How to measure the degree of PIT-ness in a credit rating system for a low default portfolio?

An alternative approach using a markovian framework

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Abstract

In order to be compliant with the Basel regulations, banks need to compute two probabilities of default (PDs): point-in-time (PIT) and through-the-cycle (TTC). The aim is to explain fluctuations in the rating system, which are expected to be affected by systematic and idiosyncratic factors. Being able to, in an objective manner, determine whether the rating system is taking the business cycle - i.e. the systematic factors - into account when assigning a credit rating to an obligor is useful in order to evaluate PD-models. It is also necessary for banks in order to use their own risk parameters and models instead of standardized models, which is desirable for most banks as it could lower capital requirements.

This thesis proposes a new measure for the degree of PIT-ness. This measure aims to be especially useful when examining a low default portfolio. The proposed measure is built on a markovian approach of the credit rating system. In order to find a suitable measure for a low default portfolio, the proposed measure takes into account credit rating migrations, the seasonal component of the business cycle and time series analysis. An analysis were performed between two different credit portfolios in order to interpret results.

The results demonstrated that the degree of PIT-ness was lower in a low default portfolio in comparison with a sampled portfolio which displayed a greater amount of rating migrations with a larger magnitude. The importance of considering relevant macroeconomic variables to represent the business cycle was mentioned amongst the most important factors to consider in order to receive reliable results given the proposed measure.

Keywords

Markov theory, Business cycle, Migration matrix, Directional mobility index, Time series analysis, Spectral analysis, Basel III, PIT-ness, PIT, TTC.
Sammanfattning


Resultaten visade att graden av PIT-ness var lägre i en kreditportfölj med få fallissemanget jämfört med en testportfölj som uppvisade en större mängd kreditbetygsmigrationer med en större magnitud. Vikten av att beakta relevanta makroekonomiska variabler för att representera affärszyklus namnades bland de viktigaste faktorerna att beakta för att få tillförlitliga resultat givet det föreslagna måttet.

Nyckelord

Markov teori, Affärszykel, Migrationsmatris, Riktningsrörelsesindex, Tidsserieanalys, Spektralanalys, Basel III, PIT-ness, PIT, TTC.
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1 Introduction

Since the recent economic downturn periods, there has been continuous advancement of the regulatory requirements. Adequate risk mitigation and overall risk management, including quantitative risk modeling, is rapidly evolving and gaining attention from regulators well as the banks. The experience with banking crises in numerous countries has also demonstrated the intricate links between deterioration in creditor quality, macroeconomic conditions, and institutional failure, highlighting the importance of evaluating the credit risk models utilized.

1.1 Background

The major source of risk for banks is the credit risk, being the risk that one of the bank’s counter parties goes into default and thereby not repaying interest and/or principal [6]. A solid framework for measuring credit risk is therefore of the utmost importance for a bank to manage and control its credit risks properly. Since the foundation of the Basel Committee in 1974, the Basel accords have been developed and fine tuned during several occasions until today’s date. The latest implementation with the purpose of enhancing the financial stability is the Basel III accords, whose two main amendments compared to the Basel II accords consists of stricter capital requirements and an introduction of regulatory capital requirements [5]. In order to become compliant with the Basel III frameworks, most financial institutions will have to develop internal models to adequately determine the risk arising from their credit exposures. The amount of risk a bank faces have an impact on the buffer capital - the regulatory capital requirement - that banks are required by regulators to put aside as a cushion in case the risks would materialise.

Under the Basel II internal ratings-based approach (IRB), banks are allowed to use their own estimated risk parameters for the purpose of calculating regulatory capital [8]. One of the main elements in doing so under the IRB approach is to determine the probability of default (PD) of their obligors, which is based on the bank’s own assessment of the PD of the individual borrowers [9]. In this
context, banks build up rating systems, which refers to the entire mathematical and technological infrastructure that a bank has put in place to quantify and assign the risk parameters. In order to adopt the IRB approach, banks must also satisfy certain requirements that they can demonstrate to the national supervisor, including a logical and documented methodology for the rating system. This is required for a bank to be allowed to use its internally created ratings systems [22]. The Basel capital requirements regulation also requires banks to take all relevant information into account when assessing an obligor’s default risk [49].

In order to explain the methodology of the rating systems used to the national supervisor under the IRB approach, bank’s seeks to use measures for explanatory and validation purposes. One aspect that bank’s seeks to clarify in their models is to which extent they are depending on the current state of the business cycle. In its latest proposal regarding the IRB approach, the Basel committee of banking supervision states: "Rating systems should be designed in such a way that assignments to rating categories generally remain stable over time and throughout business cycles. Migrations from one category to another should generally be due to idiosyncratic or industry-specific changes rather than due to business cycles" [7]. The aim is to explain fluctuations in the rating system, which are expected to be affected by systematic and idiosyncratic factors. The former factor expresses rating grades that excludes migrations due to the business cycle, which leads to a congruous definition for through-the-cycle (TTC) rating grades. The latter expresses the point-in-time (PIT) rating grades which explains the rating grades that depends on changing macroeconomic conditions.

The two factors, TTC and PIT, are also mentioned in the International Financial Reporting Standard 9 (IFRS9). IFRS9 requires banks to be able to be aware of the rating philosophy of their rating systems and explicitly be able to model "point-in-time, forward looking PDs". In order to describe the rating system to a third party, a measure of how sensitive the rating system’s estimate is to the systematic influence of the business cycle is valuable. This measure is called the degree of point-in-time-ness (PIT-ness) in the rating system. IFRS9 also implicate that an indicator which shows the PIT-ness of the rating system should be used in order to choose the right method for comparative purposes [31]. The information
regarding how a rating system is responding to an economic expansion and an economic recession is therefore highly relevant from this perspective and can also be used for calibration purposes. This document also includes the expectation that banks are aware of the rating philosophy of their rating systems (i.e. the level of PIT-ness), given the following statements about the measure of the degree of PIT-ness:

- "It should be able to analyze the appropriateness of the philosophy underlying the rating or pool assignment in terms of how institutions assign exposures, obligors or facilities to risk buckets according to appropriate risk drivers”.
- "It should decide the rating philosophy”.
- "The choice of rating philosophy must be taken into account in the calibration”.
- "Care needs to be taken in the use of information from another rating system that has a different rating philosophy”.

Intuitively, the degree of PIT-ness is a characteristic of the bank’s rating system that tells us how sensitive the rating system estimates are to the systematic influence of the business cycle. However, matters become more complicated if the bank obtains its PD estimates not directly, but rather in two consecutive steps: first, by assigning rating grades to customers on the basis of the ordinal rating score (i.e. rating assignment), and, second, by estimating rating-grade PDs (i.e. calibration). In this case an additional dimension of the problem is to be considered, namely rating migration. The latter splits the impact of the business cycle into two parts, so that one can talk about the degree of PIT-ness of a rating system (i.e. the degree of rating migration in response to changing macroeconomic conditions) [44]. The approach to be investigated in this thesis uses the credit ratings and its migrations to assess the PIT-ness of the rating system. Credit ratings are set by rating agencies such as Standard and Poor’s (S&P) or Moody’s, but larger banks and financial companies often have their own internal rating system used on its counterparties [52].

As the first contribution in this study, this thesis seeks to estimate rating migrations using a Markovian framework for a low default portfolio, meaning
that an obligors credit rating can be regarded as a Markov chain. This approach requests that the rating system considered uses some sort of ordinal rating scale. The markovian approach is specifically relevant for a low default portfolio due to the fact that such a portfolio consists of few actual default observations. Thus applying structural-form based approaches, that will be presented in greater detail in section 3.1.1, could be problematic. Credit ratings are considered as an evaluation of the credit risk of an obligor and thus provide an implicit forecast of the likelihood of an obligor defaulting [45]. Since a PIT-system re-grades obligors more actively than a TTC-system, the degree of PIT-ness, which provides a measure of the rating philosophy, can be determined by measuring the mobility of transition matrices. The mobility of a transition matrix can be measured by a single number, namely a mobility metric or index. There are two classes of mobility metrics, eigenvalue based metrics and norm based metrics. The eigenvalue based metrics measure the amount of mobility inherent in a particular transition matrix because they can be used to derive the future composition of credit ratings distribution. The norm based metrics measure the distance or magnitude of the change in the current distribution of ratings implied by a particular transition matrix. This thesis seeks to measure the mobility of transition matrices using a norm based metrics. One interpretation is that is also is a measure of the degree of rating migration. Hence, it will be a characteristic of the rating grades, pertaining to the assignment step rather than to a particular calibration technique. Thus, this thesis propose a new way of defining and estimating the degree of PIT-ness of a rating system that will take into account the direction and the magnitude of the credit rating migrations, well as considering the distribution of the obligors’ credit ratings. Furthermore, this thesis seeks to estimate the business cycle. This will be done in order to define how sensitive the rating system is considering the business cycle, which will be important given the measure of the degree of PIT-ness presented in this thesis. By evaluating time series, reflecting the directional mobility of transition matrices and the seasonal component of the estimated business cycle, this thesis aims to provide a new measure, assessing the degree of PIT-ness of a rating system. Consequently, this work can hopefully improve the accuracy of the assessment of credit risk well as being beneficial for all banks with an internal rating system.
1.2 Problem

To adopt the IRB approach and its continued use, a bank must satisfy certain minimum requirements that it can demonstrate to the regulatory supervisor. A rating system refers to the entire mathematical and technological infrastructure a bank has put in place to quantify and assign the risk parameters. Banks are allowed to use multiple ratings systems for different exposures, but the methodology of assigning an exposure to a particular rating system must be logical and documented; banks are not allowed to use a particular rating system to minimize regulatory capital requirements.

According to the Basel Framework, the standard risk weights for credit risks are very conservative compared to the historically observed loss levels. Banks with larger credit portfolios with lower risk usually have a business case of calculating their own risk parameters in their credit risk models, since this will decrease their capital requirements. The relevance for a bank to use its own credit risk model could therefore be inherited from the fact that the demands on capital requirements is based on the credit risk models, either provided by Basel regulations and IFRS9 or by the banks [23]. If the bank could provide an own credit risk model to calculate the capital required that is approved by the national supervisor, the demand on capital requirements could be optimized based on the individual bank’s demands. If a bank finds that it could be valuable for them to optimize their capital allocation, then they would like to create their own credit risk models. For this to be approved, the bank has to cater the national supervisor with a methodology for why they believe that their own risk parameters in their credit risk models could replace the risk parameters in the regulatory capital credit risk model. To grant further relevance to the model created by the bank, measures for how their credit risk models are operating could be used for evaluation. An attempt to create such a measure for evaluation would be to measure the degree of PIT-ness in a rating system. Intuitively, the degree of PIT-ness is a characteristic of the bank’s rating system that tells us how sensitive the rating system is to the systematic influence of the business cycle.

To focus the scope by having clear constraints, an important aspect of the measure is to ensure its feasibility when applied. Hence, the following research question
was posed for this thesis:

*How to measure the degree of PIT-ness in a credit rating system for a low default portfolio?*

To answer the main research question of this thesis and in order to produce further understanding in the subject, the following sub-questions are identified as:

- How is a business cycle determined, what are relevant macroeconomic factors to consider?
- How should the credit portfolio evaluated be taken into account?
- How is it made sure that the measure provides a link between the rating system and the business cycle?
- How is the degree of PIT-ness interpreted given the proposed measure?

### 1.3 Purpose

This study aims to investigate to which degree a rating system using an ordinal rating scale is considering the business cycle when determining the ratings of its obligors. Since this thesis intends to investigate a low default portfolio, the objective specifically aims to provide a measure that can simplify the degree of PIT-ness in such portfolios that generally are more complicated evaluating with other approaches, such as structural-based approaches.

### 1.4 Goal

The goal is to find a measure that can be used for any financial institute assigning ordinal credit ratings to their obligors in order to determine whether their rating system is taking into consideration the business cycle when determining the ratings. Under this approach, banks are assumed using hybrid rating system, adjusted to both PIT and TTC methodologies when assigning their obligors credit ratings.

The measure proposed has a number of features and seeks to fulfill the Institute of International Finance (IIF) requests for a measure of the degree of PIT-ness [42]:

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[42]: Institute of International Finance (IIF) request for a measure of the degree of PIT-ness.
• The measure should be able to compare rating systems independently from the portfolio it is measured on. Therefore, the measure should not be influenced by the portfolio composition (e.g. with different rating distributions or different default rates).

• The measure should also not be influenced by changes in the portfolio composition (e.g. exits, new borrowers).

• The measure should have a floor and a ceiling.

• The measure should include a link between macro economy and the rating (or PD) development. The default rate could be one indicator for the macro economy.

• In case of a stable macro economy, the measure should not be defined (or zero). As then, one could not be able to distinguish between a PIT or a TTC rating system.

