Doctoral Thesis in Physics

## Gamma-ray Spectroscopy of Neutron-rich ${ }^{11} \mathrm{Mo},{ }^{85,87} \mathrm{Ge}$ and Self-Conjugate ${ }^{88}$ Ru Far From Stability

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# Gamma-ray Spectroscopy of Neutron-rich ${ }^{11} \mathrm{Mo},{ }^{85,87} \mathrm{Ge}$ and Self-Conjugate ${ }^{88}$ Ru Far From Stability 

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Academic Dissertation which, with due permission of the KTH Royal Institute of Technology, is submitted for public defence for the Degree of Doctor of Philosophy on U1 KTH campus \& 15 Jun 2021 kl 16:00


#### Abstract

The neutron-deficient self-conjugate ( $\mathrm{N}=\mathrm{Z}$ ) nucleus ${ }_{44}^{88} \mathrm{Ru}_{44}$ was populated via the heavy ion fusion evaporation reaction ${ }^{54} \mathrm{Fe}\left({ }^{36} \mathrm{Ar}, 2 n\right){ }^{88} \mathrm{Ru}$ in an experiment performed at the GANIL accelerator laboratory in France. Using the AGATA $\gamma$-ray spectrometer together with ancillary detectors, prompt $\gamma-\gamma-2 n$ coincidence and charge particle anticoincidence analysis was performed for the low-lying energy spectrum of ${ }^{88} \mathrm{Ru}$. The results confirm the previously assigned $\gamma$-ray cascade and extend it to the $14^{+}$level. The level scheme is consistent with a deformed rotational system. However, the rotational frequency of the alignment of the valence nucleons has a significantly higher value than what is predicted by theoretical calculations performed without isoscalar neutron-proton pairing. By including isoscalar pairing, an agrement is obtained with the experimentally observed delayed rotational alignment.

Excited states in the neutron-rich nuclei ${ }^{109} \mathrm{Mo}$ and ${ }^{111} \mathrm{Mo}$ were studied following nucleon knock-out reactions. Seven $\gamma$-ray transitions, some of them in prompt mutual coincidence, were identified for the first time in ${ }^{111}$ Mo using the DALI2 and MINOS detector systems at the BigRIPS and ZeroDegree electromagnetic fragment separator at the RIBF, RIKEN, Japan. Total Routhian surface (TRS) and Particle-Plus-Rotor calculations have been performed to investigate the predicted shape coexistence and its effect on the structure of nuclei in this region of the nuclear chart. Following the results of the calculations, theoretical level schemes are proposed for positive and negative parity states and compared with the experimental findings.

Gamma-ray transitions have been identified for the first time in the extremely neutron-rich $(N=Z+25)$ nucleus ${ }^{87} \mathrm{Ge}$ following nucleon knockout reactions studied at the RIBF, RIKEN, Japan. Previously unknown $\gamma$-ray transitions between excited states in ${ }^{85} \mathrm{Ge}$ were also observed and placed in a tentative level scheme. The results are compared with large-scale shellmodel calculations and potential energy surface calculations based on the total Routhian surface formalism. The neutron-rich titanium isotopes have been studied, and preliminary results are presented in this work. For the oddeven ${ }^{57,59,61} \mathrm{Ti}$ isotopes several gamma-ray transitions has been identified for the first time. For the even-even isotopes ${ }^{56,58,60} \mathrm{Ti}$ the previously known decays from $2^{+}$and $4^{+}$spin-parity states, are confirmed with the current preliminary analysis.


## Sammanfattning

Den neutronfattiga atomkärnan ${ }^{88} \mathrm{Ru}$, med lika antal protoner och neutroner, producerades genom fusion-evaporationreaktionen ${ }^{54} \mathrm{Fe}\left({ }^{36} \mathrm{Ar}, 2 n\right){ }^{88} \mathrm{Ru}$ vid ett experiment som utfördes vid acceleratorlaboratoriet Grand Accélérateur National dIons Lours (GANIL) i Frankrike. Den experimentella uppställningen bestod av gammaspektrometern AGATA samt detektorsystem för detektion av neutroner och laddade partiklar. Händelser bestående av prompta koincidenser av två gamma och två neutroner och som är i antikoincidens med laddade partiklar analyserades för studier av lågt liggande energinivåer i ${ }^{88} \mathrm{Ru}$. Resultaten konfirmerar den tidigare observerade gammakaskaden $\mathrm{i}^{88} \mathrm{Ru}$ och utökar kaskaden till en energinivå med spin och paritet $14^{+}$. Nivåschemat är konsistent med ett deformerat roterande system. Rotationsfrekvensen för upplinjering av valensnukleonernas spin längs kärnans rotationsaxel är emellertid betydligt högre än vad som erhålls med teoretiska beräkningar gjorda utan isoskalära neutron-proton parkorrelationer. Genom att inkludera sådana parkorrelationer erhålls en överensstämmelse med den experimentellt observerade fördröjda upplinjerningen.

Exciterade tillstånd i de neutronrika kärnorna ${ }^{109}$ Mo och ${ }^{111}$ Mo studerades genom nukleon "knock-outreaktioner. Sju gammaövergångar, en del av vilka befanns vara i prompt koincidens med varandra, observerades för första gången $\mathrm{i}^{111} \mathrm{Mo}$. Experimentet utfördes med detektorsystemen DALI2 och MINOS och med de elektromagnetiska fragmentspektrometrarna BigRIPS + ZeroDegree vid RIBF acceleratorn vid RIKEN i Japan. Total Routhian Surface"(TRS) och partikel-rotor beräkningar utfördes för att undersöka den teoretiskt förutspådda samexistensen av olika kärnformer och dess effekt på kärnstrukturen i detta område av nuklidkartan. Med hjälp av resultaten från beräkningarna kunde teoretiska nivåcheman för tillstånd med positiv och negativ paritet konstrueras, vilka jämfördes med de experimentellt observerade energinivåerna.

För första gången observerades gammaövergångar i den neutronrika kärnan ${ }^{87} \mathrm{Ge}$ genom nukleon knock-out reaktioner vid experiment utförda vid RIBF, RIKEN. Tidigare okända gammaövergångar mellan exciterade tillstånd i ${ }^{85} \mathrm{Ge}$ observerades och placerades i ett preliminärt nivåschema. Resultaten jämfördes med storskaliga skalmodellsberäkningar samt med beräkningar av kärnornas potentialenergiytor baserade på TRS-formalismen.

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## List of appended papers

I Isospin properties of nuclear pair correlations from the level structure of the self-conjugate nucleus ${ }^{88} \mathrm{Ru}$.
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## XIII Collective rotation of an oblate nucleus at very high spin.

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#### Abstract

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## List of Acronyms

| AGATA | Advance GAmma Tracking Array |
| :--- | :--- |
| BDC | Beam Drift Chamber |
| BigRIPS | Big RIKEN projectile-fragment separator |
| CCPSA | Charge Comparison Pulse Shape Analysis |
| DALI2 | Detector Array for Low Intensity radiation 2 |
| DWIA | Distorted-Wave Impulse Approximation |
| FDC | Forward Drift Chamber |
| FOM | Figure Of Merit |
| FWHM | full width at half maximum |
| GANIL | Grand Accelerateur National d'Ions Lours |
| GTS | Global Trigger and Synchronization |
| HPGe | High-Purity Germanium |
| LSSM | large-scale shell-model |
| MicroMegas | Micro-mesh gaseous structure detector |
| NEDA | NEutron Detector Array |
| OFT | Orsay Forward Tracking |
| PES | Potential Energy Surface |
| PID | Particle IDentification |
| PPAC | Parallel Plate Avalanche Counters |
| PPR | Particle Plus Rotor |
| PSA | Pulse Shape Analysis |
| PSD | Pulse Shape Discrimination |
| QCD | Quantum Chromodynamics |
| RIBF | Radioactive Isotope Beam Facility |
| SAMURAI | Superconducting Analyzer for MUlti-particle RAdio Isotope beam |
| SEASTAR | Shell Evolution And Search for Two plus energies At RIBF |
| STQ | Superconducting Triplet Quadrupole |
| TEGIC | Tilted Electron Gas Ionization Chamber |
| TOF | Time-of-Flight |
| TPC | Time Projection Chamber |
| TRS | Total Routhian Surface |
|  |  |

