Acoustic Modelling and Testing of Advanced Exhaust System Components for Automotive Engines

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Abstract

The increased use of the diesel engine in the passenger car, truck and bus market is due to high efficiency and lower fuel costs. This growing market share has brought with it several environmental issues for instance soot particle emission. Different technologies to remove the soot have been developed and are normally based on some kind of soot trap. In particular for automobiles the use of diesel particulate traps or filters (DPF:s) based on ceramic monolithic honeycombs are becoming a standard. This new exhaust system component will affect the acoustics and also work as a muffler. To properly design exhaust systems acoustic models for diesel particulate traps are needed. The first part of this thesis considers the modelling of sound transmission and attenuation for traps that consist of narrow channels separated by porous walls. This work has resulted in two new models an approximate 1-D model and a more complete model based on the governing equations for a visco-thermal fluid. Both models are expressed as acoustic 2-ports which makes them suitable for implementation in acoustic software for exhaust systems analysis. The models have been validated by experiments on clean filters at room temperature with flow and the agreement is good. In addition the developed filter models have been used to set up a model for a complete After Treatment Device (ATD) for a passenger car. The unit consisted of a chamber which contained both a diesel trap and a Catalytic Converter (CC). This complete model was also validated by experiments at room temperature. The second part of the thesis focuses on experimental techniques for plane wave decomposition in ducts with flow. Measurements in ducts with flow are difficult since flow noise (turbulence) can strongly influence the data. The difficulties are also evident from the lack of good published in-duct measurement data, e.g., muffler transmission loss data, for Mach-numbers above 0.1-0.2. The first paper in this part of the thesis investigates the effect of different microphone mountings and signal processing techniques for suppressing flow noise. The second paper investigates in particular flow noise suppression techniques in connection with the measurement of acoustic 2-ports. Finally, the third paper suggests a general wave decomposition procedure using microphone arrays and over-determination. This procedure can be used to determine the full plane wave data, e.g., the wave amplitudes and complex wave numbers $k_+$ and $k_-$. The new procedure has been applied to accurately measure the sound radiation from an unflanged pipe with flow. This problem is of interest for correctly determining the radiated power from an engine exhaust outlet. The measured data for the reflection coefficient and end correction have been compared with the theory of Munt [33] and the agreement is excellent. The measurements also produced data for the damping value (imaginary part of the wavenumber) which were compared to a model suggested by Howe [13]. The agreement is good for a normalized boundary layer thickness less than 30-40.

Keywords: Acoustic modelling, testing, two-port, diesel particulate filter, after treatment device, diesel engine, two microphone, plane wave, flow, signal-to-noise ratio, microphone array, open pipe, reflection coefficient, end correction, damping
This thesis consists of the following papers and an introduction with a summary

Paper I
Acoustic Modelling and Testing of Diesel Particulate Filters.
(Submitted to Journal of Sound and Vibration)

Paper II
On Sound Propagation in An array of Narrow Porous Channels with Application to Diesel Particulate Filters. (Submitted to Journal of Sound and Vibration)

Paper III
Modelling and Testing of After-Treatment Devices.

Paper IV

Paper V
Methods for Accurate Determination of Acoustic Two-Port Data in Flow Ducts.

Paper VI
Full Plane Wave Decomposition Using Microphone Arrays in Ducts.

Division of the work between the authors
The theoretical and experimental work presented in the thesis have been done by Sabry Allam under the supervision of Mats Åbom for Papers I, II, III, and VI. Paper IV was done under the supervision of Hans Bodén and Paper V under the supervision of Hans Bodén and Mats Åbom.
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I. Introduction

Sound can be divided into two main categories wanted and unwanted sound. Unfortunately most sounds produced by our machines must be classified as unwanted and a particular important source today is transportation noise. Noise from road transports originate from two main sources tyre noise and noise from the powertrain. The powertrain is particularly important at lower speeds and during acceleration, i.e., in residential areas. One part of the powertrain noise comes from the exhaust system and it is particular issues related to this part that is investigated in this thesis. The purpose of an exhaust system is primarily to remove the exhaust gases from the engine. Due to the high level pressure pulsations from the engine outlet valves sound reducing devices or mufflers must be introduced in an exhaust system to control the sound radiation. During the last 10-15 years devices which reduce the emission of harmful exhaust gases have also become a standard component in the exhaust system. For instance catalytic converters for cleaning the exhaust gases on Otto engines were introduced during the 1990’s. Today the focus is on the diesel engine where catalytic converters combined with soot particulate traps are introduced. As a component of the exhaust system both catalytic converter and a diesel particulate trap, so called after treatment devices, will affect the acoustic performance. Therefore acoustic models for after treatment devices are needed in particular for diesel particulate traps which is a new component.

