\bar{b}(m_{i,t})$ is linear in parameters, equation (22) can be estimated by running a hedonic regression using OLS and estimate the following equation:

$$TP_{i,t} = \hat{b}(m_{i,t}) + \hat{\varepsilon}_{i,t}^{regr} \tag{24}$$

$\hat{b}(m_{i,t})$ is the predicted part of the transaction price and $\hat{\varepsilon}_{i,t}^{regr}$ are the residuals.

Expression (23a) or (23b) is in general not zero while the residuals from estimating (22) using OLS by construction has zero mean. Hence, the intercept of the hedonic regression is estimated with bias¹⁴.

¹⁴ Only the intercept is estimated with bias, not the slope coefficient(s) (Bierens, 2007). This is because the selection process is governed by μ_t , $\varepsilon_{i,t}^s$ and $\varepsilon_{i,t}^b$ which are all uncorrelated with $m_{i,t}$ by assumption.

However, the expectation in (23a) or (23b) can be estimated using a Maximum-likelihood estimator based on expression (13) or (17) in order to provide the bias correction term for the intercept.

Denote the means of the buyer and seller reservation price distributions in expressions (13) or (17) at time t , $E_t[B]$ and $E_t[S]$, with μ_t^b and μ_t^s respectively. When substituting zero for μ_t^b and $\mu_t = \mu_t^s - \mu_t^b = \bar{s}_t - \bar{b}_t$ for μ_t^s , the transaction price distribution given by (13) or (17) corresponds to the probability density function of the error term $\varepsilon_{i,t}$ in (22). Hence, by calculating $E_t[\varepsilon_{i,t}]$ from expression (14) or (18) and estimating $\varepsilon_{i,t}$ from the relationship $\hat{\varepsilon}_{i,t} = \hat{\varepsilon}_{i,t}^{regr} + E_t[\varepsilon_{i,t}]$, all model parameters and the bias correction term (23a) or (23b) can be estimated using the residuals in (24) as input for the Maximum-likelihood estimator based on (13) or (17).

Expressions (13) and (17) are very general allowing for basically any well-behaved reservation price distributions. In our simulations we assume normal distributions, $B \sim N(0, \sigma_t^b)$ and

$S \sim N(\mu_t^s, \sigma_t^s)$, thereby limiting the number of moments to estimate to two for the buyer and seller reservation price distribution, respectively. We also have to add the Poisson parameter yielding in total five parameters to estimate in the auction case. In the case of sequential arrival and negotiation, the bargaining power parameter must also be estimated, which is rather demanding from a computational point of view. However, since we only need to estimate the *distance* between the means of the buyer and seller reservation price distributions in order to obtain the bias correction term we can drop one dimension. This is done by the substitution of zero for μ_t^b and μ_t for μ_t^s in expression (13) or (17). An estimate of μ_t^b is retrieved by subtracting the bias correction term (23a or 23b) from the mean quality adjusted transaction price.

The Maximum likelihood-estimator has the following form in the case of sequential arrival:

$$\text{Max } \prod_{i=1}^{Q_t} f_t^{TP} \left(\hat{\varepsilon}_{i,t}^{regr} + E_t[\varepsilon_{i,t}] \right) \quad (25)$$

$$\text{w.r.t. } \mu_t, \sigma_t^b, \sigma_t^s, \lambda_t, w_t.$$

In the auction case we have:

$$\text{Max } \prod_{i=1}^{Q_t} f_t^{Bmax} \left(\hat{\varepsilon}_{i,t}^{regr} + E_t[\varepsilon_{i,t}] \right) \quad (26)$$

$$\text{w.r.t. } \mu_t, \sigma_t^b, \sigma_t^s, \lambda_t.$$

The fact that the estimator works without information about the size of the stock and the turnover rate implies an advantage compared to previous models since such information often is difficult to obtain. For example, in the empirical application of their model, Goetzmann and Peng (2006) caution their results since they lack information about the actual size of the stock of properties and the trading volume and therefore must use estimates of these parameters in order for their model to work.

4. Model simulations

In the following we perform simulations in order to 1), show how the proposed estimator improves on previous research and 2), show how our richer model modifies the conclusion in previous research that an index tracking movements of buyer reserves is more volatile than a traditional index tracking observed transaction prices. Due to the ease of computation of probability density function (17) compared to probability density function (13), and the large amount of simulations we make, the auction market case is used in all simulations.

