A Transient Model of WICSC and ISCAD Machines Based on Permeance Networks

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Abstract—Today, multiphase electric machinery is considered in automotive traction applications. Some multiphase machine topologies also enable the possibility to change the pole and/or phase number during operation, an option which can be of benefit thanks to the large speed range that an electric machine for traction applications typically operates with. In this paper, an analytical model of the current dynamics for the wound independently-controlled stator coils (WICSC) and the intelligent stator cage drive (ISCAD) machines is presented. The model, based on setting up a permeance network in the stator and rotor and a rotor-angle dependent connection between the two networks, allows for predicting current transients during pole and phase changes. Thus, it is suitable for the design of control methods exploiting both the multiphase nature of the machine as well as performing pole-number changes during operation. The presented model is validated by comparing results from a corresponding finite-element based model of a WICSC machine during a pole change.

Index Terms—Induction machine, multi-phase machine, permeance networks, phase changing, pole changing.

NOMENCLATURE

$\mathbf{C}$	Flux-coupling matrix
$i_{r,k'}$	k' th rotor-bar current
$i_{s,k}$	k th stator-coil current
L_a	Active machine length
n_s	Number of turns per stator slot
$\mathcal{P}$	Extended permeance matrix
$\mathcal{P}_{k'}^r$	k' th rotor permeance column vector
$\mathcal{P}_k^s$	k th stator permeance column vector
$\mathcal{P}_\lambda$	Permeance leakage matrix
Q_r	Number of rotor bars
Q_s	Number of stator slots
$\hat{r}$	Radial unit vector
r_r	Rotor radius
$r_{s,in}$	Inner stator radius
R_{bar}	Rotor-bar resistance
R_s	Coil resistance
T_e	Electromagnetic torque
$v_{conv,k}$	k th converter leg voltage with respect to dc-link midpoint
$v_{s,k}$	k th stator-coil voltage

δ	Air-gap length
θ_r	Rotor angle
θ_s	Stator angle
$\phi_{k'}^r$	k' th rotor-coil flux
ϕ_k^s	k th stator-coil flux
$\phi_{\delta,k'}^r$	k' th rotor-air gap flux
$\phi_{\delta,k}^s$	k th stator-air gap flux
μ_r	Relative permeability
ϕ	Coil flux vector
ϕ_δ	Extended air-gap flux vector
ω_m	Rotor angular speed

I. INTRODUCTION

Thanks to the possibility of using lower rated power semiconductors and a smaller dc-link capacitor, the ability to increase the torque density using current-harmonic injection, and the possibility of operation under certain fault conditions, multiphase drives are of interest also in automotive applications [1]. Some machine topologies also enable the possibility to change the pole and/or phase number during operation, an option which can be of benefit thanks to the large speed range that an electric machine for traction applications typically operates with. Examples of synchronous machines having the possibility of changing the number of poles include [2]–[4].

For asynchronous machinery, in the intelligent stator cage drive (ISCAD) machine, the currents in each stator slot (comprising of a single conductor) can be controlled freely under the constraint that the sum of the stator-slot currents must equal zero (a condition realized by the short-circuit ring mounted on one axial end of the stator and connected to each stator bar) [5]. When connecting each stator bar to a corresponding inverter-phase leg, the number of poles and phases can be changed during operation. In [6], the wound independently-controlled stator coils (WICSC) machine topology is presented which mainly differs from the ISCAD concept in that each stator slot contains a coil with n_s turns where the coil is wound in the stator slot and back again outside the stator. A star point is formed by connecting together each coil end on one of the axial ends.

Since both the number of poles and number of phases can be changed, both the ISCAD and WICSC machines are pole-phase modulation (PPM) induction machines [7]. In [9], a transient model of a PPM induction machine is presented where the winding layout allows for a certain number of different combinations of phases and poles. The mutual inductance between any two coils are determined by integrating products of winding functions. Another recent

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concept allowing for pole and phase changes in combination with a squirrel-cage rotor is presented in [10].

