KTH ROYAL INSTITUTE
OF TECHNOLOGY

Licentiate Thesis in Structural Engineering and Bridges

On the dynamics of footbridges

A theoretical approach and a comparison between running and walking loads

DANIEL COLMENARES

Stockholm, Sweden 2021

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Academic Dissertation which, with due permission of the KTH Royal Institute of Technology, is submitted for public defence for the Degree of Licentiate of Engineering on Friday the 26 March 2021, at 1:00 p.m. in M108, Kungl. Tekniska högskolan, Brinellvägen 23, Stockholm.

Licentiate Thesis in Structural Engineering and Bridges
KTH Royal Institute of Technology
Stockholm, Sweden 2021

TRITA-ABE-DLT-215
ISBN 978-91-7873-274-6

Printed by: Universitetsservice US-AB, Sweden 2021

Abstract

The dynamic behaviour of lightweight footbridges is often susceptible to Human Induced Loads (HILs). Generally HILs are taken into account as moving harmonic functions in which the loading frequency represents the step frequency of the pedestrians. In this way, there may be resonance if the loading frequencies fall within the range of the natural frequencies of the bridge, potentially compromising the serviceability limit state of the structure. Therefore, it is important to understand how to address and model HILs in the context of lightweight and slender structures. Furthermore, interesting effects can be considered in the field of footbridge dynamics, such as the Human Structure Interaction (HSI) effect. The HSI effect can be understood within a framework in which pedestrians behave as Tuned Mass Dampers (TMDs), possibly modifying the dynamic behaviour of the footbridge. In addition, the evaluation of the dynamic response of a footbridge is usually made through a time consuming dynamic analysis using the Finite Element Method (FEM). Most of the analysis of this type of slender structures rely on a prescribed stationary harmonic loading scenario, and this is usually done in the context of a walking crowd event and not much attention is given to running load events.

The aims of this research project are to study the influence of running and walking loads on the dynamic response of footbridges as well as to investigate and develop a closed-form method in order to simulate the dynamic behaviour of footbridges subjected to HILs. This has been achieved by comparing different approaches in order to simulate running load events for a small number of pedestrians with respect to experimental results (Paper I). In addition, the simply supported beam and the clamped-clamped beam (Paper II) are studied when subjected to a moving harmonic load in a closed-form framework. Then, a comparison between normal walking and normal running conditions is made. Finally, a general closed-form solution for the moving harmonic load problem (Paper III) is developed using the 2D Bernoulli–Euler beam theory for a continuous beam system on elastic supports.

The results from the study indicate that running is more critical than walking for a single pedestrian crossing, despite the fact that it is easier to achieve a steady state condition in a normal walking event than in a normal running event. Finally, the general solution of the moving harmonic load problem is found and it can be used to solve any load spectra in the time domain, with its static component, for a general multi-span beam system.

Keywords: footbridges, dynamics, human induced loads, human structure interaction, harmonic load, continuous beam.

Sammanfattning

Slanka och lätta gångbroar är ofta känsliga för dynamisk belastning från fotgängare. Dessa laster betraktas ofta som harmoniska funktioner där lastfrekvensen beror på stegfrekvensen. Resonans kan uppstå om stegfrekvensen sammanfaller med någon av brons egenfrekvenser vilket potentiellt kan överskrida föreskrivna vibrationsnivåer. Kännedom om dynamisk fotgängarlast är därför viktig, framförallt för dynamiskt känsliga konstruktioner. Samverkan mellan fotgängare och bro kan också ge upphov till intressanta samband. Fotgängarna kan i detta sammanhang liknas med en massdämpare som kan ändra brons dynamiska egenskaper. Dynamiska analyser av gångbroar utförs ofta med FEM-analyser som kan vara tidskrävande. Vanligen baseras analyserna på föreskrivna stationära harmoniska laster, ofta baserat på gånglaster och sällan med beaktande av löparlaster.

Syftet med denna uppsats är att undersöka inverkan av löpar- och gånglasters inverkan på gångbroars dynamiska respons samt att utveckla en analytisk metod för att simulera dessa laster och dess respons på broar. Detta har utförts genom att jämföra olika sätt att simulera löparlaster och jämföra broresponsen med experimentell data (artikel 1). En analytisk lösning för rörliga harmoniska laster redovisas för fallet fritt upplagd och fast inspänd balk (artikel 2), med vilken inverkan av gång- och löparlaster jämförs. En mer generell analytisk lösning för rörliga harmoniska laster (artikel 3) baseras på Bernoulli-Euler balkteori för kontinuerliga balkar på eftergivliga upplag.

Resultaten från föreliggande arbete visar att för en enskild fotgängare är fallet med löparlast mer kritiskt än gånglast, trots att det är lättare att uppnå ett fortvarighetstillstånd för gånglaster jämfört med löparlaster. Den generella lösningen för rörliga harmoniska laster som redovisas kan användas för att lösa godtyckliga lastspektra i tidsdomän, inklusive dess statiska komponent, för generella balkar.

Nyckelord: gångbroar, dynamik, fotgängarlast, samverkan mellan fotgängare och bro, harmonisk last, kontinuerlig balk.

Preface

The research presented in this thesis was conducted at the Department of Civil and Architectural Engineering, at the KTH Royal Institute of Technology. The funding for this project, provided by the Swedish Transport Administration (Trafikverket), is gratefully acknowledged.

The research was supervised by Professor Raid Karoumi and Dr. Andreas Andersson, to whom I express my sincere gratitude for their continuous support and guidance. A special thanks also to Professor Jean-Marc Battini for reviewing this thesis. I also wish to thank my friends and colleages at the department for a memorable time. Last but not least, I would like to thank my family for their endless support.

Stockholm, January 2021
Daniel Colmenares

Publications

The current thesis is based on two conference proceedings and one journal paper, labelled **Paper I–III**.

Appended papers:

Paper I Colmenares, D., Andersson, A., Karoumi, R. and Ülker-Kaustell, M. ”Pedestrian bridge evaluation and modelling subjected to running load cases”. *Footbridge Madrid 2021 International Conference.*

Paper II Colmenares, D., Andersson, A. and Karoumi, R., ”Closed-form solution of single pedestrian induced load for clamped-clamped bridges”. *EURODYN 2020, XI International Conference on Structural Dynamics.*

Paper III Colmenares, D., Andersson, A. and Karoumi, R., ”Closed-form solution for continuous beam systems on elastic supports under moving harmonic loads”. Submitted 2020 to *Journal of Sound and Vibrations.*

All papers were planned, implemented and written by Daniel Colmenares. The co-authors provided guidance throughout the work and reviews before submission.

Contents

Chapter 1

Introduction

1.1 Background

The dynamic behaviour of lightweight footbridges is often susceptible to Human Induced Loads (HILs). This is specially the case for footbridges with low inherent damping and closely spaced modes. Usually, HILs are taken into account as moving harmonic functions in which the loading frequency represents the step frequency of the pedestrians. In this way, they may cause resonance if the loading frequencies fall within the range of the natural frequencies of the bridge, potentially compromising the serviceability limit state of the structure.

HILs can be represented as Fourier series, characterized through their harmonic nature. In this way, HILs are defined by parameters such as the step frequency, the Dynamic Load Factor (DLF) and the load phase angle. Many have measured and estimated the values of the DLFs in order to improve the definition the deterministic force model. Furthermore, this is one of the initial attempts in order to characterize HILs applied not only in the field of floor vibrations but also in the context of pedestrian bridge dynamics.

Interesting effects can be considered in the field of footbridge dynamics. Such is the case of the Human Structure Interaction (HSI) effect. This phenomena can influence the modal properties of the system, i.e. its natural frequency and damping characteristics. The HSI effect can be understood within a framework in which pedestrians behave as Tuned Mass Dampers (TMDs), possibly modifying the dynamic behaviour of the footbridge.

In addition, footbridges can have time varying properties, given a specific loading event. Focusing on vertical vibrations, if a moving crowd of pedestrians is considered using a Couple Crowd Model (CCM), the mass, damping and stiffness matrix that define the equations of motion of the system will vary in time and linear dynamics is no longer applicable to describe this event. On the other hand, considering the load as idealised standing pedestrians, the system matrices will remain constant and linear dynamics provides the necessary tools to address this problem.

