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On the solar corona.

By

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With 4 figures in the text.

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I.

§ 1. Introduction.

In order to explain the mysterious phenomenon known as the solar corona many different hypotheses have been proposed. One of the most simple of these is that the corona constitutes a sort of atmosphere around the sun. But if this atmosphere — in analogy with that of the earth — were in equilibrium under the influence of gravitation alone, the enormous extension of it would require a temperature of about a million degrees. At the first sight this possibility seems to be easy to rule out, because it seems absurd that so high a temperature could exist not far outside the photosphere, whose temperature is only 6000°. However, as radiation losses in the very thin corona are small and thermal conductivity usually is of little importance in stellar atmospheres, the existence of so high a temperature is not physically impossible. Further, many arguments for the existence on the sun of particles with high energies have accumulated from recent investigations on different lines, and it is also possible to understand how high energy particles can be produced. Therefore it seems worth while to investigate whether the corona might perhaps consist of high energy particles, which is the same as to say that it has a very high temperature.

§ 2. Arguments for the existence of high energy particles on the sun.

Let us consider the more and less definite arguments for the existence on the sun of high energy particles.

A. The continuous spectrum of the corona has the same energy distribution as that of the photosphere, which according to Schwarzschild can be explained only if it consists of photospheric light scattered by free electrons in the corona. But in the inner corona no Fraunhofer lines are observed, which very probably is a result of a smoothing through the Doppler effect produced by the swift motion of the scattering electrons. This explanation is supported by the discovery made by GROTRIAN[1] that the depression of the photosphere intensity in the environment of the H and K lines is reproduced in the corona spectrum, but considerably broadened. From the difference in the breadths of the depressions GROTRIAN concludes that the mean velocity of the scattering electrons amounts to $7.5 \cdot 10^8$ cm sec^{-1}. In a later publication[2] he revises this value to $4 \cdot 10^8$ cm sec^{-1}. An electron with the velocity $7.5 \cdot 10^8$ cm sec^{-1} has an energy of 160 electron volts, whereas the latter value gives an energy of 45 *e*-volts. As the density in the inner corona is large enough to ensure thermal equilibrium (see § 3 and 10), it is of interest to calculate the corresponding temperatures. We find $1.2 \cdot 10^6$ and $0.35 \cdot 10^6$ degrees.

B. In the flash spectrum there exist emission lines with excitation or ionisation potentials far above those which are possible at thermal equilibrium. For example, UNSÖLD[3] has shown that the observed ratio He^{++}/He^{+} is 38.5 powers of ten too high. This means that it is necessary to assume the existence on the sun of electrons or quanta with energies far above thermal energies.

C. Recently EDLÉN[4] has identified some of the coronal emission lines with forbidden transitions in spectra of very highly ionized atoms (especially Fe atoms). The ionisation potentials are several hundred volts. This indicates the existence of a temperature of the same order of magnitude as found in A.

It is of interest to observe that according to LYOT[5] the line breadth of the coronal emission lines corresponds to a

[1] W. GROTRIAN, ZS. f. Astrophysik *3* p. 199 (1931).
[2] W. GROTRIAN, ib. *8* p. 155 (1934).
[3] A. UNSÖLD, Physik der Sternatmosphären p. 420. Berlin 1938.
[4] B. EDLÉN, Private communication.
[5] B. LYOT, C. R. *202* p. 1259 (1936). UNSÖLD, loc. cit. p. 452.

mean velocity of $23-26 \cdot 10^5$ cm sec^{-1}. If the emitting atoms have the mass 56 (= Fe), their kinetic energies amount to 150—190 *e*-volts, which is the same order of magnitude as found in A.

D. From observations of ionisation in the upper atmosphere during solar flares T. H. JOHNSON and S. A. KORFF[1] conclude that X-ray radiation in the wave-length region between 0.1 and 1.5 Å. U. is emitted from the sun. If the radiation is an impulse radiation produced by electrons, these must have an energy of at least 10^4-10^5 *e*-volts. Even if we admit that *so* high energies are produced only through the abnormal conditions occurring during a solar flare, it is very likely that processes of the same kind under more normal conditions produce particles with high energies.

E. According to a recent theory of magnetic storms and aurorae[2], these phenomena can be explained only if we assume the emission from the sun of an ion stream consisting of particles (electrons and ions) with high energies. The properties of the stream when it has reached the neighbourhood of the earth can be determined with some certainty out of observational data from magnetic storms and aurorae. The extrapolation to the properties of the stream when leaving the sun is of course rather precarious, and the very high energy value (10^7 *e*-volts) mentioned in the cited paper may be too high, as shown by later calculations. However, an energy of at least the same order of magnitude as found by JOHNSON and KORFF (see D) is very probable.

F. We have seen that there are several observational evidences — of varying degree of definiteness — for the existence on the sun of high energy particles. It is also possible to understand theoretically how they can be produced. In a recent paper[3] it is pointed out that solar prominences could be explained as electrical discharges. Motion of solar matter in magnetic fields on the sun (especially the vortical motion in a sunspot) must bring about potential differences between different points of the solar surface, and it was shown that under certain conditions this gives rise to discharges above the surface of the sun. Calculations indicate that the electromotive force can be as high as 10^7 volts, so that even if charged particles are usually accelerated only by a small fraction of this potential, they attain rather high energies.

[1] T. H. JOHNSON and S. A. KORFF, Terr. Mag. *44* p. 23 (1939).
[2] H. ALFVÉN, Kungl. Sv. Vetensk.-Ak:s Handlingar, III Bd 18 N:o 3 (1939); Bd 18 N:o 9 (1940).
[3] H. ALFVÉN, Ark. f. Mat., Astr. Fysik, Bd 27 A N:o 20 (1940),

The process is most conspicuous in the prominences, where, consequently, we can expect a very intense production of high energy particles. As the mechanism is of a very general character the same process is likely to take place very frequently on a smaller scale. If — as many authors mean[1] — we can regard the chromosphere as a multitude of small prominences, it is likely that a production of high energy particles takes place almost everywhere on the solar surface or in some layer above it.

§ 3. The density of the corona.

