Enhancing ESG-Risk Modelling

A study of the dependence structure of sustainable investing

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The interest in sustainable investing has increased significantly during recent years. Asset managers and institutional investors are urged to invest more sustainable from their stakeholders, reducing their investment universe. This thesis has found that sustainable investments have a different linear dependence structure compared to the regional markets in Europe and North America, but not in Asia-Pacific. However, the largest drawdowns of an sustainable compliant portfolio has historically been lower compared to the a random market portfolio, especially in Europe and North America.

**Keywords:** ESG, Sustainable Investing, Dependency Structure, Correlation, Risk, Random Matrix Theory, Eigenvalue, Eigenvalue Decomposition, Minimum Variance Portfolio
SAMMANFATTNING

Intresset för hållbara investeringar har ökat avsevärt de senaste åren. Fondförvaltare och instutitionella investerare är, från deras intressenter, manade att investera mer hållbart vilket minskar förvaltarnas investeringsuniversum. Denna uppsats har funnit att hållbara investeringar har en beroendestructur som är skild från de regionala marknaderna i Europa och Nordamerika, men inte för Asien-Stillahavsregionen. De största værdeminskningarna i en hållbar portfölj har historiskt varit mindre än værdeminskningarna från en slumpmässig marknadsporftölj, framförallt i Europa och Nordamerika.

**Nyckelord:** ESG, Sustainable Investing, Dependency Structure, Correlation, Risk, Random Matrix Theory, Eigenvalue, Eigenvalue Decomposition, Minimum Variance Portfolio
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Stockholm, January 2020

Edvin Berg & Karl Lange
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Part I

INTRODUCTION TO THE ANALYSIS
INTRODUCTION

In this section, a description of the background for the thesis is presented. The background will lead to the problem statement, which then will be followed by the aim of the thesis and a definition of the scope. Lastly, the disposition of the thesis will be presented to provide the reader with an overview of the thesis’ structure.

1.1 BACKGROUND

The interest for Environmental, Social and Governance (‘ESG’) investments has seen a surge in the past couple of years and has experienced a global momentum recently [1]. Following this momentum, there are now several asset managers using ESG as a criteria in their investment decision process. The underlying cause for the growth in ESG could be explained by the global focus on sustainability as well as risk aversion for asset managers. The global focus on sustainability has put pressure on asset managers from their stakeholders to implement an ESG-tilt in their portfolios. Furthermore, recent studies have shown that the traded stocks of ESG leading companies have a lower systematic risk than their non-ESG leading counterparty [2, 3].

A white paper published by Deutsche Bank noted that current research has found some correlation on the risk-adjusted returns of ESG investments. Furthermore, the paper states that the current research need to put more focus on finding out if there exist strict causality in their findings [2]. However, a more recent article published in July 2019 investigated the causality of the risk-adjusted returns of ESG investments and found that an upgrade in ESG rating indeed should increase the valuation of the underlying company and thus increase the risk-adjusted return for said company [4]. This was derived by investigating the cost of capital of a company that has seen an upgrade in their ESG-rating, using the beta as a proxy for cost of capital. The authors found that the beta was decreased after an upgrade in ESG-rating, which would lead to a lower cost of capital and in turn a higher valuation.

The ESG-rating is a measure of how well a company performs in the three substitute factors for sustainability: Environmental, Social and Governance. Each company is given a score for each factor by a rating institution and then an aggregated score that measures how well the company complies with the ESG factors. The two rating agencies which are the most widely used in the industry are MSCI (Morgan Stanley Capital International) and Sustainalytics (Morningstar). The rating institutions uses both quantitative and qualitative methods to assign each company a score in all underlying ESG factors. One or several analyst employed by the rating agencies are behind each company ESG-rating, making the rating prone to human error and bias. This bias can be seen by comparing ESG scores from the two different vendors, MSCI and Sustainalytics, where a recent study found that the correlation of the rating scores for the leaders between MSCI and Sustainalytics’ ESG scores were only 0.53 [5].
Furthermore, the ESG-metric can be difficult to translate between different sectors. Each rating agency tries to amend this problem by assigning different weights to the three factors depending on the sector in which the company operates, to better measure the true sustainability of the company. Each of the E, S and G factors have a direct connection to the risks of an investment. In an article published in 2017, the authors give a few examples of the risks associated with each dimension of the ESG score: "[...], a firm that produces high levels of emissions during a manufacturing process may be exposed to potential future legislation that might impose a carbon tax; a firm poorly treating its employees or suppliers may face a backlash from its consumers and sees its sales plummet; a firm with poor governance may get involved in a scandal that ultimately causes its downfall." [6].

However, the growing demand for ESG investment and the pressure on asset managers from stakeholders to focus on sustainable investments has made the investment decision process more complex than before. Asset managers need to decide how to address sustainable investments, what factors they should look at and how are they supposed to implement these into their investment strategies. This is a complex problem in its nature, and several aspects need to be investigated in order to maintain a stable risk-adjusted return. To investigate this, models need to be expanded and the dynamic of ESG investments need to be established and analysed to support asset managers with the correct foundation for fair ESG investment decisions.

1.2 RESEARCH QUESTION

The increased interest in ESG investments and the intense influx of money to ESG funds has potentially increased the valuation of ESG stocks. The valuation risk may stem from a concentration risk, where asset managers and institutional investors are under pressure from their shareholders to invest in ESG leading companies and sell their holdings with a low ESG score. If the institutional investors has to invest solely in companies with an adequate ESG rating, their investment universe decreases. As a result, there could potentially arise a discrepancy in the dependence structure of ESG leading investments. To be able to test if this is actually true, the following research question aims to be answered and discussed:

- Is there a different dependency structure amongst ESG investments compared to the market?

Furthermore, this gives rise to several questions that need to be answered before the research questions can be evaluated:

- Is there a different correlation between ESG investments compared to the market that is driven by the underlying ESG score?

- Is the tail dependence for ESG investments different from the tail dependence for the underlying companies in a market portfolio?

- How can the result be interpreted and how should they be compared?
1.3 AIM AND DEMARCATION

This thesis aims to answer the question if the dependency structure amongst ESG investments are different compared to the market. Furthermore, this thesis will try to help asset managers to understand the behaviour of ESG investments and if there are certain aspects the asset managers should consider if they want to enforce an ESG-tilt in their portfolio as requested from their stakeholders. The methods to find and define a different dependency structure will be expressed later in the thesis. The ultimate aim of the thesis is to assist Skandia Investment Management ('Skandia') on defining the behavior of ESG investments. Furthermore, this thesis and the models developed to answer the research questions will guide Skandia in their future investment decision process for ESG investing. However, the main focus of the thesis is the exploration of the dependency structure amongst ESG investments compared to the market and not how asset managers should implement ESG into investment decisions. The results are however interpreted and potential implementation strategies are discussed to interpret the results found in this thesis.

Regarding the demarcation and the scope of the thesis, the primary subject that is analysed and covered are stocks in the global equity market. To get a reasonable size and structure of the project, and to add as much value as possible for Skandia, this thesis will not investigate the bond market and e.g. green bonds which is a cornerstone of sustainable investing. The bond market in the Nordics is not as liquid as the stock market and as a result it is more beneficial to investigate the stock market for this specific research question.

1.4 THESIS DISPOSITION

This thesis is divided into three main chapters: Introduction to the analysis, Understanding of the principles and Results and discussion. The first chapter, Introduction to the analysis, commence with an introduction which is followed by a literature review where previous research on ESG investing, portfolio theory, correlations, behavioral finance, co-movement in the financial market and random matrix theory are covered.

The second chapter, understanding of principles, will commence with a theoretical background in which the fundamental ideas of Sustainalytics' ESG metric and the Global Industry Classification Standard will be presented. The theoretical background continues with a mathematical background of dependency structure, dependency structure analysis, bootstrapping and portfolio optimisation. The chapter continues with a methodology section where the structure and methods of the analysis are described.

Finally, the results and discussion chapter have three sections; results, discussion and conclusion. In the result section the most important findings are presented in tables and graphs, followed by a discussion section where the results and implications are discussed. Lastly, the thesis is concluded and areas of future research are suggested. The thesis is thereafter finalised by a list of references.
In this section, previous research regarding subjects of relevance for the thesis are presented. Most of the research on the subject of ESG investments has been done in recent years since company specific ESG ratings is a fairly new phenomena. As a result, not much research has been done on the dependence structure of ESG investments. This section will be divided into five parts. The review commences by focusing on ESG investments in general, followed by portfolio theory, correlation, the (in)efficient market hypothesis, co-movement in financial markets and ends with a review on random matrix theory.

2.1 ESG

Previous research has highlighted that companies with good ESG risk ratings (henceforth "ESG leaders") have a lower systematic risk, often derived from a lower cost of capital for ESG leaders compared to the aggregated market [3, 6, 7]. To find the effects of the ESG risk rating in valuation, it is important to differentiate between correlation and causality in order to determine if the ESG factors do provide a lower risk profile. In a study from 2018, the authors proved that there exists a correlation between ESG score and valuation by examining three transmission channels within a standard discounted cash-flow model [7]. Furthermore, the same study proved that the ESG score was transmitted into both their valuation and performance through their systematic risk profile. However, a recent meta study found that "most studies find correlation rather than specifically trying to find causality." [2].

Moreover, since the valuations of ESG leaders are generally higher, the potential valuation risks should be considered when making investment decisions in ESG leaders [6, 8, 9]. A recent study concluded that the risks in ESG leading companies are different from the aggregated market since the ESG rating agencies can affect the underlying valuation of a company by changing the ESG risk rating [9]. Other research papers have found that the ESG metric, when applied in a Fama-French factoring investment model, can be redundant and impair the performance of the portfolio since much of the effect of the ESG metric already is captured by the existing factors in a Fama-French model [10].

2.2 PORTFOLIO THEORY

Portfolio optimisation is a widely researched subject, with the modern portfolio theory rooting back to 1952, mainly focusing on the trade-off between risk (variance) and return of portfolios [11]. This is an extensively established theory, both in the academic and industrial field, seen in the number of citations on google scholar, almost reaching 40,000 [12]. This theory posed by Markowitz has laid the foundation to other fundamental capital market theories. For example, Sharpe uses the results and further develops the theories from
Markowitz' portfolio optimisation theory. This was used when he developed the Capital Asset Pricing Model 'CAPM' which has had a great impact in financial theory [13].

However, Fama et. al noted in a study from 1992 that the CAPM posed by Sharpe has several contradiction when assessing the cross-section expected common stock return [14]. In this study, Fama et. al found that there are three variables explaining the expected return of a stock, the market beta, similar to CAPM, and the two additional factors, small [market capitalisation] minus big ('SMB') and high [book-to-market ratio] minus low ('HML').

Furthermore, in 1993 Fama et. al. expanded this theory and proved that this can be used for both stocks and bonds [15]. Furthermore, they used time-series regression instead of cross-section regression to prove this. In their study from 2004, Fama et. al. concludes that "The version of the CAPM developed by Sharpe (1964) and Lintner (1965) has never been an empirical success." [16].

Finally, Fama et. al. expanded their three-factor model even further after their findings that "the [three-factor model] is an incomplete model for expected return because its three factors probably do not capture the relations between expected return and expected profitability and investment." [17]. This is based on their findings that the SMB factor is a "noisy proxy for expected return because the market value of the stock also reflects forecasts of profitability and investment". Therefore, they expand their three-factor model to a five-factor model by adding profitability and investment factors. In the same article, Fama et. al. notes that it is a possibility to add a momentum factor to the model, but states that the correlation among the variables are likely to result in poor diversification. Both the three- and five-factor model are widely used in the industry and several research papers has investigated if other factors, such as an ESG-[10], a momentum-[18] or a volatility factor[19] is explanatory for the performance of capital markets assets.

A current working paper investigated the decomposed effect of ESG in a factor investment model and concluded that the E factor proved to have the largest explanatory power of market anomalies among E, S and G. Furthermore, the paper investigated how the market prices ESG risk and concluded that the price of risk is negative close to -0.2% monthly. This could explain why ESG investments, with emphasis on the E dimension, shows lower volatility and low beta anomalies [8].

2.3 Correlation

Modern portfolio theory, presented in Section 2.2, uses correlation to estimate the risk in a portfolio. A paper from 1995 found that the correlation matrices in international equity indices that returns were unstable over time [20]. Furthermore, the authors found that the correlations rises in times of higher volatility. This study used time-series regression models to investigate the correlation matrices, in which they found that these models capture some but not all of the correlation structure. Furthermore, they conclude that 'the methodology in this paper could be a useful basis for a more detailed study of the international integration of financial markets. However such conclusions cannot be reached by looking at the correlations alone and an international asset pricing model must be explicitly used.'
In an updated article, published in 2001, the authors further developed the studies and proved that the correlation do increase in bear markets but not in bull markets [21]. A study from 2013 found two different dynamics in the correlation structure of industrial indices in U.S. equity market. One slow dynamic with a time dynamic longer than 5 year and one short with a time dynamic shorter than 3 months. They also found two examples of fast variations in the correlation structure, the dot-com bubble (1999-2001) and the subprime crises (2008-2009) [22].

2.4 THE (IN)EFFICIENT MARKET HYPOTHESIS

The efficient market hypothesis (EMH) was presented by Fama in the 1960 and states that the current asset prices reflect all available information in the market [23]. The EMH has been subject to criticism for not explaining market anomalies [24]. The anomalies can partly be explained by behavioral finance models. In an influential paper by Schafenstein et. al. the authors modeled herd behaviour as an explanation of the observed anomalies in the financial markets [25].

