Accelerated test for vehicle body durability based on vehicle dynamics simulations using pseudo damage

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A thesis presented for the degree of Master of Science in Vehicle Engineering by

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Vehicle body durability evaluation strongly relies on the possibility of testing a real prototype on different testing surfaces, such as proving grounds, test rigs and real roads. Although many efforts have been made to reduce time required for testing, this still remains one of the main resource-demanding phases in a vehicle development. As direct consequence, more and more companies aim to optimise and to improve vehicle development by means of CAE tools.

This master thesis is a step towards virtual testing of a vehicle body, aiming to investigate and to select the most important measurements for a body durability evaluation and to reproduce an invariant excitation that could be applied to other vehicle models for new durability assessments without the need of real measurements from other models. Moreover, a comparison between frequency-based methods and time-based methods and their differences were highlighted and the validity of ISO8608:2016 discussed.

The method relied on small sets of simple and easy-to-get internal measurements, called desired signal, that allowed back-calculation of wheel hub displacements and other excitations by means of the product of the model’s transfer function and desired signal. Then, the iteration procedure allowed to drastically reduce the error between the desired signal and the computed one and it proved to be essential in the process. Thanks to this procedure, damage information of also the not-measured signals could be computed and their damage assessed and thus available for durability purposes. Moreover, the chance of applying the same measurements taken from a real vehicle to a model of a newer generation was investigated. This would avoid the need of building a running prototype, allowing a more accurate durability assessment already available in the pre-design phase.

As a result, using a specific set of 8 measurements, other 22 other forces, displacements and velocities of several components were precisely reproduced, showing that not all measurement are equally valuable in a durability evaluation. A method for the measurement selection, called Observability Method, was then developed and compared with a set of measurements selected by means of experience, showing a better convergence of the pseudo damage and similar pseudo damage values. Eventually a small set of measures from an older car was applied to a newer version. It was proved that a detailed knowledge of the car model is needed, in order to successfully back-calculate the relevant measurements.
Sammanfattning

Utvärderingen av fordonskarossens tålighet fölitar sig starkt på möjligheten att testa en verklig prototyp på olika testytor, såsom övningsfält, testriggar och riktiga vägar. Även om flertalet insatser gjorts för att reducera testtiden som krävs, kvarstår denna som en av de mest resurskrävande faserna vid produktutvecklingen av ett fordon. Som en direkt följd strävar fler och fler företag mot att optimera och förbättra fordonsutvecklingen med hjälp av CAE-verktyg.


Som ett resultat av detta, med användning av en specifik uppsättning av 8 mätsignaler, kunde 22 andra, förskjutningar och hastigheter hos flera komponenter exakt reproduceras, vilket påvisar att inte alla mätsignaler är lika värdefulla i en tålighetsbedömning. En metod för urvalet av mätsignaler, kallad Observability Method, utvecklades därefter och jämfördes med uppsättningar av mätdata, utvalda utifrån erfarenhet. Den utvecklade metoden visar en bättre konvergens av pseudoskador och liknande pseudoskadevärden. Slutligen applicerades en liten uppsättning mätdata från en äldre bil till en nyare version. Resultaten visade att en detaljerad kunskap om bilmodellen behövs för att framgångsrikt kunna beräkna de relevanta mätsignalerna.
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1 Introduction

In real life any vehicle drives on a variegate set of different roads with very different roughness, filled of potholes, bumps and other impact loads that heavily affect its life durability. Therefore manufacturers invest lots of resources to test their products before releasing them to the public. As a matter of fact, roughly 30 percent of the total development cost of a car is due to testing equipment [1] and, although several testing roads have been developed to reduce the distance amount driven by the testers, validation process turns out to be still one of the most resource-demanding in the production workflow. As a consequence, nowadays more and more companies have adopted several Computer Aided Engineering (CAE) software to reduce time and expenses, but so far life assessment of the vehicle body is still conducted mainly afterward a prototype is up and ready for testing. Moreover, durability evaluation is strictly related to the vehicle under investigation with no chance of using new data to study another one. This is because present methods highly rely on variant excitation, which by definition depends on model specifications such as mass, suspension geometry, tyres and so on. Instead, if the study of invariant excitation, extrapolated from an initial test case scenario where variant excitation had been measured and analysed, was possible, this knowledge could be applied to many new models with no need of a dedicated prototype, allowing durability assessment already in the design process.

Therefore, the method developed in this thesis investigates relationship amongst variant excitations, which are easily measurable by means of different kinds of sensors, and its invariant excitation, considered to be the wheel hub displacement, in order to apply it to similar models, for example to a loaded/unloaded truck, or a following version of a car model.

This first introductory chapter gives a brief explanation about the project’s workflow, its goals and methods adopted. In the second chapter a theoretical background about fatigue definition and methods for fatigue assessment in a component has been treated to clarify what main problems lie in durability evaluation and what solutions were adopted to face them. In chapter 3 road models and car models have been explained and some considerations on Adams Solver have been provided. Chapter 4 deals with the choice of measurements based on experience and through the definition of the Observability Method and its related discussion and result. Lastly, chapter 5 includes conclusions, limitations of the project and future work.

1.1 Objective and Method

This thesis aims to achieve several goals related to the vehicle durability. In detail, those are:
- Create an invariant excitation that can be reproduced by iteration from internal measurements.

- Evaluate the accuracy level of different response signals and assess a comparison amongst body durability prediction errors.

From road profile and tyre interaction, wheel forces propagate through the wheel system and consequently to the suspension, eventually the body. In order to assess a durability evaluation, measurements of the road profile and the tyre forces are needed. However, since measurement procedures imply many difficulties in the set-up stage, they would still involve an amount of uncertainty, in spite of their cost. On the other hand, internal measurements, which in this project are called as system response or output channels \((y)\), can be quite easily extrapolated from gauge or displacement sensors positioned on many vehicle components such as springs, dampers, lower control arms, and these are used to calculate external loads.

Therefore, the starting point in this process was choosing which measurements were to be used and gathering what was named desired signal from either real testing on a predefined road or from a multi-body simulation. The desired signal is a wide set of measurements and this was used as reference and comparison in the process. Firstly, a known white noise was created and used as input \((u)\) for running a first multi-body simulation of a vehicle model on a testrig, so that the output to the white noise was registered. Whenever a system is excited from a signal, it responds uniquely to it, according to its own characteristics: the set of these characteristics defines the so called transfer function. A fully-defined transfer function would include all measurements of a system, but this arises difficulties due to the immense quantity of data to be gathered and the number of sensors ideally needed on the vehicle. Therefore, in this project a transfer function was defined using a reduced number of signals. It is worth noticing that real systems are non linear, but this fact was neglected and the multi-body system was considered linear, so that its transfer function could be easily evaluated as ratio of output to input and the inverse transfer function directly calculated from it. Then a drive input signal was created as product of the desired (measured) one and the inverse transfer function: once again the system was approximated as linear. Lastly, thanks to system’s inverse transfer function, measured signals and precalculated loads from the drive signal, the iteration process took place and, whenever successful, it converged so that same signals were reproduced, despite the model’s non linearities. From the previous iteration procedure, iterated response signals were given as result and compared with the desired ones: the closer the response is to the desired signal, the more reliable and accurate the iteration has been, the more accurate the durability evaluation could be, since there is a direct correlation between a signal and the damage evaluated from it. As a result, a smaller set of measurement can describe accurately signals affecting vehicle durability, proving others’ superfluity. In this project the
pseudo damage method was used to relate signal to damage and consequently to durability.

The two main software used for this procedure:

- Adams/Car is a multi-body software in which the car model was built.
- Femfat Lab VI is a software used for back calculating external excitation and the damaging channels.

A general overview of the project’s workflow is depicted in figure 1.