• A rating system which reacts faster and in the right direction to the changes in the macro economy should get a higher value.

• The measure should not be influenced by the accuracy of the rating system.

1.4.1 Expected contribution

With IFRS 9 recently being implemented as of January 2018, and Basel IV being implemented continuously until 2027, many studies have been presented during the past decade on the topic of macroeconomic factors in relation to credit risk evaluation and the measure of the degree of PIT-ness; see e.g. [23]. However, the focus on the macroeconomic impact on the credit risk of low default portfolios, and the evaluation of the measure of PIT-ness in the rating systems utilized, is found to be limited in previous studies. Especially, research concerning low default portfolios has not been identified by the authors in scientific publications. Also, studies that seeks to evaluate the degree of PIT-ness of rating systems which make use of structural-form models might not be favorable for low default portfolios. By using a markovian approach in order to define a measure on the degree of PIT-ness, this thesis specifically seeks to provide a new method of evaluating low default portfolios principally.

The direct beneficiaries of the thesis are banks and financial institution’s credit
risk modeling teams with the need to assess their credit risk exposure well as for consultants. It will provide the credit risk modeling team with analysis about the assessments and determination of the important terms of the relationship between macroeconomic factors and the PIT-ness of their rating systems. The benefit of a better credit risk assessment is twofold. First it gives banks a better control over the risks they are facing and can be a support for business decisions. Secondly, internal models typically results in lower risk measures and thereby lower capital requirements. Since capital is costly, this is a direct benefit for a bank. As the study is principally focusing on a low default portfolio analysis, it aims to make use of relationships and theories concerning rating systems in relation to the macro economy and apply it to a quantitative analysis on the rating migrations of these low default obligors.

### 1.5 Methodology

In order to define a measure for the degree of PIT-ness in a credit rating system for a low default portfolio, the following methods are adopted. To measure to which extent the rating system is considering the business cycle, the business cycle had to be defined. The approach used in this thesis to define the business cycle is based on creating time series of different macroeconomic variables that could be used to represent a business cycle, such as *Final consumption expenditure (% of GDP)* and *Unemployment rate*. Specifically, the seasonal component of the business cycle is of certain interest, as it could reflect the cyclicality in the business cycle time series considered. The seasonal component of the business cycle time series is then interpreted as the reference time series displaying 100% PIT-ness.

Generally, rating systems are built as a combination of PIT and TTC rating philosophies. A PIT rating philosophy account for the current macroeconomic factors, and as a result, such a rating philosophy will closely track the business cycle. A TTC rating philosophy accounts only for the idiosyncratic factors, and as a result, a rating system applying a TTC philosophy will not change largely due to changes in the macro economy. The TTC-part of the rating philosophy is, just as the name gives the hint of, some sort of stable value through at least one business cycle. To increase the accuracy of the measure of the degree of PIT-ness, a long
time series is coveted. Although, given this approach, most importantly was that the time series created ran through at least one business cycle.

![Figure 1.1: Display of the PD of a portfolio depending on the rating philosophy utilized during different periods of the business cycle.](image)

The aim is to be able to model an arbitrary rating system using an ordinal scale and to put this into relation with the seasonal component of the business cycle to determine the degree of PIT-ness. In order to do so, this thesis make use of Markov theory and migration matrices to determine the mobility of the rating system. This is done by identifying obligors credit rating migrations as Markov chains, fulfilling the Markov property. By calculating the transition probabilities with a cohort approach, the migration matrix between two measure points is decided, making it possible to evaluate the dynamics of the migration matrix using a directional mobility index. Creating a time series of the values provided by the directional mobility index enables further evaluation. By examining the relationship between the standardized seasonal component of the business cycle time series and the standardized directional mobility index time series, this thesis hope to describe the degree of PIT-ness of the rating system. In order to simplify the interpretation of the results for the low default portfolio, a sampled portfolio is created. The degree of PIT-ness is also computed for this portfolio and compared to the results for the low default portfolio.
1.6 Limitations

The main focus on the study is to design a measure that provides a link between the business cycle and the rating system to be able to determine the degree of PIT-ness in the rating system. This thesis is limited to studying credit rating migrations, Markov theory and time series analysis.

The study was limited to generated data with ratings of banks in a low default portfolio. The data are generated to mimic S&Ps described rating methodology. Macroeconomic data was sourced from The World Bank. Hence market behaviors were expected to differ across countries, so was the macro economy. To conduct this study, an evaluation of the credit portfolio had to be performed in order to provide an appropriate link between the rating system and the macroeconomic conditions. The macroeconomic factors utilized in this thesis have been given on a country-specific level, which might not be representative for the ingoing obligors in the credit portfolio as they specifically represents one business sector.

As mentioned in section 1.4, the measure proposed in this paper search to fulfill IIFs requests for a measure of the degree of PIT-ness, which required some limitations for the proposed measure [42].

- The measure should be able to compare rating systems independently from the portfolio it is measured on. Therefore, the measure should not be influenced by the portfolio composition (e.g. with different rating distributions or different default rates).

This measure was developed using a markovian approach that made use of the ratings set using an ordinal rating scale. This is the commonly used manner for both rating institutes and bank’s internal methods to assign their obligor’s with a credit rating. The approach presented is flexible in using different ordinal ratings scales in order to calculate PIT-ness. Regarding the portfolio composition, principles are defined, which are mentioned in the following request.

- The measure should also not be influenced by changes in the portfolio composition (e.g exits, new borrowers).

This request was solved by defining distinct principles for the portfolio
composition. These are the following:

– Obligors must have been assigned ratings throughout the full business cycle.
– Obligors that will be used when computing the transition matrices must have non-defaulted rating grades throughout the full business cycle.

For explanatory purposes

\[ t=1 \quad t=2 \quad t=3 \]

The tinted part displays the intercept between the portfolios consisting all observations fulfilling the declared principles.

Credit portfolio used = \( |P_{t=1} \cap \ldots \cap P_{t=n}| \)

Figure 1.2: Portfolio composition.

• The measure should have a floor and a ceiling

This request was solved by introducing a quotient in the measure of the degree of PIT-ness. Given that the measure introduced will equal 1, it will suggest that that the rating system is 100% PIT. If the measure instead equals 0, it will suggest that the rating system is 0% PIT.

• The measure should include a link between macro economy and the rating (or PD) development. The default rate could be one indicator for the macro economy.

The directional mobility index, on which the time series of investigation is based on, includes a sign-function that will assign rating downgrades with a negative value and assign rating upgrades with a positive sign. S&Ps
credit ratings express a forward-looking opinion about the capacity and willingness of an entity to meet its financial commitments as they come due. But also the credit quality of an individual debt issue, such as a corporate or municipal bond, and the relative likelihood that the issue may default [46]. By reviewing credit rating migrations for individual obligors only, one could not tell whether this would be due to idiosyncratic factors or if it is due to systematic factors. As the number of obligors increase, the idiosyncratic factors that affects the credit ratings migrations of the different obligors will subsequently take each other out and one will only be left with the systematic factor that affects credit rating migrations. Hence credit ratings and credit ratings migrations are providing information regarding the business cycle. The approach and the measure presented in this thesis will therefore include a link between macroeconomy and the rating development.

- **In case of a stable macro economy, the measure should not be defined (or zero). As then, one could not be able to distinguish between a PIT or a TTC rating system.**

The measure proposed in this thesis make use of an estimate of the seasonal component of the business cycle during at least one business cycle. The measure requires not just a single observation in order to be computed. Therefore, a stable macro economy would in this thesis refer to the estimate of the seasonal component of the business cycle being flat throughout the examined period of time. The proposed measure in this thesis make use of a min-max normalization to be able to compare the directional mobility index time series and the time series representing the seasonal component of the business cycle. Given that the time series are normalized on the interval $[0, 1]$, a flat time series would not be defined due to singularity. Consequently, the measure is not defined in case of a stable macro economy, whereas it would not be possible to distinguish between a PIT or a TTC rating system.

- **A rating system which reacts faster and in the right direction to the changes in the macro economy should get a higher value.**

The measure presented is based on a directional mobility index that includes a jump condition and weight condition. The jump condition will emphasize
major changes in credit rating grades, giving these a larger magnitude in the calculations of the directional mobility index. The weight condition will reflect the effort required of making transitions between different rating grades based on the starting rating grade.

- **The measure should not be influenced by the accuracy of the rating system.**

  This part of IIF’s requests will be outside of the scope of this thesis as external data are used from S&P in order to calculate the transition matrices. Credit ratings are generally set by rating agencies such as Standard and Poor’s or Moody’s, but larger banks and financial companies often have their own internal rating system for its counter parties. There is the possibility to construct own rating buckets and PD estimates in order to create own rating grade outcomes. Due to the fact that this thesis has a time limit, and this part would require extensive work that could be prohibitive, it was decided to use the ratings provided by S&P. Hence, this thesis will not try to determine the accuracy of their rating system and in order to fulfill the requests given by IIF fully, the creation of self-made rating buckets for example, would be necessary.
2 Theoretical background

In this chapter, the relevant background and related theory is presented to give context to the analysis.

2.1 Business cycle

There is no universal method of defining the length of a business cycle, according to Burns and Mitchell [14] it can be described as follows:

"Business cycles are a type of varying rotations that can be found in aggregated economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own".

Given this definition, one can identify four distinct phases of a business cycle; trough, expansion, peak and contraction. Luvsannyam et al. [39] describes the characteristics of the different phases accordingly; through is the turning point when the contraction transforms into the expansion phase and the peak is the turning point when the expansion phase transforms into the contraction phase.
Petrov & Rubstov [44] presents different methods in order to estimate the business cycle. They mention that one can obtain an estimate of the business cycle by conditioning rating transition probabilities on macroeconomic variables. In their work, they proclaim that some requirements have to be fulfilled to be able to estimate a business cycle. Amongst these are the following:

- **It must be external - otherwise it will be incorrect to base both estimation and validation on the same data.**
- **The data needs to be forecastable.**
- **The actual numbers, not just the forecasts, must be observable.**
- **The length of the time-series used should be long enough to capture an economic downturn period.**

Petrov & Rubstov clarifies that there are many data sets that could potentially satisfy these requirements. They argue that typically, macroeconomic indexes closely followed by the financial market, such as GDP growth or inflation, are of interest.
2.2 Markov theory

In this section, definitions, properties and aspects of the Markov chain theory utilized in this thesis are presented in accordance with the works of Enger and Grandell [21].

Observations of an event that is suspected to be in-part randomly driven, can at each discrete point in time $t \in T$ be mathematically described by a stochastic variable $X(t)$. Moreover, the chain of events can be described by a discrete time stochastic process. In this thesis the discrete time stochastic processes known as Markov chains are of special interest.

**Definition 1:** A family of stochastic variables $\{X(t); t \in T\}$, where $T \in [0, \infty)$ is the index set of a the process, is called a discrete stochastic process.

In order to define the Markov chain, the Markov property has to be defined first. It is defined as:

**Definition 2:** A stationary, discrete Markov chain is satisfying the Markov property if for all stages $n$ and all states $x_0, x_1, ..., x_{n+1},$

$$P(X_{n+1} = x_{n+1} | X_0 = x_0, X_1 = x_1, \ldots, X_n = x_n) = P(X_{n+1} = x_{n+1} | X_n = x_n).$$

 Explicitly, the stochastic process at stage $n + 1$ is only dependent on its value at stage $n$.

Furthermore, the Markov chain can shift between different states. The set that considers all different states that the Markov chain can move between is referred to as the state space. It is defined as follows:

**Definition 3:** A finite or countable set $S$ forms the state space of a Markov chain. I.e each possible outcome $x_i \in S$ is called a state and the set of possible outcomes is defined as $X_i$.

Given this, the Markov chain can be defined as:

**Definition 4:** A Markov chain is a stochastic process, $\{X_i\}_{i>0}$, with a sequence of stochastic variables with outcomes $x_0, x_1,...$ on the set $S$ that satisfies the Markov property.
Due to the Markov property, the Markov chain is often referred to as being "memory-less". Furthermore, given that the Markov chain is defined, other concepts that are related with the Markov chain can subsequently be defined. One of these is the transition probability which is defined as:

**Definition 5:** The transition probability, $p_{ij}$, in a time-homogeneous Markov chain is defined as,

$$p_{ij} = P(X_n = j | X_{n-1} = i),$$

i.e the probability to go from state $i$ to $j$ in one time step.