In order to fully exploit the possibilities and challenges with phase and pole changes, a controller should be designed such that changing between different phase and/or pole combinations can be realized fast and without unnecessary transients. In order to develop such a controller, a simple machine model that predicts the current dynamics in each stator slot and rotor bar is necessary, also during transient phases (such as during a phase and/or pole change). Since winding functions cannot directly be applied in the case of the ISCAD or WICSC machines, the approach in [8] is not applicable.

In this paper, an analytical model of the current dynamics for the WICSC and ISCAD machine topologies is presented. The model is based on setting up a permeance network in the stator and rotor and a rotor-angle dependent connection between the two networks. The model neglects effects such as magnetic saturation, the skin effect, and the impact of stator- and rotor-slotting effects, but it should be sufficient for studying controller behavior during, e.g., transient conditions. The presented analytical model is compared to corresponding results obtained using a finite-element (FEM) based model during a pole change.

The paper is outlined as follows. In Section II, the adopted model geometry is presented and expressions for the air-gap flux density due to the stator-slot and rotor-bar currents are presented. In Section III and Section IV, the resulting air-gap and coil fluxes are determined, respectively. The resulting currents from a voltage excitation are determined in Section V and how the torque can be computed is outlined in Section VI. A scheme for how the transient model can be implemented using forward Euler discretization is presented in Section VII. Finally, Section VIII presents a comparison between the presented model and corresponding results from a FEM-based model.

II. AIR-GAP FLUX DENSITY

The adopted geometry for the WICSC machine exemplified with $Q_s = 4$ stator slots (each containing n_s turns) and $Q_r = 4$ rotor bars is depicted in Fig. 1. To each rotor bar, a fictitious rotor bar carrying the opposite current but located close to $x = y = 0$ in Fig. 1 is associated. Since the sum of the rotor-bar currents must equal zero, the fictitious rotor bars do not contribute to the flux density.

Remark: For the ISCAD machine, $n_s = 1$ and the coil resistance R_s (see Section V) then corresponds to the stator bar resistance (if the voltage drop in the short-circuit ring on one axial end of the stator is neglected).

Remark: A transient model of conventional (wound) induction machines including winding distribution harmonics for both the stator and rotor is presented in [11].

A. Air-Gap Flux Density Due to Stator Currents Only

First, it is assumed that the relative permeability of the stator and rotor laminations tends to infinity ($\mu_r \rightarrow \infty$).

Fig. 1: Winding arrangement exemplified for $Q_s = Q_r = 4$. Each stator coil is wound with n_s turns. Note the four fictitious rotor bars located close to $x = y = 0$.

Further, assuming that the flux-density distribution in the air gap is radially oriented only, the field intensity in the air gap can be expressed as $\mathbf{H} = H_r(\theta_s)\hat{r}$. From Fig. 1 and using Ampère's law (neglecting displacement currents), $H_r(\theta_s)$ can be expressed as

$$H_r(\theta_s) = \frac{n_s}{\delta} \sum_{n=1}^k i_{s,n} + H_r(0^-), \quad (k-1)\frac{2\pi}{Q_s} \leq \theta_s < k\frac{2\pi}{Q_s} \quad (1)$$

where

$$k = 1, 2, \dots, Q_s. \quad (2)$$

From $\nabla \cdot \mathbf{B} = 0$, we further have

$$\frac{n_s}{\delta} \sum_{m=1}^{Q_s} \sum_{n=1}^m i_{s,n} + Q_s H_r(0^-) = 0. \quad (3)$$

By inserting (3) into (1), the flux density in the air gap $\mathbf{B} = B_r\hat{r}$ is found as

$$B_r(\theta_s) = \frac{\mu_0 n_s}{\delta} \underbrace{\left(\sum_{n=1}^k i_{s,n} - \frac{1}{Q_s} \sum_{m=1}^{Q_s} \sum_{n=1}^m i_{s,n} \right)}_{\triangleq B_k}, \quad (k-1)\frac{2\pi}{Q_s} \leq \theta_s < k\frac{2\pi}{Q_s} \quad (4)$$

where the (constant) air-gap flux density in the Q_s angular segments B_k has been defined.

