Analysis of wall-mounted hot-wire probes

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ADALBERTO PEREZ
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by

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Technical report from
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Abstract:
Flush-mounted cavity hot-wire probes have been around since two decades, but have typically not been applied as often compared to the traditional wall hot-wires mounted several wire diameters above the surface. While the latter suffer from heat conduction from the hot wire to the substrate in particular when used in air flows, the former is believed to significantly enhance the frequency response of the sensor. The recent work using a cavity hotwire by Gubian et al. (2019) came to the surprising conclusion that the magnitude of the fluctuating wall-shear stress $\tau_{w,rms}^{+}$ reaches an asymptotic value of 0.44 beyond the friction Reynolds number $Re_{+} \sim 600$. In an effort to explain this result, which is at odds with the majority of the literature, the present work combines direct numerical simulations (DNS) of a turbulent channel flow with a cavity modelled using the immersed boundary method, as well as an experimental replication of the study of Gubian et al. in a turbulent boundary layer to explain how the contradicting results could have been obtained. It is shown that the measurements of the mentioned study can be replicated qualitatively as a result of measurement problems. We will present why cavity hot-wire probes should neither be used for quantitative nor qualitative measurements of wall-bounded flows, and that several experimental shortcomings can interact to sometimes falsely yield seemingly correct results.

Descriptors:
Turbulent Boundary layer; Turbulent Channel Flow; Hot Wire Anemometer; Direct Numerical Simulations; Immersed Boundary Method.
Acknowledgments

Many thanks to Philipp Schlatter for the patience to read through my lengthy emails and for the constant clarification on Linux and how to make things work. I am very thankful for all the knowledge provided on how simulations should be done and tested.

Thanks to Ramón Pozuelo, who provided me with a base for the codes to run the simulations at the SNIC clusters. That would have taken me a very long time to figure out (If even).

Thanks to Ramis Örlü and Alex Alvisi for being understanding and having my back when I made everyone rush (you know when).

And more importantly, thanks to God and my parents. They have been my support for my whole life, and the thesis is not the exception. Without them, I wouldn’t be here at all. Adalberto Perez

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Thanks to Adalberto Perez for the collaboration. It was interesting to see how simulations and experiments merge in one single work.

Thanks to Yushi Murai for patiently introducing me to the magic craft of welding hot-wire probes and for checking on me throughout the whole process. Again, a lot of knowledge I could not find easily on books.

Thanks to André Weingärtner for teaching me how to solder hot-wire anemometers step-by-step (and for fixing all the probes I killed at the beginning of my journey).

Thanks to Antonio Segalini for all the important tips he gave me and for all the chats. I felt closer to home in a laboratory emptied by the increased remote working due to the ongoing pandemic.

Thanks to my parents for supporting me in this journey. Alex Alvisi

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tbody>
<tr>
<td>$U$</td>
<td>Mean quantity</td>
<td></td>
</tr>
<tr>
<td>$u_{\text{rms}}$</td>
<td>Fluctuating quantity</td>
<td></td>
</tr>
<tr>
<td>$u^+$</td>
<td>Viscous scaled quantity</td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>Spectral representation of term</td>
<td></td>
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### Roman letters

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<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>corr($f$, $g$)</td>
<td>Correlation between two signals</td>
<td>[-]</td>
</tr>
<tr>
<td>$C_x$</td>
<td>Streamwise length of the cavity</td>
<td>[mm]</td>
</tr>
<tr>
<td>$C_y$</td>
<td>Spanwise length of the cavity</td>
<td>[mm]</td>
</tr>
<tr>
<td>$C_z$</td>
<td>Wall-normal length (depth) of the cavity</td>
<td>[mm]</td>
</tr>
<tr>
<td>$D_{ij}^p$</td>
<td>Pressure transport of Reynolds Stress</td>
<td>[m$^2$/s$^3$]</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency</td>
<td>[1/s]</td>
</tr>
<tr>
<td>$F_N$</td>
<td>Nyquist wave number</td>
<td>[1/m]</td>
</tr>
<tr>
<td>$F_s$</td>
<td>Sampling wave number</td>
<td>[1/m]</td>
</tr>
<tr>
<td>$h$</td>
<td>Channel half-height</td>
<td>[m]</td>
</tr>
<tr>
<td>$n$</td>
<td>Index number related to discretized quantity</td>
<td>[-]</td>
</tr>
<tr>
<td>$N_x$, $N_y$, $N_z$</td>
<td>Number of grid points in the corresponding coordinate direction</td>
<td>[-]</td>
</tr>
<tr>
<td>$N_p$</td>
<td>Number of total grid points</td>
<td>[-]</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number</td>
<td>[-]</td>
</tr>
<tr>
<td>$Re_\theta$</td>
<td>Reynolds number based on momentum thickness and free-stream velocity</td>
<td>[-]</td>
</tr>
<tr>
<td>$Re_{\delta}$</td>
<td>Reynolds number based on boundary-layer height and free-stream velocity</td>
<td>[-]</td>
</tr>
<tr>
<td>$Re_{k}$</td>
<td>Reynolds number based on Kolmogorov length and velocity scales</td>
<td>[-]</td>
</tr>
<tr>
<td>$Re_{\theta}$</td>
<td>Reynolds number based on momentum thickness and free-stream velocity</td>
<td>[-]</td>
</tr>
<tr>
<td>$Re_h$</td>
<td>Reynolds number based on channel half height and wall velocity</td>
<td>[-]</td>
</tr>
<tr>
<td>$Re_{\tau}$</td>
<td>Reynolds number based on friction velocity</td>
<td>[-]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td></td>
</tr>
<tr>
<td>$Re_x$</td>
<td>Reynolds number based on distance from leading edge and free-stream velocity</td>
<td></td>
</tr>
<tr>
<td>$S_u$</td>
<td>Skewness factor</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>Time [s]</td>
<td></td>
</tr>
<tr>
<td>$u, v, w$</td>
<td>Streamwise, spanwise, wall-normal velocity component [m/s]</td>
<td></td>
</tr>
<tr>
<td>$U_{wall}$</td>
<td>Wall velocity [m/s]</td>
<td></td>
</tr>
<tr>
<td>$U_\infty$</td>
<td>Free-stream velocity [m/s]</td>
<td></td>
</tr>
<tr>
<td>$u_\tau$</td>
<td>Friction velocity [m/s]</td>
<td></td>
</tr>
<tr>
<td>$x, y, z$</td>
<td>Cartesian coordinates [m]</td>
<td></td>
</tr>
<tr>
<td>FS</td>
<td>&quot;Full Scale&quot; [-]</td>
<td></td>
</tr>
</tbody>
</table>
### Greek letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
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<tbody>
<tr>
<td>$\delta$</td>
<td>Boundary-layer thickness</td>
<td>[m]</td>
</tr>
<tr>
<td>$\delta^*$</td>
<td>Displacement thickness</td>
<td>[m]</td>
</tr>
<tr>
<td>$\delta_{99}$</td>
<td>Boundary-layer thickness (from wall to 99% of $U_\infty$)</td>
<td>[m]</td>
</tr>
<tr>
<td>$\Delta x$, $\Delta y$, $\Delta z$</td>
<td>Grid resolution</td>
<td>[m]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Inner-scaled wall-normal component for Couette flow</td>
<td>[m]</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Momentum thickness</td>
<td>[m]</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>von Kármán constant</td>
<td>[-]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Dynamic viscosity</td>
<td>[Pa s]</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
<td>[m$^2$/s]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Mass density</td>
<td>[kg/m$^3$]</td>
</tr>
<tr>
<td>$\tau_{wall}$</td>
<td>Shear stress at wall</td>
<td>[Pa]</td>
</tr>
</tbody>
</table>

### Indices: Subscripts and Superscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$n$</td>
<td>Discretized quantity</td>
</tr>
<tr>
<td>rms</td>
<td>Root mean squared quantity</td>
</tr>
<tr>
<td>tot</td>
<td>total</td>
</tr>
<tr>
<td>wall</td>
<td>Wall</td>
</tr>
<tr>
<td>$i$, $j$, $k$</td>
<td>Indices for spectral transformations</td>
</tr>
<tr>
<td>$xyz$</td>
<td>Cartesian components</td>
</tr>
<tr>
<td>$0$</td>
<td>Initial or specified quantity</td>
</tr>
<tr>
<td>$*$</td>
<td>Quantity that is inner-scaled</td>
</tr>
</tbody>
</table>

### Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transformation</td>
</tr>
<tr>
<td>DNS</td>
<td>Direct Numerical Simulation</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transformation</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transformation</td>
</tr>
<tr>
<td>KTH</td>
<td>KTH Royal Institute of Technology</td>
</tr>
<tr>
<td>LES</td>
<td>Large Eddy Simulation</td>
</tr>
</tbody>
</table>
PSD  Power Spectral Density
RANS  Reynolds Averaged Navier Stokes
ZPG TBL  Zero-Pressure Gradient Turbulent Boundary Layer
2D, 3D  Two-, three-dimensional

**Abbreviations**

e.g.  *Exempli gratia*; For example
i.e.  *Id est*; That is
et al.  *Et alia*; And others
CHAPTER 1

Introduction

Turbulence is a multi scale phenomenon where energy is introduced into the flow by larger scales and dissipated by viscous means in the smallest ones. In wall-bounded flows, in particular, viscosity becomes especially important close to walls, as it is the viscous effects that make the flow satisfy the no slip and impermeability boundary conditions imposed by the presence of external geometry constrains.

For most applications, it is this multiscale interaction where the biggest interest lies, as in the vicinity of the wall is where many important phenomena such as drag and heat transfer occur, and as such, a good understanding of the flow in this region is needed in order to make accurate predictions of its behaviour.

Over the years, many methods for the measurement of the flow characteristics in the immediate vicinity of the wall have emerged, each with its own limitation, and it is in the evaluation of one such method that the interest of this report lies.

1.1. Literature Review

A particular quantity defined exclusively in the immediate vicinity of the wall and of great importance in industry and the modelling of turbulence is the wall shear stress.

The straightforward importance of the mean wall shear stress is given by its direct connection to drag, and as stated by Örlü & Schlatter (2011), the instantaneous values of the wall shear stress can give information on the structure of the boundary layer in the vicinity of the wall and even the effects of large scale flow structures can be seen as modulations in the wall-shear stress signal.

According to Alfredsson et al. (1988), the normalized wall-shear stress fluctuations can be calculated by means of the expansion of the stream wise velocity, i.e.

\[
\frac{\tau_{w,\text{rms}}}{\tau_w} = \lim_{y \to 0} \frac{u_{\text{rms}}(y)}{U(y)}. \tag{1.1}
\]
where $\tau_{w,\text{rms}}^+\text{,}$ $u_{\text{rms}}$ as well as $U$ being the fluctuating and mean stream wise velocity, respectively.

The ideal way to measure the fluctuating wall-shear stress would be by obtaining the instantaneous force variation in an infinitesimal section of the wall, however this is not experimentally feasible when high spatial and temporal resolution needs to be achieved. It is for these cases where the expansion (1.1) is of great convenience to perform indirect measurements.

These quantities can be measured in different ways. Traditional methods involve more intrusive instruments such as hot-wire anemometers or less obstructive techniques such as Laser Doppler Velocimetry (LDV). The former presenting certain limitations due to the characteristics of the viscous sublayer such as its small thickness and slow velocities. Among the most noted difficulties are the following:

1. The probe itself changes the flow field and incorrect measurement are performed. This characteristic is also known as intrusivity and aerodynamic blockage.
2. Heat transfer from the probe to the neighboring regions can also produce incorrect measurements, this effect is particularly strong in air flows.
3. Temporal resolution effects for particle-based techniques as it need to be ensure that the tracer accurately follows the flow.
4. Spatial resolution effects produced by the use of finite length probes that have as an effect, the averaging or attenuation of the velocity fluctuations over the sensing element. Similar effects are seen in laser-based optical techniques.

The latter has been extensively investigated and many correction methods have been proposed as that put forward by Smits et al. (2011).

In spite of limitations, Alfredsson et al. was able to compile the results of experiments performed with different methods and came to the conclusion that $\tau_{w,\text{rms}}^+\sim 0.4$ for most cases.

Further studies of the behaviour of the wall-shear stress have been performed with Direct Numerical Simulations. The compilation by Örlü & Schlatter shows a weak but clear $Re$ dependence of the values of $\tau_{w,\text{rms}}^+$, such that:

$$\tau_{w,\text{rms}}^+ = 0.298 + 0.018 \ln Re\tau. \quad (1.2)$$

The behaviour represented by (1.2) is associated to the imprints of large-scale outer-layer structures in the wall shear-stress fluctuations. It is notable to mention that the trend is more evident in DNS than experiments, as the former suffer from a certain scatter which can, in most cases, be explained by the experiments suffering with measurement difficulties already exposed in this document.
1.2. MOTIVATION

A more recent study by Gubian et al. (2019) was performed using flush mounted hot wires to measure the wall shear-stress fluctuations at a narrow range of Reynolds numbers. Their finding was that $\tau^+_{\text{rms}}$ appears to reach an asymptotic value of 0.44 beyond $Re_\tau = 600$.

The mentioned study, possesses very interesting claims, among the most notorious ones are the following:

1. Statistical moments, PDF and power spectra are independent on Reynolds number after a threshold value.
2. The probe used in the experiments is free of temporal resolution effects.
3. Higher values than those previously recorded by other researchers are explained by stating that the probe of their experiments resolves the full range of wall shear stress fluctuations. Never recorded before.
4. The probe does not suffer from spatial resolution effects.

Some of these claims are supported by the design of the measurement procedure used in the experiments, which consist of a flush mounted hot-wire on top of a small cavity in the lower surface of the channel. It is stated that the cavity increases the frequency response of the probe by reducing the heat transfer effects from the substrate. Table 1.1 shows a summary of the inner scaled cavity sizes in the study.

<table>
<thead>
<tr>
<th>$Re_\tau$</th>
<th>$x^+$</th>
<th>$y^+$</th>
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<tbody>
<tr>
<td>232</td>
<td>7.7</td>
<td>0.8</td>
</tr>
<tr>
<td>336</td>
<td>11.2</td>
<td>1.1</td>
</tr>
<tr>
<td>449</td>
<td>14.9</td>
<td>1.5</td>
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<tr>
<td>486</td>
<td>16.2</td>
<td>1.6</td>
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<tr>
<td>564</td>
<td>18.8</td>
<td>1.9</td>
</tr>
<tr>
<td>677</td>
<td>22.5</td>
<td>2.3</td>
</tr>
<tr>
<td>788</td>
<td>26.3</td>
<td>2.6</td>
</tr>
<tr>
<td>895</td>
<td>29.9</td>
<td>3</td>
</tr>
<tr>
<td>954</td>
<td>31.8</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Table 1.1. Cavity size in the study by Gubian et al. (2019)

1.2. Motivation

Considering the previous statements, it is very important to possess methods that accurately measure the variables associated to $\tau^+_{\text{rms}}$. The effectiveness of such experimental practices is usually tested against the findings of Alfredsson et al. (1988) among others, i.e. $\tau^+_{\text{rms}} \approx 0.4$. However due to recent works, contradictions in the value $\tau^+_{\text{rms}}$ and its dependence on the $Re_\tau$ behaviour have arisen. It is for this reason that the current project is undertaken.
1. INTRODUCTION

While Gubian et al. (2019) have stated the beneficial effects for the data acquisition when using cavities for the measurement of wall shear stress fluctuations, the inherent effect of the cavity on the flow was not investigated. Direct numerical simulations and experiments are used to evaluate how the flow behaves with the presence of cavities and how it is measured with flush mounted probes. The main objectives of this analysis is to determine if the presence of the cavity itself or certain spatial resolution problems in the experiments have an effect in the reading of wall shear-stress fluctuations using flush mounted hot-wire probes.
CHAPTER 2

Theoretical Background

In this chapter the theory foundations of the present study are presented. For the simulations, the concepts discussed are limited to those needed to understand the methodology followed by the solver, however more information regarding methods used for the spectral approximations and solutions for systems of partial differential equations can be found in appendix A.

2.1. Velocity Vorticity formulation of the Naiver Stokes Equations

Direct Numerical Simulations in this project are done by means of the KTH developed code Simson. As expected, the main problem to solve are the Navier Stokes equations with given boundary and initial conditions, however, due to numerical efficiency an alternative formulation of the problem is presented. In this section a brief explanation on the derivation of the equations - based on Chevalier et al. (2007) - is given.

The method used for the development of the code is the Velocity-Vorticity formulation of NS equations, which according to Speziale (1987) and Gatski (1991) have certain advantages over the solution in primitive variables, such as:

1. If a non-inertial frame of reference was to be chosen, all the non inertial effects enter the solution through the implementation of boundary and initial conditions, thus the structure of the problem is not changed. This gives the formulation a certain generality that is appealing.
2. No pressure equation is solved, which means that no pressure boundary condition need to be defined, which is an advantage as the definition of these values is more problematic than velocity and vorticity conditions.
3. More equations need to be solved (which mean more computational cost) however more information about the flow field is obtained in a direct manner.

It is important to note that some of the difficulties related to the numerical solution of Navier Stokes equations in incompressible flows is the fact that the pressure is not considered a thermodynamic quantity and can not be used in the equation of state of the fluid to relate it to density and temperature. In
these cases it is a quantity that establishes itself instantaneously such that the
velocity field remains divergence free (continuity equation remains valid), for
this reason the pressure term can be dealt with in many ways, always consid-
ering its function in the system. This is the reason that, as it was mentioned
before, boundary conditions for the pressure are somewhat more vague to fix.

2.1.1. Derivation of the Velocity-Vorticity formulation

To accomplish the derivation, the process begins from the non-dimensional
Navier Stokes Equations:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p_i}{\partial x_i} + \frac{1}{Re} \nabla^2 u_i + F_i,$$

$$\frac{\partial u_i}{\partial x_i} = 0.$$  \hspace{1cm} (2.1)

The next matrix relation is introduced in order to get a rotational inter-
pretation of NS equations:

$$u_j \frac{\partial u_i}{\partial x_j} = u_j \omega_k + \frac{\partial}{\partial x_i} \left( \frac{1}{2} u_j u_j \right).$$ \hspace{1cm} (2.2)

Introducing (2.3) into (2.1) and expressing the momentum equation in a
rotating frame of reference, only considering the Coriolis accelerati
$$\left( \frac{1}{2} u_j u_j \right)$$

$$H_i = -\epsilon_{ijk} u_j (\omega_k + 2\Omega_k) + F_i.$$ \hspace{1cm} (2.4)

Where $\epsilon_{ijk}$ is the permutation tensor, an operator with the next character-
istics:

\[
\begin{cases}
1 & \text{if } (i, j, k) = (1, 2, 3) \text{ or } (i, j, k) = (2, 3, 1) \text{ or } (i, j, k) = (3, 1, 2) \\
-1 & \text{if } (i, j, k) = (3, 2, 1) \text{ or } (i, j, k) = (1, 3, 2) \text{ or } (i, j, k) = (2, 1, 3) \\
0 & \text{Otherwise, } i = j \text{ or } j = k \text{ or } k = i
\end{cases}
\]

Up to this stage, the equations can be summarized as:

$$\frac{\partial u_i}{\partial t} = -\frac{\partial p_i}{\partial x_i} + H_i - \frac{\partial}{\partial x_i} \left( \frac{1}{2} u_j u_j \right) + \frac{1}{Re} \nabla^2 u_i,$$

$$\frac{\partial u_i}{\partial x_i} = 0,$$

$$H_i = -\epsilon_{ijk} u_j (\omega_k + 2\Omega_k) + F_i.$$ \hspace{1cm} (2.6)
2.1. VELOCITY VORTICITY FORMULATION OF THE NAIVER STOKES EQUATIONS

As it has been noted before, there is no wish to solve explicitly for pressure. For this purpose, a Poisson equation for the pressure can be found by taking the divergence of the momentum equation (first equation of (2.6)):

\[ \nabla^2 p_i = \frac{\partial H_i}{\partial x_i} - \nabla^2 \left( \frac{1}{2} u_j u_j \right). \]  
\( (2.7) \)

By applying the Laplace operator on both sides of (2.6) and introducing (2.7), a fourth order equation for the velocity can be found. For this formulation, there is only interest in the wall normal component:

\[ \frac{\partial}{\partial t} \nabla^2 v = h_v + \frac{1}{R_e} \nabla^4 v, \]  
\( (2.8) \)

\[ h_v = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) H_2 - \frac{\partial}{\partial y} \left( \frac{\partial H_1}{\partial x} + \frac{\partial H_3}{\partial z} \right). \]  
\( (2.9) \)

By applying the curl of (2.6), the vorticity transport equation can be found. The second component of such equation is:

\[ \frac{\partial}{\partial t} \omega = h_\omega + \frac{1}{R_e} \nabla^2 \omega, \]  
\( (2.10) \)

\[ h_\omega = \frac{\partial H_1}{\partial z} - \frac{\partial H_3}{\partial x}. \]  
\( (2.11) \)

Equations (2.8) and (2.10) represent the sought formulation. The goal is to solve the problem with appropriate boundary conditions to get \( v \) and \( \omega \). The other components of velocity are then gathered by using the continuity equation (2.2) and the definition of normal vorticity shown in equation (2.12).

\[ \omega = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}. \]  
\( (2.12) \)

After defining the main system of equations, the problem is closed by defining appropriate boundary conditions, which, for the channel flow are the impermeability and no slip conditions. Expressed mathematically they are given in equation (2.13).

\[ v|_{wall} = 0, \]
\[ \frac{\partial v}{\partial y}|_{wall} = 0, \]
\[ \omega|_{wall} = 0. \]  
\( (2.13) \)
2.2. Geometry generation: Immersed Boundary Method

To solve the Navier Stokes equations, the flow is usually discretized. Sections with no fluid, i.e. bodies or surfaces are not meshed, and boundary conditions such as no slip and impermeability are imposed in the interface between them and the fluids. However, there are other ways to introduce bodies into a fluid domain, one such method is explained in this section.

The immersed boundary method is described extensively by Goldstein et al. (1993). It is based in the fact that in an equilibrium and isothermal condition, the fluid perceives the presence of a body through the shear and pressure forces that exist along its surface. Keeping this in mind, it is possible to model the presence of a boundary condition if a correct set of forces that simulate the no-slip and non-penetration conditions are introduced to the numerical model.

The method is implemented in the current problem by means of the forcing term $F_i$ in equation (2.6), where an appropriate set of forces is added to the field at each time-step and calculated with the proportional integrator feedback control given by:

$$F_i(x, y, z, t) = \alpha u_i(x, y, z, t) + \beta \int_0^t u_i(x, y, z, t) dt.$$  \hspace{1cm} (2.14)

In this case the input to the control, i.e. the error function is the same as the velocity because in the type of body to be introduced no-slip and non-penetration is wished for, thus the target velocity is 0 (error = $u - 0$). $\alpha$ and $\beta$ are constants that need to be tuned and that could be seen as relaxation times of the method. Since the boundary condition is treated as a control problem, by performing certain simplifications it can be seen that $\beta$ represents the spring constant of the system, while $\alpha$ the damping ratio, for this reason the parameter $\beta$ must be chosen to be high enough such that it can correctly track and control the velocity fluctuations i.e the natural frequency of the system must be higher than the most energetic flow frequencies for an adequate control, however this value will also be limited by the numerical method stability and the numerical tools used to evaluate the integral in (2.14). For the present study, the integral is evaluated through a Riemann sum:

$$\int_0^t u_i(x, y, z, t) dt = \sum_{j=1}^N u_i(x, y, z, j) \Delta t.$$ \hspace{1cm} (2.15)

In general the scheme time step must be reduced as the constants of the control loop increase.