The matrix that is constructed by all possible transition probabilities is defined as the transition matrix. The transition matrix has the following properties:

**Definition 6:** The transition matrix, $P$, is defined as the matrix $(p_{ij})_{ij} \in S$ which consists of transition probabilities such as:

$$P = \begin{pmatrix}
    p_{11} & p_{12} & p_{13} & \cdots \\
    p_{21} & p_{22} & p_{23} & \cdots \\
    p_{31} & p_{32} & p_{33} & \cdots \\
    \vdots & \vdots & \vdots & \ddots
\end{pmatrix}.$$  

**Theorem 1: (Properties of the transition matrix)**

a) $\sum_{j=1}^{n} p_{ij} = 1$ for $i = 1, \ldots, n$.

b) $p_{ij} \geq 0, \forall i, j = 1, 2, \ldots, n$.

The concept of stationarity is of interest when studying Markov processes in order to define under which circumstances the distribution of $X(t)$ is converging, and when the limit distribution is independent of the starting state.

**Definition 7:** A distribution, $\pi = (\pi_0, \pi_1, \ldots)$, is a stationary distribution to a Markov chain with the corresponding transition matrix $P$ if:

$$\pi P = \pi.$$
This is sometimes referred to as time-homogeneity, which imply to the definition of the Markov chain that,

\[ P(X_{n+1} = a|X_n = b) = P(X_n = a|X_{n-1} = b). \]

### 2.2.1 Discrete-time Markov Chain

In this thesis, the Markov chain in discrete time will be utilized. I.e. each time step can be counted as a natural number 0, 1, 2,.. in which each stochastic process has its outcomes within a finite state space, \( S = \{i_k, k = 1, 2, ..., N\} \). Worth mentioning is that the time step \( \Delta t_k \) is constant, well as the time settings in the Markov chain are referred to as stages.

### 2.2.2 Properties of the Markov Chain

Markov chains has some special properties that will be defined in the upcoming section.

**Accessibility:** A state \( j \) is said to be accessible if there is a non-zero probability eventually moving to this state for a system starting in state \( i \).

**Communication:** A state \( i \) is said to lead to state \( j \) if it is possible to go from \( i \) to \( j \) in zero, one or several time steps, it is denoted \( i \to j \). Two states are said to communicate if \( i \to j \) and \( j \to i \), it is denoted \( i \leftrightarrow j \).

**Irreducibility:** A set of states which are communicating with each other is called an irreducible set of states. A chain for which its state space is irreducible is called a irreducible chain.

**Transiency:** In a Markov chain, a state \( i \) is said to be transient if there is a non-zero probability that the Markov chain will never return to state \( i \). A state is said to be recurrent if it is not transient.

**Absorbing:** A state \( i \) lead to state \( j \) if it is possible to in a finite amount of time steps get from \( i \) to \( j \). A state is absorbing if the chain always remains in the state. This means that \( i \) is a absorbing state if and only if \( p_{ii} = 1 \).
**Periodicity:** Let $D_i$ be the set of integers $n$ such that it is possible to from state $i$ return to this state in $n$ time steps. The period, $d_i$, is referred to as the greatest common divider to the integers in $D_i$. If $d_i = 1$, it is called an aperiodic state.

### 2.2.3 The cohort model

The cohort model can be applied in order to compute the transition probabilities for all possible transition given a specified set $S$. The transition probabilities can be utilized in order to create a migration matrix $M(\Delta t_k)$, as they constitute the indexes in this. Migration matrices can for example be utilized to describe transition probabilities for several obligors’ credit rating migrations given one time step.

Let $t_0, t_1, \ldots, t_n$ be discrete time points such that the arbitrary time interval $t_{k+1} - t_k = \Delta t_k$, and $\Delta t_k$ is constant. The estimator of $p_{ij}(t_k)$ over a time interval is then,

$$\hat{p}_{ij}(t_k) = \frac{N_{ij}(\Delta t_k)}{N_i(t_k)},$$

(3)

where $N_i(t_k)$ is the number of obligors in state $i$ at time $t_k$. The factor, $N_{ij}(\Delta t_k)$, refers to the number of obligors that have migrated from state $i$ to state $j$ between time $t_k$ and $t_{k+1}$ [27].

The estimations of $p_{ii}$ and $p_{ij}$, form the migration matrix $M(\Delta t_k)$ for the time interval $\Delta t_k$. There is no time-homogeneity assumption, meaning that different migration matrices will be computed for every $\Delta t_k$ throughout the business cycle.
2.3 Directional mobility index

Let $\mathcal{P}$ be the set of transition matrices:

$$\mathcal{P} = \{ \mathbf{P} = (p_{ij}) \in \mathbb{R}^{k \times k} | p_{ij} \geq 0, \sum_{j=1}^{k} p_{ij} = 1, \forall i = 1, \ldots, k \}.$$

A directional mobility index can be defined as a function, $I : \mathcal{P} \rightarrow \mathbb{R}$, chosen in order to provide a suitable and synthetic description of the mobility. It can be utilized in order to evaluate the prevailing direction of the dynamics in the migration matrix. Ferratti et al. [24] presents a directional mobility index originating from a migration matrix, which is the one that is utilized in this thesis. The proposed directional mobility index is defined as follows:

$$I_{\text{dir}}(P) = \sum_{i=1}^{n} w_i \sum_{j} p_{ij} \text{sign}(i - j)v(|i - j|),$$

(4)

where the function $\text{sign}$ is defined as:

$$\text{sign}(x) := \begin{cases} 
-1, & \text{if } x < 0 \\
0, & \text{if } x = 0 \\
1, & \text{if } x > 0.
\end{cases}$$

The $\text{sign}$ function is supposed to give an indication whether the migration is to a positive or negative direction within the matrix. In their proposed directional mobility index, the initial states are weighted by the factor, $w_i$, for every initial state $i \in \{1, \ldots, k\}$ such that $w_i > 0$ and $\sum_{i=1}^{n} w_i = 1$. In their proposal, $w_i$ corresponds to the percentage of obligors starting from $i$. The parameter $v$, denoted as the jump parameter, estimates the magnitude of the credit rating migrations from state $i$ to state $j$, for each $i, j = 1, \ldots, k$. It can be assigned in multiple ways, and in their work, Feretti & Ganugi [24] presents the following alternatives for $v$.

$$1, \quad \log_{10}(|i - j| + 1), \quad \sqrt{|i - j|}, \quad |i - j|, \quad |i - j|^2, \quad e^{|i-j|} - 1.$$

Table 2.1: Alternative choices of the jump parameter, $v$.  

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As an illustration, selecting \( v = |i - j| \), a migration from state \( i \) to state \( i + 2 \) would be given twice the magnitude in the directional mobility index as a migration from state \( i \) to state \( i + 1 \). Linear measures, such as \( v = |i - j| \), are generally a reasonable choice when the variable under study is qualitative, which is the case when evaluating credit rating migrations [25]. Though, the selection of \( v \) is strictly related to the specific dynamics of the variable evaluated.

### 2.3.1 Properties of the mobility index

The directional mobility index fulfills four important properties; boundedness, perfect mobility, immobility and monotonicity. The following propositions are presented in order and will provide the validity of these properties for the family of directional mobility indexes, including the one presented by Ferratti et al.

**Proposition 1** For every choice of \( w_i \) and \( v \), and for every \( P \in \mathcal{P} \) we have,

\[
m_1 \leq I_{\text{dir}}(P) \leq m_2,
\]

where \( m_1 \leq 0 \) and \( m_2 \geq 0 \) are not depending on \( P \) and defined by:

\[
m_1 = \sum_{i=1}^{n} w_i v(1 - i); \quad m_2 = \sum_{i=1}^{n} w_i v(n - i).
\]

**Proposition 2** For every choice of \( w_i \) and \( v \) the index, \( I_{\text{dir}} \), satisfies \( I_{\text{dir}}(P) = m_1 \), if and only if, \( P = P_- \) (perfect positive mobility) and \( I_{\text{dir}}(P) = m_2 \), if and only if, \( P = P_+ \) (perfect negative mobility). Then the index is said to be strongly perfect mobile.

**Proposition 3** The directional index \( I_{\text{dir}} \) satisfies the following properties:

a) if \( w_i = \frac{1}{n} \) for every \( i = 1, \ldots, n \) and \( P \) is a symmetric matrix, then \( I_{\text{dir}}(P) = 0 \).

b) for every choice of \( w \), if \( P \) is a matrix such that for every \( i = 1, \ldots, n \) and for every \( l = i - 1, \ldots, n - i \) it holds \( p_{ii-l} = p_{ii+l} \), then \( I_{\text{dir}}(P) = 0 \).

In this sense \( I_{\text{dir}}(P) \) satisfies the weak immobility.

**Proposition 4** For every \( P, Q \in \mathcal{P} \) such that \( p_{ij} \leq q_{ij} \), and for every choice of \( w_i \) and \( v \), the directional index satisfies \( I_{\text{dir}}(P) \leq I_{\text{dir}}(Q) \).
2.4 Time series analysis

A time series is a set of observations \( \{x_t\} \), each one being recorded at a specific time \( t \). A discrete-time time series is one, in which the set \( T_0 \) of times at which observations are made is a discrete set [13]. In this thesis, time series models for the discrete case are evaluated.

2.4.1 Time series model

**Definition 8:** A time series model for the observed data \( \{x_t\} \) is a specification of the joint distributions (or possibly only the means and covariances) of a sequence of random variables \( \{X_t\} \) of which \( \{x_t\} \) is postulated to be a realization [13].

A time series model is a dynamic system that is identified to fit a given signal or time series data. Modeling a time series can for example make it easier to identify patterns in the examined data set.

2.4.2 Additive decomposition

Decomposing time series is a statistical task that refers to the deconstruction of a time series into several components, each representing an underlying category of patterns. An additive decomposition of a time series is illustrated as follows,

\[
X_t = m_t + s_t + \epsilon_t,
\]

where \( m_t \) is the trend component, \( s_t \) is the seasonal component and \( \epsilon_t \) is a random noise term with zero mean [13]. The additive method is often utilized when the seasonal variation is relatively constant over time, i.e, the tendency of the data repeat itself of every \( L \) period [35].
2.4.3 Stationarity

A time series, \( \{X_t, t = 0, \pm 1, \ldots\} \), is said to be stationary if its statistical properties is similar to those of the "time-shifted" time series \( \{X_{t+h}, t = 0, \pm 1, \ldots\} \) for each integer \( h \) [13]. Limited attention will be given to those properties that depend only on the first- and second-order moments of \( \{X_t\} \) in this thesis. Based on this, the following definitions are presented.

**Definition 9:** Let \( \{X_t\} \) be a time series with \( E[X_t^2] < \infty \), then the mean function of \( \{X_t\} \) is \( \mu_X(t) = E[X_t] \). The covariance function of \( \{X_t\} \) is \( \gamma_X(r, s) = Cov(X_r, X_s) = E[(X_r - E[X_r])(X_s - E[X_s])] \) for all integers \( r \) and \( s \).

**Definition 10:** \( \{X_t\} \) is (weakly) stationary if

(i) \( \mu_X(t) \) is independent of \( t \).

(ii) \( \gamma_X(t + h, t) \) is independent of \( t \) for each \( h \).
2.4.4 Stationarity tests

There are different tests to determine whether a time series is stationary or non-stationary.

2.4.4.1 Augmented Dickey-Fuller test (ADF-test)

The augmented Dickey-Fuller tests, examines the null hypothesis that a unit root is present in a time series sample. The test starts with a hypothesis test for an auto-regressive (AR) time series, which refers to a time series model that uses observations from previous time steps as input to a regression equation to predict the value at the next time step. The ADF-test is conducted accordingly:

\[ X_t = \phi X_{t-1} + \epsilon_t; \quad H_0 : \phi = 1, \; H_1 : \phi \leq 1, \quad (\ast) \]

where, if \( H_0 \) is true then the time series is said to be non-stationary.

If \( X_{t-1} \) is subtracted from both left-hand side and right-hand side of the equation, one receives:

\[ X_t - X_{t-1} = (\phi - 1)X_{t-1} + \epsilon_t, \text{ and set } (\phi - 1) = \delta. \]

\[ \Delta X_t = \delta X_{t-1} + \epsilon; \quad H_0 : \phi = 1, \; H_1 : \phi \leq 1. \]

If \( H_0 \) is true, then the time series is said to be non-stationary.

By using the \( t \)-ratio, \( \tau = \frac{\hat{\delta}}{SE(\delta)} \), the augmented Dickey-Fuller test rejects the null hypothesis of a unit root at significance level \( \rho \) (often 0.05). If \( \tau < t_\rho \), the time series is said to be stationary. This can also be done on a general level for an AR(p) model.