Concerning modeling of sound transmission in exhaust systems two main approaches exist, non-linear time domain or linear frequency domain models [1-2]. In both cases normally 1-D wave propagation is assumed in the main exhaust pipe. Most of the work on the acoustics of intake or exhaust systems is based on the linear model which more easily can handle 3-D effects, complex damping and boundary conditions. The non-linear 1-D models are today used as a standard tool for the analysis of engine gas exchange and performance. Concerning the linear models they are normally formulated as acoustic 2-ports or four-poles [2-4]. A number of software packages to analyse muffler systems regarded as a network of 2-ports, exist for
example the SID code developed at MWL/KTH [4]. The first half of this thesis will focus on the development of 2-port models for after treatment devices in particular diesel particulate traps. Besides theoretical models experimental procedures to validate our models and to determine input for data our calculations are important. In particular there seems to be a need to further investigate experimental procedures [5] [6], e.g., microphone mounting, and signal processing techniques which will enable accurate measurements beyond Mach-numbers equal to 0.2. The second half of the thesis, therefore, treats experimental procedures for the accurate measurement of 2-port data, reflection coefficients and wave numbers (for damping) in flow ducts.
II. After-Treatment Devices

The increased penetration of diesel engines in the passenger car, bus and truck markets is due to their higher efficiency and thereby reduction of fuel costs. The growing market share has brought several environmental effects. It has been observed how diesel engines are responsible for small particle (soot) as well as $NO_x$ emissions.

As a way to control emission and fulfil the environmental laws, USA and Europe are using a suitable After Treatment Device (ATD), [Figure 1] This means that in addition to the classical mufflers there is an extra unit introduced into the exhaust line. This new unit will also affect the acoustic performance of the exhaust system. A typical ATD consists of an expansion chamber where a Catalytic Converter unit is mounted in series with a Diesel Particulate Filter or trap. The Area of the expansion chamber is typically around 10 times the inlet exhaust pipe to reduce the pressure drop.

![Figure 1](image.png)

1. Flexible inlet.  
2. Inlet pipe.  
3. Diverging conical duct.  
4. Straight pipe.  
5. Straight pipe.  
6. Catalytic converter  
7. Straight pipe.  
8. Diesel Particulate Trap.  
10. Straight pipe.  
11. Converging conical duct.  
12. Outlet pipe.

**Figure 1.** Photograph of an ATD for a passenger car together with a drawing of the various parts.
Conventionally, a catalytic converter element (6) in Figure 1 consists of a ceramic brick with a uniform set of small, parallel, open channels of square cross-section running along the length of the brick. A more recent alternative is to use a metallic rather than a ceramic substrate for which the cross-section of the channels is not square. In either case, a washcoat is applied to the surface of the channels and then the catalyst is spread over the surface of the washcoat. The catalyst creates an exothermic reaction within the exhaust gas flowing through the channels as oxygen is stripped from the \( NO_x \) and is used to completely burn the carbon monoxide \( CO \) and the hydrocarbons \( HC \).

A Diesel Particulate Filter (DPF) element (8) in Figure 1 can reduce exhaust gas emissions from hydrocarbons and soot particles. Extruded ceramic monolithic honeycombs are the standard substrates, and these substrates are produced in different cell density and wall thickness configurations. Monoliths used in DPF:s are known as wall-flow monoliths in which the exhaust gases are forced to flow through the porous walls as illustrated in Figure 2. From an acoustic point of view the main difference between a catalytic converter and a diesel particulate filter is the fact that the acoustic waves must pass through the porous walls in DPF:s. This will create an additional damping to the damping created by the narrow channels.