B. Air-Gap Flux Density Due to Rotor Currents Only

In a similar way as described above, the flux density in the air gap due to rotor currents only can now be expressed

as

$$B_r(\theta_s) = \frac{\mu_0}{\delta} \underbrace{\left(\sum_{n=1}^{k'} i_{r,n} - \frac{1}{Q_r} \sum_{m=1}^{Q_r} \sum_{n=1}^m i_{r,n} \right)}_{\triangleq B_{k'}} \quad (5)$$

$$(k' - 1) \frac{2\pi}{Q_r} \leq \theta_s - \theta_r < k' \frac{2\pi}{Q_r} \quad (5)$$

where

$$k' = 1, 2, \dots, Q_r. \quad (6)$$

III. AIR-GAP FLUXES

First, we define the current vector $\mathbf{i}$ (having $Q_s + Q_r$ elements) as

$$\mathbf{i} \triangleq [i_{s,1} \ i_{s,2} \ \dots \ i_{s,Q_s} \ i_{r,1} \ i_{r,2} \ \dots \ i_{r,Q_r}]^T. \quad (7)$$

Using (4) and (5), the air-gap fluxes penetrating the stator core $\phi_{\delta,k}^s$ (see Fig. 2 (a)) can be expressed as

$$\phi_{\delta,k}^s = L_a r_{s,\text{in}} \int \frac{k 2\pi}{(k-1) 2\pi} \frac{Q_s}{Q_s} (B_k + B_{k'}) d\theta_s \quad (8)$$

where $r_{s,\text{in}}$ and L_a denote the inner stator radius and active length, respectively. By inserting (4) and (5) into (8), $\phi_{\delta,k}$ can be expressed as

$$\phi_{\delta,k}^s \triangleq \mathcal{P}_k^s \mathbf{i} \quad (9)$$

where the $Q_s + Q_r$ elements in the permeance row vector $\mathcal{P}_k^s$ (relating stator and rotor currents to the k th air-gap flux penetrating the stator core) are functions of the rotor position θ_r .

For certain angles θ_r , the derivative of $\mathcal{P}_k^s$ with respect to θ_r will change stepwise (since the air-gap flux density varies in steps with this modelling approach). To circumvent this stepwise change (potentially, risking to result in transients in the predicted currents), each of the $Q_s + Q_r$ elements of $\mathcal{P}_k^s$, here denoted $\mathcal{P}_k^s(\cdot)$, is approximated as the Fourier series

$$\mathcal{P}_k^s(\cdot) \approx D_0^s + 2 \sum_{n=1}^{N_{\text{harm}}^s} |D_n^s| \cos[n\theta_r + \arg(D_n^s)] \quad (10)$$

$$D_n^s = \frac{1}{2\pi} \int_0^{2\pi} \mathcal{P}_k^s(\cdot) e^{-jn\theta_r} d\theta_r \quad (11)$$

where N_{harm}^s is the number of harmonics to be taken into consideration (excluding the fundamental). With this assumption, the derivative of $\mathcal{P}_k^s(\cdot)$ will be defined for all angles θ_r .

Similarly, the air-gap fluxes penetrating the rotor core $\phi_{\delta,k'}^r$ (see Fig. 2 (b)) are found as

$$\phi_{\delta,k'}^r = L_a r_r \int_{\theta_r + \frac{Q_r}{Q_s}}^{\theta_r + \frac{k' 2\pi}{Q_s}} \frac{k' 2\pi}{(k' - 1) 2\pi} (B_k + B_{k'}) d\theta_s \triangleq \mathcal{P}_{k'}^r \mathbf{i} \quad (12)$$