2.5 CO-MOVEMENT IN FINANCIAL MARKETS

In Section 2.4 the concept of herd behaviour was presented which is an example of a co-movement model in the financial markets. In Section 2.3 theories were presented that states that correlations rises in bear markets but not in bull markets. That is also an example of a model of co-movement in the financial markets. Since then, new research on the topic has been published. The amount of co-movement affects the diversification of a portfolio through correlation. However, linear correlation is not a satisfactory measure for correlations in equity markets because of several reasons. Firstly, linear correlations assumes that marginal and joint distributions are elliptical which often is not the case in the financial markets. Second, the linear correlations are not invariant under nonlinear strictly increasing transformations meaning that prices could be uncorrelated whereas returns are correlated or vice versa [26].

2.6 RANDOM MATRIX THEORY

Random matrix theory can be applied to financial models to model noise in large matrices. The theory can be applied in many settings but for the scope of this thesis the focus is on correlation matrices. A random matrix approach to assessing the correlation structure of portfolios rely heavily on mathematical theory which is presented in Section 3.4.1. The theory can be used to construct portfolios with a stable risk-return ratio by distinguishing real correlations from apparent correlations which could be modeled in a random matrix [27]. Random matrix theory is an important cornerstone in a wider mathematical framework that enables correlation modelling by investigating the eigenvalues and distinguishing noise from information by a filtering process, and modelling them in different ways. A study on several markets modeled different portfolios and tested the results to a random matrix in order to differentiate significant results from random noise and also found that a market portfolio or a fund that is aimed at following the market movement, i.e. a index fund, should be
modeled on the market-correlation portfolio rather than weighting by market cap [28]. If the filtering process is not to be conducted the results of the weights of the portfolio would be very unstable and hence make the results unreliable [29].
Part II

UNDERSTANDING OF PRINCIPLES
THEORETICAL BACKGROUND

In this section the frameworks and theories used will be defined and explained thoroughly to understand the analysis of this thesis. The section starts with explanation of Sustainalytics’ ESG metric and their methodology for applying an ESG risk rating to a company, followed by an introduction to the Global Industry Classification Standard. Thereafter a review of the mathematical definitions and theories used to measure and analyse the dependence structure amongst random variables are presented. This is followed by an explanation of bootstrapping and the Section is finalised with a thorough description of portfolio optimisation.

3.1 SUSTAINALYTICS’ ESG METRIC

As the interest for sustainable investments increases the market needs a well-defined measure of a sustainable investment. This phenomenon gave rise to the two rating institutions, MSCI and Sustainalytics as described in Section 1.1. Sustainalytics’ presented a new ESG measure in September 2018 [30]. The new ESG measure is described below and will be used in this thesis. Sustainalytics’ ESG Risk Rating measures 'the degree to which a company’s economic value is at risk driven by ESG factors OR, more technically speaking, the magnitude of a company’s unmanaged ESG risks.' [31]. Furthermore, Sustainalytics divides the companies into five risk categories (negligible-, low-, medium-, high- or severe risk), measuring the unmanaged ESG risk in a company. They also state that one point of risk should be equivalent regardless of which company it applies to, these risks thereafter adds to create an overall score. They define this score as 'a single currency of ESG risks.'.

3.1.1 Sustainalytics’ Two Dimensions of ESG Risk Rating

Sustainalytics calculates two scores, Exposure and Management, described more thoroughly below, in order to create an unmanaged risk score for each material ESG issue. A material ESG issue is an ESG issue that has a material impact on the company’s enterprise value, for example a company’s carbon emission. In turn, this material ESG issue is used to create an overall unmanaged risk score for each company, producing the final output, the ESG Risk Rating score. Based on these, the companies are placed into one of the five ESG risk categories.
### Overall score | Risk | Description
--- | --- | ---
0 – 9.99 | Negligible | Enterprise value is considered to have a **negligible risk** of material financial impacts driven by ESG factors
10 – 19.99 | Low | Enterprise value is considered to have a **low risk** of material financial impacts driven by ESG factors
20 – 29.99 | Medium | Enterprise value is considered to have a **medium risk** of material financial impacts driven by ESG factors
30 – 39.99 | High | Enterprise value is considered to have a **high risk** of material financial impacts driven by ESG factors
≥ 40 | Severe | Enterprise value is considered to have a **severe risk** of material financial impacts driven by ESG factors

Table 1: Sustainalytics’ five ESG risk categories [31]

*Exposure*

Exposure measures the degree of which a company is exposed to material ESG risks, affecting the overall rating for a company for each **material ESG issue**. Hence, the total exposure is described as a set of several issues. Each issue is assigned a weight that contributes to the overall score. An issue with a high exposure will receive a high weight and vice-versa. That is, if an issue is financially material to a company it will weigh more heavily in the balance of a company’s weighting. The exposure score per issue primarily varies between 1-10, but Sustainalytics allow for higher variations in extreme cases. Sustainalytics’ analysts determine each company exposure through the following steps.

1. **Subindustry Exposure Assessment** - Determine exposure of companies operating in a given subindustry, conducted by Sustainalytics’ sector teams

2. **Issue Disabling** - An analyst determines if a specific issue is applicable for a given company. If not, the analyst disables that specific issue

3. **Beta Assessment** - For identified issues, a beta assessment on a company level is made, reflecting the company-specific deviations from the subindustry norm

4. **Issue Exposure Score Calculation** - Exposure score is multiplied by the issue beta to arrive at final exposure score for a company on each material ESG issue

One of the most crucial steps for determining the material ESG exposure for a company is the **beta assessment**. This step makes the ESG Risk Rating company specific and is the tool provided to analysts in order for them to use their own judgment to influence a company’s final rating. Hence, this is also the step that is most prone to human error. The
main objective with the beta assessment is to "determine a company's exposure to an ESG issue relative to its subindustry's exposure to the same issue." [31].

Sustainalytics sets the betas in a range between 0 and 10. A beta of 1 means that the company’s exposure to a certain issue is the same as the subindustry’s exposure to that issue, i.e. the same as the market average. In contrast, a beta < 1 indicates that the systematic risk of one certain issue exposure is lower than the market average and a beta > 1 indicates that the systematic risk is higher than the market average.

Each issue is composed by up to four beta components which are equally weighted and averaged to calculate the quantitative beta for a company. Furthermore, Sustainalytics can apply a qualitative overlay on each quantitative beta for factors that is not included in the quantitative beta to arrive at the final beta of a company.

Management

Management is the second dimension of Sustainalytics' ESG risk rating. Sustainalytics defines this as a set of commitments and actions issued by the company that demonstrates how a company approaches and handles ESG issues. Hence, this dimension is used to indicate if and how companies are managing their ESG risks. The management score is set between [0, 100], with 0 signaling no evidence of management of a certain issue and 100 signals a very strong management of a certain issue.

The management score is derived from management indicators, i.e. policies, certifications, etc. as well as an event indicator. Sustainalytics defines an event indicator as: 'An indicator that provides a signal about a potential failure of management through involvement in controversies. Events have a discounting effect against the company's management score on an issue. An event indicator for a material ESG issue has a management score of 0, and its weight within an issue increases as the event category rises.' [31].

Furthermore, Sustainalytics’ analysts performs the management assessment in the following steps:

1. **Indicator Selection** - Select the management indicators that best signals a company’s management of material ESG issues

2. **Indicator Disabling** - Analyst disables indicators that are not applicable to the certain company

3. **Indicator Weighting** - Analyst weights the indicators according to their significance on the certain company and issue

4. **Indicator Assessment** - Analyst assesses the indicators on the information available for a company

5. **Issue Management Score Calculation** - Final score calculated by aggregating the weighted individual indicator scores
3.1.2 Sustainalytics’ Risk Decomposition

After determining the exposure and management scores, Sustainalytics calculates the unmanaged risk through a risk decomposition to arrive at the final ESG risk rating. Sustainalytics’ final ESG risk rating score is used to determine a company’s unmanaged ESG risk that could have a material financial impact on the enterprise value of a company. The unmanaged ESG risk has two components: unmanageable risk, which cannot be addressed by company initiatives, and the management gap, which is the risks that could be managed by a company but may not yet be managed.

![Sustainalytics’ Risk Decomposition Diagram](image)

Sustainalytics starts at the exposure of the company and thereafter decompose the exposure into various types of risks. Several of the risks are manageable, such as on-the-job injuries which can be managed through promoting a safe culture, whereas some risks are not fully manageable, i.e. carbon emissions of airplanes in flight. Some of the risks can be managed, by e.g. modernising aircrafts, but the companies cannot manage all risks and hence the airline companies have some unmanageable risk on that issue. The other component of the unmanaged risk is the management gap which is the part of the material ESG risks that a company is facing that could be managed by the company but currently is not. The final unmanaged risk score is calculated by Sustainalytics’ analysts by the following steps.

1. **Manageable Risk Assessment** - The share of overall exposure of the material ESG issue that could be managed by a company in a given subindustry

2. **Overall Management Score Assessment** - The degree of which the company has managed their manageable risk based on the management risk assessment

3. **Final Unmanaged Risk Score Calculation** - Subtracting managed risks from a company’s overall exposure on each material ESG issue
3.2 GICS - GLOBAL INDUSTRY CLASSIFICATION STANDARD

The Global Industry Classification Standard (GICS) structure is an industry taxonomy standard developed by MSCI and Standard & Poor’s in 1999 that classifies companies on four levels by sorting them into categories and sub-categories for each level ranging from Sector level which has 11 categories to Sub-Industries with 158 sub-categories. A Sub-Industry is a subset of one and only one Industry, which in turn is a subset of one and only one Industry Group which is finally a subset of one and only one Sector. Hence, if only a company’s Sub-Industry is known, you can determine its Industry, Industry Group and Sector. The hierarchy of the GICS is illustrated in Table 2 below.

![GICS Hierarchy Diagram]

| 11 Sectors | 24 Industry Groups | 69 Industries | 158 Sub-Industries |

Table 2: GICS Hierarchy

3.3 DEPENDENCY STRUCTURE OF RANDOM VARIABLES

In this section, theories to analyse the dependency structure of data and random variables is described. Since this thesis aims to answer the question if ESG investments has a different dependency structure in comparison to the market, it is key to analyse the dependency structure among the underlying company in an ESG portfolio compared to the entire market.

3.3.1 Linear dependence structure

When analysing dependence structures, a natural first step is to analyse the linear dependence. This is done by first looking at the variance, also known as second moment, and thereafter analysing the covariance and correlation of the underlying companies, i.e. the relationship in the second moment.

Variance and covariance

Variance is a central tool in probability theory and statistics which measures the expectation of the squared deviation of a random variable from its mean, also known as the second moment. Since the aim of this thesis is to investigate if ESG investments has a dependence structure that is different to the dependence structure of the market, variance in general and covariance in particular, will be a foundation of this thesis and several tools to investigate the variance and covariance between different sets will be described below.

Covariance is a measure of the joint variability (variance) of two random variables, i.e. covariance is a measure of the linear dependency structure between two random variables, or bi-variate data. The dependency structure can be causal, but the covariance does not indicate causality. However, covariance indicate how the variables relate to each other which can be
used to make predictions from data. In financial markets, asset covariance is something that portfolio managers has to consider when constructing portfolios. However, the covariance is not easy to interpret since it is not normalised and hence depends on the magnitudes of the variance.

**Covariance matrix**

For multivariate data, a covariance matrix $\Sigma$ is a matrix whose element in position $(i, j)$ describes the covariance between the $i$:th and $j$:th variable in a random vector. The covariance matrix is used to describe the entire variance and covariance for a large data set or a multivariate random variable. The following properties always hold for a covariance matrix:

1. $\Sigma$ is positive-semidefinite, i.e. $a^T \Sigma a \geq 0 \forall a \in \mathbb{R}$
2. $\Sigma$ is symmetric, i.e. $\Sigma^T = \Sigma$

**Correlation**

Linear Correlation is a normalisation of covariance. Using a normalised measure instead of covariance is usually preferred since it is easier to interpret the result. Linear Correlation explains the linear co-movement and show the magnitude of the linear relationship in bivariate data.

The most common correlation measure is the Pearson Coefficient which measures linear correlation. There are other measures which are more robust in non-linear settings but since this thesis focuses on linear correlation, the Pearson Coefficient is sufficient. The Pearson correlation coefficient is obtained by first calculating the covariances of the two random variables and the standard deviation of each of the random variables and then dividing the covariances with the product of the standard deviations.

$$\rho_{X,Y} = \text{corr}(X,Y) = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y} \quad (1)$$

The absolute value of the correlation coefficient is always $\leq 1$. A correlation of 1 is called perfect correlation and means that the variables are directly linear dependent and similarly a correlation coefficient of $-1$ indicates a perfect inverse linear correlation (anti-correlation). The correlation coefficient spans between $(-1, 1)$. Furthermore, a correlation coefficient of 0 does not indicate independence but independent variables have a correlation of 0. This is true since the correlation coefficient only measures linear dependencies.

For the scope of this thesis, the correlation between ESG leaders and the correlation between the underlying companies in a market index is calculated. This is done to understand if the two correlations are different from each other to understand the concentration risk portfolio managers could be exposed to when they tilt a portfolio to ESG compliant companies.
Sampled Correlations

The Pearson correlation coefficient $\rho_{X,Y}$ can be estimated by the sample correlation coefficient $r_{X,Y}$. The sample correlation of a series of $n$ measurements of the random variables $(X_i,Y_i)$, where $i = 1,\ldots,n$ and is calculated by

$$r_{X,Y} := \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}$$

where $s_x, s_y$ are the corrected sample standard deviations and $\bar{x}, \bar{y}$ are the sample means of $X$ and $Y$. A sample correlation can be used to estimate the population correlation parameter with the benefit of not having to know the true population size. Furthermore, the sample correlation can also be used if the population size is large, since the estimation will be stable making it unnecessary to calculate the correlation for the whole population. However, under some conditions i.e. heavy noise between the random variables the sample correlation approach may yield a bad estimation of the true correlation for the population.