Also, running loads may have an interesting effect when applied to footbridges. If a running load event is considered using the CCM, the flying phase, i.e. the period of time in which the runner is not in contact with the bridge, may also add non-linearities to the dynamic problem due to the time varying properties of the system. More research is needed to understand the influence of the HSI effect and the flying phase in the dynamic response of the system to a running load event.

Furthermore, even though there are many design codes and recommendations that address the dynamic performance of such systems, most of them rely on Finite Element Analysis (FEA) based on loading scenarios such as: i) a single moving resonant pedestrian simulated as a moving harmonic load, ii) a stream of pedestrians taken into account as a standing single resonant pedestrian simulated as a harmonic load multiplied by partial correction coefficients that take into account the correlation between the pedestrians under a steady state regime and iii) loading scenarios based on the concept of an equivalent number of synchronized pedestrians such that, the steady state response of a loading scheme that follows the mode of the structure allows evaluating the dynamic performance of the system. Moreover, the main loading scenario considered is the walking load event. Not much attention has been given to running loads, given their transient nature. Furthermore, running loads are significantly higher than walking loads when both of their corresponding DLFs are compared. However, more loading cycles are applied to a footbridge by walking pedestrians than running ones. Therefore, a comparison is needed in order to understand which load case is more demanding in terms of the serviceability limit state of the structure.

Finally, this licentiate thesis aims to address the modelling strategies of running load events, make a comparison between running and walking loads and develop a closed-form solution of the moving harmonic load problem for a continuous beam system on elastic supports, in the context of footbridge dynamics.

1.2 Aims and scope

The aim of this study is to investigate the dynamic behaviour of footbridges under human induced loads and give an insight into the system's behaviour from an analytical perspective. The specific objectives are:

- Investigate the different modelling aspects of running loads on footbridges.
- Investigate the influence on footbridges of the running loads compared to the walking loads.
- Provide a full comprehensive insight into the moving harmonic load problem for continuous beam systems.

The following limitations apply. The comparison between running loads and walking loads is made in the framework of a single moving pedestrian load case. The pedestrian load is taken into account as a moving harmonic forces with a constant speed, following a 2-D dynamic analysis. The HSI is taken into account by assuming a CCM in a running load event.

1.3 Scientific contribution

The research project presented, with the appended papers resulted in the following scientific contributions:

1. Design and assessment curves for running and walking load cases for both simply supported and clamped-clamped beam systems.
2. Insight into the trade-off between the DLF and the number of loading cycles with regard to the dynamic response of a beam system.
3. A comparison of the different modelling strategies for the load case of a small amount of runners in footbridges, including the HSI effect and the flying phase of runners.
4. A full insight into the moving harmonic load problem over Bernoulli–Euler beam systems in a closed-form solution reference frame.

1.4 Outline of the thesis and the appended papers

The thesis consists of a theoretical introduction and three appended papers. The first part provides an overview of the research field as well as a compressive summary of the appended papers. Chapter 2 presents a literature review focused on the research into the DLFs of the pedestrian load model and the moving harmonic load problem for continuous beam systems. Chapter 3 presents the modelling aspects concerning the solution methods and factors involved in footbridge dynamics, such as the HSI effect and the flying phase of running pedestrians. An extended summary of the appended papers is presented in Chapter 4. Finally, conclusions and suggestions for future research are given in Chapter 5.

The content of the different papers is shown in Figure 1.1. Paper I focuses on the running load event for a small number of runners, as well as the different strategies in order to address consequent dynamic problem. Paper II presents a study of pedestrian walking loads vs. running loads applied to a clamped-clamped beam system. The study was made by adopting the Fourier series decomposition of the pedestrian load model. Finally, Paper III studies the moving harmonic load problem in a general framework using a continuous approach, for general elastic boundary conditions applied to continuous beam systems in the context of footbridge dynamics. Figure 1.1 shows the connections between the different papers. With the experiments and simulations reported in Paper I, the case of pedestrian tied arch bridges subjected to a moving harmonic load was studied in the context of a clamped-clamped Bernoulli–Euler beam. Finally, the general solution for a moving harmonic load applied to general continuous beam systems was developed in Paper III. A more detailed description of each appended paper is presented as follows:

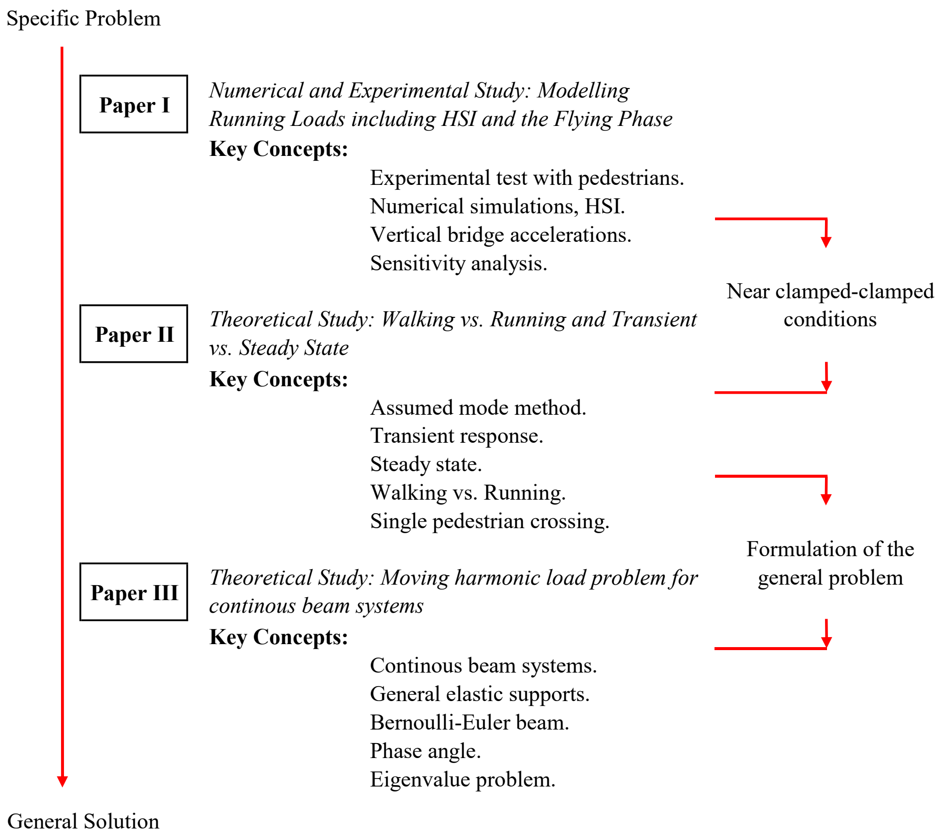

Figure 1.1: Connection between Papers I-III.

Paper I presents an experimental and numerical study with the aim of comparing different strategies for modelling running loads applied to two case studies. One of the bridges has near clamped-clamped boundary conditions. The second bridge has two closely spaced modes. The paper considers a Finite Element (FE) model, a moving force model and the CCM, in order to take into account the HSI effect. Additionally, the flying phase of the runners is taken into account in the CCM by uncoupling the equations of motion. Finally, a simplified sensitivity analysis is presented using the CCM around the vicinity of the performed experiments in order to investigate the HSI in the context of a small number of runners.

Paper II provides an analytical solution of the moving harmonic load problem for the clamped-clamped beam system. The method is based on the first mode approximation and the selection of a suitable trigonometric function that well estimates the modal properties of the system and satisfies the imposed boundary conditions. After the decomposition of the original problem into the modal domain, the transient event is quantified and compared to an equivalent steady state condition of the system. Finally, a walking load scenario and a running load scenario are compared through the formulation of the minimum linear mass parameter in order to satisfy a general acceleration serviceability state.

Paper III presents a closed-form solution of the moving harmonic load problem for continuous Bernoulli–Euler beam systems, including the phase angle in the load definition. In the closed-form formulation, the phase angle is taken into account in order to maintain the time continuity condition of the load time history and the dynamic response of the system. Three numerical examples are presented. Additionally, it is demonstrated how the general solution of the moving harmonic load problem includes the moving constant load problem. Using this solution, any load spectra applied to any continuous beam system on general elastic supports can be solved in the time domain.