## Chapter 1

## Introduction

As nuclear physicists, our aim is to understand the behaviour of one of the most complex physical systems, the atomic nucleus. It has fascinating properties like shape deformation, shape coexistence and exotic decay modes. The complexity of the nuclei comes from the competition between the electromagnetic, strong, and weak interactions. Since E. Rutherford discovered the nucleus more than a hundred years ago [1], enormous experimental and theoretical efforts were spent in order to understand its properties. The atomic nucleus is a quantum many-body system and in order to understand this system it is essential to define the properties of the strong nuclear force. Until today, there has been considerable progress in defining the nuclear force from the theory of Quantum Chromodynamics (QCD) $[2,3]$. The nuclear force can be modeled using the lattice $\mathrm{QCD}[4]$, which is the first principle approach to solve the QCD equation numerically, or chiral effective field theory $(\chi \mathrm{EFT})$ [5]. One advantage of such theories is that they allow us to derive theoretical uncertainties. A disadvantage is that they have complicated algorithms and need powerful computers for exact solutions. Alternatively, instead of using such complex algorithms, the nuclear force can be defined by phenomenological potentials. In many such solutions, the first approximation is that the nucleons which are free, spin $1 / 2$ particle obeys the Pauli principle, considered to follow long mean-free paths inside the nucleus. One of the most important discoveries in order to developed today's shell model system for a nucleus is the discovery of sudden changes in the binding energies for some nucleids. This irregularity for nuclei having specific numbers of protons and/or neutrons, brings the first definition of the magic numbers by W. Elsasser [6-8]. He suggested that if the nucleus has a closed shell of protons and neutrons, it is more bound. In this early version of the shell model, each nucleon moves in a common potential well at degenerate energy levels without interacting with other nucleons. Later, this model was developed further to reproduce experimental information. In 1949, M. Goeppert Mayer [9], and O. Haxel, J. Jensen and H. Suess [10] showed that by including the spin-orbit term to the potential, it is possible to reproduce all experimental energy-gaps that
is corresponding to shell gaps at magic numbers. This model is still one of the best that fits experimental results and can give the understanding of the fundamental nuclear structure, the next chapter is dedicated to theories on the nuclear structure.

With the simultaneous development of theoretical shell-model calculations, and experimental research we gained more information on the outer edges of the Segrè chart by the proton dripline and the region towards the neutron dripline. Moreover, one of the outcomes of the research on this region is the occurance of the nuclear magic numbers far from stability. In this work, regions of interest in the neutron-rich side around $\mathrm{N}=50$ and $\mathrm{N}=70$. This regions play vital roles in the nucleosynthesis process in astrophysics. Most of the elements heavier than iron (also some lighter elements) are produced by neutron capture nucleosynthesis. The astrophysical neutron capture process proceed in two different way as slow and rapid neutron capture process. If the neutron capture rates are slower than the $\beta$-decay rates, it is called the slow-neutron process or s-process, and it produces the isotopes close to the stability line. To synthesize isotopes far from the valley of stability, the neutron capture must progress much faster than the $\beta$ decay, this is called the rapid-neutron capture or r-process. The r-process continues to build up neutronrich isotopes until the waiting points are reached or until the equilibrium is achieved in-between photo-disintegration ( $\gamma, \mathrm{n}$ ) and neutron capture ( $\mathrm{n}, \gamma$ ) rates. At such a waiting point, the nucleid can $\beta$ decay to the next element. This continuous process of neutron capture and $\beta$-decay creates the r-process path far from stability [11]. The calculated r-process path around $\mathrm{N}=50$ to $\mathrm{N}=82$ is shown in Fig. 1.1. On the rprocess path, where the equilibrium is achieved and the waiting points are reached, correspond to regions with increased stability. At magic number nuclei have longer halflife and increased nuclear abundances. Therefore experimental studies of magic nuclei will directly contribute to our understanding of the nucleosynthesis process.

Another critical region on the Segrè chart is the nuclei around the $\mathrm{N}=\mathrm{Z}$ line. The stable nuclei on the Segrè chart follows the $\mathrm{N}=\mathrm{Z}$ line until the ${ }_{20}^{40} \mathrm{Ca}_{20}$ isotope, and than they start bending towards to $\mathrm{N}>\mathrm{Z}$ region, due to Coulomb repulsion. The available data for the heavier $\mathrm{N}=\mathrm{Z}$ nuclei is limited; the heaviest nucleid with known decay mode and half life information is the ${ }_{50}^{100} \mathrm{Sn}_{50}$ isotope. The $\mathrm{N}=\mathrm{Z}$ nuclei are a good testing ground to understand the effect of neutron-proton correlations inside the nucleus since the valence protons and neutrons occupy identical shell model orbitals. In this region shape deformation for the $\mathrm{N}=\mathrm{Z}$ isotopes masks the effect of $n p$ pairing on the nuclear structure. Despite detailed studies in the interaction between rotation and like-particle pairing interaction in deformed nuclei to understand the isovector pairing mode $\mathrm{T}=1$, the interplay between rotation and unlike-particle pairing needs more studies to understand the isoscalar $\mathrm{T}=0$ pairing. According to theoretical calculations, nuclei heavier than mass number 80 is one of the best places to look for isoscalar pairing [13]. The signature effect of the isoscalar pairing will be visible in the band crossing frequency of the even-even rotational nuclei with spins up to $16 \hbar$. The isotope ${ }_{44}^{88} \mathrm{Ru}_{44}$ is an excellent candidate to study the competition between rotation and isoscalar pairing with 44 neutrons and protons. In this work an experimental study of the self-conjugate ${ }^{88} \mathrm{Ru}$ isotope is reported in


Figure 1.1: The isotopes that were studied in this thesis are marked with red square in the Segrè chart. The color code is based on half-life values. The inset figure shows r-process path calculated with the relativistic mean-field (RMF) mass model for the $\mathrm{Z}=20-50$ region[12].

Paper I. Here, the results are discussed in terms of isoscalar pairing and delayed rotational frequency in comparison to neighboring isotones. The experimental data for this study is part of the AGATA-NEDA-DIAMANT campaign in The Grand Accélérateur National d'Ions Lourds (GANIL).

The neutron-rich nuclei around $\mathrm{A} \approx 110$ in the neutron-rich side of the Segrè chart are predicted to exhibit rapid ground state shape transition in-between prolate, oblate, and triaxial shapes with increasing neutron numbers [14]. The molybdenum isotope chain is a good testing ground to study shape coexistence with increasing neutron numbers. The recent results from ${ }^{107,109}$ Mo studies suggest that a shape transition from prolate to oblate deformation take place, and this transition could also be seen for $\mathrm{N} \geq 67$ odd-A neutron-rich isotopes [15]. The study on the ${ }_{42}^{111} \mathrm{Mo}_{69}$ isotope with $\mathrm{N}=69$ is a crucial step to understand the shape coexistence and to get a complete picture of the region. In the current analysis, which is given in Paper II, we observe $\gamma$-ray transitions for ${ }^{111} \mathrm{Mo}$ for the first time. The ground state of this nucleid was known previously [16], however knowledge on the excited states were missing. We also performed theoretical calculations based on the particle rotor model in order to see the dominant Nilsson orbit configurations for the ground states corresponding to different deformed shapes.