Correlation Matrix

In a correlation matrix $C$, pairwise correlation of the $n$ random variables $X_1,\ldots,X_n$ are listed. In $C_{i,j}$ the correlation $corr(X_i,X_j)$ is found. Since the correlation of $corr(X_i,X_j) = corr(X_j,X_i)$ the matrix is symmetric with ones along the diagonal since the correlation to the variable itself always is 1. Of course, since the covariance and correlation matrix are closely related to one another, the correlation matrix can be expressed as a function of the covariance matrix, i.e.:

$$C = \left(\text{diag}(\Sigma)\right)^{-\frac{1}{2}} \Sigma \left(\text{diag}(\Sigma)\right)^{-\frac{1}{2}}$$

where $\Sigma$ is the covariance matrix and $\text{diag}(\Sigma)$ is the diagonal of the covariance matrix, i.e. the variances.

3.3.2 Nonlinear dependence structure and tail risk

To understand the entire dependence structure of a random variable or a data set of random variables, one needs to consider the nonlinear dependence structure. These are often seen in the tails of the data and hence, they are key to understand in times of higher volatility. Therefore, a thorough mathematical background on how to analyse nonlinear dependence and tail behavior will be presented in this subsection.

Skewness

Skewness is the third normalised central moment and is a measure of the lopsidedness/asymmetry of a probability distribution of a random variable about its mean. For different probability distributions the value of the skewness $\gamma$ can be $> 0$ (right skew), $< 0$ (left skew), 0 (balanced) or undefined. If the skew is 0 for a probability distribution it could be symmetric, but it could also have one tail that is long and thin while the other one is short and fat, the value 0 just represent that the tails balance each other.
The skewness of a random variable $X$ is defined as:

$$
\gamma := E \left[ \left( \frac{X - \mu}{\sigma} \right)^3 \right] = \frac{E \left[ (X - \mu)^3 \right]}{(E \left[ (X - \mu)^2 \right])^{3/2}} \quad (4)
$$

The skewness indicates the relative magnitude and direction of a distribution’s deviation from the normal distribution. Furthermore, with a high skewness, standard statistical inference procedures such as confidence interval will perform poorly since they will result in unequal error probabilities on each quantile. Furthermore, skewness can further be used to obtain approximate probability and quantiles of distributions, and will be key when investigating the extreme values of the data.

**Kurtosis**

Kurtosis is the fourth normalised central moment and is a measure of the heaviness of the tail in a distribution or data. Similarly to skewness, the kurtosis describes the shape of a probability function. The kurtosis for a random variable $X$ is defined as:

$$
Kurt[X] = E \left[ \left( \frac{X - \mu}{\sigma} \right)^4 \right] = \frac{E[(X - \mu)^4]}{(E[(X - \mu)^2])^2} \quad (5)
$$

Kurtosis is the expected value of standardised data to the fourth power. Furthermore, any standardised value that is less than 1 falls within one standard deviation and hence will have no or little impact on the kurtosis. Hence, kurtosis measures the values outside the peak of a distribution (values outside 1 standard deviation of the distribution) and thus kurtosis measures outliers only. When analysing the kurtosis, the analyst usually looks at the *excess kurtosis* which is defined as kurtosis minus 3 since it measures the excess kurtosis in relation to the normal distribution which has a kurtosis of 3.

### 3.4 Analysis of Dependence Structure

There are several tools to analyse a multivariate random variables dependence structure. For example, an eigenvalue analysis can be conducted to analyse the dynamics of correlation or covariance matrices in depth and investigate how many factors that actually affect the matrices and how many underlying factors that are noise. Furthermore, analysis of the tails are necessary to determine how the data behaves in extreme events, since this is often different than how it behaves in 'normal' events. In this section, the theory of the analysis of covariance and correlation matrices will be presented.

#### 3.4.1 Analysis of linear dependence

As previously stated, the exploration of dependency is divided into two parts, first a linear analysis which is followed by a non-linear dependency analysis. The linear analysis is based on an analysis of the correlation matrices, where correlation is calculated by the methods described in section 3.3.1. The analysis is commenced by an investigation of the underlying eigenvalues and eigenvectors for the different correlation matrices. The theory of eigenvalues and eigenvectors are presented first since it is the main underlying theory which the analysis
is based upon. The theory is then connected to the more well known Principal Component Analysis (PCA). The focus is then directed to matrix theory which is introduced by the famous Cholesky decomposition of a matrix. Then a introduction to Random Matrix Theory is presented. The section is concluded by the Marchenko-Pastur Distribution which is a distribution function for eigenvalues from a large random matrix.

Eigenvalues & Eigenvectors

Eigenvectors and Eigenvalues of the correlation matrix can be used to model linear dependencies by applying a linear transformation and then analysing the new vector space which is constructed by the eigenvectors that are scaled by the eigenvalues.

This can be expressed as \( T(\mathbf{v}) = \lambda \mathbf{v} \) where \( T \) is a linear transformation of the vector \( \mathbf{v} \) which is the eigenvector. The eigenvector is than scaled by the scalar \( \lambda \) which is the corresponding eigenvalue associated with the eigenvector. As shown by the equation, the direction of the eigenvector is preserved after the transformation but it is scaled by its corresponding eigenvalue, an important property that will be used in the analysis part of this thesis. Note that the analysis of eigenvalues and eigenvectors only is possible if:

1. the matrix is a \( n \times n \) square matrix
2. the matrix is positive semi-definite

If these two criteria are fulfilled, the eigenvalue decomposition yields an orthogonal basis from the eigenvectors. The advantage with eigenvectors is that they maintain their direction after the linear transformation, they are only scaled by the eigenvalue. The largest eigenvalue hence scales its corresponding eigenvector more than the others which could be interpreted as an ordered explanation power of the data from the largest \( \lambda_i, \ldots, \lambda_j \) where \( i \leq j \). Furthermore, the eigenvectors are independent which enables further analysis which for example is used in Principal Component Analysis (PCA).

Principal Component Analysis (PCA)  PCA is an orthogonal transformation of data where the eigenvectors acts as the new basis for the transformation. The largest eigenvalue indicates the largest variance and serves as the first basis for the transformation. The eigenvectors with a high eigenvalue are the principle components of the data and similarly the eigenvalues with a low eigenvalue do not explain the distribution of the data very well, and hence has a limited explanation power. This framework can be very effective in reducing the number of dimensions of the data. It can also be of interest to compare the eigenvalues of two different matrices since a higher eigenvalue indicates higher variance. In this thesis PCA could help to understand how the ESG metric affects the portfolios by comparing the eigenvalues of the correlation matrices stemming from the ESG and the market portfolio.

Cholesky Decomposition  A Hermitian positive semi-definite matrix can be decomposed to reconstruct a covariance matrix from the eigenvalue analysis. The classical Cholesky decomposition for a covariance matrix \( \Sigma \) is expressed as:

\[
\Sigma = \mathbf{PD} \mathbf{P}^T = \mathbf{P}\sqrt{\mathbf{D}}\sqrt{\mathbf{D}} \mathbf{P}^T = \mathbf{A} \mathbf{A}^T
\]
Where the matrix $P$ denotes the matrix with the eigenvectors from the covariance matrix as columns and $D$ is a diagonal matrix with the eigenvalues as entries.

**Random Matrix Theory**  A large square random matrix constructed by $W = XX^T$ where $X$ is a $N \times T$ matrix and the entries in the matrix $X$ are i.i.d random normal variables, $x_{ij} \sim N(0, 1)$ is called a Wishart matrix [32]. The results from the Wishart distribution can be used to estimate noise in correlation matrices which is of importance in this thesis. Log-returns from stocks approximately satisfy the requirements in the Wishart distribution and if $T \geq N$ and $N$ is large, where in this case, $T$ is number of observations i.e. trading days with daily returns and $N$ is number of stocks the eigenvalues of a Wishart matrix will approximately follow a Marchenko-Pastur distribution. It is however only an approximation since the dimensions of the matrix must tend to infinity in order to fully satisfy the theorem [33].

**Marchenko-Pastur Distribution**  The Marchenko-Pastur distribution describes the theoretical distribution of eigenvalues for a random matrix. For example, given a large matrix, with $N$ stocks and $T$ returns series for each stock resulting in a $N \times T$ matrix. If the entries are i.i.d with mean 0 and with finite variance $\sigma^2 \leq \inf$ and we let $Y_n$ be defined as $Y_n = \frac{1}{n}XX^T$. Then, with the eigenvalues denoted as, $\lambda_1, \ldots, \lambda_n$, this can viewed as random variables and we get the density of the eigenvalues from the Marchencko-Pastur distribution as:

$$
\rho(\lambda) = \frac{Q}{2\pi\sigma^2} \frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{\lambda} 
$$

Where $\lambda_\pm$ denotes the largest and smallest eigenvalue which is calculated by:

$$
\lambda_\pm = \sigma^2 \left( 1 \pm \sqrt{\frac{T}{Q}} \right)^2, \quad Q := \frac{T}{N}
$$

$\rho(\lambda)$ is the Marchenko-Pastur density function and is a function that describes the density of the eigenvalues for a large random matrix. The matrix $W = XX^T$ will denote the empirical correlation matrix which is important to distinguish from the the true correlation matrix. If the matrix $X$ would be composed by a small number of stocks, for example, $N = 4$ and the time series of daily returns is large $T = 10^6$ the ratio $q = 1/Q = N/T$ is very small compared to one which will yield a good estimation of the true correlation. However, in many financial settings, if $T$ is large so is also $N$ which yields a $q$ which is not very small in which the Marchenko-Pastur distribution can be of help to estimate the density of the eigenvalues from the correlation matrix [29, 27]. Since the Marchenko-Pastur distribution is a function of the largest eigenvalues of a large random matrix, the eigenvalues that falls outside of the distribution function could be interpreted as information whereas the eigenvalues within the distribution could be interpreted as noise. This property is applied in this thesis in order to filter the correlation matrices. Similarly, the largest eigenvalues explains the most information in the data set and are thus important to understand the data.

The eigenvalue measures the volatility explained by each eigenvector, a large eigenvalue hence explains a large proportion of the total volatility whereas a small eigenvalue only explains a small proportion of the total volatility. The eigenvector in turn indicates the
3.4 Analysis of Dependence Structure

The total set of eigenvectors and eigenvalues hence explain all the volatility in the data but a model based on all eigenvectors would perform poorly out of sample due to overfitting.

3.4.2 Analysis of nonlinear dependence and tail risk

There are many ways to analyse the nonlinear dependence between variables, in this thesis the actual dependence structure is of less importance, instead emphasis is put into the investigation of if the two portfolios differs in the tails which might not be captured by linear dependence models. First, it could be of interest to compare the histograms of returns of the two portfolios in order to see if the returns are approximately equally distributed. Thereafter, the outliers of the data will be examined, by comparing Quantile-Quantile plots ("QQ-plot") between two sets and see if these two data sets seems behave in the same way in the tails.

Histogram of log returns

The shape of the log returns can initially be analysed in a naïve approach by plotting histograms of the log returns. The histograms provide guidance from the shape of the data. If two histograms are compared and one data set has a higher tail risk than the other, the structure of the histograms would expose the higher tail risk in one of data sets. Furthermore, the histograms gives an indication of if the data is symmetric or skewed and if the data is mesokurtic, platykurtic or leptokurtic. A data set is mesokurtic if the excess kurtosis is 0 which means that the data does not produce outliers compared to the family of normal distributions. Similarly, a dataset is platykurtic if the excess kurtosis is < 0, meaning that the data has slimmer tails than the family of normal distributions and is leptokurtic, i.e. excess kurtosis > 0, if the data has fatter tails compared to the family of normal distributions. One example of a leptokurtic distribution is the Student’s t-distribution.

Empirical Quantile-Quantile Plot

Empirical Quantile-Quantile Plots ("QQ-plots") is usually used to compare the tails of a data set to a fitted distribution. In this thesis, the plots will be adjusted to match our research question. Instead of plotting the tails of the data to a fitted distribution, the quantiles of two different data sets will be plotted. This is done to determine if the two data sets behave similarly in the extreme cases. If the data fall on a straight line with a slope approximately equal to 1, the conclusion can be drawn that the data seem to behave similarly in the tails. Furthermore, if the data does not create a straight line, the conclusion can be drawn that the two data sets does not behave in the same way in the tails.

Empirical Value at Risk

The value at risk (VaR) is a measure of the risk of loss for an investment and *the VaR of a position with value X at time 1 is the smallest amount of money that if added to the
3.5 Resampling and Simulations

Position now and invested in the risk-free asset ensures that the probability of a strictly negative value at time 1 is not greater than \( p \).\[^{[34]}\] Furthermore, the VaR is defined as:

\[
\text{VaR}_p(X) = \min\{m : P(m R_0 + X < 0) \leq p\}
\] (9)

where \( R_0 \) is the risk-free rate. With \( X = V_1 - V_0 R_0 \), i.e. \( X \) is the net gain from an investment, and by setting \( L = -X/R_0 = V_0 - V_1/R_0 \) then \( L \) naturally is interpreted as the discounted loss of an investment. Yielding that:

\[
\text{VaR}_p(X) = \min\{m : P(L \leq m) \geq 1 - p\}
\] (10)

and \( \text{VaR}_p(X) \) can be interpreted as the smallest amount of risk capital \( m \) an investor should hold to negate the largest potential loss with probability \( 1 - p \).