The analysis performed in this project is done with a pseudo spectral approach, which have the particular characteristic that the non linear term is evaluated in the physical space, as is further explained in section A.5. The forcing term is part of the non linear section of the equations to solve, and
as such it is evaluated on the dealiasing grid which means that the body created with the set of forces is correctly defined only in physical space (in the expanded grid) and depending on shape and resolution, it might be needed to introduce smoothing processes to correctly identify the body.

One particular difficulty of the IBM is that the control force field introduces a discontinuity in the physical space equations. Spectral methods possess a problem under these circumstances, named the Gibbs phenomenon, which is discussed and seen in figure A.1 and that consists on the introduction of nonphysical oscillations into the system. It is important to note that the oscillations produced by the method can be more or less pronounced depending on the gradient in the profile at the point of discontinuity. In figure 2.1 a representation of the approximation of a laminar flow, which posses a parabolic profile, is presented. It can be seen that the effect of the Gibbs phenomenon in this case is less pronounced than the one for a step function in A.1.

For turbulent flow, the gradients at the wall are known to be larger than in laminar flows. The bigger they get, the more inaccuracies are introduced to the spectral solution when using the immerse boundary method, which imposes a big limitation for the implementation. The oscillations can be eliminated by taking into consideration more frequencies which would reduce the energy content in the higher ones, however, if resolution becomes unpractical, other methods can be used.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{gibbs_phenomenon_laminar_flow.png}
\caption{Visualization of Gibbs phenomenon in a laminar flow. The black line is the actual parabolic profile. The different colored profiles represent spectral approximations with different number of retained frequencies.}
\end{figure}
2.2.1. Oscillation control

2.2.1a. **Smoothing: Low pass filter.** Spectral smoothing can be used for the control of oscillations by means of multiplying the coefficients of the term $H_i$ in (2.6) at every time step by:

$$e^{-\left(\frac{n_x}{N_x}\right)^{\lambda}} e^{-\left(\frac{n_y}{N_y}\right)^{\lambda}} e^{-\left(\frac{n_z}{N_z}\right)^{\lambda}},$$  \hspace{1cm} (2.16)

with $\lambda$ being a decay coefficient selected to have a sharp cut off of the highest modes, $(n_x, n_y, n_z)$ are the grid point indices in the $(x,y,z)$ directions and $(N_x, N_y, N_z)$ the total number of modes in said directions. With this type of approach, the energy content at the higher frequency is artificially reduced, thus it is important that the natural energy cascade doesn’t have a big amount of energy at said frequencies i.e. all scales have been solved, otherwise inaccuracies are introduced to the solutions.

2.2.1b. **Smoothing: Reversed force.** An interesting method is proposed by Goldstein et al. (1993). If a virtual flat plate is introduced to the model by means of the IBM, and its location is very close to one of the original boundaries (say, the lower one in a channel), it has been seen that the flow between the lower boundaries will approach zero, while that of the region between the virtual and upper wall follows the usual velocity profile of a channel.

Under these conditions, the discontinuity in the field is evident in the location of the immersed boundary: Just below of it, $\frac{\partial u}{\partial y}$ will be close to zero, as there is no flow, but on top of it, a turbulent channel flow is developed and as such, the gradient will be very high, hence a discontinuous profile.

For this case, the flow of interest is that located on top of the immersed boundary, and should not be modified. However, if the gradient below of it is artificially altered, such that there is a smooth transition between the two sections of the channel, the discontinuity should disappear and no oscillations are introduced into the model. This can be done by adding an additional forcing to the term $F_i$ in (2.6) which creates a reverse flow that, although does not affect the main area of interest, can create a smooth transition of the velocity profiles.

2.2.1c. **Smoothing: Diffuse Force application.** A simple way to smooth the gradient in the solution is to introduce the control force not only in the position where the immerse boundary would be, but keep it’s presence (with a reduced magnitude) in some additional grid points after the desired position of the surface. This will diffuse the results in the vicinity of the immune boundary, but it can significantly reduce the oscillations in the solution. One way to achieve this is to use a smooth step function, that for this study has the next form:

$$F_i(x, y, z, t) = F_i(x_s, y_s, z_s, t)e^{-\left(\frac{x-x_s}{\Delta x}\right)^2}. \hspace{1cm} (2.17)$$
In equation (2.17) only smoothing in the y direction is done, since it is in this orientation that the stronger discontinuity lies and \( (x_s, y_s, z_s) \) are the points where the surface is desired. This form of smoothing allows to have 100% of the control force in the desired point of application and the force will decay exponentially the farther away the grid points are from the point of interest.

2.3. Turbulence Characteristics

There is no unanimous and well-defined definition for turbulence or turbulent flows, however there is a set of properties which is common for such flows:

1. Fluctuating fields in space and time.
2. High dissipation, mixing capacity and diffusivity.
3. High Reynolds number.
4. Multi-scale phenomenon.

In particular, for wall-bounded flows, the characteristics of turbulence change due to the introduction of the no-slip condition at the solid walls. If compared to a laminar flow, the fluctuations of the turbulent flow tend to reduce the gradients in the velocity field but in the near-wall region, where the effect of viscosity becomes important as it forces the velocity to satisfy the boundary conditions. Here in this area the gradient is higher than that that a laminar flow would have. Wall turbulence can be said to be characterized by:

1. Two layers: the inner layer closer to the wall is dominated by viscous effects, while the outer layer towards the free-stream is dominated by turbulence.
2. Complex structures: the strong shear generates complex structures in the vicinity of the wall, such as streaks.
3. Anisotropic behaviour in the wall vicinity.
4. Energy distribution dominated by the velocity components parallel to the wall.

2.3.1. Inner and outer layers

The only physical parameter that enters the governing equations of the channel flow is the kinematic viscosity \( \nu \) through the Reynolds number \( Re = U_\infty \delta / \nu \). The wall boundary condition introduces the wall shear-stress \( \tau_w \) and the maximum size of the eddies in the flow is restricted by its characteristic length \( \delta \), therefore it can be said that these three physical parameters govern the wall bounded turbulent flow and from them, two characteristic length scales are constructed:

1. The kinematic viscosity \( \nu \).
2. The friction velocity:
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\[ u_\tau = \sqrt{\frac{\tau_w}{\rho}} = \sqrt{\nu \frac{\partial u}{\partial y}}. \]

3. The inner length scale: \( \ell^* = \nu / u_\tau \).
4. The outer length scale such as the boundary layer thickness \( \delta \) or the channel half-width \( h \).

2.3.1a. The inner layer. As it was mentioned, the inner layer is dominated by viscous effects and the flow is assumed not to be affected by the outer length scale (geometry constrain). In this order of ideas, a normalized wall distance is defined as:

\[ y' = \frac{y}{\ell^*} = \frac{yu_\tau}{\nu}. \] (2.18)

And the law of the wall:

\[ u^+ = \frac{u}{u_\tau} = \Phi_1(y^+), \] (2.19)

\[ -\frac{\overline{u'v'}}{u_\tau^2} = \Phi_2(y^+). \] (2.20)

2.3.1b. The outer layer. For this region, the viscous stresses are negligible compared to the turbulent effect, the appropriate normalized wall distance is defined as:

\[ Y = \frac{y}{\delta}, \] (2.21)

And the velocity defect law which considers a deviation from the free stream velocity:

\[ \frac{U_\infty - u}{u_\tau} = \Psi_1(Y), \] (2.22)

\[ -\frac{\overline{u'v'}}{u_\tau^2} = \Psi_2(Y), \] (2.23)

2.3.2. Overlap Region and Logarithmic Law

For high Reynolds numbers, it can be assumed that there are regions where:

\[ \ell^* << y << \delta. \] (2.24)

In other words, both laws apply at the same time. For this to be possible, their relative derivatives must be independent on length scale, hence constant. By imposing:
\[
\frac{y}{u} \frac{\partial u}{\partial y} = \text{const} = \frac{1}{k}.
\] (2.25)

Taking the definitions of \(u\) from (2.19) and (2.25) and the consideration in (2.25) the next is obtained:

\[
y^* \frac{d\Phi_1}{dy^*} = -Y \frac{d\Psi_1}{dY} = \frac{1}{k}.
\] (2.26)

Where \(k\) is known as the Von Kàrmàn constant and is an experimentally found quantity.

2.3.2a. Logarithmic law. In the overlap region, a linear behaviour (in a log scale) of the velocity with respect to the non dimensional wall distance can be found. This is known as the logarithmic law and comes directly from previously stated relations in (2.25):

\[
\Phi(y^*) = \frac{1}{k} \ln y^* + B,
\] (2.27)

\[
\Psi(Y) = -\frac{1}{k} \ln Y + C.
\] (2.28)

It has been observed that the larger the Reynolds number is, the larger the overlap region becomes.

2.4. Measuring Turbulence: Wind-Tunnel Facilities

2.4.1. Introduction

Wind tunnels are experimental facilities meant to observe and study the behaviour of fluids and their interaction with other fluids or solid surfaces. The first rudimentary wind tunnels were developed in the first half of the nineteenth century in an attempt to better understand how the air moves around airfoils, leading in the near future to the birth of modern aeronautics. As yet the design of wind tunnels made giant leaps and today the experimentalists can rely upon an extended spectrum of facilities that better suit their purposes.

2.4.2. Main components

The design parameters of a wind tunnel facility depend on the kind of experiments expected to be run and the available budget and space. For this reason every wind tunnel is different from another one, nonetheless it is possible to point out a recurrent architecture regardless the facility being open-loop or closed-loop.
2.4.2a. Test section. The test section is where everything happens, hence it must be carefully designed. The designer should take into consideration that the flow takes some space and time to settle, thus the test section must be long enough to get a fully developed flow pattern inside of it. A thumb rule is to avoid using the first 0.3-0.5 heights of the test section. The wake produced by the bodies and the instrumentation installed inside the section should shut before the divergent mounted at the bottom of the test section to avoid reducing the inlet section of the divergent, hence exposing the fluid to a higher expansion, thus higher head losses.

2.4.2b. Divergent and convergent. Divergents and convergents are installed to control the velocity of the fluid. This is done to set the desired quality of the flow inside the test section or to reduce the head losses in other components of the circuit.

2.4.2c. Honeycomb and nets. The honeycomb is a net-like structure with hexagonal mesh placed before the inlet of the test section to straighten the air flow. Unfortunately, the presence of the honeycomb produces low-damping vortices that might jeopardize the quality of the flow, hence nets placed in series are to be installed after the honeycomb to overcome this issue and to help making the flow more homogeneous.

2.5. Measuring Turbulence: Hot-Wire Anemometry

2.5.1. Introduction

Hot-wire anemometers are indirect local time-resolved velocity measurement instruments. Their versatility and fast frequency response, make them the most trustworthy option in the experimental research of fluid turbulence. Even today, thermal anemometers remain the best choice when it comes to validation for turbulence models or scaling laws, albeit more advanced tools have been developed. Historically this was the first instrument able to tackle the measurement of the turbulent fluctuations of the velocity field of moving fluids and it has been used for many decades so far, hence gaining a well-established reliability.

2.5.2. Working Principle

The working principle of thermal anemometers is based on their capability of detecting the change in heat-convection between the electrically heated wire and the fluid of interest, which is strongly related to the velocity field of the flow. The other types of heat transfer mechanisms are to be neglected for now being forced convection dominant, but it will be observed in the next sections that natural convection has to be accounted for when dealing with near-wall measurements, which happens to be the case in this study. In more detail:
Radiation losses can be neglected as they usually account for less than 0.1% of the convective losses (this can be explained recalling that the wire radiates only about 10% as much as a black body).

Wire-prongs heat conduction is small if compared to forced convection, but in general it is embodied in the calibration coefficients of the probe, rather than fully neglected.

Natural convection (buoyancy effects) can be neglected by the experimentalist in the majority of classical turbulence experiments. Attention in turbulent flows must be paid anywhere where in the near-wall region, where the velocity tends towards zero due to the no-slip condition.

A handy relationship between the forced convection and the Joule heating of the hot-wire can be obtained from the steady-state thermal balance equation between the fluid and the wire. Later, it can be further extended to the unsteady case to open up the discussion of the possible modes of operation of the probe.

The heating power of the wire is given by:

\[ P = IE = I^2 R_w = E^2 / R_w, \]  

where \( E \) is the voltage drop (V) across the wire, \( I \) is the current (A) passing through the wire and \( R_w \) is the resistance (Ω) of the wire.

On the other hand, the heat-flux linked to the forced convection reads as follows:

\[ \dot{Q} = h A_w (T_w - T) = h \pi D L (T_w - T), \]  

where \( h \) is the heat transfer coefficient (W/m²K), \( A_w = \pi D L \) is the heat exchange area (m²), \( T_w \) is the wire temperature (K) and \( T \) is the surrounding environment temperature (K).

The influence of the heat transfer mechanism can be merged in the Nusselt number, which dependencies take into account a crowd of fluid parameters:

\[ Nu = \frac{h D}{k_f} = f(Re, Pr, Ma, Gr, Kn, L, \frac{L}{D}, \alpha_T, \gamma, \theta, ...), \]  

where \( k_f \) is the thermal conductivity (W/mK) of the fluid.

It is relevant to examine each parameter on which the Nusselt number depends on to figure out which assumptions can be brought to the table to reduce the complexity of the problem.

The Reynolds number \( (Re = UD/\nu) \) is the index of how much the viscous and the inertial forces influence the motion of the fluid. Broadly speaking, the higher the Reynolds number the more the flow tends to a turbulent state. The present study highly depends on the Reynolds number.
2. THEORETICAL BACKGROUND

- The **Prandtl number** \( Pr = \frac{\nu}{a} \), where \( a \) is the thermal diffusivity, tells on which extent thermal diffusivity dominates on momentum diffusivity and vice-versa. Low values of \( Pr \) indicate that the behaviour of the fluid is dominated by thermal diffusivity.

- The **Mach number** \( Ma = \frac{U}{c} \), where \( c \) is the speed of sound, dictates if the flow is subsonic, transonic, supersonic or hypersonic. In this study the flow is clearly subsonic, thus compressibility effects can be safely neglected.

- The **Grashof number** \( Gr = g\beta \Delta T D^3/\nu^2 \), where \( g \) is the gravitational acceleration and \( \beta \) the thermal expansion coefficient, quantifies the buoyancy effects acting on the flow. Acceptable values of \( Re \) to neglect natural convection effects are higher than \( Gr^{1/3} \). Roughly, in this case \( Gr^{1/3} \approx 0.015 \) while \( Re \approx 2 \) (at the lowest free-stream velocity analyzed of about 5 m/s and based on the wire diameter of 5 \( \mu m \)), therefore natural convection can be neglected.

- The **Knudsen number** \( Kn = \frac{\lambda}{D} \), where \( \lambda \) is the mean free path between the fluid molecules, suggests whether the continuum hypothesis is valid or not in the current study. No experiment has been performed in vacuum conditions, hence \( \lambda \approx 70 \text{ nm} \) and the fluid can be considered as a continuum, being the smallest scale associated to the instrumentation of higher orders of magnitude.

- The **Aspect Ratio** \( L/D \) of the hot-wire is a useful value to understand if the problem should be treated as 3D or not. It also sets a reference value to limit possible averaging effects on the measurements. For tungsten wires it should be \( L/D \approx 200 \).

- The **Overheat Ratio** \( a_T = \frac{(T_w - T_0)}{T_0} \) is an important setting parameter for the operation of the probe. It enhances the responsiveness of the sensor when increased. It can be expressed also in terms of wire resistance: \( a_R = \frac{(R_w - R_0)}{R_0} \), where the subscript 0 always stands for the cold-state, i.e. the reference state, while the subscript \( w \) denotes the current value attained by the quantity of interest. Common values range between 0.70 and 0.80.

With all being said, \( Nu \) dependencies can be safely restricted to \( Re \) and \( a_T \) (or \( a_R \)) only, simplifying considerably the problem. Equation (2.30) can be multiplied and divided by \( k_f \) and then compared to (2.29) to establish the balancing equation:

\[
E^2/R_w = h\pi L k_f (T_w - T) Nu \tag{2.32}
\]

The Nusselt number equation under the mentioned assumptions can be written as:

\[
Nu = [A''(a_T) + B''(a_T)Re^n](1 + a_T/2)^m. \tag{2.33}
\]
2.5. MEASURING TURBULENCE: HOT-WIRE ANEMOMETRY

By replacing $Re$ with its definition, implementing Equation (2.33) into (2.32) and enclosing all case-dependent constants into the calibration constants $A'$ and $B'$:

$$E^2 = (A' + B'U^n)(T_w - T).$$

(2.34)

By including the thermal effects into the calibration constants, Equation (2.34) leads to the well-known King’s Law:

$$E^2 = A + BU^n.$$  
(2.35)

2.5.3. Modes of Operation

Equation (2.35) can be generalized to the unsteady case, recalling that:

$$\frac{dQ}{dt} = cm \frac{dT}{dt} = P(I, T) - W(U, T).$$  
(2.36)

Equation (2.36) opens up to the possible modes of operation of the hot-wire probe. Being (2.36) undetermined, one of the variables has to be kept constant to find an explicit expression.

2.5.3a. Constant Temperature Anemometry (CTA). The temperature of the wire, thus its resistance, is kept constant by a differential feedback amplifier which bypasses the thermal inertia of the system and relates the effect of the forced convection to the change in the current fed into the wire. In this case the second term of the right hand side of (2.36) cancels out and the dynamic balance equation reads the same as the static one.

2.5.3b. Constant Current Anemometry (CCA). In CCA mode the current passing through the wire is kept constant, hence a change in the cooling velocity is captured as a change in the wire resistance and so in the voltage between the ends of the wire.

2.5.4. Calibration

2.5.4a. Motivation. All the implicit and explicit assumptions made to simplify the relation governing the functioning of the thermal anemometer have to be taken into account in the calibration process. Being hot-wire anemometers quite sensitive to perturbations and as the flow inside the test section may change with respect to the purposes of the experiment, the probes need to be calibrated before every experimental session. Not even all the calibrations are the same and the steps may vary according to the type of hot-wire anemometer used as it will be observed in the present study, where two different kind of calibration procedure are used for the boundary layer probe and the cavity probes respectively. The calibration of the probe can be split into static and
dynamic calibration. Static calibration, in short, is the collection of the mean velocity and voltage pairs to be interpolated to find the calibration curve governing the working state of the anemometer. The dynamic calibration is done beforehand, right after the set up of the probe parameters, and it is meant to test the stability and responsiveness of the anemometer.

2.5.4b. Setting of the probe parameters and dynamic calibration. After connecting all the wires between the probe and the acquisition system (the A/D converter, the oscilloscope, the computers and the micromanometer), the total resistance \( R_t \) and the resistance of the support and the cables altogether \( R_{sc} \) is evaluated. From them the cold resistance of the wire is extracted and used to quantify the current resistance of the wire while functioning given the user defined overheat ratio \( a_R \). There is not a straight guideline regarding the setting of the overheat ratio, although a common reference value is somewhere between 70% and 80%. No substantial improvement is generally observed for higher overheat ratios in terms of responsiveness of the probe system and it is usually recommended not to exceed the threshold of 100% when experimenting in gaseous fluids as the wire may easily burn. However, higher values may be explored (being careful) if the wire requires to be pre-aged faster.

To test dynamically the anemometer, it should be put in a perturbed velocity field that spans the whole range of fluctuations that are expected to be encountered during the experiment. For instance, this can be done by means of ultrasounds. Unfortunately, as it may be expected, it is not always so easy to know a priori what that range will be, therefore other solutions should be explored. An option is to simulate the perturbations by shaking the wire while keeping unaltered the flow, but this turns out to be cumbersome to perform. A more handy and common solution indeed is to simulate the perturbed flow field by doing a square-wave test on the system and checking whether the probe rejects satisfactorily the disturbance or not. The A/D converter can be connected to an oscilloscope while performing the test to check if other sources of disturbance are affecting the outcome (for example electromagnetic and/or acoustic fields in the room). Unwanted mechanical oscillations must be avoided, hence the anemometer has to be sealed to the apparatus firmly and carefully and it should not vibrate due to aerodynamical oscillations when the wind tunnel is working. The cabling of the acquisition system has to be carefully unfolded as the inductance and the capacitance coming from the generating magnetic fields due to folding can affect the quality of the signal. Moreover, the cabling must be the same during the whole measurement session as its inherent properties are taken into account in the calibration coefficients found in the static calibration, meaning that for different tools the user would obtain different calibration curves.

2.5.4c. In-situ vs. ex-situ calibration. The calibration can be performed outside of the test section (ex-situ) or inside of it (in-situ). Usually the latter
is preferred, because the disturbances caused by the probe and its support and holder are the same in both the calibration and the measurement phases, hence the calibration coefficients will take into account the inherent characteristics of the flow. In-situ calibration is always possible when the flow is stable and homogeneous and it is highly recommended when performing experiments in a wind tunnel or a jet coming from a high contraction ratio nozzle, because the probe can be placed in the free stream or in the potential core respectively, which velocity profiles are theoretically known. The probe is always calibrated against a highly reliable velocity measurement instrument such as a Prandtl tube, which is located close to the probe. In case of the presence of a jet, the probe can also be calibrated against Bernoulli’s theorem if the contraction ratio is high enough to develop a suitable core region in the jet. If the mentioned conditions are not available, then ex-situ calibration is preferred. Ex-situ calibration consists in creating a known flow condition - such as a jet - with an external calibration facility, where the probes will be placed to be calibrated as described above before being placed again inside the measurement facility. It should be also pointed out that sometimes the need for in-situ calibration is so high that the user may want to accept the fact that the flow is not suitable for it, but proceed in any case in that way by taking the right precautions.

2.5.5. Spatial Averaging Correction

Hot-wire anemometers are generally judged to have good spatial and temporal resolution properties and this has been well confirmed through time, yet at relatively high Reynolds numbers and in the near-wall region the smallest scales dictated by the Kolmogorov scale $\eta$ make the measurements challenging. Being the wire finite in length, it is more correct to say that it senses an averaged value of the turbulent fluctuations $u(t)$, which can be expressed as follows:

$$u_m(t) = \frac{1}{L} \int_{0}^{L} u(s, t) dt,$$

where $s$ is the scalar coordinate along the wire direction and $u_m$ is the measured quantity.

The averaging effect is already substantially evident for wires 20 viscous lengths long (the error is about 10% of the turbulence intensity), hence very low $L+$ values are to be aimed for. A brief set of guidelines listed by Hutchins et al. (2009) is the following:

- Keep $L+ \leq 20$.
- Keep $L/D \geq 200$ (too small $L/D$ give a similar effect of too high $L+$).
- $\ell^* < 3$ should be resolved (hence the wire should be responsive enough, this is done by reducing the wire diameter $D$ and applying the suitable low-pass filter in the pre-setting phase).
A suitable correction scheme for the variance has been proposed by Smits et al. (2011) and it reads as follows:

$$u_c^{'2*} = u_m^{'2*} [1 + M(L^*) f(y^*)],$$  \hspace{1cm} (2.38)

where $M$ and $f$ describe the dependence of the variance to the spatial resolution and the wall distance respectively, with:

$$M(L^*) = \frac{A \tanh (\sigma_1 L^*) \tanh (\sigma_2 L^* - E)}{\max (u_m^{'2*})},$$  \hspace{1cm} (2.39)

and

$$f(y^*) = \frac{15 + \ln 2}{y^* + \ln (e^{(15-y^*)} + 1)},$$  \hspace{1cm} (2.40)

where $A = 6.13$, $E = -1.26 \times 10^{-2}$, $\sigma = 5.6 \times 10^{-2}$, $\sigma = 8.6 \times 10^{-3}$. 
Experimental Set-up

3.1. NT2011 Wind Tunnel

All the experiments were performed in the NT2011 wind tunnel at the Fluid Physics Laboratory of the Engineering Mechanics Department of KTH. Being an open-loop facility, the air is sucked by the inlet and released by the outlet in the same closed environment, hence some re-circulation is expected. Moreover, as the ambient conditions in the room are not controlled, it is important to record carefully the ambient temperature and pressure together with each experiment.

