Probabilistic reactive power capability charts at DSO/TSO interface

Stefan Stanković, *Student Member, IEEE* and Lennart Söder, *Senior Member, IEEE*

Abstract—Hosting an increasing share of distributed generation and flexible consumers, distribution systems are being more often considered as providers of ancillary services among which are reactive power support and voltage control. For the purpose of reliable and efficient planning and operation of a transmission system, information about reactive power capabilities of connected distribution systems would be beneficial. These capabilities are subject to uncertainties especially with a presence of intermittent distributed generation and stochastic loads. To capture these uncertainties, this paper introduces probabilistic capability charts using the concept of probabilistic curves. These curves analytically represent reactive power support limits of distribution systems as a family of random variables with their associated probabilistic density functions. As such, they carry more information about states of the system than deterministic curves. The proposed capability charts can be easily applied in probabilistic studies of the overlaying transmission system. By providing analytical expressions of stochastic power injections of connected distribution systems, the use of these charts can speed up time and memory demanding probabilistic calculations of transmission systems planning and operation studies. Not containing sensitive system information, the charts can be used to exchange data between distribution and transmission system operators.

Index Terms—reactive power support, distribution networks, probabilistic analysis, capability charts, optimization

I. INTRODUCTION

POWER systems nowadays and in the future are going to experience changes as a result of energy transition and increasing electrification of societies. To keep the systems stable and secure, their operation and control would need to be adapted. In these, interactions between coupled distribution and transmission systems are going to play a major role [1]. To cope with these interactions, a number of joint analysis methods have been proposed. Some of them are dealing with optimal power flows [2], [3]. Others are giving solutions on voltage stability assessment [4]. In all, new roles of distribution system operators (DSO) and transmission system operators (TSO) are assumed.

Hosting an increasing amount of generation and flexible consumers, DSOs would get new roles not just as distributors of electrical energy but also as aggregators and possible providers of ancillary services [5]. Reactive power support of the transmission system at the point of common coupling (PCC) is one of these services.

TSOs would need to adapt their operation based on the changes in the energy mix that could lead to relocation of the controllable reactive power resources to the lower voltage level grids [6]. This can lead to the lack of controllable reactive power devices connected to the transmission grid which can furthermore result in voltage control problems. Assuming

availability of reactive power from the underlaying distribution systems, TSOs would need less investments in controllable reactive power resource equipment to keep the system voltage stable and secured. In order to do so, TSOs would benefit from reliable information on distribution systems reactive power capabilities.

Reactive power support from distribution systems has been widely recognized in the literature as a viable option [7]–[22]. Different solutions on how reactive power can be aggregated and controlled at the PCC have been proposed [7]–[16]. Numerical [17]–[21], [23] and analytical [22] approaches have been used to assess distribution systems' reactive power capabilities for a number of case studies taking into account different criteria and operating conditions. Capability charts of distribution systems have been proposed by [17]–[19], [23]. These charts define a reactive power characteristic in the similar way as it is defined for any other generating unit like synchronous or doubly-fed induction generator (DFIG).

However, the previously defined capability charts for distribution systems do not take into account stochastic phenomena. They represent the capability chart as a set of curves deterministically defining the reactive power limits solely as a function of active power exchange at the PCC (P_{pcc}). But for the same value of P_{pcc} , the system can have multiple operating points leading to different values of reactive power limits. To describe this behavior, we introduce new probabilistic reactive power capability charts using the concept of probabilistic curves. As a difference to deterministic curves, a probabilistic curve is a family of random variables with their associated probability density functions (PDF) parametrized by real numbers. The PDFs are used to describe a distribution of reactive power limits values for a certain value of P_{pcc} .

The proposed probabilistic capability charts give detailed analytical description of the distribution systems ability to provide reactive power support to the overlaying transmission system. The benefit of analytical approach is that it takes only a few parameters to describe the PDFs of reactive power limits for any value of P_{pcc} . Furthermore, since PDFs can be parametrized with more than one real number, probabilistic curves carry more information about the reactive power limits and generally the state of the distribution system than deterministic curves. This fact is analyzed in this paper by comparing the shape of the probabilistic curve and values of Lagrangian multipliers corresponding to different operating limits of the system. Results indicate that the shape of the curves can be correlated to the stress level of the system.

With increased stochastic phenomena in power systems, probabilistic analyses are gaining more importance [24]. The use of proposed probabilistic capability charts can shorten

the execution time and decrease memory burdens of these analyses. These facts are the direct consequence of analytical representation of the capability chart and its use in aggregation of parts of the system, e.g. distribution grids.

While the proposed charts give a great deal of useful information for planning and control of the overlaying transmission system, they do not unveil sensitive, detailed information about the distribution system itself. As such, they also provide a way to exchange data between DSO and TSO for the purpose of enhanced planning and operation of both transmission and distribution systems.

As a summary, the main contributions of this paper are:

- Proposition of new probabilistic capability charts giving analytical description of utmost reactive power provision boundaries of distribution systems at PCC in the presence of uncertainties;
- New method on how the probabilistic capability charts can be obtained from raw operational statistics data;
- Proposition on how probabilistic capability charts can be used for probabilistic studies of power systems.

II. MODELING OF CAPABILITY CHARTS

A component's operating limits in the PQ plane are referred to as a capability chart, reactive power capability curve or PQ chart of the component. These charts are defined by physical limits of internal states of the component [25]–[28]. For example, a synchronous generator capability chart is defined by the following limits:

- maximum stator current,
- maximum rotor current,
- maximum mechanical power and
- under-excitation limit.

Since the internal states and parameters of the synchronous generator are not subject to any uncertainty, the mentioned limits are mapped into the PQ-plane as deterministic curves. Similar curves are obtained for other components such as DFIGs [28] or full converter interfaced generators [27].