2.4.4.2 Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test

The Kwiatkowski-Phillips-Schmidt-Shin test, examines the stationarity of a time series model. The procedure is similar to the ADF-test, as the null hypothesis \( H_0 : \phi = 1 \) is tested against the hypothesis \( H_1 : \phi \leq 1 \). Unlike the ADF-test, the KPSS-test takes into consideration a trend component \( m_t \) in the right-hand side of \( (\ast) \). Using the \( t \)-ratio, the null hypothesis is rejected if \( \tau < t_\rho \).
2.4.5 Spectral density

Any signal that can be represented as a variable that varies in time has a corresponding frequency spectrum, which can be calculated using the Fourier transform. The Fourier transform decomposes a function (often a function of time) into its constituent frequencies that can be characterized by cosines and sines. The spectral representation of a stationary time series, \( \{X_t\} \), is the essential method to decompose \( \{X_t\} \) into a sum of sinusoidal components with random coefficients that are uncorrelated, in order to analyze the time series in the frequency domain. Hence, spectral decomposition is an equivalent method for evaluating stationary processes of the Fourier transform.

Given that \( \{X_t\} \) is a zero-mean stationary time series with autocovariance function (ACVF) \( \gamma(\cdot) \), satisfying \( \sum_{h=-\infty}^{\infty} |\gamma(h)| < \infty \), the spectral density of \( \{X_t\} \) is the function \( f(\cdot) \) defined by [13],

\[
f(\lambda) = \frac{1}{2\pi} \sum_{h=-\infty}^{\infty} e^{-ih\lambda}, \quad -\infty < \lambda < \infty, \tag{6}
\]

where \( e^{i\lambda} = \cos(\lambda) + isin(\lambda) \) and \( i = \sqrt{-1} \). The summability of \( |\gamma(\cdot)| \) implies that the series converges absolutely. Both \( \cos \) and \( \sin \) have the period \( 2\pi \), which implies that it is sufficient to confine \( f(\cdot) \) to the interval \((-\pi, \pi]\). Some basic properties of \( f(\cdot) \) are:

a) \( f \) is even, i.e, \( f(\lambda) = f(-\lambda) \).

b) \( f(\lambda) \geq 0 \quad \forall \lambda \in (-\pi, \pi] \).

c) \( \gamma(k) = \int_{-\pi}^{\pi} e^{ik\lambda} f(\lambda) d\lambda = \int_{-\pi}^{\pi} \cos(k\lambda) f(\lambda) d\lambda \).

**Definition 11:** A function \( f(\cdot) \) is the spectral density of a stationary time series \( \{X_t\} \) with ACVF, \( \gamma(\cdot) \), if

i) \( f(\lambda) \geq 0 \quad \forall \lambda \in (0, \pi] \).

ii) \( \gamma(h) = \int_{-\pi}^{\pi} e^{ih\lambda} f(\lambda) d\lambda \) for all integers \( h \).

Its frequency, \( f \), can be described as the amount of waves that pass a fixed location in a certain time. The relation between the period, \( T \), and the frequency is defined as \( T = \frac{1}{f} \). In finance, spectral analysis are especially relevant to processes that contains inherently cyclic components, such as business cycles etc. [54]. It
can also be used in order to determine whether there are any dominant cyclical components contained in a time series, \( \{ X_t \} \). Kamen et al. [36] defines the term, ”dominant”, as any sinusoidal component whose amplitude in \( \{ X_t \} \) is much larger than the amplitudes of most of the other sinusoidal components included in \( \{ X_t \} \). The dominant sinusoidal component is of interest if one would like to represent the cyclical component in a time series given only one sinusoidal component.

### 2.4.5.1 Periodogram

A periodogram is a method for estimating the spectral density of a time series. The periodogram can be utilized in order to receive an easy to understand illustration for which frequencies that are dominant in a time series.

![Example of a periodogram](image)

Figure 2.3: Example of a periodogram, the y-axis corresponds to the level of spectral density.

If \( \{ X_t \} \) is a stationary time series with the corresponding ACVF \( \gamma(\cdot) \) and spectral density \( f(\cdot) \), then the periodogram, \( I_n(\cdot) \), can be regarded as a sample analogue of \( 2\pi f(\cdot) \). The periodogram of \( \{ x_1, x_2, \ldots, x_n \} \) is given as [13]:

\[
I_n(\lambda) = \frac{1}{n} \left| \sum_{t=1}^{n} x_t e^{-it\lambda} \right|^2.
\]
2.4.6 Standardization

Feature scaling is a method used to normalize the range of independent variables or features of data. One such method is the min-max normalization. The method provides a linear transformation of the original range of the data. The technique can be used to fit the data on a predefined boundary. The original formula on the arbitrary interval \([a, b]\) is given as:

\[
x' = a + \frac{(x - x_{\text{min}})(b - a)}{x_{\text{max}} - x_{\text{min}}},
\]  

(7)

For the interval \([0, 1]\), the formula is given by:

\[
x' = \frac{x - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}},
\]  

(8)
3  Literature review

This chapter presents relevant literature aided to deepen the knowledge on the topics treated in the thesis. Previous studies within the field of credit risk related to the application of Markov chains in credit-risk-modeling, measures of the degree of PIT-ness well as studies covering the special case regarding low default portfolios are presented.

3.1  Previous studies

3.1.1  Structural-form models and reduced-form models

The credit risk models built by banks can generally be grouped into two main categories: the structural-form models and the reduced-form models. The difference between these two categories of models is the implicit assumption they make about managerial decisions regarding their capital structure. The structural-form models are also called the asset value models for assessing credit risk, typically of a corporation’s debt. These models are generally based on the principle of pricing option in the Black-Scholes model and a more detailed model developed by Merton [12][40]. The Merton model uses the Black-Scholes-Merton option pricing methods and is called structural as it provides a relationship between the default risk and the capital structure of the firm. In spite of the extensions of Merton’s original framework, these models still suffer some drawbacks. As the firm’s value is not a tradable asset, nor is it easily observed, the parameters of the structural form models are difficult to estimate consistently. Also, there are some inclusions of some frictions like tax shields and liquidation costs. Another drawback is that corporate bonds undergo credit downgrades before they actually undergo default, but structural form models cannot incorporate these credit-rating changes. Reduced form models attempt to overcome these shortcomings of structural form models; unlike structural form models, reduced form model make no assumptions at all about the capital structure of the borrowers. In contrast, reduced form models can extract credit risks from the actual market data and are not dependent on asset value and leverage.
3.1.1.1 Markovian approach and transition matrices

There is one strand of the credit-risk-modeling literature that makes use of a matrix of transition probabilities to explain the migration of creditor quality, as measured by proxies such as credit ratings. These reduced-form models, based on rating migrations show the evolution of creditor quality for broad groups of obligors with the same approximate likelihood of default. This approach utilizes matrices of transition probabilities that can be used as an input to model credit evolution, which summarizes a broad range of possible creditor dynamics in a simple and coherent fashion.

Papers such as the one written by Jafry and Schuermann [33] suggested different approaches of comparing credit migration matrices to each other. In their seminal work on credit spread, Jarrow et al. derived the risk premium for the credit risk process from a Markov chain on a finite state space. In this paper, the estimation of PD were derived from transition matrices given a Markov chain [34]. There are also some attempts to identify the impact of the business cycle using rating transition matrices. The works of Wilson [11], Belkin et al. [10], Alessandrini [1], Kim [37], Nickell et al. [41] tried to identify the relationship between the business cycle and rating transition matrices. Rikkers and Thibeault [43] concluded in their studies that capital requirements under a PIT rating method are on average lower than under a TTC rating approach, even if the average PD over the period is the same. However, this study concluded that PIT ratings lead to very volatile and procyclical capital requirements. Cesaroni [16] showed that ex-post PD smoothing is able to remove business cycle effects on the credit risk estimates and to produce a mitigation of obligors’ migration among risk grades over time. This research also concluded that rating scale choice also has a significant impact on rating stability.

Against this convenience of adopting a markovian approach in credit risk modeling is mounting evidence of non-markovian behavior of the rating process. Altman and Kao [3], Carty and Fons [15], Altman [2], Nickell et al., Bangia et al. [4], Lando and Skødeberg [38], Hamilton and Cantor [30] have shown the presence of non-markovian behavior such as ratings drift and industry heterogeneity, and time variation due in particular to the business cycle.
Christensen et al. [18] considered the possibility of latent "excited" states for certain downgrades in an effort to address serial correlation of ratings changes. Giampieri et al. [28] made use of a hidden Markov model to deduce the state of the economy from rating dynamics although their model focuses specifically on default prediction. Stefanescu et al. [48] considered a simulation-based Bayesian approach that allowed for some ratings momentum. Feretti et al. [24] applied an extension of a Markov chain model, the Mover-Stayer model, in order to determine the migration risk of small and medium enterprises. They found that banks are over-estimating their credit risk resulting in excessive regulatory capital, reinforcing the importance for banks to have well functioning credit risk models in order to be profitable.

3.1.2 Measure of the degree of PIT-ness

There are previous studies proposing different measures of the degree of PIT-ness. In their paper, Petrov & Rubtsov [44], suggests a decomposition of portfolio-level default rates into three components; a TTC-part, a PIT-part and a part that accounts for the natural improvement in portfolio quality due to defaults of the worst obligors. These three components are then obtained for every year in the bank’s historical sample and the degree of PIT-ness is proposed to be:

\[
\text{Degree of PIT-ness} = \frac{\text{Std dev.}(\Delta ADF_{PIT})}{\text{Std dev.}(\Delta ADF_{PIT}) + \text{Std dev.}(\Delta ADF_{TTC})}
\]

The PIT component reflects the standard deviation of the change in the portfolio distribution due to credit rating grade migration, assuming constant default rates per credit rating grade. The TTC component reflects the standard deviation of the change in default rates per credit rating grade assuming no migration.

Another proposal for the degree of PIT-ness was presented by the Italian financial industry risk managers association (AIFIRM) [20], and analyzes the rating models capacity to attribute the volatility of the overall default rate to class migrations. In this proposal, the volatility of the default rate is divided into two components; the volatility of default rate for each credit rating class adjusted for the class migrations, and the overall volatility of the default rate. The relative weight
between these two components is the sought property of the rating system, for which the suggested degree of PIT-ness is defined as follows:

\[
\text{Degree of PIT-ness} = \frac{\text{Std dev of the default rate adjusted for the class migrations}}{\text{Std dev. of the default rate}}.
\]

Jafry & Schuermann [33] proposed a mobility index to calculate the degree of PIT-ness by evaluating transition matrices. In their proposal, the index used is calculated as the Euclidean distance between an observed transition matrix and the time homogenous transition matrix. The homogenous transition matrix is the identity matrix where the ratings do not change with time. Given an observed transition matrix \( T \), a mobility matrix \( \hat{T} \) is defined as:

\[
\hat{T} = T - I,
\]

where \( \hat{T} \), defines the degree of concentration of the transition matrix along its diagonal that deviates from an identity matrix. The larger the probability of transitioning to a different rating, the less concentrated will the transition matrix be along its diagonal. The average single value of a transition matrix is given by \( M_{SVD} \) and is defined as:

\[
M_{SVD} = \frac{\sum_{i=1}^{N} \lambda_i (\hat{T}^T \hat{T})}{N}.
\]

In this proposal, \( \lambda_i \) is the \( i \):th eigenvalue of \( \hat{T}^T \hat{T} \) and \( N \) is the dimension of the mobility matrix. A higher value of \( M_{SVD} \) is suggesting a higher probability of transitioning to a different rating in the rating system.

### 3.1.3 Low default portfolio

A substantial issue arises in certain credit portfolios consisting of low default obligors, as for higher rating classes practically no defaults are observed, yielding default probabilities of zero. Merton’s model, that uses actual default frequencies (ADF) to calculate the PD will not be effective. A second relevant question relates to the estimation of rating migration risk for the banks’ economic capital: to be effective, internal rating models should be designed coherently not only with the actual borrowers’ standing, but also with their expected assessment pattern [26]. Wilson [53] displayed that transition probabilities change over time as
the state of the economy evolves. In his approach, default probabilities were a function of macro variables such as unemployment, interest rate, the growth rate, government expenses, and foreign exchange rates. Wilson then used macro variables in order to derive the business cycle. The Basel Committee on Banking Supervision also emphasizes the importance of the business cycle, which may improve the accurate assessments of credit risk [6].
4 Data

In this chapter, the data sets utilized are presented and the process of handling data will be described. The process of adjusting the original data in order to establish a final data set will also be described in greater detail.

4.1 Handling data

The data set in its original form contained 11625 observations from 964 obligors, distributed over the years 1980 – 2018. The original data set did not contain any missing values. The sections below describe for which reason the original data set was adjusted and how this was executed.