Thus, we have found that several observational facts indicate the existence on the sun of particles (electrons and ions) having energies far above that which corresponds to the temperature of the photosphere. An energy of the order of magnitude of 100—200 *e*-volts is indicated. Further, we have seen that there is no theoretical difficulty in explaining the production of particles with such energies. We are now going to investigate whether the corona could be regarded as an atmosphere consisting of high energy particles.

If we assume that no other force than gravitation acts upon the corona, we can calculate the temperature in each point of the corona from the density function. The latter has been derived by BAUMBACH[2] from all available photometric observations, under the assumption that the coronal light consists mainly of photospheric light scattered by free electrons in the corona. For the mean electronic density at the height $\eta = R/R_\odot$, BAUMBACH gives the empirical formula

$$N = 10^8 (0.036\,\eta^{-1.5} + 1.55\,\eta^{-6} + 2.99\,\eta^{-16})\ \text{cm}^{-3}. \tag{3.1}$$

Of course the charge of the electrons must be compensated by the same amount of positive charge (from positive ions). In this paragraph we assume that all the ions in the corona are hydrogen ions (leaving the general case to be discussed in § 11). Thus, the number of protons per cm³ amounts also to N.

As at least in the inner corona the density is high enough to ensure thermal equilibrium between the »molecules» (but of course not between molecules and quanta!), we can apply

[1] See A. UNSÖLD, loc. cit. p. 437.

[2] S. BAUMBACH, Astr. Nachr. *263* p. 121 (1937). (In one or two cases his table and his empirical formula do not agree exactly. In such cases the formula has been used in this paper.)

the common laws of kinetic gas theory. We assume that the mean energy of the »molecules» (in our case electrons and protons) amounts to $E = \frac{3}{2} k T$. As there are $2N$ molecules the gas pressure is

$$p = \frac{2}{3}\, 2\, N E. \tag{3.2}$$

If $m_H = 1.66 \cdot 10^{-24}$ g is the mass of a hydrogen atom and $g_\odot = 2.74 \cdot 10^4$ cm sec^{-2} is the acceleration at the sun's surface, the gravitational force acting upon a cubic centimeter is $g_\odot N m_H \eta^{-2}$. As we have assumed that this force is compensated by the pressure gradient, we have

$$\frac{d\,p}{R_\odot\, d\,\eta} = -\frac{g_\odot N m_H}{\eta^2}. \tag{3.3}$$

Differentiating (3.2) we obtain from (3.2) and (3.3)

$$\frac{d}{d\,\eta}\left(\frac{E}{E_0}\right) + \frac{1}{N}\frac{d\,N}{d\,\eta}\frac{E}{E_0} = -\frac{1}{\eta^2} \tag{3.4}$$

where

$$E_0 = \frac{3}{4} g_\odot R_\odot m_H = 2.38 \cdot 10^{-9}\ \text{erg} = 1.49 \cdot 10^3\ e\text{-volts}. \tag{3.5}$$

From (3.4) we obtain

$$\frac{E}{E_0} = -\frac{1}{N}\int \frac{N}{\eta^2}\, d\,\eta \tag{3.6}$$

or, according to (3.1)

$$\frac{E}{E_0} = \frac{\frac{0.036}{2.5}\eta^{-2.5} + \frac{1.55}{7}\eta^{-7} + \frac{2.99}{17}\eta^{-17}}{0.036\,\eta^{-1.5} + 1.55\,\eta^{-6} + 2.99\,\eta^{-16}}. \tag{3.7}$$

The value of E/E_0 from this formula for different η-values is shown in Fig. 1. The values for $\eta > 5$ cannot be expected to be very reliable, for the material treated by BAUMBACH includes only two observations in this region.

As seen from the figure E/E_0 is almost constant in spite of the fact that the density varies by a factor of more than 1000. In particular a high degree of constancy is noted for

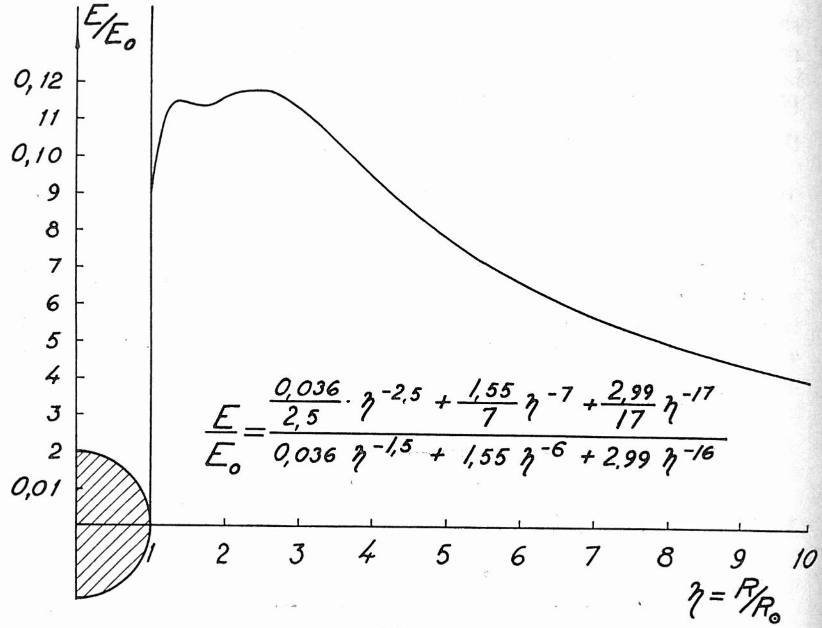

Fig. 1.

$1.2 < \eta < 3.0$, where the value of E is close to $0.12\,E_0$ or 180 e-volts. This would mean that *the temperature in the corona is (more or less) constant over a large region.*

§ 4. Discussion.

The question now arises whether this result is a mere coincidence or whether it has a physical meaning. It is striking that the temperature corresponding to 180 e-volts is of the same order of magnitude as found in § 2 A and C. On the other hand it would seem to be very absurd to think of a solar atmosphere being more or less isothermic at so high a temperature. But we must not forget that the corona is very thin so that the radiation losses are small. However, we must study the problem in more detail.

For the temperature distribution in the corona it is of importance *where* the high energy particles are supplied, in other words, where the »heating» takes place. According to § 2 F there are good reasons to believe that the high energy particles are supplied by the prominences or by a prominence-like activity in the chromosphere, so that the »heating» occurs

in a rather low layer in the corona. But even independently of any theory of the heating mechanism we must assume that the input of energy takes place not far from the surface of the sun, because it is difficult to imagine how it could be supplied directly to some point high up in the corona.