A capability chart of a distribution or sub-transmission system is mapping utmost possible values of reactive power exchange at the PCC (Q_{pcc}) for different values of P_{pcc} assuming certain controllability of reactive power resources in the system. Each mapped point should belong to the feasible operating region of the system. This region is defined as a set of points satisfying system constraints:

- grid buses' voltage limits,
- grid lines' current limits,
- limits defined by capability charts of the components connected to the grid.

In contrast to the component states, the system states and parameters corresponding to the utmost values of Q_{pcc} are subject to uncertainties. This, furthermore, means that for the same value of P_{pcc} we may have a number of different utmost values of Q_{pcc} . These uncertainties come from different factors including:

- varying levels of consumption and production,
- seasonal changes in network configuration,
- etc.

Because of this, the capability chart of distribution or sub-transmission grids cannot be drawn as a set of deterministic curves. Instead, the set of utmost values of Q_{pcc} for different values of P_{pcc} is represented as a scatter plot in the PQ plane. The difficulties arise when determining exact value of Q_{pcc} for certain value of P_{pcc} .

The solution from [18] proposes that the capability curve should be drawn in such a way that the utmost values of Q_{pcc} are guaranteed for the "worst case" scenario. This approach conservatively puts the reactive power capability envelop near the inner border of the scatter plot allowing a limited number or none of the data points to be inside the region defined by the envelope. By approximating the geometric position of the data points with deterministic curves defining the envelope, substantial information about dispersion of the data points is lost. One of the drawbacks of this approach is that for the certain operating conditions of the grid, the reactive power capability may be well underestimated. Also, the dispersion of the data points can give a substantial information about the state of the system.

This paper proposes the use of probabilistic capability charts to more accurately describe reactive power capabilities of the distribution or sub-transmission systems. These charts can be obtained by fitting the probabilistic curves, introduced by *Definition 1*, to the data points representing the operating states when the system is consuming or providing the utmost value of reactive power at the PCC.

Definition 1. A probabilistic curve is a mapping that assigns to each value p from an interval $I = [a, b]$ a random variable Q_p . Furthermore, this mapping is smooth, and in particular PDFs of random variables Q_p can be represented by a smooth function $f_Q(\cdot, p)$.

For notational convenience we will sometimes instead of $f_Q(y, x)$ write $f_Q(y; \Theta)$, where Θ will denote a set of N_g parameters which are given by smooth functions $g_i: I \rightarrow \mathbb{R}$. Certain similarities can be observed between definition of a probabilistic curve and mathematical definition of a stochastic process. While both are defined as families of random variables, the stochastic process is defined over the time dimension while probabilistic curve is defined over the range of P_{pcc} values.

A probabilistic capability chart is defined by two probabilistic curves. The first one $q_{\max}$ defines the upper boundary (maximum provision) and the second one $q_{\min}$ is defining the lower boundary (maximum consumption) of the capability chart. Probabilistic curves $q_{\min}$ and $q_{\max}$ are defined on the domain $I_P = [\min(P_{pcc}), \max(P_{pcc})]$. Their codomain is the set of continuous random variables $Q(p); p \in I_P$ and their associated probability density functions (PDF) $f_Q(\cdot, \Theta)$.

The process of obtaining the probabilistic capability charts and their associated upper and lower probabilistic curves can be divided into the following steps:

- obtaining datasets of utmost operating points,
- analysis and clustering of the datasets,
- fitting probabilistic curves to the identified data clusters.

The listed steps are explained in the following subsections.

A. Obtaining the PQ datasets

The PQ datasets describing utmost values of reactive power exchange at the PCC can be found by solving a sequence of problems (1) for varying operating conditions of the system:

$$\begin{aligned}
 \min / \max \quad & Q_{pcc} \\
 \text{s.t.} \quad & \sum_l P_{il}^{inj} = g_{Pi}(x, par), \forall i \in \text{Bus}, \\
 & \sum_l Q_{il}^{inj} = g_{Qi}(x, par), \forall i \in \text{Bus}, \\
 & \underline{U}_i < U_i < \bar{U}_i, \forall i \in \text{Bus}, \\
 & \underline{Q}_j < Q_j < \bar{Q}_j, \forall j \in \text{DG},
 \end{aligned} \tag{1}$$

where Bus is a set of the system buses and $\text{DG} \subset \text{Bus}$ is a set of distributed generators hosted in the system. The equality constraints comprise the power balance at each bus of the system. The injected active and reactive powers at bus i are denoted with P_{il}^{inj} and Q_{il}^{inj} while $g_{Pi}(x, par)$ and $g_{Qi}(x, par)$ are expressions defining powers flowing from the bus i into the lines. The active powers of the loads and generators included in (1) by P_{il}^{inj} are considered non-controllable in this analysis so they are given as parameters in (1). The decision variables of (1) are reactive powers of the generators Q_j . The reactive power exchange at the PCC, $Q_{pcc} = f(x, par)$ is a multi-variable, nonlinear function of system state variables x and parameters par . The inequality constraints define physical limits of the system:

- voltage limits on the buses ($\underline{U}_i, \bar{U}_i$),
- reactive power limits of the generating units ($\underline{Q}_j, \bar{Q}_j$).

Reactive power limits $\underline{Q}_j, \bar{Q}_j$ depend on the active power of generator P_j being considered as a parameter in this optimization problem. Therefore, $\underline{Q}_j, \bar{Q}_j$ can be pre-calculated for each instance in the sequence based on the value of P_j and the generators capability charts. Depending on the type of the generator, its capability chart is given by [14], [27], [28].

In order to obtain a PQ dataset of the utmost operating points, (1) is solved in a sequence for a series of parameter-set values representing change of the grid's operating state during defined time period. Such changes may include variations in levels of active power consumption and production in the grid but also periodical changes in the grid's configuration. DSOs may be able to obtain these data from SCADA measurements recorded over some period of time in hourly or even smaller resolution. For the purpose of the case studies in this paper, we used dataserries from Nord Pool [29] representing hourly variations of active power consumption and production during one year for Swedish regions.