We generate samples of transaction prices from a stock consisting of 100 000 properties, which with our base-case parameter values results in about 5 000 observations, corresponding to a turnover rate of 5 percent. One time period is simulated, time indexes are therefore dropped. As in Goetzmann and Peng (2006) properties are assumed to vary in one dimension m , where m is randomly generated from a uniform distribution $[1, 4]$. We may think of m as the size of the property.

Buyers' (or bidders') and sellers' reservation prices are generated according to expression (20) and (21) respectively, where the market value of a property, $\bar{b}(m_i)$, equals m_i and where the error terms ε_i^b and ε_i^s are normally distributed with standard deviations equal to $\sigma^b = \sigma^s = 0.1$ in the base case¹⁵. The search intensity parameter, λ , equals 0.2 meaning that each seller meets an expected 0.2 number of buyers during a time period. The mean of buyer reserves (for one unit of m) is 1 and the distance between the means of the buyer and seller reserves, μ , is 0.1 corresponding to a mean of seller reserves (for one unit of m) equal to 1.1.

Transaction prices are generated by allocating to each seller a random number of bidders (Poisson distributed with $\lambda = 0.2$) with reservation prices randomly drawn from the buyers' reservation price distribution. A transaction occurs if at least one of the allocated bidders has a reservation price higher than the seller's. Since we simulate the auction case buyers bargaining power is set to zero, with the implication that the transaction price equals the reservation price of the highest bidder and (dropping the time index) expression (23b) simplifies to $E[\varepsilon_i] = E[\varepsilon_i^b | \varepsilon_i^b \geq \mu + \varepsilon_i^s]$.

After generating transaction prices we use the proposed Maximum likelihood estimator given by expression (26) to estimate the parameters of the reservation price distributions as well as the Poisson parameter. The parameters are found by grid-searching all combinations of reasonable parameter values to find the values that satisfy (26). Variation in the attribute m is controlled for by running regression (24). We run 100 rounds of simulations for each set of parameter values that we want to estimate.

Since it is not feasible to calculate numerically expression (26) for all possible parameter value combinations we have limited the parameter values included in the grid search to intervals with endpoints that are sufficiently far from the true parameter values. The intervals were chosen as follows: $0 \leq \mu \leq 0.25$, $0.02 \leq \sigma^b \leq 0.25$, $0.02 \leq \sigma^s \leq 0.25$, $0.025 \leq \lambda \leq 0.75$. More sophisticated numerical methods than our grid search method may of course allow search over unlimited intervals, but optimizing the numerical estimation method is not within the scope of this research.

¹⁵ To obtain the same setting as in Fisher et al. (2003) and Goetzmann and Peng (2006) we may simply interpret the reservation prices as the log of the actual reservation prices.

An observation that may be related to possible shortcomings of our numerical method is that the following bootstrapping approach lowers the variance of our parameter estimates. For each round of simulation we draw 100 random subsamples consisting of 10 percent of the original sample. Then, for each subsample, we estimate the parameters of interest using our grid search method. After that we take, for each parameter, the median of the 100 subsample estimates. This procedure is repeated for each of the 100 original samples. When comparing the variance of the 100 estimates of the parameters without bootstrapping with the variance of the 100 median estimates of the parameters resulting from the bootstrapping approach, the latter produces in general lower variance of the estimated parameters. Due to the cumbersome calculations involved in maximizing expression (26) no exhaustive comparison was made between parameter estimates with and without bootstrapping, but for the various parameter combinations we did test the same pattern remained. The point of making inference based on the medians within each original sample instead of making inference from the $100 \times 100 = 10\,000$ subsamples directly is that in an application using real data there is only one “simulation round”. In this context our observation that bootstrapping improves the variance of the parameter estimates compared to the “raw” estimates from the original sample alone may be of interest. We, however, caution the above result since it may simply be an artifact of our crude estimation method and errors when calculating expression (17) numerically. That is, it may be the case that uncorrelated errors in individual sub-samples due to inaccuracy of the numerical calculations are “diversified” when taking averages of sufficiently many subsamples.

We run simulations for four cases, A-D. In case A and B it is demonstrated that the Maximum likelihood estimator’s performance is satisfactory. In case A data is generated using equal variance for buyer and seller reserves while in case B their variance differ. 100 rounds of simulations are run for each case. In this way it is possible to study the performance of the estimator by the mean outcome of the 100 estimates as well as the standard deviation of the estimates¹⁶.