Figure 2. Photograph of a typical ceramic diesel particulate filter unit with illustration of the flow path through the filter. The flow enters one channel plugged at the downstream end and then passes through the porous walls into the four neighboring channels plugged at the upstream end. At the inlet side of the filter close to 50% of the channels are plugged. A typical width of the quadratic channels is 1-2 mm with a wall thickness of a few tenths of a millimeter.
The removal of soot particulates is accomplished by a filtration process in which the pores filter out the particulate. Two parameters are important concerning the effect of a DPF system on the efficiency of a diesel engine. Firstly, the effect of backpressure created by the pressure increase necessary to force the exhaust flow through a filter. The backpressure will also increase as the filter becomes loaded with soot particles. A high backpressure is undesirable, since it increases fuel consumption and reduces available power. Secondly, the necessity of regular filter regeneration, i.e., combustion of trapped particles. Combustion of soot particles without catalytic aids will start at temperatures between 500-600 °C, depending on the exhaust flow rate and the oxygen content. The DPF creates an exothermic reaction within the exhaust gas flowing through the porous walls which burn the hydrocarbons. The after treatment concept has been the focus of intensive research and development activities around the world, and a variety of systems are offered by various manufactures [7-11]. Of course this new part will influence the acoustic performance and thereby change the design of the complete exhaust system. So it is very important to study the acoustic performance of the after treatment device and its effects on the performance of the exhaust system.

### III. Damping of Acoustic Waves

Generally the propagation of acoustic waves at rest in a homogeneous thermo-viscous fluid, unbounded in all directions involves reactive and dissipative processes that can be characterized, in the frequency domain, by a complex valued wave number. The imaginary part of which is proportional to the shear and bulk viscosity coefficients and the heat conduction coefficient. The wave number can also include dissipation processes due to molecular relaxation via a complex specific heat ratio [12][14]. In a bounded domain (pipe or cavity), reactive and dissipative processes at rigid walls arise from interactions between the acoustic field and the entropy (diffusion of heat) and the vorticity fields (diffusion of shear waves), created on the boundary walls and extracting energy from the acoustic wave [12][14]. The entropy and vorticity fields decay exponentially in the direction normal to the walls and, with the exception of low frequencies and narrow ducts, only play a role in a thin acoustic boundary layer region. When this is the case the resulting reactive and dissipative effects can still be
handled by introducing a complex wave number. For the case of interest here with sound propagation in the narrow channels of a DPF (width around 1 mm) the boundary layers will extend over the cross-section and a full analysis of the coupled field problem is necessary.

The case of sound waves in a visco-thermal fluid inside a cylindrical, rigid and narrow pipe with no flow has been solved exactly by Kirchhoff [14]. During the last decade there has been an interest to extend this solution to other cross-section shapes and to account for the effects of a mean flow. One reason for this interest has been the efforts to model automobile catalytic converters [16-21]. Peat [16] and Astley and Cummings [17] presented FEM solutions, based on simplified equations for waves in a visco-thermal fluid, for the problem of sound propagation in capillary tubes. The analysis is for laminar mean flow with a parabolic velocity distribution and a quadratic cross-section. Dokumaci [18] using the same set of simplified equations showed that for the case of a plug flow and a circular cross-section an exact solution is possible. Using this model an acoustic 2-port for a catalytic converter unit was derived. In a later work Dokumaci [21] extended his earlier work [18] to the case of rectangular narrow tubes with a plug flow.

For the case of diesel particulate filters there is a need to analyse wave propagation in narrow tubes (width 1-2 mm) taking into account the effect of wall porosity. All the studies discussed above are for the case of tubes with rigid and non-porous walls and are therefore not applicable.

### IV. Acoustic 2-Ports

A two-port is a linear system with an input and output. It can be noted that many authors use the word four-pole instead of two-ports originating from electrical network theory. The properties of acoustical two-ports can be determined either by theoretical models or by measurements.

The relation between the input and the output states of a time-invariant, linear and passive two-port can, in the frequency domain, be written

\[ X = T Y \] (1)
where, \( \mathbf{X/Y} \) are the state vectors at the input/output as shown in Figure 3 and \( \mathbf{T} \) is a \([2 \times 2]\)-matrix.

![Figure 3. Black box relating two pairs of state variables x and y](image)

Any pair of state variables, i.e. a state vector, belonging to a two-port defines a linear 2D state-space. This means that from a given state vector an infinite set of alternative state vectors can be generated by linear transformations. A consequence of this is that there is a large freedom in the choice of the state vectors. In practice, however, only a few of the many choices are useful [22]. In this thesis models for sound propagation in exhaust systems will be investigated. The main part of the acoustic energy in an exhaust system comes from the engine pulsations and are low frequency. In particular the wavelength for the pulsations will normally be much larger than the diameter of the main exhaust pipe. This implies that the sound field in an exhaust system will primarily consist of plane waves. Acoustic 2-port models is then an appropriate formalism and a common choice of state variables is the plane wave acoustic pressure \( p \) and volume velocity \( q \).