Chapter 2

Literature review

The problem of footbridge dynamics can be understood in the context of three aspects: (i) the loading model, (ii) the structural system and (iii) the dynamic response of the system. An important aspect to be considered is the HSI effect which introduces a feedback into the system that may affect the structural response. Finally, the serviceability assessment must also be considered.

In this chapter, a literature review of the time-domain deterministic load model is presented. The load model represents one of the first attempts in order to characterize HILs applied not only in the field of floor vibrations but also in the context of pedestrian bridge dynamics. Also, a literature review focussed on the moving load problem in the context of railway dynamics and pedestrian bridge dynamics is presented, considering the application of the moving constant force and time varying moving force on the structural systems.

2.1 Dynamic load factors

Vertical dynamic induced pedestrian forces can be expressed, using the Fourier series, as a summation of harmonic functions in the form of:

$$P(t) = G + \sum_{i=1}^{h} G\alpha_i \sin(\varphi_i + \Omega_i t) \tag{2.1}$$

where G stands for the pedestrian's weight, h is the number of considered harmonic components, α_i is the Dynamic Load Factor (DLFs) corresponding to the harmonic i, φ_i is the associated phase shift angle and Ω_i is the acting circular frequency of the harmonic load. Many have measured and estimated the appropriate values of the DLFs in order to improve the definition the deterministic force model. The following section presents a review of previous research focused on force measurements and identification of ground reaction forces concerning the dynamic load factors in the pedestrian load model.

In 1970, one of the first measurements of pedestrian-induced forces was done by Galbraith and Barton [1]. Their experiments aimed to quantify the load time histories of the ground reaction forces from the footsteps. This was done using two aluminum plates approximately 30.48 cm × 45.75 cm in size and 2.54 cm thick. Within the plates, four force gauges were installed and a 4.87 m runway was built around it so that the subjects could take several steps and attain steady pace before stepping onto the measuring surface. Since the aim of the experiments was to bound the problem and evaluate the parameters that had the greatest influence on the load time history, the study was limited to three subjects moving at rates ranging from slow walking to running. The experiments were carried out in four series in order to test different kinds of surfaces. The first two series were performed on a hard surface for each subject at different step frequencies. The third series was done using a sand surface and the fourth series was on sand over a rubber pad, trying to simulate the elasticity of a considerable depth of soil beneath the sand surface. The footstep rate was determined using a stopwatch to measure the elapsed period of time of the test for a given number of steps. Finally, 79 load time histories were measured and it was concluded that neither the surface nor the footwear had any substantial effect on the load time histories. The weight and the footstep rate were the most important parameters.

In 1972, Jacobs, Skorecki and J. Charnley [2] developed an apparatus for recording the vertical component exerted on the ground while walking over a distance. The aim was to study the vertical force component applied during walking by normal and pathological subjects and identify the different waveforms of gait. Regarding the normal sample, two sets of experiments were carried out: (i) at the normal cadence and (ii) at an imposed cadence of 112 steps per min. They confirmed the consistency between the normal shape of the load time history and the harmonic spectrum.

In 1977, Blanchard [3] was one of the first to quantify the DLF of the first harmonic component of the deterministic harmonic load model. Two scenarios were considered. If the fundamental frequency of the structure was below 4 Hz, then resonance would occur due to the first harmonic component. The value of $\alpha_1 = 0.257$ was determined with reference to a pedestrian weight of 71 kg. And, if the fundamental frequency of the structure was in the range of 4 Hz to 5 Hz, then resonance would occur due to the second harmonic component of the pedestrian-induced load. In order to address this latter case, a reduction factor was introduced.

In 1986 Rainer and Pernica [4] were able to measure the forces from walking and running pedestrians by using load cells on a 17 m floor strip. The measurements found the harmonic components which make up the dynamic force exerted by pedestrians. Three male subjects participated in the test. The tests were performed by playing a pre-recorded pulse at specific step frequencies by loudspeakers and by a free pacing chosen by the subjects. The load measuring device consisted of a simply supported joist with a span of 17.04 m, covered by 2.13 m × 1.19 m pre-cast concrete panels with a thickness of 100 mm. At the centre of the joist a piezoelectric force transducer was placed between the temporary support and the bottom chord preloaded with 4 kN. The signals of the transducers were added, amplified and recorded. Finally, a high pass filter was applied at 0.2 Hz to eliminate the low frequency drift as the subject moved across the floor strip. The lowest resonance frequency of the structure was 12 Hz, so that the results were limited to frequencies from 0.2 Hz to 10 Hz, avoiding resonance problems. They concluded that the forces consist of harmonic components and the low frequency harmonics make the most important contributions. They further developed that study and concluded that the DLFs were decreasing for the higher harmonic components [5].

Baumann and Bachmann [6] conducted experiments aiming to study the walking, running and jumping loads using large prestressed concrete beams. Eight persons participated for each load case, performing the test at least three times for each step frequency. The tests were performed on two prestressed beams. Beam 1 had a span of 19 m and beam 2 had a span of 14 m. Intermediate supports were placed so that the natural frequencies of the prestressed beams could be changed in order to study the resonance of the system. Two series of tests were done. Series 1 used force plates placed on the crown of the prestressed beams. The bottom of the beam was linked to a force plate attached to the rigid surface (ground) of the laboratory. In this way, the forces in a flexible system (the beams) and the forces on a rigid floor were evaluated. The accelerations and displacements of the beam crown were also measured. Series 2 was performed by placing a treadmill over the prestressed beams. Stationary walking and running (reported problems) load cases were studied. The tests were performed using a metronome. The walking test was done varying the step frequency from 1.6 Hz to 2.4 Hz in increments of 0.1 Hz. In the running test, the step frequency was varied from 2 Hz to 3 Hz in increments of 0.2 Hz. In the jumping test, the jumping frequency varied from 1.4 Hz to 3.4 Hz in increments of 0.2 Hz. In addition, untimed walking, running and jumping tests were performed. Moreover, the running styles were studied, as well as the influence of soft and hard shoes on the load time history. The study found the dynamic load factors for the different load cases and no differences were found between rigid floor and flexible floor in the load time histories for the running load case. HSI was not studied. In 1987, Bachmann and Ammann [7] determined the Fourier coefficients of the first five harmonic components of the pedestrian load model. They reported that the vertical DLF for the first harmonic component will vary between 0.4 and 0.5 between the frequency range 2.0 Hz and 2.4 Hz in a walking activity and reported a DLF of 0.2 for the second harmonic component around a step frequency of 2.0 Hz. Additionally, it is important to mention [8] in which consistent results were found with respect to [7].

In 1990, Pernica [9] conducted a study regarding 5 rhythmic activities: (i) walking, (ii) jogging, (iii) jumping, (iv) stride jumps and (v) running on the spot. The variation of the dynamic load factors was studied with respect to the footstep rate and the group size. The tests were performed both by playing a prerecorded pulse at the wanted step frequency by loudspeakers and by a free pacing chosen by the subjects. Twenty-two people participated in the study in order to determine the dynamic load factors for the different activities.

Ebrahimpour et al. [10] studied the loads imposed by moving crowds while walking. The dynamic forces generated by moving crowds of people while walking as well as individual loads were studied. Three tests were developed, with different objectives. Tests of one person were used to characterize the individual loads. Tests with two people were used to evaluate the coherency of motion. Tests with four people were used to verify the group load modelling. Tests were performed for free walking, fixed step frequency, marching and running (the results of the last two were not reported). The experimental setup consisted of a force platform approximately 14.2 m $\times$ 2 m. It was constructed in such a way that the subjects had 3 m of acceleration ramp and 1.2 m of flat transition to achieve their steady state motion before the instrumented module. The force platform was composed of six 9 cm $\times$ 81 cm independent force plates with instrumented strain gauges. The first natural frequency of the system was at 100 Hz. A metronome was used in the study to aim at specific step frequencies. This study can be considered as one the first to include randomness in the context of pedestrian-induced loads.