![Project workflow](image)

Figure 1: Project workflow

Once accurate vehicle body durability simulations are proved to be solid and externally valid, they could ideally replace all expensive and time consuming field testing. This thesis is a step towards this direction.
2 Fatigue and Durability

Fatigue is a degradation phenomenon that occurs whenever a time-variant load is applied to a mechanical component, creating a crack at first, enlarging it throughout time until material resistance reaches its stress limit and the structure eventually collapses. In order to accurately predict a complete fracture, information about loads and their application time functions is thus needed. As a consequence, durability is a component’s property that can be defined as its capacity to resist fatigue, which means in other words to survive the predicted usage over a suitable-long time period or number of cycles. Foreseeing failures allows to minimise downtime and accident costs, improving at the same time safety for users. Analytically, durability can be seen as number of cycles before fracture and its simplest and most common model is defined using the Wöhler curve, described by the Basquin’s equation (1)

\[
N = \begin{cases} 
\alpha S^{-\beta}, & \text{for } S > S_f \\
\infty & \text{otherwise}
\end{cases}
\]  

(1)

where \(N\) indicates the number of cycles to fatigue failure, \(S\) is the stress amplitude of the applied load, \(\alpha\) describes the fatigue strength of the material, \(\beta\) is the damage exponent, also called kurtosis coefficient and \(S_f\) the fatigue limit.

Vehicles are formed by a huge number of mechanical components that need to cooperate together to ensure the correct driveability and safety performances during their entire usage, thus they need to be designed accordingly, in such way that the following structural requirements are fulfilled [2]:

- high torsional and bending stiffness to react static and dynamic loads avoiding excessive deformation during the drive: this is essential to improve vehicle’s drive performance.
- strength (or resiliency) to survive plenty of load cycles without failure.
- good deformation under impact to improve safety, minimising passengers’ or other users’ injuries: the market is becoming more and more sensitive about this topic.

This project deals with loads included in the second point, whose loads are by definition of dynamic nature.

It is quite intuitive to understand that magnitude of load cycles is strongly dependent on the vehicle in consideration: different mass, material, stiffness characteristic, suspension layout are only few examples of parameters affecting durability. In fact, as it could be expected, one vehicle is not as reliable as another, although both are driven
on the same road by the same driver. In order to predict their durability, intended as the study of body’s structure and components behaviour in a prolonged service life, including environmental aspects such as corrosion and wear that decrease the load carrying capacity, several methods for experimental testing are applicable:

- in-service field testing
- accelerated proving ground tests
- laboratory testing

In-service field testing is usually the most realistic prevision, but it unfortunately results to be expensive and extremely time consuming. In fact, for a well-designed vehicle, failure due to fatigue should not occur before millions of loading cycles, unless any kind of misuse was conducted and this leads to a difficulty in following market’s demand, since a fully tested vehicle might be obsolete once into production.

In proving ground tests, shown in figure 2, which can be considered as a misuse form, vehicles are driven on roads defined by ISO in order to create and apply specific excitation, so that the whole amount of time to failure is substantially reduced. Some examples are the cobblestone road and the out-of-phase washboard.

Eventually laboratory experiments allow to effectively control different variables and to excite the whole vehicle or only an investigated subsystem through the so called test rigs, formed by actuators connected to either the wheel spindle or placed under the tyre.

![Figure 2: Example of proving ground, courtesy of Siemens](image)

Lab testing allows to sensibly reduce testing time and overall cost, but equipment generally requires high capital investment and maintenance costs.

### 2.1 Methods for Load Analysis and Fatigue Estimation

In this section a general overview of different methods used for load analysis and assessing fatigue is briefly given, in order to compare main advantages and drawbacks.
The workflow is depicted in figure 3.

To predict fatigue life of a component, applied stresses need to be evaluated from a measured, usually under the form of time signal, time series or load history. Depending on the application, there are different load aspects that need to be taken into account, thus several methods to store related data, which divided in two main groups: rate independent methods (or amplitude-based methods) depend only on the sequence of the local extreme values in the signal without considering the shape/behaviour between them, while rate dependent methods (or frequency-based methods) study the energy of a signal distributed over a range of frequencies. These methods are used to describe important load characteristics for load analysis and a complete list of them can be found in [3].

![Figure 3: Workflow for fatigue assessment](image)

2.1.1 Rate-Independent methods

Fatigue is a phenomenon greatly lead by stress and by strain and a good model for representing their relationship in locally uni-axial stress state is the Masing memory model that relies on cyclic stress-strain curve \( \epsilon = g(\sigma) \), and the Ramberg-Osgood relation [3]:

\[
g(\sigma) = \frac{\sigma}{E} + \frac{\sigma}{K'} \left( \frac{\sigma}{\sigma} \right)^{n'}
\]  

which is an empirical model valid in uni-axial stress-strain state for small loads compared to material’s static limit, \( E \) is the Young’s modulus, \( K’ \) and \( n’ \) are constant depending on the material, \( \epsilon \) and \( \sigma \) are respectively strain and stress.

Hysteresis models are usually applied to local stress \( \sigma(t) \) or strain \( \epsilon(t) \), but when it comes to load analysis, measurements or calculated loads \( L_i(t) \) are studied, like for
example forces. If a linear static elasticity relationship between outer loads acting on the component and local stresses $\sigma(t)$ and strains $\epsilon(t)$ on a local spot $s$ belonging to the same component is considered, the elastic stress (also called pseudo stress) is given by the sum of all the product between the acting loads and their linear coefficients:

$$\sigma_e(s, t) = c_1(s) \cdot L_1(t) + c_2(s) \cdot L_2(t) + \cdots + c_n(s) \cdot L_n(t)$$  \hspace{1cm} (3)

In order to better estimate the local stress, Neuber’s formula in [4] is thus used to avoid the overestimation caused by the linear elastic hypothesis when local yielding occurs

$$\sigma_e = \frac{\sigma^2}{E}$$  \hspace{1cm} (4)

Given these assumptions, load-strain cycles behave similarly to stress-strain cycles, meaning that the correspondent cycles open and close at the same time and every "local" cycle can be referred to its "outer" counterpart and vice versa.

Amplitude based methods neglect load frequencies of the signal but consider the sequence of local extreme values, called turning points, the global range of the signal and the number of mean crossings to collect information on the kind of load, relying on cycle counting algorithms to translate variable amplitude time signals into amplitude range and cycles. One of the most common method for high cycle fatigue is the rainfall counting, consisting in counting hysteresis loops inside the load. In other words, load history is turned into a sequence of peaks and valleys, or reversals, the total amplitude range is divided in classes and from each loop, or hysteresis cycle, a quantitative damage is evaluated and represented thanks to histograms. If $\phi$ is a monotonic transformation from outer loads to stress valid at least approximately, so that $\sigma(t) = \phi(L(t))$, then it is possible to back-transform local stress into loads. There are several algorithms to evaluate rainfall cycles, but for the matter of conciseness only the 4-point algorithm is explained [3]. From a sequence of discretised turning points $z_k$, for $k=1, \ldots, N$, the values $1, \ldots, n$ are taken so that discretisation and min-max filtering are already occurred. The Rainflow Matrix (RFM) and the Residual Array (RES) are defined as follows:

- RFM includes a pair of values if they form a cycle
- RES includes values which do not form a cycle

In other words, the algorithm can be explained as follows: initialising the first 4 points of the signal $s = [s_1 = z_1, s_2 = z_2, s_3 = z_3, s_4 = z_4]$ , the counting rule is applied:

$$\text{if } \min(s_1, s_4) \leq \min(s_2, s_3) \text{ and } \max(s_2, s_3) \leq \max(s_1, s_4)$$  \hspace{1cm} (5)
then the pair \((s_2, s_3)\) is a cycle.

In case of a first cycle is counted, this is stored in the \(RFM(s_2, s_3) = RFM(s_2, s_3) + 1\) and the two points are removed from the stack, which needs to be refilled. Refilling the stack respects the memory rules from the hysteresis model, this means including also some points from the past, or from the RES array, so these rules can be written as \((k\) stands for the next value of the signal \(z)\):

- **r1**: if \(r = 0\), \([s_1 = s_1, s_2 = s_4, s_3 = z_k, s_4 = z_{k+1}]\), and \(k = k + 2\)
- **r2**: if \(r = 1\), \([s_1 = RES, s_2 = s_1, s_3 = s_4, s_4 = z_k]\), and \(k = k + 1\), and \(r = 0\)
- **r3**: if \(r \geq 2\), \([s_1 = RES_{r-1}, s_2 = RES_r, s_3 = s_1, s_4 = s_4]\), and \(r = r - 2\)
- **r4**: if \(r = 1\), \(RES_r = s_1\), \([s_1 = s_2, s_2 = s_3, s_3 = s_4, s_4 = z_k]\), and \(k = k + 1\)

This algorithm is repeated till the last point of the time signal is reached and the counting rule (5) can not be applied anymore.