Thus, schematizing the conditions, it is of interest to investigate the following problem. Suppose that in a layer at the height h above the photosphere the energy ε erg cm^{-2} sec^{-1} is produced. What is the temperature distribution in the corona?

As the density of the corona is small we neglect the radiation losses. Then the entire energy input ε is dissipated by thermal conduction downwards to the surface of the sun and upwards into interplanetary space. The temperature distribution can be calculated in the following way. As $h \ll R_\odot$ the temperature T at the height $R - R_0 (< h)$ is given by

$$\varepsilon_1 = \varkappa \frac{d\,T}{d\,R} \tag{4. 1}$$

where ε_1 is the energy transported per cm^2 and sec from the heated layer to the sun's surface, and $\varkappa$ is the thermal conductivity. As $\varkappa$ is proportional to $T^{1/2}$, we can write

$$\varepsilon_1 = \frac{\varkappa_0}{\sqrt{T_0}} \sqrt{T} \frac{d\,T}{d\,R} \tag{4. 2}$$

which gives, after integration

$$T = \left[\frac{R - R_0}{h}\left(T_h^{3/2} - T_1^{3/2}\right) + T_1^{3/2}\right]^{2/3} \tag{4. 3}$$

where T_h and T_1 are the temperatures at the height h and at the sun's surfac. Below h we can treat the problem as conduction through a plane. Above h we must take account of the spherical shape so that we must substitute (4. 2) by

$$\varepsilon_2 = \varkappa \left(\frac{R}{R_0 + h}\right)^2 \frac{d\,T}{d\,R} \tag{4. 4}$$

where ε_2 is the energy transported upwards.

Whether ε_2 is different from zero or not is a dubious question. If the density in interplanetary space is zero, the solar corona is »vacuum insulated» outwards and no energy transport outwards is possible. In this case ε_2 is zero and the temperature is constant outside h. Consequently, the temperature distribution is

$$T = \left[\frac{R - R_\odot}{h}(T_h - T_1)^{3/2} + T_1^{3/2}\right]^{2/3} \quad \text{if } R - R_\odot < h \quad (4.5)$$

$$T = T_h \quad \text{if } R - R_\odot > h \quad (4.6)$$

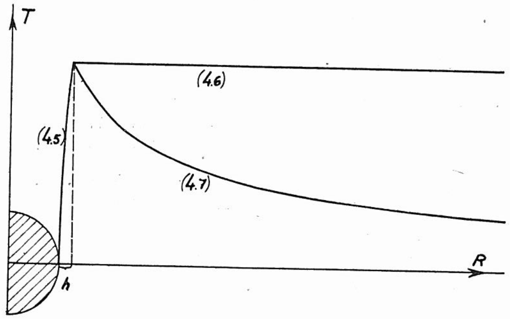

Fig. 2.

On the other hand, if the density in interplanetary space is large enough to ensure the validity of the formulae for thermal conduction, we obtain from (4.4)

$$T = T_h\left(\frac{R}{R_0 + h}\right)^{-2/3} \quad \text{if } R - R_\odot > h \quad (4.7)$$

which replaces (4.6). When integrating we have put $T = 0$ for $R = \infty$.

The temperature distributions according to (4.5) and (4.6), and according to (4.5) and (4.7) are plotted in Fig. 2.

Comparing this with the results obtained in § 3, we see that the temperature distribution expected theoretically is in rather good agreement with what is derived from BAUMBACH's empirical formula. The discrepancy at $\eta = 1$ ($R = R_\odot$) is of course due to the fact that the empirical formula does not hold for this value. The temperature distribution according to (4.5) and (4.6) gives perhaps the better agreement over the region where a comparison is really significant. On the other hand the empirical values seem to decrease when η becomes large, which would agree better with (4.7). The problem is discussed more closely in next section.

§ 5. The energy necessary to heat the corona.

The energy ε which is necessary to maintain this temperature distribution can be calculated from (4.1) and (4.4). From the ordinary formulae of kinetic gas theory we find

$$\frac{\varkappa_0}{\sqrt{T_0}} = c_1 \frac{1}{\pi\sigma^2}\frac{k^{3/2}}{m^{1/2}} \quad (5.1)$$

where $\pi\sigma^2$ is the cross-section and m the mass of the molecules, $k = 1.37 \cdot 10^{-16}$ is BOLTZMANN's constant, and c_1 a numerical constant which is approximately 0.6.

As in our coronal gas the thermal conduction is mainly due to the electrons, we put $m = 9.04 \cdot 10^{-28}\, g$ (mass of the electron) and $\pi\sigma^2 = 10^{-17}$ cm², which at least is of the right order of magnitude for the energies concerned. This gives

$$\frac{2}{3}\frac{\varkappa_0}{\sqrt{T_0}} = 2.2 \cdot 10^6 \text{ erg cm}^{-1}\text{ sec}^{-1}\text{ degree}^{-3/2}$$

so that we have (as $T_1 \ll T_2$)

$$\varepsilon_1 = 2.2 \cdot 10^6 \frac{T_h^{3/2}}{h}. \quad (5.2)$$

Here T_h is of the order of magnitude of 10^6 degrees. The height h of the layer where the heating occurs is difficult to estimate. According to Fig. 1 the temperature begins to drop rapidly when η becomes smaller than 1.3, which corresponds to a height of $2 \cdot 10^{10}$ cm. But probably the main part of the heating takes place at a much lower height because the prominences are not usually so high. If tentatively we put $h = 10^{10}$ cm, we find

$$\varepsilon_1 = 2.2 \cdot 10^5 \text{ erg cm}^{-2}\text{ sec}^{-1}.$$

As $\varepsilon_2 = 0$ or $\varepsilon_2 = \frac{h}{R_\odot}\varepsilon_1 \ll \varepsilon_1$, this gives the order of magnitude of $\varepsilon = \varepsilon_1 + \varepsilon_2$. This energy, which is necessary to maintain the high temperature of the corona, is only about 10^{-5} of the energy radiated by the sun.