The flow - corrected by the honeycomb and the nets - is accelerated by the contraction at the beginning of the tunnel and liberated in the test section, which is 0.5 m high, 0.4 m wide and 1.4 m long. The 15 kW DC fan can pull the air up to about 20 m/s, a passable range of velocity if the user needs to perform experiments at low-subsonic regimes for research or educational purposes.

The test section can be modified according to the aim of the experiment, i.e. different kind of top, side and bottom walls are available. In this study a simple flat wall to hold the flat plate was installed at the bottom, the sides were made of plexiglass to let the user check inside the test section and the top was equipped with a wall with a track to give the traversing system one degree of freedom in the longitudinal direction of the section.

Two out of three parts of the experiment, namely the boundary layer characterisation on top of the flat plate and the measurements inside the cavity via the boundary layer probe, were done by installing the probe in the traversing system at the top of the test section. The calibration was performed by placing the anemometer hanged to the traversing system at the beginning of the test section in the free-stream velocity area close to the Prandtl tube. In this way matching values between the two probes are achieved and the hot-wire anemometer can be calibrated upon the Prandtl probe. On the other hand, the wall and the wall shear-stress measurements were performed at $x = 0.550$ m from the leading edge of the plate.
3.2. Flat plate

3.2.1. Before

A 4-legged 2cm-thick flat plate the length and the width of the test section is screwed on the bottom wall. A flat PVC insert is symmetrically centred at 0.550 m from the leading edge of the flat plate. The boundary layer anemometer is vertically moved closer to it to take the measurements required to establish the calibration curves for the wall shear-stress probes. The same insert will be cut to obtain the holes to hold the cavity probes and the cavities where the wall shear-stress will be measured.

3.2.2. After

The four cavities extruded in the insert of the plate are summed up in Table 3.1. At the centre of each cavity there is a round hole to hold the wall shear-stress probes. Ideally, the anemometers are expected to be placed in the hole so that the ceramics is flush-mounted to the base of the cavity and the wire to the plate insert surface. This also means that the protruding prongs of the cavity probes are expected to be long as much as the cavity depth.
3.2. FLAT PLATE

Figure 3.2. Close up of the divergent of the NT2011 wind tunnel from the outlet. At the centre, the 15kW DC fan in its holder.

<table>
<thead>
<tr>
<th>Code</th>
<th>Length $C_x [mm]$</th>
<th>Width $C_y [mm]$</th>
<th>Depth $C_z [mm]$</th>
<th>AR ($C_x/C_z$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B02</td>
<td>4</td>
<td>20</td>
<td>0.2</td>
<td>20</td>
</tr>
<tr>
<td>B04</td>
<td>4</td>
<td>20</td>
<td>0.4</td>
<td>10</td>
</tr>
<tr>
<td>S02</td>
<td>2</td>
<td>20</td>
<td>0.2</td>
<td>10</td>
</tr>
<tr>
<td>S04</td>
<td>2</td>
<td>20</td>
<td>0.4</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3.1. Dimensions of the cavities cut in the flat plate insert. The aspect ratio AR is defined as the ratio between the streamwise length $C_x$ and the depth $C_z$ of the cavity. This quantity will be employed in the last chapter in the conclusions of the study.

Figure 3.3. Flat plate insert before (left) and after (right) the re-design process. The air is expected to flow from right to left. The streamwise length of the upper surface of the insert is 100 mm. The spanwise length is 300 mm. See Figure 3.4 for a close-up of the cavities.
3. EXPERIMENTAL SET-UP

3.3. Instrumentation

The available instrumentation consists of:

- **Micrometer** – The micrometer is held by a sliding rack which moves along the longitudinal direction of the top wall of the test section. By rolling the micrometer the user can move up or down the stick where the probe is installed, hence it is possible to set its distance from the flat plate wall.

- **Thermometer** – The thermocouple thermometer is employed to measure the air temperature inside the test section by hanging the sensor in the free-stream velocity area. The displayed value is reported manually by the user into the software.

- **Barometer** – The barometer is used to evaluate the atmospheric pressure inside the laboratory. Being the wind tunnel open-looped, that equals the ambient pressure inside the test section.

- **Micromanometer** (Furness Control Ltd, FCO12 Model 3) – The differential pressure microtransducer is connected to the test section via plastic tubes and sends a reading in pressure difference directly to the LabVIEW software used to acquire the data. The pressure range goes from \(-199.9\) to \(199.9\) Pa when operated with \(10\%\Delta p\) resolution and from \(-1999\) to \(1999\) Pa when it is \(100\%\Delta p\). This corresponds to a velocity
range that goes between 0 and 56 m/s. The accuracy of the instrument is ±0.5% FS (±1 digit).

- **A/D Converter** (Dantec Dynamics StreamLine 90N10 Frame) – The hot-wire anemometers are connected to the A/D Converter. The analog signal is converted into a digital one and sent to the computers where it is processed.

- **Oscilloscope** (Tektronix 2225) A 50 MHz oscilloscope is kept connected to the system to check the behaviour of hot-wire probes while the wind tunnel is operated. This helps the user to check in real-time in which zone of the boundary layer flow the sensor is according to the oscillatory features. For example, at the edge of the boundary layer oscillations will start to perturb the flat signal, while very close to the wall the fluctuations will grow only upwards, in the direction of the free-stream, since they cannot go under the physical limit imposed by the presence of the flat plate itself.

Data acquisition and post-processing has been done with the following programs:

- **LabView** – All the acquisition software is made of three LabVIEW routines. One for the acquisition of the calibration points of the hot-wire anemometer, one for their preliminary interpolation and the last one for the main data acquisition. All the input quantities are user defined. The ambient conditions are inputted by the user in the program before starting.

- **Dantec StreamWare Pro** – The A/D Converter software is used to set-up the anemometers before their use. Namely, it is used to set the overheat ratio, the offset and the gain and to perform the square-wave test to check for dynamic stability of the system.

- **MATLAB** – MATLAB has been used to post-process the data and produce all the plots presented in the experimental sections of this work.

### 3.4. Probe Manufacturing

#### 3.4.1. Generalities

Hot-wire anemometers are usually commercially bought and in this case their repair is totally left to the customer service. Besides making the process more long-lasting and expensive, it is limiting for the experimentalist, who is left with a narrower tool choice with respect to what it is better for the experiment. The usage of an in-house hot-wire probe manufacturing and repair station is recommended to overcome this issue.

Generally speaking, a hot-wire anemometer is made of the sensing wire which is welded or soldered on top of two aerodynamically shaped prongs. To electrically insulate the prongs, they are placed inside ceramic tubes with the help of super or two-component glues or they are hold by an epoxy housing.
The probe may be placed in a robust steel supporting tube. Note that this kind of probe is usually multi-body, meaning that it is made of different components put together and that can be replaced according to the needs of the experiment.

### 3.4.2. Wire materials and geometry

Usually the wire is made of tungsten (W) or platinum (Pt) and its alloys, sometimes nickel and nickel-titanium alloys. Commonly, the diameter is of 2.5 µm or 5 µm but nowadays the available technology makes it possible to go even below those diameters. In particular, platinum wires are available in smaller diameters because they can be made by the Wollaston process, i.e. they are covered by a sheet of silver and then drawn to a smaller diameter. Those wires are packed and shipped with the silver coating still around the wire and it can be etched only as much as needed.

The sensing length of the wire is extended for several hundreds of diameters to reduce the effect of the conductive losses to the prongs. The habit when using tungsten wires is to keep an \( L/D \) ratio of at least 200. However, due to spatial resolution prerequisites the length of the wire \( L \) should be as small as possible. A possible path is to reduce the diameter of the wire, without forgetting that the finer the wire the more fragile and the more prone to drift it will result.

### 3.4.3. Soldering versus welding

The hot-wire probe prongs are important not only because they support the wire, but also because they unsettle the flow. Note that their contribution is more dominant than the one coming from the support of the probe system, hence they need to be properly manufactured. The diameter of the prongs usually stand between 0.2 and 0.5 mm and the spacing between the two tips is recommended to be at least 10 times their diameters. Sand paper can be used to taper the tips of the prongs, so that they perform better both aerodynamically and in terms of conduction quality.

What comes into play regarding this matter is how the wire is fastened to the prongs. It can be either soldered or spot-welded to them and this depends on the type of wire that is employed and/or the type of experiment that is performed.

In practical terms, the difference stands in how the connection is obtained. While spot-welding is done by joining the two elements by melting the contact surface between the wire and the prongs through the discharge of a capacitor by means of a silver or copper electrode, soldering is a process that joins two elements with the help of a soldering tin. The melted tin embeds the wire and the prong and creates a bond between them. Correct alignment is reached when the wire is normal to the probe axis.

Welding works well when the probe is immersed in a high stagnation temperature flow, where the tin would otherwise melt. It takes several hours of
practice to learn how to weld properly, but once it is mastered it is considerably faster than soldering as there is no waiting time in-between. To have decent control of the welding process observing and listening are essential. At the moment of the discharge, a good feedback for contact between the parts is a hollow tick sound together with a small spark and sometimes a tiny plume of smoke from the contact surface. High-quality welding is reached when the discharge is released at the exact point of contact between the electrode, the wire and the prongs. Given the small orders of magnitude in play, this may be cumbersome. Besides a favorable prong shape, some help can be found in enhancing the contrast between the components and the background observed at the microscope. Working in a dark environment and highlighting the wire and the prongs with a high-intensity illuminator is helpful, but in case this is not possible a common torch and a dark piece of paper underneath the components work well to obtain enough contrast. A small dull red spark when the discharge is released indicates proper welding. Impurities can be removed with acetone.

Figure 3.5. Welding close up of a boundary layer probe. With the help of a micromanipulator. The user first check for proper alignment of the prongs to the wire, then the same procedure is done for the electrode. When the wire is trapped by the contact between the electrode and the prong the discharge can be triggered. The contact should not be neither too tight - the wire would be cut - nor too loose to avoid voltage drops that may burn the wire. When this happens a bright spark bursts. When welding is complete on both prong tips the rest of the wire can be removed by pulling or cutting it.
According to the type of wire used, soldering can be considered as an alternative or the only choice available to make contact between the wire and the prongs, namely the latter path has to be followed when employing Wollaston wires, which are thin platinum wires clad in silver and which cannot be welded. Being more schematic, soldering is easier to master, but it involves more waiting because the wire must be etched with nitric acid ($H_2NO_3$) at first. At high concentrations (about 65%) this takes up to 15 minutes. The user has to pay attention when using acids, as spilling some inside the prongs holder will corrode them jeopardizing the proper conduction of electricity. Extra acid on the prongs can be removed with the help of acetone.
3.4. PROBE MANUFACTURING

Figure 3.6. Soldering station at the Fluid Physics Lab of the Mechanics Department of KTH, Stockholm, Sweden. The probe is hold tight by means of a vise. The user first applies some melted soldering tin on top of the prongs and then attaches the wire to one prong at a time. When the tin is cooled, the rest of the wire can be detached by pulling or cutting it. To spread and to make adhere better the tin to the prongs zinc chloride flux is used (Effekto 4).

Soldering is done with the help of soldering tin, which is first melted by means of a soldering iron and poured on top of the prongs tips. Then the wire is placed on top of them one prong at a time so that the tin engulfs it. After the tin cools down, the rest of the wire can be pulled away. Acetone can be used again to get rid of impurities such as small particles. To check if the circuit has been successfully closed, the user can look at the resistance of the circuit with the help of an ohmmeter and check whether it is below a hundred of Ohms or not depending on the wire diameter, length and material.
3. EXPERIMENTAL SET-UP

3.4.4. Aging and drift

The hot-wire probes need to undergo a process known as pre-aging before use. As a matter of fact, after soldering or welding, the brand-new wire properties are not stable, hence the probe needs to be operated for an extended period of time. There is not such a rule that specifies how long exactly the anemometer has to pre-age, but half a day is a reasonable amount of time. The overheat ratio should not be so high to burn the wire. For sake of comparison, in this study the overheat ratio has never exceeded $a_R = 1.1$. To know if the hot-wire is ready to go or not, one can check if its voltage at zero-velocity $E_0$ or its cold resistance $R_{w0}$ are constant in a fair amount of time. This entails the fact that this kind of probes will always age through time but at a different pace, therefore the voltage and the resistance acquired will not always be the same. This situation is called drift and it can be observed by acquiring two sets of calibration points at different times, for instance before and after the experiment. If the two curves overlap, then no drift is affecting the data. If this does not apply the acquired points are damaged and they should be taken again. Sometimes drift happens because the probe did not age enough, therefore more pre-aging is required to solve the issue, but some other times this is due to a faulty probe or to exceptional conditions inside the wind-tunnel. In the latter scenario the probe has to be repaired or replaced.

3.4.5. Wall shear-stress probes

The flush mounted hot-wire anemometers employed to evaluate the wall shear-stress have been developed and designed in-house. The design is inspired by the work of Spazzini et al. (1999), Khoo et al. (1996) and Khoo et al. (1999), who developed special hot-wire probes to perform near-wall measurements. The concept is very simple. The probe is made of a 2-mm ceramic cylinder case that holds two metal wires that act as electrical conductors. On one side of the case the metal wires protrude slightly to obtain two tiny prongs (about 0.2 mm and 0.4 mm depending on the cavity where the probe will be installed), while on the other about 1.5 cm, which serves as the connection to the insulated copper wire which links the hot-wire to the rest of the cabling system. The metal wire is sealed to the ceramic case pouring superglue both at the top and at the bottom sides. Since the probe has to be mounted by channelling it through the 2 mm cavity holes, the hot-wire must be firmly attached to the prongs to sustain a potential collision with the hole walls. This is done by welding it. On the other hand the other connections – such as those between the probe and the cables or the cables and the golden pins – can be soldered.
4.1. Motivation

The mean wall shear-stress is calibrated against the free-stream velocity at the streamwise location of the cavity. This can be further extended by using the Reynolds number $Re_x$ instead of $U_\infty$ only, which includes the ambient conditions via the kinematic viscosity $\nu$. The relationship found will stand for the calibration curve of the cavity probes.

4.2. Boundary-Layer Probe Measurements

4.2.1. Setting

As presented in the previous sections, the reading of an hot-wire anemometer depends on a wide set of phenomena whose effects are enclosed into the parameters of its calibration law. Being the free-stream in the test section stable enough, it is possible to proceed with the in-situ calibration of the probes. The data, as mentioned in the previous chapter, is acquired by means of three LabVIEW codes that help the user in collecting the calibration points, interpolate them and use the found rule to take the main measurements. The codes were already available as they were used for educational purposes within the Engineering Mechanics department of KTH. The built-in interpolation law is the King’s Law, but it will be changed in the post-processing phase with more suitable ones when dealing with cavity probe measurements. The ambient conditions are measured and inputted in the software by the user.

4.2.2. Calibration of the Boundary-Layer Probe

Before starting the main measurements, the boundary layer probe has to be calibrated according to a more reliable measurement instrument, which is a Prandtl tube in this case. The Prandtl probe is located at the test section inlet, it is aligned to the longitudinal axis of the flat plate at the leading edge and it is elevated to about half the height of the test section. The hot-wire is located right next to the Prandtl probe and it is maintained fixed in that position throughout all the calibration procedure, while the velocity is spanned from the lowest to the highest values that the wind tunnel can provide. The first voltage signal acquired is the one at rest condition $E_0$. It is followed by
the voltage reading at the highest possible speed and then the fan capacity is lowered gradually down to the zero-velocity again. The loss of coherence in the sequence of acquisition points at velocities close to zero is a red flag for natural convection dominance over forced convection.

The above mentioned backward procedure is handy to investigate in real-time if the calibration points converge to the one at quiescent condition sensed at the beginning. If this does not happen, the anemometer may be affected by drift. If the probe is severely affected by drift, this should come to light immediately by repeating the measurement at least twice. If the data points scatter downwards that may be the cause.

For higher accuracy at lower speeds, the calibration process is split into two stages. The first batch of points is acquired at "high speeds", which in this case it applies to all the velocity points higher than 10 m/s. In this part the micromanometer resolution is set to 100% of the pressure difference \( \Delta p \). At velocities below 10 m/s the resolution is set to 10%\( \Delta p \), so that significant figures are not lost. The measurements can be repeated and overlapped to the former ones to double check for the presence of drift. This second acquisition sequence is merely qualitative, hence it can be quicker, meaning that there is no need for splitting the process into two, and the data points can be sparser than those of the main procedure. The experimentalist should aim for the overlapping of the curves as displayed in Figure 4.1.
4.2.3. Measurements

4.2.3a. Setting. After the calibration, the probe is aligned with the position where the flush-mounted probes will be located, in this study 0.550 m behind the leading edge of the flat plate. The boundary layer is expected to be 2D, hence it is not required to repeat the measurements at different spanwise locations and the evaluation of the boundary layer characteristics at the x position on the longitudinal center line of the flat plate is enough satisfactory.

In contrast with the calibration phase, the measurements are carried out at fixed velocity and variable height from the wall y. This is done with the help of the user-controlled micrometer that sets the height of the instrument from a reference location, in this case the wall. The height value is then reported by the user directly into the LabVIEW VI. The velocities explored are in a range that goes between 5 and 23 m/s.

4.2.3b. Complications near the Wall. Turbulent boundary layers are characterised by a steep velocity profile close to the wall (approximately from 40% U_∞ and down), hence higher velocity gradients are present in this area, which is of interest when dealing with the evaluation of the wall shear-stress. From a practical point of view, this translates into the need of a narrower data point mesh close to the wall. Thus, extra care must be payed by the experimentalist when taking measurements here, as the slightest change in step-size can lead to substantial changes in the streamwise velocity reading. On the other hand, technical difficulties rise when approaching the hot-wire to the wall. Lower involuntarily the micrometer just a couple of extra micrometers may lead to the wire to touch the wall and to break it. Given such a tiny order of magnitude, undesired or unexpected vibrations in the wind tunnel may let oscillate the probe enough to make it touch the wall and damage it. With all being said, the user should aim anyway for a well populated velocity profile especially in the near wall region. A safe reference value where to stop lowering the probe can be at 40% U_∞. Positions below this value are possible, but riskier.

4.2.3c. Estimation of the Wall Position. The accurate estimation of the friction velocity is an important task being it the scaling parameter used to convert the physical quantities in inner units. On top of that, the exact location of the hot-wire from the wall is often not known a priori or with adequate accuracy. As a matter of fact, the probe holder and the traversing system bend due to the aerodynamic forces developing when the wind tunnel is functioning, which is also the condition when the distance of the hot-wire from the wall should be measured. This may be cumbersome to perform, therefore an indirect method based on an iterative procedure is preferred and carried out while post-processing the data. It consists of defining a constrained minimization problem for a residual function that takes into account both the friction velocity $u_\tau$ and
the offset \( y_0 \). The function to be minimized is made of two terms, one referred to the available experimental points, the other obtained from a TBL profile model. In this study the model proposed by Chauhan et al. (2009) has been used and it is displayed in Equation 4.1. The model is composite, meaning that it comes from the combination of an inner and an outer velocity profile model. They have inside the log-law as a common part and are defined through a wake function \( \omega \) and a wake parameter \( \Pi \).

\[
U_{\text{comp}}^+ = U_{\text{inner}}^+ + \frac{2\Pi}{\kappa} \omega \left( \frac{y^+}{\delta^+} \right), \tag{4.1}
\]

where \( \Pi = 0.4 \) is the wake parameter and \( \omega \) the wake function.

The velocity points from the experiment \( U(y) \) are normalized in wall units \( U^+(y^+) \) and compared to the composite model in Equation 4.1. The comparison is done via the residual function in Equation 4.2, which is minimized to check for convergence between the experimental points and the expected profile.

\[
f = \sum_{i=0}^{N} \left( U_i^+(y_i^+) - U_{\text{comp},i}^+(y_i^+) \right)^2, \tag{4.2}
\]

where \( U_i^+(y_i^+) \) is the i-th measured velocity point in wall units located at \( y_i^+ \) and \( U_{\text{comp},i}^+(y_i^+) \) is the model velocity at the \( y_i^+ \) location. The domain is re-defined iteratively until \( U_\tau \) and \( y_0 \) (the offset) converge. The convergence is reached under a user-defined tolerance value. In this case they are \( 10^{-6} \) m/s and \( 10^{-6} \) \( \mu \)m for the friction velocity \( U_\tau \) and the offset \( y_0 \) respectively.

The experimental data is corrected at each iteration until it collapses onto the reference profile. To trigger the recursive procedure initial values for \( U_\tau \) and \( y_0 \) are requested. In this case \( U_\tau \) is set to 1, while \( y_0 \) to 0 to force the no-slip condition at the wall. Lower and upper bound constraints can be applied. For the friction velocity \( U_\tau \) the lower bound is set to 0 and the upper one to 5. In the same fashion, the bounds for the offset \( y_0 \) are \( -\infty \) and \( +\infty \) respectively, which are the expected entries when no specific bound is declared. The wall shear-stress \( \tau_w \) can be easily computed from \( u_\tau \) by recalling that:

\[
\tau_w = \rho u_\tau^2 \tag{4.3}
\]

4.2.3d. Matching with the Theory. The acquired data needs to be analyzed to seek for spatial averaging or near wall effects (one among others the heat conduction from the hot-wire to the wall substrate). Being the diameter of the hot-wire 5 \( \mu \)m and the smallest viscous length scale \((\ell^* = \nu/U_\tau)\) about 15 \( \mu \)m, the resolution in the wall-normal direction does not affect the correctness of the experimental points. On the other hand, more attention must be paid in the spanwise direction. It is known from the literature that sensing elements larger than 20\( \ell^* \) in the spanwise direction can lead to spatial averaging effects and this intensifies when in the near wall region. In this case at the
highest free-stream velocity for a 1 mm wire the spanwise length is about $65\ell^*$, therefore spatial averaging effects are to be expected.

To assess this the combination of the mean velocity profile, the root mean square and the diagnostic plot are used. More in detail, the diagnostic plot - which is reported here in its modified version with the wall normal position in inner units $y^*$ on the $x$-axis - is a useful tool introduced by Alfredsson & Örlü (2010). It helps the experimentalist in understanding if the data close to the wall is affected by any near wall effect or spatial averaging. In the ideal scenario $u_{rms}/U_\infty$ approaches the wall position constantly, but this may not be the case in reality due to the above-mentioned disturbances. The mean velocity is usually overestimated as a consequence of the fact that heat is transferred from the functioning wire to the substrate, which acts like a thermal sink. This affects the velocity reading being the wire colder than it should be, hence the velocity is higher than its actual value. At the same time, it has been observed the tendency of the velocity root mean square to be underestimated in the viscous sublayer in thermal anemometry measurements near the wall leading overall to the inclination of the $u_{rms}/U_\infty$ ratio to decrease while approaching the wall position. These plots can be used together to identify the possible outliers and the faulty data points in the near-wall region to be cleaned up before moving further. The iterative procedure previously described must be started after having cleaned the data.