Simulations C and D are constructed to illustrate some aspects of how our richer model and estimation procedure improves on previous research. Since the model used in case A and B collapses into the Goetzmann and Peng (2006) model when restricting the auctions to contain exactly one bidder bidding his reservation price, and also restricting the buyer and seller reservation price distributions to be identical except for the means, it is convenient to use their model as comparison benchmark.

Transaction prices in simulation C and D are generated in the same manner as in simulation A and B, i.e. allowing for a search intensity parameter and for buyer and seller reserves to have different variance. We then estimate parameter values using the method of Goetzmann and Peng (2006) where it is assumed that all sellers meet exactly one buyer and that buyer and seller reserves have identical variance. As in case A, transaction prices in case C are generated from buyer and seller distributions with equal variance. In case D the distributions have different variance. In all cases each seller meets an expected 0.2 number of buyers. The results from simulation of case A-D are reported in *Table 1*.

¹⁶ It is not within the scope of this paper to find an optimal method of finding the solution to the maximization problem in (26). We demonstrate the results using one possible method. The particular results in terms of precision are thus in part determined by the particular numerical calculation method that we use.

Table 1. Results from simulations testing the performance of the proposed maximum likelihood estimator.

	Expected number of buyers per seller, λ	Mean of buyer reserves, μ^b	Std. of buyer reserves, σ^b	Mean of seller reserves, μ^s	Std. of seller reserves, σ^s	"Traditional" hedonic price index
Case A: The parameters are estimated using the ML-estimator given by expression (26).						
True value	0.2	1	0.1	1.1	0.1	
<i>Bootstrapping</i>						
Mean of estimates	0.234	1.002	0.097	1.110	0.105	1.094
Std. of estimates	0.117	0.025	0.006	0.041	0.013	0.002
<i>No Bootstrapping</i>						
Mean of estimates	0.360	0.986	0.101	1.111	0.103	1.094
Std. of estimates	0.239	0.043	0.009	0.047	0.016	0.002
Case B: The parameters are estimated using the ML-estimator given by expression (26).						
True value	0.2	1	0.08	1.1	0.12	
<i>Bootstrapping</i>						
Mean of estimates	0.298	0.991	0.081	1.131	0.127	1.060
Std. of estimates	0.111	0.013	0.003	0.042	0.018	0.002
<i>No Bootstrapping</i>						
Mean of estimates	0.344	0.990	0.081	1.119	0.125	1.060
Std. of estimates	0.222	0.024	0.005	0.057	0.028	0.002
Case C: The parameters of the buyer and seller reserve distributions are estimated with the restriction that each seller meets exactly one buyer and σ^b and σ^s are restricted to be equal.						
True value	0.2	1	0.1	1.1	0.1	
Mean of estimates		0.941	0.104	1.188	0.104	1.094
Std. of estimates		0.003	0.002	0.003	0.002	0.002
Case D: The parameters of the buyer and seller reserve distributions are estimated with the restriction that each seller meets exactly one buyer and σ^b and σ^s are restricted to be equal.						
True value	0.2	1	0.08	1.1	0.12	
Mean of estimates		0.924	0.092	1.143	0.092	1.060
Std. of estimates		0.003	0.001	0.002	0.001	0.002

The number of simulations for each case is 100. "True values" means the parameter values that were used when simulating the samples. Bootstrapping means that mean and standard deviation of parameter estimates are calculated using the median parameter estimates (for each round of simulation) from 100 random sub-samples, where each sub-sample equals 10 percent of the original sample.

The estimated parameters in case A and B are fairly close to their respective true values (the parameters used when generating the data) and the standard deviation of the 100 estimates in each case is also rather small. In comparison, the Goetzmann and Peng (2006) model also estimates the standard deviation of the reserves fairly accurately in case C. However, the buyer reserve mean is downward biased by six percentage points and the seller reserve mean is biased upwards by nine percentage points. Reserve means are thus estimated far apart relative to their true values. This

stems from the fact that Goetzmann and Peng (2006) assume that all properties are on the market in each time period. In order for their model to obtain a turnover rate on the order of 5 percent in a world where all properties are on the market, the distance between buyers and sellers must be large and/or variance small.