Another neutron-rich doubly-magic nucleus that attracted attention recently is ${ }_{28}^{78} \mathrm{Ni}_{50}$. Experimental and theoretical studies show that ${ }^{78} N i$ exhibit structural features consistent with a doubly-magic "stronghold" against nuclear deformation, unlike the neighboring neutron-rich systems where there is a prediction of disap-
pear of the $\mathrm{N}=50$ and $\mathrm{N}=28$ magic numbers [17]. The orbital migration between the $f_{5 / 2}, p_{3 / 2}$ and $f_{7 / 2}$ proton orbitals may cause a reduction of the $\mathrm{N}=28$ gap, while the rearrangement of the $s_{1 / 2}, d_{5 / 2}$ and $g_{7 / 2}$ neutron orbitals affects the $\mathrm{N}=50$ neutron gap. Therefore the evolution of the single-particle levels with increasing neutron numbers is an important playground to test nuclear models. Paper III is focused on the topic to understand the single-particle structures and neutron orbit migrations at the $\mathrm{N}=53,55$ germanium isotopes. In this work, the level scheme of the isotope ${ }^{85} \mathrm{Ge}$ is extended, and the exclusive parallel momentum distribution for $1 / 2^{+}$spin state is compared with distorted-wave impulse approximation (DWIA) calculations for the neutron knockout reaction to examine the valence neutron orbitals. In addition, excited states for ${ }^{87} \mathrm{Ge}$ was observed for the first time.

The data used for Papers II and III are part of one of the large-scale experimental campaigns that aimed to measure low-spin $\gamma$ spectra via knockout reactions of various exotic nuclei. The experimental campaign named SEASTAR (acronym to Shell Evolution And Search for Two plus energies At RIBF) covers the region from $\mathrm{N}=32$ to $\mathrm{N}=70$, with the aim to measure $2+$ energies of even-even isotopes. The experiments were performed using the DALI2 and MINOS detector arrays coupled with the BigRIPS and the ZeroDegree spectrometers at the Radioactive Isotope Beam Facility (RIBF), RIKEN, Japan. The campaign was split up to three experiment to three different mass regions. The first experiment hold in 2014 focused on the region around ${ }^{78} \mathrm{Ni}$ with main channels ${ }^{66} \mathrm{Cr},{ }^{70,72} \mathrm{Fe}$ and ${ }^{78} \mathrm{Ni}$. The second experiment, 2015 focused on the beyond $\mathrm{N}=60$ with the main channels ${ }^{82,84} \mathrm{Zn},{ }^{86,88} \mathrm{Ge},{ }^{98,100} \mathrm{Kr}$ and ${ }^{110} \mathrm{Zr}$. The results presented in Paper II and III are part of the second campaign. The third sub-campaign explores the lighter region around $\mathrm{N}=32$ with the main channels ${ }^{52} \mathrm{Ar},{ }^{56} \mathrm{Ca}$, and ${ }^{62} \mathrm{Ti}$.

This doctoral thesis is divided into six chapters: following the introduction, chapter 2 gives a brief overview of the theoretical background and models used to interpret the experimental results. Chapter 3 covers a detailed description of the experimental set-ups. In chapter 4 the data analysis methods are explained. After a brief discussion of the results given in chapter 5, a summary of Papers I, II and III is given in Chapter 6 together with the information about the author's contribution to the work.

## Chapter 2

## Theoretical Framework

This section is a summary over the scientific models that aim to explain the current experimental measurements. In nuclear physics, structure and properties of the nuclei are described by both macroscopic and microscopic models. In this work the nuclear shell model with the Total Routhian Surface (TRS) calculation were utilized to interpret experimental results for germanium isotopes. Subsequently, both the Particle Rotor model and TRS calculations were performed to describe the ground- and low-lying excited states of Mo isotopes. These three models are explained briefly in this chapter as well as a short description of the direct reaction mechanism and the electromagnetic radiation following the nuclear reactions.

### 2.1 Nuclear Shell Model

The first information of shell structure comes from magic numbers which are 2,8 , $20,28,50,82$, and 126 for both protons and neutrons. Magic nuclei has several common features, such as; the nuclei with these numbers are more stable compared to other nuclei like exhibiting larger binding energy compared to other nuclei. Furthermore, the number of isotopes and isotones for magic nuclei are larger than the other stable nuclei. These common features of magic numbers are the main confirmation of shell structure in nuclei. In the shell model, a nucleon moves independently in an average mean-field potential created by the interactions of all other nucleons. The potential is relatively constant inside the nucleus and goes to zero outside the nuclear surface [18]. The Hamiltonian corresponding to $\mathrm{A}=\mathrm{N}+\mathrm{Z}$ nucleons inside the nucleus is:

$$
\begin{equation*}
H=\sum \frac{p_{i}^{2}}{2 m_{i}}+\sum V_{i j} \tag{2.1}
\end{equation*}
$$

where $m_{i}$ is the mass of the $i^{\text {th }}$ nucleon and $V_{i j}$ is the nucleon-nucleon interaction. Solving this Hamiltonian and finding exact eigenvectors and eigenvalues is not possible today and even in the foreseeable future. The main reason is that we do not
know how to calculate nucleon-nucleon interaction starting from the underlying Quantum Chromo Dynamic (QCD) theory. Even if we can define the interaction exactly, calculating a medium-heavy nucleus will be overwhelming for today's computing technology. One of the most common approaches to this problem is using an approximate solutions to eigenvectors and eigenvalues of the nuclear Hamiltonian in Eq. 2.1 and is based on out experimental knowledge on the nuclear magic numbers and double magic nuclei. As mentioned above, the double magic nucleus is stable with respect to other nuclei, and the particle outside the core with a closed shell does not affect the internal structure. Therefore we can assume this double magic core is a frozen and it induces a central potential field $\left(U\left(r_{i}\right)\right)$. This field only depends on the distance between the particle outside the core and the core's center ( assuming the spherical symmetric potential).The Hamiltonian can be written as $H=T+U+\nu$ with a residual interaction $\nu=\sum V_{i j}-U$ and new central field $H_{0}=T+U$ for pure single-particle Hamiltonian. Eq. 2.1 becomes a shell model Hamiltonian in Eq.2.2,

$$
\begin{equation*}
H=H_{0}+\nu \tag{2.2}
\end{equation*}
$$

where, the nucleons outside the core move under the effect of $H_{0}$ field, with obeying the Pauli exclusion principle [19].

The potential $H_{0}$ in Eq. 2.2 can be approximated by a harmonic oscillator potential and even better by a potential of Woods-Saxon type ( $\mathrm{V}_{W S}$ ), that more closely follows the density of nuclei. Such a potential can explain the observed spins of magical nuclei $+/-1$ nucleon.

$$
\begin{equation*}
V_{W S}(r)=-\frac{V_{0}}{1+e^{\left(r-R_{0}\right) / a}} \tag{2.3}
\end{equation*}
$$

where $V_{0}$ is the potential depth, a positive constant for the attractive potential, the radius of nuclei is $R_{0} \approx 1.2 \mathrm{fm} A^{1 / 3}$, and $a$ is the diffuseness or surface thickness in fm unit. Here $r=|\vec{r}|$ is the magnitude of the radius vector, from the center of spherical potential to the position of the nucleon. Wood-Saxon potential can not give the magic numbers above 20 correctly, likewise harmonic oscillator. In order to describe excited states or states having more nucleons outside closed shell, we have to introduce a model, where different configurations can interact among each other to generate the observed spins and parity of the nuclei we aim at describing. In 1949 Goeppert-Mayer [9] and independently Jensen, Haxel and Suess [10] introduced the spin-orbit potential as $f(r) \vec{l} \cdot \vec{s}$ to obtain the larger nuclear magic numbers. Maria Goeppert-Mayer and Jensen got the Nobel prize in Physics for their contribution to nuclear theory in 1963. The spin-orbit potential introduced by Goeppert-Mayer and Jensen et al. depend on the $l$ the orbital angular momentum and $s$ intrinsic spin of the nucleon. Although this potential gives shell structure correctly, it has a weakness that it cannot be used in obtaining analytical forms of the wave function, it is however used to determine the wave function numerically. The eigenstates of the spin-orbit term determined by the quantum numbers: $j=|l \pm 1 / 2|, n, l, s$, and $m$. Apart from predicting the magic numbers, using the valence nucleons or
holes (particles / holes outside the magic core), spin-parity values of the ground and excited states and nuclear electromagnetic moments of the nuclei can be predicted within the nuclear shell model.