B. Analysis and Clusering of PQ Datasets

After plotting the values of active and reactive power exchange at the PCC for each operating point, a PQ scatter plot such as the one shown in Fig. 4 is obtained. Next step in obtaining probabilistic capability chart would be to fit probabilistic curves to data points in PQ scatter plots getting a compact, analytical description of probabilistic reactive power exchange limits. Such descriptions would be possible to obtain

for PQ datasets where the data points belong to one cluster that can be accurately enough represented with single probabilistic curve. Examples of such cases are given later in this paper in case studies A and B.

If the data points belong to multiple clusters, the modeling of probabilistic capability charts with one probabilistic curve per boundary would not be accurate. Fig. 8 shows such a scenario for the case study C described in the Section III-C. The solution to this would be to use multiple probabilistic curves per boundary, each resembling the points from one cluster. Multiple probabilistic curves can represent a boundary of the chart using two different methods:

- 1) activation method,
- 2) probabilistic curves mixture method.

The first method activates the probabilistic curve giving the reactive power limit at the PCC based on the current state of the system. This method requires that each cluster of data points is associated with the specific subset of operating states of the system. These subsets may represent specific seasonal changes of the system ("hot summer day", "windy autumn day", etc.) or more techical representation of system states ("low wind power production in configuration X", "high PV production in configuration Y", etc.).

The second method describes probabilistic boundary as a mixture model of multiple probabilistic curves. Similarly to the first method, probabilistic curves are attributed to the each cluster of the data points after which the boundary is obtained as a mixture of these curves. This can be done in a similar way as it is done for other mixture distributions [30]. While the first method provides more accurate representation of the boundaries, the second one describes them in a more concise way.

C. Fitting of probabilistic curves

The PQ scatter plots provide discrete statistical information about the distribution of PQ points for the analyzed time period in the case of utmost provision or consumption of reactive power at the PCC. This information is given as raw data not providing concrete, analytical relation between P_{pcc} and Q_{pcc} suitable for use in probabilistic planning studies described later in Section V. In order to obtain a compact, analytical description of data points distribution, probabilistic curves are fitted and attributed to each data points cluster.

As described in Section II, probabilistic curves return continuous random variables Q_p for the input real values p . Each random variable Q_p has its associated PDF with the parameters that are smooth functions of $p \in [\min(P_{pcc}), \max(P_{pcc})]$. The choice of the family of PDFs as well as parameter's functions will influence the fitness of probabilistic curve to the given data. For the purpose of the case studies in this paper, family of skew-normal distributions [31] is used. Such a choice is made based on the observation of the raw data points showing skewed distribution properties. Furthermore, having three degrees of freedom, skew-normal distribution can be fitted better over the range of p values than other distributions described by less than three parameters. Skew-normal distribution is

given by (2) using direct parametrization. The parameters $\epsilon(p), \omega(p), \alpha(p)$ are chosen to be cubic spline functions.

$$f_Q(\cdot; \epsilon(p), \omega(p), \alpha(p)) = \frac{e^{-\frac{(q-\epsilon(p))^2}{2\omega(p)^2}} \operatorname{erfc}\left(-\alpha(p) \frac{(q-\epsilon(p))}{\sqrt{2\omega(p)^2}}\right)}{\sqrt{2\pi\omega(p)^2}} \quad (2)$$

The fitting of skew-normal distributions using direct parametrization has been reported to be problematic [32], [33]. Taking into account that the parameters are also spline functions of p , the problem of fitting a probabilistic curve to scatter plot data would become very tough to solve. To overcome this issue, centered parametrization proposed by [33] is used when fitting PDF of skew-normal distribution. Therefore, (2) is reformulated such that direct parameter splines $\epsilon(p), \omega(p), \alpha(p)$ are expressed as a functions (3) of centered parameters splines $\mu(p), \sigma(p)$ and $\gamma(p)$.

$$\begin{aligned} \epsilon(p) &= \mu(p) - \sigma(p) \left(\frac{2\gamma(p)}{4-\pi} \right)^{1/3} \\ \omega(p) &= \sigma(p) \sqrt{1 + \left(\frac{2\gamma(p)}{4-\pi} \right)^{2/3}} \\ \alpha(p) &= \left(\frac{2\gamma(p)}{4-\pi} \right)^{1/3} / \sqrt{\frac{2}{\pi} + \left(\frac{2}{\pi} - 1 \right) \left(\frac{2\gamma(p)}{4-\pi} \right)^{2/3}}. \end{aligned} \quad (3)$$

Two sets of data points (P_i, Q_i) are obtained using (1) corresponding to maximum reactive power provision and maximum reactive power consumption at the PCC. In order to find probabilistic curves $q_{\max}$ and $q_{\min}$, respectively describing previously mentioned sets, maximum likelihood estimation (MLE) method is used. Applying MLE on the problem of probabilistic curve fitting to the set of N_p data points (P_i, Q_i) , optimization problem (4) is formed:

$$\begin{aligned} \text{obj.} \quad & \max \prod_{i=1}^{N_p} f_{Q_i}(Q_i; \mu(P_i), \sigma(P_i), \gamma(P_i)), \\ \text{s.t.} \quad & S(\mu, \sigma, \gamma) = 0, \end{aligned} \quad (4)$$

where $S(\mu, \sigma, \gamma)$ is the set of constraints coming from the definitions of splines $\mu(p), \sigma(p)$ and $\gamma(p)$. The solution of the problem (4) gives optimal values of the splines' parameters which define the probabilistic curves $q_{\max}$ and $q_{\min}$.

The goodness of the probabilistic curve fit may vary at different values of $p \in [\min(P_{pcc}), \max(P_{pcc})]$. Given the definition of objective in (4), an optimisation algorithm will choose the spline function parameters such that the best fit is achieved around the $p = p_{best}$ values with the highest concentration of data points. The constraints S defining splines will limit how much distribution parameters $\mu(p), \sigma(p)$ and $\gamma(p)$ can change in some vicinity of $p = p_{best}$. This means that for some other values of p , the fit may not be as good as in $p = p_{best}$. How good the fit would be depends on the set of constraints S describing boundary conditions at spline knots.