The richer framework we propose allows for the search intensity parameter to partly account for the turnover rate. This also allows the intuitively appealing interpretation that not all properties are on the market in every time period (see Novy-Marx, 2009, for a discussion of time varying market tightness) and that buyers and sellers may have fairly similar views of the values of properties without that resulting in a huge turnover. In contrast, Goetzmann and Peng (2006) estimate vast differences between buyer and seller reserves in their empirical application using real transaction data from the Los Angeles housing market. For example, seller reserve mean is 86 % higher than buyer reserve mean at one point while buyer reserve mean is almost twice the seller reserve mean at another point, which are arguably not realistic results. These results are not discussed in the paper but one may speculate that their model specification is too simplified in that it does not capture important features of the price formation in property markets, such as time varying search intensity and non-equal moments of the buyer and seller reservation price distributions, as well as time varying bargaining power of buyers and sellers.

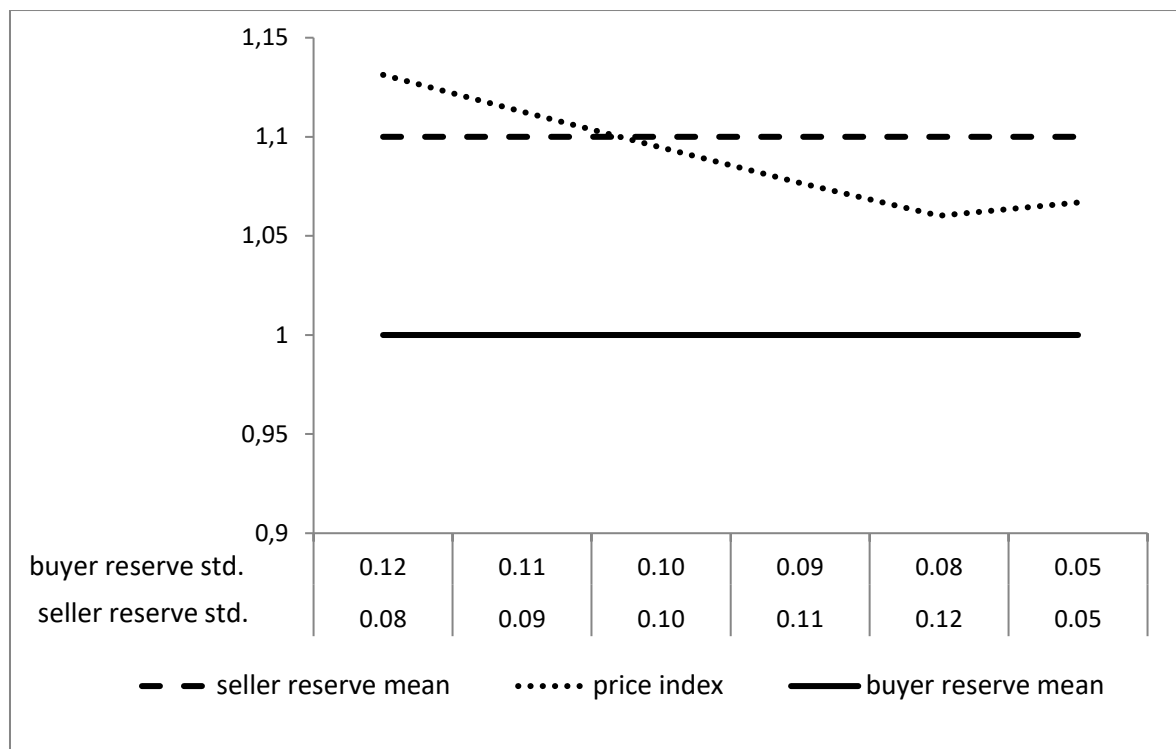
In case D, where prices are generated from buyer and seller reservation price distributions with unequal variance, the common standard deviation estimated by the Goetzmann and Peng (2006) model ends up between the true standard deviations. Buyer and seller means are lower than those estimated in case C. For the seller mean this happens to be an improvement from case C with the particular parameters used, but for the buyer the estimated mean is further from the true value.

Table 1 also displays the value of a traditional hedonic price index for each case. This shows an important point: reserve means of buyers and sellers are the same in all simulations, yet the price index changes. In other words, even if underlying demand and supply are constant over time in terms of reserve means, observed transaction prices may change significantly. An index tracking transaction prices may thus be more volatile than an index tracking underlying demand (or supply). This result represents an important difference from the results in Fisher et al. (2003) and Goetzmann and Peng (2006), where changes in observed property prices and turnover rate are solely driven by differential movements of the means of buyers' and sellers' reservation price distributions¹⁷. With this restriction transaction prices are always less volatile than the underlying demand.