Another important concept in the nuclear shell model is isospin which is effective in nucleon-nucleon interactions. In this formalism, protons and neutrons are distinguished by an isospin quantum number for $\mathrm{A}=\mathrm{N}+\mathrm{Z}$ nucleons. The isospin operator is defined as $\vec{t}=\frac{\vec{r}}{2}$ still obeying the Pauli principle with Pauli isospin matrices $\vec{\tau}\left(\tau_{x}, \tau_{y}, \tau_{z}\right)$. The sum of the isospin vectors gives the total isospin of the valence nucleons The pairing of the two valence nucleons can be done in two ways. The nucleons can be coupled with anti-parallel spins, isovector $\mathrm{T}=1$, where each nucleon pair is coupled to 0 angular momentum. In such cases, only even J values are allowed in the wave function. On the other hand, one can also have isoscalar $\mathrm{T}=0$ neutron-proton pairing with a parallel spin resulted in only odd $J$ angular momentum values. Therefore the wave function that describes the nuclear system constructed in terms of the isospin vectors and the angular momentum couplings results in all J values from 0 to $2 j$. $\mathrm{N}=\mathrm{Z}$ isotopes are a special interest due to the fact that, neutron-proton pairing can be studied; since the neutrons and protons occupy the same orbitals with the same quantum numbers, and their Fermi levels are close to each other.

When the nucleus has many valence particles/holes, it gets harder to calculate the residual interaction $\nu$ in full shell model Hamiltonian in Eq. 2.2. In this case, the residual interaction between these many valence nucleons can be calculated with a deformed potential description instead of spherical symmetric potential. The expected energy levels for such a system would shift with the shape of the potential. As a result of this deformation with excessive valence nucleons, in the exotic sides of the Segrè chart, it became clear that the gaps for the magic numbers squeeze and new large gaps appear giving rise to new magic numbers. One of the deformations is known as Quadrupole deformation, and it can describe asymmetric shapes. The triaxially distorted potential governed by the $\gamma$ shape degree of freedom describes the deformation effect at the right angle to the major nuclear axis. $\gamma=$ $0^{\circ}$ corresponds to prolate, $\gamma=60^{\circ}$ oblate and $\gamma=30^{\circ}$ gives the triaxial shape deformations. The model for the axially symmetric nonspherical nuclei is called as Deformed Shell Model (Nilsson model) [20]. The parameters for the deformation and effects will be explained in the TRS section of this chapter.

### 2.1.1 Shell model parameters in the ${ }^{78} \mathrm{Ni}$ region

In order to understand structure of the highy neutron rich Ge isotopes large-space shell model calculations were performed. For nuclei close to ${ }^{78} \mathrm{Ni}$, the experimental information on the excited states gathered from the present work are compared to the theoretical LSSM calculations. In the model space the ${ }_{28}^{78} \mathrm{Ni}_{50}$ is used as a core, and the different orbits are performed for the protons and neutrons. The two type of calculations were used to understand the neutron-rich Ge isotopes. First set of the calculations were performed using proton $\pi\left(1 p_{1 / 2}, p_{3 / 2}, f_{5 / 2}, g_{9 / 2}\right)$ and neutron
$\nu\left(g_{7 / 2}, d_{3 / 2,5 / 2}, s_{1 / 2}, h_{11 / 2}\right)$ valence orbitals with the ${ }_{28}^{78} \mathrm{Ni}_{50}$ core. More information on the effective interactions on proton-proton [21], neutron-neutron [22], and neutron -proton [23] can be found in the corresponding references. For comparison, calculations were also carried out in the model space $\pi \nu\left(1 p_{1 / 2}, p_{3 / 2}, f_{5 / 2}, g_{9 / 2}, g_{7 / 2}, d_{5 / 2}\right)$ (denoted as fpg ) in order to investigate the influence of the $N=50$ core breaking effect. That expanded model space includes all orbitals between the $N=Z=28$ and $N=Z=64$ subshells. The LSSM calculations were performed for ${ }^{83,85,87} \mathrm{Ge}$ isotopes, and the results are presented in Paper III.

### 2.2 Total Routhenian Surface (TRS) Calculations

Deformed nuclei can be described by a deformed shell model, where the potential is not any longer spherical symmetric, and hence violates that symmetry. The advantage of the deformed shell model is that one can easily obtin the equilibrium shape by minimizing the potential energy with respect to relevant deformation parameters, $\beta_{2}$ and $\gamma$. One may either calcualte this in terms of fully self consistent HartreeFock and density functional methos or by using the Strutinsky shell correction method, that combines the macroscopic energy of deformed liquid drop with the shell correction coming from the deformed shell model. The latter method is well established and offers an computational efficienct approach. For a rotational nucleus with Z proton and N neutron numbers at rotational frequency $\omega$, the total routhanian is defined with deformation parameters $\hat{\beta}=\left(\beta_{2}, \beta_{4}, \gamma\right)$ as :

$$
\begin{equation*}
E^{\omega}\left(Z, N, \hat{\beta}=E_{\text {macro }}^{\omega}(Z, N, \hat{\beta})+\delta E_{\text {shell }}^{\omega}(Z, N, \hat{\beta})+\delta E_{\text {pair }}^{\omega}(Z, N, \hat{\beta})\right. \tag{2.4}
\end{equation*}
$$

where, first term is from the macroscopic model(liquid drop), second term comes from microscopic shell model and third term is due to the pairing correction. Using the Strutinsky shell correction method from Ref. [24] Eq. 2.4 can be reorginazed and the $\omega=0$ and rotational frequency depended terms can be grouped as :

$$
\begin{equation*}
E^{\omega}\left(Z, N, \hat{\beta}=E^{\omega=0}(Z, N, \hat{\beta})+\left[\left\langle\psi^{\omega}\right| H^{\omega}\left|\psi^{\omega}\right\rangle-\left\langle\psi^{\omega=0}\right| H^{\omega}\left|\psi^{\omega=0}\right\rangle\right] .\right. \tag{2.5}
\end{equation*}
$$

As mentioned above $E^{\omega=0}(Z, N, \hat{\beta})$ term consist of liquid drop and shell correction and pairing energies at zero frequency. The second term in the brackets corresponds to the energy change induced by the rotation. The total routhian $E^{\omega}(Z, N, \hat{\beta})$ is minimized and transformed in Cartesian coordinates, $X=\beta_{2} \cos (\gamma+$ $30^{\circ}$ ) and $Y=\beta_{2} \sin \left(\gamma+30^{\circ}\right)$, to determine the equilibrium deformation in a certain rotational frequency. The calculated minima of the routhian in a fixed frequency gives the deformation parameters for a yrast state. The results are generally shown in the contour maps of the energy in the $\beta_{2}-\gamma$ plane.


Figure 2.1: The calculated single neutron levels for universal Woods-Saxon potential. Positive (negative) parity is indicated by solid (dashed) lines

### 2.3 Particle Plus Rotor (PPR) calculations

The particle-plus-rotor model was proposed by Bohr and Mottelson using the coupling of a few valence nucleons outside a rotating rigid core[25]. The model is developed to obtain the angular momentum of the odd nucleon, which is in the collective rotational motion. The total angular momentum is not a conserved quantum number anymore for the rotational nuclei. Therefore, the core system must have total angular momentum coupled with a valence nucleon for the conserved angular momentum. This explained in detail in Ref..[19,25]. This model and calculations are used in Paper I to shed a further light on the lowest active configurations of ${ }^{109,111}$ Mo isotopes.

Relavent single neutron levels were calculated usding the woodd-saxon potential, and shown in fig 2.2. In order to calculate a nuclear ground state, one has to introduce shape deformation parameters into the system.