With more spline knots in some interval $I_p \subset [\min(P_{pcc}), \max(P_{pcc})]$, better dynamic change of distribution

parameters in that interval can be achieved. Such a dynamic change in I_p would be justified for providing better fit of distributions if the amount of data points in that interval carries sufficient information on such changes. If not, too many knots would result in overfitting of the splines and not accurate representation of data points distribution. On the other hand, if the interval I_p does not contain too many data points, the number of knots can be decreased resulting in less dynamic change of distribution parameters $\mu(p), \sigma(p)$ and $\gamma(p)$ over that interval. As a consequence, distribution profile over the interval I_p would be more monotone but also more robust taking into account that the monotone profile of the distribution is fitted from the data points information over the whole I_p interval.

Another way to influence goodness of the fit for specific p values would be to introduce weight coefficients $w(p)$ in the objective of (4):

$$\text{obj.} \quad \max \prod_{i=1}^{N_p} w(P_i) f_{Q_i}(Q_i; \mu(P_i), \sigma(P_i), \gamma(P_i)). \quad (5)$$

By appropriate design of a function $w(p)$, the optimization algorithm would prioritize goodness of fit for p values with higher $w(p)$ than for those where $w(p)$ has a lower value.

III. CASE STUDIES

A process of obtaining probabilistic capability charts is illustrated on the three test systems:

- A) modified IEEE 69 bus system with integrated DFIG wind turbines,
- B) modified IEEE 14 + 2 bus system with integrated Full-scale converter wind turbines,
- C) modified IEEE 14 + 2 bus system with integrated DFIG wind turbines.

To get PQ scatter plots, varying operating conditions have to be applied on the selected systems. These conditions can be emulated from the operational statistics obtained from SCADA or other records which may include generation and consumption variation but also discrete actions like switching of capacitor and reactor banks or reconfigurations of the network over some period of time. For the purpose of analysis in this paper, consumption and generation levels in the system are varied over one year period. The hourly data from year 2016 for the varying wind power production and consumption in the south of Sweden (SE4 area) is taken from Nord Pool database [29]. This data are combined with the given test systems data in the following way:

- 1) Generation levels are linearly scaled to correspond to variations in Nord Pool data;
- 2) Consumption levels are linearly scaled to correspond to variations in Nord Pool data. It is assumed that the given original values for consumption levels correspond to 80% consumption levels in Nord Pool data.

It is assumed that all the generators vary their active power in the same manner, simultaneously. The same assumption is made for the loads where they simultaneously change the active power keeping the constant original power factors.

Fig. 1. IEEE 69 bus test system with integrated DFIG wind turbines

Fig. 2. Capability chart of DFIG wind turbine

A. IEEE 69 system

For the purpose of the analysis, IEEE 69 test system shown in Fig. 1 with the base power of 10 MVA is complemented with eight DFIG wind turbines with nominal powers ranging from 1 to 2.5 MW connected to the buses 5, 15, 23, 30, 38, 42, 53 and 60. The capability chart of each DFIG turbine is shown in Fig. 2. Voltage limits in the grid are assumed to be $\pm 10\%$. Big industry loads whose power exceeds 0.3 MW are assumed to have constant consumption. All reactive power loads are scaled to 30 % of their original value.

The probabilistic capability chart of the modified IEEE 69 bus system is represented in Fig. 4(a) with the upper boundary q_{max} and Fig. 4(b) showing the lower boundary q_{min} . The dashed red line on the figures represents the mean value $\mu(p)$ while the full lines correspond to the isolines connecting the same values (indicated by colorbar on Fig. 4) of PDFs f_Q for different P_{pcc} . Values of Lagrangian multipliers corresponding to the bus voltage and reactive power constraints from (1) are

Fig. 3. Modified IEEE 14+2 bus test system

shown in Fig. 5(a) for q_{max} and Fig. 5(b) for q_{min} .

The relevance of Lagrangian multipliers analysis is in comparing the shape of probabilistic curves and the values of multipliers. Doing so, it is also possible to identify the critical elements of the distribution system limiting higher reactive power support to the overlaying grid. Fig. 5 is divided into two subplots. The upper one shows the values of Lagrangian multipliers corresponding to the grid constraints (bus voltage limits). The bus numbers N_{bus} are indicated on the ordinate. The lower subplot shows the value of Lagrangian multipliers corresponding to constraints representing generators reactive power limits. Generator numbers N_g are indicated on the ordinate.

Comparing the shapes of q_{max} and q_{min} with the corresponding values of Lagrangian multipliers, interesting observations can be made:

- 1) For the values of P_{pcc} where no grid limits (bus voltage limits) have been hit, the shape of q_{max} and q_{min} will look like blurred shape of capability charts of the generator units (Fig. 4(a)). In this case, they look like DFIG chart from Fig. 2. If the bus voltage limits are not hit, the Lagrangian multipliers corresponding to these constraints are equal to zero. Lagrangian multipliers for reactive power limits are larger than zero indicating that all the generating units are operating at their reactive power limits;
- 2) For the values of P_{pcc} where grid limits are hit, the distortion of the original shape of DFIG occurs. This can be observed in Fig. 4(b) for $P_{pcc} \in (-0.45, -0.1)$. By analyzing the values of Lagrangian multipliers, it can be seen that bus 31 and bus 64 are reaching their voltage limits since their corresponding Lagrangian multipliers are bigger than zero. Lagrangian multiplier for bus 64 has higher value than the one for bus 31 indicating that bus 64 is more critical in limiting reactive power support at the PCC;
- 3) If the grid limits are hit, the skewness of the probabilistic curve defining the boundary will increase. The increase can be observed on enlarged area of the lower boundary q_{min} in Fig. 4(b). If the grid limits are not hit, the skewness

Fig. 4. Probabilistic capability chart of modified IEEE 69 bus system

Fig. 5. Lagrangian multipliers of the modified IEEE 69 bus system constraints

will not be so pronounced. This can be seen on enlarged area in Fig. 4(a). This furthermore indicates that the shape of the probabilistic curve is correlated to the state of the system in case of the utmost provision of reactive power at the PCC.