In statistical terms, \( \text{VaR}_p(X) \) is the \((1 - p)\)-quantile of \( L \). Thus, the empirical VaR is given by the empirical quantiles from the \( n \) ordered samples \( L_{1,n}, \ldots, L_{n,n} \) of independent copies of \( L \), giving the empirical estimate:

\[
\hat{\text{VaR}}_p(X) = L_{\lfloor np \rfloor + 1,n}, \quad \text{where} \quad L_{1,n} \geq \cdots \geq L_{n,n}.
\] (11)

3.5 Resampling and Simulations

There are three main applications of resampling in statistics, the first one being estimating the precision of a sample statistic by estimating through subsets of the available data. The drawing of subsets without replacement is called jackknifing and with replacement is called bootstrapping. Bootstrapping will be presented in more detail since it will be used in this project. The second method of resampling is used to conduct certain tests which will not be presented nor used in this thesis. The third and last method is validation of models by using random subsets which also can be a type of bootstrapping method or a cross-validation method.

3.5.1 Bootstrapping

In order to estimate the properties of an estimator, bootstrapping measures the properties by sampling from an approximating distribution function many times, about 100 times are recommended to improve the estimation of standard errors, a higher sample frequency then 100 lead to negligible improvements of the estimation of the standard errors \(^{[35]}\). The approximating distribution can for example be the empirical distribution function \( \hat{F}_n(t) \) of the observed data. If the data can be assumed to be independent and identically distributed a resampling procedure can be applied to estimate the properties of the estimator. Bootstrapping can also be used for hypothesis testing which often is used as an alternative to statistical interference when parametric interference is (or is nearly) impossible. A bootstrap approach could be:

1. Begin with a sample from a population with sample size \( n \)

2. Draw a sample from the population with replacement \( k \) times. Each draw will result in a bootstrap sample which will result in a total of \( k \) bootstrap samples in total once done.
3. Evaluate the estimator for each of the \( k \) bootstrap samples.

4. Construct a sampling distribution from the \( k \) bootstrap estimators and use it to make further statistical inference tests for example estimating the standard error or obtaining confidence intervals.

### 3.6 Portfolio Optimisation

A portfolio manager’s objective would always be to create the optimal portfolio to cover the portfolio mandate. There are several theories on how to create the optimal portfolio with Markowitz Portfolio Theory being one of the most famous ones. This section is commenced with a short review of portfolio theories and is then followed by a review on how the filtration process is applied to the minimum variance portfolio constructed in this thesis.

#### 3.6.1 Markowitz Modern Portfolio Theory

Modern Portfolio Theory was one of the earliest quantitative portfolio models available as it was introduced in the paper 'Portfolio Selection' in 1952. Markowitz modelled rate of returns from assets as random variables and focuses on linear portfolios and showed that it is possible to use diversification to optimise a portfolio for a given level of risk. [11]. Modern portfolio theory assumes that the investor is risk averse and models returns of assets as a random variable with finite mean and variance. The risk in the portfolio is measured through variance.

Variance can be used to compare the risk in different portfolios i.e. using the variance as a proxy for risk. Using the variance as a proxy for risk is sufficient in most applications. However, the variance has several flaws, one being that two different density functions can have the same variance but different shapes. For this thesis, variance will however be a good proxy of risk since log-returns usually have similar shapes [34].

#### 3.6.2 Minimum Variance Portfolio

Since variance is usually used as proxy for risk it is often of great interest to minimise the variance when optimising a portfolio. If one is forced to invest all capital in the assets available it is clear that \( 1^T w = 1 \) where \( w_i \) denotes the weight allocated in asset \( i \). The solution to the minimum variance portfolio optimisation problem is:

\[
    w = \frac{\Sigma^{-1} 1}{1^T \Sigma^{-1} 1},
\]

where \( \Sigma \) is the covariance matrix of the returns of the assets. It is stated previously that the covariance matrix can be decomposed by Equation 6. A property of the eigenvalues are that the largest eigenvalues is explanatory for the majority of the variance in the decomposed covariance matrix, whereas the small eigenvalues is primarily due to noise. The eigenvectors
are not of any interest since they only describe the direction of the eigenvalues. Thus, the minimum variance weights can be modified as a function of the eigenvalues, i.e.

\[
\mathbf{w}(\lambda) \approx \frac{D^{-1} \mathbf{1}}{1^T D^{-1} \mathbf{1}} = \left[ \frac{\lambda_1^{-1}}{\sum_{k=1}^{n} \lambda_k^{-1}} \ldots \frac{\lambda_n^{-1}}{\sum_{k=1}^{n} \lambda_k^{-1}} \right]^T
\]

(13)

Using the information above that the eigenvalues are explanatory for the covariance and the weights from Equation 13, a proxy for variance as a function of the number of eigenvalues included in \( D \), i.e.

\[
\tilde{\text{var}}(\mathbf{w} X)(\lambda) = \mathbf{w}(\lambda)^T D \mathbf{w}(\lambda) = \frac{(D^{-1} \mathbf{1})^T D (D^{-1} \mathbf{1})}{(\sum_{k=1}^{n} \lambda_k^{-1})^2} = \frac{1}{\sum_{k=1}^{n} \lambda_k^{-1}}
\]

(14)

As seen in Equation 3, the sole difference between the covariance and correlation matrix is that the correlation matrix is normalised. Using this, one can motivate that Equation 14 can be used for the decomposed correlation matrix as well. Note that using the eigenvalues from the correlation matrix will not yield the same numerical results as the ones from the covariance matrix. However, both techniques will yield the same result if the adjusted minimum variance is only used to compare several simulations.

### 3.6.3 Filtered minimum variance portfolio

The Marchenko-Pastur distribution can be applied to the distribution of eigenvalues to filter information from noise. The filtration process is applied by removing all eigenvalues that falls within the distribution function, only keeping those eigenvalues that lies outside of the right side of the distribution, i.e. removing all eigenvalues that are noise and does not actually explain the variance. By doing this and applying it to the matrix \( D \) using the eigenvalues outside the theoretical Marchenko-Pastur distribution and setting the rest to 0 we get the following:

\[
\tilde{D} = \begin{bmatrix}
\lambda_1 & 0 & \cdots & \cdots & 0 \\
0 & \ddots & \ddots & \vdots & \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & 0 & \ddots & 0 & 0 \\
0 & \cdots & 0 & \ddots & \ddots \\
\end{bmatrix}
\]

(15)

where \( \lambda_1 \) is the largest eigenvalue and \( \lambda_i \) is the smallest eigenvalue larger than the maximum value of the theoretical Marchenko-Pastur distribution. Note that some very small eigenvalues will fall outside of the distribution function which would imply that they are not random and hence contain information. The observation is correct but the smallest eigenvalues would still be treated as noise in this model since they correspond to long-short portfolios constructed of assets that at the time of the sample had a high correlation and hence provided a very low volatility in the constructed portfolio. However, since the extreme correlations typically are time dependent they are treated as noise in order to create a more robust model [29].
Thereafter, the filtered matrix $\hat{D}$ obtained in Equation 15 can be used to get a filtered covariance matrix from the Cholesky Decomposition in Equation 6, i.e.:

$$\hat{\Sigma} = P \hat{D} P^T$$  \hspace{1cm} (16)

This filtered covariance matrix $\hat{\Sigma}$ cannot be used to calculate the actual variance of a portfolio. However, it will better capture the true dependence structure of a portfolio since it will inherit the properties of the initial covariance matrix, but with reduced noise. Hence, the filtered covariance matrix can be used to calculate the portfolio weights for a minimum variance portfolio similarly to Equation 12, i.e.:

$$\hat{\mathbf{w}} = \hat{\Sigma}^{-1} \mathbf{1} \left( \hat{\Sigma}^{-1} \mathbf{1} \right)^T$$  \hspace{1cm} (17)

Hence, these weights given from the filtered covariance matrix should have reduced the noise compared to Equation 12 and as a result improve the out of sample variance for the portfolio.

### 3.6.4 Jensen’s alpha

Jensen’s alpha is used to determine abnormal returns of a portfolio of assets over the theoretical expected return [36]. The theoretical return is predicted by a market model CAPM model explained in Section 2.2. The Jensen’s alpha is defined as

$$\alpha_j = R_i - \left[ R_f + \beta_{i,M} \cdot (R_M - R_f) \right],$$  \hspace{1cm} (18)

where $R_i$ is the realised return of the portfolio, $R_M$ is the market return, $R_f$ is the risk-free rate of return and $\beta_{i,M}$ is the beta of the portfolio.
METHODOLOGY

In this section, the methodology to answer the research question is presented. The section begins with a description on how the data was gathered and manipulated to get it in an adequate form. The data section is followed by a description of the portfolio construction, which was applied to filter out regional and sectoral effects. These effects were filtered out since they could otherwise have an impact on the portfolio correlations that would not be attributable to the ESG Risk Rating. Lastly, this section describes the methodology behind the investigation of dependence structure amongst the simulated portfolios.

4.1 DATA COLLECTION

All data used in this thesis has been provided by Skandia. The selection and filtration of the data have been done in dialogue with supervisors from both Skandia and KTH. Certain data, such as the ESG data and the procedure to match the data was already available in a SQL-database provided by Skandia, which was accessed through a script in Python. All equity return data was downloaded from Bloomberg to an excel spreadsheet which was loaded into a dataframe in Python and combined with the ESG data from the SQL-database.

The return data was composed of daily cumulative total return from October 2018 to end of September 2019. The relative short time frame is due to new Sustainalytics ESG measure which was introduced in 2018. These cumulative total returns were however transformed to daily log returns. Daily log returns are usually preferred over cumulative returns since they:

1. have approximately zero mean
2. are approximately time independent
3. approximately symmetrical around 0

Hence, they replicate i.i.d samples from a normal distribution better than cumulative returns. The ESG data provided by Skandia is based on Sustainalytics ESG data. The GICS data was downloaded from Bloomberg but is provided by MSCI.

4.1.1 Data handling

The reference index for all markets are constructed from regional MSCI indices, i.e. MSCI Europe, MSCI North America and MSCI Asia Pacific. These are constructed to capture the equity markets as they are today, i.e. the companies in the index changes over time [37]. Because of this, some returns series are incomplete. The reason why a company has an incomplete return series can be due to, e.g. corporate actions, change of ticker, dropped from the index, etc. In order to attain a high data quality, all return series where at least 20% of the returns were 0 were dropped. This procedure deleted 115 return series from the
reference portfolio, as the return series shrunk from \( n = 1640 \) to \( n = 1525 \). Here \( n \) is the number of companies with 258 daily returns for each company and this represents a loss of \( \approx 7\% \).

4.2 PORTFOLIO CONSTRUCTION

Two portfolios are constructed for each region. These will henceforth be called the ESG leaders ("leaders") and the reference portfolio ("reference"). The two portfolios are constructed for three regions: Europe, North America and Asia-Pacific which results in a total of six portfolios. The reference portfolios are constructed from the three MSCI regional indices.

The leader portfolio for each region is constructed by first calculating the sector distribution in the MSCI index corresponding to the same region. Thereafter, the leader portfolio mimics the sector distribution of the reference portfolio such that both portfolios have approximately the same sector distribution. This is an important step in order to avoid to model sector effects instead of the ESG effect. Once the sector distribution is determined, leaders, in terms of highest ESG score defined by Sustainalytics, are picked from each sector. A flowchart of the entire process is presented below:
4.2 Portfolio Construction

To create these reference portfolios, random picks without replacements are made until the reference portfolio has the same amount of underlying companies as the leader portfolio. A flowchart of the steps to create the reference portfolios are presented below:
To remove company specific effects in the reference portfolio and to reduce the randomness, we create 10 different reference portfolios of equal size. Furthermore, to remove the time dependence bootstrapping is used, the leaders are bootstrapped 100 times and the 10 reference portfolios are bootstrapped 10 times, resulting in 100 simulations of both portfolios. The bootstrap picks 516 daily log returns at random from our sample of approximately 280 daily log returns in total.

The data was categorised into the three regions; Europe, North America and Asia Pacific, in order to better isolate the regional ESG effects. The regional ESG effects will thereafter be analysed such that regional conclusions can be drawn. The regional effects are analysed since different regions could have adapted differently towards ESG investments.
4.2.1 Europe

The European market is interesting for the scope of this thesis since the European Union has expressed interest in regulating the financial institutions to require ESG aspects to be taken into account in investment decisions [38]. The regulatory addition of a ESG metric in Europe is unique and before any decisions can be made, the taxonomy of ESG must be established which is the first step in the process. The first effects of legislative regulation are expected to earliest 2022 in the EU.

The sector distribution in the European markets are presented in 3 and shows that the portfolio represents all of the eleven GICS Sectors, the weight in percent is also presented for each sector in the table.

<table>
<thead>
<tr>
<th>Gics Sector</th>
<th>%</th>
<th>Gics Sector</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td>8.1</td>
<td>Consumer Discretionary</td>
<td>12.8</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>8.8</td>
<td>Energy</td>
<td>3.3</td>
</tr>
<tr>
<td>Financials</td>
<td>17.1</td>
<td>Health Care</td>
<td>8.1</td>
</tr>
<tr>
<td>Industrials</td>
<td>20.2</td>
<td>Information Technology</td>
<td>3.8</td>
</tr>
<tr>
<td>Materials</td>
<td>9.8</td>
<td>Real Estate</td>
<td>3.0</td>
</tr>
<tr>
<td>Utilities</td>
<td>5.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: European Portfolios - Sector Partition

Leaders

The leaders in the European portfolio is composed of the ≈ 50 companies with the highest ESG scores. Furthermore, the leader portfolio has approximately the same sector split as the MSCI Europe index.

