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# Migration plan of Risky Total Return Swap to Bond Return Swap


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## Abstract

Since the 2008 crisis, the hedging instruments have gained popularity with financial institutions. This is the case of the total return swap that is used today by major institutions like Goldman Sachs or J.P. Morgan. Murex is a software provider for financial institutions. The company already had a total return swap product, the RTRS (for Risky Total Return Swap), but with the growing demand Murex decided to develop a new product, the BRS (Bond Return Swap). So now they have two bond total return swaps.

This master thesis aims to analyze total return swap and highlight the improvement of the BRS. After a theoretical analysis of the total return swap, a test campaign is realized. For different types of bond and different configurations of total return swap, formulas are derived to be compared to the returned values. The results given by the RTRS are good on basic bonds. If the bond is more complex, for instance a bond with credit risk or an amortized bond, the values returned by the RTRS are not reliable if not wrong. On the other hand, the BRS performs well in every situation and positions itself as the best total return swap proposed by Murex.

## Sammanfattning

Sedan finanskrisen 2008 har hedginginstrument blivit allt viktigare för finansinstitut. Detta är fallet med det så kallade Total Return Swaps (TRS) som används idag av stora institutioner såsom Goldman Sachs och J.P. Morgan. Murex är en mjukvaruleverantör som redan hade en TRS produkt, den RTRS (Risky Total Return Swap) . Men med den växande efterfrågan beslutade fretaget att utveckla en ny produkt, den så kallade BRS (Bond Return Swap). Så nu har de två TRS:ar.

Denna uppsats syftar till att analysera Total Return Swap och belysa de förbättringar som tillförs av BRS. Efter en teoretisk genomgång av TRS realiseras en serie tester. För olika typer av obligationer och olika konstellationer av TRS härleds formler och jämförs deras värden. Resultaten från RTRS verkar vara bra på basobligationer. Om obligationen är mer komplex, till exempel en obligation med kreditrisk eller en amorterad obligation, är RTRS returnerade värden inte tillförlitliga om inte fel. Å andra sidan presterar BRS bra i alla situationer och positionerar sig som det bästa Total Return Swap som föreslagits av Murex.

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Louis Maziere

# Contents

# 1 Introduction

Swaps agreements are very common in finance. They are used on a large range of underlying in order to enable two parties to exchange cash flows. The most common swaps are the interest rate swaps. They enable two parties to exchange usually a floating rate with a fixed rate. These contracts are a way for investors to get exposure on markets with the idea to make profits whereas the other part wants to be protected from the volatility of the market.

Like interest rate swaps, total return swaps are an financial instrument to gain or lose exposure to the returns of a security without having to own it. Even if this financial product is not new, there has been a growing demand for it, especially since the last financial crisis. Big institutions like Goldman Sachs or J.P. Morgan are building new strategies based on total return swaps while European institutions are using it to hedge against the credit risk.

## 1.1 Environment

The master thesis was done in the Security Finance team at Murex. Murex is an internationally renowned software developer specializing in the development and implementation of decision support systems and position management systems in the field of financial derivative instruments. Murex now has more than 2,200 employees in 17 offices all over the world. Hundreds of institutions are using it including sixties of the world's top banks.

The Security Finance team is a Front Office desk managing products whose purpose is to manage the securities inventories, to finance new positions and to generate additional revenues. The team is in charge of products like repository or security lending. Security Finance is also in charge of the products called "synthetic" because, unlike the repository for instance, they do not imply the exchange of security in the contract. The synthetic products are total return swaps on bonds, stocks or baskets of assets.

## 1.2 Principle of the Total Return Swap

A Total Return Swap is a contract in which one party gets exposure on the performance of an asset and the other party receives interests following an index or determined from the forecast of this future performance.

The underlying asset can be of different type. It can be a bond, an equity, an index or a basket of the previous assets.

The performance includes the cash flows of the assets like the dividends or the coupon flows. But what is interesting and gives to the total return swap its full meaning is that in the performance includes the variation of price of the underlying. The variation of price of the underlying is paid (or received depending on the ) on previously determined periods.

A schedule is agreed in advance between both parties with periods on which the performance flows are computed, more particularly on which a flow is paid on the variation of price. The schedule contains lots of information like the settlement delays after each payments or the dates on which the dividends or the coupons are paid over the period.

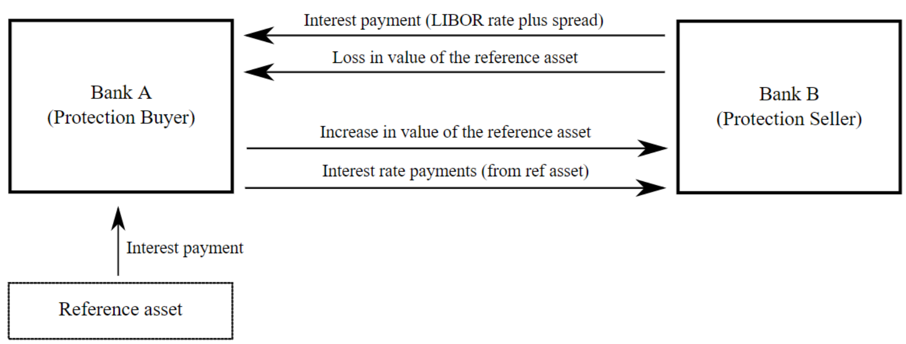


Figure 1: Principle of the Total Return Swap

In the figure 1, the principle of total return swap is synthesized. The performances of the bonds are transferred to the protection seller (also called performance buyer) and the protection buyer gets interests based on a floating rate (here the Libor) and a spread (the TRS rate) and protection flows that cover the loss in value of the underlying and in the case of a bond a recovery flow if default.

## 1.3 Aims of the Total Return Swap

The total return swap is first a way to get a full exposure to an underlying asset without having to buy it. On this purpose it joins other Security Finance products like the security lending (this contract enables one party to get a stock in exchange of a fee and a collateral asset). But contrary to a security lending, the TRS is a synthetic product, the total return receiver does not get the underlying asset, only the performance (and the performance seller does not have to own the underlying). First it can have some accounting advantage but this point will not be developed here. Then it enables to get access to closed markets or simply to avoid fees and long processes. To give an example, many trading website do not propose the users to buy stocks but they propose them to buy total return swaps on the stocks. The users can trade stocks more quickly, more easily and on international markets with low fees.

An other important aspect of the TRS is that by selling the performance of the asset, the party can get protection against risks. Indeed, the payment of the difference of price enables to protect against the market risks. For example if an underlying stock price falls, the total return seller is covered by the performance flow.

Furthermore in the case where the underlying is a bond, the total return seller is protected against a default of the bond. The TRS works as a Credit Default Swaps. The future cash flows are estimated with the credit risk in order to adjust the interest payments. And a recovery flow is paid to the protection buyer if the bond defaults.

Also as explained before, the total return swap is a synthetic product which means that there is no exchange of the underlying asset and the performance seller does not have to own the underlying asset. On this point of view, the TRS is only a contract agreed by two parties to exchange cash flows indexed on an underlying asset.

The total return swap is like many derivative. It can be used as a protection tool to hedge a portfolio against both the market risk and the credit risk or it can be used as a speculative tool on underlying assets not owned by any part of the contract. That is why this instrument grew in popularity over the last years.

## 1.4 Introduction of the RTRS and the BRS

Murex has developed financial instruments to cover the total return swaps. Originally two products were developed to cover the TRS on bonds and on stocks. The RTRS, for risky total return swap, is the instrument dedicated to bonds. This product begins to be old. Murex wanted to be effective and reliable on simple transactions. However with the growing demand for total return swaps over the last years, the RTRS was not good enough to cover all the client's needs and not reliable enough facing the increasing complexity of the trades.

In this context, Murex decided a few years ago to develop new products to cover total return swaps on stocks, bonds and an additional one for baskets. In this master thesis, only the bond total return swap will be studied. The new total return swap provided by the company is called bond return swap or BRS. The BRS is more adapted to the client's needs, more ergonomic and more efficient.

Now that the BRS is operational, there is no point in keeping the two products. However, no testing campaign has been done. Therefore in this thesis, the total return swaps on bonds are studied. Theoretical results will be compared to the outputs of the BRS and the RTRS in order first to attest to the quality of the BRS and to highlight the improvements of the BRS compared to the RTRS.

## 1.5 Project Goals

The Murex TRS are built with models given by clients. Murex goal is to provide solutions to the client's needs. The master thesis aims to check that the behaviors of the two products are in line with the theory and to do a comparaison between the RTRS and the BRS on the most used bonds.

The project goals of the master thesis are to:

- derive the mathematical models behind the TRS on bond to check the formulas and verify that the Murex TRS are in line with the theoretical results.
- identify the weaknesses of the RTRS.
- highlight the improvements made on the BRS compared to the RTRS.
- provide a documentation on bond TRS and the associated Greeks.
- perform a test campaign on the main bonds used by the clients.

## 1.6 Thesis Disposition

In Section 2 the mathematical model is defined and the pricing formulas are derived. The formulas are theoretical. At the end of this Section, the formulas are alleviated in order to get more understandable results. In Section 3, the expressions of the Greeks are derived from the pricing formula already defined.

After the mathematical analysis, the TRS flows and Greeks can be computed. The formulas need to be adapted to the different bonds. Section 4 presents the bonds chosen for the test campaign. The theoretical formulas are defined for each of them. Before looking at the results, a global comparison is made between the two Murex TRS in Section 5. With the data from Murex the theoretical values are computed and the results are compared to the returned values of the RTRS and the BRS in Section 6.

# Part I

# Mathematical Background

The aim of this part is to derive the mathematical formulas to price the total return swap and to derive the Greeks. The obtained formulas will not be used directly under this form for the following parts. At the end of this part, the formulas derived here will be alleviated in order to be more meaningful and practical for the next sections.

## 2 Mathematical Model

### 2.1 The Heath–Jarrow–Morton Framework

Before to begin, let us introduce the mathematical environment to develop the financial mathematics behind the total return swap.

The environment is an uncertain economy defined as a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ satisfying the hypothesis of completeness and non-arbitrage. Under the measure $\mathbb{P}$ the discounted bond prices are martingales. The filtration without default $\mathcal{F}_t$ represents information about the interest rate and $\mathcal{G}_t$ the filtration with default. Let $\mathcal{H}_t = \mathcal{F}_t \bigvee \mathcal{G}_t$. $\mathbb{P}$ is directly taken as the risk-neutral pricing measure. Under $\mathbb{P}$ on filtration $\mathcal{H}_t$, the expectation at time $t$ is denoted by $E$.

