

Stability analysis and control design of spatially developing flows

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May 2008
Technical Reports from
Royal Institute of Technology
Department of Mechanics
SE-100 44 Stockholm, Sweden

Akademisk avhandling som med tillstånd av Kungliga Tekniska Högskolan i Stockholm framlägges till offentlig granskning för avläggande av teknologie licentiatexamen torsdagen den 05 juni 2008 kl 14.00 i D41, Kungliga Tekniska Högskolan, Lindstedtsvägen 17, 1tr, Stockholm.

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Universitetsservice US-AB, Stockholm 2008

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Abstract

Methods in hydrodynamic stability, systems and control theory are applied to spatially developing flows, where the flow is not required to vary slowly in the streamwise direction. A substantial part of the thesis presents a theoretical framework for the stability analysis, input-output behavior, model reduction and control design for fluid dynamical systems using examples on the linear complex Ginzburg-Landau equation. The framework is then applied to high-dimensional systems arising from the discretized Navier–Stokes equations. In particular, global stability analysis of the three-dimensional jet in cross flow and control design of two-dimensional disturbances in the flat-plate boundary layer are performed. Finally, a parametric study of the passive control of two-dimensional disturbances in a flat-plate boundary layer using streamwise streaks is presented.

Descriptors: Global modes, transient growth, model reduction, feedback control, streaks, Tollmien–Schlichting waves

Preface

This thesis considers the stability and control of spatially developing flows. In the first part a short review of the basic concepts and methods is presented. The second part consists of the following papers:

Paper 1. BAGHERI S., HEPFFNER J., SCHMID P.J. AND HENNINGSON D.S., 2008

“Input-output analysis and control design applied to a linear model of spatially developing flows”, Accepted for publication in Applied Mechanics Reviews

Paper 2. BAGHERI S., BRANDT L. AND HENNINGSON D.S., 2008

“Input-output analysis, model reduction and control design of the flat-plate boundary layer”, Submitted to the Journal of Fluid Mechanics

Paper 3. BAGHERI S., SCHLATTER P. AND HENNINGSON D.S., 2008

“The three-dimensional global stability of the jet in crossflow”, Internal report

Paper 4. BAGHERI S. AND HANIFI A., 2007

“The stabilizing effect of streaks on TS-waves: A parametric study”, Published in Physics of Fluids, **19**, 078103

Division of work between authors

The research project was initiated by Prof. Dan Henningson (DH) who also acted as supervisor. Co-advisors were Dr. Luca Brandt (LB) and Dr. Philipp Schlatter (PS). Collaborates were Prof. Peter Schmid (PJS), Dr. Ardeshir Hanifi (AH) and Dr. Jérôme Hoëpffner (JH).

Paper 1

The code development and calculations were done by SB with feedback from JH. Most of the paper was written by SB with input from PJS, JH and DH. JH wrote the “Stochastic forcing” section, PS wrote parts of the “Input-output behavior” section and the introduction was written by PJS and DH.

Paper 2

The control and model reduction algorithms were implemented by SB with feedback from LB. The computations were done by SB. The mathematical formulation in the paper was done by SB and DH. The paper was written by SB and LB with input from DH.

Paper 3

The eigenvalue solver and the jet in crossflow were implemented by SB with feedback from PS. The simulations were done by SB. The paper was written by SB and PS with input from DH.

Paper 4

The streaks were implemented by SB in a code developed at FOI with feedback from AH. The parametric studies were done by SB. The paper was written by SB and AH.

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Part I

Introduction

CHAPTER 1

Introduction

The combined efforts of scientists and engineers in fluid mechanics have strongly contributed to milestones in technological developments. However, some of these technological successes, such as airplanes and spacecrafts, are also contributing to global warming and the draining of earth's limited resources. For example, the worldwide shipping consumes about 2.1 billion barrels of oil per year (Corbett & Koehler 2003) whereas the airline industry consumes about 1.5 billion barrels per year (Kim & Bewley 2007).

If the existing technological solutions can be improved, it can help to reduce the world's oil consumption and preserve the earth's resources. Engineers have used the physical principles of fluid mechanics, established by scientists in last two centuries, to increase the efficiency of their applications. At the same time a mathematically well-established field, systems and control theory, has emerged which given a set of constraints and an objective, it provides the "best solution". The incorporation of systematic methods from systems and control theory in fluid mechanics can make a significant difference in efficiency of various applications.