where r_r is the rotor radius and the $Q_s + Q_r$ elements in the permeance column vector $\mathcal{P}_{k'}^r$ (relating stator and rotor currents to air-gap fluxes penetrating the rotor core) are functions of the rotor position θ_r . Each of the $Q_s + Q_r$ elements of $\mathcal{P}_{k'}^r$, denoted $\mathcal{P}_{k'}^r(\cdot)$, is also approximated as a Fourier series as

$$\mathcal{P}_{k'}^r(\cdot) \approx D_0^r + 2 \sum_{n=1}^{N_{\text{harm}}^r} |D_n^r| \cos[n\theta_r + \arg(D_n^r)] \quad (13)$$

$$D_n^r = \frac{1}{2\pi} \int_0^{2\pi} \mathcal{P}_{k'}^r(\cdot) e^{-jn\theta_r} d\theta_r \quad (14)$$

where N_{harm}^r is the number of harmonics to be taken into consideration.

Fig. 2: Air-gap and coil fluxes exemplified for $Q_s = Q_r = 4$.

Remark: When approximating $\mathcal{P}_k^s(\cdot)$ and $\mathcal{P}_{k'}^r(\cdot)$ using (10) and (13) (in order to determine $\mathcal{P}$ in (36)), the number of harmonics to take into consideration (N_{harm}^s and N_{harm}^r) must be selected. Here, it is recommended to select $N_{\text{harm}}^s = Q_s/2$ and $N_{\text{harm}}^r = Q_r/2$.

Remark: Using Matlab¹, the coefficients D_n^s and D_n^r are readily obtained using the `fft` command.

IV. COIL FLUXES

From Fig. 2 and using Kirchoff's current law, the relation between $\phi_{\delta,k}^s$ and ϕ_k^s and the relation between $\phi_{\delta,k'}^r$ and $\phi_{k'}^r$ can be expressed as

$$\phi_{\delta,k}^s = \begin{cases} \phi_k^s - \phi_{k+1}^s, & k < Q_s \\ \phi_k^s - \phi_1^s, & k = Q_s \end{cases} \quad (15)$$

$$\phi_{\delta,k'}^r = \begin{cases} \phi_{k'}^r - \phi_{k'+1}^r, & k' < Q_r \\ \phi_{k'}^r - \phi_1^r, & k' = Q_r. \end{cases} \quad (16)$$

Now, we define $\mathcal{R}_s$ as the reluctance in the stator lamination between two consecutive coils. Using Kirchoff's voltage law for the magnetic circuit represented by the red circle in Fig. 2 (a), we obtain

$$0 = \mathcal{R}_s \phi_1^s - n_s i_{s,1} + \mathcal{R}_s \phi_2^s - n_s i_{2,s} + \mathcal{R}_s \phi_3^s - n_s i_{3,s} + \mathcal{R}_s \phi_4^s - n_s i_{4,s}. \quad (17)$$

Generalized to Q_s number of coils, (17) can be expressed as

$$\mathcal{R}_s (\phi_1^s + \phi_2^s + \dots + \phi_{Q_s}^s) = n_s (i_{s,1} + i_{2,s} + \dots + i_{s,Q_s}). \quad (18)$$

¹Matlab is a registered trademark of The Mathworks Inc. Natick, MA, U.S.

In practice, the value of $\mathcal{R}_s$ is very low since a high-permeable stator lamination is assumed (in fact, it is assumed to be identically zero in Section II). However, if the coils are connected in a star arrangement, we have

$$i_{s,1} + i_{s,2} + \dots + i_{s,Q_s} = 0. \quad (19)$$

Hence, from (18) and (19), we obtain that

$$\phi_1^s + \phi_2^s + \dots + \phi_{Q_s}^s = 0 \quad (20)$$

which holds for any non zero value of $\mathcal{R}_s$. Using an analogous reasoning for the magnetic circuit in Fig. 2 (b), we obtain

$$\phi_1^r + \phi_2^r + \dots + \phi_{Q_r}^r = 0. \quad (21)$$

Eq. (15) and (16) together with the constraints given by (20) and (21) can be expressed in matrix form as