One of the first studies that suggested the phenomenon of HSI was made in 1997 by Pimentel [11]. The aim was to evaluate the dynamic performance of pedestrian bridges due to human loads and compare it with the design guidelines to improve the understanding and applicability of the serviceability criteria, focusing on the cases of walking and jumping loads. Two test subjects participated in the experiment. One of the measurement objects consisted of a composite cross section footbridge of a 2.16 m wide concrete deck connected to a steel box girder. It had a span of 19.9 m and was simply supported on elastomeric bearings. The first vertical natural frequency was 3.6 Hz and the second vertical natural frequency was 13.2 Hz. A hammer test was performed in order to characterize the structure. A metronome was employed in all tests. The first test consisted of comparing the time response of the body accelerations of a test subject walking on a rigid surface and on a footbridge in resonance conditions. The accelerometer was attached at the waist of the test subject using an L-shaped plate in such a way that it remained in a vertical position. Another accelerometer was placed at the center of the structure. Higher accelerations were obtained on the pedestrian when crossing the footbridge than on the ground. After a sequence of tests attaining resonance conditions, the experiments indicated that the load changes while being applied to the structure when the pedestrian is walking due to the difficulties of keeping in phase with the vibration of the structure and the HSI. By comparing RMS of the simulated and experimentally obtained accelerations, significantly lower dynamic load factors were obtained than those of [4].

Kerr, as part of his PhD thesis [12], and then with Bishop [13], investigated the differences between HILs on a floor while ascending or descending a staircase. Force plate testing was conducted in order to quantify the impact load applied by the test subjects when walking along a horizontal platform and ascending or descending a staircase. Fourier analysis was used to determine the harmonic amplitudes and frequency content. A force plate was used to make the measurements. Its dimensions were 610 mm $\times$ 380 mm, with a thickness of 30 mm. The natural frequency of the plate was 650 Hz, eliminating concerns about inducing higher order harmonic resonances. Within the plate, 4 piezoelectric transducers were placed. The walkway provided 4 meters of 'lead in' and 1 m 'lead out' so that subjects could attain a normal stride before reaching the force plate. The frequencies determined by the walking tests varied from 1 to 3 Hz. Over 1000 individual traces were recorded from 40 subjects. Each subject started at the beginning of the walkway using a metronome set to the specific frequency being studied. The DLFs were determined and the results coincided very closely with the results of [4]. However, according to [14], the disadvantage of the work was the fact that the data cannot represent intra-subject variability, i.e. it was not taken into account that a single pedestrian is incapable of reproducing two identical force signals.

In 2000, Ellis [15] investigated load models of pedestrians and evaluated the structural response of floors using data measured for the case of walking. The floor was 9 m $\times$ 6 m with a fundamental resonance frequency of 8.5 Hz. The accelerations and displacements were determined at the centre of the floor. It was determined that the system had a modal stiffness of 1.94 MN/m. The walking tests were undertaken by asking a person to walk either down the centreline of the floor or across a diagonal, at different step frequencies varying between 1.7 Hz and 2.4 Hz. The pacing rate was achieved through the use of a portable computer that gave an audible signal at each selected frequency. For the 3^{rd} to the 6^{th} Fourier coefficients, a large spread and no noticeable trend was found; the average values are: 0.07, 0.08, 0.07, and 0.06 respectively. It was confirmed that a critical situation occurs when the floor's fundamental frequency is an integer multiple of the pacing frequency and resonance occurs. The DLFs were determined for the walking case.

Kala [16] conducted a study in order to confirm the conclusions of previous research regarding DLFs. Three pressure sensors were used, placed in a rigid platform. The predestrians were moving at 1.4 m/s with a step frequency of 1.55 Hz. The reported results were in agreement with those of previous studies.

In 2002, Yoneda [17] proposed a DLF for the first harmonic component and the concept of the correction coefficient was introduced in order to provide a relation between the step frequency and the speed of movement of the pedestrian.

Young [18] and Wilford [19] carried out a wide ranging study for modelling walking forces. Assembling data from different publications in the literature, they performed a statistical regression and proposed mean and design values for DLFs for the first four harmonic components. In their paper, the proposed of design values have a

25% probability of exceedance of the considered data. A summary of the previous mentioned studies can be seen in Tables 2.1 and 2.2. A similar investigation can be found in [20].

In 2014, Racic and Monit [21] investigated, through data-driven modelling the vertical dynamic excitation of bridges induced by people running. The aim was to obtain data regarding the load applied on a bridge due to running excitations. Over 458 vertical running force time histories were recorded, involving 45 persons. The tests were performed on two instrumented treadmills. All components of each treadmill were mounted on a rigid frame connected to the ground by 4 triaxial piezoelectric force sensors. The whole system was mechanically isolated, i.e. the sensors measured only the external running forces. Different test protocols were carried out in the Vibration Engineering Section Laboratory of the University of Sheffield (UK) and Laboratoire de Physiologie de l'Exercice of the University Jean Monnet Saint-Etienne. During the tests, the participants were asked to run at a fixed speed. The pacing rate was not prompted by any device such as a metronome. Then, the running speed was increased by 0.5 km/h up to 10 km/h at Sheffield, and up to 20 km/h at Saint-Ettiene. Each test ended after 64 successive footfalls were recorded. No probability distribution could be assigned to the scattered data of the DLFs. Finally, a robust data-driven mathematical model was developed to generate random signals which can replicate both time and spectral features of the running load. The key parameters that played the main role in the generation of a running force time history signal are: (i) shape of the running footfall, (ii) the equivalence in the step frequency and (iii) the equivalent energy from each cycle.

Table 2.1: Outline of DLFs for walking.

Authors	***DLFs***	***Comments***
Blanchard [3]	$\alpha_1 = 0.257$	In the range of frequencies between 4 and 5 Hz the DLF is reduced
Yoneda [17]	α_1	Frequency dependent and correction coefficient
Bachmann and Ammann [7]	$\alpha_1 = 0.4$ - 0.5 and $\alpha_2 = \alpha_3 = 0.1$	In the range of 2.0 Hz -2.4 Hz for α_1 and $\alpha_{2,3}$ around 2.0 Hz
Baumann and Bachmann[6]	α_1, α_2 and α_3	Contact time dependent
Pernica [9]	Maximum values reported: $\alpha_1 = 0.56$, $\alpha_2 = 0.28$, $\alpha_3 = 0.16$, $\alpha_4 = 0.09$	Frequency dependent
Rainer and Pernica [4], [5]	α_1, α_2, α_3 and α_4	Frequency dependent
Ebrahimpour [10]	α_1	Frequency dependent. Also dependent on the number of people
Bachman [8]	$\alpha_1 = 0.4/0.5$, $\alpha_2 = \alpha_3 = 0.1$	at 2.0 Hz/2.4 Hz
Kerr [12], [13]	α_1, $\alpha_2 = 0.07$, $\alpha_3 \simeq 0.6$	α_1 is frequency dependent
Ellis [15]	α_1, α_2, α_3=0.07, α_4=0.08, α_5= 0.07 and α_6= 0.06	α_1, α_2 are frequency dependent
Kala [16]	α_1= 0.32, $\alpha_2 = 0.09$, $\alpha_3 = 0.12$ and α_4=0.02	At around 1.55 Hz
Willford [19], Young [18]	α_1, α_2, α_3 and α_4	Reported mean values

Table 2.2: Outline of DLFs for running.

Authors	*DLFs*	*Comments*
Yoneda [17]	α_1	Frequency dependent and correction coefficient
Bachmann [8]	$\alpha_1 = 1.6$ and $\alpha_2 = 0.7$, $\alpha_3 = 0.2$	In the range of 2.0 Hz - 3.0 Hz
Pernica [9]	Maximum values reported: $\alpha_1 = 1.57$, $\alpha_2 = 0.58$, $\alpha_3 = 0.26$, $\alpha_4 = 0.15$	Frequency dependent
Rainer and Pernica [4], [5]	α_1, α_2, α_3 and α_4	Frequency dependent

2.2 The moving load problem

Many have studied the moving load problem applied to bridge systems. In this section, a review of the previous research focused on the different formulations of the moving constant force and the moving time varying force models applied to continuous beam systems is presented. Also, the work is presented in the context of railway dynamics and pedestrian bridge dynamics, going from Bernoulli–Euler continuous beam formulation through semi-analytical proposals in order to find the dynamic response of the different beam systems.