Choosing \( p \) and \( q \) as the components of the input and output state vectors the 2-port is obtained in the so called transfer matrix form

\[
\begin{bmatrix}
\hat{p}_1 \\
\hat{q}_1
\end{bmatrix} =
\begin{bmatrix}
t_{11} & t_{12} \\
t_{21} & t_{22}
\end{bmatrix}
\begin{bmatrix}
\hat{p}_2 \\
\hat{q}_2
\end{bmatrix}.
\]

The major advantage of the transfer matrix form is the simplicity with which the transfer matrix for a system with 2-ports in series (a cascade) is generated, for instance the ATD unit shown in Figure 1. Assuming continuity of \( p \) and \( q \) at each of the element interfaces the total 2-port is simply obtained from

\[
T_{ATD} = \prod_{j=1}^{12} T_j.
\]

The continuity of \( p \) and \( q \) is only strictly valid when the interface between two elements is in a straight duct section where a pure plane wave field exists. At sudden area expansions or at locations where strong mean flow gradients exist \( p \) and \( q \) can be
discontinuous. At such locations special coupling 2-ports must be introduced to handle the discontinuity [2-4]. For the ATD device in Figure 1 this means that at each area discontinuity a coupling 2-port should be introduced. This would not change the formalism in equation 3 but merely means that the number of 2-ports we must include in the product increases.

V. Experimental Techniques for Flow Ducts

According to ISO 10534 [23,24] there are two standards methods for measuring induct acoustic properties; the standing wave ratio method (SWR) and the two-microphone method (TMM). The SWR method using pure tone excitation is the classical acoustic measurement method in ducts with a plane wave propagation. The method works in the manner that it uses a movable microphone inside an impedance tube, to determine the location and magnitude of successive maxima and minima of the standing-wave pattern in the tube. From these maxima and minima the reflection coefficient magnitude can be calculated. The disadvantage of the SWR method is that it is relatively time consuming, since a discrete frequency is normally used, and it is difficult to use in the presence of flow [25].

During the 1980’s when Fourier systems became generally available Seybert and Ross [26], Chung and Blaser [27,28] and Seybert [29] developed the so called two-microphone method. From the measurements of the cross- and auto-spectrum or the transfer function between two microphones they demonstrated how the plane wave pressure components could be obtained. Once these components are known everything of interest can be calculated, e.g., the reflection coefficient or the acoustic power in the positive direction. By using the two-microphone method (TMM) with broadband excitation, acoustic properties can be determined over a very wide frequency range and the measurement can be much faster than with the SWR method. With wall mounted pressure probes to pick up the plane wave acoustic pressure the TMM is also better adapted for use in investigating sound in ducts with flow. A problem for this case is however the local flow noise (turbulence) pressure fluctuations that are picked up by a pressure probe thus reducing the signal-to-noise ratio.
Regarding the flow noise suppression, the most common way of reducing wind noise is to use a suitable windshield [30]. The effect of this type of device is most marked at velocities below 12 m/s. With higher flow rates the transducers will pick the sum of the acoustic and turbulent pressure fluctuations. A problem with this type of arrangement where the pressure probe or microphone is shielded, is that this will normally affect the frequency response in both amplitude and phase. In particular the phase shift can be critical for the TMM since this method requires probes which are phase calibrated. An efficient way of suppressing turbulent pressure fluctuations is to use a reference signal uncorrelated with the disturbing noise in the system and related linearly to the acoustic signal in the duct [31][32]. For measurement situations were a controllable source is used, say, a loudspeaker, a possible choice for this reference signal is the electric signal driving the loudspeaker. A detailed investigation of this technique plus other issues related to the application of the two-microphone method in flow ducts is presented in papers IV and V.

The traditional TMM is a plane wave decomposition method which assumes that the wave numbers are known. For the case of plane wave sound propagation in gas filled pipes with a circular cross-section the effect of damping and wall vibrations can normally be neglected. There is then only the well known convective effect that must be included when calculating the wave numbers. However, for very accurate measurements or when the microphones are separated over longer distances than normal the effect of damping must be included. Of course also for applications in ducts with flexible walls, e.g., ventilation ducts with rectangular cross-sections, the wave numbers are not accurately known. There is therefore a need to develop a general plane wave decomposition procedure which works without a pre-knowledge of the wave numbers. This problem is treated in the last paper of the thesis, paper VI. As a test of the new procedure the reflection of sound from an unflanged pipe opening, with flow, has been investigated. This problem is of interest both for the calculation of sound radiation from the tailpipe of an exhaust system but is also a classical problem in duct acoustics. A theoretical solution has been presented by Munt [33] but the results presented here of both amplitude and phase is the first published complete experimental validation of Munt’s theory.
VI. Summary of the Papers and Main Results