$$\phi_\delta = \mathbf{C}\phi. \quad (22)$$

Here, the *extended air-gap flux vector* ϕ_δ is defined as

$$\phi_\delta = [\phi_{\delta,1}^s \phi_{\delta,2}^s \dots \phi_{\delta,Q_s-1}^s \textcolor{red}{0} \phi_{\delta,1}^r \phi_{\delta,2}^r \dots \phi_{\delta,Q_r-1}^r \textcolor{red}{0}]^T \quad (23)$$

where the zeros marked in red represent the conditions (20) and (21). Also, the *non zero* elements in the *flux-coupling matrix* $\mathbf{C}$ representing the conditions given by (15) and (16) are found as

$$\mathbf{C}(i, i) = 1, \quad i = 1, 2, \dots, Q_s - 1 \quad (24)$$

$$\mathbf{C}(i, i+1) = -1, \quad i = 1, 2, \dots, Q_s - 1 \quad (25)$$

$$\mathbf{C}(i, i) = 1, \quad i = Q_s + 1, Q_s + 2, \dots, Q_s + Q_r - 1 \quad (26)$$

$$\mathbf{C}(i, i+1) = -1, \quad i = Q_s + 1, Q_s + 2, \dots, Q_s + Q_r - 1. \quad (27)$$

Further, the additional non zero elements in $\mathbf{C}$ representing the conditions given by (20) and (21) (marked in red) are found as

$$\mathbf{C}(Q_s, i) = \textcolor{red}{1}, \quad i = 1, 2, \dots, Q_s \quad (28)$$

$$\mathbf{C}(Q_s + Q_r, i) = \textcolor{red}{1}, \quad i = Q_s + 1, Q_s + 2, \dots, Q_s + Q_r. \quad (29)$$

Finally the *coil flux vector* ϕ is defined as

$$\phi = [\phi_1^s \phi_2^s \dots \phi_{Q_s}^s \phi_1^r \phi_2^r \dots \phi_{Q_r}^r]^T. \quad (30)$$

V. COIL AND CONVERTER VOLTAGES

From Faraday's law, the coil voltages $v_{s,k}$ and coil fluxes $\phi_{s,k}$ are related as

$$v_{s,k} = R_s i_{s,k} + n_s \frac{d\phi_k^s}{dt} \quad (31)$$

where R_s is the coil resistance. Similarly, for the rotor circuit, we have

$$0 = R_{\text{bar}} i_{r,k'} + \frac{d\phi_{k'}^r}{dt} \quad (32)$$

where R_{bar} is the rotor-bar resistance and the voltage drop in the short-circuit ring has been neglected. The resulting fluxes

given the excitation $v_{s,k}$ can now be determined from (31) and (32) as

$$\phi_k^s = \frac{1}{n_s} \int_{-\infty}^t (v_{s,k} - R_s i_{s,k}) d\tau \quad (33)$$

$$\phi_{k'}^r = \int_{-\infty}^t (-R_{\text{bar}} i_{r,k'}) d\tau. \quad (34)$$

A. From Converter Voltages to Coil Voltages

It is assumed that each of the Q_s coils are excited with a converter leg outputting the voltage $v_{\text{conv},k}$. It is straight forward to show (see, e.g., [6]) that the converter voltage $v_{\text{conv},k}$ and coil voltage $v_{s,k}$ are related as

$$v_{s,k} = v_{\text{conv},k} - \frac{\sum_{k=1}^{Q_s} v_{\text{conv},k}}{Q_s}. \quad (35)$$