It is of interest to see how rapidly a stationary temperature in the corona is attained if the energy production starts suddenly. This depends upon the thermal capacity of the corona. According to BAUMBACH (loc. cit.) the total number of electrons above a square cm of the solar surface amounts to $4 \cdot 10^{18}$. If the number of protons is the same, and each of the particles has an energy of about 100 *e*-volts, the total energy is $8 \cdot 10^{20}$ *e*-volts $\mathrm{cm}^{-2} = 1.3 \cdot 10^{9}$ erg cm^{-2}. If our value of ε is correct, this energy is produced in about two hours. Consequently, we can expect that a stationary state is attained already after some hours.[1]

II.

§ 6. On the forces acting upon the corona.

The theory developped in the preceding section is able to account for several of the phenomena observed in the corona. However, we have introduced only one force, the gravitation, as acting upon the corona. In order to make a complete theory it is necessary to take account of all forces. The ionized matter constituting the corona is likely to be subjected not only to gravitation but also to electromagnetic forces: radiation pressure and forces from electrostatic and magnetic fields.

Several authors have supposed that *the radiation pressure* is of fundamental importance not only for the structure of the corona but also for the motions of prominences and — generally speaking — for all phenomena which can not be explaned through gravitation. Ordinary calculations show, however, that the effect of radiation pressure in the corona is likely to be negligible. According to UNSÖLD[2] more than one tenth of the total solar radiation must be absorbed in the corona if the radiation pressure should be able to carry it. As the absorbtion is probably of the order of magnitude of 10^{-6}, radiation pressure is likely to be five powers of ten too small. Consequently, without introducing very artificial assumptions, we cannot suspect the radiation pressure to be of any importance at all.

On the other hand the forces due to *electric and magnetic fields* must have a considerable influence upon the ionized matter of the corona. We shall first discuss the effect of *magnetic fields* only under the assumption that *no electric fields* are present. The effect of electric fields are discussed in § 13.

[1] As the thermal capacity (being proportional to the density) decreases very rapidly when the height increases, but the thermal conductivity (being independent of the density) remains constant, this holds also for the outer parts of the corona.

[2] A. UNSÖLD, loc. cit. p. 453.

§ 7. Influence of magnetic fields.

Investigations by HALE and collaborators have shown the existence on the sun of strong local magnetic fields (up to several thousand gauss), especially in the sunspots. Moreover, the sun has a general magnetic field, the character of which is likely to be not very far from that of a dipole field.[1] The most probable value of the dipole moment seems to be $4.2 \cdot 10^{33}$ gauss cm^3 which means that on the solar surface the field amounts to 25 gauss at the heliomagnetic poles and 12.5 gauss at the equator. At a distance of 10 solar radii, the field is 0.001 of this. As the sunspot fields do not spread very far, we need only to take account of the general magnetic field (except in some cases concerning the inner corona).

The motion of electrons and ions is affected fundamentally by the magnetic field. In general, they spiral around the magnetic lines of force. Suppose that a particle with the mass m and charge e has a velocity v, the components of which parallel to and perpendicular to the magnetic field H are $v_{\parallel}$ and $v_{\perp}$. Putting $E_{\parallel} = \frac{1}{2} m v_{\parallel}^2$ and $E_{\perp} = \frac{1}{2} m v_{\perp}^2$, the total kinetic energy is $E = E_{\parallel} + E_{\perp}$.

If the magnetic field is homogeneous, the motion of the particle is composed of a rectilinear motion with the constant velocity $v_{\parallel}$ in a direction parallel to the magnetic field and a circular motion with the velocity $v_{\perp}$ perpendicularly to the magnetic field. The radius of curvature of the circular motion is

$$\varrho = \frac{1}{H} \sqrt{\frac{2\, m c^2}{e^2} E_{\perp}} \tag{7.1}$$

(c = velocity of light).

If the particles have an energy $E = 100$ *e*-volts, the maximum radius of curvature (if $E_{\perp} = E$) in the corona amounts to:

	Magnetic field	Radius of curvature Electrons	Protons
$\eta = 1$ (sun's surface)	20 gauss	1.7 cm	72 cm
$\eta = 10$ (outer corona)	0.02 »	$1.7 \cdot 10^3$ cm	$7.2 \cdot 10^4$ cm

[1] The sun's general magnetic field is often supposed to decrease extremely rapidly with the height in solar atmosphere. The observational evidence for this is very weak, and theoretically it would be very difficult to understand such a phenomenon.

On the other hand using the density values given by BAUMBACH we find easily (see § 10) that even in the inner corona the mean free path of the particles is not less than 10^8 cm.

Consequently, *the radius of curvature of the paths of the particles in the corona is always several powers of ten less than their mean free paths.*

In order to illustrate this let us take an example. A proton in the inner corona must travel more than 10^8 cm = 1000 km before it is likely to hit another particle. But during all this time it cannot elongate more than about *one meter* from that »line of force» around which it is spiralling. Consequently, it can move almost exclusively along the magnetic lines of force.

Thus, *in the absence of electric fields transport of matter as well as of electric charge can take place (almost) only in the direction of the magnetic field.*

We have treated the magnetic field as being homogeneous. This is permitted to a first approximation, because — as we have seen — the radius of curvature ϱ is much smaller than the extension of the field. However, taking account of the inhomogeneity of the magnetic field we must introduce a force acting upon the particle in the direction of the magnetic field.[1] (Moreover, the particles are subjected to a drift perpendicular to the magnetic field, but this is very slow. We shall not take it into consideration until in § 13). The force due to the inhomogenity of the magnetic field amounts to

$$f^{(m)} = -\frac{E_\perp}{H}\frac{dH}{dz} \tag{7.2}$$

if the z-axis is parallel to the magnetic field. In the case of a dipole field the angle α between the vector radius $R = R_\odot \eta$ and the magnetic field is given by

$$\cot \alpha = 2 \operatorname{tg} \varphi \tag{7.3}$$

where φ is the »magnetic latitude» $\left(\frac{\pi}{2} - \varphi \text{ is the angle between the vector radius and the dipole}\right)$. Through elementary geometrical considerations we find (see loc. cit. p. 12, equation 8.9)

$$f^{(m)} = \frac{3 E_\perp \vartheta}{R_\odot \eta} \cos \alpha \tag{7.4}$$

[1] H. ALFVÉN, Ark. f. Mat., Astr. Fysik. Bd 27 A. N:o 22 (1940).

where

$$\vartheta = 1 + \frac{\cos^2 \varphi}{2(1 + 3 \sin^2 \varphi)}. \tag{7.5}$$

It is of interest to point out that the absolute value of H does not enter in (7.4).