The rainflow matrix includes all the closed cycles, which are divided in two different groups, shown in figure 4: the first one includes cycles whose maximum point occurs before minimum point inside of it and they are named hanging cycles, while the second is formed by cycles whose the minimum point occurs before the maximum point and these are called standing cycles. It is worth noticing that according to this algorithm a hanging cycle is also defined when \(s_4\) is the maximum of the stack, or the stress-strain curve crosses itself in higher part of the close cycle. On the other hand, in case of standing cycles \(s_4\) is the minimum point in the stack and the stress-strain curve crosses itself in the lower part of the closed cycle. As a consequence, hanging cycles are positioned below and standing cycles above the RFM diagonal.

![Figure 4: Handing and standing cycles in the stress-strain plane](image)

The residual array includes all the other turning points which do not form any hysteresis cycle in the time signal and its task is to map the hysteresis component of the load history. In other words, RES allows to distinguish which cycles, assigning
them an order, so that it is possible to back calculate the signal amplitude with respect of time. These are also the strongest advantage of the rainflow counting along with the 4-point algorithm against other counting methods. For further details on different methods, references [3] and [5] are addressed.

2.1.2 Rate-Dependent methods

To achieve a complete comprehension of a full time signal as local response of a complex (non-linear) system, different properties from amplitude-based methods need to be considered too. Frequency-based methods are generally useful for load analysis of the system, rather than local loads as for time-dependent methods, and focus on how energy of a general signal is distributed on a spectrum of frequencies.

One of the most used method in signal analysis is the Power Spectrum Density (PSD) function and the Periodogram, which are also used in the ISO16750-3:2012 [6] to describe exciting loads on a mechanical system and in ISO8608:2016 [7] to define road surface profiles.

For a generic non-periodic signal \( x(t) \) included in \( L_2 \) a Power Spectral Density is defined as:

\[
\hat{P}_x(f) = c \cdot |\hat{x}(f)|^2
\]  

(6)

where \( c \) is a constant and \( \hat{x}(f) \) is the Fourier transform of the signal. In case of periodic signals, the PSD can be defined using coefficients \( c_k \) of the Fouries series by means of the spectral decomposition of the signal:

\[
\hat{P}_x(f_j) = c \cdot |c_j|^2, \quad f_j = \frac{j}{N\Delta t}, \quad j = 0, ..., N - 1
\]  

(7)

where \( N \) is the number of samples and \( \Delta t \) is the sampling rate. Thanks to the Discrete Fourier Transformation of the signal with coefficients \( \hat{x}_j \), the periodogram is expressed as:

\[
I_x(f_0) = \frac{1}{N^2} |\hat{x}_0|^2
\]  

(8)

\[
I_x(f_i) = \frac{1}{N^2} (|\hat{x}_j|^2 + |\hat{x}_{N-j}|^2), \quad \text{for} \quad j = 1, ..., \frac{N}{2} - 1
\]  

(9)

\[
I(f_{N/2}) = \frac{1}{N^2} |\hat{x}_{N/2}|^2
\]  

(10)

It is worth noticing that the Periodogram is an estimator of the PSD applied to the observed signal and since its spectrum is noisy, its consistency is low. To solve this matter, other PSD estimators have been defined, an example in Matlab and Signal Processing Toolbox Release 2018a [8], so that the whole time load history can be mapped in frequency domain. Last but not least, PSD is expressed only as
real number, which means that no phase information is included in it. A comparison between the Periodogram and the Welch estimators can be found in [9].

2.2 Fatigue analysis

In order to increase model accuracy, different numerical methods are adopted by several software to reproduce trustworthy local stress-strain histories from external fatigue cycles acting inside the component. In this section, some methods for internal loads assessment from complex models are briefly introduced and compared.

2.2.1 Quasi-Static Superposition

This model can be described in time-dependent terms from the general equation of motion:

\[ M \ddot{u}(t) + D \dot{u}(t) + Ku(t) = L(t) \]  

(11)

where \( M, D, K \) are respectively the mass, damping and stiffness matrices, \( u \) the external displacement and its derivatives with respect to time, and \( L \) a general external load, which is valid if a linear material behaviour relates strain and stress, as already assumed in equation (3). The general equation of motion can be reduced, in case of constant or almost constant forces, so that the phenomenon can be described by only

\[ K \cdot u(t) = L(t) \]  

(12)

that can always solved uniquely for an arbitrary \( L \), since matrix \( K \) is not singular, which implies a rigid constraint between component and "ground". A good way to solve equation (12) is to decompose \( K \) in a lower triangular matrix and an upper triangular matrix, so that finding a solution for the set of \( n \) results to be almost as fast as solving a single static equation.

For \( n \) number of section forces acting on the component and a force vector \( e_i \) so that the \( i^{th} \) component of the section forces is one and all other components are zero, the independent force vector \( L(t) \) can be written in the form

\[ L(t) = \sum_{i=1}^{n} l_i(t) \cdot e_i \]  

(13)

with \( l_i \) is a scalar time signal representing the \( i^{th} \) section force. Consequently,

\[ K \cdot u_i = e_i, \quad i = 1, \ldots, n \]  

(14)

which depicts the so called unit load cases, in fact \( u_i \) includes all the displacements for every node in the structure. Eventually, equation (12) can be rewritten as:
that represents the so called principle of Quasi-Static Superposition (QSS) and where \( u_i(x) \) represents displacements only for the point \( x \). This method relies on inertia relief technique, which consists in reliving the model of its inertial effects so that the static equilibrium is satisfied [10].

The main advantage of QSS is the possibility of assessing any load by applying new time signals in (15), once the unit loads are already known from an initial Finite-Elements (FE) analysis.

### 2.2.2 Modal Superposition

The modal superposition is a time-based model used whenever the derivative terms in the general equation of motion (11) can not be neglected because of the vibrational motion of the system. Given an ideal structure, with very small internal damping and with no external forces applied, substituting the solution \( u(x, t) \) with \( \phi(x)e^{i\omega t} \), equation (11) can be rewritten as

\[
-\omega^2 M \phi + K \phi = 0
\]  

and

\[
\det(K - \omega^2 M) = 0
\]

are the solutions of the systems, also called normal modes. Each of these modes can be related to a specific frequency \( f \) corresponding to the eigenvalue \( \omega^2 \) so that \( 2\pi f = \omega \). In case of an unconstraint system, the first six modes are related to its degrees of freedom (DOF) in the space and their corresponding frequencies are zero, otherwise constrained normal modes are defined and they are usually numbered for increasing frequency so that \( f_1 < f_2 < f_3 \ldots \) and ordered in the modal matrix \( \phi = [\phi_1, \phi_2, \ldots, \phi_m] \).

Given an arbitrary deformation of the component \( u(x) \), this is expressible as linear combinations of all of the modes so that \( u = \phi \cdot q \). In practice this is usually difficult to achieve due to the great number of DOF (and thus modes) of the system, therefore only a subset of modes is taken into account and the deformation \( u(t) \) can be approximated as:

\[
u(x, t) = \sum_{i=1}^{m} \phi_i(x) \cdot q_i(t)
\]

for a time dependent deformation of the component.
It is also worth noticing, that modes are orthogonal with respect to the mass and the stiffness matrix, thus by means of a mass normalisation:

\[
\phi^T \cdot M \cdot \phi = I \tag{19}
\]

\[
\phi^T \cdot K \cdot \phi = \Lambda \tag{20}
\]

where \(I\) stands for identity matrix and \(\Lambda\) for the diagonal matrix whose elements are the eigenvalues of the system \(\omega_i^2 \ [3]\).