B. Flux-Current Relationship

Given the fluxes ϕ_k^s and $\phi_{k'}^r$, the resulting currents $i_{s,k}$ and $i_{r,k'}$ need to be determined. Using (8) and (12), the total extended permeance matrix $\mathcal{P}$ is defined as

$$\mathcal{P} \triangleq \begin{bmatrix} \mathcal{P}_1^s \\ \mathcal{P}_2^s \\ \vdots \\ \mathcal{P}_{Q_s-1}^s \\ \textcolor{red}{1} \quad \textcolor{red}{1} \quad \dots \quad \textcolor{red}{1} & \textcolor{red}{0} \quad \textcolor{red}{0} \quad \dots \quad \textcolor{red}{0} \\ \textcolor{red}{Q_s \text{ ones}} & \textcolor{red}{Q_r \text{ zeros}} \\ \mathcal{P}_1^r \\ \mathcal{P}_2^r \\ \vdots \\ \mathcal{P}_{Q_r-1}^r \\ \textcolor{red}{0} \quad \textcolor{red}{0} \quad \dots \quad \textcolor{red}{0} & \textcolor{red}{1} \quad \textcolor{red}{1} \quad \dots \quad \textcolor{red}{1} \\ \textcolor{red}{Q_s \text{ zeros}} & \textcolor{red}{Q_r \text{ ones}} \end{bmatrix} \quad (36)$$

where the ones and zeros marked in red represent the conditions that the sums of the stator-coil currents and rotor-bar currents both equal zero. The vector ϕ_δ on the left-hand side of (22) can now be expressed as

$$\phi_\delta = \mathcal{P}\mathbf{i} \quad (37)$$

where the *permeance matrix* $\mathcal{P}$ needs to be updated for each rotor position θ_r . To approximately take into account stator- and rotor-slot leakage fields, $\mathcal{P}$ can be replaced with

$$\mathcal{P}' = \mathcal{P} - \mathcal{P}_\lambda \quad (38)$$

where the *non zero* elements in the *permeance leakage matrix* $\mathcal{P}_\lambda$ representing stator-slot and rotor-slot leakage fields are found as

$$\mathcal{P}_\lambda(i, i+1) = \mathcal{P}_\lambda^s, \quad i = 1, 2, \dots, Q_s - 1 \quad (39)$$

$$\mathcal{P}_\lambda(i, i) = -\mathcal{P}_\lambda^r, \quad i = Q_s + 1, Q_s + 2, \dots, Q_s + Q_r - 1. \quad (40)$$

Here, $\mathcal{P}_\lambda^s$ and $\mathcal{P}_\lambda^r$ represent the stator-slot leakage and rotor-slot leakage permeances, respectively. For a rectangular stator

slot with a width w_{slot}^s and height h_{slot}^s containing n_s turns, the leakage permeance can be approximated as (see, e.g., [12])

$$\mathcal{P}_\lambda^s \approx \frac{\mu_0 n_s L_a h_{\text{slot}}^s}{3w_{\text{slot}}^s} \quad (41)$$

where L_a is the active length of the machine. Similarly, for a rectangular rotor slot with a width w_{slot}^r and height h_{slot}^r , $\mathcal{P}_\lambda^r$ can be approximated as

$$\mathcal{P}_\lambda^r \approx \frac{\mu_0 L_a h_{\text{slot}}^r}{3w_{\text{slot}}^r}. \quad (42)$$

Using (22) and (37), the current vector $\mathbf{i}$ can now be solved for as

$$\mathbf{i} = (\mathcal{P}')^{-1} \mathbf{C}\phi. \quad (43)$$

VI. TORQUE COMPUTATION

Using Lorenz force law, the torque acting on the *rotor* can be determined as

$$T_e = -L_a r_{s,\text{in}} \sum_{k=1}^{Q_s} B_r \left(\theta_s = (k-1) \frac{2\pi}{Q_s} \right) i_{s,k}. \quad (44)$$

Alternatively, the energy stored in the air gap can first be approximated as

$$W_m \approx \frac{\delta L_a r_{s,\text{in}}}{2\mu_0} \int_0^{2\pi} B_r^2 d\theta_s. \quad (45)$$