Some industry groups for a certain sector, e.g. Commercial & Professional Services being an industry group for the sector Industrials, is more represented than the other two; Capital Goods and Transportation, as seen in Table 4. This gives the portfolio an industry group tilt, even though the sector tilt is removed as explained in Section 4.2. The leader portfolio includes companies from many European countries with the most (12 companies) represented from United Kingdom. The complete list of countries is listed below in table 5.
4.2 Portfolio Construction

<table>
<thead>
<tr>
<th>Gics Industry Group</th>
<th>%</th>
<th>Gics Industry Group</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Automobiles &amp; Components</td>
<td>2.0</td>
<td>Banks</td>
<td>2.0</td>
</tr>
<tr>
<td>Capital Goods</td>
<td>3.9</td>
<td>Commercial &amp; Professional Services</td>
<td>13.7</td>
</tr>
<tr>
<td>Consumer Durables &amp; Apparel</td>
<td>5.9</td>
<td>Diversified Financials</td>
<td>11.8</td>
</tr>
<tr>
<td>Energy</td>
<td>3.9</td>
<td>Food &amp; Staples Retailing</td>
<td>2.0</td>
</tr>
<tr>
<td>Food, Beverage &amp; Tobacco</td>
<td>5.9</td>
<td>Health Care Equipment &amp; Service</td>
<td>7.8</td>
</tr>
<tr>
<td>Insurance</td>
<td>3.9</td>
<td>Materials</td>
<td>9.8</td>
</tr>
<tr>
<td>Media &amp; Entertainment</td>
<td>7.8</td>
<td>Real Estate</td>
<td>3.9</td>
</tr>
<tr>
<td>Retailing</td>
<td>3.9</td>
<td>Technology Hardware &amp; Equipment</td>
<td>3.9</td>
</tr>
<tr>
<td>Transportation</td>
<td>2.0</td>
<td>Utilities</td>
<td>5.9</td>
</tr>
</tbody>
</table>

Table 4: European ESG Leaders - Industry Group Partition

<table>
<thead>
<tr>
<th>Country</th>
<th>%</th>
<th>Country</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denmark</td>
<td>5.9</td>
<td>Finland</td>
<td>3.9</td>
</tr>
<tr>
<td>France</td>
<td>19.6</td>
<td>Germany</td>
<td>5.9</td>
</tr>
<tr>
<td>Ireland</td>
<td>2.0</td>
<td>Italy</td>
<td>2.0</td>
</tr>
<tr>
<td>Netherlands</td>
<td>7.8</td>
<td>Norway</td>
<td>2.0</td>
</tr>
<tr>
<td>Portugal</td>
<td>2.0</td>
<td>Spain</td>
<td>7.8</td>
</tr>
<tr>
<td>Sweden</td>
<td>5.9</td>
<td>Switzerland</td>
<td>9.8</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>25.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: European ESG Leaders - Country Partition

**Reference Index**

The proxy for the European market is the MSCI Europe index which has a skew towards companies with a high market capitalisation, but also smaller companies are included in the index [39]. The index has a geographical diversification in order to capture the European financial markets in an aggregate. The geographical distribution of the index is shown in Table 7. The index is well distributed over sectors as shown in table 6. Furthermore, as stated previously, the index is continuously evaluated to remain a relevant proxy of the European markets. The industry group distribution can be found in Table 6. Note that even though the reference and leader portfolio has the same sector split, the industry group distribution differ. For example, the reference portfolio has a skew towards Capital Goods whereas the leader portfolio has a skew towards Commercial & Professional. Both of these belong to the sector **Industrials**.
### Table 6: European Reference Portfolio - Industry Group Partition

<table>
<thead>
<tr>
<th>Gics Industry Group</th>
<th>%</th>
<th>Gics Industry Group</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Automobiles &amp; Components</td>
<td>3.8</td>
<td>Banks</td>
<td>6.3</td>
</tr>
<tr>
<td>Capital Goods</td>
<td>13.1</td>
<td>Commercial &amp; Professional Services</td>
<td>3.5</td>
</tr>
<tr>
<td>Consumer Durables &amp; Apparel</td>
<td>4.8</td>
<td>Consumer Services</td>
<td>2.3</td>
</tr>
<tr>
<td>Diversified Financials</td>
<td>4.8</td>
<td>Energy</td>
<td>3.3</td>
</tr>
<tr>
<td>Food &amp; Staples Retailing</td>
<td>2.5</td>
<td>Food, Beverage &amp; Tobacco</td>
<td>5.0</td>
</tr>
<tr>
<td>Health Care Equipment &amp; Service</td>
<td>3.5</td>
<td>Household &amp; Personal Products</td>
<td>1.3</td>
</tr>
<tr>
<td>Insurance</td>
<td>6.0</td>
<td>Materials</td>
<td>9.8</td>
</tr>
<tr>
<td>Media &amp; Entertainment</td>
<td>3.8</td>
<td>Pharmaceuticals, Biotechnology</td>
<td>4.5</td>
</tr>
<tr>
<td>Real Estate</td>
<td>3.0</td>
<td>Retailing</td>
<td>2.0</td>
</tr>
<tr>
<td>Semiconductors &amp; Semiconductor</td>
<td>1.0</td>
<td>Software &amp; Services</td>
<td>2.0</td>
</tr>
<tr>
<td>Technology Hardware &amp; Equipment</td>
<td>0.8</td>
<td>Telecommunication Services</td>
<td>4.3</td>
</tr>
<tr>
<td>Transportation</td>
<td>3.5</td>
<td>Utilities</td>
<td>5.0</td>
</tr>
</tbody>
</table>

The European portfolio however seems to include some companies from countries outside of the region as shown in table 7. The weight of these companies are however small, the reason to why those companies are included in the European index could be, for example, due to the stock being listed on multiple exchanges in different regions. Due to the small number or companies affected by this the problem it is omitted since manipulating the reference index could include unnecessary bias.

### Table 7: European Reference Portfolio - Country Partition

<table>
<thead>
<tr>
<th>Country</th>
<th>%</th>
<th>Country</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.5</td>
<td>Austria</td>
<td>1.3</td>
</tr>
<tr>
<td>Belgium</td>
<td>2.0</td>
<td>Denmark</td>
<td>3.8</td>
</tr>
<tr>
<td>Finland</td>
<td>2.5</td>
<td>France</td>
<td>17.9</td>
</tr>
<tr>
<td>Germany</td>
<td>14.4</td>
<td>Ireland</td>
<td>2.5</td>
</tr>
<tr>
<td>Italy</td>
<td>4.5</td>
<td>Luxembourg</td>
<td>1.5</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.3</td>
<td>Netherlands</td>
<td>4.8</td>
</tr>
<tr>
<td>Norway</td>
<td>2.5</td>
<td>Portugal</td>
<td>0.8</td>
</tr>
<tr>
<td>South Africa</td>
<td>0.3</td>
<td>Spain</td>
<td>4.3</td>
</tr>
<tr>
<td>Sweden</td>
<td>5.3</td>
<td>Switzerland</td>
<td>9.3</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>21.4</td>
<td>United States</td>
<td>0.3</td>
</tr>
</tbody>
</table>
4.2.2 North America

There is currently no legislative initiative active in the US similar to the European-style reporting standards and the upcoming regulations in the European Union. The US Congress rejected a proposal to introduce the European-style reporting in the US for ESG matters in July 2019 [40]. The Council of Institutional Investors which is a nonprofit, nonpartisan organisation of public, corporate and union employee benefit funds and with more than $35 trillion in assets under management are in favor of some sort of regulation that would impose mandatory ESG disclosure for all companies [41]. There are already over 600 companies in the US that are voluntarily using the GRI-standard for ESG disclosure [40].

The sector distribution in the North American portfolios shown in Table 8 demonstrates a similar sector distribution as in the European market.

<table>
<thead>
<tr>
<th>Gics Sector</th>
<th>%</th>
<th>Gics Sector</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td>4.8</td>
<td>Consumer Discretionary</td>
<td>11.6</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>6.1</td>
<td>Energy</td>
<td>6.7</td>
</tr>
<tr>
<td>Financials</td>
<td>15.8</td>
<td>Health Care</td>
<td>11.0</td>
</tr>
<tr>
<td>Industrials</td>
<td>13.2</td>
<td>Information Technology</td>
<td>14.0</td>
</tr>
<tr>
<td>Materials</td>
<td>5.3</td>
<td>Real Estate</td>
<td>6.4</td>
</tr>
<tr>
<td>Utilities</td>
<td>5.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8: North American Portfolios - Sector Partition

Leaders

Similarly to the European portfolio, the leaders in the North American portfolio are composed of the ≈ 50 companies with the highest ESG scores, with a sector split approximately equal to the reference index. The sector distribution is well distributed with some sectors having a higher weight which is to be expected since different regions in a globalised economy often specialise in different sectors. The industry group distribution in the leader portfolio is constructed is presented in Table 9.
Table 9: North American ESG Leaders - Industry Group Partition

The country partition in the leader portfolio is listed in Table 10 and shows that most companies, 72%, are represented from the United States and the remaining 28% from Canada. Furthermore, this shows that all companies represented in the leader portfolio are North American companies.

<table>
<thead>
<tr>
<th>Country</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>28.0</td>
</tr>
<tr>
<td>United States</td>
<td>72.0</td>
</tr>
</tbody>
</table>

Table 10: North American ESG Leaders - Country Partition

Reference Index

The reference index for North America is constructed from the MSCI North America index. The index is supposed to capture the market dynamics in the region by incorporating both large and small companies. Most companies are, however, large since they have the most impact on the market [42]. The reference index for North America consists of approximately 670 companies from the region.
4.2 Portfolio Construction

<table>
<thead>
<tr>
<th>Gics Industry Group</th>
<th>%</th>
<th>Gics Industry Group</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Automobiles &amp; Components</td>
<td>1.0</td>
<td>Banks</td>
<td>4.2</td>
</tr>
<tr>
<td>Capital Goods</td>
<td>8.3</td>
<td>Commercial &amp; Professional Services</td>
<td>2.1</td>
</tr>
<tr>
<td>Consumer Durables &amp; Apparel</td>
<td>2.8</td>
<td>Consumer Services</td>
<td>2.8</td>
</tr>
<tr>
<td>Diversified Financials</td>
<td>6.1</td>
<td>Energy</td>
<td>6.7</td>
</tr>
<tr>
<td>Food &amp; Staples Retailing</td>
<td>1.3</td>
<td>Food, Beverage &amp; Tobacco</td>
<td>3.6</td>
</tr>
<tr>
<td>Health Care Equipment &amp; Service</td>
<td>5.5</td>
<td>Household &amp; Personal Products</td>
<td>1.2</td>
</tr>
<tr>
<td>Insurance</td>
<td>5.5</td>
<td>Materials</td>
<td>5.3</td>
</tr>
<tr>
<td>Media &amp; Entertainment</td>
<td>3.6</td>
<td>Pharmaceuticals, Biotechnology</td>
<td>5.5</td>
</tr>
<tr>
<td>Real Estate</td>
<td>6.4</td>
<td>Retailing</td>
<td>4.9</td>
</tr>
<tr>
<td>Semiconductors &amp; Semiconductor</td>
<td>2.7</td>
<td>Software &amp; Services</td>
<td>7.9</td>
</tr>
<tr>
<td>Technology Hardware &amp; Equipment</td>
<td>3.4</td>
<td>Telecommunication Services</td>
<td>1.2</td>
</tr>
<tr>
<td>Transportation</td>
<td>2.8</td>
<td>Utilities</td>
<td>5.2</td>
</tr>
</tbody>
</table>

Table 11: North American Reference Portfolio - Industry Group Partition

As in the European reference portfolio some companies in the index are listed as originating from other parts of the world other than North America as seen in Table 12. The majority of the companies however, 95.4%, are from the United States or Canada. The reason to why some companies are listed as originating from different regions than the North America is unknown but no manipulation of the index is made.

<table>
<thead>
<tr>
<th>Country</th>
<th>%</th>
<th>Country</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>0.1</td>
<td>Bermuda</td>
<td>0.6</td>
</tr>
<tr>
<td>Canada</td>
<td>12.3</td>
<td>Ireland</td>
<td>1.8</td>
</tr>
<tr>
<td>Singapore</td>
<td>0.1</td>
<td>Switzerland</td>
<td>0.4</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1.5</td>
<td>United States</td>
<td>83.1</td>
</tr>
</tbody>
</table>

Table 12: North American Reference Portfolio - Country Partition

4.2.3 Asia-Pacific

The Asia-Pacific area, and China in particular, have been laggards historically in ESG aspects. This could partly be explained by the lack of legal requirements of ESG disclosure for companies active in the region and the large manufacturing sector in the region that tend to have a lower ESG score compared to e.g. the services industries [43]. The sector distribution in the region is similar to the European sector partition with Industrials being the largest sector in the market. The overall dispersion over the sectors is, however, more concentrated in the Asia-Pacific area than in Europe as shown in Table 13.
Table 13: Asia Pacific - Sector Partition

Leaders

The leaders portfolio is composed of the ≈ 50 companies with the highest ESG scores and with an approximately equal sector split as the reference index. The industry group partition in the leader portfolio is presented in Table 14 and shows a quite even distribution across the industry groups.

Table 14: Asia Pacific ESG Leaders - Industry Group Partition

The country partition shown in Table 15 display a tilt towards Japan.

Table 15: Asia Pacific ESG Leaders - Country Partition
Reference Index

The reference index for Asia-Pacific is constructed from MSCI Pacific Index which is constructed of companies with a large and middle market capitalisation across five developed markets in the pacific region [44]. The index is tilted towards Japan which constitutes approximately 60% of the index. The reference index consists of approximately 450 companies.