§ 8. Conditions for equilibrium.

Thus, we have found that the effect of a magnetic field upon the corona is:

1. The particles can move only in the direction of the magnetic field (»along the magnetic lines of force»).
2. A force given by (7.4) acts upon the particles in the direction of the magnetic field.

These results compell us to modify the theory of the corona developped in section I, because to the gravitational force we must add the force due to the inhomogeneity of the solar magnetic field.[1] As we have found that the particles can move only parallel to the magnetic field, in the first place we are interested to know the components of the forces along the magnetic lines of force.

Consider a point at the distance $\eta R_\odot$ from the centre of the sun and at the heliomagnetic latitude φ. The sun's general magnetic field, which — as said earlier — can be assumed to derive from a central dipole, makes the angle α with the vector radius. Upon a cubic centimeter containing N electrons and N protons the following forces act in the direction of the magnetic field:

Pressure gradient: $$-\frac{dp}{R_\odot\, d\eta} \cos \alpha. \tag{8.1}$$

Gravitation: $$-\frac{g_\odot N m_H}{\eta^2} \cos \alpha. \tag{8.2}$$

Magnetic gradient: $$2N \frac{3 E_\perp \vartheta}{R_\odot \eta} \cos \alpha. \tag{8.3}$$

As the »molecules» (electrons and protons) of our corona gas are subject to mutual collisions, their mean energy E will be

[1] Since the corona must rotate with the same angular velocity as the sun, the centrifugal force ought to be added. However, even in the outer corona it does not exceed a few percent of the gravitation and consequently it can be neglected.

equally distributed over the three degrees of freedom, so that on an average the kinetic energy along each of the coordinate axes is $\frac{1}{3}E$. Consequently, we have

$$E_{\perp} = \frac{2}{3}E \tag{8.4}$$

$$E_{\parallel} = \frac{1}{3}E \tag{8.5}$$

Introducing (3.2), (3.5), and (8.4) into (8.1), (8.2), and (8.3), and adding, we obtain the total force $F_{\parallel}$ acting in the direction of the magnetic field.

$$F_{\parallel} = \frac{4}{3}\frac{\cos\alpha}{R_{\odot}}\left[-\frac{d}{d\eta}(NE) - \frac{E_0 N}{\eta^2} + \frac{3NE\vartheta}{\eta}\right]. \tag{8.6}$$

If the corona is in equilibrium, $F_{\parallel}$ must vanish everywhere.

The force due to the magnetic gradient compensates the gravitation exactly if

$$\frac{E}{E_0} = \frac{1}{3\vartheta\eta}. \tag{8.7}$$

If E surpasses this value, the gas is expelled from the sun

§ 9. Calculation of the temperature.

If we assume that the corona is in equilibrium, we can now calculate the energy E (= the temperature) if we know the density function N.

In (8.6) ϑ depends upon the latitude φ according to (7.5). At the pole $\left(\varphi = \frac{\pi}{2}\right)$ it equals 1 but increases to 3/2 at the equator. This means that the lifting action of the magnetic field is larger near the equator than at the pole. Consequently, we must expect the density of the corona to be larger at low latitudes than at high latitudes, which seems to be in accordance with observations.

The density function N given by BAUMBACH is an average for all latitudes. In order to be able to use this function we must assume that the corona is approximately spherically symmetrical. Hence we use for ϑ the average value 5/4 (which is very close to the mean taken over the surface of the sphere,

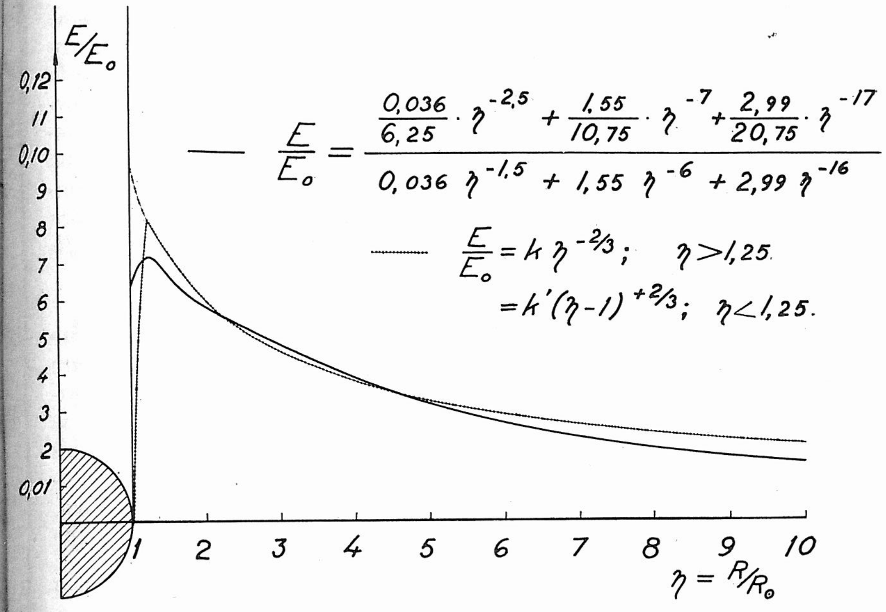

Fig. 3.

which is $\bar{\vartheta} = \int_0^{\frac{\pi}{2}} \vartheta \cos\varphi \, d\varphi = \frac{5}{6} + \frac{2\pi}{9\sqrt{3}} = 1.237$). Putting $F_{\parallel} = 0$ we can calculate E from (8.6). We obtain

$$\frac{E}{E_0} = \frac{\eta^{3\vartheta}}{N}\int N\eta^{-3\vartheta-2}\,d\eta \tag{9.1}$$

where $\vartheta = 5/4$. E_0 is given by (3.5). Introducing (3.1) we obtain *the temperature E of the corona if it is in equilibrium under the influence of gravitation and the magnetic gradient and has the density found empirically by* BAUMBACH.

$$\frac{E}{E_0} = \frac{\frac{0.036}{6.25}\eta^{-2.5} + \frac{1.55}{10.75}\eta^{-7} + \frac{2.99}{20.75}\eta^{-17}}{0.036\,\eta^{-1.5} + 1.55\,\eta^{-6} + 2.99\,\eta^{-16}}. \tag{9.2}$$

This function is plotted in Fig. 3.