4.3. The Calibration Plots

4.3.1. Friction Velocity vs. Free-Stream Velocity

The friction velocity $u_\tau$ found via the previously described recursive method relates to the free-stream velocity inside the wind tunnel $U_\infty$. As depicted in Figure 4.4, $u_\tau$ is linearly proportional to $U_\infty$.

4.3.2. Free-stream velocity vs. Boundary Layer Parameters

The boundary layer parameters – i.e. the boundary layer thickness $\delta_{99}$, the displacement thickness $\delta^*$ and the momentum thickness $\theta$ – are derived having recourse to the turbulence intensity $T_u = u_{rms}/U(y)$ scaled by the square-root of the boundary layer shape factor $H_{12} = \delta^*/\theta$. It can be observed that when that quantity is about 0.02, the mean velocity $U(y)$ is about 99% $U_\infty$. By testing if there is any entry less or equal to 0.02, it is possible to mark the velocity entry $U_i(y_i)$ corresponding to $T_u/\sqrt{H_{12}} = 0.02$ as the last velocity value inside the boundary layer. Its position defines the BL limit and its value is used to derive all the BL parameters according to their definition.

As shown in Figure 4.7, the BL parameters decrease as long as the free-stream velocity $U_\infty$ becomes higher. This behaviour is not surprising being the boundary layer squeezed on the wall at higher free-stream velocities. The effect is more evident for the boundary layer thickness $\delta_{99}$, which becomes about 5
mm thinner when going from the lowest to the highest velocity reached by the wind tunnel. The same trend but weaker is observed for $\delta^*$ and $\theta$, which go down about 1 mm between 5 m/s and 23 m/s.

By incorporating the free-stream velocity $U_\infty$ and the kinematic viscosity $\nu$ (obtained via the Sutherland law using the ambient temperature $T$ and pressure $p$ acquired with the thermometer and the barometer respectively) one retrieves the corresponding Reynolds numbers. As shown in Figure 4.6, all of them increase monotonically with respect to $U_\infty$, being proportional by definition. The experimental points of both Figures 4.7 and 4.6 are interpolated using a power law, whom coefficients are listed in Table 4.1.

4.3.3. $Re_x$ vs. Wall Shear-Stress

The calibration of the wall shear-stress probes totally relies on the relationship between the mean wall shear-stress $\tau_w$ and the free-stream velocity $U_\infty$ at the probe location in no-cavity condition. The relationship is found iteratively...
4.3. THE CALIBRATION PLOTS

Figure 4.3. Turbulence intensity ($T_u = u_{t rms}^*/U^*$) versus the distance from the wall in inner units $y^*$. 5.0 m/s (⋆), 8.5 m/s (▽), 12.1 m/s (⋆), 15.9 m/s (△), 18.9 m/s (○), 23.3 m/s (◻).

with the method above mentioned and it is further extended by implementing the effect of the room temperature and pressure by defining the Reynolds number $Re_x$ based on the distance of the probe from the plate leading edge ($Re_x = U_∞x/ν$). The result is shown in Figure 4.8. It is clear that $τ_w$ increases monotonically whether the wind tunnel speed rises up or the air kinematic viscosity goes down. The interpolating function is quadratic.

4.3.4. How to extract data from the cavity probes

Now that the relationship between $τ_w$ and $Re_x$ is available, the wall shear-stress measurements can be performed using the cavity probes after installation. The modified Reynolds number $Re_x$ is obtained using the free-stream velocity reading $U_∞$ coming from the Pitot tube mounted at the beginning of the test section and the kinematic viscosity. In parallel, each $U_∞$ is coupled to the corresponding voltage time series $E(t)$. Using the calibration plot in Figure 4.8 the mean wall-shear stresses $τ_w$ corresponding to the new $Re_x$ values are extracted. Then, the obtained $τ_w$ are paired to the mean voltages $E$ from the voltage time-series, obtaining a final and effective calibration curve for each
4. FLOW CHARACTERISATION

The yellow filled circles are the experimental points. The dashed line is the interpolating curve: $u_\tau = AU_\infty + B$, where the slope is $A = 0.0410$ and the intercept is $B = 0.0527$.

Since the wall shear-stress measurements are performed at low-speed, the modified King’s Law (4.4) has been used to interpolate the experimental points. The main feature that it endows is the inclusion of the voltage at the zero-velocity $E_0$. For sake of comparison, the trend obtained by means of the former King’s Law (4.5) and a polynomial degree of 4th order (4.6) are depicted in Figure 4.9. As it can be observed, including $E_0$ in the King’s Law modifies substantially the trend below $\tau_w \approx 0.2$, but also for higher values the match with the experimental points is not satisfying. A better overlap between the modified King’s Law and the polynomial curve is obtained at higher values, but when the curve approaches zero the polynomial function tends to overestimate the actual mean wall shear-stress $\tau_w$. The coefficients of (4.4), (4.5) and (4.6) are listed in Table 4.2.
4.3. THE CALIBRATION PLOTS

Figure 4.5. The turbulence intensity scaled by $\sqrt{H_{12}}$ is about 0.02 when the mean velocity $U(y)$ is 99% $U_\infty$, i.e. the boundary layer limit definition employed. The vector entry corresponding to the BL limit can be found exploiting this relationship. Data from flow characterisation at 25 m/s taken just for qualitative considerations.

\[
\tau_w = k_1 (E^2 - E_0^2)^{(1/\alpha)} + k_2 (E - E_0)^{1/2} \tag{4.4}
\]

\[
E = \sqrt{k_1 + k_2 \tau_w^n} \tag{4.5}
\]

\[
E = k_1 \tau_w^4 + k_2 \tau_w^3 + k_3 \tau_w^2 + k_4 \tau_w + k_5 \tag{4.6}
\]
4. FLOW CHARACTERISATION

Figure 4.6. Boundary layer thickness $\delta_{99}$ (●), displacement thickness $\delta^*$ (■), momentum thickness $\theta$ (▲). The dashed lines are the interpolating power laws.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>a</th>
<th>b</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary layer thickness $\delta_{99}$</td>
<td>0.02981</td>
<td>-0.008732</td>
<td>0.264500</td>
</tr>
<tr>
<td>Displacement thickness $\delta^*$</td>
<td>0.004984</td>
<td>-0.001534</td>
<td>0.231400</td>
</tr>
<tr>
<td>Momentum thickness $\theta$</td>
<td>0.002289</td>
<td>-0.0002174</td>
<td>0.528800</td>
</tr>
<tr>
<td>Friction Reynolds no. $Re_\tau$</td>
<td>-331.5</td>
<td>361.7</td>
<td>0.317</td>
</tr>
<tr>
<td>Displacement thickness Reynolds no. $Re_{\delta^*}$</td>
<td>-166.8</td>
<td>389</td>
<td>0.6322</td>
</tr>
<tr>
<td>Momentum thickness Reynolds no. $Re_{\theta}$</td>
<td>-203.8</td>
<td>295.7</td>
<td>0.6188</td>
</tr>
</tbody>
</table>

Table 4.1. Parameters of the power laws used to interpolate the experimental points of the BL parameters and Reynolds numbers. The form of the function reads as follows: $y = a + bx^n$, where in both cases $x$ is the free-stream velocity $U_\infty$, while $y$ is either a thickness parameter $\delta$ or a Reynolds number $Re$. 
4.3. THE CALIBRATION PLOTS

Figure 4.7. Boundary layer thickness Reynolds number $Re_{\delta}$ ($\bullet$), displacement thickness Reynolds number $Re_{\delta^*}$ ($\blacksquare$), momentum thickness Reynolds number $Re_{\theta}$ ($\triangle$). The dashed lines (---) are the interpolating power laws.

<table>
<thead>
<tr>
<th></th>
<th>modified KL</th>
<th>KL</th>
<th>4th order pol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>30.41 (25.23, 35.6)</td>
<td>3.673 (3.648, 3.697)</td>
<td>0.0118</td>
</tr>
<tr>
<td>$k_2$</td>
<td>1.04 (0.9317, 1.148)</td>
<td>0.259 (0.2312, 0.2868)</td>
<td>0.0034</td>
</tr>
<tr>
<td>$k_3$</td>
<td></td>
<td></td>
<td>-0.0797</td>
</tr>
<tr>
<td>$k_4$</td>
<td></td>
<td></td>
<td>0.1285</td>
</tr>
<tr>
<td>$k_5$</td>
<td></td>
<td></td>
<td>1.9177</td>
</tr>
<tr>
<td>$n$</td>
<td>0.373 (0.3526, 0.3934)</td>
<td>0.5869 (0.4664, 0.7074)</td>
<td></td>
</tr>
<tr>
<td>$E_0$</td>
<td></td>
<td></td>
<td>1.918</td>
</tr>
</tbody>
</table>

Table 4.2. Coefficients of the interpolating laws displayed in Figure 4.9. The values in brackets are the confidence intervals (where available).
4. FLOW CHARACTERISATION

Figure 4.8. Wall shear-stress $\tau_w$ vs. Reynolds number $Re_x$. The yellow filled squares (∎) are the experimental points. The dashed line (---) is the interpolating curve, which is a polynomial function of degree 2: $\tau_w = AU_x^2 + BU_x + C$, where $A = 1.0863 \times 10^2$, $B = 0.058205 \times 10^{-5}$ and $C = -7885.3 \times 10^{-5}$. 
4.3. THE CALIBRATION PLOTS

Figure 4.9. Power-law calibration curve for the flush-mounted cavity probe. The employed law is the modified King’s Law (-----), which includes the voltage at zero-velocity $E_0$. The former King’s Law (-----) does not take it into account. For comparison purposes, interpolation with a 4th order polynomial (——) has been attempted, as well. The data (○) comes from the case of the cylindrical cavity S04LWnc and it is just for qualitative considerations.
4. FLOW CHARACTERISATION
Numerical Set-up

In this chapter, an explanation of the process that the numerical code follows in order to solve the problem presented in chapter 2 is given. From showing how the algebraic equations to be solved numerically are obtained, to the tools used for the interpretation and processing of the results.

5.1. Equation Discretization

The equations derived in previous sections must be discretized in time and space in order to be numerically solved, this section is based on the manual written by Chevalier et al. (2007) and provides a final version of the set of equations that are solved by Simson. Note that (2.8) can be efficiently solved by splitting it into two second order equations as shown next:

\[
\frac{\partial \phi}{\partial t} = h_v + \frac{1}{Re} \nabla^2 \phi \quad (5.1)
\]

\[
\nabla^2 v = \phi \quad (5.2)
\]

This section relies in connections with Appendix A, where a detailed explanation of certain numerical schemes for the solution of PDE is given. If the reader, however, is only interested in the final form of the equations solved, the information contained in this section is sufficient.

5.1.1. Temporal Discretization

It can be seen that (2.10) and (5.1) have the same structure described in (A.43), therefore the methods exposed in that section can be implemented. For this case the linear term is also discretized by Crank-Nicolson and the non linear one by Runge-Kutta of third order. A similar procedure than the one used in the derivation of (A.46) is followed to obtain the next set of equations:

\[
\left(1 - \frac{a_n + b_n}{2Re} \nabla^2 \right) \phi^{n+1} = a_n h_v^n + b_n h_v^{n-1} + \left(1 + \frac{a_n + b_n}{2Re} \nabla^2 \right) \phi^n \quad (5.3)
\]

\[
\nabla^2 v^{n+1} = \phi^{n+1} \quad (5.4)
\]

\[
\left(1 - \frac{a_n + b_n}{2Re} \nabla^2 \right) \omega^{n+1} = a_n h_\omega^n + b_n h_\omega^{n-1} + \left(1 + \frac{a_n + b_n}{2Re} \nabla^2 \right) \omega^n \quad (5.5)
\]
5. NUMERICAL SET-UP

5.1.2. Spatial Discretization

The spatial discretization is done by using Fourier series in the horizontal directions and Chebyshev transformation in the wall normal one.

5.1.2a. Horizontal Discretization. The way that the Fourier series is formally expressed (see Appendix A) is valid for a domain in the interval $[0, 2\pi]$. If the domain has a different size, then a mapping must be done. This is easily visualized in the next equation:

$$u(x) = \sum_{n=-N_x}^{N_x-1} \hat{u}_k e^{ikx} \quad (5.6)$$

It can be appreciated that the sum is done for $n$ instead of the wavenumber $k$, and:

$$k = \frac{2\pi n}{x_l} \quad (5.7)$$

Where $x_l$ is the size of the domain. From (5.7), if $x_l = 2\pi$ then $n = k$ and the same form of equation (A.9) is valid.

The stream-wise and span-wise discretization is done considering the mapping just mentioned and the fact that the variables have already been discretized in time, therefore, no time dependence is evidenced such that:

$$u(x, y, z) = \sum_{l=-N_y}^{N_y-1} \sum_{m=-N_z}^{N_z-1} \hat{u}_{\alpha, \beta}(y) e^{i(\alpha x + \beta z)} \quad (5.8)$$

with

$$\alpha = \frac{2\pi l}{x_l}, \quad \beta = \frac{2\pi m}{z_l}, \quad k^2 = \alpha^2 + \beta^2 \quad (5.9)$$

Applying (5.8) to (5.3) and taking into account the orthogonality of the Fourier series as seen in (A.51) yields:

$$\left( 1 - \frac{a_n + b_n}{2Re} \left( \frac{d^2}{dy^2} - k^2 \right) \right) \hat{\phi}^{n+1}(y) = a_n \hat{h}_n^\alpha(y) + b_n \hat{h}_n^{n-1}(y) + \left( 1 + \frac{a_n + b_n}{2Re} \left( \frac{d^2}{dy^2} - k^2 \right) \right) \hat{\phi}^n(y) \quad (5.10)$$

for:

$$\alpha = \frac{\pi N_x}{x_l}, \ldots, \frac{\pi N_x}{x_l}, \quad \frac{2\pi}{x_l} \quad (5.11)$$

$$\beta = \frac{\pi N_z}{z_l}, \ldots, \frac{\pi N_z}{z_l}, \quad \frac{2\pi}{z_l}$$

$$k^2 = \alpha^2 + \beta^2$$
Note that all the coefficients also have dependence in $\alpha, \beta$ for example 
$\hat{\phi}_{n+1}(y) = \hat{\phi}_{n+1}^{\alpha, \beta}(y)$, which means that there is a version of the equation (5.10) for each combination of $\alpha$ and $\beta$ which is taken into account in the value of $k$ and therefore there are as many coefficients $\hat{\phi}$. In other words, if $\hat{\phi}$ is to be found, a value for $\alpha$ and $\beta$ must be set and then the equations solved. The process is then repeated for a new $\alpha$ and $\beta$ until all combinations have been evaluated. All this notation, however, will be dropped for the sake of simplicity along with the $y$ dependence. Doing the same procedure with the rest of equations, the next system is found:

$$
\left( \frac{d^2}{dy^2} - \lambda^2 \right) \hat{\phi}_{n+1} = \hat{f}^n_v
$$

$$
\left( \frac{d^2}{dy^2} - \lambda^2 \right) \hat{\phi}_{n+1}^{\alpha, \beta} = \hat{\phi}_{n+1}^{\alpha, \beta}
$$

$$
\left( \frac{d^2}{dy^2} - \lambda^2 \right) \hat{\phi}_{n+1} = \hat{f}^n_{\omega}
$$

with

$$
\lambda^2 = k^2 + \frac{2Re}{a_n + b_n}
$$

$$
\hat{f}^n_v = \hat{p}^n_v - \frac{2Re a_n b_n}{a_n + b_n}
$$

$$
\hat{f}^n_{\omega} = \hat{p}^n_{\omega} - \frac{2Re b_n}{a_n + b_n}
$$

Defining the partial right hand side as:

$$
\hat{p}^n_v = -\hat{f}^{n-1}_v - \left( \frac{2Re}{a_{n-1} + b_{n-1}} + \frac{2Re}{a_n + b_n} \right) \hat{\omega}^n - \left( \frac{2Re b_n}{a_n + b_n} \right) \hat{p}^{n-1}_v
$$

$$
\hat{p}^n_{\omega} = -\hat{f}^{n-1}_{\omega} - \left( \frac{2Re}{a_{n-1} + b_{n-1}} + \frac{2Re}{a_n + b_n} \right) \hat{\omega}^n - \left( \frac{2Re b_n}{a_n + b_n} \right) \hat{p}^{n-1}_{\omega}
$$

5.1.2b. Boundary conditions. As can be seen, the problem described by (5.12) is the same type of ODE described in (A.49) which can be solved in the $y$ direction by the Chebyshev tau method. However the boundary conditions for the problem must be defined, as they do not apply explicitly to the quantities that the differential equations are dependent on. The approach followed by the solver takes advantage of the next property of linear systems:

**Particular and Homogeneous solutions** For a linear system:

$$
Au = b
$$

(5.15) is homogeneous if $b = 0$ and inhomogeneous otherwise. A particular solution $u_p$ is a solution of the inhomogeneous equation such that:

$$
Au_p = b
$$

(5.16)
5. NUMERICAL SET-UP

All solutions of the general problem (5.15) can be found by finding homogeneous solutions and then adding the particular one, such that:

\[ A(\mathbf{u}_h + u_p) = 0 + b = b \] (5.17)

Following the previous result, the equations in (5.12) will be solved for particular and homogeneous cases. The final solution for the problem will be obtained from a combination of those previously obtained cases. For reference see Kim et al. (1987) and Chevalier et al. (2007).

According to this information and considering only the equations for \( \hat{\phi} \) and \( \hat{v} \), the particular and homogeneous systems that will be solved in place of (5.12) will be shown next. It is important to keep in mind that combination of the solutions will be tuned in order to satisfy the real boundary conditions of the problem:

1. Particular solution with homogeneous boundary conditions:

\[
\begin{align*}
\frac{d^2}{dy^2} - \lambda^2 \hat{\phi}^{n+1}_p &= f^n_v \\
\frac{d^2}{dy^2} - \lambda^2 \hat{v}^{n+1}_p &= \hat{\phi}^{n+1}_v \\
\hat{\phi}^{n+1}_p(0) &= 0, \quad \hat{\phi}^{n+1}_p(y_l) = 0 \\
\hat{v}^{n+1}_p(0) &= 0, \quad \hat{v}^{n+1}_p(y_l) = 0
\end{align*}
\]

(5.18)

2. Homogeneous solution non homogeneous B.C:

\[
\begin{align*}
\frac{d^2}{dy^2} - \lambda^2 \hat{\phi}^{n+1}_{ha} &= 0 \\
\frac{d^2}{dy^2} - \lambda^2 \hat{v}^{n+1}_{ha} &= \hat{\phi}^{n+1}_v \\
\hat{\phi}^{n+1}_{ha}(0) &= 1, \quad \hat{\phi}^{n+1}_{ha}(y_l) = 1 \\
\hat{v}^{n+1}_{ha}(0) &= 0, \quad \hat{v}^{n+1}_{ha}(y_l) = 0
\end{align*}
\]

(5.19)

3. Homogeneous solution considering only \( v \)

\[
\begin{align*}
\frac{d^2}{dy^2} - \lambda^2 \hat{v}^{n+1}_{hb} &= 0 \\
\hat{v}^{n+1}_{hb}(0) &= 1, \quad \hat{v}^{n+1}_{hb}(y_l) = 1
\end{align*}
\]

(5.20)

A particular characteristic of the current problem, is the fact the equations can be solved simultaneously by eliminating the same banded matrix with different right hand sides, which contain the boundary conditions.
5.1.2c. Normal Discretization. The system of equation that needs to be solved has already been defined. However, as for the Fourier space, a mapping needs to be done from $[0, y_I]$ to $[-1, 1]$ which is the interval where Chebyshev polynomials are orthogonal. This is done by considering $y' = \frac{2y}{y_I} - 1$. Only a generalization of (2) is shown for $\phi$ after such scaling is done (ignoring the scaling apostrophe), but the same process must be applied for all variables:

$$\left( \frac{d^2}{dy'^2} - \nu \right) \hat{\phi}^{n+1} = \hat{f}_n$$  \hspace{1cm} (5.21)

with $\nu = \frac{\lambda^2 y_I^2}{4}$. The next step is to apply the Chebyshev transformation (A.15) to the differential equations and the boundary conditions:

$$\hat{\phi}_{\alpha, \beta}^{n+1}(y) = \sum_{j=0}^{N_y} \hat{\phi}_j^{n+1} T_j(y)$$  \hspace{1cm} (5.22)

$$\hat{f}^n(y) = \sum_{j=0}^{N_y} \hat{f}_j^n T_j(y)$$  \hspace{1cm} (5.23)

$$\hat{\phi}_{\alpha, \beta}^{n+1}(1) = \sum_{j=0}^{N_y} \hat{\phi}_j^{n+1}$$  \hspace{1cm} (5.24)

$$\hat{\phi}_{\alpha, \beta}^{n+1}(-1) = \sum_{j=0}^{N_y} (-1)^j \hat{\phi}_j^{n+1}$$  \hspace{1cm} (5.25)

It is once again noted that by this stage $\alpha$ and $\beta$ have already been set and their value affects the equations thought $\nu$. The structure of the Chebyshev approximation of equation (5.21) is the same as that of (A.52):

$$\hat{\phi}_j^{(2)} - \nu \hat{\phi}_j = \hat{f}_j \hspace{1cm} j = 0, ..., N_y - 2$$  \hspace{1cm} (5.26)

where the super index $n$ has been dropped for convenience.

(5.26) can be solved by means of Chebyshev derivation recurrence (Chebyshev Tau approximations) as seen in the derivation of (A.55). The system to be solved is:

$$\frac{c_j - 2\nu}{4j(j-1)} \tilde{\phi}_{j-2} + \left[ 1 + \frac{\nu \beta_j}{2(j^2 - 1)} \right] \tilde{\phi}_j + \frac{\nu \beta_{j+2}}{4j(j+1)} \tilde{\phi}_{j+2}$$

$$= \frac{c_j - 2\nu}{4j(j-1)} \tilde{f}_{j-2} - \frac{\beta_j}{2(j^2 - 1)} \tilde{f}_j + \frac{\beta_{j+2}}{4j(j+1)} \tilde{f}_{j+2}$$  \hspace{1cm} (5.27)
5. NUMERICAL SET-UP

Reminder: This system of equation exist for every combination of $\alpha$ and $\beta$ (horizontal wave numbers) and must be solved for each case to find all coefficients $\hat{\phi}_{\alpha,\beta}^{n+1}(y)$.

Once the solution for $\phi$ has been found, the equation for $v$ is solved. This process is then repeated for every particular and homogeneous system defined previously and the acquired solutions are linearly combined as follows:

$$\hat{v}^{n+1}_p = \hat{\nu}^{n+1}_p + C_{v1}\hat{\nu}^{n+1}_{ha} + C_{v2}\hat{\nu}^{n+1}_{hb} \quad (5.28)$$

where the coefficients are chosen to satisfy the boundary condition:

$$\frac{\partial \hat{v}^{n+1}}{\partial y}(0) = 0 \quad \frac{\partial \hat{v}^{n+1}}{\partial y}(y_l) = 0 \quad (5.29)$$

After knowing the values of $\hat{v}$ and $\hat{\omega}$ the rest of the variables are found through continuity equation and the definition of vorticity as was mentioned in previous sections.

5.2. Solver Methodology

In the present Section, an account of the main methodology followed by the solver is presented accompanied by the characteristics of the elements introduced to the numerical scheme for the analysis of cavities. A great amount of the contents of the chapter are discussed in depth by Chevalier et al. (2007).