As explained in section II there is a need for new models to predict the sound transmission in an After Treatment Device (ATD) for diesel engines. The aim of the first two papers of this thesis is to develop theoretical models for the acoustic 2-port of a Diesel Particulate Filter (DPF). In the third paper a 2-port model for a complete diesel ATD unit is developed and validated at cold conditions with flow. In the fourth paper, experimental methods to improve the signal-to-noise ratio in flow duct experiments either by different microphone holder configurations or by using different signal processing techniques are tested. The results of paper four have been used in paper five to investigate the accurate measurement of 2-ports for mufflers at Mach-numbers typical for exhaust systems (0.2-0.3). In paper five the effect on the signal-to-noise ratio from using multiple loudspeaker configurations is also tested. In paper six an experimental technique has been developed to perform a full plane wave decomposition which determines the wave amplitudes as well as the wave numbers. A short summary of each paper is presented in the following sections.

A. Paper I: Acoustic Modelling and Testing of Diesel Particulate Filters

This paper presents a first attempt to describe the acoustic behaviour of DPF’s and presents two models which allow the acoustic two-port to be calculated. The simplest model neglects wave propagation and treats the filter as an equivalent acoustic resistance modeled via a lumped impedance element. The resistance is obtained by measuring the steady flow pressure-drop over a DPF which is assumed to follow Darcy’s law with Forchheimer’s extension [15]. From the linear "viscous" flow resistance and the quadratic flow resistance the acoustic resistance can be calculated. This simple model gives a constant frequency independent transmission loss and agrees well with data measured on a typical filter (length 250 mm) up to 200-300 Hz at room temperature. The validity of this model depends on the ratio between the wave length and the filter length and under hot conditions the model could be valid up to 400-500 Hz.
In the second model the filter is divided into five parts as shown in Figure 4; the inlet cross-section (IN), a short narrow pipe with hard impermeable walls (I); the filter section consisting of narrow pipes with porous walls (II); a short narrow pipe with hard impermeable walls (III) and the outlet cross-section (OUT). In the plane wave range these sections can be described via 2-port transfer matrices (T). The resulting transfer matrix for a filter unit is then simply

$$T_{DPF} = T_{IN} T_{I} T_{II} T_{III} T_{OUT}$$  \hspace{1cm} (4)$$

The ceramic filter monolith part (II) is described as a system of coupled porous channels carrying plane acoustic waves and the coupling between the channels through the porous walls is described via Darcy’s law \([7]\). The wall resistance can be calculated from the porosity and thickness of the wall material or from measured pressure drop data using the linear "viscous" flow resistance. The IN and OUT sections have been treated as lumped impedances using the quadratic flow resistance which is divided equally between the inlet and outlet sections. The short narrow pipes, parts (I) and (III), can be added as a mass plug end correction or can be modelled as a pipe with impermeable walls. For the cold case this model gives a frequency dependent transmission loss and agrees well with measured data for clean filters in the entire plane wave range. One minor difference in values calculated using the 1-D model (using the measured wall resistance and adiabatic speed of sound) is the small
oscillations present, see Figure 5. This is related to the fact that the 1-D model does not correctly account for the damping along the channels. Regarding the hot case the wall resistance is calculated from the physical properties of the wall material and the soot. The flow and the temperature distribution through the cannels have been taken into account according to theory available in references [7,8].

For the hot case, i.e., for the practical application it has been found that: (i) the effects of mean flow through channels are small and can be neglected (error less than 0.2 dB); (ii) flow is only important for the estimation of the quadratic pressure drop at the inlet and outlet sides; (iii) concerning the exact temperature distribution tests show that a very good approximation for the damping, (the error is typically around 0.3 dB) is obtained by using a constant temperature equal to the average; (iv) a large effect of soot loading on the filter damping is also observed, which is related to a pressure drop increase due to the soot layer.

Figure 5. Measured and predicted transmission loss for a DPF at $M=0.02$ and $T=293^\circ$ K before the filter inlet.