Since the selection of companies are made on sector basis in the GICS, some industry groups are represented in the reference portfolio but not in the leader portfolio. This is seen by comparing Table 14 and Table 16

<table>
<thead>
<tr>
<th>Gics Industry Group</th>
<th>%</th>
<th>Gics Industry Group</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Automobiles &amp; Components</td>
<td>4.0</td>
<td>Banks</td>
<td>5.9</td>
</tr>
<tr>
<td>Capital Goods</td>
<td>11.6</td>
<td>Commercial &amp; Professional Services</td>
<td>2.0</td>
</tr>
<tr>
<td>Consumer Durables &amp; Apparel</td>
<td>3.3</td>
<td>Consumer Services</td>
<td>3.1</td>
</tr>
<tr>
<td>Diversified Financials</td>
<td>3.3</td>
<td>Energy</td>
<td>2.2</td>
</tr>
<tr>
<td>Food &amp; Staples Retailing</td>
<td>2.0</td>
<td>Food, Beverage &amp; Tobacco</td>
<td>4.8</td>
</tr>
<tr>
<td>Health Care Equipment &amp; Service</td>
<td>3.1</td>
<td>Household &amp; Personal Products</td>
<td>1.8</td>
</tr>
<tr>
<td>Insurance</td>
<td>2.6</td>
<td>Materials</td>
<td>9.0</td>
</tr>
<tr>
<td>Media &amp; Entertainment</td>
<td>2.6</td>
<td>Pharmaceuticals, Biotechnology</td>
<td>3.7</td>
</tr>
<tr>
<td>Real Estate</td>
<td>10.3</td>
<td>Retailing</td>
<td>3.5</td>
</tr>
<tr>
<td>Semiconductors &amp; Semiconductor</td>
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<td>Software &amp; Services</td>
<td>2.2</td>
</tr>
<tr>
<td>Technology Hardware &amp; Equipment</td>
<td>4.8</td>
<td>Telecommunication Services</td>
<td>2.0</td>
</tr>
<tr>
<td>Transportation</td>
<td>6.4</td>
<td>Utilities</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Table 16: Asia Pacific Reference Portfolio - Industry Group Partition

Similarly to the North American and European case, the country split for the reference portfolio in Asia-Pacific has some flaws. For example, one sees that the country partition for the reference portfolio found in Table 17 has Bermuda and Ireland included, which are not part of Asia-Pacific.

<table>
<thead>
<tr>
<th>Country</th>
<th>%</th>
<th>Country</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>13.2</td>
<td>Bermuda</td>
<td>0.4</td>
</tr>
<tr>
<td>China</td>
<td>0.4</td>
<td>Hong Kong</td>
<td>8.8</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.2</td>
<td>Japan</td>
<td>69.7</td>
</tr>
<tr>
<td>Macau</td>
<td>0.7</td>
<td>New Zealand</td>
<td>1.3</td>
</tr>
<tr>
<td>Singapore</td>
<td>5.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 17: Asia Pacific Reference Portfolio - Country Partition
4.3 INVESTIGATION OF DEPENDENCE STRUCTURE

For each region an investigation of the dependence structure is conducted by comparing the leader and reference portfolios. For each region, several aspects of dependence structure is computed which then is analysed to answer the research questions. The main focus is the analysis of linear dependency structure which is conducted through a thorough investigation of the correlation structures in the different portfolios. The tails of the distributions are then further investigated in the results which can be found in section 5 where some conclusions regarding non-linear dependencies can be drawn.

4.3.1 Linear dependence

To investigate the dependence structure of two subsets and see if they stem from the same distribution is a logical first step is to compare the linear dependence. This is done by comparing the correlation of the simulated portfolios and thereafter trying to determine if they are approximately equal. In this thesis, this is done by calculating the covariance and correlation for the leader portfolio and the ten random reference portfolios for each region. Moreover, a bootstrapping procedure is performed on the portfolios to include approximately twice as many daily samples. Each reference portfolio is bootstrapped 10 times and for each bootstrap of the reference portfolio, the leader portfolio is bootstrapped, totaling 100 simulations for the reference and leader portfolios for each region.

Eigenvalues

With the correlation and covariance matrices in place, methods to investigate if these are approximately equal needs to be evaluated. One efficient method is to compare the eigenvalues for the correlation and covariance matrices between the leader and reference portfolio. The analysis of eigenvalues is done in the following steps:
Input Correlation matrix from bootstrapped log returns

Calculate eigenvalues for Correlation matrix

Plot eigenvalues ($\lambda$) as a cumulative distribution function for each simulation

Does $\lambda_{ref}$ and $\lambda_{lead}$ follow the same distribution?

- yes
  - Compare eigenvalues with the theoretical MP-distribution
  - Decompose $\Sigma = PDP^T$ and calculate minimum variance portfolio as Equation 14 w.r.t. $k$, $k = \#\lambda$
  - Plot variance of decomposed $\Sigma$ as a function of $k$, break at maximum theoretical MP-distribution
  - Keep eigenvalues outside MP-distribution and set all other to 0
  - Filter correlation and covariance matrix w.r.t. eigenvalues outside MP-distribution
  - Calculate a portfolio with the reconstructed covariance matrix and evaluate the results

- no

Stop

Figure 4: Analysis of Eigenvalues
cumulative distribution of eigenvalues  It is established from Section 3.4.1 that the eigenvalues should follow a Marchenko-Pastur distribution given a set of constraints. Hence, to determine if the eigenvalues are approximately equally distributed, the sampled cumulative probability function for the eigenvalues are plotted and analysed. If they look equal, the conclusion can be drawn that they come from the same distribution and that there is no major difference in the linear dependence structure from the two samples.

Figure 5: Europe: Cumulative distribution function for Correlation matrix’s eigenvalues

In the European case, from Figure 5 one notes that the eigenvalue from the simulations of the leader portfolios seems to be different from the reference portfolio. The eigenvalue distributions are different since the simulations of the cumulative distribution function for the leader portfolio does not lie in the same space as the simulated reference portfolio. This can be assumed to be a first indication that these may not have the same linear dependence structure.

Figure 6: North America: Cumulative distribution function for Correlation matrix’s eigenvalues
Similarly to the European portfolios, the American portfolios show a difference in the distribution of the eigenvalues for the leader and reference portfolios, seen in Figure 6. Thus, the same conclusions as in the European case can be drawn from the American portfolios.

![Figure 7: Asia Pacific: Cumulative distribution function for Correlation matrix’s eigenvalues](image)

For the Asian portfolios however, we do not see any difference in the cumulative distribution function of the eigenvalues for the simulated leader and reference portfolios, as seen in Figure 7. Asian markets are laggards in the field of ESG and sustainable investing compared to e.g. Europe, evidenced by their low scores compared to other regions [43]. This could explain why there is no approximately no difference between the portfolios.

**Comparison of Eigenvalues vs Theoretical Marchenko-Pastur Distribution** The theoretically fitted Marchenko-Pastur distribution for the European portfolios are shown in Figure 8 as the red line. The blue bars are the eigenvalues from the correlation matrix. The distribution show a quite good fit for the bulk of the values and some values that falls outside of the distribution function. The values within the distribution function are interpreted as noise as described in Section 3.4.1 and similarly the values outside of the distribution are interpreted as information.

The largest eigenvalue in the European portfolios is larger in the reference portfolio than in the leader portfolio. The largest eigenvalue is also called the market eigenvalue which means that it explains the variance in the portfolio in relation to market movement. If the aggregated market moves in a certain direction, one particular asset is more probable to move in the same direction so it describes the co-movement in the stock market. Hence, the market eigenvalue explains the volatility caused by the aggregated market volatility. The reference portfolio has a larger market eigenvalue in Europe which translates to that the market portfolio is more sensitive to market movement in terms of volatility than the leader portfolio which is to be desired since the reference portfolio is supposed to be an approximation of the market.
4.3 Investigation of Dependence Structure

(a) Leader Portfolio

(b) Reference Portfolio

Figure 8: Europe: Empirical eigenvalue distribution vs theoretical Marchenko-Pastur distribution

The North American portfolios show the approximately same characteristics as the European portfolios where the fitted Marchenko-Pastur distribution provide a good fit for the bulk of low eigenvalues as seen in Figure 9. Once again the reference portfolio exhibit a larger market eigenvalue which is an indication that the portfolio proxies the market better than the leader portfolio which is desirable. The difference is slightly smaller than in the European portfolio but the two portfolios are still significantly different from each other. The distribution of the other large eigenvalues are similar in the two American portfolios in size but bear in mind that it does not imply that the corresponding eigenvectors are identical. The eigenvalues only scale the eigenvectors in terms of volatility but the eigenvectors provide the direction.

(a) Leader Portfolio

(b) Reference Portfolio

Figure 9: North America: Empirical eigenvalue distribution vs theoretical Marchenko-Pastur distribution

The Asia Pacific portfolios shows a different characteristic than both the European and American which was discussed in relation to Figure 7 earlier. The fitted theoretical Marchenko-Pastur distribution function as seen in Figure 10 confirms the conclusion that the two distribution probably are approximately the same in the leader and reference portfolio. Firstly, we note that the market eigenvalue seems to be approximately equal in both portfolios, implying that the leader and reference portfolios are affected by market volatility in a similar manner. Secondly, the second and third largest eigenvalue are almost identical in both portfolios. While this alone does not imply that the eigenvectors between the two portfolios are the same, it shows that they are scaled by an approximately equal magnitude. An interpre-
4.3 Investigation of Dependence Structure

The investigation of this is that the even though the eigenvectors may not be equal, the eigenvectors between the two portfolios describes equally much of the variance. Since every eigenvector that is not noise should correspond to a factor that explains the correlation. Hence, it is not improbable that the factors between the two portfolios are the same factor. However, the shape of the eigenvectors need to be investigated further in order to deduce the exact characteristics. However, if that proves to be true that could imply that ESG scores simply doesn’t provide any information in the Asian markets as of today.

![Figure 10: Asia Pacific: Empirical eigenvalue distribution vs theoretical Marchenko-Pastur distribution](image)

In summary, the eigenvalues of the correlation matrix for the European and American markets share most of the same characteristics, i.e. that the leader portfolio is different from the reference portfolio. However, in Asia-Pacific no major difference between the two portfolios were found. The fit of the theoretical Marchenko-Pastur distribution which was applied to filter out noise from the data shows a good fit in all markets, as suspected. Some of the smallest eigenvalues are not captured in the distribution function since they are smaller than the theoretical smallest value $\lambda_-$. This is probably due to that some stocks have a very large sampled correlation and hence by buying one and selling the other efficiently decouples the volatility. Leading to a very small variance, a result shown in [27]. These results are however not stable over time and are hence treated as noise [29].

**Adjusted Variance of Decomposed Minimum Variance Portfolio** The analysis of the eigenvalues in paragraph *cumulative distribution of eigenvalues* indicated that the leader and reference portfolios seemed to differ from each other in distribution in Europe and North America. However, no clear difference was found in Asia-Pacific. The results from the eigenvalues for Europe and North America proved that the leader portfolio had a larger concentration of small eigenvalues in contrast to the reference portfolio and hence the empirical cumulative probability distributions of the eigenvalues had different characteristics.
Figure 11 represent the adjusted variance for the minimum variance portfolio in Europe as derived in Equation 14 on the y-axis and the number of $\lambda$ included on the x-axis. The eigenvalues are sorted in descending order i.e. e.g. when $x = 0$ the only eigenvalue included is the market eigenvalue. Furthermore the y-axis starts at the theoretical highest value for the Marchenko-Pastur distribution as defined in Equation 7.

Since every eigenvector is a potential portfolio to which asset weights can be allocated and the corresponding eigenvalue scales the variance of that portfolio. A smaller eigenvalue hence implies a lower variance in that portfolio to which asset weights can be allocated and the total portfolio can therefore be diversified to a lower level of variance. The adjusted variance for the simulations is usually higher for the reference portfolio when only the market eigenvalue is used. The interpretation from this is that the market eigenvalue for the reference portfolio describes more of the total variance for the underlying stocks (is larger) compared the leader portfolio. In contrast, the adjusted variance is often lower with the addition of one more eigenvalue, i.e. the reference portfolio decreases faster than the leaders portfolio. Hence, the leaders second largest eigenvalue is often larger than the reference second largest eigenvalue, i.e. $\lambda_{2}^{lead} > \lambda_{2}^{ref}$, which can be interpreted in two ways:

1. The leader portfolio have one factor (eigenvalue) that describes a large part of the variance in addition to the market eigenvalue

2. Since $\lambda_{2}^{lead} > \lambda_{2}^{ref}$, the portfolio corresponding to the second eigenvalue for the reference portfolio can diversify away more variance compared to the leader portfolio
4.3 Investigation of Dependence Structure

Similarly, in Figure 12 the adjusted minimum variance for the North American portfolio is presented. Compared to Europe, the adjusted variance for the North American portfolio of ESG leaders is lower than the reference index for both $k = 1$ and $k = 2$. The conclusion from this is that the North American ESG leaders can diversify away more variance both when they have access to the portfolio corresponding to the market eigenvalue only or several portfolios corresponding to several eigenvalues. However, the adjusted variance in Figure 12 seems to be more spread out in the simulations compared to the European case as seen Figure 11. Since the simulations are based on a random picks of different daily log returns, the North American portfolios may differ more than the European portfolio on a day to day basis regarding the variance and correlation.

For the Asian comparison, similar conclusions can be drawn as as the ones presented in paragraph cumulative distribution of eigenvalues, i.e. that there is no large difference between the leader and reference portfolios. This is clear since the leader portfolio is clearly in the
middle of the reference portfolio and no major deviations in the shape of the adjusted variance is seen.

**Filtering of Correlation and Covariance Matrix**  The correlation matrices are filtered to deduce the information and filter out the noise as described earlier. The filtering process is based on Random Matrix Theory and hence relies on how eigenvalues from a large random matrix are distribution. This is thereafter applied to the correlation matrices in order to filter out noise. As described in the summary of the *comparison of eigenvalues vs theoretical marchenko-pastur distribution* paragraph, some of the smallest eigenvalues are not fitted by well by the theoretical distribution. This is the case since they are smaller than the theoretically smallest value $\lambda_-$ from a random matrix, since some highly correlated assets could offset the variance if they are bought and sold simultaneously. Since such correlations are subject to sampling error and also due to the expected return of zero they are filtered out in this thesis and the focus is put into the largest eigenvalues.