The state variable studied here is the interest rate $r_t$. In order to model the interest rates, the Heath–Jarrow–Morton (HJM) framework will be used. This framework goes further than models like short rate models and describes fully the dynamics of the forward rate curve.

Under the risk-neutral measure $\mathbb{P}$, the stochastic differential of the forward rate $f(t,T)$, for $t \leq T$, is given by:

$$df(t,T) = \mu(t,T)dt + \sigma(t,T)dW_t$$

where $W$ is a Brownian motion under $\mathbb{P}$ and with the $\mathcal{H}_t$-adapted processes $\mu(t,T)$ and $\sigma(t,T)$.

Under the measure $\mathbb{P}$, the drift term in the HJM framework can be expressed as a function of the volatility:

$$\mu(t,T) = \sigma(t,T)\int_t^T \sigma(t,s)^T ds$$

The aim here is not to dig deeply in the HJM framework. Now that the main model is set and the forward rate is defined, the bond prices can be expressed. For a zero-coupon bond paying 1 at the maturity $T$, the forward price at $t$ is linked to the forward rate by:

$$p(t,T) = e^{-\int_t^T f(t,u)du}$$

The instantaneous interest rate is derived from the forward rate:

$$r_t = f(t,t)$$

At the interest rate $r_t$ the discounted process is :

$$D(t) = e^{-\int_0^t r_u du}$$

The HJM framework enables to have the following relations:

$$p(t,T) = e^{-\int_t^T f(t,u)du} = E\left[e^{-\int_t^T r_u du} \mid \mathcal{H}_t\right]$$

and

$$p(0,T) = e^{-\int_0^T f(0,u)du} = E\left[e^{-\int_0^t r_u du}\right]$$

This price of the zero-coupon bond is the discounted price of 1\$ paid at T. From now on, the price $p(0,T)$ will be denoted by $DF(T)$, for discount factor. The above relation with the notations becomes:

$$DF(T) = E[D(T)]$$

On a fixed set of time $T_1$, ... , $T_n$, the Libor forward rate is defined on the period $[T_{i-1}, T_i]$ by:

$$\text{For t in } [T_{i-1}, T_i],\ \ L(t,T_i) = \frac{1}{\delta_i}\frac{p(T_{i-1},t) - p(t,T_i)}{p(t,T_i)}$$

where $\delta_i = \frac{T_i - T_{i-1}}{360}$.

## 2.2 Default Model

The particularity of bonds is the risk of default. Total return swaps are used to hedge a portfolio against credit risk or on the contrary to gain exposure.

For the bond issuer, the time of default is denoted by $\tau$. This is the first time when the default occurs. This random variable is linked to the counting process N. N is incremented when a default occurs. $N(t)$ is the number of default by time t and $\tau$ is the first time of default when N becomes equal to 1. N is assumed to follow a inhomogeneous Poisson distribution (this is a common assumption for default bond, it will not be discussed here):

- $P[N(0) = 0] = 1$
- $P[N(t + dt) - N(t) = 1] = \lambda_t dt$
- $P[N(t + dt) - N(t) > 1] = 0$

The increments of N are independent:

$$\begin{aligned} P[N(t+dt) = 0] &= P[N(t) = 0].P[N(t+dt) - N(t) = 0] \\ &= P[N(t) = 0](1 - \lambda_t dt) \\ \frac{dP[N(t) = 0]}{dt} &= -\lambda_t P[N(t) = 0] \\ P[N(t) = 0] &= e^{-\int_0^t \lambda_s ds} \end{aligned}$$

The non-default probability is defined by:

$$NDP(t) = P(\tau > t) = P[N(t) = 0] = e^{-\int_0^t \lambda_s ds} \tag{1}$$

## 2.3 Pricing Formula of the Total Return Swaps

The TRS is a contract with maturity $T$ such as the TRS ends before the bond. The contract is an exchange of flows at the time $T_1$, ... , $T_n$ between 0 and $T$ with $T_0 = 0$ and $T_n = T$. The variation $\delta_i = \frac{T_i - T_{i-1}}{360}$ will be considered fixed: $\delta_i = \delta$.

On the one hand, the total return receiver gets the performance of the bond. The cash flows are denoted by $C_i$ at time $T_i$. They are exchanged to the condition that there is no default before the time $T_i$. $C_i$ includes all the performance flows (the coupons and variations of price). The performance leg is the sum of the discounted cash flows of this bond. With the discounting term $D(t)$ and the non-default condition $\mathbb{1}(\tau > T_i)$, the value of the performance leg is equal to:

$$E[CF_{perf}] = E\left[\sum_{i=1}^{n} D(T_i).C_i.\mathbb{1}(\tau > T_i)\right] \tag{2}$$

On the other hand, the interest payments are the combination of a floating rate, here taken as the Libor rate $L(T_{i-1}, T_i)$, and the TRS *rate* at time $T_i$, to the condition that there is no default before the time $T_i$. To simplify, the schedule is in line with the performance leg schedule. $N$ is the current nominal of the TRS : $N =$ Quantity of underlying $\times$ Current Price of the bond . Over the period $[T_{i-1}, T_i]$, the nominal is $N_{i-1} = Q.P_{i-1}$ with Q the quantity of underlying and $P_{i-1}$ the price at $T_{i-1}$.

As the TRS is also a credit-protection instrument, a recovery amount is paid in case of a default over the period. For a recovery rate $RR$, the capital recovered is $RR.N_{bond}$ with $N_{bond}$ the nominal of the bond itself (not the nominal of the TRS, it is defined as the quantity times the reference price of the bond). As for the performance leg, the interest leg is the sum of the discounted interest flows:

$$E[CF_{interest}] = E\left[\sum_{i=1}^{n} D(T_i).\delta.N_{i-1}(L(T_{i-1}, T_i) + rate)\mathbb{1}(\tau > T_i)\right] + E\left[D(\tau).N_{bond}.RR.\mathbb{1}(\tau < T)\right] \tag{3}$$

Under the non-arbitrage condition holds the following equation:

$$\begin{aligned} E[CF_{perf}] &= E[CF_{interest}] \\ E\left[\sum_{i=1}^{n} D(T_i).C_i.\mathbb{1}(\tau > T_i)\right] &= E\left[\sum_{i=1}^{n} D(T_i).\delta.N_{i-1}(L(T_{i-1}, T_i) + rate)\mathbb{1}(\tau > T_i)\right] \\ &\quad + E\left[D(\tau).RR.N_{bond}.\mathbb{1}(\tau < T)\right] \end{aligned} \tag{4}$$

The pricing formula can be derived from this formula. Pricing the total return swap is equivalent to finding the TRS rate:

$$\begin{aligned} \Pi_{BRS} &= rate \\ &= \frac{E\left[\sum_{i=1}^{n} D(T_i).\mathbb{1}(\tau > T_i)(C_i - \delta.N_{i-1}.L(T_{i-1}, T_i))\right]}{E\left[\sum_{i=1}^{n} \delta.N_{i-1}.D(T_i).\mathbb{1}(\tau > T_i)\right]} - \frac{E\left[D(\tau).RR.N_{bond}.\mathbb{1}(\tau < T)\right]}{E\left[\sum_{i=1}^{n} \delta.N_{i-1}.D(T_i).\mathbb{1}(\tau > T_i)\right]} \end{aligned} \tag{5}$$

Let us compute the different components of this pricing equation:

$$
\begin{aligned}
&E\left[\sum_{i=1}^{n} D(T_i).\mathbb{1}(\tau > T_i)(C_i - \delta.N_{i-1}.L(T_{i-1},T_i))\right] \\
&\qquad = \sum_{i=1}^{n}(C_i - \delta.N_{i-1}.L(T_{i-1},T_i)).E\left[e^{-\int_0^T r_u du}\right].E\left[\mathbb{1}(\tau > T_i)\right] \\
&\qquad = \sum_{i=1}^{n}(C_i - \delta.N_{i-1}.L(T_{i-1},T_i)).DF(T_i).P(\tau > T_i) \\
&\qquad = \sum_{i=1}^{n}(C_i - \delta.N_{i-1}.L(T_{i-1},T_i)).DF(T_i).NDP(T_i) \qquad (6)
\end{aligned}
$$

Let us recall that the formula of the non default probability is:

$$NDP(t) = e^{-\int_0^t \lambda_s ds}$$

Similarly, the second formula gives :

$$E\left[\sum_{i=1}^{n} \delta.N_{i-1}.D(T_i).\mathbb{1}(\tau > T_i)\right] = \sum_{i=1}^{n} \delta.N_{i-1}.DF(T_i).NDP(T_i) \qquad (7)$$

Finally, the last formula gives:

$$
\begin{aligned}
E\left[D(\tau).RR.N_{bond}.\mathbb{1}(\tau < T)\right] &= RR.N_{bond}.E\left[D(\tau).\mathbb{1}(\tau < T)\right] \\
&= RR.N_{bond}.E\left[E[D(\tau).\mathbb{1}(\tau < T) \mid \mathcal{H}_\tau]\right] \\
&= RR.N_{bond}.E\left[DF(\tau).\mathbb{1}(\tau < T)\ \right] \\
&= RR.N_{bond}\int_0^T DF(s).d\mathbb{P}(\tau < s) \\
&= -RR.N_{bond}\int_0^T DF(s).dNDP(s) \\
&= RR.N_{bond}\int_0^T DF(s).NDP(s).\lambda_s ds \qquad (8)
\end{aligned}
$$

Now let us insert it in the pricing formula:

$$\Pi_{BRS} = \frac{E\left[\sum_{i=1}^{n} D(T_i).\mathbb{1}(\tau > T_i)(C_i - \delta.N_{i-1}.L(T_{i-1},T_i))\right]}{E\left[\sum_{i=1}^{n} \delta.N_{i-1}.D(T_i).\mathbb{1}(\tau > T_i)\right]} - \frac{E\left[D(\tau).RR.N_{bond}.\mathbb{1}(\tau < T)\right]}{E\left[\sum_{i=1}^{n} \delta.N_{i-1}.D(T_i).\mathbb{1}(\tau > T_i)\right]},$$

to obtain:

$$\Pi_{BRS} = \frac{\sum_{i=1}^{n}(C_i - \delta.N_{i-1}.L(T_{i-1},T_i)).DF(T_i).NDP(T_i)}{\sum_{i=1}^{n}\delta.N_{i-1}.DF(T_i).NDP(T_i)} - \frac{RR.N_{bond}\int_0^T DF(s).NDP(s).\lambda_s ds}{\sum_{i=1}^{n}\delta.N_{i-1}.DF(T_i).NDP(T_i)} \tag{9}$$

## 2.4 Convenient Formula

Above a mathematical model has been built around the Total Return Swaps. But to be used the formula has to be more accessible.