There have been many studies of the moving constant and harmonic load problem. In 1972, Fryba [22] presented a study of the moving constant and harmonic load problem applied to simply supported beams, in the context of railway dynamics, through the use of the Laplace–Carson transform. Matsumoto [23] studied the dynamic performance of 5 bridges in Tokyo by characterizing the pedestrian load as a moving harmonic load; the contrast between the design of footbridges following a static approach and the dynamic approach is shown, as well as how the dynamic approach was more suitable for the evaluation of the acceleration serviceability limit state of such systems. Wheeler [24] introduced a design method based on the numerical solution of the equation of motion applied to footbridge dynamics; the relationship between the excitation load, the dynamic response of the system and the serviceability limit state based on the effect on the pedestrians was emphasized using a moving harmonic load model. Rainer [5] studied the dynamic behaviour of single span bridges by determining the Fourier amplitude coefficients of the pedestrian load and computing the dynamic load factor at a resonance condition of the equivalent single degree of freedom model. Moreover, Rainer [5] introduced one of the first simple assessment tools for footbridges based on accelerations, noticing that the DLF of a moving harmonic load is smaller than the dynamic amplification factor of a steady state excitation.

Zibdeh [25] studied the vibrations of beams with general boundary conditions under a series of random moving pulse loads, obtaining a closed-form solution. Furthermore, Abu-Hilal and Zibdeh [26] studied the vibrations of beams with general boundary conditions subjected to a single deterministic moving force, considering the effects of accelerating, decelerating and constant velocity schemes of motions of the load was considered. Bryja and Smiady [27] treated the dynamic analysis of a suspension bridge subjected to a series of loads. Using influence functions, closed-form expression for the variance and the expected value of the deflection of the system were found. Abu-Hilal and Mohsen [28] treated the vibrations of a single span beam with general boundary conditions subjected to a moving harmonic load. They considered the effects of accelerating, decelerating and constant velocity schemes of motions of the load.

Garinei [29] treated the vibrations of a single span simply supported beam bridge under moving harmonic loads, focusing on the influence of the velocity, frequency and phase of the load on the maximum deflection of the system. Also, Garinei and Risitano [30] studied the vibrations of railway bridges under moving varying loads applied to simply supported bridges. The influence of the speed, frequency and phase of the harmonic components were studied. Ricciardelli [31] introduced the concept of the transient frequency response function for the fundamental case of a simply supported beam. The number of load cycles and the damping of the structure dominate the response at the resonance condition of the system in the transient event of a single moving harmonic load. Catal [32] studied Bernoulli – Euler beams under forced vibration conditions using the Differential Transformed Method (DTM), taking into account different boundary conditions. Piccardo and Tubino [33] treated the vibration of Bernoulli–Euler beams under resonant moving harmonic loads by calculating the convolution integral. Using the Multiple Scale Method (MSM), the solution for a simply supported beam with rotational springs at both ends was found. Also, Piccardo and Tubino [34] introduced simplified methods for the assessment of the serviceability of footbridges based on the proposal of the Equivalent Amplification Factor (EAF) and the Equivalent Synchronisation Factor (ESF). The first one indicates the ratio between the maximum dynamic response of the actual loading scenario and a single resonant pedestrian. The latter one indicates the ratio between the maximum dynamic response of the actual loading scenario and a uniformly distributed resonant pedestrian load.

Dmitriev in [35] and [36] studied the vibrations of two-span and three-span bridges. The results were further extended to continuous multi-span beams subjected to a moving force. However, these results were presented under the assumption of no damping, with equally long beam spans and with the same sectional properties. Yin [37] proposed an alternative approach in order to investigate the vibrations on simply supported beam bridges due to pedestrian loading by the use of a rectangular moving pulse. Attar [38] studied the dynamic response of cracked Timoshenko beams on elastic foundations subjected to moving harmonic loads. A a semi-analytic approach was developed using the Transfer Matrix Method (TMM) in combination with the modal approach in order to find the dynamic response of Timoshenko beams supported by shear elastic foundations. Hamada [39] treated the dynamic analysis of a beam subjected to a moving force using a double Laplace Transform applied to both time and spatial coordinates of the beam system. The transient response of the system was obtained as a Fourier series. Wang [40] studied the dynamic performance of Timoshenko beams under a moving constant force. The effect of the shear deformation, the rotatory inertia and the number spans on the dynamic performance of a multi-span beam was investigated and compared against the Euler–Bernoulli beam model. Martinez-Castro [41] presented a semi-analytical solution for non-uniform multi span Bernoulli–Euler beams under moving loads by using cubic Hermitian polynomials.

Johansson [42] presented a general framework for multi-span beam bridges under concentrated moving loads for general elastic conditions. The mass normalization scheme was adopted and this will be further used in the present work. Xu [43] studied the dynamic response of multi-span bridges subjected to moving loads with emphasis on the coupling conditions within the spans. It was found that the bridge performance can be improved through the proper joint and coupling parameter adjustments and configurations, significantly changing the dynamic performance of a multi-span bridge.

With regard to a free vibration analysis applied to multi-span beam systems, it is worth mentioning the work of [44], where it is studied the reduction of the response of the system produced by increasing the number of spans. The case of multi-span beams with intermediate flexible constraints is treated in [45]. The case of Timoshenko beam systems with intermediate elastic supports, flexible attachments and discontinuities is treated in [46] and [47]. The forced vibration of Timoshenko beams under general stationary or moving loads is studied in [48].

Finally, with regard to the use of the frequency domain approach, [49] has used the frequency domain spectral element method in order to treat the moving load problem applied to beam systems.

Chapter 3

Modelling aspects

3.1 Crowd modelling

The HSI effect has been studied both experimentally and numerically [50, 51]. The CCM used in this work is based on the hypothesis that the pedestrian can be modelled as a mechanical system consisting of a moving spring-mass-damper (SMD) degree of freedom, as shown in [52]. In this model, the bridge response is represented in the modal domain and is coupled to the SMD pedestrian system according to its location on the bridge. Considering the bridge response as a linear combination of N modes,

$$y(x,t) = \sum_{n=1}^{N} \phi_n(x) q_n(t) \tag{3.1}$$

in which $\phi_n(x)$ denotes the displacement normalized nth mode shape function of the structural system and $q_n(t)$ denotes the corresponding generalized modal coordinate, the equation of motion of the SMD pedestrian system can be written as

$$m_{\mathrm{p},r}\ddot{z}_r + c_{\mathrm{p},r}\left[\dot{z}_r - \dot{y}\right] + k_{\mathrm{p},r}\left[z_r - y\right] = 0$$

$$m_{\mathrm{p},r}\ddot{z}_r + c_{\mathrm{p},r}\left[\dot{z}_r - \sum_{n=1}^{N}\phi_n(x)\dot{q}_n(t)\right] + k_{\mathrm{p},r}\left[z_r - \sum_{n=1}^{N}\phi_n(x)q_n(t)\right] = 0 \tag{3.2}$$

where $m_{\mathrm{p},r}$, $c_{\mathrm{p},r}$, $k_{\mathrm{p},r}$ are the mass, damping, and stiffness of pedestrian r. The vertical displacement, velocity and acceleration of the SMD pedestrian system are denoted as z_r, $\dot{z}_r$ and $\ddot{z}$, respectively. Taking into account that the interaction forces $G_{\mathrm{p},r}(x,t)$ for the general pedestrian r can be written as

$$G_{\mathrm{p},r}(x,t) = \left[F_{\mathrm{p},r}(t) - m_{\mathrm{p},r}\ddot{z}_r\right]\delta(x - v_{\mathrm{p},r}t) \tag{3.3}$$

where $F_r(t)$ denotes the applied load and δ denotes the Dirac function, one has that the nth modal equation of the bridge system can be written as

$$m_{\mathrm{b},n}\ddot{q}_n + c_{\mathrm{b},n}\dot{q}_n + k_{\mathrm{b},n}q_n + m_{\mathrm{p},r}\phi_n(v_{\mathrm{p},r}t)\ddot{z}_r = \phi_n(v_{\mathrm{p},r}t)F_{\mathrm{p},r}(t) \tag{3.4}$$