Figure 14 show the filtered correlation matrices in Europe. Note that these are no longer are correlation matrices but approximations for the correlation matrices that share the similar dynamics. There are no longer ones along the diagonal but the diagonal elements show a higher value and hence appear more yellow in the figure.

![Figure 14: The filtering effects on the correlation matrices on the European portfolios](image)

The North American portfolios shown in Figure 15 shows a similar result as the European portfolios.
The Asian Pacific portfolio once again show a slightly different structure than the European and American portfolios as seen in Figure 16. One can note that the correlation in the reference portfolio is lower than the leader portfolio but the overall correlation is a little higher in the Asian portfolios.
Part III

RESULTS AND DISCUSSION
RESULTS

In this section the results of the thesis is presented. The section is divided into three parts, one for each region. The resulting filtered minimum variance portfolios derived from the leader and reference data is analysed for each region. The analysis commence with a discussion regarding the weights derived from filtered and unfiltered leader and reference portfolios. Thereafter, standard measures used in probability theory and statistics are presented to compare the filtered leader and reference portfolio as well as histograms of the daily returns. Finally, each region is concluded with an analysis of the tails using a QQ-plot, comparing the filtered reference and leader portfolio returns.

5.1 EUROPE

As seen in the Methodology Section - Section 4 - there seemed to be a difference between the European leader and reference portfolios. In this section, the final results and an analysis of the results will be presented.

5.1.1 Portfolio composition

The results from both the unfiltered and filtered portfolios are presented in Figure 17. The unfiltered portfolios will be able to achieve an artificially low volatility with a lower diversification since it can allocate weight to the eigenvectors that later is filtered out as noise. The filtered portfolio show a higher level of diversification which is to be expected. The number and magnitude of short positions are smaller in the filtered portfolios which also is to be expected since the linear dependencies should be lower. Hence, the need for short positions are lower in order to achieve the lowest possible variance within the portfolio. It is also noted that the filtered leader portfolio holds fewer short positions than the filtered reference portfolio. The weight of each company in the portfolio is shown in the y-axis direction in decimal form and add up to a total weight of one. The companies, about 50 in each portfolio are listed along the x-axes.
5.1.2 Portfolio return structure

The distribution of the returns for all simulated, bootstrapped, filtered minimum variance portfolios are presented as histograms in Figures 18, 21, 24.

In the histograms for the European region, in Figure 18, there seems to be a clear signs of a difference between the two returns. The returns for the filtered minimum variance reference portfolio looks fairly similar to ordinary stock returns, while the leader portfolio has several peaks. These peaks in the return series could indicate that there is stronger correlation among the companies in the leader portfolio as compared to the reference portfolio.
5.1 Europe

5.1.3 Mean & Variance

The mean of the two filtered portfolios are similar but the reference portfolio show a slightly higher mean in absolute terms but almost 7 times higher in relative terms as shown in Table 18. The variance and hence the standard deviation is however lower in the leader portfolio compared to the reference portfolio but the difference is again very small. Furthermore, the reference portfolio has a slightly higher empirical value at risk which is what one would expect from Figure 18. Finally, the skewness of the two portfolios are the same, but the reference portfolio has a higher excess kurtosis, i.e. the reference portfolio is more leptokurtic and has fatter tails than the leader portfolio.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Leaders</th>
<th>Measure</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>$2.9 \cdot 10^{-5}$</td>
<td>Variance</td>
<td>$3.4 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>$5.4 \cdot 10^{-3}$</td>
<td>Standard Deviation</td>
<td>$5.8 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>Mean</td>
<td>$9.5 \cdot 10^{-6}$</td>
<td>Mean</td>
<td>$6.4 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>VaR$_{0.05}$</td>
<td>$-1.3%$</td>
<td>VaR$_{0.05}$</td>
<td>$-1.4%$</td>
</tr>
<tr>
<td>VaR$_{0.001}$</td>
<td>$-2.3%$</td>
<td>VaR$_{0.001}$</td>
<td>$-2.6%$</td>
</tr>
<tr>
<td>Skewness</td>
<td>$-1.9$</td>
<td>Skewness</td>
<td>$-1.9$</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>$0.13$</td>
<td>Excess Kurtosis</td>
<td>$0.40$</td>
</tr>
</tbody>
</table>

Table 18: European Filtered Minimum Variance Portfolios - Probability measures
5.1.4 Q-Q Plot - Portfolio comparison in the tails

The left tail, which corresponds to losses, seems to be lower in the leader- than the reference portfolio as shown in Figure 19. The returns from the filtered minimum variance reference portfolio is plotted along the y-axis and the filtered minimum variance leader portfolio along the x-axis. The largest drawdowns in the portfolios are larger in the reference portfolio compared to the leader portfolio, as there seem to be a break point around $-2.0\%$. Hence, the returns from the two portfolios are approximately linear except for after the breakpoint in the left tail. This indicates that the distribution of returns for the reference portfolio has a heavier left tail than the distribution of returns for the leader portfolio.

![Figure 19: Europe: Empirical QQ-plot of filtered portfolio returns](image)

5.2 NORTH AMERICA

Similarly to the European case, the North American ESG leader portfolio seem to have a different correlation structure compared to the reference portfolio from what was captured in Section 4. In this section, the final findings of the paper for the North American portfolio will be presented and an analysis of the findings will be conducted.

5.2.1 Portfolio composition

The North American portfolios show the same general characteristics as its European counterparts but with some exceptions. The filtration process have the same effect on the portfolios and the number of short positions are fewer in the filtered portfolios once
again. However, the filtered leader portfolio is less diversified than both the European portfolios and the American filtered reference portfolio. The number of short positions in the two portfolios are similar this time.

![Investment weights in the two minimum variance portfolios](image)

**Figure 20: Investment weights in the two minimum variance portfolios**

5.2.2 Portfolio return structure

The return structure in the North American portfolios are similar but less extreme than its European counterpart as seen when comparing Figure 18, to the North American leader portfolio in Figure 21 which has one larger peak. This makes the samples look like that they do not seem to follow a some known distribution function, but rather that there is a correlation among the simulations that is not captured in the reference portfolios. The North American reference portfolio seems to have fatter and longer tails compared to the leader portfolio.
5.2.3 Mean & Variance

In the North American case, we can conclude that the leader portfolio has a higher mean, lower variance and slightly shorter tails and hence a lower empirical value at risk. This seems to indicate that the leader portfolio has an $\alpha$ over the reference portfolio. Here $\alpha$ is defined as Jensen’s alpha as explained in Section 3.6.4, which is returns that exceed the theoretical expected return i.e. abnormal returns [36]. However, we note from Table 10 and Table 12 that there is a clear difference in the country split for our leader portfolio and reference portfolio. Hence, the result may stem from the fact that the markets in general in Table 10 has outperformed the markets in Table 12 or that the ESG leaders has an actual $\alpha$ compared to the market. The leader portfolio has a lower absolute skewness compared to the reference portfolio, indicating that the leader portfolio has less of a left skew. Furthermore, the leader portfolio also has a lower kurtosis, indicating that the portfolios of ESG leaders has thinner tails.
### Table 19: North American Filtered Minimum Variance Portfolios - Probability measures

<table>
<thead>
<tr>
<th>Measure</th>
<th>Leaders</th>
<th>Measure</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
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<td>Variance</td>
<td>$3.3 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>$5.6 \cdot 10^{-3}$</td>
<td>Standard Deviation</td>
<td>$5.8 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>Mean</td>
<td>$8.7 \cdot 10^{-4}$</td>
<td>Mean</td>
<td>$4.7 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>VaR&lt;sub&gt;0.05&lt;/sub&gt;</td>
<td>$-1.2%$</td>
<td>VaR&lt;sub&gt;0.05&lt;/sub&gt;</td>
<td>$-1.3%$</td>
</tr>
<tr>
<td>VaR&lt;sub&gt;0.01&lt;/sub&gt;</td>
<td>$-3.1%$</td>
<td>VaR&lt;sub&gt;0.01&lt;/sub&gt;</td>
<td>$-3.4%$</td>
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<tr>
<td>Skewness</td>
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<td>$-0.7$</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>$0.8$</td>
<td>Kurtosis</td>
<td>$2.1$</td>
</tr>
</tbody>
</table>

5.2.4 *Q-Q Plot*

In Figure 22 it is clear that the reference portfolio actually has fatter tails than the leader portfolio. The two return series looks to be approximately equal in the center, but the tails between the two distributions seems to follow different probability distributions. The relationship between the right tails seems to be approximately linear for more samples compared to the left tail. However, the right tails between the simulated portfolios deviate from one another to a higher extent compared to the deviations in the left tail.

![Figure 22: North America: Empirical QQ-plot of filtered portfolio returns](image-url)
5.3 ASIA-PACIFIC

In this section the primary results from our investigated of the dependence structure for ESG leaders in Asia-Pacific will be presented. The result from Section 4 indicated initially that the correlation among the ESG leaders and reference portfolios in Asia-Pacific does not seem to differ. In this section, our finalised findings will be presented and an analysis will be conducted.

5.3.1 Portfolio composition

Similarly to the European and North American portfolios, the minimum variance portfolio based on the filtered correlation matrices seem to have reduced the number of short positions, as seen in Figure 23. The short positions are significantly reduced in the leader portfolio, but only partly reduced the reference portfolio. Furthermore, the filtered portfolios look more diversified, with smaller absolute investment weight to specific underlying stocks. The reference and leader portfolio seem to be approximately as diversified, with no absolute investment weight in the filtered portfolios of more than c. 6%, compared to the unfiltered portfolios who has several investment weights in the region around 15 – 20%.
5.3.2 Portfolio return structure

Comparing the histograms of the returns in Figure 24, the two histograms do not seem to deviate from one another in any significant way. Furthermore, both portfolio returns seem to have approximately as thin tails.

Figure 24: Histogram of the return structure from the two minimum variance portfolios
5.3.3 Mean & Variance

When comparing the different measures in Table 20 there seem to be no major difference in the variance and standard deviation of the portfolios. However, the mean of the leader portfolio is twice the mean for the reference portfolio but the difference in absolute terms is very slight. The general interpretation is that the results indicates that it may have been more profitable to invest in ESG leaders compared to a reference index and that these may have an $\alpha$ over the market. For the higher degree moment measure, Skewness and Kurtosis, both portfolios are the same, thus no portfolio seem to have significantly higher skew or fatter tails than the other one.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Leaders</th>
<th>Measure</th>
<th>Reference</th>
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<tbody>
<tr>
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<td>Variance</td>
<td>$3.1 \cdot 10^{-5}$</td>
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<tr>
<td>Standard Deviation</td>
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<td>Standard Deviation</td>
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</tr>
<tr>
<td>Mean</td>
<td>$2.6 \cdot 10^{-4}$</td>
<td>Mean</td>
<td>$1.3 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>VaR$_{0.05}$</td>
<td>$-1.3%$</td>
<td>VaR$_{0.05}$</td>
<td>$-1.4%$</td>
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<td>VaR$_{0.001}$</td>
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<td>VaR$_{0.001}$</td>
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<tr>
<td>Skewness</td>
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<td>Skewness</td>
<td>$-0.3$</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>$0.6$</td>
<td>Excess Kurtosis</td>
<td>$0.6$</td>
</tr>
</tbody>
</table>

Table 20: Asia Pacific Filtered Minimum Variance Portfolios - Probability measures

5.3.4 Q-Q Plot

Similarly to the result in Section 4 and Figure 24, the reference and leader filtered minimum variance portfolios does not seem to deviate from one another in a significant degree based on the Q-Q plot in Figure 25. There are minor deviations in the tails of the leader and reference simulations which may be due to the fact that ESG leaders should have a lower systematic risk, however, no extreme deviations are found.
Figure 25: Asia Pacific: Empirical QQ-plot of filtered portfolio returns
DISCUSSION

In this section, a discussion of the results found in the thesis will be presented. The discussion section is outlined by four parts: Regional differences, Flaws & Potential errors, ESG valuation risk and is finalised with implementation aspects of our findings.

6.1 REGIONAL DIFFERENCES

Some of the regions used, e.g. North America, has a lower number of underlying countries compared to e.g. Europe. As a result, Europe will have a stronger "country diversification" than North America and hence a wider spread in how the underlying countries in each respective region are ESG compliant or not. However, if a country analysis was conducted instead of a regional analysis several other flaws would arise. For example, some countries are much larger than others, some countries are not ESG compliant at all, etc. Because of these flaws, a regional analysis for this thesis was chosen.

As noted in the Section 4 the ESG investments in Europe and North America has a dependence structure that is different to the reference market, whereas in Asia Pacific there seem to be no difference. The fact that there is no difference in Asia Pacific is in line with current research, derived from the fact that the investors in Asia Pacific are not as mature regarding sustainable investing as North America and Europe [43]. In the future, if/when Asia Pacific becomes more mature regarding sustainable investing, the characteristics seen in the European and North American markets may be transferred to the Asia Pacific market.

On one hand, both the European and North American ESG investments seem to be different from the reference market in general. However, they are differentiated in separate ways. For European investments, one notes from the QQ-plot in Figure 19 that right tail is approximately equal between the leader and reference portfolio. However, in the left tail the ESG leader portfolio has a significant lower drawdown compared to the reference market. That is, the reference portfolio has a fatter left tail than the leader portfolio, while the right tail is approximately equal.