The pricing formula is only a technical tool to price TRS. The most interesting formula is the payoff equation:

$$\begin{aligned} E[CF_{perf}] &= E[CF_{interest}] \\ E\left[\sum_{i=1}^{n} D(T_i).C_i.\mathbb{1}(\tau > T_i)\right] &= E\left[\sum_{i=1}^{n} D(T_i).\delta.N_{i-1}(L(T_{i-1},T_i) + rate)\mathbb{1}(\tau > T_i)\right] \\ &+ E\left[D(\tau).RR.N_{bond}.\mathbb{1}(\tau < T)\right] \end{aligned}$$

Using the previous calculation, the following equation holds:

$$\begin{aligned} \sum_{i=1}^{n} C_i.DF(T_i).NDP(T_i) &= \sum_{i=1}^{n} \delta.N_{i-1}.(L(T_{i-1},T_i) + rate).DF(T_i).NDP(T_i) \\ &+ RR.N_{bond}\int_0^T DF(s).NDP(s).\lambda_s ds \end{aligned} \tag{10}$$

The notation $C_i$ (the performance flows) includes the coupon flows and the variations of the prices over the period $[T_{i-1}, T_i]$. The coupons are assumed to be paid at the end of the period, at $T_i$ along with the flow for the variation of price. $CPN_{i-1}$ is defined as the sum of the coupons over the period $[T_{i-1}, T_i]$, $P_i$ the price of the bond at time $T_i$ and $Q$ the quantity of underlying bonds. Then,

$$\begin{aligned} C_i &= CPN_{i-1} + Q.(P_i - P_{i-1}) \qquad \text{if } i < n \\ C_n &= FV \qquad \text{the face value if } T_n\text{, the TRS maturity, is also the maturity of the bond} \end{aligned}$$

From now on the TRS is assumed to end before the maturity of the bond. So the bond flows will only be of type: $CPN_{i-1} + Q.(P_i - P_{i-1})$. The equation becomes:

$$\sum_{i=1}^{n} CPN_{i-1}.DF(T_i).NDP(T_i) + \sum_{i=0}^{n-1} Q.(P_{i+1} - P_i).DF(T_{i+1}).NDP(T_{i+1})$$
$$= \sum_{i=1}^{n} \delta.N_{i-1}.(L(T_{i-1}, T_i) + rate).DF(T_i).NDP(T_i) + RR.N_{bond} \int_0^T DF(s).NDP(s).\lambda_s ds \quad (11)$$

This formula is already more concrete. Until here a bond with credit risk and interests based on a financial index was considered. To go further a more basic example is studied, without credit risk where interest are only based on a fixed rate, the above formula is now simply:

$$\sum_{i=0}^{n-1} CPN_i.DF(T_{i+1}) + \sum_{i=0}^{n-1} Q.(P_{i+1} - P_i).DF(T_{i+1}) = \sum_{i=0}^{n-1} \delta.N_i.rate.DF(T_{i+1}) \quad (12)$$

Pricing the bond total return swaps is simply reduced here to isolate the rate and compute it.

# 3 Greeks

The Greeks represent the sensitivity of the financial product to parameters. For total return swaps with an underlying bond, the product is dependent on interest rates (DV01), credit risk (CR01)... The DV01 is the most important Greek since interest rate is the most impacting parameter on bonds. But it is also more complex to compute. The CR01 will be more developed. Additional Greeks could be used, but only the DV01 and the CR01 will be analyzed in this section.

## 3.1 DV01

The DV01 is the Greek which represents the sensitivity of the TRS to the interest rate. It is the change of value measured per basis point (not in percentage). For a bond TRS, the interest rate is the most impacting parameter. The DV01 is therefore the main Greek to be considered.

The NPV formula derived before is:

$$NPV = \sum_{i=1}^{n} -C_i.DF(T_i).NDP(T_i) + \sum_{i=1}^{n} \delta.N_{i-1}.(L(T_{i-1},T_i) + rate).DF(T_i).NDP(T_i) \\ + RR.N_{bond} \int_0^T DF(s).NDP(s).\lambda_s ds \qquad (13)$$

with $C_i = CPN_{i-1} + Q.(P_i - P_{i-1})$

The integral can be decomposed as follow:

$$\int_0^T DF(s).NDP(s).\lambda_s ds = \sum_{i=0}^{n-1} \int_{T_i}^{T_{i+1}} DF(s).NDP(s).\lambda_s ds$$

In practice, there is the DV01(zero) which represents the sensitivity to the zero-coupon rates of the interest rate curve and the DV01(par) which is the sensitivity linked to the market quote. If $R_t$ is the zero-coupon rate at t, the DV01(zero) is the derivative of the NPV with respect to $R_t$:

$$DV01(zero)_t = \frac{\partial NPV}{\partial R(t)} \qquad (14)$$

$$DV01(zero)_t = \sum_{i=1}^{n} -\frac{\partial(C_i.DF(T_i))}{\partial R(t)}.NDP(T_i) + \sum_{i=1}^{n} \delta.N_{i-1}.(L(T_{i-1},T_i) + rate).\frac{\partial DF(T_i)}{\partial R(t)}.NDP(T_i) \\ + RR.N_{bond} \sum_{i=1}^{n} \frac{\partial}{\partial R(t)} \left[ \int_{T_i}^{T_{i+1}} DF(s).NDP(s).\lambda_s ds \right] \qquad (15)$$

The DV01(par) is the derivative with regards to the market quote. It is linked to the DV01(zero) by:

$$DV01(par)_t = \left[\frac{\partial R_t}{\partial MR}\right]^t .DV01(zero) \tag{16}$$

However it is not easy to compute directly because of the derivative of the integral. What is done in Murex, the zero-coupon rates are discretized in pillars and the values are interpolated between the pillars. For a sample of time $t_k$ for k=1, ... ,n between 0 and T (the TRS maturity) and $R_k$ the zero-coupon rate at time $t_k$, the DV01(zero) is defined as:

$$DV01(zero) = \begin{pmatrix} DV01(zero)_1 \\ \vdots \\ DV01(zero)_m \end{pmatrix} = \begin{pmatrix} \frac{\partial NPV}{\partial R(t_1)} \\ \vdots \\ \frac{\partial NPV}{\partial R(t_m)} \end{pmatrix} \tag{17}$$

Using the above formula, the DV01 could be derived. However the calculation is complex because too many parameters are impacted. The CR01 is more detailed.

## 3.2 CR01 (zero)

The CR01 is the credit risk Greek. It represents the effect of a change of the credit curve on the value. As with the DV01, before to derive the Greek expression the CR01(zero) and the CR01(par) need to be differentiated. The CR01(zero) is the sensitivity to the default spreads of the credit curve. The CR01(par) is the sensitivity to the market quotes.

The NPV of the TRS is defined by:

$$\begin{aligned} NPV = \sum_{i=1}^{n} -C_i.DF(T_i).NDP(T_i) + \sum_{i=1}^{n} \delta.N_{i-1}.(L(T_{i-1}, T_i) + rate).DF(T_i).NDP(T_i) \\ + RR.N_{bond} \int_0^T DF(s).NDP(s).\lambda_s ds \end{aligned} \tag{18}$$

with $C_i = CPN_{i-1} + Q.(P_i - P_{i-1})$

Let us recall that :

$$NDP(s) = e^{-\int_0^s \lambda_u du}$$

The default spread is defined as:

$$DS(s) = \frac{\int_0^s \lambda_u du}{s}$$

Then

$$NDP(s) = e^{-DS(s).s}$$

The CR01 is computed like the DV01. It is the derivative of the NPV with respect to the default spread $DS$:

$$CR01(zero) = \frac{\partial NPV}{\partial DS(t)} \tag{19}$$

$$\begin{aligned} CR01(zero) = \sum_{i=1}^{n} -C_i.DF(T_i).\frac{\partial NDP(T_i)}{\partial DS(t)} + \sum_{i=1}^{n} \delta.N_{i-1}.(L(T_{i-1},T_i)+rate).DF(T_i).\frac{\partial NDP(T_i)}{\partial DS(t)} \\ +RR.N_{bond}\sum_{i=1}^{n} \frac{\partial}{\partial DS(t)}\left[\int_{T_i}^{T_{i+1}} DF(s).NDP(s).\lambda_s ds\right] \end{aligned} \tag{20}$$

Similarly, the default spreads are discretized in a few pillars and the values are interpolated between the pillars. With $DS(t_k)$ the pillar of the credit curve at $t_k$, the sensitivity to the default spread is directly projected on the credit curve pillars:

$$CR01(zero) = \begin{pmatrix} CR01(zero)_1 \\ \vdots \\ CR01(zero)_m \end{pmatrix} = \begin{pmatrix} \frac{\partial NPV}{\partial DS(t_1)} \\ \vdots \\ \frac{\partial NPV}{\partial DS(t_m)} \end{pmatrix} \tag{21}$$

The $CR01(zero)_k$ are as before:

$$CR01(zero)_k = \frac{\partial NPV}{\partial DS(t_k)} \tag{22}$$

$$\begin{aligned} CR01(zero)_k = \sum_{i=1}^{n} -C_i.DF(T_i).\frac{\partial NDP(T_i)}{\partial DS(t_k)} + \sum_{i=1}^{n} \delta.N_{i-1}.(L(T_{i-1},T_i)+rate).DF(T_i).\frac{\partial NDP(T_i)}{\partial DS(t_k)} \\ +RR.N_{bond}\sum_{i=1}^{n} \frac{\partial}{\partial DS(t_k)}\left[\int_{T_i}^{T_{i+1}} DF(s).NDP(s).\lambda_s ds\right] \end{aligned} \tag{23}$$