However, the complexity of the governing equations of fluid mechanics have until recently hindered the use of many of these methods on a full scale in applications. The Navier–Stokes equations consist of a four-dimensional nonlinear partial differential equation (PDE). Since there exist analytical solutions only for a few flow configurations, these equations are approximated numerically giving rise to well above one million ordinary differential equations. In systems and control theory the most elegant results require the solution of various matrix equations, such as the Riccati or Lyapunov equations. Even with the use of supercomputers it is prohibitively expensive to solve these equations for large systems.

The present thesis is part of a long-term project with the aim of applying stability, systems and control theoretical tools to systems of very large dimension arising from various fluid dynamical situations. The most fundamental tool is a Navier-Stokes solver, which given a flow field at certain time, the solver provides a field at a later time.

Theoretical background

2.1. Hydrodynamic stability theory

The incompressible Navier–Stokes equations given by

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u}, \quad (2.1a)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (2.1b)$$

$$\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0, \quad (2.1c)$$

govern the evolution of the flow field $\mathbf{u}(\mathbf{x}, t) = [u, v, w]^T$ and pressure field $p(\mathbf{x}, t)$ in space $\mathbf{x} = (x, y, z)$ and time t . The equations are non-dimensionalized with the characteristic velocity scale U , the length scale L and the kinematic viscosity ν . The Reynolds number is defined as $\text{Re} = UL/\nu$.

In general, hydrodynamic stability theory is concerned with characterizing the behavior of infinitesimal disturbances \mathbf{u}' to a base flow \mathbf{U} , which is a steady solution to Navier–Stokes equations (2.1). The governing equations of these disturbances are found by inserting $\mathbf{u} = \mathbf{U} + \epsilon \mathbf{u}'$ and $p = P + \epsilon p'$, where p' is the pressure perturbations, into (2.1) and neglecting the terms of order ϵ^2 . The resulting linearized Navier–Stokes equations are,

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{U} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u}, \quad (2.2a)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (2.2b)$$

$$\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0, \quad (2.2c)$$

where the superscript $'$ of the disturbance fields has been omitted. The base flow in our case is a steady solution to (2.1), but it can also be a time-periodic solution or a time-averaged turbulent flow. To solve equations (2.1) and (2.2) various boundary conditions depending the physical domain can be imposed.

If the PDE (2.2) is discretized in space and projected on a divergence-free subspace it can be approximated by the initial-value problem

$$\frac{\partial \mathbf{u}}{\partial t} = A \mathbf{u} \quad (2.3a)$$

$$\mathbf{u}(0) = \mathbf{u}_0, \quad (2.3b)$$

where $\mathbf{u}(t)$ now denotes the state vector, comprising the divergence-free amplitude functions of the perturbations. The discretized and linearized Navier–Stokes equations including boundary conditions are represented by the action

of the matrix A on \mathbf{u} . The solution to (2.3) is given by

$$\mathbf{u}(t) = e^{At}\mathbf{u}_0. \quad (2.4)$$

The matrix exponential is the key to stability analysis and also to input-output analysis and control design discussed in subsequent sections. However, this function also poses the greatest computational challenge due its dimension. The dimension of the linearized operator depends on the number of non-homogeneous spatial directions of the base flow. In table 2.1 we list the base flows studied in this thesis. They are all spatially developing, *i.e.* the direction which the disturbances travel in is inhomogeneous. We observe that the dimension of A for flows with two or more inhomogeneous directions becomes prohibitively large to allow for an evaluation of the exponential matrix.

Except for one-dimensional base flows the exponential matrix must be approximated. The most common methods (Molder & Van Loan 2003) require that all elements of the matrix can be stored in memory. For fluid systems this requirement cannot always be met. Instead, the recognition that the action of e^{At} simply represents integrating the Navier-Stokes equations in time, the exponential matrix can be approximated with a direct numerical simulation (DNS) code, also referred to as a time-stepper. In what follows the reader should equate $e^{A(t+T)}\mathbf{u}(t)$ with a DNS simulation starting with an initial condition $\mathbf{u}(t)$ and providing $\mathbf{u}(t+T)$. In this so called ‘‘timestepper approach’’, matrices are never stored and storage demands in memory are of the same order as a small number of flow fields.