It is of interest to compare the temperature function E with the temperature distribution if (as supposed in § 4) the heating of the corona takes place in a low layer. In § 4 it was found that below the »heated layer» h the temperature function was given by (4. 5). Above h it was difficult to decide whether the formula (4. 6) or (4. 7) was the correct one. However, as formula (8. 7) gives an upper limit to E, which decreases as η^{-1}, it is evident that the temperature cannot be constant. Consequently, we can be pretty sure that the temperature must obey (4. 5) and (4. 7). These functions are plotted in Fig. 3 where tentatively we have put $h = 1.25\, R_0$ and $\frac{3}{2} k\, T_h = E_h = 0.082\, E_0$.[1].

The agreement between the function E found from the emperical density formula and the theoretical temperature distribution is quite satisfactory. Only close to the sun's surface ($1 < \eta < 1.25$) the curves disagree. This is certainly due to an inaccuracy of the empirical density function, which does not give the true density in the chromosphere. If terms corresponding to the density gradient in these layers are added, the energy obtained from the formula must go down to the temperature of the photosphere.

In order to check the result, let us calculate the theoretical density of an atmosphere, the temperature of which obeys (4. 7). We have

$$\frac{d}{d\eta}(N E) = \frac{3\, N E \vartheta}{\eta} - \frac{E_0 N}{\eta^2} \tag{9.3}$$

and

$$\frac{E}{E_0} = 3\, k\, \eta^{-2/3} \tag{9.4}$$

where $3\,k$ is an arbitrary constant. This gives after integration

$$\log N/N_0 = (3\,\vartheta + 2/3) \log \eta + \frac{1}{k\, \eta^{1/3}} \tag{9.5}$$

where $\vartheta = 5/4$. This function is plotted in Fig. 4 with arbitrary values of N_0 and k. For $\eta > 1.25$ it agrees with BAUMBACH's empirical function within the limits of error.

Close to the sun's surface the temperature cannot be expected to obey (4. 7).

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[1] This curve is intersected by the curve given by (8. 7) at $\eta \sim 20$. The density at this distance is very small so that the escape of matter has no great influence.

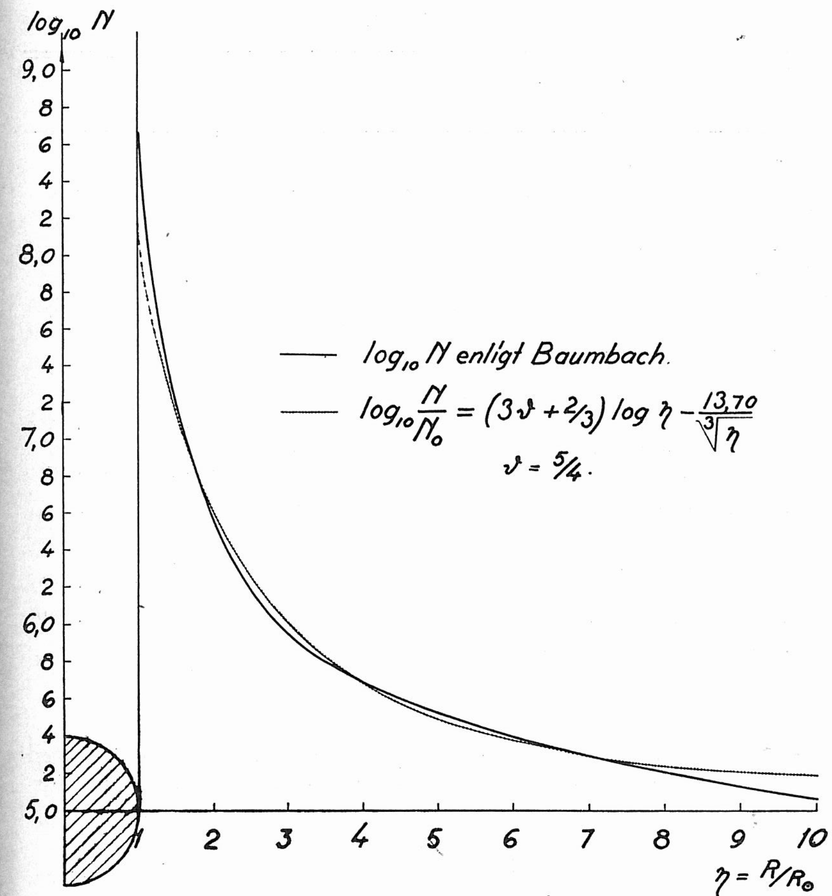

Fig. 4.

§ 10. **Numerical results.**

We have seen that the empirical density of the corona agrees very well with the theoretical density of an atmosphere which is under the influence of gravitation and the sun's magnetic field and is heated near its base to a very high temperature ($\sim 10^6$ degrees). It seems worth while to compute the characteristic properties of such an atmosphere from the data obtained in the preceding paragraphs. The result is shown in Table 1. All data (except the magnetic field) are calculated from BAUMBACH's density function with the help of the for-

Table 1.