4.1.1 Credit data

The original data set consisted of credit ratings retrieved from an internal database at Nordea in the beginning of each year. The credit ratings were set in a way that mimic how S&P would rate the customers in accordance with their described rating process [47]: "For corporate, government, and financial services company or entity (collectively referred to as “C&G”) Credit Ratings, the analysis generally includes historical and projected financial information, industry and/or economic data, peer comparisons, and details on planned financing’s. In addition, the analysis is based on qualitative factors, such as the institutional or governance framework, the financial strategy of the rated entity and, generally, the experience and credibility of management".
<table>
<thead>
<tr>
<th>year</th>
<th>id</th>
<th>rating</th>
<th>country</th>
<th>sub_region</th>
<th>region</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>238806</td>
<td>A+</td>
<td>US</td>
<td>Northern America</td>
<td>Americas</td>
</tr>
<tr>
<td>2007</td>
<td>239083</td>
<td>A+</td>
<td>US</td>
<td>Northern America</td>
<td>Americas</td>
</tr>
<tr>
<td>2007</td>
<td>239342</td>
<td>AA</td>
<td>US</td>
<td>Northern America</td>
<td>Americas</td>
</tr>
<tr>
<td>2007</td>
<td>239691</td>
<td>A+</td>
<td>CA</td>
<td>Northern America</td>
<td>Americas</td>
</tr>
<tr>
<td>2007</td>
<td>239705</td>
<td>AA-</td>
<td>CA</td>
<td>Northern America</td>
<td>Americas</td>
</tr>
<tr>
<td>2007</td>
<td>239710</td>
<td>A</td>
<td>CA</td>
<td>Northern America</td>
<td>Americas</td>
</tr>
<tr>
<td>2007</td>
<td>250824</td>
<td>AA</td>
<td>GB</td>
<td>Northern Europe</td>
<td>Europe</td>
</tr>
<tr>
<td>2007</td>
<td>271413</td>
<td>AA</td>
<td>FR</td>
<td>Western Europe</td>
<td>Europe</td>
</tr>
<tr>
<td>2007</td>
<td>271442</td>
<td>AA</td>
<td>DE</td>
<td>Western Europe</td>
<td>Europe</td>
</tr>
<tr>
<td>2007</td>
<td>271618</td>
<td>AA-</td>
<td>ES</td>
<td>Southern Europe</td>
<td>Europe</td>
</tr>
</tbody>
</table>

Table 4.1: Sample of the original dataset provided by Nordea.

The data fields that existed in the data set provided by Nordea - representative for each observation - was given as follows:

- **year**: The year for which the rating was assigned.
- **id**: An identification code used for identifying a specific obligor.
- **rating**: The credit rating grade assigned for the obligor.
- **country**: The country code in which the obligor has its tax residency.
- **sub_region**: The subregion in which the country is located.
- **region**: The continent in which the subregion is located.

The ordinal scale provided by S&P, used as the credit ratings were given as:
The credit rating grades are monotonically increasing w.r.t credit risk. That is, going from one credit rating to another implies an increase/decrease in credit risk. The original data set provided by Nordea also consisted the credit rating grades: NR, R, SD. NR corresponds to not rated obligors, R and SD corresponds to different errors when assigning ratings for the obligors. These ratings can be assigned to an obligor if, for example the different credit rating grades have certain conditions that has to be met in order to receive a rating. These conditions can for instance be based on the size of the obligor or, if the obligor is a counterparty with outstanding loans or trades, and therefore withholds an exposure risk threshold. These are not official credit rating grades for which they were not included in the ordinal scale above.

Each of the data fields mentioned above were considered to be relevant in different parts of this thesis. When determining the business cycle for the evaluated credit
portfolio, the data fields; \textit{id}, \textit{year}, \textit{sub\_region} and \textit{region}, were utilized. The reason for this was to declare in which geographical regions the ingoing obligors in the credit portfolio evaluated were present. In order to calculate the transition matrices, the data fields; \textit{year}, \textit{id} and \textit{rating}, were utilized.

The database, from which the data set used for evaluation in this thesis were provided, contained a lot of information. A first round of filtering was made in order to receive what is referred to as the original data set. The original data set was then adjusted in order to fulfill the requests that a measure of PIT-ness should fulfill provided by IIF, mentioned in chapter 1.4.

4.1.1.1 Non-rated states
As the data set were examined, there were obligors that had been assigned with the credit rating grades: \textit{NR}, \textit{R}, \textit{SD}. These credit rating grades were considered non-rated states and were not considered to contribute any information to this thesis. Transitions to and from non-rated states were out of scope of the rating migrations under examination in this thesis. More formally, these types of transitions do not occur within the state space, \( S \), in accordance with the principles for portfolio composition as defined in chapter 1.6. One of these defined principles states that obligors must have been assigned ratings throughout the full business cycle. The obligors assigned with these rating grades at least once were removed from the data set, totalling 211 obligors.

4.1.1.2 Recovering defaults
Due to the Markov process assumption made in the thesis, the default state, \( D \), was assumed to be an absorbing state for which an obligor will remain in during the whole business cycle. Consequently, obligors that occurs at least once in the absorbing state were removed from the sample. Furthermore, the transition matrices did not contain the state \( D \), meaning that they will only contain states for which obligors were assigned approved ratings throughout the full business cycle. This was an important part of this thesis, as it seeks to evaluate a homogeneous data set throughout the time frame considered. The obligors assigned with this rating grade were removed from the data set, totalling 3 obligors.
4.1.1.3 Mapping ratings

The ratings provided originate from S&P, thus the credit rating scale used in this thesis replicated the one utilized by S&P. The highest credit rating is AAA and then each credit rating follow in descending order to CCC. Though, during the time period for investigation, there were no obligors with credit rating CCC, CCC+ and B−, thus the lowest credit rating was B. In order to create the transition matrices used for calculating the mobility index at each point in time, the credit ratings were ordered accordingly to the ordinal credit rating scale as provided by S&P in R, see 4.2.

4.1.1.4 Clustering ratings

Referring to the properties of the migration matrix mentioned in section 2.2, the sum of each row in the migration matrices should equal to 1. For this condition to be fulfilled in each time step it was necessary that at least one obligor starts in the i:th rating grade for every t. However, as the data were examined it was found that this was not the case. In order to fulfill the property:

\[ \sum_{j=1}^{n} p_{ij} = 1 \text{ for } i = 1, \ldots, n, \]

some ratings had to be clustered to compute the migration matrices using the same rating grades throughout the full business cycle. For the given data set, there were some missing cases of the rating grades BB+, BB−, B+ and B− for the examined time frame. Due to this, to be able to calculate the migration matrices, the rating grades BB+, BB and BB− were clustered into a single rating grade, BB. This was also applied to the rating grades B+, B and B− that were clustered into the single rating grade, B.

4.1.1.5 Portfolio composition principles

After treating the data, the defined business cycle ran between the years 2003 – 2018. The obligors considered had to be assigned approved credit ratings throughout this period, in accordance to the portfolio composition principles stated in section 1.6. The amount of obligors that fulfilled this criteria summed up to 126.
4.2 Credit portfolios

4.2.1 Low default portfolio

The portfolio examined in this thesis consisted of sampled and generated data points, replicating a low default portfolio of major banks distributed all over the world. The original obligors used in sampling, were anonymized and an unknown amount of the data was generated using skewed bootstrap for security reasons by Nordea in order to avoid identification of the counter parties and the underlying credit portfolio. During the last period of recession - following the global finance crisis - several major banks were bailed out by the governments in their respective countries, leading to defaults in this sector never being realized as they could have an undesirable effect on society \[32\]. The global finance crisis in 2008 also led to stricter regulations for banks where actions, such as guarantees to prevent bank runs, were established to maintain the financial market’s stability. Consequently, the political actions employed to prevent defaults in this sector have historically contributed to even less cases of defaults in this type of credit portfolio. Banks from the United States represented a major part of the portfolio, as approximately 60% of the total obligors had their domicile in the US. Aside from these obligors, there were banks from Europe, Asia etc., which implied a diversification in the portfolio. Due to this diversification in the portfolio, there were macroeconomic factors influencing the credit ratings of the specific obligors that could not be governed by a single country or a single continent.

4.2.1.1 Data handling, low default portfolio

The obligors retained their country code throughout the observed time period, e.g., a obligor that started with \textit{US} as country code, retained this country code for all computations. It seemed reasonable to have at least 100 obligors in the portfolio to be able to generate migration matrices. Too few observations would not be representative of the portfolio, as each migration would have a significant impact on the migration matrix. But at the same time, not having a sufficient long time series could have negative a impact on the further analysis. This reasoning induced that the observation period was set between the year, 2003 – 2018, as before year 2003, the amount of observations were scarce.
4.2.2 Sampled portfolio

To be able to interpret the results given the created measure, this thesis searched to compare two different credit portfolios against each other. The main idea was to create a credit portfolio that differed from the low default portfolio in order to draw conclusions about the proposed measure. As there was no data provided for another credit portfolio, a sampled portfolio was created, originated from the low default portfolio presented in section 4.2.1. The intention was to generate a portfolio displaying more migrations with different magnitudes, as it would differ from the low default portfolio. A desired behaviour would be that the rating system was more sensitive to the business cycle when assigning credit ratings to the obligors. This would hopefully lead to that the rating system becoming more PIT than TTC, which ultimately would lead to a higher degree of PIT-ness.

4.2.2.1 Data handling, sampled portfolio

In order to retrieve the desired behaviour in the portfolio, explained in section 4.2.2, the low default portfolio was manipulated accordingly. Between every $\Delta t_k$, approximately 10% of the ratings were manipulated. As the portfolio only consisted of 126 obligors, manipulating 10% of these was considered to have a distinguishable impact on the portfolio. This would hopefully also provide a noticeable difference in the resulting degree of PIT-ness as the different portfolios were compared. One intention was to fulfill Proposition 4 mentioned in section 2.3.1 during the business cycle, giving the directional mobility index a greater volatility. Arbitrary obligors were chosen and the ratings were changed manually in order to replicate desired behaviour of the portfolio. The ratings were manipulated such that obligors, which in the original data set had the same rating between two years, $t$ and $t+1$, received a different rating in one of these two years. An example of the procedure can be demonstrated as follows:

- Given an arbitrary obligor assigned the credit rating $AA$ in the low default portfolio at both time $t$ and time $t+1$, the same obligor was given an arbitrary credit rating, let’s say $A+$ at time $t+1$ in the sampled portfolio.
Applying this methodology on 10% of the arbitrary chosen obligors, the amount of migrations and the magnitude of these, became greater in the sampled portfolio than in the low default portfolio.

4.3 Business cycle

The data utilized to create the business cycle measures were provided by The World Bank. The time series utilized in order to represent the business cycle in this thesis are Unemployment rate and Final consumption expenditure (% of GDP).

4.3.1 Unemployment rate

The unemployment rate is the share of the labor force that is jobless, expressed as a percentage. A low unemployment rates could paradoxically disguise substantial poverty in a country, while high unemployment rates can occur in countries with a high level of economic development and low rates of poverty. This is because countries without unemployment or welfare benefits people eke out a living in vulnerable employment. In countries that has well-developed safety nets, workers can afford to wait for suitable or desirable jobs. High and sustained unemployment rates although, generally indicates serious inefficiencies in resource allocation. Unemployment is a key measure to monitor whether a country is on track to achieve inclusive and sustainable economic growth.

The data on Unemployment rate were taken from The World Bank, International Labour Organization, ILOSTAT’s database, the government of Bermuda and the CIA [19][29][51]. It consisted of data from the countries represented in the credit portfolio evaluated between the years 1991 – 2018.

4.3.2 Final consumption expenditure (% of GDP)

Final consumption expenditure (FCE) is the sum of household final consumption expenditure, i.e private consumption, and general government final consumption expenditure that could be interpreted as general government consumption. Government final consumption expenditure is made for collective consumption or for individual consumption in the form of social transfers in kind to households.
Consumer spending is a key driving force in the economy and a critical concept in economic theory. Consumer spending can simplified as the demand side of "Supply and demand" whereas the production can be simplified as the supply side. Many economists, especially those in the tradition of John Maynard Keynes, believe consumer spending is the most important short-run determinant of economic performance and is a primary component of aggregate demand [17]. If consumers spend too much of their income now, future economic growth could be compromised because of insufficient savings and investment. If consumers provide fewer revenues for a given business or within a given industry, companies must adjust by reducing costs, wages, or innovating and introducing newer and better products and services.

The data on *Final consumption expenditure (% of GDP)* were provided by the World Bank national accounts data and OECD National Accounts data files. It consisted of data from the countries represented in the credit portfolio evaluated between the years 1970 – 2018.