In case an external load is applied to the system, the general equation of motion is written as:

\[
M \ddot{u} + Ku = L \tag{21}
\]

which combined with equation (20), gives

\[
\ddot{\tilde{q}} + \Lambda q = \phi^T L = \tilde{L} \tag{22}
\]

Eventually, since the damping matrix does not depend only on the material, but also on other components like bearings and bushings, and it is not diagonal, meaning that the computational effort to solve the equations is quite demanding, its definition is not taken into consideration in this report. For further investigation reference [11] is addressed.

### 2.2.3 Spectral method

The spectral method is a frequency-based approach that relies on the general equation of motion defined in equation (11) transformed in frequency domain:

\[
(-(2\pi f)^2 \tilde{M} + i2\pi f \tilde{D} + \tilde{K}) \cdot \tilde{u}(f) = \phi^T \tilde{L}(f) \tag{23}
\]

where

- \(\tilde{M} = \phi^T M \phi\) is the modal mass matrix
- \(\tilde{D} = \phi^T D \phi\) is the modal damping matrix
- \(\tilde{K} = \phi^T K \phi\) is the modal stiffness matrix

The external load acting on the system in not defined by means of its time signal, rather by its Power Spectral Density (PSD). The system’s transfer function is thus written as:

\[
T(f) = (-(2\pi f)^2 \tilde{M} + i2\pi f \tilde{D} + \tilde{K})^{-1}\phi^T \cdot l(f) \tag{24}
\]
where \( l(f) \) are the load cases vectors \([12]\), and the results of (23) is:

\[
\tilde{u}(f) = T(f) \cdot L(f)
\]

(25)

Eventually, because of the linear relation between load and local stress, this can be expressed as the result of:

\[
\tilde{\sigma}(f) = c \cdot \tilde{u}(f) = c \cdot T(f) \cdot L(f)
\]

(26)

### 2.2.4 Comparison of the methods for Fatigue Analysis

The main benefit concerning the spectral method is its low computational effort, since it allows to determine any external load applied to the system through only one FEM analysis \([12]\). Although every method provides accurate results when gaussian loads are applied to the system, in case of non-gaussian load the PSD misjudges its severity, leading to a load underestimation and thus to an overestimation of the component durability. This happens because PSD does not take into account statistical characteristics such as crest factor, kurtosis coefficient and most importantly phase information in the signal \([13]\) and it outputs only the damage value in the signal.

The Modal Superposition and the Quasi-Static Superposition share the computational cost as largest drawback. In fact, in the first method the more the modal shape numbers are, the more modal stresses need to be evaluated and superposed. It is also important noticing that a modal transient analysis has to be computed according to a proper time-step, in order to evaluate an accurate \( q_i(t) \), which implies choosing a sampling rate large enough with respect to the load signal, thus its computational effort can not be sensibly reduced without an unwanted loss of information. In the QSS the biggest effort is finding the system responses \( \sigma(s, t) \) in equation (3) from the given unit forces \( e_i \), which also strongly depend on sampling frequency.

This projects aims to minimise the computation effort derived from the time signal analysis defining the minimum amount of information, decreasing as much as possible matrices’ dimensions and thus measured signals needed, from which back calculate all of the required loads for a durability assessment.

### 2.2.5 Critical Plane Method

In order to evaluate the accumulated damage of a component in durability applications, an estimation of the stress cycles and an equivalent stress value are needed. Since the Von-Mises equation can not be applied in this situation because it would output only positive values of stress that would lead to a wrong cycle counting, the critical plane
method is thus used. At the surface of a component, stress can be considered to be a only bi-dimensional tensor, with two stress components $\sigma$ and a shear component $\tau$. Once the applied load generates a crack, this will have a certain orientation, therefore a critical plane normal to the surface can be defined and an equivalent stress can be expressed as:

$$\sigma_{eq} = \text{sign}(\sigma_n) \sqrt{\sigma_n^2 + \left(\frac{\sigma_a}{\tau_a}\right)^2 \cdot \tau_n^2}$$  \hspace{1cm} (27)$$

where

- $\sigma_n$ is the amplitude of the normal stress in the critical plane
- $\tau_n$ is the amplitude of the shear stress in the critical plane
- $\sigma_a$ is the alternating tension or compression strength
- $\tau_a$ is the alternating shear stress limit

In order to assess the orientation of the critical plane, the full stress history needs to be considered and $\sigma_{eq}$ is evaluated on several planes and only the most damaging is elected as critical [14].

\subsection{2.2.6 Pseudo damage}

The pseudo damage definition relies on the Wöhler curve, the rainflow cycle counting and the Palmgren-Miner rule, that defines damage as sum of all of the contributions of the load cycles:

$$D = \sum_j \frac{n_i}{N_i}$$  \hspace{1cm} (28)$$

where $M$ is the number of load levels, $n_i$ is the number of load cycles for the $i^{th}$ level and $N_i$ is the life predicted from the Wöhler curve. The damage $D$ reaches value 1 when the fracture occurs. This can be implemented with the Basquin’s equation (1), so that damage is expressed as

$$D = \frac{1}{\alpha} \sum_j S_i^{\beta}$$  \hspace{1cm} (29)$$

When comparing severity from several loads, the material coefficient $\alpha$ has got no relevance, thus it can be neglected. As a consequence pseudo damage dependency from the material is considered only inside the coefficient $\beta$, which is experimentally extrapolated, and pseudo damage is defined as

$$d = \sum_j S_i^{\beta}$$  \hspace{1cm} (30)$$
where \( i = 1, \ldots, M \) describes the range of all measurements and \( j = 1, \ldots, N \) represents the channel number. It is worth noticing that the lengths \( l_i \) of the measurements can differ, thus it is useful to define a normalised pseudo damage \( \tilde{d} \), as ratio of the original pseudo damage \( d \) to travelled distance, usually in kilometer.

Since pseudo damage is often difficult to interpret in a physical manner, it is usually translated in equivalent load amplitude or in equivalent mileage. A simple evaluation of equivalent mileage on a predefined road is given by

\[
M_{eq} = \frac{d}{d_{ref}}
\]

More complicated models for equivalent loads can be found in *A statistical approach to multi-input equivalent fatigue loads for the durability of automotive structures* [15].

### 2.2.7 Invariant loads estimation methods

In the last decade, thanks to the development of many software and higher computational capacity, a virtual proving ground (VPG) approach has been on the rise, since it does not need a real prototype of neither the car, nor its subsystems and allows to save a substantial amount of time to assess an accurate life prediction. Every VPG approach strongly relies on precise fatigue data of the component, often in Wöhler S-N curve form, and on robust algorithms for assessing the equivalent fatigue life under multiple-axle dynamic loads, since different methods provide quite different life previsions [2]. Although these methods are still in use because needed in any CAE approach validation, they do not allow to obtain any information in the early stage of production process, nor to use information from previous models to predict the next generation’s durability, despite similarities between the two vehicles under investigation. This is because fatigue analysis is based on internal responses of an external excitation and those are typical of a specific system, with no possibility of extend knowledge to a different one.

Therefore, in order to be able to use information from previous tests on future models, researchers shifted their attention from internal or variant loads, to external or invariant loads.

So far, three methods are found to evaluate invariant excitations from variant loads.

On one hand, MBS models and FE models rely on an accurate road and tyre knowledge, as for the interaction between them. As starting point for the so called Digital Road method, a full multi-body system (MBS) and its relative tyres are already defined. Assuming a specific driver’s behaviour on pre-built road profiles, the model can be excited and its load data can be consequently derived. The main issue related to this method comes from the tyre model, since its parameters are designed
to describe force transfer behaviour of the physical tyre rather than geometric and material characteristics, thus they can hardly be transferred between a model that differ in geometry (for example tyre sizes), despite their structure similarities.

FE models are much more capable than MBS models of capturing characteristics about tyre geometry, including advanced details (belt, rubber, carcass...), thus ideally they could be transferred from one to another model. However, for one model to be built accurately, material parameters are required and this point turns out to be relatively difficult to achieve because of many non-linear affecting factors, like temperature dependencies and hysteresis deterioration.

Finally, back-calculation of invariant method relies on assessing an invariant load acting on the new model based on an old model’s data. An example of this method is using an 'effective vertical road profile', evaluated from an old model and a relatively simple tyre model. It is possible to set this effective vertical road profile so that important target quantities are maintained and used to excite the new model, since any tyre influence is already taken into account in it [3].