Graph 1 shows our opposing result and how price is influenced by the difference in buyer and seller reserve variance. For example, the hedonic price changes from 1.131 in the case when $\sigma^b = 0.12$ and $\sigma^s = 0.08$ to 1.060 when $\sigma^b = 0.08$ and $\sigma^s = 0.12$, keeping the other parameters constant. But price also changes when buyers' and sellers' *common* variance changes. When both buyers' and sellers' reserve variance changes from 0.1 to 0.05, price changes from 1.094 to 1.067. Hence, unless there are compelling reasons why the second and higher moments of the reservation price distributions should be equal and constant over time, imposing restrictions on the moments may significantly bias estimates of distribution parameters.

¹⁷ Krainer (2001) also derive the result that prices vary less than buyers' valuations although his result is driven by different causes.

Graph 1. Comparison of a hedonic price index with the mean of the buyers' and sellers' reservation price distributions when standard deviations of the reservation price distributions changes according to case A-D in *Table 1*.



λ is 0.2. Turnover is 4.6-4.7 percent for all the parameter cases in *Graph 1* except for the case where standard deviation is 0.05 for both buyers and sellers where turnover is 1.5 percent.

Why would the second and higher moments be time varying and/or differ between buyers and sellers? Firstly, one may hypothesize that due to the price discovery process the buyers' reservation price distribution is skewed to the left and the sellers' reservation price distribution is skewed to the right. That is, only a smaller fraction of potential buyers are prepared to pay much more than observed prices while many potential buyers (those who are not actively searching) have reservation prices significantly below observed prices and vice versa for potential sellers¹⁸. Secondly, if we consider the characteristics of different phases in a property cycle one would typically expect that turnover increases in an upturn, giving buyers and sellers more price information which would lead to smaller variance in reserves compared to when turnover is lower. Thirdly, in a down-turn sellers' reservation prices may become more dispersed than the buyers' as some sellers are reluctant to, or cannot, lower their reservation price (e.g. due to mortgages restrictions, see discussion in e.g. Springer, 1996), while other sellers choose to lower their reservation prices significantly in order to sell quickly e.g. due to financial distress. The point here is not to give an exhaustive discussion on why the second and higher moments of the reservation price distribution would be unequal among buyers and sellers and time varying, but to argue that this idea is not just of theoretical interest.

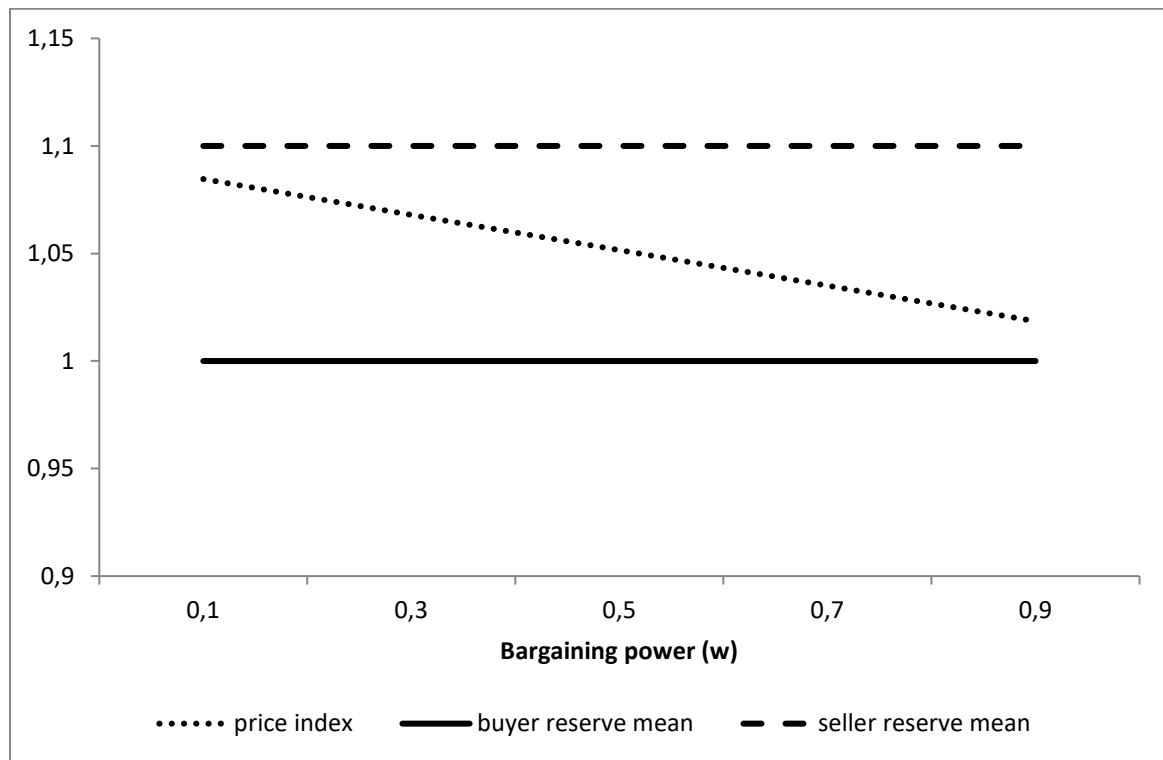
In the context of our model of sequential search not only movements in reserve means and variation in reserve variance affect the level of transaction prices significantly. Also variation over time of the bargaining power parameter, w , will act in the direction of increasing the volatility of observed transaction prices compared to that of the underlying reservation price distributions. For this to be the case buyers bargaining power should vary counter-cyclically, i.e. buyers (sellers) bargaining