To compute the derivative, the default spread is interpolated from the credit curve. Linear interpolation is used assuming a forward default spread constant. At time t between $[t_k, t_{k+1}]$ the interpolation of the default spread gives:

$$t.DS(t) = \frac{t_k - t}{t_{k+1} - t_k}.t_k.DS(t_k) + \frac{t - t_{k+1}}{t_{k+1} - t_k}.t_{k+1}.DS(t_{k+1})$$

The default intensity can be derived as:

$$\lambda(t) = \frac{t_{k+1}}{t_{k+1} - t_k} DS(t_k) - \frac{t_k}{t_{k+1} - t_k} DS(t_{k+1})$$

At time $T_i$ between $[t_k, t_{k+1}]$ the derivative of the non default probability with regards to $DS(t_k)$ is:

$$\begin{aligned}
\frac{\partial NDP(T_i)}{\partial DS(t_k)} &= \frac{\partial NDP(T_i)}{\partial DS(T_i)} \frac{\partial DS(T_i)}{\partial DS(t_k)} \\
&= -\frac{T_i}{360} NDP(T_i) \frac{\partial DS(T_i)}{\partial DS(t_k)} \\
&= -\frac{T_i}{360} NDP(T_i) \frac{t_k - t}{t_{k+1} - t_k} \frac{t_k}{T_i} \qquad (24)
\end{aligned}$$

For the integral, the interval $[T_i, T_{i+1}]$ is assumed to be included in $[t_k, t_{k+1}]$.

$$\begin{aligned}
&\frac{\partial}{\partial DS(t_k)} \int_{T_i}^{T_{i+1}} DF(s).NDP(s).\lambda_s ds \\
&= \int_{T_i}^{T_{i+1}} DF(s) \frac{\partial NDP(s).\lambda_s}{\partial DS(t_k)} ds \\
&= \int_{T_i}^{T_{i+1}} DF(s) \left[ \frac{\partial NDP(s)}{\partial DS(t_k)} \lambda_s + \frac{\partial \lambda_s}{\partial DS(t_k)} NDP(s) \right] ds \\
&= \int_{T_i}^{T_{i+1}} DF(s) \frac{s}{360} NDP(s) \frac{t_{k+1} - s}{t_{k+1} - t_k} \frac{t_k}{s} \left[ \frac{t_{k+1}}{t_{k+1} - t_k} DS(t_{k+1}) - \frac{t_j}{t_{k+1} - t_k} DS(t_k) \right] ds \\
&\quad - \int_{T_i}^{T_{i+1}} DF(s).NDP(s) \frac{t_j}{t_{k+1} - t_k} ds \qquad (25)
\end{aligned}$$

These two expressions inserted in the definition of the CR01 enable to compute the value.

## 3.3 CR01(par)

For the CR01(par), let us recall that it is the sensitivity to the market rates. It can be derived from the CR01(zero):

$$CR01(par) = \left[ \frac{\partial DS}{\partial MR} \right]^t .CR01(zero) \qquad (26)$$

For the market rates at time $t_k$ $MR_k$, the matrix $\frac{\partial DS}{\partial MR}$ is defined by:

$$\frac{\partial DS}{\partial MR} = \begin{pmatrix} \frac{\partial DS_1}{\partial MR_1} & 0 & \cdots & 0 \\ \frac{\partial DS_2}{\partial MR_1} & \frac{\partial DS_2}{\partial MR_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial DS_m}{\partial MR_1} & \cdots & \frac{\partial DS_m}{\partial MR_{m-1}} & \frac{\partial DS_m}{\partial MR_m} \end{pmatrix} \tag{27}$$

The matrix is triangular because the default spreads depend only on the previous market rates. From this matrix and with the expression of the CR01(zero), the CR01(par) can be derived.

## 3.4 In Practice

The DV01 and CR01 expressions derived are theoretical formulas that can be used by a computer to compute the Greeks. But they are not very convenient to verify the values. From now on the Greeks will be computed more simply. The interest rate curves and credit curves of the TRS can be bumped (it means that they are shifted of one basis point) and the parameters are updated automatically. The Greek is approximated to the variation of the net present value of the trade. It will be developed in the next section.

# Part II

# Methodology

The aim of the project is to analyze and compare the two bond TRS developed by Murex. The most effective way to do it is to analyze different types of bond and check the data returned by the two Murex products. In order to achieve it, the results must be compared to the theoretical values expected for each bond. In this part, the theoretical formulas for flows and Greeks will be derived on several bonds.

## 4 Test Campaign

The different examples studied in the tests come from Murex database. For confidentiality reasons, Murex does not want information or data to be written explicitly. The bonds used will therefore not be named.

To do the comparison, the RTRS and BRS are set with the same underlying bond and the same schedule. So they represent the same TRS contract. The data used are also the same: the prices, the discount factors and all the other factors. Using these data, the theoretical flows and Greeks can be computed and compared to the returned values of the Murex products.

### 4.1 Flows

The flows formula has already been derived in the first section. The formula with the performance flows, the coupon flows, the interests and including the credit risk is:

$$\sum_{i=0}^{n-1} CPN_i.DF(T_{i+1}).NDP(T_{i+1}) + \sum_{i=0}^{n-1} Q.(P_{i+1} - P_i).DF(T_{i+1}).NDP(T_{i+1})$$

$$= \sum_{i=0}^{n-1} \delta.N_i.(L(T_i, T_{i+1}) + rate).DF(T_{i+1}).NDP(T_{i+1}) + RR.N_{bond} \int_0^T DF(s).NDP(s).\lambda_s ds \quad (28)$$

For the tests done in the following sections, the TRS will not have indexed interests (so no Libor rate) and the integral term is globally computed by Murex, it will be denoted by:

$$DP(T) = \int_0^T DF(s).NDP(s).\lambda_s ds$$

The convenient formula is:

$$\begin{aligned}
&\sum_{i=0}^{n-1} Q.CPN_i.NDP_{i+1}DF_{i+1} + \sum_{i=0}^{n-1} Q.(P_{i+1} - P_i).DF_{i+1} \\
&= \sum_{i=0}^{n-1} \delta.N_i.rate.NDP_{i+1}.DF_{i+1} + RR.N_{bond}.DP(T) \qquad (29)
\end{aligned}$$

All the complex expressions are hidden behind parameters that are more meaningful. The formula is here only an exchange of performance and coupons with interests and recovery amount in case of default.

The aim here is to simplify the formula in order to compute easily the flows for each bond of the test campaign. Therefore, the TRS trades are booked with only one period for the flows, one coupon over the period and one interest payment. It enables to avoid the sum.

## 4.2 Greeks

The Greeks have also been derived in the first section but the formulas are not convenient. In Murex, the prices and the other factors are derived from data curves like the interest rate curves, the credit curve for the credit parameters, the inflation curve...

In practice, the Greeks are the difference of the net present value (NPV) of the trade before and after the "bump" of the curve. A bump of the curve is a shift of this one corresponding to a variation of one basis point. The Greeks are approximated by differences. The DV01 for instance is given by:

$$\begin{aligned}
DV01 &= \frac{\partial NPV}{\partial R} \\
&\approx \frac{\Delta NPV}{\Delta R} \\
&\approx \frac{\Delta NPV}{1} \qquad (30)
\end{aligned}$$

The Greeks are simply variations of the NPV due to the bumps of the curve. Knowing the prices and other parameters after a bump of a given curve, the Greeks can be easily computed.

## 4.3 Cases Studied

The cases studied in the following cases are the most used bonds and configurations by the Murex TRS clients. It is important for the bond TRS proposed to cover these different situations:

- TRS in NPV evaluation on a basic bond: the TRS flows are evaluated by discounting the future cash flows.
- TRS in Accrual evaluation on a basic bond: the TRS flows are the accrued flows until today. The evaluation mode is used a lot, especially in accounting.
- Bond with Credit risk: This bond is very important because of the hedging purpose of the TRS. It also enables to check the CR01 for the RTRS and the BRS.
- Cross-currency TRS: the bond used is a basic bond in NPV evaluation, the TRS flows are computed in a currency different from the one of the bond.
- Inflation bond: the bond is linked to the inflation, the prices are moving along with the inflation curve.
- Amortized bond: the face value is paid during the bond contract along with the coupons and not at maturity.
- Convertible bond: the bond can be converted in a certain amount of shares. This bond is not as used as the other one, but it enables to check the Greeks of the option applied to the TRS.

## 4.4 TRS in NPV Evaluation on a Basic bond

To begin, the first bond to be analyzed will be a simple case. The bond will be risk-free, without any particularity (inflation...). The aim here is first to check the good behavior of the RTRS and the BRS on a simple case and to have an increasing complexity of the bonds on which the bond TRS are tested.

This bond is only linked to one interest rate curve. It is important to note that the bond could have more than one rate curve assigned. The Greek would be the sum of the effects for each curve: $DV01_{total} = DV01_{curve1} + \cdots + DV01_{curveN}$. For this case to be simple, a bond with only one rate curve assigned is used. The only sensitivity analyzed will be the DV01.

The TRS contract has the following properties:

- Evaluation mode: NPV. The flows are computed using the net present values of the future cash flows.
- start date: today. Today is the current date of the system. Since it is a test version, it is not the current day. It is only that after this date the prices are forward prices and historical prices before.

- Number of periods: one. It means that the coupons and the performance flows are paid along with the interests, at the end of the contract.
- maturity: one year. As the period is over the whole contract (one year), the factor ( $\frac{length of period}{360}$ ) is equal to 1.
- Prices: clean. The prices do not include the accrued coupon. This point do not affect the formulas. If the prices include the accrued coupon, the formulas are the same, only the values of the prices are changed.

### 4.4.1 Flows Formulas

As there is only one period and the trade starts today, the formulas are much more ”light”. The only prices involved in the calculation are the today price $P_0$ and the price at the maturity of the contract, $P_T$ (the one-year forward price). The discount factor between today and the maturity $T$ is denoted by $DF$. For a quantity $Q$, the performance flow is:

$$Q.(P_T - P_0).DF$$

The coupon flows are denoted by $CPN$ (they already comprise the quantity, for instance if the coupon is 1% and the quantity represents 1000000\$, $CPN = 10000\$$). The discounted flow is:

$$CPN.DF$$

The interest payment is:

$$Nominal.rate.DF$$

The nominal is given here by $Nominal = Q.P_0$. The rate is the bond TRS rate. In the arbitrage-free model, it is computed as explained before by:

$$-Q.(P_T - P_0).DF - CPN.DF + Nominal.rate.DF = 0 \quad (31)$$

This formula corresponds to the equations described above but very simplified. It enables to compute the theoretical values easily, to compare with the returned values of the Murex products. This is a simple exchange of interests for the cash flows of the bond. The TRS here is used to hedge against the market risk without default risk. The signs do not matter but they are taken here from the performance seller point of view: the performance are paid and the interest are received.