B. Modified IEEE 14+2 system

While a radial distribution system was used in the previous case study, here we apply the concept of probabilistic capability charts on the case of a meshed grid similar to the sub-transmission grids found in Sweden. For this purpose, modified IEEE 14 test system shown on Fig. 3 with integrated type 4 wind turbines and base power of 100 MVA is used. The loop is added between buses 9 and 13 containing two additional buses: bus 15, on which a 10MW wind farm is connected and bus 16 on which the load ($P=10\text{MW}$, $Q=3\text{MVar}$) is connected. The lines creating the loop have R/X ratios equal to one. The lines connecting buses 13-15 and 16-9 have reactance of 0.1 p.u. while the line 15-16 has reactance of 0.2 p.u.. Other wind farms are connected to buses 3 (50 MW), 6 (15 MW), and

10 (10 MW). The generator at the bus 2 is removed. Loads at buses 3 and 4 are scaled down 50%. Bus 1 is considered to be the PCC with the higher voltage grid.

The probabilistic capability chart of modified IEEE 14+2 test system is shown on Fig. 6. Similarly to the IEEE 69 bus case, certain correlation between the shape of the probabilistic curves on Fig. 6(a) and Fig. 6(b) and the values of corresponding Lagrangian multipliers from Fig. 7(a) and Fig. 7(b) can be drawn. These indicate the following:

- 1) For the case of a meshed grid, the shape of probabilistic capability does not correspond to the shape of the capability charts of the generators connected to the grid as was the case with the radial grid.
- 2) As the voltage reaches the limit on more buses, the skewness and standard deviation of the probabilistic curve increases. This can be observed on enlarged areas in Fig. 8(a) and Fig. 8(b) as the dashed red line representing the mean value starts to move away from the top of the contour plots. For the same range of P_{pcc} values, voltages begin to reach their limits on more and more buses. In the case

Fig. 6. Probabilistic capability chart of modified IEEE 14+2 bus system

Fig. 7. Lagrangian multipliers of the modified IEEE 14+2 bus system constraints

of upper boundary shown on Fig. 6(a), those are buses 3, 4, 5 and 6 since Lagrangian multipliers from Fig. 7(a) corresponding to the limits on these buses have values larger than zero. Bus 3 is here the most critical one since the value of its Lagrangian multiplier is the highest. In the case of lower boundary shown on Fig. 6(b), limiting buses are 2, 8 and 15 where the most limiting one is bus 8 since its Lagrangian multiplier value from Fig. 7(b) is the highest.

C. Modified IEEE 14+2 system with integrated DFIGs

The previous two case studies show that the shape of the probabilistic capability chart depends on the topology of the system. Besides the topology of the grid, the shape might also depend on the type of controllable reactive power resources installed in the grid. Fig. 8 shows the scatter plot data for the upper and lower boundary of probabilistic capability chart in case of modified IEEE 14+2 system with DFIGs. It can be observed that the data points belong to two clusters, first one colored in dark blue color and another colored in light green. The reason for this grouping is the sharp change in DFIG capability chart on Fig. 2 for provision of reactive power

during low active power production (light green lines). The light green color points on Fig. 8 correspond to the cases of low active power production from DFIGs. Similar segregation of the data points into different clusters might be expected if the system during observed period experiences sharp or discontinuous changes including:

- 1) changes in a topology of the grid,
- 2) switching of big capacitor or inductor banks,
- 3) connection/disconnection of generating units for the purpose of maintenance.

If the data points belong to multiple clusters, the modeling of probabilistic capability charts with one probabilistic curve per boundary would not be accurate. The solution to this would be to use multiple probabilistic curves per boundary, each resembling the points from one cluster as described in Section II-B. For this study case, each boundary is described using two probabilistic curves. The first curve describes the points from the light green cluster and the second one the points from the dark blue cluster. The final boundaries could be assembled by applying activation or mixture method described in Section II-B.

Fig. 8. Scatter plots of modified IEEE 14+2 bus system with DFIG turbines

IV. JOINT PQ PROBABILISTIC CAPABILITY

The probabilistic curves, defined and explained in the previous sections, are giving the PDFs of Q_{pcc} limits for a given value p of P_{pcc} : $f_Q(\cdot; \Theta(p))$. In contemporary and future power systems, P_{pcc} would also have a stochastic nature due to the presence of non-controllable stochastic active power loads and intermittent distributed generation. Therefore, the stochastic nature of P_{pcc} would need to be included in the capability chart to provide full information needed to run probabilistic studies of the overlaying power system. One of the solutions might be a capability chart that shows joint probability density function of P_{pcc} and Q_{pcc} limits.

Joint probability density function $f_{P_{pcc}, Q_{pcc}}$ of two random variables P_{pcc} and Q_{pcc} can be expressed as:

$$f_{P_{pcc}, Q_{pcc}}(p, q) = f_{Q_{pcc}|P_{pcc}}(q|p) \cdot f_{P_{pcc}}(p), \quad (6)$$

where (p, q) are given values of active and reactive power exchange at the PCC, $f_{P_{pcc}}(p)$ is a marginal distribution of P_{pcc} and $f_{Q_{pcc}|P_{pcc}}(q|p)$ is a conditional distribution of Q_{pcc} . The concept of probabilistic curves, introduced by Definition 1, represents a set of conditional distributions $f_{Q_{pcc}|P_{pcc}}(q|p)$ where $p \in I_P = [\min(P_{pcc}), \max(P_{pcc})]$. Having already found the probabilistic curves q_{max} and q_{min} , conditional distribution $f_{Q_{pcc}|P_{pcc}}(q|p)$ is known while a marginal distribution $f_{P_{pcc}}(p)$ would need to be found. This distribution can be easily found by assessing the dataset values of P_{pcc} and fitting a certain probability distribution to them.