As time tends to infinity the disturbance approaches the least stable eigenmodes, ϕ_i , of A ,

$$A\phi_i = \lambda_i\phi_i. \quad (2.5)$$

Even with iterative methods it is in general not possible to explicitly solve the above eigenvalue problem, since A cannot be stored in memory. Instead we make use of our DNS code and a time-stepper technique (Barkley *et al.* 2002) by noting that the eigenmodes are invariant under the transformation e^{At} (for fixed time t),

$$e^{At}\phi_i = \sigma_i\phi_i, \quad |\sigma_1| > \dots > |\sigma_n|. \quad (2.6)$$

The asymptotic stability of disturbances as $t \rightarrow \infty$ is determined by the largest magnitude of σ_1 ,

$$|\sigma_1| > 1 \quad \text{asymptotically unstable,} \quad (2.7a)$$

$$|\sigma_1| \leq 1 \quad \text{asymptotically stable.} \quad (2.7b)$$

The eigenvalues of A can be recovered from $\lambda_i = \log(\sigma_i)/t$ in order to obtain the growth rate and frequency of the associated eigenmode ϕ_i .

For many open shear flows where fluid is continuously entering and leaving the physical domain the matrix A is non-normal (Trefethen & Embree 2005)

	Base Flow	Inhomogeneous direction(s)	Dimension of $\mathbf{u}(t)$	Storage of A
Ginzburg-Landau	$\mathbf{U}(x)$	1D	10^2	1 MB
Blasius	$\mathbf{U}(x, y)$	2D	10^5	25 GB
Jet in crossflow	$\mathbf{U}(x, y, z)$	3D	10^7	500 TB

TABLE 2.1. Spatially developing flows investigated in this thesis.

($AA^* \neq A^*A$). Here, the superscript $*$ denotes the complex-conjugate operation¹. As a consequence, the disturbance can experience large transient energy growth, although all eigenvalues λ_i have negative real parts. The amplification of the initial disturbance \mathbf{u}_0 is then given by,

$$G(t) = \|e^{At}\mathbf{u}_0\|^2. \quad (2.8)$$

In particular, if there is an initial condition (with unit norm) that results in the *maximum* energy amplification, this function ϕ_i^o is an eigenmode of $e^{A^*t}e^{At}$ corresponding to the largest eigenvalue of

$$e^{A^*t}e^{At}\phi_i^o = \sigma_i^o\phi_i^o, \quad \sigma_1^o \geq \dots \geq \sigma_n^o \geq 0. \quad (2.9)$$

The condition for a short-time energy amplification becomes

$$\sigma_1^o > 1 \quad \text{transient growth,} \quad (2.10a)$$

$$\sigma_1^o \leq 1 \quad \text{no transient growth.} \quad (2.10b)$$

The matrix exponential e^{A^*t} is approximated by solving the adjoint Navier-Stokes equations numerically using an adjoint time-stepper.

The most common approach to modern linear stability theory is the quest for eigenmodes of A and A^*A , referred to as global modes and optimal disturbances respectively. The calculation of these eigenmodes is computationally tractable for very large systems using time-steppers (Barkley *et al.* 2002) in combination with Krylov subspace methods. In Paper 3 the global eigenmodes of jet in crossflow are computed using the Arnoldi method (Trefethen & Bau 1997). In Paper 4 optimal disturbances for the Blasius boundary layer are computed using power iterations (Andersson *et al.* 1999).

2.2. Linear systems theory

Under realistic conditions the flow system is continuously forced with external disturbances and entire instantaneous velocity fields are not available for analysis. The natural extension to the previous section is to include inputs and outputs,

$$\begin{aligned} \dot{\mathbf{u}} &= A\mathbf{u} + Bw, \\ z &= C\mathbf{u}. \end{aligned} \quad (2.11)$$

¹We assume for simplicity that the norm is defined as $\|\mathbf{u}\|^2 = \mathbf{u}^*\mathbf{u}$. See Paper 1 and 2 for other definitions.

The column vector B and the row vector C govern the type and location of the input $w(t)$ and output $z(t)$, respectively. In the context of aerodynamic flows, the input can represent the effects of free-stream turbulence, wall roughness or impingement acoustic waves and the output can represent measurements of pressure or friction at the boundaries of the flow domain.

For a stable system the equations (2.11) have the formal solution

$$z(t) = C \int_0^t e^{A(t-\tau)} B w(\tau) d\tau \quad (2.12)$$

where $\mathbf{u}_0 = 0$. For input-output analysis it is useful to define a mapping from past inputs to future outputs,

$$(Hw)(t) = C \int_0^\infty e^{A(t+\tau)} B w(\tau) d\tau. \quad (2.13)$$

Notice that if the input $u(t) = w(-t)$ for $t < 0$ then the output for $t \rightarrow \infty$ will be $z(t) = (Hw)(t)$ (Glover 1999).