### 4.4.2 Greeks

The bump of the interest rate curve affects the discount factors and the forward prices. The prices $P$ are denoted after bump by $P^r$ and the discount factors $DF^r$. The total DV01 is the variation of the last formula after the bump of the interest rate curve:

$$DV01 = \Delta(-Q.(P_T - P_0).DF - CPN.DF + Nominal.rate.DF)$$
$$DV01 = -Q(P_T^r DF^r - P_T.DF) - Q.P_0.(DF^r - DF) - CPN(DF^r - DF) + Nominal.rate.(DF^r - DF) \quad (32)$$

Expressed as a variation the DV01 is easily computed but behind this variation it is a derivative.

## 4.5 TRS in Accrual Evaluation

Many TRS are computed in accrual evaluation. Only the accumulated revenues over time until now are considered. It means that the evaluation is based on the current positions but not the future cash flows. The same risk-free bond, without any particularity (inflation...) is used.

The TRS contract has the following properties:

- Evaluation mode: Accrual.
- start date: one month ago.
- Number of periods: one. The coupons and the performance flows are paid along with the interests at the end of the contract. The coupons have not been emitted yet, they are future cash flows. But the interests are accrued as for the performances.
- maturity: one year.
- Prices: clean. The formula would be the same with the dirty prices.

The contract starts in the past to have accumulated flows (if it starts today the accrued values would be 0).

### 4.5.1 Flows Formulas

In Accrual, the flows are considered if they are not already past or in the future. Therefore there is no coupon flows. If today the price of the bond is $P_t$ and the start price $P_0$, the pricing formula is simply:

$$-Q.(P_t - P_0) + Nominal.rate.\delta = 0 \tag{33}$$

with $\delta = \frac{\text{today}-\text{start date}}{360}$

In Accrual evaluation, few parameters impact the trade, which makes the equation very simple. There is no discount value. The interests are computed based on the current situation of the asset and not on the prevision of future flows. The performance part of the equation is the current state of the position on the bond. This is why this evaluation mode is useful for accounting.

#### 4.5.2 Greeks

As the flows formula is really simple, so is the DV01. After the bump, the today price is the only parameter impacted, denoted by $P_t^r$:

$$\begin{aligned} DV01 &= \Delta(-Q.(P_t - P_0) + Nominal.rate.\delta) \\ DV01 &= -Q.(P_t^r - P_t) \end{aligned} \tag{34}$$

Note that the today price $P_t$ varies whereas in NPV evaluation the today price is considered fix ($P_0$ does not vary in the NPV DV01).

### 4.6 Credit Risk Bond

A big issue with the bonds is the management of the risk. The credit risk bond is a very important case to analyze. The bond used has similar characteristics to the basic bond except that a risk of default is considered. The evaluation mode is NPV.

The TRS contract has the following properties:

- Evaluation mode: NPV. The flows are computed using the net present values of the future cash flows.
- start date: today.
- Number of periods: one. The coupons and the performance flows are paid along with the interests, at the end of the contract.
- maturity: one year. As the period is over the whole contract (one year), the factor $\frac{lengthofperiod}{360}$ is equal to 1.

- Prices: clean. The prices do not include the accrued coupon. This point do not affect the formulas. If the prices include the accrued coupon, the formulas are the same, only the values of the prices are changed.
- Particularity: Credit Risk.

#### 4.6.1 Flows Formulas

The formula for credit bonds is the main formula derived in the theoretical part. In Murex, the prices of the bond already include a non-default probability. So the variation of price does not need the non-default probability anymore:

$$\sum_{i=0}^{n-1} Q.CPN_i.NDP_{i+1}DF_{i+1} + \sum_{i=0}^{n-1} Q.(P_{i+1} - P_i).DF_{i+1} - RR.N_{bond}.DP_n = \sum_{i=0}^{n-1} \delta.N_i.rate.NDP_{i+1}.DF_{i+1} \tag{35}$$

For one period the formulas are the same than the basic bond ones, with the non-default probabilities $NDP$ and the value of the integral defined in part 1 $DP$. Using the same parameters, the pricing formula is:

$$-Q.(P_T - P_0).DF - CPN.DF.NDP + RR.N_{bond}.DP + Nominal.rate.DF.NDP = 0 \tag{36}$$

Here the TRS is a protection against both the market risk with the variation of price and the credit risk with the recovery flow and the flows paid under the condition of non-default. The TRS rate is a mix of the TRS rate without credit and a credit default swap rate, the part $RR.N_{bond}.DP$ is the recovery flow of a CDS and interests already include CDS premiums.

#### 4.6.2 Greeks

The bump of the interest rate curve affects the discount factors and the forward prices. The prices $P$ are denoted after bump by $P^r$ and the discount factors $DF^r$. The DV01 is almost the same than in the first case. The expression is:

$$\begin{aligned} DV01 &= \Delta(-Q.(P_T - P_0).DF - CPN.DF.NDP + RR.N_{bond}.DP + Nominal.rate.DF.NDP) \\ DV01 &= -Q(P_T^r DF^r - P_T.DF) - Q.P_0.(DF^r - DF) - CPN(DF^r - DF).NDP \\ &\quad + RR.N_{bond}.(DP^r - DP) + Nominal.rate.(DF^r - DF).NDP \end{aligned} \tag{37}$$

Also for credit bond the sensitivity to assess the credit risk is the CR01. As the bond is linked to the credit risk curve, the CR01 is as for the DV01 the variation after the bump of the curve. This credit curve impacts the forward price of the bond and naturally the non-default probabilities and the default flow. The price after bump of the credit curve is denoted by $P^c$ and the non-default probabilities $NDP^c$. The CR01 is given by:

$$\begin{aligned} CR01 &= \Delta(-Q.(P_T - P_0).DF - CPN.DF.NDP + RR.N_{bond}.DP + Nominal.rate.DF.NDP) \\ CR01 &= -Q(P_T^c - P_T).DF - CPN.(NDP^c - NDP).DF \\ &\quad + RR.N_{bond}.(DP^c - DP) + Nominal.rate.DF.(NDP^c - NDP) \end{aligned} \tag{38}$$

## 4.7 Cross-Currency Total Return Swap

An other important aspect for the TRS to cover are the cross-currency exchanges. The TRS are a way to get access to a security without having to own it. It is a powerful tool to get exposure on restrained markets. Therefore it is very current for the trade to be composed of different currencies. In this case the bond is in the currency $C1$ and all the flows are paid in the currency $C2$. The forex rate is defined by $FX = \frac{C2}{C1}$. More complex situations can be handled (for instance the interest paid in a third currency), but the aim here is to stay in a simple situation.

The TRS contract has the following properties:

- Evaluation mode: NPV. The flows are computed using the net present values of the future cash flows.
- start date: today.
- Number of periods: one. The coupons and the performance flows are paid along with the interests, at the end of the contract.
- maturity: one year. As the period is over the whole contract (one year), the factors $\frac{lengthofperiod}{360}$ are equal to 1.
- Prices: clean. The prices do not include the accrued coupon. This point do not affect the formulas. If the prices include the accrued coupon, the formulas are the same, only the values of the prices are changed.
- Particularity: Cross currency.

### 4.7.1 Flows Formulas

The flows are almost like the ones in the basic example with the forex rates inserted. In the trade currency ($C2$), the expression is the same than before, there is no change, but the parameters include the forex rates.

$$-Q.(P_T(C2) - P_0(C2)).DF - CPN(C2).DF + Nominal(C2).rate.DF = 0 \tag{39}$$

By changing the currency, this expression is equivalent to the below equation with the FX factors:

$$-Q.(P_T(C1).FX_T - P_0(C1).FX_0).DF - CPN(C1).FX_T.DF + Nominal(C1).FX_0.rate.DF = 0 \tag{40}$$

### 4.7.2 Greeks

An other point changed with the cross-currency trades. Now the trade does not have only one interest rate curve, but there is one for each market. Indeed, in Murex a curve is assigned for the bond in the currency $C1$ and one for the TRS in $C_2$. Actually, the TRS and the bond always have their own curve but in the previous examples, this curve is the same (to keep only one curve). In this situation, the curves are linked to the currency so they cannot be the same.

The DV01 has the same expression in both cases but it will be computed separately for each currency.

$$\begin{aligned} DV01 &= \Delta(-Q.(P_T(C1).FX_T - P_0(C1).FX_0).DF - CPN(C1).FX_T.DF + Nominal(C1).FX_0.rate.DF) \\ DV01 &= -Q(P_T^r(C1).FX_T^r.DF^r - P_T(C1).FX_T.DF) + Q.P_0(C1).FX_0.(DF^r - DF) \\ &\quad - CPN(C1)(FX_T^r.DF^r - FX_T.DF) + Nominal(C1).FX_0.rate.(DF^r - DF) \end{aligned} \tag{41}$$

For cross-currency trades, the trade is sensitive to the change of currency. This sensitivity is measured by the FX Delta. Like the DV01 it is a variation of the total market value of the trade to a bump, here of the FX rate. The bump of the FX rate is not one but 0.01, so the FX delta is not exactly a variation: $FXDelta = \frac{\Delta(NPV)}{0.01}$. The impacted parameters are the forward prices and the forward FX rates. After the bump, the FX is denoted by $FX^{fx}$:

$$\begin{aligned} FXDelta &= \frac{\Delta(-Q.(P_T(C1).FX_T - P_0(C1).FX_0).DF - CPN(C1).FX_T.DF + Nominal(C1).FX_0.rate.DF)}{0.01} \\ FXDelta &= \frac{-Q.P_T(C1).(FX_T^{fx} - FX_T).DF + CPN(C1)(FX_T^{fx} - FX_T).DF}{0.01} \end{aligned} \tag{42}$$

## 4.8 Inflation Bond

The inflation bond is a bond indexed on the inflation or the deflation. This type of security represents an important proportion of the bond issued. Long-term investors that cannot predict the future inflation use them to benefit from the stability of bonds without having to worry about the inflation.