2.2.8 Back-calculation of invariant load using Iterative Learning Control

Assuming \( x = x(t) \) defined as the time dependent vector of state variables, which in a MBS model is usually the vector including all displacements and velocities (potentially other variables may be included), and \( u = u(t) \) used as representation of the time dependent vector of input variables acting on the system, identified as driver’s input such as steering wheel angle, acceleration, braking, in order to simplify the road profile model, which actually is a complex non-linear contact constraint, only vertical excitation along a straight forward road at constant speed is considered. Neglecting driver input allows to define \( u(t) = \xi(t \cdot v) \) as input signals, where \( v \) is velocity; therefore \( \xi \) can be written in the form \( \xi(t \cdot v) = \xi(s) \), where \( s \) is the position on the road and \( \xi(s) \) is the vertical road profile. The general mathematical expression is of the form:

\[
F(x, \dot{x}, u, t) = 0
\] (32)


Variables of interest may not be only displacements and velocities, but also other quantities \( y \) that derive from \( x \), so that:

\[
y = g(x, \dot{x}, u, t)
\] (33)

The problem of forward simulation is to solve equation (32) with respect to \( x \) given the input \( u \) and then calculate \( y \) in a post-processing step. To shorten the formality, \( y \) is thus written directly as:

\[
y = S(u)
\] (34)
where the operator $S$ includes initial conditions, solves equation (32) and evaluates the response based on equation (33). Back-calculation consists in finding input $u$ that generates a predefined desired output response $y = y_{ref}$ after evaluating the inverse $S^{-1}$.

Unfortunately, since $S(u)$ is very often complex and non-linear when considering only observation of output $y$ given input $u$, a Newton-type procedure for linearisation is commonly used to face the calculation.

Firstly, a set of points $y_0 = S(u_0)$ is determined, then a white noise signal, which is defined for all of the frequencies in the spectrum with the same amplitude, is added to the set of points and its response evaluated:

$$y_{noise} = S(u_0 + u_{noise}) - y_0$$  \hspace{2cm} (35)

Consequently the linearised system $H = H(u_0)$ is evaluated from the response and its related input.

Finally, thanks to an iterative process, the corresponding input to a desired output $y_{ref}$ is found, so that the update tends to close the distance between the current output response $y_i$ and the desired (reference) output $y_{ref}$:

$$u_{i+1} = u_i + \Delta u_i$$  \hspace{2cm} (36)

$$H : \Delta u_i = y_{ref} - y_i$$  \hspace{2cm} (37)

For deeper insights about the iteration procedure and method comparison, *Guide to Load analysis* [3] is pointed as reference.

Femfat Lab Virtual Iteration (VI) was the software used in order to accomplish this iteration procedure. It computes relative damages of the iterated channels with respect to the measured (desired) signal, which could come from either real experiments or simulated through MBS: the software’s purpose is to recreate the desired signal as much as possible from the given input. More in detail, from a known MBS model, Femfat Lab VI creates a white noise signal, then it evaluates the output channel values and calculates the MBS transfer function and its inverse. The last step consists in creating a first drive signal as product of the desired (test-road) signal and the inverse transfer function and consequently iterating for several cycles in order to reduce the error in computing output signals. Eventually, by means of the rainflow counting, pseudo damages of both desired and iterated signal are assessed and compared to check: if two pseudo damages of one signal assume the same value, their relative pseudo damage is 1. From iterated pseudo damage Femfat Lab VI back calculates the desired input given to the system.
One particular mention is dedicated to the additional input, which will be treated in Section 4.3. These are measured signals used only in the iteration process, added to $u_n$ in figure 5, which shows the workflow followed in this part of the project, and are not used in the transfer function evaluation and help the iteration converge, increasing the constraint level in the model, imposing as reference measured forces.

\[ u_{n+1} = u_n + M^{-1}(y_{\text{desired}} - y_n) \]

Figure 5: Femfat Lab VI workflow
3 Multi-body model description

In this chapter the Adams/Car road models and vehicle models used for the research will be briefly explained, together with some information about Adams/Solver.

3.1 Input load case

A road vehicle is affected differently whenever subjected to different kinds of roads. Although knowing exactly what loads will affect the vehicle once it will be delivered to the final user is impossible, several road categories were standardised in ISO8606:2016 [7]. Therefore, sixteen road models were modelled according to it in Damage and equivalent load definition for durability of vehicle [10] and thus available for this project’s purposes. Roads can be classified as explained in figure 6:

Figure 6: Load classification

Cobblestone and roads A, B, C, D are examples of random loads, while pothole, bump, curb island, in-phase washboard, out-of-phase washboard are deterministic. More insight about their definitions, which strongly rely on the Power Spectrum Density concept, can be found in [16] and in [17]. Tables in figure 27 in Appendix A show road profiles at predefined vehicle velocities.

It is worth discussing also an important aspect in the definition of load given in the ISO8606:2016 [7]. As things stand today, loads are defined by means of their PSD. As stated in Section 2.2.4, this fact leads to inaccurate definitions though, as proved in figure 7, which depicts two PSDs of an in-phase washboard and an out-of-phase washboard in black and in red respectively and their summed pseudo damage from investigated channels plotted in figure 8, from Damage and equivalent load definition for durability of vehicle [10]. These figures show that load case 7 corresponding to the out-of-phase washboard is 1000 more damaging than load case 6 related to in-phase washboard, despite their very similar PSD. For this reason, all measurements from simulations should be analysed in time-domain, rather than frequency.

Since Femfat Lab VI creates a white noise signal and inputs it to the car model, two methods were investigated to achieve this purpose: a general actuation analysis and a four-post test rig analysis. Both methods can be used, but each of them requires its own expedient to work properly. In particular, the first allows the vehicle model to
be unconstrained, therefore the vehicle body position and orientation can change. The main drawback is that a vehicle rollover might occur when a force excitation is applied to the wheel spindle, but real wheel force transducers can be accurately reproduced. Moreover, actuators are already defined in Adams/Car models as force transducers and defining motion transducers may lead to constraint errors in the Full-Vehicle simulation later on.

On the other hand, test rig simulation actually constrains the vehicle position and orientation with respect to the rig actuators. As a consequence, it is not possible to accurately reproduce wheel force transducers (WFT), but the risk of rollover is extinguished, since the excitation is a displacement, rather than a force. For vehicle
body durability purposes, excitations of interest are the ones damaging the body, which implies only suspension loads to the body need to be investigated.

Therefore, for this project’s purposes, the four-post test rig was chosen for its readiness in setting up different vehicles because of the predefined wheel hub motion setting and the choice to input the external excitation beneath tyres or directly at wheel spindles.

As last note, when testing a vehicle on a post rig in real laboratory experiment, an example is shown in figure 9, this is constrained by the rig itself, which allows only small lateral movements. In Adams/Car, in order to avoid overconstraining of the car model, the connection joint between rig pad and wheel hub is an in-plane type and a bushing is placed between them, allowing small movements.

Figure 9: Durability investigation using a real test rig, courtesy of MTS Systems Corporation

3.2 Car model

In total, 5 different car models were assembled and driven on different roads in Adams/Car to reproduce invariant excitations.

The first model, named 'Template', was used as reference to develop the method. It consists of a front MacPherson and a rear twist-beam, whose suspension, bushing and tyre characteristics were unknown. Experimental road data of a rough D road had been measured on a real vehicle and excitation were input on the wheel spindle in the Adams/Car model.

A second model, called 'Compact', was built in order to be as similar as possible to the template model. Similar hardpoints were assigned to the suspensions and the body was defined using the same mass and inertia.

The third model, 'Rigid', was based on an Adams/Car template, properly tuned in bushings, springs and damper characteristics using values from a real car. This
time, since no real road data were available, wheel system were needed to drive the vehicle on a simulated road, thus a simple tyre model (PAC2002) was used.