5.2.1. Original solver process

The solver can be used for a variety of flow types and boundary conditions. Having set that the case of interest are channel flows, then there are two main subroutines that can be considered as the bulk of Simson since they deal with necessary data needed to get a solution.

5.2.1a. Initialization. The initial conditions are generated from the subroutine bls. In all cases evaluated in this project, the starting state consist of a laminar flow to which perturbations, such as noise, are introduced in order to trigger a transition to turbulence as time steps are advanced.

Under these circumstances, the subroutine must be compiled for a given resolution and executed with the inputs shown in Table 5.1.

5.2.1b. Solution. The main program that solves the equations derived in section is named bla. This subroutine must also be compiled for a given resolution, but additional parameters need to be specified depending on the type of computing strategy that is to be used: Serial, OpenMP, MPI.

The inputs to the program itself are similar to those of bls, with some additional flags that need to be specified in case of requiring particular functions
5.2. SOLVER METHODOLOGY

Reynolds Number \( R_e \)

<table>
<thead>
<tr>
<th>Domain size in ( x )</th>
<th>( x_l )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain size in ( y )</td>
<td>( 2h )</td>
</tr>
<tr>
<td>Domain size in ( z )</td>
<td>( z_l )</td>
</tr>
</tbody>
</table>

Table 5.1. Bls inputs.

such as the introduction of volume forces, which are in fact used for the creation of the cavity. The main storage of the variables is done as shown in table 5.2.

<table>
<thead>
<tr>
<th>Direction</th>
<th>Storage Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>X direction</td>
<td>Fourier space</td>
</tr>
<tr>
<td>Y direction</td>
<td>Physical space</td>
</tr>
<tr>
<td>Z direction</td>
<td>Fourier space</td>
</tr>
</tbody>
</table>

Table 5.2. Variable storage space.

This means that even though the initial conditions are known, which in normal circumstances would allow a solver to predict the next time step by solving the system of equations, the complete right hand side of equation (5.12) can not be efficiently constructed due to the convolution sum present while evaluating \( h_v \) and \( h_\omega \) in Fourier space, as explained in section A.5. For this reason, only the partial right hand sides defined in (5.14) are initially calculated.

On each time step, the process explained in section A.5 is used to obtain the non linear term in the equations, a simple summary is presented here:

1. Transform the velocities and vorticities back into physical space on a dealiasing (padded) grid.
2. Calculate the non linear term \( H_i \) in physical space.
3. Add volume forces \( F_i \) in physical space to the non linear term.
4. Transform back into Fourier-Physical space.

Having done this process, the solver transforms the non linear term into the Fourier-Chebysev space and builds the complete R.H.S presented in equations (5.13). The Chebyshev integration method shown in (A.55) is used to solve the equations and the variables are transformed back into Fourier-Physical space in order to be stored. A final set up of certain parameters is done for the next time step, and the process is repeated until the targeted simulation time, defined in the inputs, is reached.
5.2.1c. Post processing. The main program prints binary statistics files and can store instantaneous fields or planes as requested in the inputs, these files can be read by in-built subroutines such as pxyst and rit respectively. However in most cases they are used to export specific fields into data that is then used by Python or Matlab scripts to be further manipulated.

5.2.1d. Solver validation. As a way to test the solver, the results from Simson are compared to validated data from Lee & Moser (2015). By doing this, confidence is generated in the accuracy of the obtained results.

The test run was executed following the procedure explained in section 5.2 with the simulation parameters given in table 5.3

<table>
<thead>
<tr>
<th>Grid Size</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_x$</td>
<td>96</td>
</tr>
<tr>
<td>$n_y$</td>
<td>97</td>
</tr>
<tr>
<td>$n_z$</td>
<td>96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Domain Size</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_l$</td>
<td>$4\pi h$</td>
</tr>
<tr>
<td>$y_l$</td>
<td>$2h$</td>
</tr>
<tr>
<td>$z_l$</td>
<td>$2\pi h$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flow Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re$</td>
<td>4200</td>
</tr>
<tr>
<td>$Re_\tau$</td>
<td>$\approx 180$</td>
</tr>
</tbody>
</table>

Table 5.3. Simulation setup.

For the comparison, logarithmic scale was used in order to better visualize the section closer to the wall of the mean velocity profiles, as can be seen in Figure 5.1. It is possible to see good accordance between the results, where the divergence in values close to the middle of the channel ($y^* \approx 180$) is suspected to be originated by the difference in resolution between both simulations as well as possible slight differences in the setting up of the mass flow. This observation comes into play by taking into consideration that due to the nature of the collocation points used for the Chebyshev approximation in the wall normal direction, the spacing between the grid points is most noticeable the farther away they are from the walls i.e. the center of the channel, which is in turn, the section were the results slightly diverge. The mean gradient was also compared and the findings can be seen in Figure 5.2.
Figure 5.1. Mean flow validation. Results by Moser et al (—). Results obtained with SIMSON (—).

Figure 5.2. Mean gradient validation. Results by Moser et al (—). Results obtained with SIMSON (—).
Based on the collected results, even at relatively low resolution, the expected behaviour of the mean velocity is captured by Simson. Thus, from this point onward in the report, results obtained from the pseudo-spectral solver will be used as a reference to test the methods implemented for the creation of the cavity, as well as for the analysis of the final results.

5.2.2. Modifications for cavity generation

In the previous section, the main workflow of the solver was explained, however some modifications where made in order to introduce the cavity into the model, such processes are explained next.

Some of the equations in this section have been presented before, however many of them are rewritten here for the sake of completeness.

5.2.2a. Control forces. It has been mentioned that the cavity is generated by introducing a set of volume forces into the system by the Immersed Boundary Method (IBM), which consist of a closed loop control scheme presented in equation (2.14) rewritten here:

\[ F_i(x, y, z, t) = \alpha u_i(x, y, z, t) + \beta \int_0^t u_i(x, y, z, t) \, dt \quad (5.30) \]

To create this force, the integral part must be calculated on each time step, and as it has also been mentioned, it is approximated by means of a Riemann sum as shown in equation (2.15), multiplying the value of the velocity field by the \( \Delta t \) at each time step and adding it to the previously stored value for the integral as follows:

\[ \int_0^t u_i(x, y, z, t) \, dt = int_i(x, y, z) = int_{i(n-1)}(x, y, z) + u_i(n)(x, y, z) \Delta t \quad (5.31) \]

Where the sub index \( n \) represents the current iteration or time step.

Having calculated this, it is possible to apply a volume force that brings the flow to rest at any point of the domain. Knowing this, the next step is to include where the cavity needs to be defined, which is done in the inputs of the bla subroutine. The parameters to be defined are:

1. \( \alpha \).
2. \( \beta \).
3. Center of the cavity in the x direction.
4. Width of the cavity.
5. Depth of the cavity.

Having done all the previous steps, the way that the non linear term in equation (2.6) is defined, can be modified to:

\[ H_i = -\epsilon_{ijk} u_j (\omega_k + 2\Omega_k) + SF_i \quad (5.32) \]
5.2. SOLVER METHODOLOGY

Where the parameter $S$ represents a flag with values:

$$ S = \begin{cases} 
0 & \text{if grid point is outside the area defined by the inputs} \\
1 & \text{if grid point is inside the area defined by the inputs}
\end{cases} \quad (5.33) $$

This means that the volume force is in reality calculated for the whole domain but only applied in the adequate places i.e. zones where velocity should be zero, depending on the inputs given to the program.

5.2.2b. Re-scaling. When introducing the cavity to the simulation, a virtual lower surface is created in the channel, which effectively reduces the free height that the flow is allowed to travel, however this is an artifact of the IBM and corrections must be made if the effective Reynolds number and mass flow is to be kept as specified in the inputs.

The simplest way to make the corrections, is to use a domain that has a higher height, such that, after introducing the immersed boundary, the free channel is 2 units high, as needed for the Chebyshev transformation to work. This is easily defined as:

$$ y_l = 2h = 2 + d \quad (5.34) $$

Where $d$ is the cavity depth and $h$ is the channel half height. It is very important however, to correctly modify the other dimensions of the domain, since failing to do so will originate scaling errors in the solver. The size of the channel is:

<table>
<thead>
<tr>
<th>Domain size in $x$</th>
<th>$x_l = 4\pi h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain size in $y$</td>
<td>$y_l = 2h$</td>
</tr>
<tr>
<td>Domain size in $z$</td>
<td>$z_l = 2\pi h$</td>
</tr>
</tbody>
</table>

**Table 5.4.** Domain sizes.

Considering that for this re-scaled case $h > 1$. All this process is needed to ensure that the channel has the same characteristics as the ones defined for a channel at $Re_\tau = 180$ with no cavity. Only this way can the results between cases with cavity and without it be compared.

5.2.2c. Mass flow and pressure gradient check. The solver does not use an additional equation in the system to solve for the driving pressure that keeps the mass flow constant, instead, it assumes an initial pressure, from the laminar flow, and adjusts it at every time step so that the flow rate remains unchanged.

To do this, the program initially solves a system of equations for the mean flow, i.e. wave number zero, which are obtained by horizontally averaging the
first and third component of (2.3). After discretizing in time and space (in x and z), a system of equations is obtained:

\[
\begin{align*}
\frac{d^2}{dy^2} - \lambda^2 \hat{u}^n_0(y) &= \hat{f}^{n}_01 \\
\frac{d^2}{dy^2} - \lambda^2 \hat{w}^n_0(y) &= \hat{f}^{n}_03
\end{align*}
\] (5.35)

Taking into consideration that the assumed driving pressure is introduced into the terms of (5.35), which are solved with the Chebysev-Tau integration method. The solution of the system, produces all the sets of Chebyshev coefficients of both quantities ie. \( \hat{u}_{0j} \) and \( \hat{w}_{0j} \) for \( j = [0, n_y] \).

Having obtained the Chebysev coefficients, it is now possible to get the mean flow in physical space, simply by transforming the current data. This is done by the relation given in (A.28) to transform from Fourier-Chebyshev to Fourier-Physical space.

\[
\hat{u}_{0,0}(y_j) = \hat{u}_{\alpha,\beta}(y_j) = \sum_{k=0}^{n_x} \hat{u}_{0k} \cos\left(\frac{\pi kj}{n_y}\right) \quad j = 0, 1, ..., n_y
\] (5.36)

Considering that \( \alpha = \beta = 0 \) (since the calculations were done for wave number zero) it is very easy to note that trying to transform the Fourier coefficients obtained by (5.36) into physical space, would provide with:

\[
\bar{u}_{\text{mean}}(y_j) = \hat{u}_{0,0}(y_j)
\] (5.37)

The mean velocity field is now used to calculate the flow rate. If the obtained value is close enough to the desired mass flow, the driver pressure is kept. If there are big deviations, the pressure is adjusted and introduced again into the system (5.35) and the same process is done to evaluate the flow rate once again.

In the last step, it is important to take into consideration that an Immersed boundary is introduced into the system. For this reason care must be made while doing the integration that computes the mass flow (a simple trapezoidal method) to only include the sections of the channel that are in the area of interest and discard anything that is bellow the IB surface. It is also important to temporally scale the channel to the real height \( (y_l = 2 + d) \) since the height above the virtual wall needs to be 2 to match the simulations without cavity.

\textbf{5.2.2d. Filters and oscillation control.} Methods for oscillation control were introduced in the code, mainly at the same moment of the definition of the volume force or after transforming all variables into the Chebyshev space. The methods have already been explained in section 2.2 thus, the reader is referred to that part of the project for a more in depth look at them.
5.2.2e. **Immersed Boundary Method testing.** The IBM is used extensively in this project, and it was found to give good solutions. The testing of the method is shown in appendix B in order to limit the main sections of the project to relevant findings related to cavities instead of IBM.

5.3. **Research methodology**

The way that the solver finds solutions to the equations for one particular run of simulations was explained, however there exist a structure in which they were scheduled in order to manage the data in the most efficient way possible. The contents of this section can be used as a guide for the subsequent explanations concerning the simulations in the report.

5.3.1. **Analysis Initialization**

In general, for every new resolution to be analyzed, a case without an immersed boundary had to be executed in order to recover certain flow characteristics such as:

1. $Re_r$
2. $u_r$
3. $\ell^*$

The cavity size of relevance is usually identified in inner (scaled) units, however the inputs of the main solver are in physical units. For this reason, a python routine called "slotsize" was run before each case, with the flow parameters and the desired cavity size as inputs, and would in turn have as outputs the appropriate re-scaled domain size and cavity geometry that needed to be input into the subroutines bls and bla.

The simulations were executed with serial, OMP or MPI methodologies according to the computing tools available at any given moment, with almost all cases running during a total of 1000 time units, where the recording and storing of statistics was performed after 500 time units, since it was observed that in most cases, the turbulent development had been complete by this stage.

Once simulations had finished their execution time, the data was extracted with the built-in post processing tools mentioned before, i.e. "pxyst" and "rit", this way more appropriate data sets where exported to python or matlab for an in depth analysis. In general, velocity profiles where processed using Python, while pictures of velocity contours where made in Matlab.
CHAPTER 6

Numerical Test Case

Based on the study done by Gubian et al. (2019), it was decided that one of the main cases of interest would be that of a cavity with a depth of 3 inner units. For this purpose, and considering the resolution used up to this point and the time limitation of the simulations, it was decided that the case would be tested under the conditions presented in Table 6.1

<table>
<thead>
<tr>
<th>Grid Size</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_x$</td>
<td>256</td>
</tr>
<tr>
<td>$n_y$</td>
<td>97</td>
</tr>
<tr>
<td>$n_z$</td>
<td>96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flow characteristics</th>
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</thead>
<tbody>
<tr>
<td>$Re_\tau$</td>
<td>182.43</td>
</tr>
<tr>
<td>$u_\tau$</td>
<td>0.043436</td>
</tr>
<tr>
<td>$l^*$</td>
<td>0.0054815</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cavity size (in inner units)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>50</td>
</tr>
<tr>
<td>$y$</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Amount of grid points in the cavity</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>7</td>
</tr>
<tr>
<td>$y$</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 6.1. Simulation set-up for x-50, y-03.

The idea of using a test case, is to determine certain characteristics to be taken into consideration while performing the simulations. The objectives of this analysis are summarized as follows:

2. General evaluation of the IBM through symmetry condition.
3. Evaluation of resolution effects.
6. NUMERICAL TEST CASE

6.1. Reference Case

An initial run of the solver under the specified conditions provided the results presented in Figure 6.1. As somewhat expected, the fact that even small cavity depths produce a difference in mean velocity can be seen in Figure 6.1. It is also apparent that oscillations are introduced into the solution. The latter is accountable to the fact that the immersed boundary is close to the channel real lower surface. Due to this condition, the flow has less space to develop, which means that the changes happen faster. This implies that there are bigger gradients close to the surface, which gives the Gibbs phenomenon more chances to appear due to stronger discontinuities being introduced into the system.

It is possible to see that said oscillations happen at locations very distant to the immersed surface. Their impact in the results close to $y^+ = 0$ remain to be analyzed. It can be seen however, that the oscillation intensity is very low and increasing resolution in the wall normal direction could potentially eliminate them all.

The behaviour of the velocity fluctuations is also presented in figure 6.2, where the same tendency to a higher quantity at the cavity center than the expected value at the surface is seen. An additional variable is shown now, specifically the value of the turbulence intensity which is defined as:

$$ T_u = \frac{u_{rms}}{U}. $$

(6.1)
This is an important variable as it is the one that allows for indirect measurements of the wall shear stress fluctuations as seen in (1.1).

**Figure 6.2.** Velocity fluctuations profile in log scale. Profile at a streamwise station without a cavity (---). Profile at the streamwise location of the cavity center (-- -).

**Figure 6.3.** Turbulence intensity profile in log scale. Profile at a streamwise station without a cavity (---). Profile at the streamwise location of the cavity center (-- -).
It was expected that mean and fluctuating velocities would increase due to the absence of a no slip boundary condition at the surface height in the cavity, however, the fact that the turbulent intensity does not remain constant, as can be seen in Figure 6.3 means that the increase of the parameters is not proportionally the same. At this stage however, it remains to be seen which quantity has the biggest impact in the final value.

Additionally to the values obtained for each variable, there are two conditions that need to be evaluated and that can further validate the quality of the results. Those are studied in the next sections.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6_4.png}
\caption{Mean velocity symmetry in log scale. Profile at a streamwise station without a cavity for the lower half of the channel, i.e. cavity side (---). Profile at the streamwise location of the cavity center for the lower half of the channel, i.e. cavity side (---). Profile at a streamwise station without a cavity for the upper half of the channel (----). Profile at the streamwise location of the cavity center for the upper half of the channel (----).}
\end{figure}

6.2. Symmetry condition
It is known that in a non modified periodic channel, the flow is symmetric between the upper and lower surface given that the boundary conditions in them are the same. In this study, the lowermost surface of the channel, i.e. the bottom of the cavity is natural, however in sections where the cavity is not present, the surface is artificial, and generated by the Immersed Boundary
Method as has been mentioned before, however, even in this case, and given that the surface emulates the boundary conditions of a rigid wall (no slip, non penetration) it is important that the symmetry is conserved. For this purpose, a set of graphs will be presented where this property is shown to be achieved by the model. This also allows for calculation in the immerse boundary ($y^+ = 0$) to be made with data from the upper channel wall instead, this might be necessary due to the mentioned fact that the discontinuities in the force field might make some quantities such as the gradient to be difficult to calculate in the immersed boundary.

Figure 6.5. Velocity fluctuations symmetry in log scale. Profile at a streamwise station without a cavity for the lower half of the channel, i.e. cavity side (---). Profile at the streamwise location of the cavity center for the lower half of the channel, i.e. cavity side (-----). Profile at a streamwise station without a cavity for the upper half of the channel (—). Profile at the streamwise location of the cavity center for the upper half of the channel (- - -)
These plots were made from mirroring the data from the upper half of the channel and superimposing it to that of the lower half. It is easy to see how the plots do not begin from the same value. This is due to the decision of presenting the results in log scale, which means that the point $y^+ = 0$ had to be excluded. This is, however, a good moment to analyze one of the disadvantages of the IBM (also explained in Figure B.2), i.e. the loss of resolution due to the distribution of the collocation points.

Although it can be seen that there is a very good agreement in the results, and it is possible to conclude that the flow behavior is indeed symmetric. It must be noted that, as remarked before, the channel domain IS NOT. The gridpoints in the upper wall are closer than those in the neighboring area around the immersed boundary, which translates into an effective reduction of the resolution. Note that this behavior will tend to worsen as the cavity depth increases, since the location of $y=0$ will be higher in the domain. (For a graphical representation, the reader is advised to go through B)
6.3. Resolution Effects

It was mentioned that the results of the reference case have some method-induced oscillations due to the discontinuity in the force field. For this reason, an analysis must be conducted at higher resolutions with 2 main purposes:

1. See if the oscillations disappear by increasing resolution (as they should)
2. Evaluating if the oscillations are affecting considerably the value of the variables in the sections of interest for the study.

For this purpose, resolution was increased in the x and y direction individually to assess their effect in the main solution.

The resolution in the wall normal and streamwise direction was increased to the values in table 6.2:

If the results are plotted in a linear scale as shown in Figure 6.7, which include the first gridpoint. It can be seen that they indeed start from the same point, however, it must be kept into consideration that "there is more information" captured close to the upper surface than in the lower one.

**Figure 6.7.** Mean velocity symmetry in linear scale. Profile at a streamwise station without a cavity for the lower half of the channel, i.e. cavity side (—). Profile at the streamwise location of the cavity center for the lower half of the channel, i.e. cavity side (---). Profile at a streamwise station without a cavity for the upper half of the channel (—). Profile at the streamwise location of the cavity center for the upper half of the channel (---).
6. NUMERICAL TEST CASE

<table>
<thead>
<tr>
<th>Grid Size</th>
<th>Gridpoints</th>
<th>Y check</th>
<th>X check</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_x )</td>
<td>256</td>
<td>512</td>
<td></td>
</tr>
<tr>
<td>( n_y )</td>
<td>161</td>
<td>97</td>
<td></td>
</tr>
<tr>
<td>( n_z )</td>
<td>96</td>
<td>96</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.2. New resolution.

6.3.1. Y resolution check

The main results are shown in log scale in order to get a picture of the channel half height. Note that it is expected that the results start from different points in this case. Not due to an apparent resolution loss of the Immersed boundary method as was shown in the previous testing procedure, but from an actual resolution difference.

From figures 6.8 to 6.10, 2 things are evident.

1. The oscillations in the Y-direction are indeed eliminated from the solution when increasing resolution.
2. Change in results occur, but they are minimal.

These two observations are critical and allow to suggest that even thought it is true that better resolution provide smoother results, it is also a more expensive process, and good enough results are acquired with a lower resolution.

6.3.2. X resolution check

For this case, the resolution of the field was increased to the values given in table 6.2. Under these conditions, the results are compiled in figures 6.11 to 6.13. It is suspected that the difference in results are mainly due to the fact that the higher resolution in x allows a better characterization of the development of the flow inside the cavity. The results are close, however it was understood that the resolution in the x direction is very important, as the solution seem to be very dependant on this variable.
6.3. RESOLUTION EFFECTS

Figure 6.8. Mean velocity profile in log scale. Profile at a streamwise station without a cavity. Reference resolution (—). Profile at the streamwise location of the cavity center. Reference resolution (---). Profile at a streamwise station without a cavity. Increased wall normal resolution (—). Profile at the streamwise location of the cavity. Increased wall normal resolution (---).

Figure 6.9. Velocity fluctuations profile in log scale. Profile at a streamwise station without a cavity. Reference resolution (—). Profile at the streamwise location of the cavity center. Reference resolution (---). Profile at a streamwise station without a cavity. Increased wall normal resolution (—). Profile at the streamwise location of the cavity. Increased wall normal resolution (---).
Figure 6.10. Turbulence intensity profile in log scale. Profile at a streamwise station without a cavity. Reference resolution (—). Profile at the streamwise location of the cavity center. Reference resolution (---). Profile at a streamwise station without a cavity. Increased wall normal resolution (—). Profile at the streamwise location of the cavity. Increased wall normal resolution (---).

Figure 6.11. Mean velocity profile in log scale. Profile at a streamwise station without a cavity. Reference resolution (—). Profile at the streamwise location of the cavity center. Reference resolution (---). Profile at a streamwise station without a cavity. Increased stream-wise resolution (—). Profile at the streamwise location of the cavity. Increased stream-wise resolution (---).
6.3. RESOLUTION EFFECTS

Figure 6.12. Velocity fluctuation profile in log scale. Profile at a streamwise station without a cavity. Reference resolution (—). Profile at the streamwise location of the cavity center. Reference resolution (—). Profile at a streamwise station without a cavity. Increased stream-wise resolution (—). Profile at the streamwise location of the cavity. Increased stream-wise resolution (—).

Figure 6.13. Turbulence intensity in log scale. Profile at a streamwise station without a cavity. Reference resolution (—). Profile at the streamwise location of the cavity center. Reference resolution (—). Profile at a streamwise station without a cavity. Increased stream-wise resolution (—). Profile at the streamwise location of the cavity. Increased stream-wise resolution (—).
6.4. Resolution refinement

Based on the results obtained in the previous section, some decisions were made in order to further investigate the behaviour of the flow in the cavity, specifically below the surface height.

Under these observations, and taking into consideration the known required resolution to correctly solve a channel with the current characteristics, the parameters used to run the next set simulations are summarized in the next table.