○○○○, Measured; ——, predicted using 1-D Model; −−, predicted using lumped Model.
Figure 6. Predicted transmission loss for the studied filter unit at $M=0.02$ and $T=700\,^\circ\,K$ before the filter inlet. ..., predicted for no flow and no soot; ---, predicted for no flow in the channels and no soot BUT with flow losses at the inlet/outlet; ——, predicted with flow and no soot; °°°, predicted with flow and soot layer.

B. Paper II: On Sound Propagation in An Array of Narrow Porous Channels with Application to Diesel Particulate Filters

An extension of the first paper has been done to overcome the approximate treatment of the viscous and thermal losses along the narrow channels. In this paper the problem is analysed in more detail. By solving the linearized equations for a visco-thermal fluid, simplified in the manner of the Zwikker and Kosten theory [34], for coupled waves in two neighbouring channels. The coupling between the channels has been treated as described in Paper I. From this solution the acoustic 2-port has been calculated to predict the sound transmission loss for an entire DPF unit. The theoretical results are compared with experimental data for clean filter units at room temperature. The agreement is very good and better than the earlier presented 1-D model especially for very low Mach-numbers. A modified 1-D model using the classical Kirchhoff exact solution [35] for a plane wave in a narrow tube is also presented. This modified 1-D model is in close agreement with the predictions of the new theory as can be seen from the examples presented in Figure 7 and Figure 8.
A comparison of the 1-D model from paper I and the improved models presented here has been done for operating (hot) conditions. The comparison gave the following conclusions: (i) all models agree in the low frequency limit; (ii) the 1-D model (assuming an isothermal speed of sound) works satisfactorily up to 600-800 Hz; (iii) the modified 1-D model is in close agreement with the new model presented here.

**Figure 7.** Measured and predicted transmission loss for a DPF at room temperature at $M=0.01$ and $T=293^\circ$ K before the filter inlet. --- Measured; —— predicted using the new model; ----- predicted using Modified 1-D Model.

**Figure 8.** Measured and predicted transmission loss for the studied filter at $M=0.02$ and $T=293^\circ$ K before the filter inlet. --- Measured; —— predicted using the new model; ----- predicted using Modified 1-D Model.
Figure 9. Predicted transmission loss for the studied filter unit with soot layer at $M=0.02$, $T=700^\circ \text{K}$ before the filter inlet and $T_v=775^\circ \text{K}$ (through the filter). ---, using the 1-D model and isothermal speed of sound presented in paper I; …., using the modified 1-D model; —, using the new model.

C. Paper III: Modelling and Testing of After-Treatment Devices

In paper III an acoustic model of a complete ATD for a passenger car has been presented, see Figure 1. The model is built up of 5 basic elements: i) flexible inlet; ii) conical inlet/outlet; iii) straight pipes; iv) CC unit and v) DPF unit. For each of these elements a 2-port model was used and with the exception of the DPF unit and the flexible inlet known models from the literature are available. For the flexible inlet there was not enough information about the acoustic properties of the material, so the acoustic 2-port has been measured. For the DPF unit the models developed in Papers I and II have been used. From the 2-ports of the constituting elements the 2-port for the complete ATD unit was calculated according to equation 3. One limitation of the model for the ATD unit is that the effects of chemical reactions on the sound field are assumed to be negligible.

As a part of the validation for the ATD model the models available for catalytic converters were examined. It has been found that the classical Kirchhoff exact
solution $[^{35}]$ for a plane wave in a narrow circular tube works well for modelling of a catalytic converter, because the Mach number is so small ($< 0.05$) that mean flow effects can be neglected. A comparison between measured data, a full CC model as proposed by Dokumaci $[^{21}]$, and the approximation obtained by using the Kirchhoff solution is presented in Figure 10.

Theoretical predictions for the entire ATD and experimental data based on measurements at cold conditions, i.e., at room temperature for various flow speeds are in good agreement. An example of the results is shown in Figure 11. One conclusion from both measurements and simulations is that the ATD is not sensitive to the effect of flow since the transmission loss changes slowly with the Mach number.

Figure 10. Transmission loss versus frequency for the Catalytic Converter at room temperature with $M= 0.02$ at the inlet. ------, Simulated using Kirchhoff exact solution $[^{35}]$; —, simulated using Dokumaci $[^{21}]$; ○○○○, measured.