The TRS contract has the following properties:

- Evaluation mode: NPV. The flows are computed using the net present values of the future cash flows.
- start date: today.
- Number of periods: one. The coupons and the performance flows are paid along with the interests, at the end of the contract.
- maturity: 4 months. The maturity is shorter to limit the number of flows otherwise the bond that is used will have several coupon flows.
- Prices: dirty. The prices include the accrued coupon. It does not change the formulas.
- Particularity: inflation.

### 4.8.1 Flows Formulas

The bond used for the test is a slightly more complex (dirty price...) but it does not affect the formulas. The inflation bond has the same flows than a basic bond with inflation factors which multiply the prices and nominals. The inflation factor at time t is denoted by $Inf_t$. The pricing formula is defined by:

$$-Q.(Inf_T.P_T - Inf_0.P_0).DF - CPN.DF + Nominal.Inf_0.\delta.rate.DF = 0 \tag{43}$$

with $\delta = \frac{\text{end date}-\text{start date}}{360}$

The coupons are still denoted by $CPN$ as if they were fixed, but they are also indexed on the inflation:

$$CPN = \text{Period interest rate}.\frac{\text{length of period}}{360}.Inf_{\text{end of coupon period}}$$

The periods for the coupons are defined with the bond. As only one coupon flow is received in this TRS, the $CPN$ term is still called as before even if it is indexed.

As the current inflation $Inf_0$ is fixed, the future impact of inflation is only included in the performance part of the equation. So the TRS hedges the performance seller against the inflation with this configuration. If there were interest flows based on a future nominal, interests would also be indexed on inflation. Nevertheless the TRS described here is a good tool for any market with a high inflation risk.

#### 4.8.2 Greeks

As the pricing formula is not very different from the one of the basic TRS, the DV01 neither:

$$\begin{aligned} DV01 &= \Delta(-Q.(Inf_T.P_T - Inf_0.P_0).DF - CPN.DF + Nominal.Inf_0.rate.\delta.DF) \\ DV01 &= -Q(P_T^r.DF^r - P_T.DF).Inf_T - Q.P_0.InfF_0.(DF^r - DF) - CPN(DF^r - DF) \\ &\quad + Nominal.Inf_0.rate.\delta.(DF^r - DF) \end{aligned} \tag{44}$$

As the TRS is sensitive to the inflation factor, a Greek is used to assess it, the inflation Delta.The inflation factors are derived from an inflation curve assigned to the bond. The inflation delta is, as the DV01, the variation of the NPV with regards to the bump of the inflation curve. The inflation curve impacts the inflation factors, denoted by $Inf^i$, the prices denoted by $P^i$ and the coupons since they are also indexed, denoted by $CPN^i$:

$$\begin{aligned} InflationDelta &= \Delta(-Q.(Inf_T.P_T - Inf_0.P_0).DF - CPN.DF + Nominal.Inf_0.rate.\delta.DF) \\ InflationDelta &= -Q.\left((Inf_T^i.P_T^i - Inf_0^i.P_0^i) - (Inf_T.P_T - Inf_0.P_0)\right).DF \\ &\quad - (CPN^i - CPN).DF + Q.(Inf_0^i.P_0^i - Inf_0.P_0).rate.\delta.DF \end{aligned} \tag{45}$$

### 4.9 Amortized Bond

Amortized bonds are an important type of security. For the previous bonds, the face value is paid at the bond maturity in one time. For amortized bonds, the face value is paid along with the interests. This type of bond is well-known because it is the same principle than a mortgage for instance.

The bond with amortization used has similar characteristics to the inflation bond. The TRS contract has the following properties:

- Evaluation mode: NPV. The flows are computed using the net present values of the future cash flows.
- start date: today.

- Number of periods: one. The coupons and the performance flows are paid along with the interests, at the end of the contract.
- maturity: 4 months. The maturity is shorter to limit the number of flows otherwise the bond that will be used will have several coupon flows and amortization flows.
- Prices: dirty. The prices include the accrued coupon. It does not change the formulas.
- Particularity: Amortized.

### 4.9.1 Flows Formulas

The TRS maturity is shorter in order to take only one flow of each type: one for the performances, one for the coupons, one for the amortization and one for the interests.

The premium of the amortized bond is defined as

$$Premium_{issue} = (P_{issue} - FV)$$

where $FV$ is the face value and $P_{issue}$ the bond issue price.

For an amortized bond, the premium is amortized during the bond life. The face value is not paid at the bond maturity but the premium is paid in the interest. Each time that a coupon is paid, a part of the face value is paid and the total value of the bond is reduced. The percentage of the face value that still must be paid at time t is the amortization factor denoted by $AF_t$. Between the coupon payments this factor is constant (the percentage does not change). The value of premium that still must be amortized is:

$$Premium_t = (P_t - FV).AF_t$$

The bond periods are defined as the period between the coupon payment. As for the inflation bond, the $CPN$ is not a fix amount but an indexed rate:

$$CPN = \text{Period interest rate}.\frac{\text{length of period}}{360}.AF_{\text{period}}$$

The coupon flow stays:

$$-CPN.DF$$

The performance flow is based on the variation of price of the bond. Using the same notation as before, the flow is:

$$-Q.(Amount_T - Amount_0).DF$$

This formula works for the standard bond with the face value paid at the bond maturity. But in the amortized case, the variation of the amount is not the variation of price but rather the variation of premium:

$$\begin{aligned}-Q.(Premium_T - Premium_0).DF &= -Q.((P_T - FV).AF_T - (P_0 - FV).AF_0).DF \\ &= -Q.(P_T.AF_T - FV.AF_T - P_0 AF_0 + FV.AF_0).DF \\ &= -Q.(P_T - P_0).AF_T.DF + Q.(FV - P_0).(AF_T - AF_0).DF\end{aligned}$$

The above formula can be divide in two. One for the variation of the price as before:

$$-Q.(P_T - P_0).AF_T.DF$$

and one which is the amortization flow:

$$Q.(FV - P_0).(AF_T - AF_0).DF$$

The interest are computed as before on the nominal of the price amortized:

$$Nominal.AF_0.rate.DF$$

with $Nominal = Q.P_0$.

The total market value is :

$$-Q.(P_T - P_0).DF.AF_T - CPN.DF + Q.(AF_T - AF_0).(FV - P_0).DF + Nominal.AF_0.rate.DF = 0 \quad (46)$$

In this form, the formula matches with the formulas of the previous bond with the variation of price of the bond, the coupon flow and an additional amortization flow which represents the variation of the reference amount.

### 4.9.2 Greeks

For the amortized bond, the only curve assigned is the interest rate curve. So the only sensitivity considered here is the DV01. It is computed as before:

$$\begin{aligned}DV01 &= \Delta(-Q.(P_T - P_0).DF.AF_T \\ &\quad - CPN.DF + Q.(AF_T - AF_0).(100 - P_0).DF + Nominal.AF_0.rate.\delta.DF) \\ DV01 &= -Q(P_T^r.DF^r - P_T.DF).AF_T - Q.P_0.AF_T.(DF^r - DF) - CPN(DF^r - DF) \\ &\quad + Q.(AF_T - AF_0).(100 - P_0).(DF^r - DF) + Nominal.AF_0.rate.(DF^r - DF)\end{aligned} \quad (47)$$

## 4.10 Convertible Bond

The last case analyzed is the convertible bond. There is many types of bonds more and more exotics. The TRS is also a tool to get access to exotic markets and possibly exotic products. The convertible bond is a bond that can be convert into shares. The bond itself is like a classic bond for the pricing formulas and DV01. The formulas will be derived in accrual to simplify. Nevertheless the bond is linked to a stock. It means that the Greeks of the option are interesting to observe.

The TRS contract has the following properties:

- Evaluation mode: Accrual.
- start date: one month ago.
- Number of periods: one. The coupons and the performance flows are paid along with the interests at the end of the contract.
- maturity: one year.
- Prices: clean.
- Particularity: convertible.

### 4.10.1 Flows Formulas

In Accrual, again the formula is:

$$-Q.(P_t - P_0) + Nominal.rate.\delta = 0 \tag{48}$$

However, to have the simplest formula possible, only the performances are kept:

$$NPV = -Q.(P_t - P_0) \tag{49}$$

The formula is very simple but the aim of this test is above all to check the Greeks of the stock computed on the BRS and the RTRS.

### 4.10.2 Greeks

The DV01 is as before:

$$DV01 = \Delta(-Q.(P_t - P_0))$$
$$DV01 = -Q.(P_t^r - P_t) \tag{50}$$

The main interest in a test with a convertible bond is to compute the Greeks of the option on the bond TRS. The Greeks for the TRS are computed as for a stock. The stock $\Delta$ is the derivative of the NPV with regards to the stock price. The derivative is approximated by differences, the stock price is shifted and the modified parameters of the TRS are used to compute the $\Delta$. The stock price shifted is denoted by $stockprice^s$ and the bond price after the shift $P^s$:

$$\Delta = -Q.\frac{P^s - P}{stockprice^s - stockprice} \tag{51}$$

The gamma is the derivative of the $\Delta$:

$$\delta = -Q.\frac{\Delta^s - \Delta}{stockprice^s - stockprice} \tag{52}$$

The $\nu$ is the derivative with regards to the volatility of the stock. The volatility is a curve assigned to the equity. With $P^v$ the bond price after bump of the volatility, the $\nu$ is given by:

$$\nu = -Q.(P^v - P) \tag{53}$$

# Part III

# Analysis and Conclusion

This part contains the returned results of the RTRS and BRS along with the theoretical values of the TRS based on the data of Murex.

Before to analyze the different tests described in the previous part, a global comparison is done to highlight some gaps between the RTRS and the BRS.

## 5 Global Comparison

The BRS has been developed more recently to answer to the growing demand for TRS products. The aim was not only to have a more reliable product but also to propose a better user-friendly tool. The BRS offers additional settings and an improved interface.

### 5.1 Schedule

The schedule is the configuration screen of the dates on which the trade is based. It includes the length of the flows periods, the number of periods before maturity, the delays between the end of the period and the payment of the flow, the coupon payment dates...

The schedule of the RTRS is functional but less configurable than the one of the BRS. The BRS schedule has more options, for instance it proposes more delays to fulfill the requirements of the clients.