<table>
<thead>
<tr>
<th>Required DNS grid size</th>
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<tbody>
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<td>$n_x$</td>
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</tr>
<tr>
<td>$n_y$</td>
<td>129</td>
</tr>
<tr>
<td>$n_z$</td>
<td>160</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Selected resolution</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_x$</td>
<td>512</td>
</tr>
<tr>
<td>$n_y$</td>
<td>129</td>
</tr>
<tr>
<td>$n_z$</td>
<td>160</td>
</tr>
</tbody>
</table>

Table 6.3. Simulation set-up for x-50, y-03.

As it can be seen, the resolution in the wall normal direction was increased to its bare minimum, considering traditional channel simulations, due to the fact that as shown in the previous section, results are not affected in a big scale by this parameter. The resolution in the span-wise direction was increased due to the fact that it is known that the velocity fluctuations are influenced by this parameter. Finally, the resolution in $x$ was greatly increased due to the fact that the solutions seem to be very dependant on this parameter. Apart than considering that for smaller cavities, higher resolution must be used in order to characterize the correct geometry since the domain in the $x$ direction is much larger than wall. At this moment, the current parameters were considered to be a good compromise between results and efficiency.

From this point onward, the simulations were executed exclusively using multiple processor parallelization in a High Performance Computing center (HPC) of the Swedish National Infrastructure for Computing (SNIC) due to their larger size.

For the purpose of analyzing the current behaviour of the flow inside the cavity, the same test case as in the previous sections of the report was chosen, i.e. cavity of size $x^* = 50$ and $y^* = 3$. A limited number of turbulence intensity contours are highlighted in the next figure:
6.4. RESOLUTION REFINEMENT

The values in the contours of Figure 6.14 are not normalized, this means that the regions colored as 1 might have values greater than that. This is simply a hard limit for visualization.

A "wave-like" behaviour is encountered in Figure 6.14, where it can be seen that for a given height ($y^+$), the turbulence intensity would increase and decrease in a wave like manner while traversing the cavity. It was considered important to determine if these oscillatory flow structures are physical or artificially generated by the method. In case of not being a real state, two main reasons for the flow to behave the way it is doing are explored:

1. There is not enough time averaging being performed in the simulation.
2. Lack of resolution is producing the current oscillations

Both possible reasons for the wave-like behaviour are very easy to evaluate, and the process is done in the next section of the report.

6.4.1. Statistics time

The way to realize if the current statistics capture time is already enough for the analyzed case, is simply to perform a larger averaging for the test case. In this instance, the simulation was continued for an extra 1000 time units. Since by this point the turbulent behaviour of the channel has developed, statistics were recorded all through the new simulation time and the results are shown in figure 6.15.
It is possible to observe that the behaviour remained the same for this case, with some slight changes, but overall not enough to say that the wave structure in the solution is originated by lack of averaging. It is then concluded that the simulations taken have a valid statistics recording time.

### 6.4.2. Resolution

The next case to evaluate is to increase resolution and see if the results changes. Up to this point, particular care has been taken to avoid or minimize the Gibbs phenomenon in the y direction, since it is that way where faster changes and higher gradients are expected, however, inside the cavity, specially for a very small one, high changes of velocity in the x direction are a possibility. For this reason it was decided that the resolution in this direction should be increased to test the problem. It was effectively doubled such that the new simulation was run with the next parameters.

Under these conditions, the next results were achieved:

From figure 6.16, it is evident that the behaviour does change and that the resolution in this direction appears to be very important. A sort of convective effect can be seen in the turbulent intensity, and the fact that there are still some "peaky" oscillations, indicate that resolution could go higher in order to correctly solve everything inside the cavity as the Gibbs phenomenon is probably affecting the solution in the stream-wise direction.

![Image](image.png)
6.5. Remarks

Given the results, it is evident that the turbulence intensity does increase in the surroundings of the cavity, however it was decided to increase the resolution in x for all cases to that shown in figure 6.4 and execute the simulations once more.

The reason for not increasing resolution further from the test case, comes from 2 main facts: simulations with larger grids will take considerable more time to solve, and second but most important: the interest of this study lies in the surface, not inside the cavity, for this reason, if results are obtained, where even with certain oscillations, the flow seems stable at the surface zone (y=0), the results were considered to be good enough for the type of conclusions expected to be made in this report. This requirement was indeed met by all cases while executing them under the new conditions.
6. NUMERICAL TEST CASE

6.6. Final Numerical Set up

After the analysis already done in the previous sections, a new set of simulations needed to be run.

It was decided that 12 cases would be used for the analysis. Initially considering a case of a free simulation (no IBM) to evaluate $Re_{\tau}$ and other parameters, after which a variety of cavity sizes was executed. The main idea of this procedure is to be able to compare how depth and aspect ratio of the cavity influence in turbulence intensity and velocities individually. A list of the characteristics of the cases is shown next:

<table>
<thead>
<tr>
<th>Study resolution</th>
</tr>
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<tbody>
<tr>
<td>$n_x$</td>
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<td>$n_y$</td>
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<td>$n_z$</td>
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<table>
<thead>
<tr>
<th>Cavity cases</th>
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<tbody>
<tr>
<td>case</td>
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<tr>
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<td>11</td>
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<td>12</td>
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</tbody>
</table>

Table 6.5. Revised simulation cases.
CHAPTER 7

Results and Discussion

In this chapter, results from the numerical simulations and the experiments are disclosed. The findings from the DNS are shown first, followed by the validation by experiments and finally a joint discussion is presented.

7.1. DNS: Channel section without a cavity

The results are initially shown on a location without a cavity in order to validate their accuracy with respect to an unmodified solver:

![Figure 7.1. Mean velocity at a stream-wise station without a cavity. In log scale. Different cavity sizes: (x* = 0, y* = 0 --), (x* = 30, y* = 3 - - - -), (x* = 50, y* = 3 - - - -), (x* = 70, y* = 3 - - - -), (x* = 100, y* = 3 - - - -), (x* = 30, y* = 6 - - - -), (x* = 50, y* = 6 - - - -), (x* = 70, y* = 6 - - - -), (x* = 100, y* = 6 - - - -), (x* = 30, y* = 9 - - - -), (x* = 50, y* = 9 - - - -), (x* = 100, y* = 9 - - - -).](image)
Figure 7.2. Velocity fluctuation at a stream-wise station without a cavity. In log-scale. Different cavity sizes: $(x^+ = 0, y^+ = 0)$, $(x^+ = 30, y^+ = 3)$, $(x^+ = 50, y^+ = 3)$, $(x^+ = 70, y^+ = 3)$, $(x^+ = 100, y^+ = 3)$, $(x^+ = 30, y^+ = 6)$, $(x^+ = 50, y^+ = 6)$, $(x^+ = 70, y^+ = 6)$, $(x^+ = 100, y^+ = 6)$, $(x^+ = 30, y^+ = 9)$, $(x^+ = 50, y^+ = 9)$, $(x^+ = 100, y^+ = 9)$.

Figure 7.3. Turbulence intensity at a stream-wise station without a cavity. In log scale. Different cavity sizes: $(x^+ = 0, y^+ = 0)$, $(x^+ = 30, y^+ = 3)$, $(x^+ = 50, y^+ = 3)$, $(x^+ = 70, y^+ = 3)$, $(x^+ = 100, y^+ = 3)$, $(x^+ = 30, y^+ = 6)$, $(x^+ = 50, y^+ = 6)$, $(x^+ = 70, y^+ = 6)$, $(x^+ = 100, y^+ = 6)$, $(x^+ = 30, y^+ = 9)$, $(x^+ = 50, y^+ = 9)$, $(x^+ = 100, y^+ = 9)$.
As expected, very good agreement between all the cases and the reference solution is seen. Note that the plots start in different locations due to the already explained resolution loss originated by the IBM.

7.2. DNS: Channel at cavity center

The main findings from this study can be found in the current section and will be presented in a similar order as those before.

7.2.1. Mean velocity

The mean velocity profiles for all the cases analyzed in the study are presented directly in figure 7.4.

It had been observed, during the analysis of the reference case, that the presence of the cavities modify the flow. This is confirmed by Figure 7.4, where the mean velocity profile is increased by the presence of the cavity for all analyzed cases. This can be explained by the fact that due to the separation of the flow at the leading edge of the cavity, the no slip condition is not longer present at $y^+ = 0$ at the center of the it.
It is also possible to observe a very interesting feature: the cases with cavity length 30, 50 and 70 presented the same velocity profile in disregard to their cavity depth, with an overall increasing velocity as cavity length does.

7.2.2. Velocity fluctuations

The results for the velocity fluctuations can be visualized in Figure 7.5. Once again, the behaviour seen in the test case is confirmed. It can be seen that for a given cavity depth, the amplitude of turbulent fluctuations steadily increases as the cavity size does. However, the fact that the biggest cavity not always presents the higher value indicates that $u_{\text{rms}}$ depends on the relation between the cavity dimensions, rather than one specific parameter. The behaviour can also be explained from a flow separation standpoint, as it is known that on separated shear layers, the velocity fluctuations tend to increase.

Even though not measured in the experiments of the current project, it is also notable to show how the span wise velocity fluctuations are affected by the cavity, such information is shown in Figure 7.6

Note that increase in the velocity fluctuations is also produced by the presence of the cavity. However, there appears to be more uniformity in the increase among the cases.

![Figure 7.5. Velocity fluctuations at the cavity center. In linear scale. Different cavity sizes: ($x^+ = 0, y^+ = 0$), ($x^+ = 30, y^+ = 3$), ($x^+ = 50, y^+ = 3$), ($x^+ = 70, y^+ = 3$), ($x^+ = 100, y^+ = 3$), ($x^+ = 30, y^+ = 6$), ($x^+ = 50, y^+ = 6$), ($x^+ = 70, y^+ = 6$), ($x^+ = 100, y^+ = 6$), ($x^+ = 30, y^+ = 9$), ($x^+ = 50, y^+ = 9$), ($x^+ = 100, y^+ = 9$).]
7.2. DNS: CHANNEL AT CAVITY CENTER

7.2.3. Turbulence intensity

Turbulence intensity is analyzed taking into consideration some of the observations already done for mean and fluctuating velocity. Note that this is the most important variable of this study, as it is in the limit as the profile reaches the wall, that the normalized wall shear stress fluctuations can be measured. The results are presented in Figure 7.7, where it can be seen that the presence of the cavity increases the turbulence intensity of the flow. However, some particular observations can be made.

In the analysis of the mean flow, it was noted that for some cases, the velocity profile remained the same, yet, in Figure 7.7 it is evident that the same does not occur. From this fact it is possible to conclude that fluctuating velocities are dominant in the state of $T_u$ in the cavity region and are a highly influential factor in its value.

It is possible to determine the region of modified turbulence intensity in the surrounding area from the cavity if contour plots are analyzed. For this effect, one case is shown in figure 7.8.
7. RESULTS AND DISCUSSION

Figure 7.7. Turbulence intensity profile. In linear scale. Different cavity sizes: \((x^+ = 0, y^+ = 0 --), (x^+ = 30, y^+ = 3 \ldots\ldots), (x^+ = 50, y^+ = 3 \ldots\ldots), (x^+ = 70, y^+ = 3 \ldots\ldots), (x^+ = 100, y^+ = 3 \ldots\ldots), (x^+ = 30, y^+ = 6 \ldots\ldots), (x^+ = 50, y^+ = 6 \ldots\ldots), (x^+ = 70, y^+ = 6 \ldots\ldots), (x^+ = 100, y^+ = 6 \ldots\ldots), (x^+ = 30, y^+ = 9 \ldots\ldots), (x^+ = 50, y^+ = 9 \ldots\ldots), (x^+ = 100, y^+ = 9 \ldots\ldots)\).

Figure 7.8. Turbulence intensity contours. case: \(x^+ = 30, y^+ = 3\).
7.3. EXPERIMENT: BOUNDARY LAYER WITH CAVITIES BY MEANS OF BOUNDARY LAYER HOT-WIRE ANEMOMETER

As expected, there is an increase in the turbulence intensity not only at the cavity interior, but in a region on top of it of roughly the same size. Only one case was presented, but the same trend was observed in all studied simulations.

For the sake of a complete analysis, turbulence intensity in the span wise direction is shown in Figure 7.9. Clear increase in the values due to the presence of the cavity can be seen. However, a particularity of this variable is the fact that contrary to the stream-wise counterpart, the turbulence intensity in the $z$ direction seems to reduce for a given depth when the length of the cavity does.

Since it was seen that as cavity length increase, so does the mean and rms values of velocity, it is reasonable to attribute the reduction on $T_u$ to the fact that the mean velocity increases more than the span-wise fluctuations do, giving $U$ a dominant behaviour for the value of the intensity in the $z$ direction.

7.3. Experiment: Boundary layer with cavities by means of boundary layer hot-wire anemometer

The data obtained by pushing the boundary layer probe into the cavity is consistent with the DNS as illustrated in Figure 7.10 and 7.11. The turbulence intensity $T_u$ spikes up approaching the wall when the cavities are present in
7. RESULTS AND DISCUSSION

Figure 7.10. Turbulence intensity $T_u$ with respect to the distance from the wall in inner units $y^+$. The free-stream velocity $U_\infty$ is 8.5 m/s. Experimental data with cavities (▽), experimental data without cavities (△), filled markers have not been considered in the iterative procedure to seek for $u_\tau$ and the offset $y_0$. DNS data in the absence of cavities (—).

the insert, while no significant effect, compared to the baseline, is observed far from it. In the case without the presence of the cavities, the obtained data looks better at the lowest speed displayed, while at 16 m/s averaging of the signal might be highlighted by the flat trend approaching the wall.

7.4. Experiment: Boundary Layer with cavities by means of wall-flush-mounted hot-wire anemometer

The results from the cavity probes are rolled out in this section, starting with the mean wall shear-stress values and moving forward with their fluctuations and the turbulence intensity seen as the limit of $\tau_{w,rms}/\tau_w$ at the wall position. The list of symbols representing each case is summarized in Table 7.1.

The mean wall shear-stress points are necessary to verify if the data is trustworthy. Being the probes calibrated upon the same characteristic curve, i.e. Figure 4.8, they should overlap and this clearly happens as Figure 7.12
7.4. EXPERIMENT: BOUNDARY LAYER WITH CAVITIES BY MEANS OF WALL-FLUSH-MOUNTED HOT-WIRE ANEMOMETER

<table>
<thead>
<tr>
<th>Cases</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>B02SWnc</td>
<td>●</td>
</tr>
<tr>
<td>B04LWnc</td>
<td>□</td>
</tr>
<tr>
<td>B04SWnc</td>
<td>▼</td>
</tr>
<tr>
<td>B04SW</td>
<td>▲</td>
</tr>
<tr>
<td>B02LW</td>
<td>⋆</td>
</tr>
</tbody>
</table>

Table 7.1. Case and symbol pairs used to plot the data acquired with the wall shear-stress probes.

demonstrates. The friction Reynolds numbers $Re_{\tau}$ explored go from about 280 to about 680, while the respective range of wall shear-stress goes from 0.1 to $1.2 \, N/m^2$.

Figure 7.11. Turbulence intensity $T_u$ with respect to the distance from the wall in inner units $y^+$. Free-stream velocity $U_\infty$ is 16 m/s. Experimental data with cavities (▽), experimental data without cavities (△). DNS data in the absence of cavities (—).
Figure 7.12. Experimental points of the mean wall shear-stress: B02SWnc (●), B04LWnc (■), B04SWnc (▼), B04SW (▲), B02LW (★).

7.4.1. Mean velocity

7.4.2. Wall shear-stress fluctuations

Figure 7.13 shows the wall shear-stress root mean square. As expected from the literature Alfredsson et al. (1988), the fluctuations tend to increase the higher friction Reynolds number \( Re_\tau \), i.e. the higher the free-stream velocity. Some deviation between the 0.4 mm-deep cylindrical cavity and full cavity cases using a "long-wire" is observed. The other cases are standing alone, thus they are not comparable to other configurations. Yet, they provide a qualitative trend of the wall shear-stress fluctuations in the presence of the cavity.

7.4.3. Turbulence intensity

It can be observed from Figure 7.14 that the turbulence intensity \( T_u \) decreases monotonically the higher the friction Reynolds number \( Re_\tau \). Please note that the data did not undergo any spatial averaging correction process. However, it can be qualitatively observed the tendency of \( T_u \) to rise the lower the friction Reynolds number \( Re_\tau \).
Similar observations can be done with respect to Figure 7.15. In both cases, again, the trend shows a decrease in turbulence intensity as $Re_\tau$ increases. Higher values are experienced by the deeper cavity, suggesting the evolution of larger eddies in the area. The same configuration shows a plateau-like behaviour at lower $Re_\tau$ values, again suggesting the need for $L+$ correction.

### 7.5. Discussion

It has been claimed that using flush mounted probes on top of cavities provide many benefits for the measurements of wall shear stress fluctuations. To test this statement, DNS and experiments have been used in order to analyze the effect of introducing such cavities in the flow.

From the results, especially by observing Figures 7.7, 7.10 and 7.11, it is evident that the turbulence intensity at the center of the cavity is generally increased. This means that the measured value of the fluctuating shear stress is also modified, as stated in (1.1).

The previous observation is an important one, as in the studies by Gubian et al. (2019), an increase in values of fluctuating shear-stress was reported.
which they attributed to the fact that their probe was solving the full range of turbulent fluctuations never solved before. The results of the present study show that the increase can be attributed to the presence of the cavity while performing the measurements instead.

Additionally, by analyzing the data presented in Figures 7.8, 7.10, 7.11, 7.14 and 7.15 it is apparent that introducing a cavity also produces an alteration of the flow in the region surrounding it, which effectively means that using a slotted surface would not only provide incorrect measurements for a flush mounted hot wire, but will also affect the accuracy of data acquisition techniques that work for other regions.

It is clear that the value of the quantities have a certain dependence on the size of the cavity. For this reason, a representation of size is introduced as Aspect Ratio (AR) which takes into account the relation of length and depth and is defined as follows:

\[ AR = \frac{C_x}{C_z}, \]  

\[ (7.1) \]

\[ \text{Figure 7.14. Turbulence intensity experimental points: B02SWnc (●), B04LWnc (■), B04SWnc (▼).} \]
where $C_x$ is the cavity stream-wise length and $C_z$ is the cavity wall-normal length (depth). Introducing this quantity allows for a better comparison of the data. The cavity size of the experiment done in Figure 7.11 corresponds to $AR = 5$. The same value (approximately) can be recovered from the simulations for the case of size $x^+ = 50$ and $y^+ = 9$. If the corresponding plot for this case in Figure 7.7 is compared to the results of 7.11, it can be seen that the simulations and experiments are not only providing qualitatively confirmation, but they produce very similar numerical results which gives more strength to the findings of this project.

Additional trends can be seen if the wall shear stress fluctuations are plotted as a function of the aspect ratio, as shown in Figure 7.16

As was mentioned before, $\tau_{w,rms}$ is definitely measured with a higher value when flush mounted probes are used in tandem with cavities. However, in Figure 7.16 there seems to be an $AR$ range where the turbulent fluctuations increase, just to begin decreasing again after a limit factor. This behaviour seems to be dependent on how the flow behaves inside the cavity as it size varies. Although not in the scope of the study, it is important to give a possible

![Figure 7.15. Turbulence intensity experimental points: B04SW (▲), B02LW (★).]
7. RESULTS AND DISCUSSION

Figure 7.16. $\tau_{w, rms}$ as function of $AR$. The value of the wall shear fluctuations at a conventional channel (---) is shown with the values obtained from the simulations of all cavity sizes. Cavities with $y^+ = 3$ (---), $y^+ = 6$ (---) and $y^+ = 9$ (---).

explanation for this phenomenon, as this would further validate the claims and findings stated therein.

The found behaviour seems to be completely dependant on the mean flow structures surrounding the cavities, as for the different ranges, the findings are presented in the following subsections.

7.5.1. Low AR

Results obtained by Catalano (1987), who conducted experiments and simulations in up scaled low aspect ratio cavities, show the same flow characteristics as the low $AR$ cases analyzed in the present study i.e the presence of a vortex core and a re-circulation area below the whole cavity, as seen in figure 7.17. The difference in size between the cavities leads to believe that this is a general characteristic of the flows under these conditions.

7.5.2. High AR

High aspect ratio slotted walls can be conveniently analyzed as two combined cases: a Backwards Facing Step (BFS) at the leading edge of the cavity and a Forward Facing Step (FFS) at the trailing edge; both being conditions broadly studied by the scientific community. To observe this, the cavity case: $x^+ = 100$, $y^+ = 3$, which posses a $AR = 33$, is analyzed.
7.5. DISCUSSION

Clear characteristics observed in BFS studies such as the one conducted by Le et al. (1997) are present in the case, as seen in Figure 7.18.

Figure 7.17. Mean flow structure for low AR cavities.

Figure 7.18. BFS-like characteristics.

Just as seen in the studies by Le et al., the presence of a re-circulation zone following the flow separation at the leading edge of the cavity was observed.
Additionally, a seemingly purely cavity-depth dependant reattachment point in the stream-wise direction was evidenced for all the cases where the shear layers were able to reattach.

In the referenced study, it was also noted that the turbulent shear layer do not appear to be recovered even 20 step heights after the reattachment zone, which can also be seen in the present study if the turbulence intensity is analyzed for the same case.

By the point that the flow reaches the FFS, a characteristic separation bubble is also seen ahead of the trailing edge (Figure 7.19), such as it was evidenced in studies done by Fang et al. (2019) and Iftekhar & Agelin-Chaab (2016), where a low intensity vortex core is seen ahead of cavity edge.

It is noted that a characteristic vortex usually seen at the wake of a FFS is not appreciated in this study. Probable reasons for this are a lack of resolution at the relevant location or the need for higher constant values in the control forces of the IBM since initially there was not a big interest to produce perfectly defined sharp corners in the cavity. The results are, however, qualitatively the same and serve as a good reference.

7.5.3. Moderate AR

There seems to be conditions that combines the previous characteristics. If the aspect ratio is in a medium range, the flow remains separated as seen in low AR cases however it does so with the presence of the two characteristic vortex cores of the BFS and FFS which interact with each other. This finding is in
accordance with studies done by Ashrafian et al. (2004). A demonstration of the characteristic can be seen in Figure 7.20

![Figure 7.20](image)

**Figure 7.20.** Mean flow structures for moderate $AR$, the presence of two vertices is apparent.

As noted by Pearson et al. (2011) the formation of the trailing edge separation bubble seems to be dependant on the upstream flow and acquires different shapes according to the cases.

### 7.5.4. Final remarks

The mean flow behaviours just discussed could explain the trends seen in Figure 7.16, i.e. why there seems to be monotonic increase in the wall shear stress fluctuations for low $AR$ (one strong vortex, separated flow), just to deviate from this linear trend as $AR$ increases (formation of a secondary vortex, flow separated) and finally a relaxation of the shear stress as the flow is allowed to reattach for larger $AR$.

Based on all the comparisons, it has been found that the obtained flow characteristics for the analyzed geometry match, at least in a qualitatively level, to those of studies performed with different purposes and scales. From this, it is possible to state that the found behaviours are inherent to the geometry and independent of size. This is an important claim, as it allows to extrapolate the conclusions stated in the report to cases not studied therein.
7. RESULTS AND DISCUSSION

7.6. Conclusions

In the body of this project, it has been demonstrated that turbulence intensity, thus, wall shear stress fluctuations increase due to the presence of cavities. Following this idea, the increase in the values reported by Gubian et al. (2019) can be attributed to the cavity effects instead of their probe solving the full range of fluctuations as was stated. Additional comments on this are provided by Örlü & Schlatter (2020)

Flush mounted hot wires along with cavities are therefore, not recommended for the measurement of this particular characteristic.