The PPR calculations in this work use the same Woods-Saxon potential as for the TRS calculations. Relevant single neutron levels were calculated using the universal Wood-Saxon potential, shown in Fig. 2.1. In ordere to calculate a nnuclear ground state, one has to introduce all shape deformation parameters into the system. The quadrupole deformation parameter $\left(\beta_{2}\right)$, the triaxial deformation

$$
\text { Theory } \begin{aligned}
\gamma=-20 & \beta=0.34 \\
& -17 / 2_{1} 1551.1
\end{aligned}
$$

$(-17 / 2) \quad 1357$


Figure 2.2: Level schemes of ${ }^{109} \mathrm{Mo}$, the experimental data are compared with Particle Plus Rotor calculations using the Woods-Saxon potential. The experimental data are taken from [15, 26]. The figure is taken from Paper II.
parameter $(\gamma)$, and the hexadecapole deformation parameter $\left(\beta_{4}\right)$ were calculated within the total routhian space, as explained in the previous section and used for the PPR calculation. [20]

As a result of the PPR calculations, one can determine the ground and excited state spins, and excited state energies as a function of deformation parameters in different values. Also, the orbital configuration of the ground state in terms of Nillson states is calculated. The calculations gave the mixing ratio of the valence neutron orbitals (possible levels for ${ }^{109,111}$ Mo isotopes shown in Fig. 2.1) to create a certain spin and parity of the ground state in different deformation levels. The calculated ${ }^{109}$ Mo level scheme for the $\gamma=-20$ and $\beta=0.34$ deformation parameters is compared with the experimental data, taken from [15,26] shown in Fig. 2.2.


Figure 2.3: schematic representation of the heavy ion fusion evaporation reaction (left) and the direct knockout reaction (right).

### 2.4 Nuclear reactions; Compound and Direct

Various nuclear reactions can occur with energetic particles impinging on a target material. The common symbolism of the reaction is :

$$
\begin{equation*}
X(a, b) Y \tag{2.6}
\end{equation*}
$$

the incident particles are $X$ and $a$, while $Y$ and $b$ describe the outgoing particles. As one can understand that the simplest reaction occurs if $Y$ and $b$ are in their ground state and $X=Y, a=b$, which is an elastic scattering. The more complex reactions occur if the particles lose kinetic energy. Two main reaction types are compound and direct reactions. If there is an intermediate nucleus created by the interaction of $X$ and $a$, it is called a compound reaction. In compound reactions, the outgoing nucleus could be totally different from the incoming one while the internal structure is different. The compound nucleus is formed by a sequence of collisions that creates complicated rearrangements of the target nucleus. The compound nucleus is in highly excited state before it decay by a particle or $\gamma$-ray emission. In a short time period, $10^{-18} \mathrm{~s}$, particles are evaporated from the hot compound nucleus, and the residue nucleus (in this notation Y) decays with a long chain of $\gamma$-rays depend on the excitation energy. One of the common compound reaction is heavy ion fussion evaporation reactions. With fusion evaporation reaction, it is possible to populate the high spin states of the outgoing nuclei. In the $\mathrm{N}=\mathrm{Z}$ AGATA-NEDA-DIAMANT experiment (Paper I), we populate the ${ }^{88} \mathrm{Ru}$ isotope with heavy ion fusion evaporation reaction, the reaction kinematics are shown in Fig. 2.3(left), and the experimental details are explained in the next chapter.

In contrast to multi-step reaction, the direct reaction can occur without an intermediate nucleus in a short period of time around $10^{-22} \mathrm{~s}$, contact between the incident particles. The simpler direct reactions requires that the final state is related to initial state by the reaction mechanism [27]. The direct reactions became a important tool for studying the low-energy structure and properties of the nuclei. However the experiments to produce radiactive exotic nuclei with direct
reactions are not practical, since the target with such a short half-lives is not easy to produce. Therefore, direct reactions in inverse kinematics is practically used in the radioactive beam facilities. In inverse kinematics, radiactive nuclei are a beam projectiles, while the target is stable material like, hydrogen. There is a number of different type of direct reactions; the simplest ones are elastic and inelastic scattering. The more complex direct reaction is the transfer reactions, which can occur by a transfer of one or a small number of nucleons between the projectile nucleus and the target. The knock-out or pickup nucleons characterize the transfer reactions. If there is more than two reaction products are observed in outgoing channel, it is called knock-out reaction or mechanism. Excited nuclei studied in Paper II and Paper III are populated by knockout reactions, one of the outgoing product is the proton from the target, and the second outgoing particle is the nucleon or nucleons from the incoming beam. As a result, both $b$ and $Y$ in Eq. 2.6 carry information about the reaction mechanism. On the other hand, in case of pickup reactions, the nucleon from the beam $X$ or from target $a$ is transferred to outgoing products $Y$ or $b$.

Since simple nucleon exchange or knockout happens on the surface of the nucleus, the change produced in the target must be a simple rearrangement of one or several nucleons or collective degrees of freedom. The schematic view of the reaction mechanism shown in the right side of the Fig. 2.3. Because of the conservation laws, the change produced in the nucleus must be reflected in the energy and angular momentum carried by the outgoing particles [20]. Therefore, the excited states are related to initial states of $X$ and $a$ by adding nucleons.

In the SEASTAR experiment (Paper II and Paper III), we are dealing with direct reactions inverse kinematics and the nucleus of interest is produced through nucleon/s-removal of the projectile, the reactions that are presented in current work listed in table 2.1.

Table 2.1: The nucleon-removal reactions that present in current work.

| residue <br> isotope | Reactions |
| :---: | :---: |
| ${ }^{85} \mathrm{Ge}$ | ${ }^{87} \mathrm{As}(p, 2 p n),{ }^{86} \mathrm{As}(p, 2 p),{ }^{86} \mathrm{Ge}(p, p n)$ |
| ${ }^{87} \mathrm{Ge}$ | ${ }^{89} \mathrm{As}(p, 2 p n),{ }^{90} \mathrm{Se}(p, 3 p n),{ }^{88} \mathrm{Ge}(p, p n)$ |
| ${ }^{111} \mathrm{Mo}$ | ${ }^{113} \mathrm{Tc}(p, 2 p n),{ }^{112} \mathrm{Mo}(p, p n)$ |

One of the advantages of the knockout reactions when the final state of the outgoing nucleus populated, it keeps the memory of the initial state before the knocking out of the nucleons. This property helps to connect direct reactions with nuclear structures, the notion of spectroscopic factor. The quantum mechanical probability of creating the certain final state of the nucleus depends on the overlap of the wave functions of the final state and the initial state of the projectile. This
probability is called spectroscopic factor $S_{k}$ and defined with the initial state of the $\mathrm{X}\left|\psi_{i}^{X}\right\rangle$ and the final state of the $\mathrm{Y}\left|\psi_{f}^{Y}\right\rangle$, for one nucleon knockout reaction, it can be written as:

$$
\begin{equation*}
\left.S_{k}=\left|\left\langle\psi_{i}^{A}\right| a_{k}^{\dagger}\right| \psi_{f}^{A-1}\right\rangle \mid \tag{2.7}
\end{equation*}
$$

Therefore, $S_{k}$ goes from 0 to 1 , if it is close to zero it means less likely that the final state $(k=(n l j))$ is a single hole state in the final nucleus. or if $S_{k}$ aproach to 1 , which means that there is larger probability that the final state is a single hole state. Spectroscopic factors can be extracted from the reaction cross-sections $(\sigma)$, since $S_{k}$ is not observable experimentaly[28]. The exclusive cross section for a given bound state with a spin $J_{f}^{\pi f}$ is calculated with a single particle cross section $\sigma_{s p}^{k}$ to remove the nucleon from the $k=(n l j)$ orbital.

$$
\begin{equation*}
\sigma_{e x c}\left(J_{f}^{\pi f}\right)=\sum_{k} S_{k} \sigma_{s p}^{k} \tag{2.8}
\end{equation*}
$$

For example if we consider the ${ }^{85} \mathrm{Ge}$ case (from Paper III) the neutron removed from ${ }^{86} \mathrm{Ge}$ (which has a ground state $J_{f}^{\pi f}=0^{+}$) can be either from $\nu 2 d_{5 / 2}, \nu 1 g_{7 / 2}$ or $\nu 3 s_{1 / 2}$ orbitals. The final state in ${ }^{85} \mathrm{Ge}$ with a spin $J_{f}^{\pi f}$ populated from the neutron removed in an orbital k with $j_{k}^{(-1)^{l_{k}}}=J_{f}^{\pi f}$. The Eq. 2.8 is simplified to one term as $S_{k}=\frac{\sigma_{\text {exc }}\left(J_{f}^{\pi f}\right)}{\sigma_{s p}^{k}}$. It is important to know the spin-parity of the state for calculating the spectroscopic factors since $\sigma_{s p}^{k}$ depends on the orbit that nucleon removed. This spin determination can be done by measuring the momentum distribution of the knocked-out nucleon. The measured momentum distribution for the single nucleon knockout reactions explained in the chapter 4, Data Analysis.