Now, using the principle of energy conservation, the torque acting on the *rotor* can be determined as

$$T_e = n_s \sum_{k=1}^{Q_s} \frac{\partial \phi_k^s}{\partial \theta_r} i_{s,k} + \sum_{k'=1}^{Q_r} \frac{\partial \phi_{k'}^r}{\partial \theta_r} i_{r,k'} - \frac{\partial W_m}{\partial \theta_r}. \quad (46)$$

VII. A TRANSIENT MODEL

With a constant sample time T_s and forward Euler discretization, the transient model for the WICSC or ISCAD machine rotating at the fixed angular speed ω_m is updated as follows:

- 1) With the converter voltages as input, the corresponding coil voltages are computed using (35).
- 2) The stator and rotor currents at $t=t_n$ (i.e., $i_{s,k}(t_n)$ and $i_{r,k'}(t_n)$) are computed using (43).

Remark: Preferably, the matrix $\mathcal{P}'$ has been computed for different values of θ_r and stored in look-up tables which are read at each sample step using interpolation.

- 3) With the currents at time $t=t_n$ determined, the air-gap flux density $B_r(\theta_s)$ is computed using (4) and (5) for the angles

$$\theta_s = \frac{2\pi m}{N_s}, \quad m = 0, 1, \dots, N_s \quad (47)$$

where N_s represents the desired angular resolution. The developed torque at time $t=t_n$ is now computed using (44) as

$$T_e(t_n) = -n_s L_a r_{s,\text{in}} \sum_{k=1}^{Q_s} B_r \left(\theta_s = (k-1) \frac{2\pi}{Q_s} \right) i_{s,k}(t_n). \quad (48)$$

Alternatively, the energy stored in the air gap is first computed using (45) as

$$W_m(t_n) = \frac{\delta \pi L_a N_s r_{s,\text{in}}}{\mu_0} \sum_{m=0}^{N_s} \left[B_r \left(\frac{2\pi m}{N_s} \right) \right]^2. \quad (49)$$

Now, the torque acting on the rotor is computed as

$$T_e(t_n) = \frac{1}{\omega_m T_s} \left(n_s \sum_{k=1}^{Q_s} [\phi_k^s(t_n) - \phi_k^s(t_{n-1})] + \sum_{k'=1}^{Q_r} [\phi_{k'}^r(t_n) - \phi_{k'}^r(t_{n-1})] - W_m(t_n) + W_m(t_{n-1}) \right). \quad (50)$$

- 4) Using (33) and (34), $\phi_k^s(t_{n+1})$ and $\phi_{k'}^r(t_{n+1})$ are now updated as

$$\phi_k^s(t_{n+1}) = \phi_k^s(t_n) + \frac{T_s}{n_s} (v_{s,k}(t_n) - R_s i_{s,k}(t_n)) \quad (51)$$

$$\phi_{k'}^r(t_{n+1}) = \phi_{k'}^r(t_n) - T_s R_{\text{bar}} i_{r,k'}(t_n) \quad (52)$$

$$\theta_r(t_{n+1}) = \theta_r(t_n) + \omega_m T_s. \quad (53)$$

- 5) The algorithm returns to step 1.

VIII. MODEL EVALUATION

The model implementation described in Section VII is now evaluated using a set of Matlab scripts with the following parameters: $Q_s = 36$, $Q_r = 28$, $n_s = 40$, $L_a = 201$ mm, $\delta = 0.6$ mm, and $R_s = 215.7$ m Ω . The slot leakage fields are included in the analysis and approximated according using (41) and (42). The resistance of a single rotor bar is 55.1 $\mu\Omega$. However, to approximately take into account the impact of the resistive voltage drop over the short-circuit ring elements on each axial end of the rotor, the effective bar resistance is determined as

$$R_{\text{bar}} = 55.1 \mu\Omega + 2R_{\text{scr}} \left(\frac{Q_r}{\pi p} \right)^2 \quad (54)$$

where $R_{\text{scr}} = 1.08 \mu\Omega$ is the resistance of a short-circuit ring element between two rotor bars [12]. Note that since the expression for R_{bar} is a function of p , the effective bar resistance is only exact during a steady-state condition.