Further studies for the deepening of the understanding of these characteristic are, among others, the following:

1. DNS with different Reynolds numbers.
2. Applying filters to DNS to simulate spatial resolution effects.
3. Having different probe length in the experiments to analyze L+ effects.
References


REFERENCES


APPENDIX A

Matematical tools for spectral methods

In this Appendix, the mathematical tools needed to understand how a spectral solver for Navier Stokes equations works are provided, along with some examples that might aid in their understanding. This chapter is strongly influenced by Canuto et al. (1988), Hussaini et al. (1984) and it serves as a compilation of the specific methods used in Simson.

A.1. Spectral Approximation

The spectral methods for the solution of partial differential equations are based in the expansion of a function \( u \) into a sequence of orthogonal functions \( \phi_k \), i.e. a linear combination of orthogonal functions. In 1 dimension, considering \( u = u(x) \) then a expansion can be given by:

\[
\hat{u}(x) = \sum_{k=-\infty}^{\infty} \hat{u}_k \phi_k(x) \quad (A.1)
\]

Note that the coefficients \( \hat{u}_k \) are not dependant on \( x \) anymore, this is a useful characteristic of this procedure. If enough coefficients are used in the expansion to represent all the essential characteristics of the function, there will be a rapid decay in their value such that truncating the infinite series after a few more terms would yield a good approximation of the function. In the best case, given certain conditions for the type of expansion, the decay will be exponential and in these circumstances, it is said that that the method posses spectral accuracy.

There exist many types of polynomials that guaranty spectral accuracy and can be used for this purpose, however the discussion in the present section will be centered around the Fourier and Chebyshev expansions.

A.1.1. Fourier Transform

The Fourier transformation method posses spectral accuracy if it is applied to an infinitely differential periodic function. The set of functions used for this approximation are:

\[
\phi_k(x) = e^{ikx} \quad (A.2)
\]
which is set of trigonometric polynomials and an orthogonal system in the interval \([0, 2\pi]\). The Fourier series of the system is then:

\[
    u(x) = \sum_{k=-\infty}^{\infty} \hat{u}_k e^{ikx} \quad (A.3)
\]

where the Fourier transform, i.e. the coefficients, are defined as:

\[
    \hat{u}_k = \frac{1}{2\pi} \int_{0}^{2\pi} u(x) e^{-ikx} dx \quad k = \pm 1, \pm 2,... \quad (A.4)
\]

It can be seen that each coefficient depends on the value of the original function in the whole field and that there is a scaling equal to the domain size.

As mentioned before, under certain conditions the coefficients will rapidly decay and the series can be truncated and still give good results, this is known as **Truncated Fourier series** and is defined as follows:

\[
    u(x) = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{u}_k e^{ikx} \quad (A.5)
\]

Where \(N\) is equal to the number of coefficients, often interpreted as frequencies, included in the analysis.

In most applications for numerical analysis, the continuous function \(u(x)\) is not available, therefore the coefficients \(\hat{u}_k\) cannot be acquired in closed form, for this reason they are approximated in what is known as **Discrete Fourier Transform**.

**A.1.1a. Discrete Fourier Transform (DFT).** In the process of performing DFT to a function, first a set of nodes or grid points are defined:

\[
    x_j = \frac{2\pi j}{N} \quad j = 0,...,N - 1 \quad (A.6)
\]

Where \(N\) is the total number of nodes and \(2\pi\) is the domain size, from which can be said that the field is divided into a set of equally spaced nodes. In these conditions, the discrete Fourier coefficients of function \(u\) are:

\[
    \hat{u}_k = \frac{1}{N} \sum_{j=0}^{N-1} u(x_j) e^{-ikx_j} \quad -\frac{N}{2} \leq k \leq \frac{N}{2} - 1 \quad (A.7)
\]

and the inversion formula is:

\[
    u(x_j) = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{u}_k e^{ikx_j} \quad j = 0,1,...,N - 1 \quad (A.8)
\]

It can be seen that \(\hat{u}_k\) depends on the \(N\) values of \(u(x_j)\), and the DFT can interpreted as the mapping of the \(N\) \(u(x_j)\) and \(\hat{u}_k\) values with \(j = 0,...,N - 1\) and \(k = -\frac{N}{2},...,\frac{N}{2} - 1\). Furthermore, it is noted that \(\hat{u}_k\) can be regarded as an
A.1. SPECTRAL APPROXIMATION

approximation of $\tilde{u}_k$ using the trapezoidal rule to evaluate the integral in (A.4) and is used instead of $\tilde{u}_k$ in (A.9) so that:

$$u(x) = \sum_{k=-N/2}^{N-1} \tilde{u}_k e^{ikx}$$  \hspace{1cm} (A.9)

A.1.1b. Fast Fourier Transform. The evaluation of DFT can usually be performed by means of the Fast Fourier algorithm that is designed to reduce the computational cost of the procedure. In most cases the operation count is of the order of $N \log N$ with most algorithms requiring $N$ to be multiple of 2, which must be kept in mind while choosing the numerical method to use.

A.1.1c. Differentiation. Differentiation in a spectral method is performed in different ways depending on the type of space the work is being done: physical or transform. The method used in this report is the transform space differentiation, however an overview of both will be given.

In Transform Space, the method is called Fourier Galerkin derivative and the differential of a function is obtained by multiplying each Fourier coefficient by the imaginary unit ($i$) and the corresponding wavenumber, such that if the Fourier transform of function $u(x)$ is $u(x) = \sum_{k=-\infty}^{\infty} \hat{u}_k e^{ikx}$, then its derivative is given by:

$$u'(x) = \sum_{k=-\infty}^{\infty} ik\hat{u}_k e^{ikx}$$  \hspace{1cm} (A.10)

In Physical Space the derivatives are obtained from the values of the function $u$ at the nodes $x_j$ given by (A.6). The process is as follows:

1. Get the discreet Fourier coefficients of $u$.
2. Multiply coefficients by $ik$.
3. Transform back to physical space. In this case the scaling factor for the Fourier coefficient is changed, the correct representation is now:

$$\hat{u}_k^{(1)} = \frac{ik}{N} \sum_{j=0}^{N-1} u(x_j) e^{-ikx_j} \quad -\frac{N}{2} \leq k \leq \frac{N}{2} - 1$$  \hspace{1cm} (A.11)

In this context, the derivative of $u$ at node $x_l$ is given by:

$$(\mathcal{D}_N u)_l = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{u}_k^{(1)} e^{ikx_l} \quad l = 0, 1, \ldots, N - 1$$  \hspace{1cm} (A.12)

(A.12) may also be represented in matrix form, where a matrix $D$ can be defined in closed form and the derivation procedure will reduce to a matrix multiplication problem rather than resorting to Fourier transforms, in this case the total amount of operations required is $2N^2$. More details of the procedure
and the definition \( D \) can be found in Canuto et al. (1988). The complete method is called in this case *Fourier collocation derivative*.

### A.1.2. Chebyshev Polynomials

Chebyshev polynomials are often used for expansions of non periodic problems. The set of polynomials is orthogonal in the interval \([-1, 1]\) and is given by:

\[
T_k(x) = \cos k\theta
\]

with \( \theta = \cos^{-1}(x) \). By expanding the polynomials and taking advantage of certain trigonometric relations, the functions can be obtained by the recursive relation:

\[
T_{k+1}(x) = 2xT_k(x) - T_{k-1}(x)
\]

with \( T_0(x) = 1 \) and \( T_1(x) = x \). The Chebyshev expansion of a function \( u(x) \) is then defined as:

\[
u(x) = \sum_{k=0}^{\infty} \hat{u}_k T_k(x) \tag{A.15}\]

and the coefficients are defined by:

\[
\hat{u}_k = \frac{2}{\pi c_k} \int_{-1}^{1} u(x)T_k(x)w(x)dx
\]

with \( w(x) = (1-x^2)^{-\frac{1}{2}} \) being known as "weight" and defined in the derivation of Chebyshev polynomials. On the other hand, \( c_k \) is defined as follows:

\[
c_k = \begin{cases} 
2 & \text{if } k = 0 \\
1 & \text{if } k \geq 1 
\end{cases}
\]

Since this set of polynomials possess spectral accuracy, the expansion can also be truncated and still be a good representation of the original function:

\[
u(x) = \sum_{k=0}^{N} \hat{u}_k T_k(x) \tag{A.18}\]

The attractiveness of the Chebyshev expansion, is that carefully choosing the independent variable \( x \) as \( x = \cos(\theta) \), the Chebyshev polynomials become cosine functions (trigonometric series). This way, many procedures intended for Fourier systems can be adapted to work for Chebyshev polynomials. In this order of ideas, if a periodic and even function \( \tilde{u} \) is defined as \( \tilde{u}(\theta) = u(\cos(\theta)) \), then as mentioned before, the expansion will be as follows:

\[
\tilde{u}(\theta) = \sum_{k=0}^{\infty} \hat{u}_k \cos k\theta \tag{A.19}
\]

where \( \hat{u}_k \) for equation (A.19) is the same as that shown in (A.16), i.e. the Chebyshev series for \( u \) corresponds to a cosine series for \( \tilde{u} \).
A.1. SPECTRAL APPROXIMATION

A.1.2a. Discrete Chebyshev Series. In a similar manner as for DFT, the coefficients are not found in closed form but approximated. For this case, the nodes and weights are obtained from Gauss-type quadrature, in particular the Gauss-Lobatto formulas are used, such that:

\[ x_j = \cos \frac{\pi j}{N} \quad j = 0, ..., N \] (A.20)

\[ w_j = \frac{\pi}{c_j N} \quad j = 0, ..., N \] (A.21)

\[ \gamma_k = \frac{\pi}{2} c_k \quad k = 0, ..., N \] (A.22)

where:

\[ c_j = \begin{cases} 2 & \text{if } j = 0, N \\ 1 & \text{if } 1 \leq j \leq N - 1 \end{cases} \] (A.23)

\[ c_k = \begin{cases} 2 & \text{if } k = 0, N \\ 1 & \text{if } 1 \leq k \leq N - 1 \end{cases} \] (A.24)

The discreet Chebyshev transform is then defined as:

\[ \tilde{u}_k = \frac{1}{\gamma_k} \sum_{j=0}^{N} u(x_j) T_k(x_j) w_j \quad k = 0, 1, ..., N \] (A.25)

By introducing all previous results, (A.25) takes the form:

\[ \tilde{u}_k = \frac{2}{N c_k} \sum_{j=0}^{N} \frac{1}{c_j} u(x_j) \cos \frac{\pi j k}{N} \quad k = 0, 1, ..., N \] (A.26)

The transformation from chebyshev to physical space is given by:

\[ u(x_j) = \sum_{k=0}^{N} \tilde{u}_k T_k(x_j) \quad k = 0, 1, ..., N \] (A.27)

and by replacing previous notation, the final form is:

\[ u(x_j) = \sum_{k=0}^{N} \tilde{u}_k \cos \left( \frac{\pi j k}{N} \right) \quad k = 0, 1, ..., N \] (A.28)

It can be seen that these are cosine series thus The FFT algorithm can be modified in such a way that the discreet Chebyshev coefficients \( \tilde{u}_k \) are obtained in an efficient way.
A.1.2b. **Differentiation.** The derivative of a function $u$ expanded in Chebyshev polynomials can be obtained in transformed and physical space just as in the Fourier approach. As in the previous case both will be outlined but it is noted that the solver used in this report implements the transform method.

In *Transformed Space* the derivative of a function $u(x)$ is called **Chebyshev Galerkin derivative** and can be expressed as follows:

$$u'(x) = \sum_{k=0}^{\infty} \hat{u}_k^{(1)} T_k$$  \hspace{1cm} (A.29)

where

$$\hat{u}_k^{(1)} = \frac{2}{c_k} \sum_{p=k+1 \text{ odd}}^{\infty} p \hat{u}_p$$  \hspace{1cm} (A.30)

Likewise, an expression for the coefficients of the second derivative would be given by:

$$\hat{u}_k^{(2)} = \frac{1}{c_k} \sum_{p=k+2 \text{ even}}^{\infty} p(p^2 - k^2) \hat{u}_p$$  \hspace{1cm} (A.31)

Due to trigonometric identities, it is possible to obtain an efficient way to differentiate a function by means of Chebyshev space. This is done by computing the coefficients in decreasing order (first those of higher order derivatives for all $k$) by following the next general recurrence relation:

$$2k^{(q-1)} = c_{k-1}^{(q)} \hat{u}_{k-1}^{(q)} - \hat{u}_{k+1}^{(q)}$$  \hspace{1cm} (A.32)

Where $q$ is the order of the derivative the coefficient corresponds to. More details in the derivation of the recurrence can be found in [1].

In *Physical space* the process is called **Chebyshev collocation differentiation** and it is accomplished by means of transform method. The process is as follows:

1. The discreet Chebyshev coefficients are obtained by means described in this section.
2. Use the recurrence (A.32) to differentiate in transform space.
3. Transform back to physical space.

It is also possible to express the collocation derivative in matrix form as:

$$(D_N u)_l = \sum_{j=0}^{N} (D_N)_{lj} u(x_j)$$  \hspace{1cm} (A.33)

Matrix $(D_N)_{lj}$ can be obtained in closed form if needed.
A.2. Method Induced Errors

There are some errors introduced to the mathematical models that have to do with the numerical scheme used and not to the physical system itself, among these, notorious ones are:

A.2.0a. Aliasing. Aliasing can happen in Fourier and Chebyshev systems, however for this case only the Fourier is analyzed as operations that introduce strong aliasing errors in the solver are only present in Fourier-discretized directions. A relation between the discreet and continuous Fourier coefficient is presented next:

\[ \tilde{u}_k = \hat{u}_k + \sum_{m=-\infty}^{\infty} \hat{u}_{k+m} - \frac{N}{2}, \quad \frac{k}{2} \leq \frac{N}{2} - 1 \]  

(A.34)

The second term of the R.H.S shows 2 features:

1. There is a deviation (error) of the discreet Fourier coefficient with respect to the continuous one.
2. It is possible to see that the \( k \)-th DFT coefficient depends not only in the \( k \)-th mode of \( u \) but in all the \((k + m)\)th frequencies which are considered as a frequency that "alias" the \( k \)-th

The latter numeral occurs because of the trigonometric polynomial characteristic that \( \phi_{k+Nm}(x_j) = \phi_k(x_j) \), this means that even though, in general, the function \( \phi_{k+Nm}(x) \) is not equal to \( \phi_k(x) \), they take the same value at the nodes \( x_j \) and are, therefore, indistinguishable. This poses an error because if the approximation to function \( u(x) \) given by the DFT is:

\[ u(x) = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \tilde{u}_k \phi_k \]  

(A.35)

Then considering (A.34)

\[ u(x) = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \left( \hat{u}_k \phi_k + \sum_{m=-\infty}^{\infty} \hat{u}_{k+m} \phi_k \right) \]  

(A.36)

Where the second term of the R.H.S is the Aliasing error, induced not by truncating the Fourier series, but by the fact that a DFT is used instead of a continuous one.

A.2.0b. Gibbs Phenomenon. The spectral approximations work well in continuous functions, but when a discontinuity point is present in the interval, The truncated or discreet Fourier series of said function will begin to present an oscillatory behaviour. Depending on the type of function, there might be an overshoot in the oscillations or not, and its location converges towards the
discontinuity point as the number of retained frequencies increases. The oscillator\al{} behaviour described by the Gibbs phenomenon is important because it influences the truncated Fourier series not only in the point of discontinuity but in the entire interval $[0,2\pi]$ which of course limits the reliability of the approximation.

In Figure A.1 a visualization of the problem is presented for different number of retained frequencies ($n$) while approximating a step function in the interval $[0,2\pi]$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{gibbs.png}
\caption{Visualization of Gibbs phenomenon. The black line represents the original signal while the different colored lines are Fourier approximations with different retained frequencies.}
\end{figure}

In concrete, the Gibbs phenomenon is related to the slow decay of the Fourier coefficients of a discontinuous function, i.e. the lack of spectral accuracy, the way to control its effects is to use procedures that attenuate the higher order coefficients, this way, the oscillations associated to these modes are damped. It must be taken into account that the structure of the coefficients carry information about the function and too strong smoothing would also cause negative effects in the approximation.
A.3. Numerical Schemes for solutions of systems evolving in time

In Computational Fluid Dynamics (CFD), there is an interest in evaluating the evolution of a system in time, starting with a known initial state $u^0$. These type of formulations can usually be described as Initial Value Problems, which are systems of ODE with initial conditions and characterized as follows:

$$\frac{du}{dt} = f(t, u(t))$$

$$u(t_0) = u^0$$ (A.37)

The main idea for the solutions of these problems is to evaluate the function $f$ at a given time $t^n$ and use that information to predict the state at a time $t^{n+1} = t^n + \Delta t$. The general representation of the schemes is given next:

$$u^{n+1} = u^n + \Delta t \Phi(t^n, u^n)$$ (A.38)

In this report two families of methods will be presented.

A.3.1. One Step Methods

One step methods depend on information on only one point in time to advance to the next. 2 popular methods are:

A.3.1a. Crank-Nicolson Method. Crank-Nicolson scheme is based on the use of an intermediate step $t^{n+\frac{1}{2}}$ from which a Taylor expansion of (A.37) is done in both directions (to $t^{n+1}$ and $t^{n-1}$). The result is then averaged and the final expression for the scheme takes the next form:

$$u^{n+1} = u^n + \frac{1}{2} \Delta t [f^{n+1} + f^n]$$ (A.39)

A.3.1b. Runge-Kutta Method. Runge-kutta methods are based on obtaining a numerical approximation of the integral $\Phi = \int_{t^n}^{t^{n+1}} f(t, y(t)) dt$ by quadrature formulas. This procedure can be seen, in a practical way, as if the scheme evaluates the functions in a multitude of intermediate steps before finally predicting the actual physical state at $t^{n+1}$.

A representation of RK3 (Scheme of third order) with 4 stages is shown next:

$$K_1 = f(t^n, u^n)$$

$$K_2 = f(t^n + \frac{\Delta t}{2}, u^n + \frac{\Delta t}{2} K_1)$$

$$K_3 = f(t^n + \Delta t, u^n + 2\Delta t K_2 - \Delta t K_1)$$ (A.40)

$$u^{n+1} = u^n + \frac{\Delta t}{6} (K_1 + 4K_2 + K_3)$$
The scheme can also be seen as follows:

\[ u^{n+1} = u^n + a_n f^n + b_n f^{n-1} + c_n f^{n-2} \]  \hspace{1cm} (A.41)

It is noted that with (A.41) each time step is defined as \( t^{n+1} = t^n + \Delta t_n \) where \( \Delta t_n \) is different each advancement. This is due to the fact that with this notation, the intermediate steps are treated as full steps but a real physical time advancement would only be produced after these intermediate steps are evaluated.

### A.3.2. Multi-step Methods

Multi-step methods are those that use information from more than one point in order predict the value of the function. Depending on the order of the method, they might not be self-starting, in these cases another method must be used for the first time steps.

#### A.3.2a. Adams Bashford

The second order Adams-Bashford (AB2) is presented next:

\[ u^{n+1} = u^n + \frac{1}{2} \Delta t n [3f^n - f^{n-1}] \]  \hspace{1cm} (A.42)

### A.3.3. Combination of Numerical Schemes

In the solutions of PDE, it is quite often that the system is reduced to an ODE with the next characteristics:

\[ \frac{du}{dt} = G + Lu \]  \hspace{1cm} (A.43)

Where the first term of the R.H.S is a non linear operator and the second term is the linear part. For computational reasons it is often required that the non linear part is discretized with an explicit method and the linear part with an implicit one. In this section an example will be shown where 2 different schemes are used for one equation.

#### A.3.3a. Example of implementation

One of the equations that needs to be solved for the simulations in [3] is presented next:

\[ \frac{\partial g}{\partial t} = h_g + \frac{1}{R_e} \nabla^2 g \]  \hspace{1cm} (A.44)

here \( h_g \) is the non linear part and is discretized with AB2 and the second term in the R.H.S is linear and discretized by Crank-Nicolson. They can be treated individually such that:

\[ g^{n+1} = g^n + \frac{\Delta t}{2R_e} \nabla^2 (g^{n+1} + g^n) \]  \hspace{1cm} \text{By using (A.39) on (A.37) and } f = \frac{1}{R_e} \nabla^2 g

\[ g^{n+1} = g^n + \frac{\Delta t}{2} (3h_g^n - h_g^{n-1}) \]  \hspace{1cm} \text{By using (A.42) on (A.37) and } f = h_g
A.4. Solutions of ODE Based on Spectral Approximations

It can be seen that the L.H.S and the first term of the R.H.S remain the same on both equations (since they represent the current and next time step to be predicted). The actual prediction that the methods do for the time advancement is given by the second term of the R.H.S in both cases. Therefore both equations can be condensed into one if the next state $g^{n+1}$ is predicted by adding their contributions to the current state $g^n$, as follows:

$$g^{n+1} = g^n + \frac{\Delta t}{2}(3h^n_y - h^{n-1}_y) + \frac{\Delta t}{2Re} \nabla^2 (g^{n+1} + g^n) \quad (A.45)$$

On the R.H.S the second and third terms represents the evolution of $g$ due to the non linear and linear parts respectively. From this expression, the final form of the equation that was used in [3] can be trivially derived:

$$\left(1 - \frac{\Delta t}{2Re} \nabla^2\right)g^{n+1} = \frac{\Delta t}{2}(3h^n_y - h^{n-1}_y) + \left(1 + \frac{\Delta t}{2Re} \nabla^2\right)g^n \quad (A.46)$$

A.4. Solutions of ODE Based on Spectral Approximations

The spectral approximation methods have been introduced because they allow to represent ODE in a convenient way for their solution. It has been seen that spectral discretizations usually lead to linear systems of the form:

$$LU = F \quad (A.47)$$

where $U, F$ are vectors consisting of the expansion coefficients the function $u, f$ and boundary conditions, while $L$ is the appropriate matrix in transform space.

A.4.1. Variables with multiple dimension and time dependence

It is important to note that if having a function that depends on time and space $u(x, t)$, by discretizing using the Fourier transform in $x$, the next result is obtained:

$$u(x, t) = \sum_{k=-N_2}^{N_2-1} \hat{u}_k(t) e^{ikx} \quad (A.48)$$

Note that the coefficient $\hat{u}_k$ is exclusively a function of time, and it does not depend on the dimension that the original variable was expanded on. In this way the order of a differential equation can be reduced by applying a transformation: a PDE in $u(x, t)$ will become a ODE in $\hat{u}_k(t)$ after a space discretization and in turn, an ODE would become an algebraic system. Something similar would happen if the function depend on two dimensions in space where $u = u(x, y)$. If such a equation is expanded only in $x$, then the spectral coefficients will still depend on $y$ such that: $\hat{u}_k(y)$
To illustrate how the transformed space is used for solution of equations, the next problem will be solved by 2 methods:

\[
\frac{d^2 u}{dx^2} - \lambda u = f \tag{A.49}
\]

### A.4.2. Fourier Approximations

For this case, all the terms of (A.49) are transformed according to (A.3) and (A.10). (A.49) becomes:

\[
- \sum_{k=-\infty}^{\infty} k^2 \hat{u}_k e^{ikx} - \lambda \sum_{k=-\infty}^{\infty} \hat{u}_k e^{ikx} = \sum_{k=-\infty}^{\infty} \hat{f}_k e^{ikx} \tag{A.50}
\]

Due to the orthogonality property of the Fourier expansion, the sums and polynomials can be discarded such that (A.50) becomes:

\[
- k^2 \hat{u}_k - \lambda \hat{u}_k = \hat{f}_k \quad k = -\frac{N}{2}, \ldots, \frac{N}{2} - 1 \tag{A.51}
\]

The only unknowns in (A.51) are the coefficients \( \hat{u}_k \), which can be obtained by solving the equation for each \( k \). It can also be represented in Matrix form with the same structure as (A.47) and can be solved efficiently by any algorithm designed for the solution of linear systems.