Linear systems theory is concerned with the response behavior of the output signal to various input signals. The amplification of the output signal at a certain time is given by $\|Hw\|^2$. In particular, the largest output response is given by the input w_i corresponding to the largest eigenvalue of H^*H ,

$$H^*Hw_i = (\sigma_i^b)^2 w_i, \quad \sigma_1^b \geq \dots \sigma_n^b \geq 0 \quad (2.14)$$

where σ_i^b are called the Hankel singular values. For an input with unit norm the Hankel singular value gives a measure of how much the output is amplified,

$$\sigma_1^b > 1 \quad \text{output amplification,} \quad (2.15a)$$

$$\sigma_1^b \leq 1 \quad \text{no output amplification.} \quad (2.15b)$$

One can associate a sequence of modes, referred to as balanced modes with the sequence of inputs w_i ,

$$\phi_i^b = \int_0^\infty e^{At} B w_i. \quad (2.16)$$

If a particular balanced mode ϕ_i^b has an associated Hankel singular value which is zero, $\sigma_i = 0$, this mode does not influence the input-output behavior. This has led to an efficient method of model reduction where the input-output behavior is preserved, called balanced truncation (Moore 1981). Balanced truncation is based on the idea of reducing the dimensions of the original system by removing the redundant states – *i.e.* balanced modes corresponding to $\sigma_i^b = 0$ – and also, in addition, removing the states that have a very weak influence on the input-output behavior, *i.e.* ϕ_i^b corresponding to $\sigma_i^b \ll 1$.

In Paper 1 and Paper 2 a time-stepper approach (Rowley 2005) is used to compute the balanced modes for the Ginzburg-Landau equation and the Blasius boundary layer. It is shown that a few balanced modes can preserve the input-output behavior of the original high-dimensional system.

2.3. Linear control theory

The next step after the analysis of the amplification behavior of a linear system to initial conditions and external excitations is to manipulate the inherent dynamics of a system or to control it. In fact the linear system written in form (2.11) is the starting point for control. One additional input representing the actuator and one additional output representing a sensor result in the system

$$\dot{\mathbf{u}} = A\mathbf{u} + B_1w + B_2f, \quad (2.17a)$$

$$z = C_1\mathbf{u} + f, \quad (2.17b)$$

$$y = C_2\mathbf{u} + g. \quad (2.17c)$$

Note that the outputs are also forced. The first output z can be regarded as the objective function,

$$\|z\|^2 = \|C_1\mathbf{u}\|^2 + \|f\|^2 = \int_0^T (\mathbf{u}^* C_1^* C_1 \mathbf{u} + f^* f) dt, \quad (2.18)$$

where it is assumed that the cross weighting between the state and control signal is zero (Zhou *et al.* 1999). The second output is forced with noise g to model the uncertainty that may exist in the measurements under realistic conditions. The so called \mathcal{H}_2 control problem can be formulated as following:

Find an optimal control signal $f(t)$ based on the measurements $y(t)$ such that the influence of the external disturbances $w(t)$ and measurement noise $g(t)$ on the output $z(t)$ is minimized.

The solution to this control problem is obtained by solving two quadratic matrix equations called the Riccati equations (Zhou *et al.* 1999). However, these equations require the linearized Navier-Stokes operator A which we do not have at our disposal in an explicit form. On the other hand, we can note that the \mathcal{H}_2 control design process amounts to the determination of control signal f given the output signal y . Therefore, it is sufficient to capture only a small fraction of the dynamics, namely the relationship between the input and output signals to design an optimal controller. It thus seems prudent to replace the large matrix A in the Riccati equations with a reduced-order matrix \hat{A} obtained by the projection of equations (2.17) onto the balanced modes. Once the control signal is obtained using the reduced-order model it is applied to full Navier-Stokes system.

The solution to the \mathcal{H}_2 in a stochastic framework (also known as Linear Quadratic Gaussian) is derived in Paper 1 and applied to Blasius flow in Paper 2 using a reduced-order model.