Fig. 9 shows an example of joint probabilistic capability for lower boundary of modified IEEE 14+2 bus system. The capability is obtained from (6) using information about $f_{Q_{pcc}|P_{pcc}}$ from the probabilistic curve q_{min} represented on Fig. 6(b). Marginal distribution $f_{P_{pcc}}$ is assumed to be from skew-normal family. It is fitted using MLE method applied to the dataset values of P_{pcc} used to find the q_{min} .

V. APPLICATION ON PROBABILISTIC STUDIES OF POWER SYSTEMS

Power systems of the future will be subject to more uncertainties since the share of renewable energy sources and stochastic loads would increase. Such uncertainties would

Fig. 9. Lower boundary of joint probabilistic chart of the modified IEEE 14+2 bus system

influence planning of active and reactive power reserves responsible for balancing and safe and secured operation of the system. This brings the importance of running probabilistic studies of power systems for the purpose of operation, planning and control [24]. Some of the methods used for this purpose include probabilistic power flows [34]–[36] and stochastic programming methods [37]–[39]. Both of these group of methods are characterized by high computational and memory burdens.

A common way to speed up the demanding computations is to create aggregate models of parts of the systems therefore reducing dimension of the original problem. Such parts are active distribution networks (ADN). Methods to represent ADNs through aggregated capability charts have been previously proposed in the literature [17]–[20], [23]. Common for all of these methods is that they are giving numerical and deterministic representations of ADNs. Compared to them, the probabilistic capability charts proposed by this paper and shown in the results of the case studies are providing analytical, probabilistic description catching stochastic behavior of ADNs components. As such, the proposed probabilistic capability charts may be easily applied in probabilistic studies of power systems, decreasing their computational and memory needs. This conclusion comes directly from the characteristics of the probabilistic capability charts:

- 1) they aggregate the part of the power system, thereby reducing the dimension of the problem to be solved;
- 2) they provide analytical expressions for stochastic active power injection and stochastic limits of reactive power injections of connected aggregated part of the system such as an ADN.

The following subsections illustrate how the proposed probabilistic charts can be applied in reality as inputs to stochastic planning studies of power systems.

A. Stochastic programming

A stochastic programming problem in power systems can be written as:

$$\begin{aligned}
 \text{obj.} \quad & \min f(x, \text{par}) \\
 \text{s.t.} \quad & \sum_l P_{il}^{\text{inj}} = g_{Pi}(x, \text{par}), \forall i \in \text{Bus}, \\
 & \sum_l Q_{il}^{\text{inj}} = g_{Qi}(x, \text{par}), \forall i \in \text{Bus}, \\
 & \underline{U}_i < U_i < \bar{U}_i, \forall i \in \text{Bus}, \\
 & \underline{Q}_j < Q_j < \bar{Q}_j, \forall j \in \text{Gen}, \\
 & \underline{Q}_k < Q_k < \bar{Q}_k, \forall k \in \text{Ren}
 \end{aligned} \tag{7}$$

where $f(x, \text{par})$ is an objective function depending on system states x and parameters par , Bus is a set of all the buses, $\text{Gen} \subset \text{Bus}$ is a set of conventional generators and $\text{Ren} \subset \text{Bus}$ is a set of distributed generators (DG) assumed to have stochastic active power injections. $g_{Pi}(x, \text{par})$ and $g_{Qi}(x, \text{par})$ are expressions defining powers flowing from the bus i into the lines. The stochastic part of the problem (7) comes from the fact that P_{ik}^{inj} , $k \in \text{Ren}$ and stochastic loads are random variables. Furthermore, reactive power limits $\underline{Q}_k$ and $\bar{Q}_k$ are functions of P_{ik}^{inj} making them random variables as well. This type of problems are usually solved using chance constraints and scenario generation methods.

Let's assume that the power system under consideration consists of ADNs and a transmission grid to which the previous ones are connected. Such a system can consist of many buses with stochastic power injections increasing the dimensionality of the problem (7). Using the results of this paper, each ADN can be modeled as a generator with a stochastic active power injection P_{ik}^{inj} and its probabilistic capability chart defining analytically probabilistic reactive power limits $\underline{Q}_k$ and $\bar{Q}_k$ as a function of P_{ik}^{inj} . In such a way, ADNs that may have up to 100 buses are represented with power injections at one bus substantially decreasing the dimensions of the problem (7).

The next step in solving problem (7) would be to address stochastic active power injections P_{ik}^{inj} . Usual way to do this is to use the scenario generation method. The information needed by this method is how probable is a particular scenario s with certain power injection $P_{ik}^{\text{inj}}(s)$. This can be obtained from PDF of the P_{pcc} of joint PQ probabilistic capability calculated for each ADN.

Assuming a scenario s with active power injection of $P_{ik}^{\text{inj}}(s) = P_{pcc}$, the reactive power limits $\underline{Q}_k(s)$ and $\bar{Q}_k(s)$ with their associated PDFs can be directly read in the analytical form from the proposed probabilistic capability charts of

ADNs. Here, it is to be observed that for each scenario s , the limits $\underline{Q}_k(s)$ and $\bar{Q}_k(s)$ are given as random variables defining probabilistic limits on ADNs reactive power injections to the transmission system. In order to solve a problem with probabilistic constraints, chance constraints method can be used. Using information on PDFs of $\underline{Q}_k(s)$ and $\bar{Q}_k(s)$, the chance constraint method converts stochastic programming problem into ordinary nonlinear programming problem by converting stochastic constraints into deterministic constraints. If information on the PDFs is given in analytical form, this conversion is much easier to be done [40].