With $Q_s = 36$ and $Q_r = 28$, the (square) extended permeance matrix that needs to be inverted during each time step in (43) has $(Q_s + Q_r)^2 = 4096$ elements. However, using an ordinary desktop computer, the resulting simulation times are satisfactory with simulations times corresponding to a few seconds taking a minute or less to execute.

The corresponding FEM-based model is implemented using Comsol Multiphysics² and is depicted in Fig. 3. As can be seen in the figure, the rotor has closed slots (the dc

²Comsol Multiphysics is a registered trademark of COMSOL AB, Stockholm, Sweden.

resistance of a single rotor bar equals $55.1 \mu\Omega$ as in the analytical model). The resulting current levels are such that magnetic saturation is not pronounced i.e., the machine is operating mainly in the linear region of the stator and rotor-core lamination BH-curves (steel grade used: M270-50A).

Pole-changing transitions are simulated while the angular speed of the rotor ω_m is kept constant. Two cases are examined, with mechanical speeds $\omega_{m,1} = 0.95 \cdot 100\pi$ rad/s and $\omega_{m,2} = 0.92 \cdot 200\pi$ rad/s. The coil voltages $v_{s,k}$ are selected as

$$v_{s,k} = \hat{v} \cos\left(\frac{p\omega_s t}{2} - \frac{p(k-1)\pi}{Q_s}\right) \quad (55)$$

where $\hat{v}_1 = 25$ V, $\hat{v}_2 = 30$ V, $\omega_{s,1} = 100\pi$ rad/s and $\omega_{s,2} = 200\pi$ rad/s. The number of poles is changed from $p=2$ to $p=4$ and $p=6$ at $t=0.25$ s and $t=0.5$ s, respectively.

Fig. 3: FEM model of the WICSC machine and air-gap mesh detail.

Using both the analytical and numerical model, the stator-coil and rotor-bar current vectors are evaluated. For the selected stator coil and rotor bar highlighted in Fig. 4, the results of the two methods are presented in Fig 5. The comparison shows a good agreement between the analytical and the numerical model, rendering the analytical approach an accurate and time-efficient method for estimating the stator-coil and rotor-bar current vectors.

In addition, the comparison between the numerically calculated torque (obtained using Maxwell's stress tensor) and the torque obtained using (50) is shown in Fig. 6. As can be seen in the figure, the results follow similar trends in both examined cases and, thus, an acceptable agreement is achieved. Determining the torque using (48) yields significantly poorer agreement with the numerically calculated torque and the results are not reported here.

IX. CONCLUSION

In this paper, a transient model of WICSC and ISCAD machines based on deriving permeance networks of the stator

Fig. 4: Selected stator coil and rotor bar (marked in yellow) for comparison of the evaluated currents $i_{s,1}$ and $i_{r,1'}$ using the numerical and analytical method.

and rotor and how these two networks connect as function of the rotor angle is presented. The model is intended for evaluating control designs during transients including phase and/or pole changes. Discretization of the presented model enables fast simulations on ordinary desktop computers. Comparing the predicted stator-slot and rotor bar currents during pole change with corresponding results from a FEM-based model where the current levels were such that magnetic saturation is not pronounced yielded a satisfactory agreement.

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Fig. 6: Comparison of the analytically estimated via the principle of energy conservation and the numerically calculated torque T_e for (a) $\omega_{m,1} = 0.95 \cdot 100\pi$ rad/s and (b) $\omega_{m,1} = 0.92 \cdot 200\pi$ rad/s.

Fig. 5: Comparison of the analytically estimated and numerically calculated stator-coil and rotor-bar current $i_{s,1}$ and $i_{r,1'}$ respectively for (a) $\omega_{m,1} = 0.95 \cdot 100\pi$ rad/s and (b) $\omega_{m,1} = 0.92 \cdot 200\pi$ rad/s.

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