From Rigid, another model was then developed in such a way that differed from the first in the following characteristics:

- Centre of gravity position
- Mass increased 15%
- Wheelbase increased by 150mm
- Spring rate increased by 10%
- Damper characteristics adjusted to the new spring rate

The last model is a flexible car model used in AVL. This could be divided into rigid subsystems which are powertrain (driving shaft, gearbox, differential), engine, steering system, and flexible subsystems such as body-in-white (BiW), torsion bar, front MacPherson suspension with flexible upright, subframe, tie rod and control arm, and rear twist-beam suspension. It is worth noting that all the components of both suspensions are flexible, including lower control arm, upper control arm, uprights and tie rods. These flexible components can be switched to a rigid setting, to save simulating time despite a lower simulation accuracy. More details on the flexibility setting can be found in *Adams User Manual* [18].

The reference system used to describe vehicle’s measurements and forces is depicted in figure 10

The next step involved setting up and applying user-defined input splines acting as both vertical wheel displacements and wheel forces. Then in order to gather the desired data from which the damage analysis would be assessed, proper requests were defined into the car model. Requests can be seen as sensors, whose list and position for each car model can be found in Appendix B. Rigid model request are listed below:

- Spring and damper displacement: 8 channels
- Spring and damper forces: 8 channels
- Damper velocities: 4 channels
- Vertical wheel centre acceleration: 4 channels
- Vertical accelerations of several positions of the body, according to the model: 12 channels
- Wheel force transducers (WFT): 24 channels
A particular note requires to be stated about their units: to help the iteration process, measurement magnitudes are required to be as close as possible to 1, normalising accelerations in g, forces in kN, torques in kNm. Moreover, it is worth noticing that no excitation in X and Y direction was considered. The reason is shown in figure 11, where the vertical force is sensibly larger than forces in other directions not only in magnitude, but also in terms of variance, which is one of the major factors affecting directly the damage counting. The torques at the top mount also were not taken in consideration because of their small values in all of the three axis, as reported in the same figure.

One last relevant consideration was assuming of main interest those signals affecting body durability and not the overall vehicle. This is useful to assess fatigue already in predesign phase, when engine and suspension systems might not be defined yet. This explains why excitation at the top mount were addressed with particular care, with respect to other forces acting on the body, such as subframe attachments to body in case of a MacPherson, or through lower control arms to body in case of a multilink suspension.

Once the car models were set, they were run on different roads and in four-post rig simulation and the investigated measurements were exported in a result file.
3.3 Adams/Car: Solver Considerations

After the car model was driven on the road model in Adams/Car, an analysis of the results have been made. Starting evaluating the deterministic loads, in order to easily understand the simulation and the different sources of error in the signals. Lots of attention was dedicated to the signal analysis because, an error in the signal would be widely amplified in magnitude due to the iteration process.

Adams/Solver takes advantage of a predictor-corrector solver that acts in four different phases [18]:

1. Predictor: The predictor relies on the previous values to estimate the following for the state vector $\dot{x}$. In case the simulation is only at the beginning, the solver sets small step sizes in order to build a starting set of data.

2. Corrector: A cost function $G(x, \dot{x}, t)$ calculates the equations representing external forces, reaction forces and user defined differential equations for position or position and velocity according to the formulation choice. Thanks to a modified Newton-Raphson algorithm the correct value is evaluated minimising the output.
of the cost function $G(x, \dot{x}, t)$. The predicted value is computed and the revised state vector for the following time step is evaluated.

3. Error check: In this phase, the solver evaluates the error value through the following inequality:

$$\Delta x < \frac{\text{error}}{1000} + \frac{\text{adaptivity}}{\Delta t}$$  \hspace{1cm} (38)

where adaptivity can be seen as a relaxing-error parameter and $\Delta t$ is the time step.

4. Preparation: The solver defines the time step size and the number of previous states used in the next prediction.

### 3.3.1 Integrator

A simulation is always to consider as a mathematical model that tries to represent the real world through a set of equations, which in some cases may not have an analytical solution, but only a numerical one. Adams/Car allows users to choose among different integrators in order to run a simulation, which are:

1. GSTIFF: Specifies that the Gear integrator is to be used for integrating the differential equations of motion.

2. WSTIFF: Specifies that the Wielenga stiff integrator is used for integrating the equations of motion.

3. HHT: Specifies that the Hilber-Hughes-Taylor integrator is used for integrating the equations of motions.

4. Newmark: Specifies that the Newmark integrator is used for integrating the equations of motion.

5. Hastiff: Specify that the Hiller Anantharaman stiff integrator is used for integrating the differential equations of motion.

In Adams/Car three types of formulations are implemented, named I3, SI2, SI1 and they affect the error control in the corrector phase. It is worth noticing that not every formulation is available for every integrator. I3 leads to faster solution, but the error control is assessed only on displacements of body equations, flex modes and state variables. SI1 for HASTIFF and SI2 control the error also on velocities of body equations; eventually SI1 for HHT and Newmark integrators takes into account also accelerations.
From equation (38), the maximum error setting has a clear meaning and helps to increase the simulation precision in the numerical solution, but also "hides" some drawbacks. For example, if solver’s maximum error is too small, some Adams/Car model might fail the simulation, which will send a Static Error back to the users, due to some instability. It also worth to notify how adaptability changes its weight in \( \tilde{x} \) prediction according to the time step size: whenever the time step size decreases, the more adaptability affects the prediction, becoming potentially even more relevant than the maximum error.

Another relevant parameter is Hmax, defined as the maximum allowed time interval between the simulated point and the following. On one hand, it is quite intuitive to grasp that the smaller this is, the more accurate the simulation is, on the other hand running time increases substantially, sometimes without being beneficial in terms of numerical precision of the solution.

### 3.3.2 Correction of errors

Neglecting statistical aspects, given a sampled signal the errors that could occur are offsets, drifts and spikes. It is called offset the error leading leading to a different neutral average value than the one expected. The drift makes the local signal average increasing or decreasing through the sampling window. Spikes occurs when a maximum or a minimum value is reached without any event affecting the signal in that moment and it is caused by a numerical approximation or a disturbing faction in a real experiment. There are some robust methods to solve offset and drift errors:

- Calculating the mean and subtracting it from the signal
- Fitting a smooth curve to the data (e.g. a linear function)

Once again, *Guide to Load analysis* [3] is addressed for further insights about correction of signal errors and signal analysis.
4 Results and discussion

In principle, the success of the iteration process depends on the load, the vehicle model and the selected channels used in the transfer function evaluation. Therefore, firstly it was decided to test the same vehicle using different transfer function on the same road. The only real measurements available were provided from Femfat Support. The measured channels were the following:

- Vertical wheel centre accelerations
- Vertical top mount accelerations (on the body)
- Axial tie rod forces
- Spring displacements
- Vertical damper forces
- Wheel force transducers (forces acting on the wheel hub in all directions)

Of these 42 channels, spring displacements and wheel centre accelerations were used to evaluate the transfer function (TF) and its inverse, which can be seen in figure 12:

![Figure 12: Zoom of inverse transfer function of the wheel centre acceleration channels](image)

From the plot it can be noticed that the transfer function was evaluated only in frequencies between 1Hz and 40Hz because the excitation was applied through a free movable test rig and very low frequency movement was suppressed because it does not affect significantly channel’s relative damage evaluation. Relative damage was defined as:

\[ d_{rel} = \frac{d_{iterated}}{d_{signal}} \]  

(39)
and its values are considered acceptable when included in the interval [0.6; 4] [12].

Since the system of equations describing the car model and its motion on the road are not linear, transfer function calculation alone could not achieve a satisfactory result and in this occasion it actually underestimated the damage caused by the signal: figure 13 highlights how spring displacement of the left front wheel directly derived from the drive signal and the transfer function (in red) differed from the measured deflection (in black) and the 10th iteration (in green).

Figure 13: Measured signal (black), signal back-calculated directly from transfer function (red) and after 10 iteration (green)

Pseudodamagewas consequently computed through the rainflow counting method and shown in figure 14, where a comparison between the relative damage values from the 1st and from the 10th iteration is highlighted. No value was in the acceptable range in the first iteration, therefore it could be inferred that the product of model’s transfer function and the desired signal alone did not allow an accurate signal reproduction because of the system’s non-linearities, thus the iteration procedure was actually needed to recreate the original signal and to reduce the amount of error in the signal back-calculation.