where $m_{\mathrm{b},n}$, $c_{\mathrm{b},n}$, and $k_{\mathrm{b},n}$ denote the modal mass, damping, and modal stiffness n of the bridge system, respectively. Equations 3.2 and 3.4 show the coupling between the SMD and the modal projection of the bridge system. Finally, taking into account N modes of vibration and N_p pedestrians, a total of $N + N_\mathrm{p}$ equations can be obtained and the coupled crowd–structure model can be written as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F} \tag{3.5}$$

where **M**, **C** and **K** denote the mass, damping, and stiffness matrix of the coupled system, as defined by

$$\mathbf{M} = \begin{bmatrix} \mathbf{M_{11}} & \mathbf{M_{12}} \\ \mathbf{0} & \mathbf{M_{22}} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{C_{11}} & \mathbf{0} \\ \mathbf{C_{21}} & \mathbf{C_{22}} \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{K_{11}} & \mathbf{0} \\ \mathbf{K_{21}} & \mathbf{K_{22}} \end{bmatrix} \tag{3.6}$$

the vector $\mathbf{q}$ is defined as $\mathbf{q} = \begin{bmatrix} q_1 & \dots & q_N & z_1 & \dots & z_{N_\mathrm{p}} \end{bmatrix}^T$. It represents the response of each modal coordinate of the structural system and the response of each SMD pedestrian degree of freedom. Taking into account the mode shape vector of each pedestrian r as $\mathbf{\Phi}(v_r t) = \begin{bmatrix} \phi_1(v_r t) & \dots & \phi_N(v_r t) \end{bmatrix}^T$, the above submatrices can be defined as shown in Table 3.1.

It is important to note that:

- This approach is valid under the reasonable assumption that the structural mode shapes of the system do not change due to the presence of pedestrians.
- There is an explicit time dependency of the system matrices **M**, **C** and **K** due to changes of the pedestrian's position, shown in the definition of the submatrices in Table 3.1.

Finally, the modal force vector **F** is defined as the superposition of the modal forces due to each individual pedestrian. In this way, the load vector is defined as

$$\mathbf{F} = \begin{bmatrix} \sum_{r=1}^{N_\mathrm{p}} F_{\mathrm{p},r}(t)\mathbf{\Phi}(v_r t) \\ \mathbf{0} \end{bmatrix} \tag{3.7}$$

Table 3.1: Submatrices definition.

Entity	***Definition***	***Size***
$\mathbf{M_{11}}$	$\mathrm{diag}(m_{\mathrm{b},n})$	$\in \mathbb{R}^{N\times N}$
$\mathbf{M_{12}}$	coupling mass matrix, each column r is $m_{\mathrm{p},r}\mathbf{\Phi}(v_r t)$	$\in \mathbb{R}^{N\times N_p}$
$\mathbf{M_{22}}$	$\mathrm{diag}(m_{\mathrm{p},r})$	$\in \mathbb{R}^{N_\mathrm{p}\times N_\mathrm{p}}$
$\mathbf{C_{11}}$	$\mathrm{diag}(c_{\mathrm{b},n})$	$\in \mathbb{R}^{N\times N}$
$\mathbf{C_{21}}$	coupling damping matrix,each row r is $-c_{\mathrm{p},r}\mathbf{\Phi}(v_r t)^T$	$\in \mathbb{R}^{N_\mathrm{p}\times N}$
$\mathbf{C_{22}}$	$\mathrm{diag}(c_{\mathrm{p},r})$	$\in \mathbb{R}^{N_\mathrm{p}\times N_\mathrm{p}}$
$\mathbf{K_{11}}$	$\mathrm{diag}(k_{\mathrm{b},n})$	$\in \mathbb{R}^{N\times N}$
$\mathbf{K_{21}}$	coupling stiffness matrix, each row r is $-k_{\mathrm{p},r}\mathbf{\Phi}(v_r t)^T$	$\in \mathbb{R}^{N_\mathrm{p}\times N}$
$\mathbf{K_{22}}$	$\mathrm{diag}(k_{\mathrm{p},r})$	$\in \mathbb{R}^{N_\mathrm{p}\times N_\mathrm{p}}$
$\mathbf{F}$	applied force vector	$\in \mathbb{R}^{(N+N_\mathrm{p})\times 1}$

3.2 Flying phase

To take into account the off-contact period of the runners in the HSI model, different modelling scenarios may arise due to the different running kinematics of pedestrians. Moreover, the leg stiffness and the parameters of the model for running pedestrians can be developed in different ways and the corresponding values can vary over a certain range, in which nonlinearities can be introduced and considered. This is shown in [53–58].

In the present thesis, an approximation has been made by decoupling the equations of motion of the coupled crowd model by multiplying the mode shape function by a square wave that alternates the coupling action between the bridge and the SMD pedestrian system. The square wave is made by assuming the negative part of the running pedestrian load time history to be null, but when the load time history is positive to be unitary. This is shown in Figure 3.1.

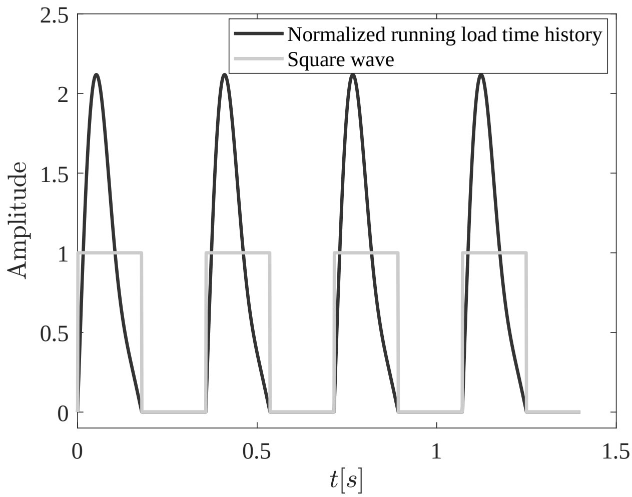

Figure 3.1: Flying phase approximation.

Chapter 4

Research work

This chapter provides extended descriptions of the topics from Paper I to Paper III. Section 4.1 presents an explanation of the methods used in Paper I, as well as a sensitivity analysis, including the flying phase as a decoupling feature of the CCM model. Section 4.2 provides an insight into the methods adopted in Paper II and its application to the case of a simply supported beam system. Lastly, Section 4.3 provides an insight into the eigenvalue problem in the context of the closed-form solution presented in Paper III.

4.1 Paper I

In Paper I the results of an experimental study of two footbridges were presented. Running tests were made and different modelling strategies for simulating these tests were compared. The modal properties of both bridges were determined by computing the spectrum of the measurements.

The paper considers a FE model, a moving force mode, and the CCM, in order to take into account HSI. For the moving force model and the CCM, the mode shape functions of both bridges were estimated by fitting trigonometric functions in the form of a Fourier series, as shown in Equation 4.1.

$$\bar{\phi}_n(x) = \sum_{i=1}^{k} A_k \sin(B_k x + C_k) \tag{4.1}$$

The general system of the CCM considers the pedestrians as SDOFs moving across the system. In the modal domain, the system can be interpreted as the equivalent SDOF of the bridge on which the pedestrians are attached, weighted by the mode shape function of the considered mode of vibration, evaluated at the current location of the pedestrian. This is shown in Figure 4.1.

In order to evaluate the results, a sensitivity analysis of the acceleration response of each system was made around the vicinity of the values of the parameters employed in the tests. The acceleration response of each system was sampled by varying the natural frequency of the pedestrian systems and the frequency of the applied load.

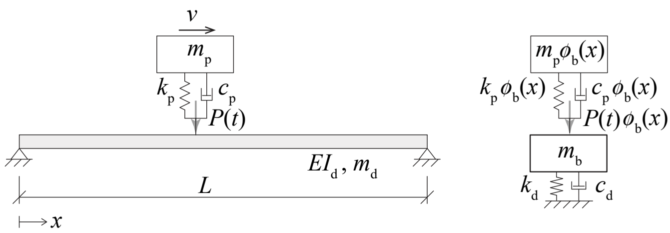

Figure 4.1: HSI model. Left: spatial model. Right: the modal model.