The exclusive cross sections can be determined from the experimental data, if one knows the branching ratios of the gamma ray transitions. Finally, the cross section of the final populated state is basicly the sum of the exclusive cross sections but it can be also calculated without the exclusive cross sections, by measuring the number of reactions in the target, as explained in Sec. 4.4.1:

$$
\begin{equation*}
\sigma_{i n c}=\sum_{J_{f}^{\pi_{f}}} \sigma_{e x c}\left(J_{f}^{\pi f}\right) \text { with } j_{k}^{(-1)^{\iota_{k}}}=J_{f}^{\pi_{f}} \tag{2.9}
\end{equation*}
$$

### 2.5 Distorter Wave Impulse Approximation (DWIA)

One of the Approximation that is used in this work to understand knock-out reaction is Distorted Wave impulse Approximation (DWIA). The effect of nuclear potential for the intermediate energies are companseted by the DWIA which is a non-relativistic model [29]. In the direct reaction calculations, one can assume that the incident nucleon acts as a plane wave until it interacts with the nucleon inside the nucleus. In reality the nucleon wave fucntion inside the nucleus is distorted
by the optical potential field of the nucleus. The optical potential contains both imaginary and real parts, and the imaginary part models the absorption of certain reaction channels. This approximation considers the reaction to the sequential process. The DWIA has been used for the neutron knockout reaction from the ${ }^{86} \mathrm{Ge}$ isotope. The approximation describes ( $\mathrm{p}, \mathrm{pN}$ ) nucleon knockout process as protonnucleon elastic scattering inside the nuclei.As an important feature of the ( $\mathrm{p}, \mathrm{pN}$ ) reactions is that energy and momentum $(\omega-q)$ transfer with the angular momentum transfer $\delta l$ is large in general [30]. DWIA has been applied to proton-induced knockout reactions in several studies. The transition matrix of ( $\mathrm{p}, \mathrm{pn}$ ) reaction is written as

$$
\begin{equation*}
T_{p p n}=K\left\langle\chi_{1}^{-} \chi_{2}^{-}\right| t_{p n}\left|\psi_{0}^{+} \psi_{n}\right\rangle \tag{2.10}
\end{equation*}
$$

where $\mathrm{t}_{p} n$ is the efective interaction between proton and knockout neutron, $\chi_{1}^{-}$ and $\chi_{2}^{-}$are outgoing nucleons (proton and neutron), $\chi_{0}^{+}$is the incoming proton distorted waves. $\psi_{n}$ is a normalized bound-state wave function of the neutron inside the nucleus. The details of the DWIA calculation is explained for neutron and proton knockout reactions with a light proton target in Ref [30].

### 2.6 Gamma-Ray transitions

Gamma-ray spectroscopy is one of the most powerful tools to study nuclear structure. An excited radioactive nucleus will decay to the ground state directly or with an intermediate lower state. The energy of the gamma-ray is equal to energy differences between initial and final states. Near to the energy that a photon carries the angular momentum, which has to be conserved in $\gamma$ decay. For this reason, the angular momentum $L$ transferred by the photon satisfy

$$
\begin{equation*}
\left|J_{i}-J_{f}\right| \leq L \leq J_{i}+J_{f} \tag{2.11}
\end{equation*}
$$

where $J_{i}\left(J_{f}\right)$ is the spin of initial (final) state. $L$ can take any integer value (except $\mathrm{L}=0$ ) that obeys Eq. 2.11. If there is a shift in the charge distribution, the $\gamma$ ray is classified as electric (E), or if there is change in the current distribution, photon calssified as magnetic(M) transition. The electromagnetic determination is also calssified by the parity change between final and initial states. When the parity between initial and final state change with $(-1)^{L}$ it is electric, and parity is changed with $(-1)^{L+1}$ if it is magnetic transition. Therefore if;

$$
\begin{align*}
\pi_{i} & =\pi_{f} \text { even electric, odd magnetic (M1 E2 M3 ...) }  \tag{2.12}\\
\pi_{i} & =-\pi_{f} \text { odd electric, even magnetic (E1 M2 E3...) }
\end{align*}
$$

The selection rules permit several multipoles for a transition. Therefore the probability for each multipoles and type can be calculated from this formula [18].

$$
\begin{aligned}
\lambda(\sigma L ; i \rightarrow f)= & \frac{8 \pi}{\hbar} \frac{L+1}{L[(2 L+1)!!]^{2}}\left(\frac{E_{\gamma}}{\hbar c}\right)^{2 L+1} B(\sigma L ; i \rightarrow f) \\
& \text { Weisskopf estimates for } B(\sigma L ; i \rightarrow f) ; \\
B(E L ; i \rightarrow f)= & \frac{1}{4 \pi}\left(\frac{3}{L+3}\right)^{2}\left(1.2 A^{1 / 3}\right)^{2 j} \\
B(M L ; i \rightarrow f)= & \frac{10}{\pi}\left(\frac{3}{L+3}\right)^{2}\left(1.2 A^{1 / 3}\right)^{2 j-2}\left(\frac{\hbar}{2 m_{p} c}\right)^{2}
\end{aligned}
$$

while $E_{\gamma}$ is the energy of the transition with $m_{p}$ is the proton mass. $B(\sigma L ; i \rightarrow f)$ is reduced transition probability. The probability of a transition is calculated in Weiskopf units (W.u.) which is the ratio between experimental value and the Weisskopf estimate. From the Weisskopf estimates it is clear that the lowest permitted multipole transitions are favoured.

## Chapter 3

## Experimental Setup

Two different experimental setups are explained in this chapter which is divided into two parts. The first part of this chapter focuses on the SEASTAR campaign from 2015, conducted at the Radioactive Isotope Beam Facility (RIBF), at RIKEN, Japan. Papers II and III are based on this experiment. An additionally setup which was used in 2017 at RIKEN is also explained briefly in this chapter. The second part focuses on the AGATA+NEDA+DIAMANT campaign from 2018, carried out at the Grand AccÃ®lÃ@rateur National dâIons Lourds (GANIL) facility in France. Paper I is based on this campaign.

## Part 1 SEASTAR Campaign

There were three SEASTAR (Shell evolution and search for two-plus energies at the RIBF) campaigns, which were investigating neutron-rich nuclei produced through nucleon knockout reactions by using $\gamma$-ray spectroscopy. The first campaign was in May 2014 and focused on the ${ }^{66} \mathrm{Cr},{ }^{70,72} \mathrm{Fe}$ and ${ }^{78} \mathrm{Ni}$. The second campaign took place in April/May 2015, with the aim of studying the nuclei ${ }^{82,84} \mathrm{Zn},{ }^{86,88} \mathrm{Ge}$, ${ }^{90,92,94} \mathrm{Se},{ }^{98,100} \mathrm{Kr}$ and ${ }^{100} \mathrm{Zr}$. The third campaign took place, in May 2017 aiming at the studies ${ }^{52} \mathrm{Ar},{ }^{56} \mathrm{Ca}$, and ${ }^{62} \mathrm{Ti}$.

The data that is analysed in this thesis ( Paper II and Paper III) comes from the second SEASTAR campaign. Preliminary results from third SEASTAR campaign are also included in this thesis.

### 3.1 Beam Production

### 3.1.1 Primary Beam Production

For the second SEASTAR campaign ${ }^{238} \mathrm{U}$ was used as the primary beam, where as for the third campaign radioactive ${ }^{70} \mathrm{Zn}$ beam was used for the same purpose. As an example, a short summary of ${ }^{238} \mathrm{U}$ beam production is given below.