### 5.2 Ergonomics

The ergonomics of the RTRS is low. The configuration of the RTRS is not convenient. Especially for tests on the products, it can be very heavy and time-consuming.

This is not a default but rather a disadvantage. Keeping in mind that the clients are paying for the right to use the Murex products, ergonomics is a criteria to consider in the development of the product.

### 5.3 Market Operations

Many operations are available for the TRS. They enable the user to make modifications on the settlements of the trade. The idea is not to make a list of all the events available, but for instance events enable to change the quantity of the underlying bond or to bring forward the maturity of the trade while the trade is ongoing.

RTRS events are complex and not user-friendly. The product is less robust and events are more likely to cause errors. The BRS also proposes additional events to fit the client's needs.

## 6 Results to the Tests

In this part, the results to the two TRS products are compared to the theoretical results for the different bonds listed in the second part.

### 6.1 TRS in NPV Evaluation on a Basic Bond

The basic bond is the bond without credit risk evaluated in NPV. It is used to have a first look at how the two Murex products return results in a standard situation.

#### 6.1.1 Flows

| | Computed Flows | BRS Flows | RTRS Flows |
|---|---|---|---|
| Performance flows | -52054.63 | -52054.63 | -52054.63 |
| Coupon flows | -409356.04 | -409356.04 | -409356.04 |
| Interest flows | 461410.67 | 461410.67 | 461410.67 |
| Total | 0 | 0 | 0 |

Table 1: Flows of the TRS in NPV evaluation on a basic bond

The flows are exactly the same, there is no difference between the three products. They all manage successfully the flows calculation for a classic bond. Let us remark that the performance flows are lower than the coupons and interests flows.

#### 6.1.2 Greeks

The DV01 can be computed separately for each flow (coupon, performance and interest ). For this first example, the detail will be computed to have an idea of the importance of each flow in the DV01.

| DV01(zero) | |
|---|---|
| Coupon flows DV01 | 41.16 |
| Performance flows DV01 | 4345.31 |
| Interest flows DV01 | -46.39 |
| | |
| Total DV01 computed | 4340.08 |
| BRS DV01 | 4339.98 |
| RTRS DV01 | 4333.04 |

Table 2: DV01(zero) of the TRS in NPV evaluation on a basic bond

Above all, an error can be observed between the DV01 computed, the BRS and the RTRS DV01. This error comes from small errors in the interpolation of the discount factors and the price derivation from the curves. The values used are not the same for each DV01 calculation. Nevertheless, the computed DV01 and the BRS DV01 are almost the same, the difference is around 0.002%, whereas the RTRS DV01 and the BRS DV01 have a difference of 0.16%. The error is more important in the case of the RTRS.

An additional point to remark is that the DV01 is almost entirely due to the performance flows. Indeed the coupon flows DV01 and the interest flows DV01 are due to the discount factor whereas the performance flows are also based on the forward prices, they are more likely to be changed when the curves are changed, even if the coupons flows are more important than the performance flows. The performance of the bond is the main source of risk in the contract.

| DV01(par) | |
|---|---|
| Total DV01 Computed | 4275.99 |
| BRS DV01 | 4275.96 |
| RTRS DV01 | 4269.18 |

Table 3: DV01(par) of the TRS in NPV evaluation on a basic bond

The DV01(par) shows the same behavior. The BRS DV01 and the computed one are almost the same whereas the RTRS DV01 is more different. As the DV01(par) is close to the zero value, for the next examples the par values will not be studied because they do not bring additional information to the zero values. Only one needs to be studied.

## 6.2 TRS in Accrual Evaluation

In accrual evaluation, the flows are computed until today. Only the accumulated flows are considered in the calculations. There is no future flow taken into account, so no coupon flows. This evaluation mode is used for example by accounting departments.

### 6.2.1 Flows

| | Computed Flows | BRS Flows | RTRS Flows |
|---|---|---|---|
| Performance flows | -88218.18 | -88218.18 | -88218.18 |
| Coupon flows | 0 | 0 | 0 |
| Interest flows | 88218.18 | 88218.18 | 88218.18 |
| Total | 0 | 0 | 0 |

Table 4: Flows of the TRS in accrual evaluation

Again the flows are the same, there is no difference between the three products. The 2 products all manage successfully the two evaluation mode.

#### 6.2.2 Greeks

| DV01(zero) | |
|---|---|
| Total DV01 Computed | 5370.19 |
| BRS DV01 | 5369.64 |
| RTRS DV01 | 5369.64 |

Table 5: DV01 of the TRS in accrual evaluation

The DV01 are almost the same. The computed DV01 presents a small error compared to the two others. Nevertheless, the DV01 are well computed in all the cases in accrual evaluation. The RTRS DV01 is even better than in NPV evaluation. This is linked to the fact that the formulas are less complicated and less parameters are implied. But at the end the results are similar.

### 6.3 Credit Risk Bond

The credit bonds are important because of the hedging purpose of the TRS. The credit risk is implied the forward prices and adds non-default probabilities to the flows. The sensitivity to the credit curve is measured by the CR01.

#### 6.3.1 Flows

The flows are the same except for the default flows. The interests of the RTRS are computed with the same TRS rate than the BRS in order to limit the differences between the products when computing

| | Computed Flows | BRS Flows | RTRS Flows |
|---|---|---|---|
| Performance flows | 246 477.13 | 246 477.13 | 246 477.13 |
| Coupon flows | -607 620.33 | -607 620.33 | -607 620.33 |
| Default flows | 59 372.31 | 59 372.31 | 65 452.05 |
| Interest flows | 301770.89 | 301770.89 | 301770.89 |
| Total | 0 | 0 | 6 079.73 |

Table 6: Flows of the TRS on the credit risk bond

the sensitivities. That is why the interest flows of the RTRS are equal to the others (otherwise the total would be 0 for the RTRS but the interest flows would be different).

The RTRS does not get the same default flow value because the formula used is not the same. Let us recall the theoretical formula used to compute the default flow:

$$\text{Default Flow} = -N_{bond}.RR.DP$$

with DP the value of the integral derived in the first section.

First the value of the integral (the DP term) used by the RTRS has a difference of 0.5% because the RTRS computes the integral with errors in the final date. But this error is small (only two days for a period of one year) so the difference is not very important. The main difference comes from that the formula used by the RTRS:

$$\text{Default Flow} = -Nominal.RR.DP$$

The $N_{bond}$ used in the default flow formula is the nominal of the bond itself. It is the reference price of the bond (set when issued) times the quantity of bonds owned. The $Nominal$ used in the RTRS formula is the current nominal of the TRS, the quantity times the current price of the bond. The $Nominal$ is the current value whereas $N_{bond}$ is the reference value. The TRS is used with credit risk bond as a Credit Default Swap, if there is a default of payment, the protection buyer gets a recovering amount based on the face value of the bond and not the current value. So the formula used by the RTRS is wrong which leads to the difference in the flows.

### 6.3.2 Greeks

The DV01 returned are almost the same despite the difference of flows. This is because the default flows have a low impact on the DV01 (only 0.2%) and because the interests flows of the RTRS is computed with the same TRS rate than the BRS to check that the interest flow DV01 is well computed for credit bonds.

The BRS manages well the bonds with credit risk, the flows and Greeks are well computed especially the CR01. The RTRS does not because of the wrong computation of the default flows.

| DV01(zero) | |
|---|---|
| Coupon flows DV01 | 61.59 |
| Performance flows DV01 | 1440.25 |
| Default flows DV01 | -2.97 |
| Interest flows DV01 | -30.59 |
| | |
| Total DV01 computed | 1468.28 |
| BRS DV01 | 1468.28 |
| RTRS DV01 | 1467.99 |

Table 7: DV01 of the TRS on the credit risk bond

| CR01(zero) | |
|---|---|
| Total CR01 computed | 1 608.14 |
| BRS CR01 | 1 608.14 |
| RTRS CR01 | 1 608.14 |

Table 8: CR01 of the TRS on the credit risk bond

## 6.4 Cross-Currency Total Return Swap

The cross-currency TRS used here has its flows in dollar with an underlying bond in euro. All the flows and interests are computed and exchanged in dollar.

### 6.4.1 Flows

| | Computed Flows | BRS Flows | RTRS Flows |
|---|---|---|---|
| Performance flows | 83 010.24 $ | 83 010.24 $ | -51 312.30 $ |
| Coupon flows | -596 785.31 $ | -596 785.31 $ | -596 785.31 $ |
| Interest flows | 513 775.08 $ | 513 775.08 $ | 648 097.61 $ |
| Total | 0 | 0 | 0 |

Table 9: Flows of the cross-currency TRS

The flows already show the problem of the RTRS to deal with cross-currency trades. Errors appear at this level and will impact later the Greeks. The difference cannot be explained like for the credit case. This time, the RTRS makes errors while converting the prices. On the other hand there is no problem with the BRS, the returned results are consistent with the expected values.

### 6.4.2 Greeks

The bond used is linked to two curves: one in euro and one in dollar. For the curve in euro, the DV01 returned are:

| DV01(zero) EUR | |
|---|---|
| Total DV01 computed | 5 129.44 € |
| BRS DV01 | 5 190.02 € |
| RTRS DV01 | 2898.27 € |

Table 10: DV01 of the cross-currency TRS in euro

The DV01 in euro of the RTRS is completely wrong. This is due to the difference of flows and the bad management of the forex rates. The BRS DV01 are good.

For the USD curve, the returned DV01 are:

| DV01(zero) USD | |
|---|---|
| Total DV01 Computed | -1 452.06 $ |
| BRS DV01 | -1 452.06 $ |
| RTRS DV01 | N/A |

Table 11: DV01 of the cross-currency TRS in usd

There is no DV01 returned for the RTRS in USD. The product considers only one curve for the trade, the euro curve. It is one source of error which leads to the bad flow computation. The RTRS and the BRS share the same database, the problem does not come from this, but rather from a mismanagement by the RTRS. The USD curve is more relevant since it is the TRS currency, the RTRS cannot be used for this type of trade.

For cross-currency trades the sensitivity to the FX rates is measured by the FX Delta:

| FX Delta | |
|---|---|
| Coupon flows | 596 785.31 |
| Performance flows | 114 409 431.42 |
| Interest flows | 0 |
| | |
| Total FX Delta Computed | 15 006 216.74 |
| BRS FX Delta | 14 990 770,18 |
| RTRS FX Delta | 596 171.02 |

Table 12: FX Delta of the cross-currency TRS

The FX Delta values are high numbers, because the variation of NPV is multiplied by 100 (see the second part). The RTRS FX Delta is wrong because, as for the DV01, the flows are wrong so the Greeks too. A difference can also be noticed between the computed FX Delta and the BRS FX Delta. This difference is only of 0.1% so the error is acceptable (even if it is more important than for the DV01) and the results are considered in line.