The sensitivity analysis presented in Paper I included the flying phase of the runners in both the load $P(t)$ and the pedestrian stiffness k_p. In this section, the sensitivity analysis is presented without taking into account the flying phase in the pedestrian stiffness k_p . In this way, the running pedestrian is always in contact with the bridge system. For the Vega Bridge, the sensitivity analysis without the flying phase is shown in Figure 4.2. It can be seen that the HSI effect occurs when the natural frequency of the bridge and the natural frequency of the pedestrians coincide, as in the analogy with the TMD, partially disrupting the dynamic amplification response of the system. Note the small shift of the step frequency that produces the highest response around the natural frequency of the bridge system. The highest response of the acceleration surface is shifted to a higher step frequency around the lower range of the natural frequency of the pedestrian system and towards a lower step frequency around the upper range of the natural frequency of the pedestrian system. This effect is increased due to the continuous contact between the pedestrians and the bridge system.

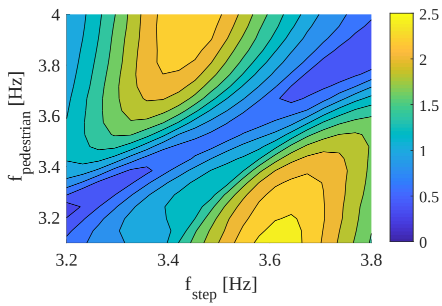

Figure 4.2: Acceleration response surface, Vega Bridge. $\left[\mathrm{m/s^2}\right]$.

For the KTH Bridge, the sensitivity analysis without the flying phase is shown in Figure 4.3. It can be seen again that the HSI effect occurs when the natural frequency of the bridge and the natural frequency of the pedestrians coincide. Additionally, not taking into account the flying phase maximises the interaction, having a more pronounced effect on the acceleration response in comparison with the analysis when the flying phase is taken into account. Also, note the small shift of the step frequency that produces the highest response around the natural frequency of the bridge system, as in the previous case. The highest response of the acceleration surface is shifted to a higher step frequency around the lower range of the natural frequency of the pedestrian system and towards a lower step frequency around the upper range of the natural frequency of the pedestrian system. This effect is increased due to the continuous contact between the pedestrians and the bridge system.

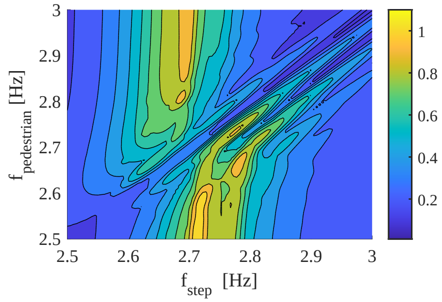

Figure 4.3: Acceleration response surface, KTH Bridge. $\left[\mathrm{m/s^2}\right]$.

From both Figure 4.2 and Figure 4.3, it can be seen that the HSI effect can also take place even if the pedestrian is modelled only as a mass-spring system in the transient event of a single crossing. Further studies need to be done in order to fully characterize running events.

4.2 Paper II

Paper II studies the clamped-clamped beam case. The method applied is based on the assumed mode method, given that it is possible to select a suitable trigonometric function that satisfies the boundary conditions and well estimates the modal parameters of the system itself. Then, the harmonic load is applied to the system and the modal force is transformed into well known solutions. The response is normalised by the equivalent steady state of the same harmonic load applied in the centre of the beam.

Four parameters are considered to study the pedestrian loading:

- The magnification factor Θ_1, that quantifies the dynamic response in the transient event of a single crossing given a certain level of damping and number of load cycles applied.

- The Λ parameter, which is the ratio of the transient dynamic response with respect to the equivalent steady state response.

- The minimum linear mass parameter m, which shows the minimum linear mass necessary to satisfy a certain level of acceleration.

- The S parameter, which is the ratio between the loading frequency and the natural frequency of the system.

The magnification factor curve is generated by sampling the magnification factor as a function of S, the damping, and the non-dimensional position of the moving harmonic load applied to the structural system in resonance condition. Then, the envelope is computed for the maximum response given a specific damping ratio.

In this section, the explained the method is applied to the simply supported beam case. The magnification factor Θ_1, the parameter Λ, and the minimum linear mass m, are presented in Figure 4.4, Figure 4.5 and Figure 4.6, respectively.

The results presented in Figure 4.4 were first presented by [4], using a numerical approach in order to quantify the maximum response of the equivalent single degree of freedom. It can be seen from Figure 4.4 that the magnification factor computed for a simply supported beam system is higher for the case of the clamped-clamped beam (see Paper II) given the same number of cycles and damping ratio. This is explained by the modulating effect of the mode shape of the simply supported case when the modal approach is adopted. However, as the damping ratio increases, this difference gradually decreases.

In addition, for both the simply supported beam system and the clamped-clamped beam system, it is easier to achieve the steady state response for the walking load case scenario than for the running load case scenario in the event of a single pedestrian crossing the beam. It can be concluded from the evaluation of Λ in Figure 4.5 that in order to achieve a steady response due to a single pedestrian, a beam system must have very low inherent damping in combination with sufficient applied loading cycles.

Lastly, similar to the case of the clamped-clamped beam, for simply supported beam systems the running load case scenario is always more critical than the walking load case scenario given the same amount of inherent damping in the system. This is shown in Figure 4.6. Moreover, it is important to note that for the case of a simply supported beam, the minimum linear mass design curve is lower than for the clamped-clamped beam. This is due to the combined effect of the corresponding modal masses and the magnification factor of each system.

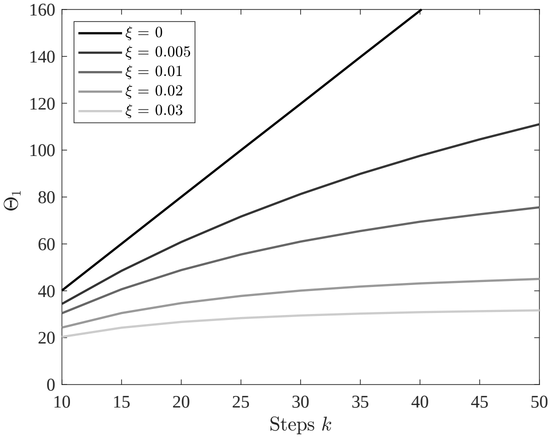

Figure 4.4: Magnification factor Θ_1.

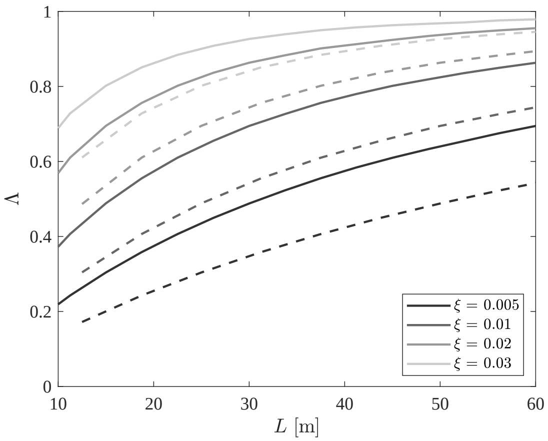

Figure 4.5: The parameter Λ as a function of span length (- -normal running —normal walking).

4.3 Paper III

Paper III is an extension of the results of Paper II to a more general case. It shows the general solution of the moving harmonic load problem applied to continuous beam systems with general boundary conditions. The general system can be characterised by its beam segments j as shown in Figure 4.7, in which $P(t)$ and v represent the time-varying load and the velocity at which the load moves. The terms EI_j, $k_{v,j}$, $k_{r,j}$ and L_j represent the bending stiffness, vertical elastic support,

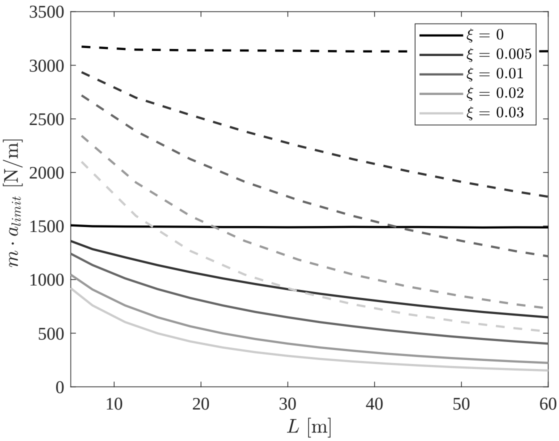

Figure 4.6: Minimum linear mass as a function of span length (- -normal running —normal walking).

rotational support and the length of the general beam segment j.