### 4.3.3 Handling missing data

When analyzing the data provided by *The World Bank*, it was found that some member states had not reported the used metrics utilized for describing the business cycle throughout the time period examined. Regarding the *Unemployment rate*, Bermuda had reported observations for the years 2009, 2010, 2012 – 2018 meaning that data were missing for the years 1991 – 2008 and 2011. Regarding the *Final consumption expenditure (% of GDP)*, Bermuda had reported observations for the years 2009 – 2013 meaning that data were missing for the years 1970 – 2008 and 2014 – 2018. Hungary had reported observations for the years 1991 – 2018 meaning that data were missing for the years 1970 – 1991. The missing data were handled by setting the concerned observations as *N.A.*
5 Method

In this chapter the process from creating migration matrices to producing the final results is described. Based on the final data set, migration matrices were computed in R. Given the migration matrices, the directional mobility indexes for each directional migration matrix were estimated and a time series were constructed. In order to measure the degree of PIT-ness of the rating system - reflected by the directional mobility index time series - representations of the business cycle were created. The seasonal component of the business cycle time series were created using spectral analysis to estimate sinusoidal waves. At last, the measure of the degree of PIT-ness was determined.

5.1 The Markov model

The framework that was utilized in this thesis states that an obligor is assigned a credit rating once a year. An obligor’s credit rating was assumed to be a discrete stochastic process over time. Another assumption made was that an obligor’s credit rating at year \( t+1 \) is only dependent by its credit rating the previous year \( t \). This was interpreted as what has happened to an obligor previously is considered in the given rating at time \( t \), why it should not be reflected in the rating given at time \( t+1 \). Hence, the stochastic process is in some sense “memory-less”, which was interpreted as that it fulfills the Markov property. Using this interpretation, applying Definition 4 in section 2.2, the stochastic process was defined to be a Markov chain.

Further, it was assumed that an obligor at time \( t \), could have an arbitrary credit rating. The credit ratings are finite and can be seen as different states to which obligors can be assigned at time \( t \). The credit rating grades are as previously mentioned monotonically increasing w.r.t credit risk. That is, migrating from one state to another implied an increase/decrease in credit risk. The state space, \( S \), consisted of the following states:

\[
S = \{AAA, AA+, AA, A-, A, A+, BBB+, BBB, BBB-, BB, B\}.
\]
An obligor can be in state $AA$ at time $t$, and then potentially migrate to state $BB$ by the time $t+1$. The probability of this event was denoted as $p_{AA,BB}$. After defining the stochastic process as a Markov chain, the cohort method, see (3), could be utilized in order to compute migration matrices $\mathbf{M}(\Delta t_k)$.

$$
\mathbf{M}(\Delta t_k) = \\
\begin{pmatrix}
p_{AAA,AAA} & p_{AAA,AA+} & \cdots & p_{AAA,B}
p_{AA+,AAA} & p_{AA+,AA+} & \cdots & p_{AA+,AA}
p_{AA,AAA} & p_{AA,AA+} & \cdots & p_{AA,AA}
& \vdots & & \vdots 
p_{BB,AAA} & p_{BB,AA+} & \cdots & p_{BB,B}
\end{pmatrix}
$$

Moreover, the states were assumed to be able to communicate with each other and were accessible in each time step. However, the state space could not be seen as an irreducible chain as there was no guarantee that each state communicated with each other. Furthermore, all states in this approach could be defined as aperiodic states, which implied that an obligor could be assigned the same credit rating in each stage. The migration matrices below displays credit rating migrations of the examined credit portfolios.

![Example of migration matrices, 2009-2010; Upper: Low default portfolio, Lower: Sampled portfolio.](image_url)
5.2 Directional mobility index

In order to calculate the directional mobility index utilized in this thesis, the metric were set up in $\mathbf{R}$ as described in chapter 2.3.

$$I_{dir}(\mathbf{M}) = \sum_{i=AAA}^{B} w_i \sum_{j} p_{ij} \text{sign}(i - j)v(|i - j|).$$

As displayed, it consists of two adjustable components, each providing a different feature to the mobility index. From Proposition 3 defined in section 2.3.1, it was stated that uniformly distributed weights could contribute to $I_{dir}(\mathbf{M})$ satisfying weak immobility. Emphasizing this, a reasonable definition of $w_i$ was: $w_i = p_0(i)$, which corresponded to the percentage of obligors starting from $i$. There are several ways of defining the jump parameter, $v$, and the values for $v$ introduced by Feretti & Ganugi were evaluated in this thesis, see table 2.1.

5.2.1 Creating the directional mobility index time series

The directional mobility index was computed as described by equation (4), providing a value for year $t + 1$, given that credit ratings for the years of $t$ and $t + 1$ were evaluated. The value determined by the directional mobility index intend to display the current state of the credit portfolio in terms of the aggregated direction and magnitude of the credit rating migrations of the obligors in the credit portfolio. The directional mobility index time series displayed below is the ones assessed in this thesis, corresponding to the credit portfolios examined.
Figure 5.2: Directional mobility index time series of the low default portfolio, using \( v = |i - j| \) and \( w = p_0(i) \).

Figure 5.3: Directional mobility index time series of the sampled portfolio, using \( v = |i - j| \) and \( w = p_0(i) \).
5.3 Determining the business cycle

The credit portfolio analyzed did not represent all member states applicable in the database, why a subset was created consisting only of the countries present in the credit portfolio. The original data set provided by Nordea were processed in order to fulfill the requests proposed by IIF, for which the limitations introduced in section 1.4 were applied. The data set was also adjusted as presented in section 4.3.3. Thus, the subset created from the original data set had the following geographical distribution.

<table>
<thead>
<tr>
<th>Country</th>
<th>Occurencies</th>
<th>(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>4</td>
<td>3,17</td>
</tr>
<tr>
<td>Austria</td>
<td>1</td>
<td>0,79</td>
</tr>
<tr>
<td>Belgium</td>
<td>1</td>
<td>0,79</td>
</tr>
<tr>
<td>Bermuda</td>
<td>4</td>
<td>3,17</td>
</tr>
<tr>
<td>Canada</td>
<td>10</td>
<td>7,94</td>
</tr>
<tr>
<td>Denmark</td>
<td>1</td>
<td>0,79</td>
</tr>
<tr>
<td>Finland</td>
<td>1</td>
<td>0,79</td>
</tr>
<tr>
<td>France</td>
<td>2</td>
<td>1,59</td>
</tr>
<tr>
<td>Germany</td>
<td>2</td>
<td>1,59</td>
</tr>
<tr>
<td>Hong Kong SAR, China</td>
<td>1</td>
<td>0,79</td>
</tr>
<tr>
<td>Hungary</td>
<td>1</td>
<td>0,79</td>
</tr>
<tr>
<td>Italy</td>
<td>3</td>
<td>2,38</td>
</tr>
<tr>
<td>Japan</td>
<td>3</td>
<td>2,38</td>
</tr>
<tr>
<td>Korea Rep.</td>
<td>1</td>
<td>0,79</td>
</tr>
<tr>
<td>Mexico</td>
<td>3</td>
<td>2,38</td>
</tr>
<tr>
<td>Netherlands</td>
<td>2</td>
<td>1,59</td>
</tr>
<tr>
<td>Norway</td>
<td>1</td>
<td>0,79</td>
</tr>
<tr>
<td>Panama</td>
<td>1</td>
<td>0,79</td>
</tr>
<tr>
<td>Singapore</td>
<td>1</td>
<td>0,79</td>
</tr>
<tr>
<td>Spain</td>
<td>2</td>
<td>1,59</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>7</td>
<td>5,56</td>
</tr>
<tr>
<td>United States</td>
<td>74</td>
<td>58,73</td>
</tr>
</tbody>
</table>

| Total:                       | 126         | 100% |

Table 5.1: The geographical composition of the credit portfolio.
The geographical distribution of the obligors, and the percentual occurrences of the obligors domiciles, was of interest in the creation of the business cycle measures as they should be representative for the analyzed credit portfolio to be relevant.

A weighted average using the ingoing countries in which the obligors had their domicile were used as an aggregation method to retrieve a single time series for each business cycle measure that could be representative for the credit portfolio. The weighted average is given as:

\[ \bar{x} = \sum_{i=1}^{n} x_i w_i, \]

where \( x_i \) corresponds to the specific domicile of the obligor and \( w_i \) corresponds to the weight that it has in the portfolio. This approach presumed that all obligors in the credit portfolio carried an equal impact in the credit portfolio.

5.3.1 Business cycle estimate - Unemployment rate

As mentioned in chapter 4.3.1, indications of the present state of the business cycle could be identified from a measure such as variations in employment rate. In periods of economic contractions, the unemployment rate generally decrease, as in periods of economic expansions the unemployment rate tend to increase.
The original data set was weighted based on the percentual occurrences of the obligors domiciles in order to create a business cycle time series that would be representative of the credit portfolio, computed as displayed in equation ??.

Figure 5.4: Original data set - Unemployment rate adjusted to the credit portfolio.

Figure 5.5: Weighted average Countries - Unemployment rate adjusted to the credit portfolio.
5.3.2 Business cycle estimate - Final consumption expenditure (% of GDP)

As final consumption expenditure is the sum of household final consumption and general government final consumption, the dynamics of the final consumption expenditure was based on the ingoing sub parts of the measure as mentioned in section 4.3.1. By letting the final consumption expenditure be representative of the business cycle instead of letting the sub parts separately be representative of the business cycle, a more comprehensive view of the consumption expenditure is hopefully reflected.

![Chart showing original dataset - Final consumption expenditure (% of GDP)](image)

Figure 5.6: Original data set - Final consumption expenditure (% of GDP).

The original data set was weighted based on the percentual occurrences of the obligors domiciles in order to create a single business cycle time series that would be representative of the credit portfolio, computed as displayed in equation ??.

The time series below is used for further evaluation.
5.3.3 Estimating the seasonal component of the business cycle time series

The data utilized in order to determine the seasonal component of the business cycle time series - which were to be used as benchmarks for the directional mobility index time series - were retrieved and processed as described in sections 5.3.1 and 5.3.2. The time series used for evaluation are displayed in the figures 5.5 and 5.7. Given the data provided on credit ratings for the credit portfolio, a time series consisting of only 15 observations were calculated using the directional mobility index. This decided the time frame of interest for the business cycle time series.

5.3.3.1 Determining the frequency using spectral density of business cycle estimates

The seasonal component was of particular interest as it could reflect the cyclicality in the business cycle time series considered. As the business cycle was assumed to be cyclical, a sinusoidal wave was given the task to represent the seasonal component of the business cycle in this thesis, in accordance with the theory stated in section 2.4.5. The important argument to determine in the sinusoidal wave for making it representative of the seasonal component of the business cycle time series was the frequency, as it defines the length of the period. The determined period was then interpreted as the length of the business cycle for the computed
Initially, stationarity tests were performed to determine whether the business cycle time series were stationary. The two tests introduced in section 2.4.4, were carried out given a significance level of 0.05.

<table>
<thead>
<tr>
<th>Test</th>
<th>Unemployment</th>
<th>FCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF ((p\text{-value}))</td>
<td>0.049</td>
<td>0.10</td>
</tr>
<tr>
<td>KPSS ((p\text{-value}))</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 5.2: Stationarity tests for the business cycle estimates

The tests demonstrated that the *Unemployment rate* time series was stationary, and that there was a trend component in the *Final consumption expenditure* time series. As the *Unemployment rate* time series was stationary - the execution of an additive decomposition was not possible - as stationarity implies the absence of a seasonal component in a time series. The *Final consumption expenditure (% of GDP)* time series could not be considered stationary based on the ADF-test. Although, the KPSS-test, which considers the trend component in a time series, implied that the time series was stationary when excluding the trend component. Based on this, it was concluded that a distinguishable seasonal component was missing in this time series. Consequently, given that a seasonal component of the business cycle was to be determined, an alternative approach considering the analysis of the spectral densities was carried out. This was done for both time series used as representations of the business cycle. The analysis of spectral densities is based on the assumption that there is some cyclic behaviour in the time series considered.

Using the built-in function *spectrum* in **R**, periodograms were created. The trend component was handled by *spectrum*, which removed the trend in the analysis for the *Final consumption expenditure (% of GDP)* time series. The periodograms could be used to determine for which frequency the spectral density attained its maximum value. The frequency spectrum provided information regarding the dominant sinusoidal component, which would provide the best depiction of the random process evaluated. The periodograms displayed below are the ones
assessed in this thesis.

Figure 5.8: Periodogram - Unemployment rate adjusted for the credit portfolio.

Figure 5.9: Periodogram - Final Consumption expenditure (% of GDP) adjusted for the credit portfolio.
Table 5.3: Frequencies and periods determined from the periodograms for the business cycle representations

<table>
<thead>
<tr>
<th>Index</th>
<th>Unemployment rate</th>
<th>FCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$ (cycle/year)</td>
<td>0.067</td>
<td>0.12</td>
</tr>
<tr>
<td>$T$ (year/cycle)</td>
<td>15</td>
<td>8.33</td>
</tr>
</tbody>
</table>

5.3.3.2 Creation of sinusoidal waves representing the seasonal component of business cycle time series

To illustrate the seasonal component of the business cycle time series, sinusoidal waves were created given the frequencies $f$ calculated in section 5.3.3.1. These were computed given the equation:

$$\text{Sinusoidal wave estimate of business cycle} = \sin(2\pi ft).$$

The sinusoidal waves were determined for the full time frame given the data found available. This corresponds to the following time frames given the macroeconomic variables:


In order to compare the business cycle time series against the directional mobility index time series, the sinusoidal waves representing the business cycles had to be truncated to the time frame, 2004 – 2018, as this was the time frame of interest for further analysis.