In order to increase the quality of acoustic measurements in flow ducts the signal-to-noise ratio has to be improved. The signal to noise ratio is here defined as the ratio of the pressure auto-spectrum of the desired acoustic signal to the pressure auto-spectrum of the (flow) noise. The signal-to-noise ratio was determined by first measuring the noise generated by the loudspeaker at the microphone position without flow. Then the loudspeaker was turned off and the microphone signal was measured again with flow. Two ways to reduce the influence of flow noise on the measured pressure auto-spectrum have been investigated, different holder configurations and signal enhancement based on a reference signal. Six different types of microphone holders were tested for flow Mach numbers from $M = 0.07$ to $M = 0.26$ and it was concluded that the reference microphone holder shown in Figure 12 with a flush mounted transducer, gave the best result as can be seen from Figure 13. Two different signal enhancement techniques were tested: synchronised time domain averaging (STDA) and cross-spectrum based frequency domain averaging (CSFDA). These two techniques were compared with the ordinary frequency domain averaging (FDA) found in most FFT analysers. The conclusion was that synchronised time domain
averaging and cross-spectrum based frequency domain averaging gave equally good results. Both gave a signal-to-noise ratio improvement of $N$ or $10 \cdot \log(N)$ dB, where $N$ is the number of averages. It was also found that when using cross-spectrum based frequency domain averaging it does not make any difference to the signal-to-noise ratio improvement, if a periodic or a random acoustic signal were used, as long as a noise-free reference signal is available. The improvement obtained when shifting from random noise excitation to for instance stepped sine excitation is due to the increase in initial signal-to-noise ratio caused by the concentration of signal energy to one frequency at a time.

**Figure 12.** Construction drawing for the reference holder.

$Y$ is the pipe wall thickness and $X$ is the distance from microphone grid to inner pipe diameter.

$X=0$ for flush mounted microphone.
E. Paper V: Methods for Accurate Determination of Acoustic Two-Port Data in Flow Ducts

The aim of this work is to investigate and present accurate methods to measure the acoustic 2-port especially at Mach-numbers typical for exhaust systems $M=0.2-0.3$. The investigation contains improvement of signal-to-noise ratio by a multiple loudspeaker configuration and by application the signal processing techniques from Paper IV. Different configurations for the loudspeaker connection to the duct, as
shown in Figure 15 have been studied at Mach numbers ranging from M=0.15 to M=0.3.

Figure 15. Different configurations for connecting the loudspeaker.

The identical set of signal processing techniques used in Paper IV have been used here. That is ordinary frequency domain averages (FDA), synchronized time domain averages (STDA) and cross spectrum based frequency domain averages (CSFDA). Different types of excitation signals have been used; stepped sine, random and saw tooth excitation. Saw-tooth excitation was used to obtain a multi-frequency deterministic excitation signal for comparing the STDA method to the other signal processing techniques. The tests have been made for three different acoustic elements: a straight duct, an expansion chamber and a commercial automotive muffler show in Figure 16. Stepped sine excitation with 400 cross spectrum based frequency domain averages was used as the reference case for comparing the other excitation signals and signal processing techniques. The STDA and CSFDA can give the same results as the reference case if a sufficient number of averages are used, see Figure 18. In these cases the electric signal exciting the loudspeaker has been used as the noise free reference required for the STDA and CSFDA methods. If a noise free reference signal is not available at each frequency the microphone with the highest signal-to-noise ratio can be used.
**Figure 16.** Photo of the commercial muffler used in the experiments.

**Figure 17.** Transmission loss versus frequency using different type of excitations at $M=0.0$ and $T=293\,^\circ K$ for commercial muffler presented in figure 16.

----, random excitation with 1000 averages (FDA); oooo stepped sine excitation with 100 averages (CSFDA); ++++, saw tooth excitation with 1000 averages (STDA).
Figure 18. Transmission loss versus frequency using different type of excitations at M=0.26 and T=293º K for commercial muffler presented in figure 16.

----, random excitation with 10000 averages (CSFDA); oooo stepped sine excitation with 400 averages (CSFDA); ++++, saw tooth excitation with 10000 averages (STDA).

F. Paper VI: Full Plane Wave Decomposition using Microphone Arrays in Ducts

Often when plane wave decomposition techniques are used it is assumed that the wave numbers are known so that only the travelling wave amplitudes in positive and negative directions ($p_z$) are needed. Without flow accurate models for the damping in a duct are available [41] but with flow the situation is more complex. There are also other situations where wall vibrations are important and influence the wave propagation and the wave numbers ($k_z$).