B. Probabilistic power flow

Another approach for handling uncertainties in the power systems are probabilistic power flows (PPF). These methods treat the active and reactive power injections as random variables. PPFs can be solved numerically using Monte Carlo simulations [41]–[43] or analytically using convolution techniques [35]. As inputs to the algorithms, these methods usually need analytical expressions of power injections PDFs. Such expressions are provided by probabilistic capability charts proposed by this paper. Furthermore, the proposed charts aggregate the part of the system (ADNs), decreasing the size of the problem and so the solution time.

While PPFs tend to be memory and time demanding, the advances in the algorithms and computational technology make them a viable and promising approach when it comes to studying more and more present stochastic phenomena in power systems. The proposed probabilistic capability charts can complement PPFs in achieving this task.

VI. DISCUSSION

A concept of capability charts for the grids have been previously introduced in the literature [17], [18]. However, this paper builds on that concept by providing more detailed analytical information on the capabilities taking into account stochastic phenomena. Analyzing the shape of the probabilistic curves, clear indication of correlation between the shape of the curves and the state of the system has been observed. The more detailed analysis of this correlation goes beyond the scope of this paper and is subject to future work.

When designing the probabilistic capability charts, the basic set of constraints described by (1) have been used. These constraints may be complemented assuming other criteria limiting reactive power provision at the PCC. Some of those may include constraints coming from controllers actions such as voltage control setpoints in the grid or economic criteria such as a limit on active power losses in the grid. From the case studies it is shown how some of the system constraints influence the charts. Future studies should be done showing how more detailed models of systems with additional operational and physical constraints may influence the shape of the proposed probabilistic capability charts.

Further improvements in design of probabilistic charts can be obtained by clustering of scatter-plot data and creation of a set of probabilistic charts each corresponding to specific operating conditions e.g. "hot summer day", "windy winter

day”, etc. A yearly variation of generation and load profiles was used when drawing capability charts in this paper. More accurate representations would have been achieved if multi year data clustered in season-specific operations have been used. Then, a set of probabilistic capability charts representing each of these seasons could have been obtained reflecting more accurately the limits of reactive power provision at the PCC.

The proposed probabilistic capability charts describe power injections at nodes of transmission system without considering their correlation. The correlation between the nodes described by proposed probabilistic charts is subject to a future work.

VII. CONCLUSIONS

This paper introduces the concept of probabilistic capability charts to describe reactive power potential at the PCC between a distribution and the overlaying transmission system. The proposed charts give analytical information on availability of reactive power support at the PCC taking into account stochastic phenomena in the system. Stochastic phenomena is captured using probabilistic curves to analytically describe the boundaries of reactive power support at the PCC. Besides representing reactive power limits for certain active power injection at the PCC, they also carry information on the state of the distribution system. While accurately representing the reactive power capability of the underlying distribution grid, the proposed charts do not reveal sensitive information about distribution grid models. As such, the charts can be used as means to exchange the information between DSOs and TSOs.

The intermittent nature of the future power systems might require increased use of probabilistic methods in power system analysis. With today’s increased availability of computational power, these methods may be an essential tool to provide more precise analyses of the stochastic and highly dynamic future power systems. The proposed charts may be easily implemented in these methods, providing the analytical models of the stochastic system and speeding-up the probabilistic computations.