$\eta = \frac{R}{R_\odot}$	1.2	2	4	8
Density $2N$ (particles/cm^3) according to BAUMBACH	$141 \cdot 10^6$	$7.38 \cdot 10^6$	$1.02 \cdot 10^6$	$0.33 \cdot 10^6$
Mean energy E (e-volts) according to Fig. 3	108	87	58	30
Temperature T (degrees)	$8.4 \cdot 10^5$	$6.7 \cdot 10^5$	$4.5 \cdot 10^5$	$2.3 \cdot 10^5$
Pressure p (dyn cm^{-2})	$160 \cdot 10^{-4}$	$6.8 \cdot 10^{-4}$	$0.63 \cdot 10^{-4}$	$0.10 \cdot 10^{-4}$
Mean velocity component $v = \sqrt{\frac{2kT}{m}}$ (cm/sec) for				
electrons	$5.0 \cdot 10^8$	$4.5 \cdot 10^8$	$3.7 \cdot 10^8$	$2.6 \cdot 10^8$
protons	$12 \cdot 10^6$	$11 \cdot 10^6$	$9 \cdot 10^6$	$6 \cdot 10^6$
Fe-ions	$16 \cdot 10^5$	$14 \cdot 10^5$	$12 \cdot 10^5$	$8 \cdot 10^5$
Average magnetic field (gauss)	12	2.5	0.31	0.04
Radius of curvature (cm) for				
electrons	2.4	10	69	380
protons	100	440	2900	$1.6 \cdot 10^4$
Mean free path l (cm)	$5 \cdot 10^8$	$1 \cdot 10^{10}$	$7 \cdot 10^{10}$	$2 \cdot 10^{11}$
Collisions per second $\sim \frac{v_{el}}{l}$	1.0	$4.5 \cdot 10^{-2}$	$5 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$

mulae given above. The radius of curvature is calculated from (7.1) with $E_\perp = 2/3\,E$. For the mean free path the formula

$$l = \frac{1}{\sqrt{2}\,\pi\sigma^2}\,\frac{1}{2N}$$

is used, where $\pi\sigma^2$ is the cross-section. The value of $\pi\sigma^2$ has been put equal to 10^{-17} cm^2 (compare § 5).

The values for the inner corona (say $\eta = 1.2$) can be compared with the following empirical values (discussed in § 2). The mean velocity component for electrons is $4.0 \cdot 10^8$ cm/sec (earlier value $7.5 \cdot 10^8$ cm/sec) according to GROTRIAN. This is in excellent agreement with our theoretical value of $5.0 \cdot 10^8$ cm/sec. According to LYOT and EDLÉN the corresponding velocity for Fe-ions is $23-25 \cdot 10^5$ cm/sec, which is in rather good agreement with the theoretical value of $16 \cdot 10^5$ cm/sec.

A serious difficulty is that in the outer corona Fraunhofer lines are observed with no detectable broadening. If they arise from light scattered by free electrons these must have a very low temperature ($< 10°$ K) which is not reconcilable with our theory. According to GROTRIAN (loc. cit.) a scattering by cosmic dust would explain the phenomenon.

§ 11. **Influence of the molecular weight.**

All our calculations have been made under the assumption that the corona consists mainly of electrons and protons. However, the existence of emission lines in the spectrum of the corona shows that it certainly contains other elements than hydrogen. Suppose that per each electron there are ν_n ions each one having the charge $\varepsilon_n e$ and the mass $\mu_n m_H$. As the total charge of all the positive ions must equal the charge of the electrons, we have

$$\Sigma\, \varepsilon_n \nu_n = 1.$$

The following changes must be made in our formulae. In (3.2) and (8.3) $2N$ must be substituted by $N(1 + \Sigma \nu_n)$. In (8.2) we must replace N by $N \Sigma \nu_n \mu_n$. The result is that the constant E_0 (see 3.5) is multiplied by

$$\alpha = \frac{1 + \Sigma\, \nu_n \mu_n}{1 + \Sigma\, \nu_n}.$$

Let us estimate the minimum and maximum values of α. The relative abundance of elements in the corona may be the same as in the photosphere — which gives the maximum value of α — but there may also be an excess of light elements. The extreme case of a corona consisting of pure hydrogen gives the minium value of $\alpha = 1$.

In order to calculate the maximum value of α, let us schematize the composition of the photosphere — as UNSÖLD[1] does — through assuming that it consists of $H\,(\mu_H = 1)$, $O\,(\mu_O = 16)$ and metals (average $\mu_{\text{metals}} = 32$) in the proportions $28:1:1$. Let us further assume that due to the very high temperature in the corona the atoms are highly ionized, say $\varepsilon_H = 1$, $\varepsilon_O = 6$, and $\varepsilon_{\text{metals}} = 10$. These assumptions give $\alpha = 1.6$.

Consequently, if the corona contains a considerable fraction of other elements than hydrogen, the values of E, T, and p in Table are increased but probably by not more than 60 %. (The velocity v increases by ≤ 30 %.)

[1] A. UNSÖLD, loc. cit. p. 348.

§ 12. On the degree of ionization in the corona.

Starting from the data given in Table 1 we can estimate the degree of ionization of matter in the corona. If we use SAHA's formula we find that ionization proceeds until an ionization potential of almost 2000 volts is reached. However, SAHA's formula gives the state of equilibrium, but this is approached very slowly because of the extremely small density of the corona. Because of the rather swift (macroscopic) motions in the corona no matter is likely to remain there long enough to reach this state. Consequently, the degree of ionization is usually much lower.

An estimation of the degree of ionization may be made in the following way. Suppose that the energy necessary to ionize a certain atom which is already $n-1$ times ionized amounts to E_n. The condition for such an ionization is that another molecule (probably an electron) having an energy of at least E_n strikes within a certain surface $\pi\sigma^2$. The number of collisions of this type is

$$Z = \pi \sigma^2 v N_\varepsilon \text{ sec}^{-1}.$$

Here N_ε means the number of particles (electrons) per unit volume which have energies above E_n. If we put $E_n = \varepsilon_n E$ (E denoting the mean energy as earlier) we have

$$n_\varepsilon = \sqrt{\frac{6}{\pi}} N \sqrt{\varepsilon_n}\, e^{-\frac{3}{2}\varepsilon_n}.$$

according to the usual formulae of kinetic gas theory. Further, $v = \frac{4}{\sqrt{3\pi}}\sqrt{\frac{\varepsilon_n E}{m}}$ is the linear velocity of these particles. We can now calculate the mean life time τ_n of an n times ionized atom. Putting $E = 100\, e$ volts $= 1.59 \cdot 10^{-10}$ erg and $m = 9.0 \cdot 10^{-28}\, g$ (mass of the electron) we obtain

$$\tau_n = \frac{1}{Z} = \frac{0.75 \cdot 10^{-9}}{\pi\sigma^2 N \varepsilon_{n+1}} e^{\frac{3}{2}\varepsilon_{n+1}} \qquad (12.1)$$

or $\varepsilon_{n-1} - 1.54 \log_{10} \varepsilon_{n-1} =$

$$= 1.54\,[9.1 + \log_{10} \pi\sigma^2 + \log_{10} N + \log_{10} \tau_n]. \qquad (12.2)$$

Let us try a tentative estimate of the maximum value of ε. As found earlier, $\log \pi\sigma^2$ is probably about -17 and $\log N$ is about $+8$ for the inner corona. According to LYOT the mean life of the structure of the corona is some days. It is likely that the time during which a certain atom remains in the corona is of the same order. If this atom has to pass several states of increasing ionization, the value of τ for each of them cannot surpass about one day. Thus putting $\log \tau_n \sim 5$ we obtain $\varepsilon \sim 9$ and $E_{n+1} = \varepsilon E \sim 900$ volts. Consequently, we should expect the matter in the corona to be ionized up to ionization potentials of about 900 volts.