¹⁸ While we are not simulating distributions that are skewed in this manner, the maximum-likelihood estimator in expression (26), which is derived in a completely general form, easily accommodate such a specification.

power should increase (decrease) in a downturn and vice versa in an upturn, which seems plausible, see e.g. Novy-Marx (2009) and Carillo (2013). Using US residential real estate transaction data Carillo (2013) empirically estimates variation over time of sellers' bargaining power during the time period 1998-2009 and obtains estimates ranging from 0.4 to 0.9 (where zero means no bargaining power and 1 means complete bargaining power).

Graph 2 shows that prices may vary significantly when bargaining power changes. When going from a market in which the seller has most of the bargaining power ($w = 0.1$) to a market in which the opposite is the case ($w = 0.9$), keeping all moments of the buyer and seller reserve distributions and the search intensity constant, the price index changes from 1.084 to 1.019. Hence, similar to the case of changes in the variance of the reservation price distributions, variation in bargaining power alone may make observed transaction prices more volatile than the underlying reservation price distributions.

Graph 2 Comparison of a hedonic price index with the mean of the buyers' and sellers' reservation price distributions when the bargaining power parameter, w , changes from 0.1 to 0.9.



The following parameter values are used: $\lambda = 0.2$, $\mu^b = 1$, $\mu^s = 1.1$, $\sigma^b = 0.1$, $\sigma^s = 0.1$.

Finally, assuming that search intensity is higher in an upturn than in a downturn (Novy-Marx, 2009), variation over time of the search intensity parameter, λ , also affects the price index in the way of making it more volatile. For example, if we change λ from 0.2 to 1 in case A, the price changes from 1.095 to 1.105 and turnover increases from 4.6 percent to 19.3 percent.

5. Conclusions

We derive a model of property transaction prices in a search market where changes in observed transaction prices and transaction volumes are driven by changes in buyers' and sellers' reservation price distributions as well as changes in search intensity and bargaining power over the property

cycle. We further derive a new Maximum-Likelihood estimator of the parameters of the reservation price distributions as well as the search intensity parameter and the bargaining power parameter. The estimator uses observed transaction prices (adjusted for property characteristics) as input.

We show that the estimator works as intended by performing simulations where the estimator successfully recovers the parameters used to generate the data in the simulations. We further show that one of the central results in the previous literature that an index tracking observed transaction prices is always less volatile than an index tracking the underlying buyer reservation price distribution does not hold in a more realistic model of search and price formation. In our richer model setting observed transaction prices may be more volatile than the underlying demand for realistic parameter values. This is an important observation since movements of the underlying demand and supply capture fundamental property market information that is not captured by observed transaction prices alone.

Whether or not our model contributes with less biased indexes tracking buyer and seller reserves compared to more restrictive models in previous research is in the end an empirical question. We believe, however, that the new features of our model have strong empirical support. Sellers do not meet exactly one buyer during a typical index period. There is no reason why buyers' and sellers' reservation price distributions should be identical except for the first moments, nor that they should be constant over time. It is unlikely that the bargaining power of buyers and sellers respectively should be constant over the property cycle. Our model does not rely on these restrictions.

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