In order to avoid letting the sinusoidal wave estimates originate in a specific phase of the business cycle - such as in an equilibrium state for instance - they were phase shifted to ensure that they originated at a starting point that was corresponding better to the original business cycle time series. The phase shift, $\phi$, was determined by minimizing the area between the standardized sinusoidal wave estimate of the seasonal component of the business cycle (SSBC) and the standardized business cycle time series, using the min-max normalization method described in section 2.4.6. The area between the curves given the different phase shift angles, $\phi = 1^\circ, 2^\circ, \ldots, 360^\circ$, were computed and the phase shift that
minimized the area between the time series was utilized as the phase shift for the sinusoidal wave.

\[ \text{Sinusoidal wave estimate of business cycle} = \sin(2\pi tf + \phi). \]

Figure 5.10: Sinusoidal business cycle estimate using frequency retrieved from unemployment rate.
The measure of the degree of PIT-ness

5.4.1 Standardization of time series

As this thesis search to compare time series against each other in order to determine the degree of PIT-ness, some preprocessing had to be done to be able to handle and evaluate time series that are displaying different information against each other. To provide the requested dynamics of the measure of the degree of PIT-ness, a transformation of the time series were performed to compare these given the same frame of reference. For this reason the directional mobility index time series and the sinusoidal wave estimates representing the seasonal component of the business cycle time series were standardized using equation (8). The objective with the standardization was to make sure that the directional mobility index time series and the sinusoidal wave estimate of the business cycle time series had the same range.
Figure 5.12: Time series of the standardized directional mobility index of the low default portfolio vs. sinusoidal wave estimate letting \textit{Unemployment rate} represent the business cycle.

Figure 5.13: Time series of the standardized directional mobility index of the sampled portfolio vs. sinusoidal wave estimate letting \textit{Unemployment rate} represent the business cycle.
Figure 5.14: Time series of the standardized directional mobility index of the low default portfolio vs. sinusoidal wave estimate letting Final consumption expenditure (% of GDP) represent the business cycle.

Figure 5.15: Time series of the standardized directional mobility index of the sampled portfolio vs. sinusoidal wave estimate letting Final consumption expenditure (% of GDP) represent the business cycle.
5.4.2 The measure of the degree of PIT-ness

Given the approach presented, the measure of the degree of PIT-ness aims to relate the standardized directional mobility index, which intends to depict the sensitivity of the rating system, and the seasonal component of the business cycle estimate. The proposed measure was developed as followed.

5.4.2.1 The development of the measure of the degree of PIT-ness

To evaluate the rating system, utilizing the SSBC as a reference for a 100% PIT rating system, the difference between the time series at time \( t \) was evaluated.

\[
yDMI_t - y_{Business\ cycle_t}.
\]

Here, \( yDMI_t \), corresponds to the standardized directional mobility index at time \( t \) and \( y_{Business\ cycle_t} \), corresponds to the SSBC at time \( t \).

The measure proposed in this thesis should be adaptive regarding what macroeconomic variable was utilized in order to represent the business cycle. As an example, consider the two different macroeconomic variables Unemployment rate and BNP growth. Intuitively, the dynamics of Unemployment rate are such that it increases in periods of economic recessions and decreases in periods of economic expansions. Regarding BNP growth, the anticipated dynamics are the opposite. If these are the characteristics of two different representations of the business cycle, then the measure should be able to handle both of these. In order to remove the dependency of the direction of the business cycle, the absolute value was utilized.

\[
|yDMI_t - y_{Business\ cycle_t}|.
\]

To be able to distinguish a PIT- and a TTC-part from the rating system, the time series had to be evaluated through at least one full business cycle. Because of this, the computed values were summarized for the time period of interest.

\[
\sum_{t=1}^{n} |yDMI_t - y_{Business\ cycle_t}|.
\]

As one of IIFs request states the following, see 1.4: "The measure should have
a floor and a ceiling”, the average value of the sum was computed. As both
time series were standardized on the interval [0, 1], the measure was automatically
given a floor and a ceiling.

\[
\frac{1}{n} \sum_{t=1}^{n} |y_{DMI_t} - y_{Business cycle_t}|.
\]

The average value of the sum constituted the minimum and maximum value to
be 0 and 1 respectively. This thesis intended to create a measure that had a floor
corresponding to a value of 0 and a ceiling corresponding to a value of 1, why the
measure of the degree of PIT-ness was defined as:

\[
\alpha = 1 - \frac{1}{n} \sum_{t=1}^{n} |y_{DMI_t} - y_{Business cycle_t}|.
\] (10)

In the measure, \( n \) refers to the total number of yearly observations.
6 Results

In this chapter the results of the degree of PIT-ness for the different credit portfolios will be presented.

In order to evaluate the proposed measure of the degree of PIT-ness, introduced in section 5.4.1, the degree of PIT-ness was calculated for both the low default portfolio and the sampled portfolio. Using equation (10), the measure of the degree of PIT-ness was computed given the different business cycles.

6.1 Degree of PIT-ness letting unemployment rate represent the business cycle

\[
\begin{array}{cccccc}
 v & 1 & log(|i - j| + 1) & \sqrt{|i - j|} & |i - j| & e^{i |i - j| - 1} \\
 \alpha & 0.49 & 0.49 & 0.49 & 0.49 & 0.48 & 0.43 \\
\end{array}
\]

Table 6.1: Results for the low default portfolio.

\[
\begin{array}{cccccc}
 v & 1 & log(|i - j| + 1) & \sqrt{|i - j|} & |i - j| & e^{i |i - j| - 1} \\
 \alpha & 0.55 & 0.57 & 0.57 & 0.58 & 0.60 & 0.68 \\
\end{array}
\]

Table 6.2: Results for the sampled portfolio.

Comparing the results of the different portfolios, letting Unemployment rate represent the business cycle, it was notable that the degree of PIT-ness was greater for every choice of \( v \) in the sampled portfolio. This result could potentially be due to the fact that the sampled portfolio contained a greater amount of migrations and that these displayed a larger magnitude. The largest deviation in the results between the two portfolios occurred when \( v \) was given by: \( v = e^{|i - j|} - 1 \). It was also notable that this choice of \( v \) generated the smallest degree of PIT-ness for the low default portfolio, whilst it returned the highest degree of PIT-ness for the sampled portfolio. For the other choices of the jump parameter, \( v \), the values remained relatively stable.
6.2 Degree of PIT-ness letting final consumption expenditure
(% of GDP) represent the business cycle

\[
v_i = \log(|i - j| + 1) \quad \sqrt{|i - j|} \quad |i - j|^2 \quad e^{|i-j|} - 1
\]

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>0.52</th>
<th>0.52</th>
<th>0.52</th>
<th>0.52</th>
<th>0.52</th>
<th>0.51</th>
</tr>
</thead>
</table>

Table 6.3: Results from the low default portfolio.

\[
v_i = \log(|i - j| + 1) \quad \sqrt{|i - j|} \quad |i - j|^2 \quad e^{|i-j|} - 1
\]

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>0.61</th>
<th>0.62</th>
<th>0.62</th>
<th>0.63</th>
<th>0.66</th>
<th>0.70</th>
</tr>
</thead>
</table>

Table 6.4: Results from the sampled portfolio.

Comparing the results from the low default portfolio and the sampled portfolio, it was observable that the degree of PIT-ness was deviating more between these credit portfolios whilst letting Final consumption expenditure (% of GDP) represent the business cycle. The largest difference between the results of the two credit portfolios was obtained using, \(v = e^{|i-j|} - 1\), as the jump parameter. One can also observe that the results were stable for every other choice of \(v\) in the low-default portfolio. The sampled portfolio was displaying a higher degree of PIT-ness for every choice of \(v\). Overall, the degree of PIT-ness resulted in values above 0.6 for all choices of \(v\) in this portfolio.
7 Discussion

This chapter aims to summarize the thesis by highlighting conclusions from previous chapters and enlightening implications of the framework. Furthermore, suggestions for further studies in the research area will be recommended.

7.1 Interpretation of results

This section will review the results and the measure of the degree of PIT-ness proposed in this thesis. Both advantages and disadvantages of the proposed measure will be evaluated.

As mentioned in sections 6.1 and 6.2, the results for the low default portfolio are relatively stable given both representations of the business cycle - regardless of the choice of the jump parameter $v$. Evaluating the extreme cases of the proposed measure, $\alpha = 1$ corresponds to a rating system being 100% PIT, whereas $\alpha = 0$ corresponds to a rating system being 0% PIT, i.e a 100% TTC-system. Consequently, 100% PIT-ness would correspond to $y_{DMI}$, and $y_{Business cycle}$, having the exact same value for each time step. Contrariwise, 0% PIT-ness would correspond to one of these values being 1 and the other 0 for each time step for the examined period of time. When observing the results of the low default portfolio for both representations of the business cycles, these could be interpreted as the rating system being a hybrid rating system, implying that it consists of both a TTC and a PIT-part. As the results were approximately 0.5, this would imply that equal parts of the different philosophies were utilized in the rating system. As the results of the sampled portfolio for both representations of the business cycles were observed, these could also be interpreted as the rating system being a hybrid rating system. As the results were approximately 0.6, this would imply that this rating system utilized a PIT philosophy to a greater extent than the rating system used for the low default portfolio.

For the evaluated choices of the jump parameter $v$, the alternative, $v = e^{\lvert i-j \rvert} - 1$, returned the largest deviation for the degree of PIT-ness between both portfolios given both representations of the business cycle. An interpretation based on this, is that the term, $e^{\lvert i-j \rvert} - 1$, grows large as the migrations demonstrates greater
magnitudes which can be demonstrated by the following example:

- Imagine that obligor X is migrating from AA to A during the time interval $\Delta t_k$ and, obligor Y is migrating from AA to AA− during the same time interval. The former being a three-scaled credit rating downgrade, and the latter being a one-scaled credit rating downgrade. The jump parameter for obligor X will be given by: $e^{3} - 1 \approx 19$, and in the case of obligor Y, the value of the jump parameter will result in: $e - 1 \approx 1.7$.

For other choices of the jump parameter, the degree of PIT-ness is stable for both representations of the business cycle, thus it is difficult to distinguish any difference between the portfolios.

From the results demonstrated in section 6.1 and 6.2, it is obvious that the degree of PIT-ness is higher in the sampled portfolio than in the low default portfolio. This could be interpreted such as that the rating system in the sampled portfolio is considering systematic factors to a greater extent than the rating system of the low default portfolio. This was also the desired behaviour for the rating system of the sampled portfolio.

### 7.2 Conclusions and implications

This section summarizes the main conclusions and implications that could be drawn from the methods, results and discussions in this thesis.

The occurrences of missing data were handled by setting the concerned observations as $NA$ in this thesis. Handling missing data in this manner, considering that a weighted average method were utilized when creating the business cycle estimates, will affect the weights in the credit portfolio composition. Due to the missing data, the business cycle estimates might not be representative for the credit portfolio given the chosen approach. Problems regarding missing data could also have been solved by imputing observations.

The original data set consisted of 964 obligors and the time series ran between 1980 – 2018. After the data had been handled, only 126 obligors remained and the examined period of time was 2003 – 2018. Given the approach presented, the longer the evaluated time series is, the better. Although, this had to be weighed
against the amount of obligors that would be included in the examined credit portfolio. The trade-off between having a sufficient amount of obligors that would be representative of the original credit portfolio and a sufficient long time series given the original data set was handled in a subjective manner and was definitely affecting the final results.

Another circumstance to consider is the fact that S&P:s rating process have been developed over time, which could imply that different credit ratings could have been applied to the same obligor given two different points in time. Although this could be problematic, due to the fact of the time limit of this thesis, further evaluation of the development of S&P rating methodology over time was considered as prohibitive.