In order to obtain the full plane wave data that includes both the wave amplitudes and wave numbers a new experimental technique has been developed in this paper. An over determined non-linear solver has been introduced to determine the four complex unknowns [$p_z$ and $k_z$] from the measured data. For the practical tests of the method an array consisting of six microphones (B&K4938) was used. The tests were performed using a steel pipe typical for a car exhaust system with circular cross-section and an inner diameter of 35 mm, a wall roughness of less than 1 µm and an outer diameter of 40 mm. The cut off frequency of the first higher order mode in this
pipe is approximately 4.5 kHz at room temperature. The microphone separation and the orientation of the microphones are shown in Figure 19. The pressure at the microphones was measured using the electric signal of the acoustic driver as a reference signal. Cross-spectrum-based frequency domain averaging was used to suppress the flow-generated noise. The measurements were repeated three times and the average value was used in the final analysis.

Figure 19. Layout of the test rig. The opening to the right represents an unflanged pipe.

The measured data have been used to calculate the magnitude of the reflection coefficient at the open unflanged end as well as the end correction. Regarding the reflection coefficient it has been found that Munt’s theory [33] gives very good agreement with the measured results if the maximum flow velocity in the centre of the duct was used to calculate the flow Mach number, see Figure 20. The same excellent agreement with Munt [33] was obtained for the end correction, see Figure 21. Regarding the low frequency behaviour the reflection coefficient magnitude has been found to approach $R_o = 1$ when the Helmholtz-number $ka \to 0$, the lowest value measured was $R_o = 1.005$ at $ka = 0.0162$. For the normalized end correction the lowest measured valued is $\delta/a = 0.265$, where $a$ is pipe radius at Strouhal numbers $St = 0.15$ which agree well with the value $\delta/a = 0.2554$ predicted for small Mach-numbers by Rienstra [33] when $St \to 0$. 

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Figure 20. Measured and predicted reflection coefficient for an unflanged pipe at different Mach numbers. +++, Measured at M=0, −, predicted at M=0 by Levine and Schwinger [39]; ○○○, Measured at M=0.05, −−−−, predicted at M=0.05 by Munt [33]; ♦♦♦, Measured at M=0.1, ••••, predicted at M=0.1 by Munt [33]; ◇◇◇, Measured at M=0.15, ••••, predicted at M=0.15 by Munt [33]; ***, Measured at M=0.2, −−−−, predicted at M=0.2 by Munt [33].

Figure 21. Measured and predicted end correction for an unflanged pipe at different Mach numbers. +++, Measured at M=0, −, predicted at M=0 by Levine and Schwinger [39]; ○○○, Measured at M=0.05, −−−−, predicted at M=0.05 by Munt [33]; ♦♦♦, Measured at M=0.1, ••••, predicted at M=0.1 by Munt [33]; ◇◇◇, Measured at M=0.15, ••••, predicted at M=0.15 by Munt [33]; ***, Measured at M=0.2, −−−−, predicted at M=0.2 by Munt [33].
The damping of the plane wave $\alpha_\pm$ (equal to minus the imaginary part of the wave number) could also be extracted from the measured data. The results have been compared with five different theoretical models [13, 33-38]. The results show that Howe’s theory [13] gives good agreement with the measurements except at high-normalized boundary layer thickness and Dokumaci [37] can be used at low normalized boundary layer thickness.

![Diagram showing damping coefficient $\alpha_\pm/\alpha_0$ versus normalized boundary layer thickness $\delta^*_n$, at $ka = 0.0323$.](image)

**Figure 22.** Damping coefficient $\alpha_\pm/\alpha_0$ where $\alpha_0$ is the damping without flow versus normalized boundary layer thickness $\delta^*_n$, at $ka = 0.0323$.

- ○○○, measured $\alpha_+/\alpha_0$,
- ·····, predicted by Howe [13],
- ----, predicted by Dokumaci [33]
- ○○○, measured $\alpha_-/\alpha_0$,
- –·–·, predicted by Howe [13],
- ⋯⋯, predicted by Dokumaci [33]
VII. Future Work

(i) The presented model for a Diesel Particulate Filter needs to be validated at engine operating (hot) conditions.

(ii) To further improve the DPF model the effect of chemical reactions on the acoustic transmission should be investigated. This comment is also relevant for the existing published models for Catalytic Converters.

(iii) Another possible extension of the work on the DPF unit is to include non-plane waves incident on the filter. This is of interest since the filter is placed in an expansion chamber which typically has a diameter 3 times larger than the main exhaust pipe.

(iv) The microphone arrays studied in Paper VI could be combined with the techniques discussed in Papers IV and V to develop methods with high flow noise suppression capability.
References