Numerical codes

3.1. Direct numerical simulations

The system of partial differential equations given in expression (2.2) for boundary-layer flows is solved numerically using spectral methods. The simulation code is described in detail by Chevalier *et al.* (2007) and employed for the simulations presented in Paper 2 and Paper 3. The spatial discretization is based on Fourier expansion in the streamwise and spanwise directions, and an expansion in Chebyshev polynomials in the wall-normal direction. The time is advanced using a four-step low-storage third-order Runge-Kutta method for the nonlinear and forcing terms, and a second-order Crank-Nicholson method for the linear terms. The code is fully parallelized for efficient use on both shared and distributed-memory systems. To retain periodic boundary conditions, which is necessary for the Fourier discretization, a fringe region is added at the end of the computational domain where a forcing is applied so that the flow smoothly changes from the outflow velocity of the physical domain to the desired inflow velocity (Bertolotti *et al.* 1992; Nordström *et al.* 1999).

3.2. Parabolized stability equations

An alternative for solving the Navier-Stokes equations in weakly spatially developing flows is the parabolized stability equations (PSE) (Herbert 1993; Bertolotti *et al.* 1992). They are based on the expansion of the disturbances into Fourier modes in the horizontal directions. However, in the streamwise direction every mode is decomposed into a slowly varying amplitude function and a wave function with slowly varying wave number. The neglect of the small second derivatives of the slowly varying functions with respect to the streamwise variable leads to an initial boundary-value problem that can be solved by numerical marching procedures.

The PSE approach is a relative fast computational method compared to DNS and is employed in Paper 4 of this thesis. The results presented in Paper 4 are calculated using the NOLOT code, developed by Hanifi *et al.* (1995). A fourth-order compact difference scheme is used to approximate the derivative with respect to the wall-normal coordinate. The derivative with respect to the streamwise coordinate is approximated by a first or second order backward Euler finite difference schemes. The nonlinear terms appear as a source on the right hand side of the equations. The calculations start with fundamental

modes initialized only. The higher modes are introduced in the calculations when the corresponding forcing is larger than a predefined threshold.

Summary of papers

Paper 1

Input-output analysis and control design applied to a linear model of spatially developing flows.

This paper presents a review of recent developments in stability, systems and control theory for a linear model of spatially developing flows. The review covers a wide variety of topics, including transient growth of non-normal systems, convective and absolute instability, global modes, linear input-output systems, model reduction, and optimal/robust design of controllers and observers. The concepts are demonstrated on a single canonical problem, the complex Ginzburg-Landau equation, in order to elucidate the theory presented. The complex Ginzburg-Landau equation is an amplitude equation which arises in the context of non-equilibrium systems and is often used to describe the dynamics near the onset of instability (Chomaz 2005). Here, a linearized version of this model is used to mimic the linearized Navier-Stokes equations. The review is divided into four parts. First, the solution of the linear system (2.4) is investigated, where the stability properties of the matrix exponential e^{At} , in terms of global eigenmodes and optimal disturbances are analyzed. The second part deals with the forced solution (2.12), where the notions of controllability and observability are introduced and the response of the system to impulse, harmonic and stochastic forcing is investigated. In the third section, model reduction techniques based on projection of the linear system on an appropriate subspace is discussed. In the last part of the review, the control design of spatially developing flows is reviewed within the $\mathcal{H}_2/\mathcal{H}_\infty$ -framework.

Paper 2

Input-output analysis, model reduction and control of the flat-plate boundary layer.

This paper considers the model reduction and control design of the flat-plate boundary layer from an input-output viewpoint. The linearized Navier-Stokes equations are written in the standard state-space form (2.17). The inputs represent external disturbances, measurement noise and actuators and the outputs represent sensors and objective functions. Using matrix-free methods, such as the snapshot method, the most controllable (or energetic) and observable modes of the linear system are computed and analyzed. For the given inputs and outputs it is found that the observable modes are located upstream in the

physical domain, where the sensitivity to forcing is the largest. The controllable modes are on the other hand located downstream in the domain where the response to forcing is the largest. These two sets of modes can be combined in order to obtain the balanced modes. These modes are computed and used for projection basis of the Navier-Stokes equations in order to construct a reduced-order model. It is shown that the reduced-order model is able to reproduce the input-output behavior of the flat-plate boundary layer with few degrees of freedom. An optimal controller within the \mathcal{H}_2 framework is designed using the reduced system and applied to the full Navier-Stokes equations. The closed-loop behavior is significantly different compared to the uncontrolled Navier-Stokes equations. The most amplified frequencies of the latter system are efficiently damped by the control.

Paper 3

The three-dimensional global stability of the jet in crossflow.