Due to the fact that only country-specific data were provided for defining the business cycles, the weighing methods for the measures were dependent on which country the obligors had their domicile within. A weighted average of the ingoing countries in which the obligors had their domicile were used as aggregation method in order to retrieve a single time series for each measure that would be representative for the credit portfolio. This approach presume that all obligors in the credit portfolio carries equal impact in the credit portfolio, which might not be the case as one obligor might be more influential compared to another. Other macroeconomic variables supplied by The World Bank were evaluated to represent the weight in equation (9). Amongst these was using *GDP per capita* and *Population* as a metric. There are apparent deficiencies using both of these macroeconomic variables. For instance, applying the metric *GDP per capita* as a method to weigh the business cycle, it was found that a country such as Bermuda - which is present in the credit portfolio - was given a considerable proportion of the business cycle index. Considering the fact that the population fluctuate around 60000 inhabitants for the examined time period - which is a fraction of the total population considering the countries present in the credit portfolio - it could be considered as a misleading metric to use. Applying the metric *GDP* instead, would give a country like Bermuda a small fraction of the business cycle index, which also might be misleading as no obligor specific information was given. This could mean that one presume that the Bermudian banks in the credit
portfolio has a smaller impact than the other banks in the portfolio. By applying a weighted average dependent on the ingoing countries in which the obligors has their domicile, this thesis presumes that all obligors carries an equal impact on the credit portfolio. It is assumed to be an appropriate approach given that no obligor specific information was given. It is important to clarify that there is no objective method to determine the business cycle and this will act as a proposal for determining the business cycle given the optional macroeconomic variables *Final consumption expenditure (% of GDP)* and *Unemployment rate*. There is a non-negligible level of subjectivity in these choices, and they are made on assumptions that they should provide some information regarding the general macroeconomic conditions in the geographic regions considered in the examined credit portfolio.

The degree of PIT-ness will be highly dependent on the business cycle estimate given the approach presented, which requires a measure that truly is representative of the credit portfolio and is capable of displaying the business cycle well for this. As the business cycle was to be considered as cyclical in this thesis, a sinusoidal wave was given the task to represent the seasonal component of the business cycle. This approach is far from optimal as it simplifies the dynamics of a business cycle given generally all macroeconomic variables there are to consider. A more advanced approach in defining the business cycle would probably be beneficial for the results. Nevertheless, the method presented in this thesis, allows different macroeconomic variables to be utilized in order to represent the business cycle. This also means that variables that exhibit different dynamics could be utilized as the representation. This could be seen as a strength in the approach presented in this thesis, as it is able to handle different types of representations of the business cycle in order to determine the degree of PIT-ness of a credit rating system.

The method of estimating the seasonal component of the business cycle as a sinusoidal wave do simplify the behaviour of the seasonality of the business cycle. First of all, this estimate is not able to reflect steeper or gradual fluctuations in the seasonal component of the business cycle. By estimating the business cycle as a sinusoidal wave, it is also assumed that the strength of the state of the business
cycle is the same for each period which is not true as the expansive phase of a business cycle will not always reach the same amplitude for example. This approach also assumes that the business cycle always will have the same length, which is not the case. Thus, the most important part of the approach is the measure of the degree of PIT-ness and that this can be computed given different representations of the business cycle.

Another aspect that could affect the computations of the degree of PIT-ness occurs when both the business cycle and the directional mobility index are standardized. As the directional mobility indexes are standardized, some values in these time series tends to fit the business cycle time series to a greater extent, causing a spurious relationship between the time series. This may also eradicate the importance of the magnitudes for both time series. Given the proposed measure in this thesis, this is something that could entail a higher degree of PIT-ness and that could potentially contribute to a high degree of PIT-ness in general given this approach. Subsequently, this could ultimately lead to a higher degree of PIT-ness independent of the amount of migrations in the examined credit portfolio.

The directional mobility index in this thesis is supposed to provide a link between the credit rating migrations and the rating system. However, there is no justified conclusion that the chosen directional mobility index is optimal in order to provide the requested link. Another problem would occur if the maximum value of the directional mobility index time series equals the minimum value of the directional mobility index time series throughout the examined period of time, meaning the directional mobility index time series would be flat. Standardizing such a directional mobility time series on the interval $[0, 1]$ would result in a singularity.

As the final data set was analyzed, it was found that there were no credit rating migrations in the low default portfolio during the three first years. The authors of this thesis finds it very surprising, and not very realistic, that no credit rating migrations occurred during this period of time. This is believed to be a defect in the data. The results were affected by this, as it would decrease the degree of PIT-ness in the examined rating system. One should recall that a TTC-system regrades obligors less actively than a PIT-system, whereas a portfolio without credit
rating migrations for a longer period of time will display a TTC-rating philosophy to a greater extent.

The jump parameter, \( v \), in the directional mobility index has the role of measuring the magnitude of credit rating migrations. As mentioned in section 2.3, there are several ways of defining the magnitude of credit rating migration, although there is no way to propose an objective method of choosing \( v \). The selection of \( v \) is strictly related to the specific dynamics of the variable evaluated, which is a clear disadvantage in the approach presented in this thesis.

Another implication regarding the directional mobility index utilized in this thesis, is that it is not able to detect momentous events in the economy due to the sparseness in time of the credit rating data. For example, given the following scenario that the business cycle utilized displays a large increase that is followed by a large decrease bringing it back to its starting point between the credit ratings assignments, the directional mobility index will not be able to reflect this sequence of events. A comparison of sparse time series, for which one of these are more sparse in its observed values than the other, could also decrease the accuracy for the results created. It can also cause difficulties in finding significant relationships between these. In this thesis, only observations from the business cycle estimates corresponding in time to the observations for the directional mobility index were utilized for instance.

Given that this portfolio consists of major banks - whose credit ratings generally can be seen as less sensitive to the business cycle compared to many other business sectors - the perception would be that such a credit portfolio would rely on a rating system considering a TTC-rating philosophy to a greater extent. Given the proposed measure in this thesis, the rating system utilized by Nordea to evaluate the low default portfolio can be identified as a hybrid rating system, utilizing both PIT and TTC rating philosophies. Analyzing the results from the sampled portfolio one can identify a higher degree of PIT-ness, than for the low default portfolio. The degree of PIT-ness in general is more than 10% higher in the sampled portfolio than in the low default portfolio. Approximately 10% of the credit ratings were manipulated, generating an increasing degree of PIT-ness in more or less the same scale. An interpretation could be that the measure of PIT-ness is proportional to
the amount of migrations and their magnitude. A conclusion from these results is that, given a greater amount of migrations within a credit portfolio, the higher will the degree of PIT-ness be. This seems reasonable given the characteristics of a PIT rating system. A higher degree of PIT-ness would implicate that the rating system is more sensitive to the business cycle which results in a greater amount of credit rating migrations.

Amongst IIFs requests, mentioned in section 1.4, it is stated that: "The measure should not be influenced by the accuracy of the rating system". This is in some sense a vague statement. As all rating systems in practice are to some degree a hybrid between PIT and TTC rating system, as both systematic and idiosyncratic risk drivers should be taken into consideration when assigning a credit rating to an obligor. Hence, the accuracy of the rating system is dependent on the degree of PIT-ness in the examined portfolio, and simultaneously, the accuracy of the measure of the degree of PIT-ness depends on the rating system. A conclusion is then that the accuracy of the rating system and the measure of the degree PIT-ness are essentially influencing each other. Thus, they are tangled up in a way that is not easy to separate.

As mentioned in section 7.1, the extreme cases of the proposed measure in this thesis demanded special circumstances to occur. The probability of $y_{DMI_t}$ and $y_{Business\ cycle_t}$ having the same value in each time step throughout the examined period of time is very low, corresponding to a 100% PIT rating system. It is also very unlikely that $y_{DMI_t} = 1$ and $y_{Business\ cycle_t} = 0$ - or vice versa - throughout the examined period of time. This is mainly due to the fact that the seasonal component of the business cycle, which in the proposed measure in this thesis is represented by a sinusoidal wave, will most probably never be either 0 or 1 for each observation. This is a requirement in order to achieve a 100% TTC rating system. The fact that these extreme values requires such specific conditions, the absolute majority of outcomes of $\alpha$ will probably have a more restricted sample space. Due to this, it would probably be more interesting to evaluate the outcomes within this, more probable sample space, in order to draw conclusions regarding the proposed measure in this thesis. This, instead of evaluating the outcomes on the sample space $\alpha \in [0, 1]$. Although, this would require the evaluation of a greater amount
of credit portfolios than were provided for this thesis.

Other measures of the degree of PIT-ness were evaluated based on the markovian approach applied in this thesis. Given that there are several measures of evaluating and explaining the behaviour of time series, different metrics were examined. Correlation and covariance analysis were performed, trying to find results that could explain the relationship between the different time series. Although, the results provided from these analyzes did not provide results that could be interpreted as a measure of the degree of PIT-ness.

7.3 Suggestions for further studies

This report has taken an application approach in determining the degree of PIT-ness of a rating system through migration matrices, and provide a broad focus on appeared issues and necessities when implementing the Markov chain framework to rating migrations in practice. Many of the application components such as defining business cycles and developing a measure given the well defined restrictions provided by IIF are research fields in their own right.

It would be of interest to evaluate the measure of the degree of PIT-ness given another credit portfolio with other characteristics of the obligors included in this. The obligors included in the credit portfolio considered in this thesis were large international banks and no obligors specific information were provided regarding these. There is a possibility that obligors in other business sectors would display more credit rating migrations, which hopefully would be reflected in the measure of the degree of PIT-ness proposed in this thesis. There would be of interest to examine whether the degree of PIT-ness is reacting to different credit portfolios as expected. Thus, in this thesis no data were provided for any other credit portfolio and a fictitious portfolio had to be created. The interpretation of the results would be given more significance if a greater amount of credit portfolios were evaluated. Hopefully, evaluating a greater amount of credit portfolios could also lead to that more conclusions could be drawn regarding what type of characteristics of the credit portfolios corresponds to specific degrees of PIT-ness.

This thesis searched to fulfill IIFs requests for a measure of the degree of PIT-ness as mentioned in section 1.4. One of these requests stated that: "The measure
should not be influenced by the accuracy of the rating system”, a request that was not satisfied in this thesis. An alternative approach of managing this was presented in section 1.6, which could make sure that the approach presented in this thesis is not influenced by the accuracy of the credit ratings set by S&P. This would be a necessary extension to this approach in order to fulfill the requests given IIF fully. Given the specific portfolio examined in this thesis - consisting of major international banks - applying a structural-form approach, such as the ones presented in section 3.1.1 were also discussed in greater detail. During the last period of recession - following the global finance crisis - several major banks were bailed out by the governments in their respective countries, leading to defaults in this sector never being realized. The political initiatives that did save these banks from defaulting are hard to extrapolate as one can not be sure that they will, or will not, repeat themselves given a similar scenario in the future. One way to handle this issue could be to model a structural-formed approach using a technical definition of default instead, meaning that as a bank violates a certain threshold of a prespecified ratio, it will considered as having defaulted.

Unfortunately, the database provided by The World Bank only supplied country-specific data that set some limitations to the approach for estimating the business cycle presented in this thesis. Other techniques could have been to examine measures that could be representative of specific business sectors, such as for instance the banking sector, in order to estimate a business cycle given the credit portfolio examined in this thesis. As for this thesis, there were no such data available. The business cycle estimate could include obligor specific features as well as general macroeconomic variables. The definition of the business cycle applying the methodology presented in this thesis is subjective and other approaches of doing this is a welcome contribution to the approach presented. This could hopefully provide better results.

As this thesis was working with country-specific data when defining the business cycle estimate, the metrics representing the business cycle were weighed depending on which country the obligors had their domicile within. Given that one had access to obligor specific data, other methods of weighing the business cycle estimate could be implemented given the approach in this thesis. One of
the more prominent methods could be to weigh the obligors in the credit portfolio according to the credit exposure in an attempt to reflect the counter party risk taken by the bank. Another method could have been to weigh the obligors based on the market capitalization, which could provide an insight of the impact each obligor carry in the credit portfolio. As the obligors in the data set provided by Nordea were anonymous, it was not possible to weigh the obligors according to obligor specific features. Given that credit risk is evaluated, weighing the business cycle based on obligor specific characteristics would probably reflect credit risk in a more prominent manner. Applying the methodology presented in this thesis, weighing the portfolio based on other premises would be an appreciated extension to this approach.

Another method could also be used in order to determine in which macroeconomic state the sinusoidal wave estimate of the seasonal component of the business cycle is originating from. One such method could be to perform a qualitative research in order to determine this. The quantitative method presented in this thesis also lacks precision due to the fixed period of the sinusoidal wave.

An initial thought in this thesis was to evaluate the seasonal components in the business cycle time series. Decomposing the time series into partial components in accordance with the decomposition method mentioned in section 2.4, could make it easier to identify a seasonal pattern. Given the data provided on credit ratings for the credit portfolio, a time series consisting of only 15 observations were calculated using the directional mobility index. Problems with so few observations includes difficulties of detecting patterns and decomposing the time series. A suggestion for further studies, given sufficient data, would be to decompose both the directional mobility index time series well as the business cycle time series and evaluate the seasonal components of both time series using the measure of the degree of PIT-ness presented in this thesis.
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