Mode (1): RILAC + RRC + (stripper2) + fRC + (stripper3) + IRC + SRC


Figure 3.1: RIBF heavy-ion accelerator system. RILAC+ RRC+ (stripper2) $+\mathrm{fRC}+$ (stripper3)+ IRC+ SRC is used for the RI-beam generation at 345 $\mathrm{MeV} /$ nucleon (fixed energy). The figure is taken from Ref. [31]

The primary beam of ${ }^{238} \mathrm{U}$ is accelerated to $345 \mathrm{MeV} /$ nucleon using the RIBF heavy-ion accelerator system, which is shown in Fig 3.1. The uranium ions are produced in the Superconducting- Electron Cyclotron Resonance Ion Source (SCECRIS). The first acceleration is done in RIKEN heavy-ion linac (RILAC) with energies up to $6 \mathrm{MeV} /$ nucleon. Following the initial acceleration in RILAC, the ${ }^{238} \mathrm{U}^{35+}$ pass through the four-ring cyclotrons at the RIBF facility. After the RIKEN Ring Cyclotron (RRC), the beam passes a helium gas stripper [32] and the charge state of the uranium ions increase up to $71+$ and reach the fixed-frequency Ring Cyclotron (FRC)[33]. The second stripper, a beryllium disk [34], is situated before Intermediate Ring Cyclotron (IRC), stripping the ions to ${ }^{238} \mathrm{U}^{88+}$ and ions finally reach to Superconducting Ring Cyclotron(SRC) [35]. After exiting the SRC, the $345 \mathrm{MeV} /$ nucleon uranium beam is sent to the experimental area.

### 3.1.2 Secondary Beam Production

One of the well-known methods to access the neutron-rich nuclei of the nuclear chart is in-flight fission of the uranium beam. The fission mechanism tends to produce medium mass fragments, which generally carry the same neutron/proton ratio as the primary isotope. For the ${ }_{92}^{238} \mathrm{U}$ beam, the neutron/proton ratio is around 1.6; therefore, the fission of uranium will create neutron-rich nuclei in the medium mass region [36] [37].

The secondary beam was produced at the F0 enterance of the BigRIPS spectrometer with impinging ${ }^{238} \mathrm{U}$ primary beam on a 3 mm thick ${ }^{9} \mathrm{Be}$ target. The average intensity of the uranium beam on the ${ }^{9} \mathrm{Be}$ target was 30 pnA for the second SEASTAR campaign. For the third campaign ${ }^{70} \mathrm{Zn}$ primary beam at $345 \mathrm{MeV} / \mathrm{u}$


Figure 3.2: Pictural view of fragmentation fission reaction, Projectile is ${ }_{92}^{238} \mathrm{U}$ while the target is ${ }^{9} \mathrm{Be}$. The drawing modified from Ref.. [36]
interacts with 10 mm thick ${ }^{9} \mathrm{Be}$ target in the entrance of BigRIPS sepectrometer to produce the secondary beam.

The collision of the primary beam and the primary target at F0 induced a fragmentation-fission reaction, as shown in Figure 3.2. Since the reaction occurs in-flight, the fission fragments continue forward with a velocity close to the primary beam velocity before the reaction. The secondary fragment beam is then identified and separated with the help of the BigRIPS spectrometer, which will be explained in the next section.

### 3.2 Beam Line Detectors



Figure 3.3: Schematic view of the BigRIPS and ZeroDegree spectrometers. Image modified from Ref.[38]

The Big RIKEN projectile-fragment separator (BigRIPS) and ZeroDegree spectrometers shown in Fig. 3.3 were used during the 2015 SEASTAR experimental campaign to identify the in-flight projectile and residues. During the $3^{r d}$ SEASTAR
campaign the residuals after the secondary target interaction were detected and measured in the SAMURAI spectrometer, see Sec. 3.2.3 instead of ZeroDegree spectrometer, see Sec. 3.2.2.

### 3.2.1 BigRIPS spectrometer

BigRIPS is a two-stage separator characterized by two main features: a large ion acceptance and a precise particle identification [39]. Along the beamline of BigRIPS, seven focal planes(F0-F7), fourteen superconducting triplet quadrupoles (STQs), and six dipoles (D1-D6) magnets are used, shown in Fig. 3.3. The first stage of BigRIPS starts from the production target (F0) and ends at the F2 focal plane, and the length is 31.6 m , while the second stage extends from F2 to F7 focal plane with a 46.6 m length [39].

With the fragmentation of the ${ }^{238} \mathrm{U}$ beam, many isotopes are produced with different mass and neutron/proton ratio, and we need to select the isotopes that we are interested in. The first stage of BigRIPS is used to separate and select the secondary beam using the momentum loss- achromatic method, which is also called $B \rho-\Delta E-B \rho$ selection.

Firstly, the trajectory of the ion which is moving through the constant magnetic field $B$ will be determined by the ion mass $A$, the charge $Q$, and the momentum $P$ throught the following expression :

$$
\begin{equation*}
B \rho=\frac{P}{Q}=\frac{A}{Q} \beta \gamma u c, \tag{3.1}
\end{equation*}
$$

where $\rho$ is the radius of curvature of the ion path, for a fully stripped ion $Q=Z e$, $c$ is speed of light, $u \approx 931.5 \mathrm{MeV}$ atomic mass unit, $\beta=\frac{v}{c}$ is the velocity of ions in terms of $c$, and $\gamma=\frac{1}{\sqrt{1-\beta^{2}}}$ is the relativistic correction factor. The first selection is done with respect to the magnetic rigidity $(B \rho)$ of the ions since the isotopes with different $A / Q$ ratios follow different paths under the magnetic field applied by the dipole magnets (D1 and D2). After the $B \rho$ selection in the D1 dipole magnet, the next selection is done using the aluminium wedge degrader at F1, which give an energy loss of a particle inside the material depending on its atomic number, $Z$, through the well-known Bethe-Bloch equation [40]:

$$
\begin{equation*}
\Delta E=\frac{4 \pi e^{4} Z^{2}}{m_{e} v^{2}} n_{a} Z_{a}\left[\ln \left(\frac{2 m_{e} v^{2}}{I}\right)-\ln \left(1-\beta^{2}\right)-\beta^{2}\right] \tag{3.2}
\end{equation*}
$$

where $n_{a}$ and $Z_{a}$ are the number density and atomic number of the absorber, and $I$ is the the average excitation and ionisation potential of the absorber which is experimentally determined for each element [40]. With the energy loss calculation on the degrader, the isotopic seperation is induced. After this degrader the D2 dipole magnet is used to make a second $B \rho$ selection and also used to align the center of the beam to the mass region that is interested for the experiment. In
summary the first part of the BigRIPS seperator use the $B \rho-\Delta E-B \rho$ technique for selecting and aligning the beam for the nuclei of interest.

At the second stage of BigRIPS between F3 to F7, the $B \rho-\Delta E-T O F$ method is used to make an event-by-event identification of the beam before it reaches the secondary target position as illustrated in Fig 3.3. This section consists of eight STQs, four dipoles (D3-D6), forming a four-bend achromatic system, and a degrader at F5 to improve the separation. Also, vertical and horizontal slits are added along the beamline to clean the unwanted ions from the beam. In this part of the measurements, the ion mass-to-charge ratio, $A / Q$, and its atomic number, $Z$, are determined by measuring the magnetic rigidity, $B \rho$, an energy loss, $\Delta E$ and time of flight TOF,

$$
\begin{equation*}
T O F=\frac{L}{\beta c} \tag{3.3}
\end{equation*}
$$

where $L$ is the flight path between plastic scintillator detectors from F3 to F7 (46.6 $\mathrm{m})$.

To measure these observables, different beam line detectors are used. As shown in Fig 3.3, there are three Parallel Plate Avalanche Counters (PPACs) located at F3, F5 and F7 in BigRIPS. These PPACs are used to measure the position of the beam to extract $\rho$ which is necessary for the $A / Q$ determination in Eq.3.1. Detailed information about PPACs can be found in Ref. [41]. Another beam line detector type is Plastic Scintillators as shown in Fig.3.4. The two plastic detectors, each coupled with Photo multiplier tubes two sides, are placed at the F3 and F7 focal planes of BigRIPS. Plastic scintillators are chosen for their fast timing properties, to measure time of flight from which we can extract the ion velocity, $v$. The tilted electron gas Ionization chamber (TEGIC) [42] is used at F7 focal plane, to detect the energy loss of the beam ions. As seen from the Bethe Bloch formula in Eq.3.2, with measuring $\Delta E$ we can deduce the atomic number $A$.