Also, the RTRS FX Delta corresponds to the coupon FX Delta. The interest FX Delta is always zero for this case because it does not depend on the forward FX rates. But it means that the FX Delta for the performance flows computed by the RTRS is zero. The error of the RTRS with the FX rates comes from the performance. Nevertheless there are too many errors with the RTRS, the product cannot be used for cross-currency trades.

## 6.5 Inflation Bond

Inflation bond is an important class of asset. These bonds are like the basic ones with the prices indexed to the inflation curve.

### 6.5.1 Flows

| | Computed Flows | BRS Flows | RTRS Flows |
|---|---|---|---|
| Performance flows | 4752.99 | 4752.99 | 4752.99 |
| Coupon flows | -122760.17 | -122760.17 | -122760.17 |
| Interest flows | 118007.18 | 118007.18 | 118007.18 |
| Total | 0 | 0 | 0 |

Table 13: Flows of the TRS on inflation bond

The RTRS works very well for inflation bonds. The three TRS results are consistent, there is no error.

### 6.5.2 Greeks

The bond used is linked to four curves aside from the inflation curve. All of them impact the prices and the discount factors. The total DV01 is the sum of the DV01 computed for each curve. The DV01 shown below is the total DV01.

| DV01(zero) | |
|---|---|
| Total DV01 Computed | -127.94 |
| BRS DV01 | -127.94 |
| RTRS DV01 | -127.94 |

Table 14: Caption

The RTRS manages well the inflation bond. The flows and Greeks are well computed.

For inflation bond, the Greek to the inflation curve is computed like the DV01 for the inflation curve:

| Inflation Delta | |
|---|---|
| Inflation Delta Computed | -1488.15 |
| BRS Inflation Delta | -1488.15 |
| RTRS Inflation Delta | -1488.15 |

Table 15: Inflation Delta of the TRS on inflation bond

The RTRS computes again without problem this Greek. The three TRS returns are in line.

## 6.6 Amortized Bond

Amortized bond is also an important class of asset. The amortization consists in paying the face value of the bond in the coupons. The principle is equivalent to a mortgage loan.

### 6.6.1 Flows

| | Computed Flows | BRS Flows | RTRS Flows |
|---|---|---|---|
| Performance flows | -18787.77 | -18787.77 | -18787.77 |
| Amortization flows | 12231.16 | 12231.16 | 12231.16 |
| Coupon flows | -95662.69 | -95662.69 | -95662.69 |
| Interest flows | 102219.30 | 102219.30 | 108 441.35 |
| Total | 0 | 0 | 6 222.04 |

Table 16: Flows of the TRS on amortized bond

While the BRS flows match the expected results, it is not the same for the RTRS. As there is only one period, the interests are computed as follows:

$$\text{Interest Flows} = \text{Nominal}.Rate.\text{Amortization Factor}.\delta.DF$$

However, the amortization factor changes over the period. It is not constant over the period (at the date where the coupon is emitted as a part of the premium is paid) and it must be considered in this formula. There are now two parts, one for the beginning of the period and one from the coupon emission to the end.

$$\begin{aligned}\text{Interest Flows} = (&\text{Nominal}_{\text{period start date}}.\text{Amortization Factor}_{\text{period start date}}.\delta_{\text{first part}} \\ &+ \text{Nominal}_{\text{coupon emission}}.\text{Amortization Factor}_{\text{coupon emission}}.\delta_{\text{second part}}).Rate.DF \end{aligned} \quad (54)$$

The BRS applies this formula (and the theoretical value is computed in this way), but the RTRS does not take into account the change of amortization factor. That is why the returned flows are not the same.

### 6.6.2 Greeks

| DV01(zero) | |
|---|---|
| Coupon flows DV01 | 0 |
| Amortization flows DV01 | 0 |
| Performance flows DV01 | -80.82 |
| Interest flows DV01 | 0 |
| | |
| Total DV01 Computed | -80.82 |
| BRS DV01 | -80.83 |
| RTRS DV01 | -81.03 |

Table 17: DV01 of the TRS on amortized bond

The DV01 is computed as before. The BRS DV01 matches with the expected result. For the RTRS, although that the flows are wrongly computed, the DV01 is good thanks to the fact that the interest DV01 is too low to impact the global DV01. Nevertheless, the RTRS DV01 would not be good if the interest DV01 impact was more important.

## 6.7 Convertible Bond

The convertible bonds are bonds that can be converted in shares of a related company. The bond is computed like a standard bond. The difference is that the bond prices are sensitive to the variation of the stock prices. It is measured by the Greeks of the stock applied to the TRS.

### 6.7.1 Flows

Here the aim is only the Greeks. The flows in accrual are only computed for the performance (that is why the total NPV is not zero).

| | Computed Flows | BRS Flows | RTRS Flows |
|---|---|---|---|
| Performance flows | 149912.96 | 149912.96 | 149912.96 |
| Coupon flows | 0 | 0 | 0 |
| Interest flows | 0 | 0 | 0 |
| Total | 149912.96 | 149912.96 | 149912.96 |

Table 18: Flows of TRS on the convertible bond

As for the accrual TRS with the basic bond, there is no problem, the three TRS are consistent.

#### 6.7.2 Greeks

The DV01 is computed like for the accrual TRS with the basic bond, it is not shown here. The main interest of the convertible bond is the Greeks of the option.

| Equity Delta | |
|---|---|
| Equity Delta Computed | -188 312.96 |
| BRS Equity Delta | -187758.57 |
| RTRS Equity Delta | -187 758.58 |

Table 19: Delta of TRS on the convertible bond

| Equity Gamma | |
|---|---|
| Equity Gamma Computed | -6 300.24 |
| BRS Equity Gamma | -6 431.65 |
| RTRS Equity Gamma | -6 431.66 |

Table 20: Gamma of TRS on the convertible bond

| Equity Vega | |
|---|---|
| Equity Vega Computed | -54 540,52 |
| BRS Equity Vega | -58 057,16 |
| RTRS Equity Vega | -58 057,16 |

Table 21: Vega of TRS on the convertible bond

The sensitivities to the stock price and to the volatility are well computed by the three products. The BRS and RTRS return almost exactly the same results while the computed Greeks show some differences with the two Murex products because of approximations of the derivatives by finite difference, the variations used by the BRS and the RTRS are more precise. Nevertheless, the errors are acceptable: 0.3% for the $\Delta$, 2% for the $\Gamma$ and 6% for the $\nu$ (even if it is not small, it is acceptable). So all the values are consistent.

### 6.8 Conclusion of the Tests

The results of the tests are summarized below in a table for the flows and a table for the Greeks.

As highlighted before, the RTRS returns wrong values for credit bonds, cross-currency TRS and amortized bonds. In the case of credit risk, the problem is above all due to a bad formula used for the default flows calculation. For cross-currency trades the errors are due to a bad management of the FX rates in every formula.

| Flows | | |
|---|---|---|
| | BRS | RTRS |
| NPV TRS | ✓ | ✓ |
| Accrual TRS | ✓ | ✓ |
| Credit Risk Bond | ✓ | ✗ |
| Cross-Currency TRS | ✓ | ✗ |
| Inflation Bond | ✓ | ✓ |
| Amortized Bond | ✓ | ✗ |
| Convertible bond | ✓ | ✓ |

Table 22: Results of the RTRS and the BRS for the flow calculation

Cross-currency trades are very important for the bond TRS. It is part of the main requirements for the TRS. Amortized bonds are also an important class of asset, but the cross-currency is a feature used a lot and that is applied on all the types of bonds. So it is very problematic for the TRS not to handle this situation. The credit bonds are also very important since the TRS has a hedging purpose. The protection buyer must be covered in case of default of the bond. It is problematic for the RTRS to have errors handling these situations.

The BRS manages successfully all the tests. The product is a good evolution compared to the RTRS. It is more adapted to the client's needs, the interface is more user-friendly and the formulas behind are reliable.

| Greeks | | |
|---|---|---|
| | BRS | RTRS |
| NPV TRS | ✓ | ✓ |
| Accrual TRS | ✓ | ✓ |
| Credit Risk Bond | ✓ | ✗ |
| Cross-Currency TRS | ✓ | ✗ |
| Inflation Bond | ✓ | ✓ |
| Amortized Bond | ✓ | ✗ |
| Convertible bond | ✓ | ✓ |

Table 23: Results of the RTRS and the BRS for the Greeks calculation

The Greeks are in line with the flows. The problems highlighted in the flows are confirmed when comparing the Greeks. There is no error added by the Greeks calculation. Again, the BRS Greeks are well computed for all the tests.

# 7 Conclusion

This master thesis enabled to do an analysis of the total return swaps on bonds. This financial instrument is gaining interest because of the double protection aspect: market protection and default protection. The product is also used as a speculative tool. Cash Flows are exchanged on the basis of an underlying that the performance seller does not have to own. It makes the access to any security cash flow very easy. The users can have access to closed markets or simply avoid a lot of fees.

Murex developed the TRS to answer the client's demand. The formulas used have been simply based on client's models. The master thesis aims to derive the mathematical models behind the TRS and to check the good behavior of the two TRS products of the company.

The results have highlighted the gaps between the RTRS and the BRS. The RTRS manages only basic bonds without any particularity. If the calculation becomes complex, the product is not reliable as shown with cross-currency trades or credit bonds. Also the RTRS is not well developed for an intense use. The product is not ergonomic and lacks configuration settings. The BRS is a good improvement, it is more recent and more adapted to the client's requirement. The returned values are reliable in all the situations studied here.

To go further, similar models can be developed on stock or basket of securities.

- For stocks, the models are easier to develop since there is no credit risk in the formulas like with bonds.
- For a basket of securities, the total return of the whole basket is exchanged with interests. The formulas are the same with additional sums which make the situation more complex.
- Another subject interesting to study is the Total Return Future or TRF. This financial instrument is completely new and get growing interest. It is at the intersection of total return swaps and futures. It replicates the performance of the TRS with an exchange-traded product.

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