### A.4.3. Chebyshev Tau Approximations

The Chebyshev Tau approximation (following a similar thought process as for the Fourier case) can be written as:

\[
\hat{u}_n^{(2)} - \lambda \hat{u}_n = \hat{f}_n \quad n = 0, \ldots, N - 2 \tag{A.52}
\]

The differentiation procedure is done by means of the recurrence formula (A.32) with \( q = 2 \):

\[
2n\hat{u}_n^{(1)} = c_{n-1} \hat{u}_{n-1}^{(2)} - \hat{u}_{n+1}^{(2)} \tag{A.53}
\]

Now the value of \( \hat{u}^{(2)} \) is recovered from (A.52) and used in (A.53) to obtain:

\[
2n\hat{u}_n^{(1)} = c_{n-1} (\hat{f}_{n-1} + \lambda \hat{u}_{n-1}) - (\hat{f}_{n+1} + \lambda \hat{u}_{n+1}) \quad n = 1, \ldots, N - 3 \tag{A.54}
\]

Note the quantity of nodes \( n \) where the equation is valid reduces, as with each derivative, more nodes are needed for its evaluation. The recursion (A.53) is called again but this time with \( q = 1 \) this way all the derivative coefficients are removed from the equation, and a linear system with the same shape as (A.47) dependant only on \( \hat{u}_n \) is acquired. Some corrections need to be done for the nodes that are dropped in each derivative, but the final result can be
expressed as follows:

\[
\frac{c_{n-2} \lambda}{4n(n-1)} \hat{u}_{n-2} + \left[ 1 + \frac{\lambda \beta_n}{2(n^2 - 1)} \right] \hat{u}_n + \frac{\lambda \beta_{n+2}}{4n(n+1)} \hat{u}_{n+2} = \frac{c_{n-2}}{4n(n-1)} \hat{f}_{n-2} - \frac{\beta_n}{2(n^2 - 1)} \hat{f}_n + \frac{\beta_{n+2}}{4n(n+1)} \hat{f}_{n+2} \quad (A.55)
\]

with \( n = 2, ..., N \) and:

\[
\beta_n = \begin{cases} 
1 & \text{if } 0 \leq n \leq N - 2 \\
0 & \text{if } n \geq N - 2 
\end{cases} \quad (A.56)
\]

(A.55) is a linear system were the even and odd \( n \) are uncoupled. It can be solved by used of direct or iterative algorithms for these purposes.

### A.5. Pseudo Spectral Approximation

Up to this section, all the required methods at work in the numerical solver used in this report have been provided, however a particular distinction need to be made on a specific step taken in the solution of the equations. It is this particularity that gives the "pseudo" spectral name to the method since, as it will be shown, not all steps are evaluated in the frequency domain.

The solver finds solutions for equations of the next form:

\[
\frac{\partial g}{\partial t} = h_g \nabla^2 g + R_e \nabla^2 g \quad (A.57)
\]

where \( h_g \) is the non linear term. The derivation of the version of (A.57) that SIMSON solves will be shown in subsequent sections, however it is important to take into consideration that (A.57) needs to be discretized in time and space. The non linear part \( h_g \) contains a term \( H_i \) that is roughly defined as:

\[
H_i = u_j \omega_k 
\]

(A.58)

Since the solver is working with the discretized version of the equations, which are in the Fourier-Chebyshev space, the variables are stored in such domain. For this reason the natural way for the numerical method to solve equation (A.58) is in the spectral domain as well. Considering that multiplications in the physical space represent convolution sums in the frequency domain, the evaluation that the solver would need to make each iteration has in reality the form of a convolution sum of the form:

\[
\hat{H}_i = \sum_{j+k=-i} \hat{u}_j \hat{\omega}_k 
\]

(A.59)

For relatively large domains, convolution sums are very expensive to do as in 3D it would take up to the order of \( N^4 \) operations, for this reason an alternative method is commonly used.
A.5.1. Pseudo spectral approach for convolution evaluation

When encountered with a convolution sum in a spectral method, depending on the domain size, it is more advantageous to transform the individual variables of the sum into physical space, perform a term by term multiplication and finally transform back into the frequency domain. A simplified procedure is shown next:

1. Identify the terms of the convolution
\[ \hat{w}_k = \sum_{m+n=k \atop |m|, |n| \leq \frac{N}{2}} \hat{u}_m \hat{v}_n \quad (A.60) \]

2. Transform the variables into physical space such that from \( \hat{u}_m \) and \( \hat{v}_n \), \( u(x) \) and \( v(x) \) are obtained.
3. Perform a term by term multiplication.
\[ w(x_j) = u(x_j)v(x_j) \quad j = 0, ... , N - 1 \quad (A.61) \]
4. Obtain the desired variable by transforming back into spectral domain using (A.4):
\[ \hat{w}_k = \frac{1}{N} \sum_{j=0}^{N-1} w(x_j) e^{-ikx} \quad k = \frac{-N}{2}, ..., \frac{N}{2} - 1 \quad (A.62) \]

This procedure requires the use of 3 FFT and \( N \) multiplications, however under the correct circumstances, it provides an advantage over directly evaluating the convolution. The method then takes the name as pseudo spectral approach, since a transformation took place and the solution is not found purely by using the frequency domain.

Care must be taken when performing this operation, since aliasing error is introduced into the system.

A.5.2. Aliasing removal

Aliasing Error can be eliminated by two basic techniques: padding and truncation however only the padding method will be explained in this report. A more in depth version of the procedure shown before is presented, in order to understand why aliasing is produced.

The Padding method consist in transforming \( \hat{u}_k, \hat{v}_k \) to physical space by using \( M \) grid points rather than \( N \), setting the coefficients for frequencies outside of the range \( (-\frac{N}{2}, \frac{N}{2}) \) to be zero, such that:
\[ u(x_j) = \sum_{k=-\frac{M}{2}}^{\frac{M}{2}-1} \hat{u}_k e^{ikx} \quad (A.63) \]
A.5. PSEUDO SPECTRAL APPROXIMATION

\[
v(x_j) = \sum_{k=-M/2}^{M/2-1} \hat{v}_k e^{ikx_j}
\]  

(A.64)

\[
\hat{u}_k, \hat{v}_k = \begin{cases} 
\hat{u}_k, \hat{v}_k & \text{if } |k| \leq \frac{N}{2} \\
0 & \text{Otherwise}
\end{cases}
\]  

(A.65)

Then the term by term multiplication is performed:

\[
 w(x_j) = u(x_j)v(x_j) = \sum_{m=-M/2}^{M/2-1} \sum_{n=-M/2}^{M/2-1} \hat{u}_m \hat{v}_n e^{ix_j(m+n)}
\]  

(A.66)

To transform back to spectral space, (A.7) is used:

\[
\hat{w}_k = \frac{1}{M} \sum_{j=0}^{M-1} w(x_j) e^{-ikx_j} - \frac{N}{2} \leq k \leq \frac{N}{2} - 1
\]  

(A.67)

with \(w(x_j)\) given by (A.66), then (A.67) becomes

\[
\hat{w}_k = \sum_{m+n=k} \hat{u}_m \hat{v}_n + \sum_{m+n=k \pm M} \hat{u}_m \hat{v}_n
\]  

(A.68)

It can be seen that the first term of the R.H.S of (A.68) is the desired value of the convolution sum as shown in (A.60) however there is an additional term that comes from all the transformations performed. This is precisely the aliasing error, and the philosophy of the padding method is that by choosing \(M\) such that the second term on the right hand side vanishes for \(|k| \leq \frac{N}{2}\), the aliasing error is effectively eliminated. It can be demonstrated that by choosing \(M \geq \frac{3N}{2}\) the desired results are obtained.
APPENDIX B

Immersed Boundary Method Testing

The IBM and the way it was implemented in the solver were introduced in sections 2 and 5.2 respectively. This chapter intends to shed light in the method efficiency at creating surfaces inside the model in order to determine the adequacy of using it for the analysis of cavities, which is the intended purpose of this study.

In this chapter, the process of choosing important simulation parameters is explained as well some of the limitation of the method are explored.

B.1. Control constants selection

As has already been explained, the IBM consist on a control loop with two constants, $\alpha$ and $\beta$, that need to be arbitrarily chosen. In the selection of the parameters, the next characteristics had to be taken into consideration:

1. The constants need to be sufficiently high to be able to bring the velocities to zero. In particular the value of $\beta$ in equation (2.14) must allow the tracking and control of the velocity fluctuations.
2. Increasing the values of the constants might render the numerical method unstable if done without measure.

In this study, they were chosen by trial and error: running multiple simulations, adjusting the constant values such that no slip condition is attained at the immersed wall. The whole procedure is omitted in this report, however the final results are shown. The values for the constants with the simulation conditions of this study are:

\[
\begin{array}{c|c}
\hline
\alpha & 10 \\
\beta & 1500 \\
\hline
\end{array}
\]

Table B.1. Control constants

B.2. Flat surface generation

In this section, results are compared between simulations with and without the use of the IBM. It is important to note that the conditions in both are the
same, i.e. mass flow and Reynolds number, and can be found in Table 5.3.
Possibilities to eliminate the oscillations originated by the Gibbs effect are also analyzed.

For the generation of the flat surface, two main procedures were explored:

1. Introducing a "virtual block" below the desired channel height such that the velocity of the flow at every grid point in these locations is brought to zero by the control force.
2. Introducing a "virtual plate" such that the velocity is brought to zero for an xz plane in the channel, letting flow develop naturally above and below of it.

The main purpose of this exploration is to see if good results are found in both cases, which way produces less oscillations, and if not easily avoidable, which method allows for an easier oscillation control.

The subdivision on methodologies do not change how the method is implemented, since the same control mechanism is applied in all cases. The only difference between them is WHERE the control forces are introduced, i.e. where \( S = 0 \) or \( S = 1 \) in equation (5.32). The point and manner of application of the control forces determines how strong the discontinuities are in the domain, thus has a direct effect on the accuracy of the solutions as it will influence on whether oscillations will appear in the system.

Before analyzing the effects of the methods, it is also important to note that there are some effects that are independent on the way that the control forces are imposed but that are introduced by using the IBM by itself.

**B.2.1. Surface introduction effect on resolution**

The surface is initially generated at a height \( y^+ = 27 \). This is an important parameter because one of the disadvantages of the IBM is the fact that some of the grid points that would normally be used to capture data, are below the immersed boundary thus, wasted.

This phenomenon has been expressed before, and its origin lies in the Chebyshev collocation points used in the discretization. Under these conditions, the gridpoints on top of the immersed surface are farther apart than they would be if they were in the real boundary of the domain. Due to these inconveniences, smallest scales that would normally be analyzed, will not be solved when the IBM is used. This effect will be felt more, the higher is the location of the immersed surface.

A visualization of what has been said can be seen in Figures B.1 and B.2. In both cases, the effective lower wall of the channel is at \( y = 0 \).
The domain discretization is the same in case IBM is applied or not, the difference falls in where the lower wall of the channel section of interest lies.
B. IMMERSED BOUNDARY METHOD TESTING

In Figure B.2 the black line represents where the immersed surface is. All the field above that point is of interest for the analysis. As can be seen, the next gridpoint on top of the effective lower surface of the channel \((y = 0)\) is farther away than it would normally be, and every data point below the virtual surface is effectively unused.

B.2.2. Virtual block method

In this particular procedure, the aim is to get all velocities below the immersed surface to be zero. A visualization of the flow field after applying this methodology can be seen in Figure B.3 where the dashed white line represents the exact position of lower wall of the channel.

It is apparent that the objective of having zero velocities at a certain point is met, however to better understand what is happening, a plot of the mean velocity in an \(xy\) plane is shown in Figure B.4, where a comparison between the profile of unmodified Simson and Simson with the IBM shows that the results are very close. In this case the surface is located at \(y = 0\) and as expected from the way the method was formulated, all velocity below this point \((y < 0)\) is zero. It can also be seen how oscillations are produced in the velocity profile, which was expected due to Gibbs phenomenon.

B.2.2a. Smoothing methods. It is apparent that the oscillations generated are not very convenient for the analysis, although it remains to be seen if they affect the particular areas of interest of this study, however, smoothing methods to get rid of the oscillations were explored. The two particular ways of eliminating the Gibbs phenomenon for the "block" implementation have already been formulated in \((2.16)\) and \((2.17)\), and they are referred in this report as: low pass filter and diffused force.

Low pass filter: The method was used with a cutout time of 20, which significantly reduces the energy level of the higher frequencies leaving all other wave numbers effectively untouched.

Figure B.3. Instantaneous velocity contours for surface generation via: virtual block method.
B.2. FLAT SURFACE GENERATION

Diffused force: For this case, the force is applied over 2 gridpoints in a sort of "half" Gaussian distribution: The control force in the immersed surface $(y = 0)$ is 100% the required value to bring the velocity to rest, while in the adjacent gridpoint has the 36% of the control force. This is done in order to create a smooth change between zones with a control force applied and regions where this is not the case. Figure B.5 shows the effects of both methods on the mean velocity profile.

As can be seen, both methods reduce the oscillations present in the field, but both come with disadvantages:

1. The low pass filter does not completely remove the oscillations and on top of it, slightly alters the flow. This is probably due to the fact that with the present resolution, the highest frequencies still carry relevant energy and information on the flow, which renders this method more appropriate for cases where even higher resolutions are used but oscillations are still present.

2. The diffused force application works very well and effectively dampens all the oscillations, this is due to the fact that the gradient at the wall is reduced, thus, there is a weaker discontinuity in the domain that can be dealt better by the spectral method. However there is a big flaw in this approach: The wall is not perfectly defined. Considering that the main subject of this study is to analyze that particular location, this methods
is the least convenient even if it is the most effective in its damping effect.

**Figure B.5.** Smoothing methods for the virtual block. Results obtained with the block approach to the IBM (---) compared with applying a low pass filter (----) and diffusing the application of the control force (-----).

**B.2.3. Virtual plate method**

For the virtual plate, the control force is only applied in one xz plane such that velocity can be non zero on both its sides. The velocity field can be seen in Figure B.6

**Figure B.6.** Instantaneous velocity contours for surface generation via: virtual plate method
A very similar behaviour of that seen in Figure B.3 is observable. However, the development of a flow in the lower region can not be easily appreciated and as such, a mean velocity profile is shown next.

![Mean velocity profile for the virtual plate method. Results obtained with unmodified simson (---) compared with those of the plate approach to the IBM (---).](image)

In Figure B.7 the difference in the approaches is evident. In this case a velocity develops also in the lower part of the Immersed boundary layer. This particular aspect makes that the discontinuity in the field is less pronounced, which practically eliminates the oscillations induced in the solution.

**Note on the mass flow:** As was already mentioned in section 5.2, it is noted that the flow below the surface is not taken into account in the calculation of the driving pressure gradient needed to keep the mass flow constant. For this reason special care was taken in order to ensure that only the flow in the channel section of interest is relevant in the calculation of pressure and mass.

**B.2.3a. Smoothing methods.** Even though the oscillations for the surface were significantly lower compared to the virtual block case, a smoothing method is analyzed for this approach as well. For this purpose, a force in opposite direction of the main flow is introduced in the lower part of the channel, this way, the gradient on top and below the IB surface are expected to have a smoother transition, eliminating completely the discontinuity and thus the Gibbs phenomenon.

As Figure B.8 shows, this method effectively removes oscillations and does not affect the definition of the immersed surface, however its main disadvantage,
B. IMMERSED BOUNDARY METHOD TESTING

Figure B.8. Smoothing methods for the virtual plate. Results obtained with the plate approach to the IBM (—) compared with applying a back force at the lower section of the channel (—).  

is that in order for the artificial flow below the surface to be created, there needs to be space. The more confined the region is, the bigger the forcing term needs to be introduced to create the flow and this can render the whole solver unstable.

B.2.4. Immersed boundary gradients

A recurrent important aspect of the immersed boundary method is the fact that there are big discontinuities in the gradient and force fields. An easy way to see this in the Figure B.9 where the overshoot from the Gibbs phenomenon and consequent oscillations are evident.

This is of particular importance since as already defined, the shear stress is dependent on the gradient of the mean flow at the wall. For practical purposes, throughout this report, quantities such as $u_\tau$ and $Re_\tau$ will be acquired from simulations where the channel does not have any immersed surface, as it was seen that the results are in good agreement. In case the gradient at the immersed boundary is of interest for a particular situation, the gradient on the upper wall will be used, as it will be shown in coming sections, the symmetry of the velocity profiles is kept in the simulations even if the there are oscillations on one side of the plot.
B.3. CAVITY GENERATION

The process to create the cavity in the channel is exactly the same as that to create only the surface, the difference once again lies in the grid points of the domain where the control force is applied.

Considering the domain size in the x direction, it was determined that resolution would probably need to be higher than that shown in table 5.3 for most cases in order to correctly define the length of the cavity. For the initial exploration, the resolution of the domain was changed to have more flexibility in this orientation, with modified parameters given in B.2.

At this point of the project, simulations with the virtual block and plate methods were still being examined. In the coming sections, the results of this testing is compiled.

B.3.1. Virtual block method

In this case every point below the surface and outside of the cavity was brought to a zero velocity state. The general behaviour of the flow is shown in Figure B.10, where an instantaneous velocity contours graph in an xy plane is presented.

The dashed white line in Figure B.10 represents the locations where the immersed boundary is. The effects inside the cavity can be partially seen,
B. IMMERSED BOUNDARY METHOD TESTING

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**Table B.2.** Simulation set-up for cavity simulation

**Figure B.10.** Instantaneous velocity contours for surface generation via: virtual block method.

however for the sake of clarity, an enhanced view of the zone neighboring the cavity is presented in **Figure B.11** with a slightly altered color map range.

**Figure B.11.** Cavity view.
In Figure B.11, it is possible to see that the flow outside the cavity is effectively zero, while in its interior a predominately negative flow forms, which might point to re-circulation zone.

To better visualize the actual order of magnitude of the results, plots of the normalized mean velocity and root mean square of the fluctuating velocity are shown in Figure B.12 and B.13.

![Figure B.12. Mean velocity profile for the virtual block method. Profile at a streamwise station without a cavity (---) compared with the profile at the streamwise location of the cavity center(----).](image)

In Figure B.12 the mean profile can be seen. It is evident that there is a flow development inside the cavity that affects the value at the surface of the channel ($y^+ = 0$), this is an important finding considering that is the location that this study is mainly concerned about.

Another particular characteristic to notice is the fact that in this case there are also oscillations induced into the system. This behaviour is evident in both: mean and rms cases. A plot of the fluctuating velocity is shown in Figure B.13, where it is noted that the values at the channel surface differ from those at the equivalent height in the cavity. An important finding from this, is the expected result that the flow will be altered in the surrounding on the cavity, but far enough from it, the velocities collapse once again into the main flow.

It is also important to note that this was a very exploratory stage, where statistics were taken even before the flow was stabilized for the sake of supervising the simulation progression. For this matter, not the most accurate results
B. IMMERSED BOUNDARY METHOD TESTING

Figure B.13. RMS of velocity fluctuations for the virtual block method. Profile at a streamwise station without a cavity (---) compared with the profile at the streamwise location of the cavity center (- - -).

are expressed in this section, as the main purpose was to see how the method is working while generating the cavity.

B.3.2. Virtual plate method

This method follows from the already explained plate generation. The main idea is to make the velocity zero through the control forces only in a given set of gridpoints that define the lower wall of the channel as well as the cavity. An instantaneous flow field is presented in Figure B.14

Figure B.14. Instantaneous velocity contours for surface generation via: virtual plate method.
Once again, the cavity can be briefly appreciated, however an enhanced version of the plot is presented in Figure B.15 to better illustrate the phenomenon.

![Cavity view](image)

**Figure B.15. Cavity view**

In this plot, different activity from that of Figure B.11 is registered, however it must be noted that these are instantaneous captures of the velocity field. While checking the averaged quantities, the difference between the methods greatly reduce.

It is also possible to notice that the velocity below the surface is not exactly zero in this case, this is due to the fact that the flow is let to develop by itself below the immersed boundary. However, contrary to the case with no cavity, where the flow had open space to develop, in this case the magnitude of the velocity below the surface is much lower, this is probably because the cavity is imposing another boundary condition on those sections of the domain, that make the flow approach rest. A very convenient way to observe this behaviour is by means of the velocity profiles, which are shown in figures B.16 and B.17.

As seen in Figure B.16, the behaviour is basically the same as shown in B.12, however it noted that there are no oscillations present in the solution even without a smoothing method in effect. This is a very strong characteristics, since adding these methods, specially the back force, can render the simulation unstable if the chosen force is too big.

In Figure B.17 the same expected behaviour is seen, noting that even thought $u_{rms}$ is not zero below the surface, the flow is effectively brought to rest at the point of interest ($y^+ = 0$).
B. IMMERSED BOUNDARY METHOD TESTING

Figure B.16. Mean velocity profile for the virtual plate method. Profile at a streamwise station without a cavity (---) compared with the profile at the streamwise location of the cavity center(-----).

Figure B.17. RMS of velocity fluctuation for the virtual plate method. Profile at a streamwise station without a cavity (---) compared with the profile at the streamwise location of the cavity center(-----).
B.3. CAVITY GENERATION

B.3.3. Method selection

Between the block and plate approaches, it was seen that plates produce less oscillation in the solution without affecting the zone close to the immersed surface, which is essential to the study. It was also seen that though the back force smoothing method works well with open surfaces, the presence of the cavity, which serve as an obstacle in the lower part of the channel, does not permit the formation of artificial flows which in turn makes the whole smoothing method obsolete. A round about to this would be to create a set of forces that make a re circulation zone, however this was not considered in the present project.

Based on the exposed remarks, it was decided to proceed to perform higher quality simulations with the un-smoothed plate method, as consistent good results where found by the use of it. It has been shown that with this approach, the velocity profile below the surface is not exactly zero, however for the sake of presenting the information in a more organized way, the velocity in these regions, which are not of interest, will be presented as zero from this point onward.
Wall Shear-Stress Measurements

The results from the cavity probe measurements are disclosed in this section.

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Table C.1. 4mm-long 0.2mm-deep cavity with short hot wire configuration, cylindrical cavity.
Table C.2. 4mm-long 0.4mm-deep cavity with short hot wire configuration, full cavity.
### Table C.3. 4mm-long 0.2mm-deep cavity with long hot wire configuration, full cavity.

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### Table C.4

4mm-long 0.4mm-deep cavity with long hot wire configuration, full cavity.

**B04LW**

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C. WALL SHEAR-STRESS MEASUREMENTS

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Table C.5. 4mm-long 0.4mm-deep cavity with long hot wire configuration, cylindrical cavity.
### C. WALL SHEAR-STRESS MEASUREMENTS

#### B04SWnc

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#### Table C.6.
4mm-long 0.4mm-deep cavity with short hot wire configuration, cylindrical cavity.
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With our signatures we declare the accuracy of these specifications.

Place, Date Alex Alvisi

Place, Date Adalberto Perez