In contrast, the returns in the North American reference and leader portfolios are different to each other, but not in the same manner as the European portfolio. This is seen by comparing Figure 19 and Figure 22. From the QQ-plot in Figure 22, one notes
that the right tail between these two portfolios seems to differ quite a lot. That is, the North American ESG leading portfolio outperforms the North American reference portfolio more than the European leader portfolio outperforms the European reference. However, the reference portfolio also have significantly fatter left tails than the leader portfolio for both regions. This is both seen in the QQ-plot and in the value of the Kurtosis from both portfolios, as found in Table 19.

6.2 POTENTIAL FLAWS & ERRORS

This section is included to give the reader an understanding of the potential flaws & errors in this report. This is by no means significant errors, but it is important for the reader to know about these limitations when interpreting the result. For example, the results presented in this thesis reflect the current state of the market in the investigated time frame. The results could change or differ depending on the choice of time one investigates. Research have shown that the correlations rise in times of higher volatility why the time frame is important [20]. Furthermore, the results rely on the ESG metric provided by Sustainalytics which is one of the largest rating institutes. Since the definition of leaders is dependent on the Sustainalytics rating, the rating has a high impact on the portfolio construction.

6.2.1 Simulation errors

There could be some flaws in the technique used to simulate the different portfolios. The results from the histograms in the Figures 18, 21 and 24 are based on the filtered minimum variance portfolios for the reference and leader portfolios. However, the leader simulations are based on 100 bootstraps (on the days) of one ESG leader portfolio, whereas the reference simulations are based on 10 different randomly picked reference portfolios, bootstrapped 10 times each.

Due to this, the outcomes of the reference portfolio will have more underlying companies, even though each simulated portfolio will have the same number of companies. As a result, it is not surprising that there seem to be a higher correlation amongst the outcomes of the leader portfolio compared to the reference portfolio. This is since all leader portfolios will have the same underlying companies (but potentially different weights due to the bootstrapping). On the other hand, the outcomes of the reference portfolios are based on 10 randomly picked reference portfolios, bootstrapped 10 times each. Thus, all simulations will not have the same underlying companies and hence there should be a lower correlation amongst the simulations.
6.2.2 The market eigenvalue dilemma

Most research strongly supports that the market eigenvalue reflects the overall correlation structure in the market which implies that the portfolio constructed by the market eigenvector tracks the market in terms of both expected value and volatility [27, 28]. The market eigenvalue is however much larger than the theoretical largest eigenvalue that random matrix theory provides which is the underlying theory for the filtering process of the correlation matrices. Thus one could argue for deletion of the largest value in order to better model the correlation structure in harmony with random matrix theory [27].

However, the reverse argument should also be considered i.e. including the market eigenvalue. Since the objective of this thesis is on the differences between ESG investments and the aggregated market, the two market eigenvalues could provide important information on how the two portfolios differ. Research have for example claimed that ESG investments have a lower systematic risk then the aggregated market due to factors such as, exposure to ESG related risks, lower cost of capital etc. If the ESG investments in fact have a lower systematic risk and thus differs from the aggregated market, removing the market eigenvalue would affect the total structure of the modeled differences. The market eigenvalue is in fact lower in both Europe and North America which indicated that the systematic risk could be lower in the ESG portfolio which in terms implies that the market eigenvalue should be included in order to model the differences between the two portfolios which is why the market eigenvalue has been included in the analysis in this thesis.

6.2.3 Country split

As noted in Section 4.2 there seem to be certain flaws in the country split for the reference index, which was used as an approximation for the regional market. This is seen in all regions, e.g. in Table 7 (Europe) there seem to be some underlying companies from Mexico, United Stats and South Africa, in Table 12 (North America) there are companies from UK, Singapore, Ireland etc. and in Table 17 (Asia-Pacific) there are underlying companies from Ireland and Bermuda.

Since the underlying companies for the reference indices are provided by MSCI, whereas the country classifier is provided by Skandia’s SQL database, there could be some errors in the data. For example, MSCI could include a company that is registered in Bermuda but has it operations in Asia-Pacific, as a company that should be included in Asia-Pacific index, whereas from the country and region classifiers we have obtained from Skandia would not have included this in Asia-Pacific. Further-
more, there could also be errors if one stock is traded on several exchanges but is still classified to one region only from Skandia’s SQL database, whereas MSCI could include it in several indices. Please note that these reasons should be taken with reservation, since these are only hypothetical explanation as we have not investigated the actual reasons in detail.

Since these outliers are fairly small, and we chose the MSCI indices as an approximation for the market, we have chosen not to manipulate these since that could lead to personal bias and incorrect assessments by us.

6.3 ESG VALUATION RISK

The focus in this thesis is concentration risks which where interpreted as correlation risk. There could of course exist other risks within the ESG investment universe that have not been covered in this thesis nor have been captured by the methods applied in the thesis. One such risk that would not have been captured with this framework are valuation risks. As mentioned in the Section 2.1, valuation risks should be considered when making a full risk assessment of the ESG investment universe. However, concluding whether or not the higher valuation of ESG companies are legitimate or not is not easy. Even though ESG leading companies have a higher valuation compared to its non-ESG leading peers, this valuation could still be fair. The valuation could be fair since ESG leading companies often have a lower systematic risk, better risk management, and lower cost of capital. Hence, they maybe should have a higher valuation since these are less risky compared to the non-ESG leading peers. [4]. There is however no clear consensus in the matter and more research must be conducted in order to fully understand the potential valuation risk in ESG leading companies.

6.4 NON-CONSTANT CORRELATIONS

It is important for the reader to note that asset correlations are not constant over time and that equity correlation rises in times of higher volatility as put forward in Section 2.3. As a result, models of time varying correlations has become popular to capture this problem [45]. This has not been applied in this paper, and hence it is important for the reader to note that one should be careful to extrapolate the results of this report to correlations outside the period of the data. Instead, this report has shown that in the last year, there has been a difference amongst the correlation of leaders compared to the market, but this may not be true in the future.
6.5 IMPLEMENTATION ASPECTS

The following section is a brief executive summary of potential implementation aspects that could be considered based on the results found in this thesis. The interpretations are only suggestions and only reflects the specific historic time period treated in this thesis, past performance is no guarantee of future returns. The results found in this thesis only reflect one specific time period and hence may or may not stay true outside of the historic time period.

The implementation of ESG aspects in investment decisions has been partly driven by the upcoming regulation in the European Union. The results in this thesis indicate that including more ESG leading companies into a portfolio generally will not result in higher risk in terms of volatility, but rather the opposite as seen in Section 5. The dynamics of the two portfolios across the three regions prove some interesting results that could be interesting to explore further when implementing an ESG strategy to a portfolio. The first part of the section focuses on how an ESG tilt could affect a portfolio depending on the region where this tilt is implemented. The second part discusses some aspects that should be considered when extending an existing strategy to capture ESG factors. Finally the last section looks ahead and discusses potential future trends in the ESG area.

6.5.1 Interpretation of results

EUROPE The European reference portfolios do indicate a significantly higher risk in the left tail, i.e. the largest losses are more severe in the reference portfolio than in the leader portfolio as seen in Figure 19. This could be interpreted as that the risks of extreme downside events are lower in the ESG leader portfolio than in the European market in general. Hence, based on the results in this thesis, including more ESG companies into portfolios in the European market could protect the portfolio from some of the largest drawdowns of the portfolio compared to investing in the full index. These results should however be interpreted cautiously as they only reflect the time period studied in this thesis. Furthermore, the expected return is lower in the leader portfolio than in the reference portfolio, this is however offset by a lower variance in the leader portfolios, evident by Figure 19 and Table 18. This suggests that the inclusion of ESG companies could provide some protection to the most extreme negative returns but in turn offer a slightly lower expected return.

NORTH AMERICA The North American portfolios on the other hand indicate that the upside potential of the leader portfolio is inferior to the reference portfolio.
but the left tail risk seems to be approximately the same as seen in Figure 22. This implies that an investment in ESG leading companies have the approximately same downside risk but the potential largest gains are lower in the ESG leading companies compared to the reference index. The mean of the leader portfolio is slightly higher but the difference is very small as seen in Table 19 and could be a result of noise in the samples.

**ASIA-PACIFIC** The Asian portfolios seems to be less influenced by ESG rating than its two other regional counterparts and the ESG rating does not seem to influence returns nor risk in the portfolios. The result was expected. The portfolio dynamics of the leader and reference portfolio in Asian regions seems to be very similar. The empirical QQ-Plots in Figure 25 follows an almost perfect straight line expect for the two tails but the deviations in the tails are minor and are probably just due to noise.

### 6.5.2 Potential implementation effects

Implementing ESG factors as a factor to a Fama-French Factor model have been proven to be problematic partly due to the overlap between traditional factors and an ESG factor [10]. The best practice of implementation of ESG factors in allocation decisions probably varies over time. Earlier research indicated that implementing social responsibility was beneficial since it seemed to yield a higher return [46, 47]. However a review of the research concludes that the returns of ESG investments are no better than the market which also is concluded in a paper published in 2015 [48]. A study of Mutual Funds however concluded that fund that have a high ESG rating performed better in the financial crisis then the funds with a low ESG score [49]. How the two portfolios behave in a financial crisis is interesting and could be an interesting topic for further testing and research. The general results from this thesis is that the leader portfolios i.e. the portfolio with the highest ESG score has a lower volatility than the reference portfolio. However, there could be other risk factors such as valuation risk, covariance risks, poor ESG metrics or some other ESG specific risk that is hard to predict and could have an impact in a large drawdown in the markets.

Furthermore, the implementation aspects differ across regions. In order to conclude how this should be interpreted one need to conduct some further research. There are many ways to implement ESG factors into investment decisions, the easiest one being a manual filtering process. This thesis however has not investigated how different levels of ESG compliance affects returns, only how the companies with the highest scores compare to a reference portfolio which is supposed to proxy a large index of the region. Hence the effects from a filtering process is not handled in this thesis. The
general results however is that a reference portfolio compared to a portfolio of ESG leading companies have different effects in different regions.

Finally, how should a portfolio manager approach ESG metrics in order to achieve a better ESG implementation process? A recent paper discussed that the quality of the ESG data is the most important step in the process and the authors found that the best strategy is to focus on ESG items that are material for the specific industry of the company [50]. That means that the new Sustainalytics measure which was applied in this thesis already has the correct focus. If a portfolio manager wishes to extend the focus it is possible to get access to the items that the Sustainability Accounting Standards Board deem important for most industries [51]. Additionally the authors found that the ESG leaders did have a higher valuation in terms of market to book ratio than the companies with a lower ESG score. The authors could however not conclude any difference of returns between the two sets of companies. The results could however be of interest if a Fama-French model is to be extended to incorporate ESG aspects. The authors further argues that the lack of difference of returns may be due to expectations from investors that the two subsets, good and bad ESG companies, are to yield the same return. That could however change over time due to regulations, taxation and public opinion on ESG matters.

6.5.3 Potential future trends

The European markets are to be regulated in terms of ESG [38]. The North American market on the other hand, are not facing any regulations yet. However, some discussions have been held which could result in a even wider effects on the North American region compared to what was found in this thesis. The Asian markets however does not seem to be affected by ESG metrics today. The reason to that could be that the ESG metrics in the Asian region does not capture the dynamics as well or that the market is less mature in terms of ESG compliance. This could imply that the Asian markets have a potential development ahead that will focus more on ESG. This could potentially reward some of the ESG leading companies in the future as capital is moved from companies with a poor rating to companies with a better rating. In order to investigate how this could affect the Asian Market, one could investigate historical data on how ESG factors affected the European and North American markets in the past. There are also some possibility of a future regulation imposed on the Asian markets, which could could act as a catalyst for the process of ESG becoming a more important metric in the investment decision process. One should keep in mind that the Asian market is composed of mainly Japanese companies in the setting of this thesis so the results could be heavily influenced by the Japanese markets. It could
therefore be interesting to further investigate how ESG metrics are approached and interpreted in Japan in order to better understand how to implement ESG factors better on the Asian markets.
CONCLUSION

7.1 CONCLUSION

This thesis has aimed to answer the research question stated in Section 1.2, and primarily if there is there a different dependency structure amongst ESG investments compared to the market?

Based on the results from Section 6 it was concluded that there seem to be a difference amongst the linear dependence structure for ESG leaders compared to the market in Europe and North America, but no such findings were found in Asia-Pacific. The results provide asset managers and other interested parties an understanding of the characteristics of ESG investments compared to the overall market. Furthermore, the results also provide an indication of the potential effects that an ESG-tilt can have on a portfolio. In the European and North American regions, the ESG companies have a lower systematic risk, i.e. lower volatility. In Europe, as seen in Table 18, this lower systematic risk is offset by a lower mean whereas in North America the portfolio of ESG leaders has both a lower systematic risk and higher mean, given from Table 19.

This thesis has shown that there is in fact a different dependence structure amongst ESG leaders compared to the market, but that does not affect the portfolio performance significantly.

7.2 FURTHER RESEARCH

It is highly recommended for researchers and asset managers to study this area further. Topics of interest for further research are for instance:

- Study different time periods and horizons
- The use of other reference indices and other portfolio construction methods, as well as other portfolio optimisation methods
- Use time series to capture the change in dependence structure over time
- Analyse ESG momentum (i.e. when a company gets an upgraded ESG Risk Rating) instead of ESG leaders
• Analyse the extreme values more, e.g. by applying a Clayton canonical vine copula to model the left tail

• Analyse the effect of including or excluding the market eigenvalue

• Study how asset managers can implement these findings rigorously

• Instead of determining the sector split of the total reference index, researchers could determine the sector split for each randomly picked reference portfolio and thereafter use this sector split to create the leader portfolios. This could make each simulation more comparable and the problem with only 1 leader portfolio but 10 reference portfolio would be removed
BIBLIOGRAPHY