This means that damage information brought by the 10th iteration channels was similar to the real measured one in black. Once the iteration process was completed, results in figure 15 were obtained.

Using a transfer function evaluated with respect to spring displacements and wheel centre accelerations, it can be observed how relative damages converge asymptotically and their values were close to one for most channels. In particular, spring displacements
adopted in the transfer function showed a very good correlation because their values were close to one. This means that damage information brought by the iterated spring channels was almost the same to the measured. The second set of channels used for the transfer function were vertical wheel centre accelerations, which showed good results on the rear axle, but only acceptable on the front, since their value was around 0.7 and slightly underestimate the damage.

The other channels were considered only as model check, since they were used in neither the transfer function evaluation nor the iteration process.

It is worth noticing the low values of the tie rods, channels 17 and 18 in figure 15 relative damages, which were around 0.27 for the left front wheel and 0.39 for the right front wheel. This could be due to the fact that measurements and inputs were taken and iterated mainly in the Z-direction of the absolute reference system described in figure 10 and not enough information were expressed in the axial direction of the tie rod, which mainly enveloped along absolute Y-axis.

Furthermore, WFT around Y-axis were not shown because their iteration brought no meaning to the results, since wheels were free to move around the axis and torque around Y evaluation would overconstrain the model, giving unrealistic results. The same reason could be applied also to explain the vertical forces overestimation: the software evaluated the vertical wheel displacements from the output channels and they were strictly related to vertical forces. Nevertheless, in figure 15 it can be seen that WFTs could be reproduced in almost all directions.

An important conclusion can be drawn from this result: measuring only some channels of the all of whole set, 4 spring displacements and 4 vertical wheel centre accelerations used in the transfer function evaluation, this procedure allows to reproduce also other signals, such as vertical body accelerations. Thus, this result proves also
measuring all the 42 channels was unnecessary in the back-calculation of the complete range of damages. In fact, despite the use of only 8 channels out of the total set in the transfer function calculation, most of the relative damages successfully converged to 1. Therefore, only 8 channels needed to be measured to find damages caused by all the others, allowing a great reduction of sensors and data manipulation. Another consequence is that evaluation of structural damage of a component could be assessed not necessarily measuring loads acting directly on it, but also acting on different points because the relative damage is unitary. In other words, when body durability is investigated, measuring WFT assumes a superfluous meaning, since there could be preferable signals to be gathered, as long as their damage information remains the same.

Another counter-intuitive result is the usage of many channels for the transfer function evaluation does not necessarily ensure a better iteration. As proof of this,
Figure 16 remarks three damage sets: grey damages from a transfer function formed by only 4 spring displacement channels, blue damages from a transfer function evaluated with respect to 4 spring displacements and 4 wheel centre accelerations and orange damages from a transfer function of 4 spring displacements, 4 wheel centre accelerations and 4 damper forces. It is clear that, although the same channels were used for all of the cases, adding more channels lead to predict wrong relative damages, in this scenario even more severe than 40 times the measured ones, affecting heavily durability assessment of the related component. On the other hand, according to experience 4 channels could not always be enough to successfully iterate required relative damages, but this actually depends on the road and on the vehicle model run on it and no pattern was drawn. Relative damage values of the channels are shown in Appendix C.

![Figure 16: Relative damage comparison for required number of channels, rigid model on rough D road](image)

The usual sensor configuration on the vehicle was placing one sensor in every corner (4) of the vehicle, as common sense would suggest. Nonetheless, figure 17 shows a comparison of the Rigid model simulation with accelerometers placed only on one side of the body, whose signals were then used to evaluate the transfer function, a configuration where sensors were placed on the shock towers and a third where sensors were positioned at the bottom of the body, as displayed in figure 17. A figure with sensor positions is provided in Appendix B. Clearly the iteration of the one-side configuration lowers considerably relative damages accuracy compared to the other two. It is of interest the fact that the 'Right side' configuration showed different results from the 'Shock tower', although two of the sensors were placed on the same
A main conclusion can be drawn from these initial results: it is of primarily importance to know how many and which channels are actually required for accurately back calculating a signal.

### 4.1 Signal Investigation and Observability Method

From the main conclusion stated in the previous chapter, it arose consequently the necessity of investigating more thoroughly which channels allowed to back calculate the highest number of signals possible with the best accuracy, or in other words, which channels were essential to correctly define the model.

Therefore, two methods were adopted to detect channel quality and accuracy. The first was trivial, running as many simulations as possible, evaluating several sets of transfer functions from signal lists in Appendix A, where they were combined and iterated and eventually their relative damages compared. After running the same model on different roads, it could be inferred that using the effective channels from one scenario to another did not actually guarantee relative damages closer to 1, meaning that channel importance depends also on the load applied to the car model.
Due to the great number and types of measurements that might have relevance to compute vehicle model’s transfer function and thus to assess invariant loads acting on the vehicle, a theoretical approach was then investigated in order to reduce drastically simulation time and number of trials.

The Observability Method developed for this project relies on creating a Generalized Popov-Belevitch-Hautus Test of Observability [19], which allows to evaluate Observability of a linear dynamical system. Given an explicit discrete time-invariant state space system:

\[
\begin{align*}
  x(t+1) &= A \cdot x(t) + B \cdot u(t), \\
  y(t) &= C \cdot x(t) + D \cdot u(t)
\end{align*}
\]  

where A, B, C, D are linear state space matrices, \( y(t) \) is the output response from the noise input \( u(t) \) evaluated in chapter 2.2.8.

In control theory literature, observability is defined as a measure of how well internal states of a system can be inferred from its external outputs.

A workflow regarding the procedure is portrayed in figure 18:

Figure 18: Workflow for assessing observability of a system

State variables were defined in Adams/View according to investigated input and output of the system and the model was then linearised at one specific point. Then system’s state space matrices were imported in Matlab and its observability calculated by means of 4 steps [20]:

- Consider the initial pool of investigated sensors.
• Remove sensors for which response amplitudes does not reach a chosen threshold.

• Remove sensors that do not show a sufficient matching degree between simulated response and measured response.

• Keep sensors that maximise a certain observability metric based on the Popov-Belovich-Hautus Observability test.

This process was repeated for several points of the simulation, paying attention not to linearise where spikes occur. Although in single event tests, like pothole, bump, or in periodic tests, such as washboard road, this might not be essential, in case of random loads model linearisation at one point or another could greatly affect the Observability test. To tackle this issue, an average observability could be evaluated for different points of the simulation. The credit of this method goes also to the author of *Accelerated test for vehicle suspension durability based on vehicle dynamics using pseudo damage*[21].
4.2 Observability: Results and Discussion

The Observability Method was used to choose the best channels of the Rigid model. Selected channels obtained as result of a Matlab script were then used to compute the Transfer Function and the iteration process to back-calculate the invariant loads for different roads. The chart in figure 19 shows the comparison of relative damage values between an iteration process by means of two different Transfer Functions. Channels are displayed and numerated according to the table 6 in Appendix. The blue damage was evaluated thanks to experience, according to which spring displacement and vertical wheel centre acceleration channels usually output a good iteration, whilst the orange damage was based on the Observability method, which selected channels 3, 4, 7, 9, 12, 22, 25, 28.

![Figure 19: Pseudo damage comparison: in blue damages from a TF chosen by experience, in orange damages from a TF chosen with Observability method](image)

Although vertical wheel centre accelerations on the rear axle resulted to be larger than 1, the Observability Method showed good results, especially in channels reproducing damper forces and damper displacement, which are signals bringing damage to the body, along with the spring displacements. Another important feature where Observability method turned out to be effective was the convergence behaviour of its iteration. In fact, trends of relative damage highlighted a more regular behaviour in all of the channels under investigation, avoiding oscillations, as it can be seen in figure 20.

As a general conclusion it was drawn that the Observability Method was effective together with the assumption of choosing proper linearisation points. A table with
the pseudo damage values is shown in figure 21.