In Paper 3 the global linear stability analysis of the jet in crossflow to three-dimensional perturbations is numerically investigated. At a velocity ratio $R = 3$, defined as the ratio of jet velocity to free-stream velocity, the flow is found to be globally linearly unstable. In this case, the temporal frequency of the most unstable global mode is in good agreement with the dominant intrinsic frequency associated with the jet shear-layer vortices observed in direct numerical simulation (DNS). In the DNS code described in section 3.1 the jet is enforced as a boundary condition with parabolic velocity distribution. Shear-layer vortices are continuously shed along the jet trajectory with a well-defined frequency. The base flow for the stability analysis is a steady solution of Navier-Stokes, obtained by damping the unstable temporal frequencies using the selective frequency damping method (Åkervik *et al.* 2006). The steady state consists of a dominant counter-rotating vortex pair in the far field emerging from the near field vorticity of the shear layer. The large eigenvalue problem is solved using the ARPACK library (Lehoucq *et al.* 1998) and the linearized DNS as a time stepper. The most unstable mode takes the shape of a localized wavepacket, wrapped around the counter-rotating vortex pair.

Paper 4

The stabilizing effect of streaks on TS-waves: A parametric study.

In Paper 4, the Parabolized Stability Equations described in section 3.2 are modified to account for the algebraic growth of streamwise elongated vortices called streaks. Using these equations, the nonlinear interaction of TS waves and steady streamwise streaks, and the stabilizing effect of the streaks on the mean flow is verified with previous DNS results (Cossu & Brandt 2002). The amplification of the TS waves is calculated in the presence of a set of streaks with varying spanwise wave numbers and fixed maximum streak amplitudes. In this case, it is found that the optimal stabilization effect is obtained for streaks with the location of the maximum amplitude close to neutral point (branch I)

of the TS wave. These streaks generate the largest total mean flow excess in the unstable streamwise region of the TS waves.

Outlook

Flow control based on systematic methods adopted from control theory is becoming a fairly mature field. Model reduction plays an important role in developing effective control strategies for practical applications, since the dynamical systems which describe most flows are discretized partial differential equations with a very large number of degrees of freedom. Balanced truncation was applied to two-dimensional disturbances in the flat-plate boundary layer in Paper 2. The model reduction procedure will be extended to more general three-dimensional disturbances in the flat-plate layer and also to other more complex geometries. The method is applicable primarily to linear systems, although there is an extensive theory on nonlinear balanced truncation (Scherpen 1993). One method of balanced truncation for nonlinear systems has been introduced by Lall *et al.* (2002), but the method is considerably more expensive than the linear method applied in Paper 2. In the future possible extension of the nonlinear method will be examined in order to make it more computationally feasible.

Stability analysis has until recently only been constrained to the most simple flows due its large memory requirements. DNS on the other hand has been performed for various complex three-dimensional flows such as ducts, diffusers, parabolic leading edge *etc.* In Paper 3, the timestepping-technique for stability analysis employed for fully 3D jet in crossflow has a cost which is comparable to direct numerical simulations. The computation of steady-states based on simple filtering techniques instead of Newton iterations is also an important factor that has enabled the extension of stability analysis to complex flows. In the future, the jet in crossflow for higher velocity ratios will be considered, in order to explore the presence of a critical velocity ratio for global instability. Recently (Barkley *et al.* 2008) the timestepper approach has been extended to optimal transient growth analysis. If a convectively unstable configuration of the jet in crossflow is found, a direct optimal growth analysis will be performed using an adjoint simulation code.

Acknowledgements

I would like to thank my supervisor Professor Dan Henningson for his guidance and generosity during these two years. Many thanks to my co-advisors Dr. Philipp Schlatter and Dr. Luca Brandt for sharing their knowledge with me. I have learned a lot from them. Thanks to my collaborators Prof. Peter Schmid, Dr. Ardeshir Hanifi, Dr. Jérôme Hœpffner, Espen Åkervik and Milos Ilak for all the discussions. I appreciate the helpful comments of Fredrik Lundell and Johan Ohlsson on this manuscript. Thanks to my roommates Lars-Uve Schrader and David Tempelmann and the others in the group Qiang Le, Antonios Monokrousos and Stefan Ivanell for making the working atmosphere so friendly. The Swedish Research council (VR) is gratefully acknowledged for the financing the project.

Finally, I like to thank my family for all their support and especially Sara for her love and encouragement.

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