The previous literature, e.g. [41, 42] focused on the general solution for the moving constant load problem, as this finds an application to railway dynamics. However, the moving harmonic load problem can also find an application to pedestrian bridge dynamics.

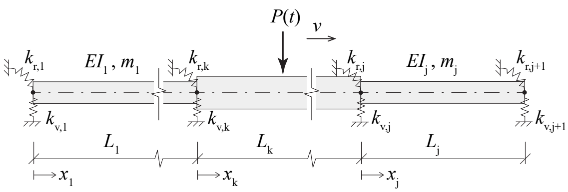

Figure 4.7: A continuous beam system on elastic supports as in Paper I.

The general problem is formulated when multiplying the harmonic force by the general mode shape functions, made out of trigonometric and hyperbolic functions. Then, the modal force is decomposed into well known solutions and the method of undefined coefficients is applied to solve the equation of motion. The solution is found by adding ten different functions that compose both the steady state response as well as the transient response of the system.

In the formulation of the general solution, a continuous beam system is considered according to its beam segments. Compatibility and equilibrium conditions between each segment are satisfied. Also, cross sectional properties are assumed constant over each beam segment. In this way, the eigenvalue problem is formulated and the mode shape of the structure becomes a step-wise function with respect to each beam segment. The dynamic interpretation of this approach is that on each segment, there is applied the moving harmonic load, modulated by the mode shape functions of the considered beam segment.

Finally, the phase angle in the definition of the load and of the already traversed beam segments is taken into account in order to maintain continuity in the dynamic response of the system.

In Paper III, the eigenvalue problem is solved in closed-form by properly assembling the characteristic matrix that contains the boundary conditions, i.e. the displacement and slope conditions and the equilibrium conditions, namely, the shear forces and bending moments of the general continuous beam system constituted by j segments. In matrix form, the eigenvalue problem can be written as

$$\mathbf{\Lambda}_{4j\times 4j}(\omega_n)\mathbf{C}_{4j\times 1} = \mathbf{0}_{4j\times 1} \tag{4.2}$$

in which $\mathbf{\Lambda}$ and $\mathbf{C}$ represent the evaluated base functions matrix and, the coefficient vector of the base functions, respectively.

In order to obtain non-trivial solutions of the mode shape coefficients, the determinant of the characteristic matrix $\mathbf{\Lambda}$ is set equal to zero. It is important to note that in the continuous formulation, the characteristic matrix $\mathbf{\Lambda}$ is made of continuous functions. This is also the case for the corresponding determinant of $\mathbf{\Lambda}$, which becomes a function dependent on ω_n, i.e. dependent on the natural frequency of the system, as shown in Equation 4.2.

In order to determine the natural frequencies of the three examples presented in Paper III, the determinant was sampled along the frequency range of interest and the roots of the function were identified, such that Equation 4.3 is satisfied, finding the eigenvalues that make the matrix $\mathbf{\Lambda}$ a singular matrix.

$$\det \mathbf{\Lambda}(\omega_n) = 0 \tag{4.3}$$

The graph of the logarithm of the determinant for the first example solved in Paper III is shown in Figure 4.8.

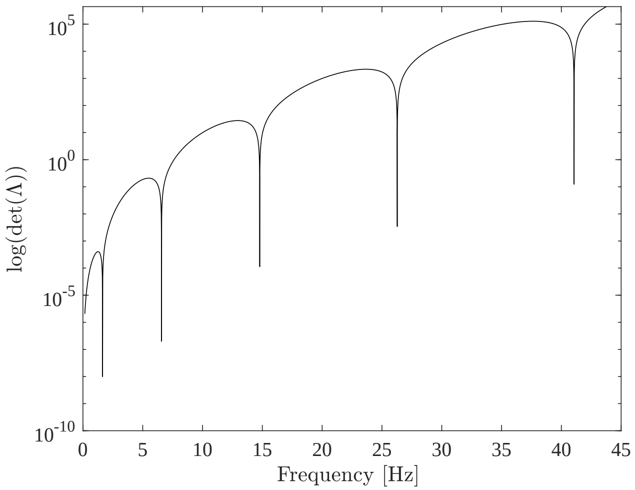

Figure 4.8: Frequencies of Example 1: Simply supported bridge.

After the eigenvalue problem has been solved, the mass normalization scheme is adopted. Then, the general formulation of the input modal force is considered to be the multiplication of the loading sinusoidal function and the trigonometric and hyperbolic functions that build the mode shapes of the system. Once the general problem has been formulated, the hyperbolic functions are written in exponential form and the method of undefined coefficients is applied in order to solve the differential equation. Finally, the phase angle is taken into account in order to maintain the time continuity condition in the load time history and in the response of the system. This is done by considering not only the phase angle of the load definition but also that obtained due to the load's traversing each beam segment. It is important to note that by considering the phase angle in the load definition it is possible to represent a moving constant load, as is more usual in railway dynamics. In this way, the general solution of the moving harmonic load problem includes the solution of the moving constant load problem.

Chapter 5

Conclusions and further research

In this chapter, the general conclusions of the thesis and the research performed during the licentiate project are given. Lastly, suggestions for future research related to the overall aim of this research project will be presented.

5.1 General conclusions

The study of the moving harmonic load problem in the context of footbridge dynamics has led to the following conclusions:

- Under normal walking and normal running load conditions it is difficult achieve a steady state response, especially for lightly damped footbridges.
- It is easier to achieve the steady state response under a normal walking conditions than it is under normal running conditions. This has been quantified in terms of the influence of the number of load cycles applied to the system and its relationship with the transient event.
- For a single pedestrian crossing, running is more critical than walking. This has been demonstrated by the minimum linear mass parameter, which shows the need to address the running load case scenario.
- If the HSI effect is negligible, the generalized degree of freedom strategy, the finite element approach, and the coupled system model with and without the flying phase can consistently simulate the dynamic response of footbridges with negligible differences.
- The HSI effect can appear in footbridges subjected to a number amount of runners even if the flying phase is taken into account in the transient loading event.
- In order to formulate a closed-form solution of the moving harmonic load problem applied to general continuous beam systems, the phase angle of the load definition and the phase angle obtained of the already traversed beam segments must be taken into account. In this way, the time continuity condition of the load time history and the corresponding response of the considered system is maintained.

- The solution proposed in Paper III can be used to find the dynamic response of any continuous Bernoulli–Euler beam system under any load spectra, including the static contribution.
- The solution of the moving harmonic load problem includes the solution of the moving constant load problem since the phase angle of the load definition has been taken into account.

5.2 Further research

Base on the findings presented in this licentiate thesis, the author has identified several topics of interest for future research:

- The HSI effect can be further studied in the framework of a closed-form solution in order to fully describe the different relations between the governing parameters and generalise its application with a simpler method.
- Characterise different crowd loading events on footbridges in the context of random vibrations, in which the phase angle of the loading scenario between pedestrians will no longer need to be taken into account. In this way, load models based on different power spectral density functions can give further insight into the system response in accordance with the defined input.
- The HSI effect can be studied in the framework of random vibrations in which the evaluation of the studied systems can be made based on the expected value of the response.
- Formulation and evaluation of a more adequate criteria with regard to the serviceability design and assessment of footbridges is required. Using the maximum acceleration response of footbridges given a certain event does not give any insight of the duration in which the acceleration limit has been surpassed nor the probability of the associated limit state function.
- The moving harmonic load problem applied to curved beams considering damping in the context of footbridge dynamics can be studied in order to give an insight into the response of such systems in which vertical, transversal and longitudinal vibrations are coupled.

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