Dr. B. EDLÉN has kindly given me the following list of the iron lines identified by him in the corona spectrum.

Spectrum	Ionization voltage	
Fe VI	100	Lines expected but not observed
Fe VII	125	» » » » »
Fe VIII	150	Not expected
Fe IX	233	» »
Fe X	261	Expected and observed
Fe XI	289	» » »
Fe XII	331	Not expected
Fe XIII	361	Expected and observed
Fe XIV	392	» » »
Fe XV	454	Not expected
Fe XVI and higher		» »

The strongest line (green corona line) belongs to Fe XIV.

Introducing the ionization values above into (12.1), we find as mean life for Fe VII 5 sec, for Fe X 20 sec, and for Fe XIV 150 sec.[1] Since all these values are much smaller than the total life ($\sim 10^5$) of the atoms in the corona, the number of atoms in the different states are proportional to the mean lifes. This would account for the facts that Fe VII is not observed and that Fe XIV gives the strongest of the spectral lines.

§ 13. On the ray structure of the corona.

Up to now we have treated the corona as being spherically symmetrical to a first approximation. On practically all photographs, however, the corona exhibits a pronounced ray structure. As has been pointed out by several authors the rays remind one very much of the lines of force from a magnetic dipole. This is in good agreement with the views developed in this paper.

Suppose that in a certain small part of the sun's surface there is a strong activity, which produces an unusually large

[1] The absolute values are of course very uncertain.

number of high energy particles or particles having an unusually high energy. Since to a first approximation the particles can move only parallel to the magnetic field, it is evident that the particles flow out along the magnetic lines of force going through the centre of activity, thus »illuminating» these. According to § 8 all the forces acting along the magnetic lines of force are proportional to $\cos \alpha = \frac{2 \sin \varphi}{\sqrt{1 + 3 \sin^2 \varphi}}$ (φ being the heliomagnetic latitude). Consequently, if the activity disturbs the equilibrium so that the expression within parentheses in (8.6) does not cancel, the force $F_{\parallel}$ becomes larger the higher the latitude. This means that the motion of matter in the prefered direction determined by the magnetic field is most pronounced towards the magnetic poles.

Close to the equatorial plane $F_{\parallel}$ vanishes. Thus the motion parallel to the magnetic field must be expected to be less pronounced here. On the other hand the drift perpendicular to the magnetic field is at a maximum at the equator. This drift is due to the combined action of gravitation, pressure gradient, and the inhomogeneity of the magnetic field and is perpendicular to the vector radius from the centre of the sun. It can be calculated from the formulae given in a recent paper.[1] Even at low heliomagnetic latitudes the drift is very slow (< 1 cm/sec) if the particles have energies of the order of 100 e volts. If the corona is spherically symmetrical it is probably of very little importance. However, if the density or (and) energy of the charged particles is unusually large in a certain (»active») region the drift gives rise to an electric polarisation, because the velocity of the drift of electrons is different from that of ions. The electric field produced in this way gives rise to another drift, which is directed outward from the sun. This process has been discussed in some details in connection with the problem of the origin of magnetic storms and aurorae.[2] Although in the paper referred to the value given for the energy of the particles is probably too high, there seems to be little doubt that such a process really occurs. As shown in the same paper it gives rise to streams of charged particles leaving the sun in the radial direction close to the equatorial plane. If their energy is large enough such streams may reach the earth and cause magnetic disturbances and aurorae.

It would carry us too far here to analyze these interesting phenomena in more detail. Our brief discussion has indicated a possible line for explaning the ray structure of the corona:

[1] H. ALFVÉN, Ark. f. Mat., Astr. o. Fysik. Bd 27 A. N:o 22, 1940.
[2] H. ALFVÉN, K. Sv. Ak. Handlingar III. Bd 18. N:o 3, 1939 and 9, 1940.

The short »brush-like» rays observed at high heliomagnetic latitudes are probable due to outflow along the magnetic lines of force from active centres. The long, more irregular rays observed at low heliomagnetic latitudes may be caused by drifts due to electric fields.

Summary.

In part I it is pointed out that several observational facts indicate the existence in the solar corona of particles (electrons and ions) having energies far above thermal energies (see § 2). It is tentatively suggested that the corona might consist altogether of such high energy particles which is the same as to say that it is heated to an extramely high temperature. This temperature is computed from the density function (derived from empirical data by BAUMBACH) under the assumption that the solar gravitation is the only force acting upon the corona (see § 3). The value obtained ($\sim 10^6$ degrees) is consistent with the values found in different ways in § 2. The supply of high energy particles to the corona (heating of the corona) may be due to the prominences and prominence-like action of the chromosphere (see § 4). The total energy necessary to maintain the very high temperature of the corona is small ($\sim 10^{-5}$ of the energy radiated by the sun) because the radiation losses of the extremely thin corona are negligible (see § 5).

In part II a more complete theory of the corona is outlined. In addition to gravitation it is necessary to introduce forces due to the action of the sun's general magnetic field upon the charged particles (electrons and ions) constituting the corona (see §§ 6, 7 and 8). From the empirical density function the temperature in the corona is calculated according to the refined theory. The result is in good agreement with the temperature distribution expected theoretically (see § 9 and Fig. 3). Further consequencies of the theory are drawn and compared with observational data (see § 10). Of special interest is that the identification by EDLÉN of several lines in the coronal spectrum is consistent with the theory (see § 12). Finally the importance of the magnetic field for the ray structure of the corona is discussed briefly (see § 13).

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