![Convergence comparison](image)

**Figure 20: Convergence comparison**

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![Relative damages](image)

**Figure 21: Relative damages: channels used in transfer function evaluation in bold**

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4.3 Invariant Excitation

Literally speaking, the only invariant excitation a vehicle is subjected to is the road, which is usually considered to be non-deformable with respect to the tyre (it would be a different situation in case of rail vehicles though), while any other excitation is internal to the system, thus it can not be treated as invariant, at least in principle. Figure 22 shows PSDs calculated from an out of phase washboard: in black the one computed from the contact patch assumed to be the road profile and in red the one from the wheel hub displacement. Two different areas of interest, where this particular tyre model reduced spindle displacement amplitude in the spectrum range of 14-40Hz filtering and dissipating part of the excitation, and below 14Hz, where hub displacement was higher than road’s.

![Power spectrum density](image)

Figure 22: PSD of contact patch (black) vs vertical wheel spindle displacement (red)

Figure 23 illustrates how hub displacement differed from road profile, proving that tyre really affected excitation acting on vehicle structure and thus vehicle durability itself. Therefore, although in theory considering the two excitations as the same would have lead to a reduction of internal validity of the project, for sake of simplicity of modelling and iteration process, these were considered the same and the goal of the process would be finding hub excitation, rather than the contact patch itself. In fact, although a simple tyre model could be considered, as in the approach in [22], where an hybrid excitation is seen as combination of tyre and road profile, durability assessment would still rely on a tyre model, which this method aimed not to depend on.

Therefore, invariant load application to different vehicle models was investigated in order to allow a durability prevision already in the pre-design phase of a newer car version. The starting point were old measurements from an older vehicle, whose model was supposed to be available and from which the newer was developed from, so that they could be used as input. If signals could accurately be back calculated through iteration in the newer version, then the same invariant load could be extrapolated and used for a complete fatigue evaluation.
For the Template model, only hardpoints and joints information were available, while suspension characteristics, bushings were unfortunately unknown, therefore the Compact model was developed to match as close as possible template’s dimensions and dynamic behaviour. Because of the differences between the models, WFTs were used to help the iteration converge and this was coherent with the fact that in a real scenario the two models would not be the same. Measurements already presented in section 3.2 were applied to the Compact model to reproduce relative damages and consequently wheel spindle invariant excitation. As shown in figure 24, only the front left damper force signal was successfully iterated out of 42 channels, while all the other channels were either too damaging or too light. As a consequence, invariant load signal could not be back calculated using this method.

The second model investigated, Rigid model, was then modified as explained in 3.2, changing suspension characteristics, masses and wheelbase, keeping the same multilink suspension configuration and old signals were targeted in the back calculation phase. Relative damages iterated are shown in figure 25, together with relative damages values. Channels are defined as in the older Rigid model and listed in Appendix B. Although not all of the channels showed unitary relative damage, the iteration process successfully iterated the excitation. In particular, channel values from 13 to 16 and 19, 20, 25, 26 are very close to 1, which correspond to damper velocities and spring displacements, as shown in figure 26, proving an accurate reproduction of those signals acting on the vehicle body and therefore the possibility of using them as internal loads for a body durability assessment of the newer model.
Figure 24: Compact model relative damage values from Template data

Figure 25: Newer rigid model relative damage values from Rigid model data

Figure 26: Newer model relative damage values

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</table>
5 Conclusions, Future Work and Limitations

Given the present difficulties in assessing vehicle durability through usual methods, mainly related to the fact that damaging excitation strongly depends on road, tyre and vehicle interaction, it was decided to consider invariant load wheel hub displacement. The main goals of this project were investigating which internal measurements allowed to reproduce an invariant excitation through back calculation, how its accuracy would affect the damage assessment and evaluate the possibility of using the same internal measurements to evaluate damage and consequently durability of a similar model, in order to tackle the necessity of creating a real prototype before assessing durability, thus anticipating it already in the pre-design phase. To achieve this, several multi-body models were created in Adams/Car defining the measurements under investigation as user-input requests and run on different predefined roads according to ISO8608:2016. Back-calculation of invariant loads were achieved by means of Femfat Lab VI, which evaluated the vehicle model transfer function with respect of user-selected channels as the ratio of the system response to a generated white noise, then used the internal measurements from multi-body simulation or from real testing to compute the road excitation. Eventually the iteration process allowed to reduce the error between the iterated signal and the measured signal whenever successful, so that iterated channels’ relative damages were as close as possible to the measured ones.

It was shown that the ISO8608:2016 [7] did not actually define with accuracy a load in time signal because PSD does not include statistical characteristics such as phase and kurtosis factor and this could lead to wrong damage assessments.

According to several simulations of different models driven on different roads the best channels that allowed the best transfer function were a combination of spring displacements and vertical wheel centre accelerations, damper forces and vertical wheel centre accelerations.

An Observability Method was developed for investigating the numerous combinations of possible channels. Compared to the channels selected by means of experience, convergence improved sensibly, while relative damages results were still in the acceptable range.

In spite of experimental road data availability, the back-calculation of invariant loads from a modified model was not achieved, due to the substantial difference between the two vehicle models. However, invariant loads were successfully reproduced changing rigid model suspension characteristics, mass and wheelbase.

Many assumptions were made in creating this method. Firstly, the lack of a validated multi-body model might lead to inaccurate simulation data, thus to inaccurate relative damages iterated during the back calculation of invariant excitation, despite the effort in setting the Adams/Solver. The ideal situation would include a real vehicle on which the investigated signals are measured when it is driven on an ISO predefined
road and its related verified multi-body model. Then, a newer car model would be
developed from the first and its invariant loads evaluated from real measurements.

Secondly and most importantly, only vertical excitations were considered. This
approach was decided mainly for two reasons: those are the most damaging for
the body structure, while lateral and longitudinal excitation affect more heavily the
suspension system; number of channels and thus investigation would arise by a factor
of 3, leading to a substantial increase of simulation time.

Another important aspect consisted in not considering real sensors accuracy.
In other words, a real sensor measures for example velocity through integration of
acceleration or derivation of displacement, or flow rate, thus the signal is not directly
related to its physical quantity, rather extrapolated from an easier-to-get measurement.
This fact lead to discrepancies with the measurements in multi-body environment
and all sensor’s characteristics such as accuracy, sensibility, response time were not
evaluated in this project, therefore the choice of the best channels might be affected
as well.

Although several roads were already defined, in general this set of proving
grounds can be quite different from the actual road a real car would withstand in real
application. A definition of a real road based on statistical research about roads in a
single country on which the vehicle drives would give a closer estimation of durability
in terms of mileage instead of damage, so that experimental and simulated data would
more easily be comparable.

Eventually, the Observability Method developed in this project and in [21]
could be investigated deeper, creating a linearisation algorithm able to maximise
channel observability and finding the proper number of linearisation points of the road
taken into consideration, in order to evaluate an average value of observability and
consequently choose which signals would bring the essential information needed for
the transfer function evaluation and its related iteration process.
References


[10] B.D. Lévai, Damage and equivalent load definition for durability of vehicle, Master’s Thesis, Linköping University, 2018


[13] A. Rissoan, Damage and Equivalent Load Definition for Mechanical Durability of Subframe, Master’s Thesis, ENSTA Bretagne, 2018


6 Appendix

Appendix A: Roads and simulation specifications
3D road profiles used for the multi-body simulations in Adams/Car are shown in figure 27

Figure 27: Road profiles
Car velocities, simulated distance and time in Adams/Car are described in figure 28

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Figure 28: Predefined velocities
Appendix B:
Template and Compact suspension configurations and their related investigated sensor locations and units are described in figure 29

Figure 29: Template and Compact’s sensors and their locations
Rigid model and newer model suspension configurations and their related investigated sensor locations and units are depicted in figure 30.

Figure 30: Rigid model and newer model sensors and their locations.
Flexible model suspension configuration and its related investigated sensor positions and units are shown in figure 31.

Figure 31: Flexible’s sensors and their locations

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Appendix C:
Comparison of rigid model relative damages for 8, 12 and 4 channel transfer functions on rough D road at their last iteration are illustrated in figure 32:

- Blue: 8-channel transfer function evaluated using spring displacements, wheel centre accelerations.
- Orange: 12-channel transfer function evaluated using spring displacements, wheel centre accelerations and damper forces.
- White: 4-channel transfer function evaluated using only spring displacements

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Figure 32: Comparison of rigid model relative damages calculated from