REFERENCES

- [1] (2018) Smartnet project. [Online]. Available: <http://smartnet-project.eu/>
- [2] A. Mohammadi, M. Mehrdash, and A. Kargarian, “Diagonal quadratic approximation for decentralized collaborative tso+dso optimal power flow,” *IEEE Trans. Smart Grid*, 2019.
- [3] Z. Li *et al.*, “Coordinated transmission and distribution ac optimal power flow,” *IEEE Trans. Smart Grid*, 2018.
- [4] —, “A distributed transmission-distribution-coupled static voltage stability assessment method considering distributed generation,” *IEEE Trans. Power Syst.*, 2018.
- [5] E. Hillberg *et al.*, “Flexibility needs in the future power system,” Int. Smart Grid Action Network (ISGAN) Annex 6 Power T&D Syst., Tech. Rep., March 2019.
- [6] *Systemutvecklingsplan (System development plan) 2018-2027*, Svenska Kraftnät, November 2017.
- [7] G. Valverde and T. V. Cutsem, “Control of dispersed generation to regulate distribution and support transmission voltages,” in *IEEE PowerTech Conf.*, 2013.
- [8] H. S. Bidgoli and T. V. Cutsem, “Combined local and centralized voltage control in active distribution networks,” *IEEE Trans. Power Syst.*, 2018.
- [9] P. Aristidou, G. Valverde, and T. V. Cutsem, “Contribution of distribution network control to voltage stability: A case study,” *IEEE Trans. Smart Grid*, 2017.
- [10] Z. Yang, Y. Li, and J. Xiang, “Coordination control strategy for power management of active distribution networks,” *IEEE Trans. Smart Grid*, vol. 10, no. 5, September 2019.
- [11] L. Chen and H. Li, “Optimized reactive power supports using transformer tap stagger in distribution networks,” *IEEE Trans. Smart Grid*, vol. 8, no. 4, July 2017.
- [12] E. Kaempf *et al.*, “Reactive power provision by distribution system operators - optimizing use of available flexibility,” in *IEEE PES Innovative Smart Grid Technol. (ISGT) Europe*, 2014.
- [13] F. Marten *et al.*, “Optimizing the reactive power balance between a distribution and transmission grid through iteratively updated grid equivalents,” in *Power Syst. Comp. Conf.*, 2014.
- [14] M. Braun, “Reactive power supply by distributed generators,” *IEEE PES General Meeting*, 2008.
- [15] F. Hinz and D. Möst, “Techno-economic evaluation of 110 kv grid reactive power support for the transmission grid,” *IEEE Trans. Power Syst.*, 2018.
- [16] P. Cuffe and A. Keane, “Voltage responsive distribution networks: Comparing autonomous and centralized solutions,” *IEEE Trans. Power Syst.*, vol. 30, no. 5, September 2015.
- [17] P. Cuffe, P. Smith, and A. Keane, “Capability chart for distributed reactive power resources,” *IEEE Trans. Power Syst.*, 2014.
- [18] —, “Transmission system impact of wind energy harvesting networks,” 2012.
- [19] A. V. Jayawardena *et al.*, “Capability chart: A new tool for grid-tied microgrid operation,” in *IEEE PES T&D Conf. Expo.*, 2014.
- [20] D. M. Gonzalez *et al.*, “Determination of the time-dependent flexibility of active distribution networks to control their tso-dso interconnection power flow,” in *Power Syst. Comp. Conf. (PSCC)*, 2018.
- [21] S. Stanković and L. Söder, “Identification of reactive power provision boundaries of a distribution grid with dfigs to a transmission grid,” in *IEEE PES Innovative Smart Grid Tech. Europe (ISGT-Europe)*, 2017.
- [22] —, “Analytical estimation of reactive power capability of a radial distribution system,” *IEEE Trans. Power Syst.*, vol. 33, no. 6, Nov. 2018.
- [23] D. A. Contreras and K. Rudion, “Improved assessment of the flexibility range of distribution grids using linear optimization,” in *Power Syst. Comp. Conf. (PSCC)*, 2018.
- [24] K. N. Hasan, R. Preece, and J. V. Milanović, “Existing approaches and trends in uncertainty modelling and probabilistic stability analysis of power system with renewable generation,” *Renewable and Sustainable Energy Reviews*, 2019.
- [25] P. Kundur, *Power System Stability and Control*. McGraw-Hill, 1994, vol. 7.
- [26] J. Dragosavac *et al.*, “On-line estimation of available generator reactive power for network voltage support,” in *8th Mediterranean Conf. on Power Generation, Transmission, Distribution and Energy Conversion (MEDPOWER)*, 2012.
- [27] G. Valverde and J. J. Orozco, “Reactive power limits in distributed generators from generic capability curves,” in *IEEE PES General Meeting*, July 27-31 2014.
- [28] T. Lund, P. Sørensen, and J. Eek, “Reactive power capability of a wind turbine with doubly fed induction generator,” *Wind Energy*, vol. 10, pp. 379–394, April 2007.
- [29] Nord pool market data. [Online]. Available: www.nordpoolgroup.com
- [30] C. M. Bishop, *Pattern Recognition and Machine Learning*. Springer, 2009.
- [31] A. Azzalini, “A class of distributions which includes the normal ones,” *Scandinavian J. of Statist.*, vol. 12, no. 2, pp. 171–178, 1985.
- [32] A. Pewsey, “Problems of inference for azzalini’s skewnormal distribution,” *J. of Appl. Statist.*, vol. 27, no. 7, 2000.
- [33] R. B. Arellano-Valle and A. Azzalini, “The centred parametrization for the multivariate skew-normal distribution,” *J. of Multivariate Anal.*, vol. 99, no. 7, August 2008.
- [34] B. Borkowska, “Probabilistic load flow,” *IEEE Trans. Power App. Syst.*, vol. PAS-93, no. 3, pp. 752–759, May 1974.
- [35] M. Fan *et al.*, “Probabilistic power flow studies for transmission system with photovoltaic generation using cumulants,” *IEEE Trans. Power Syst.*, 2012.
- [36] P. Chen, Z. Chen, and B. Bak-Jensen, “Probabilistic load flow: A review,” in *Third Int. Conf. on Elect. Utility Deregulation and Restructuring and Power Technol.*, 2008.
- [37] A. Kargarian, Y. Fu, and H. Wu, “Chance-constrained system of systems based operation of power systems,” *IEEE Trans. Power Syst.*, 2016.
- [38] C. Hamon, M. Perninge, and L. Söder, “A stochastic optimal power flow problem with stability constraints - part i: Approximating the stability boundary,” *IEEE Trans. Power Syst.*, 2013.
- [39] M. Perninge and C. Hamon, “A stochastic optimal power flow problem with stability constraints - part ii: The optimization problem,” *IEEE Trans. Power Syst.*, 2013.

- [40] S. Kataoka, "A stochastic programming model," *Econometrica*, vol. 31, no. 1-2, pp. 181–196, 1963.
- [41] A. M. L. da Silva and A. M. de Castro, "Risk assessment in probabilistic load flow via monte carlo simulation and cross-entropy method," *IEEE Trans. Power Syst.*, 2019.
- [42] M. Hajian, W. D. Rosehart, and H. Zareipour, "Probabilistic power flow by monte carlo simulation with latin supercube sampling," *IEEE Trans. Power Syst.*, 2013.
- [43] Z. Q. Xie *et al.*, "Quasi-monte carlo based probabilistic optimal power flow considering the correlation of wind speeds using copula function," *IEEE Trans. Power Syst.*, vol. 33, no. 2, March 2018.

Stefan Stanković (S'17) received the Dipl. Ing. and M.Sc. degrees in electrical engineering from University of Belgrade, Belgrade, Serbia, in 2014 and 2015, respectively. He is currently a PhD candidate at KTH Royal Institute of Technology, Sweden, Stockholm. His current research interests include integration of wind and solar power into power systems, reactive power support and voltage control of distribution and transmission grids, power systems control and stability.

Lennart Söder (M'91-SM'10) was born in Solna, Sweden, in 1956. He received the M.Sc. and Ph.D. degrees in electrical engineering from the KTH Royal Institute of Technology, Stockholm, Sweden, in 1982 and 1988, respectively. He is currently a Professor of Electric Power Systems with the KTH Royal Institute of Technology. His current research interests include power system operation and planning, integration of wind and solar power, hydropower system modelling, demand side management, distribution systems, power system reliability, power markets and regulation.

bility, power markets and regulation.