### 3.2.2 ZeroDegree spectrometer

The ZeroDegree spectrometer is made of six STQs and two Dipole magnets of the same type as those mentioned in the BigRIPS section. The ZeroDegree has four focal plane positions F8 to F11 with a total length 36.5 m , also shown in Fig 3.3. For the SEASTAR-2 campaign the ZeroDegree was used in its large acceptance mode, more detailed information can be found in Ref. [43]. For different settings ZeroDegree is focused for different isotopes. For the ZeroDegree, the $B \rho-\Delta E-$ TOF method is used for the identification, similar to the second stage of BigRIPS. This time, TOF is measured from two plastic scintillators located at F8 and F11 focal plane, while the trajectory is detected by three PPACs at F8, F9 and F11, and the $\Delta E$ is determined with the ionization chamber (TEGIC) placed at F11 focal plane. The photograph of the beam line detectors with their positions in the BigRIPS and ZeroDegree spectrometers and the observable parameters are shown in Fig.3.4


Figure 3.4: Standart SEASTAR-2 campaign beam line detectors. The beam line positions are also mentioned inside the figure. The photographs are modified from [31].

### 3.2.3 SAMURAI Spectrometer

A large-acceptance multiparticle spectrometer SAMURAI (Superconducting Analyzer for MUlti-particle RAdio Isotope beam) is designed primarily for kinematically complete experiments such as for invariant-mass spectroscopy of unbound states in exotic nuclei. The SAMURAI system is quite different from the ZeroDegree and BigRIPS spectrometers. The large superconducting dipole magnet, beamline detectors, heavy fragment detectors, neutron and proton detectors are the main components of the system [44]. The schematic drawing of the SAMURAI system is shown in Fig. 3.5.

As shown in Fig. 3.5, for the SEASTAR-3 setup, two beam drift chambers (BDC1 and BDC2) were placed in front of the target position. Beam drift chambers were a part of the SAMURAI setup, and they were used to measure beam trajectories before interacting with the $\mathrm{LH}_{2}$ target. One difference between the two SEASTAR campaigns was the ion chambers that were used in the setup. Instead of TEGIC ion chambers in BigRIPS and Zerodegree in the SEASTAR-2 campaign, ICB and ICF ion chambers were used to measure charge distributions in SEASTAR3. ICB is used before the target, and ICF is used after the SAMURAI magnet close to FDC2.

The secondary beam products first interacted with the Forward Drift Chamber 1 (FDC1), placed between the target and the SAMURAI magnet. The FDC1 detected the heavy fragments from the knockout reaction, and it measureed the


Figure 3.5: Schematic drawing for SEASTAR-3, SAMURAI magnets and detectors.The figure is modified from [44].
emission angle of the fragments. The SAMURAI magnet was rotated by $30^{\circ}$ with respect to the incoming beamline to increase the neutron angle coverage. The fragments passed into the SAMURAI magnet, which has a central magnetic field of 2.7 T , and the charged and uncharged particles separated with the help of a strong magnetic field. The superconducting magnet was placed inside the vacuum chamber, which had two exit windows for the charged particles and the neutrons. The studies of unbound neutrons are another subject of interest for the SEASTAR3 project which aims to measure invariant masses and unbound neutrons in exotic neutron-rich nuclei. For the unbound or free neutrons, NEBULA and NeuLAND detector arrays were placed in the beam direction, as shown in Fig. 3.5. The detailed information for the NEBULA and the NeuLAND neutron detector arrays can be found in Ref. [45, 46] [47].

The data set that was analyzed in this study did not contain the neutron data from NEBULA and NeuLAND arrays. We were only interested in the charge particle window and the corresponding detectors. We measured the particles that passed the charged particle window and were detected by the second Forward Drift Chamber (FDC2) and then continued to the Hodoscope fragment plastic scintillator. The energy loss and ToF information of the fragments were measured in the hodoscope. The previously explained ToF- $B \rho-\delta E$ method was used for the event by event fragment identification. More information on the SAMURAI system can be found in Ref. [44] and the SAMURAI collaboration webpage Ref. [48]

### 3.3 Detectors for in-beam $\gamma$-ray spectroscopy

For the in beam $\gamma$-ray spectroscopy the SEASTAR (DALI2+MINOS) setup was located in F8 focal plane before the Zerodegree spectrometer during the second campaign. For the third campaign the same setup was placed in F13 focal plane in BigRIPS+SAMURAI setup. The schematic view of 186 NaI DALI2 detectors coupled with the MINOS TPC in the target frame is shown in Fig 3.6. Parts of the in beam detectors are explained in this section.

### 3.3.1 MINOS

MINOS(MagIc Number Off Stability) is a device, that is placed in-between the BigRIPS and ZeroDegree spectrometers at the center of DALI2 array. It is composed of a liquid-hydrogen $\left(L_{2}\right)$ target and the time-projection chamber (TPC) to track proton for vertex reconstruction [49]. The schematic view of this device given in Fig. 3.7. The knockout reaction occurs inside the $L H_{2}$ target. The knockout proton and the induced protons from the target are tracked in the TPC around the $\mathrm{LH}_{2}$ target. With the proton vertex reconstruction, one can point out the reaction position inside the target, and obtain a better Doppler correction as compared to passive target. With the current TPC around the $L H_{2}$ we can have a target length up to 150 mm . The target length is measured to be around 100 mm for the $2^{\text {nd }}$ campaign, and around 150 mm for $3^{\text {rd }}$ campaign. The extra information about the $\mathrm{LH}_{2}$ target holder and construction can be found in [49,50].

### 3.3.1.1 Vertex Tracker - Time Projection Chamber-TPC

The MINOS vertex tracker is used to make 3D reconstructions of the tracks of the scattered protons to determine the reaction point inside the $L_{2}$ target. The Time Projection Chamber is the decendant of the drift chamber and the multiwire proportional chamber. In 1969 Charpak, Bressani, Rahm and Zupancic realized that the time of a signal could be useful for the coordinate determination of charged particle $[52,53]$. The first drift chamber was developed in 1971 [54] for particle physics experiments and they are still used for their economic read-out, high accuracy and large area coverage properties. In 1974, the TPC was introduced at Berkeley laboratories by David Nygren, and since then it is widely used in particle and nuclear physics expeirments. The basic idea was to create a large sensitive volume that is filled with a gas and kept in an electric field. By recording the drift times of the electrons that are forced during ionization, tracks of protons are produced. Three dimensional measurements was performed by pick-up electrodes on the x and y axes and drift-time measurement for the z axis.

The TPC of 300 mm length, with a inner diameter 80 mm and outer diameter 178.8 mm , surrounds the target chamber. To construct the electric field cages, inner surface of the outher cylinder and outer surface of the inner cylinder coated with a Kapton foil.The TPC volume was filled with a gas mixture of $\operatorname{Ar}(82 \%), C F_{4}(15 \%)$,
and iso $C_{4} H_{10}(3 \%)$ at room temperature and atmospheric pressure, which is optimized for maximum electron transport. The Micro-MEsh GAseous Structure detector (MicroMegas) technology which was developed in 1996 is used in the MINOS TPC [55]. A micromegas detector consist of a thin micro-mesh placed above


Figure 3.6: 3D drawing of the SEASTAR setup at F8 area in RIBF. The setup consist of DALI2 array red squares surronding the MINOS target system. The yellow cylinder is the MINOS TPC, $\mathrm{LH}_{2}$ target is inside the TPC. Note that half of the DALI2 detectors are removed to see cross section of the whole system, and beam direction is from left to right. Image source, Ref.[49].


Figure 3.7: Schematic view of the MINOS device from front and side. A (p,2p) reaction is demonstrated with the two protons tracked in TPC. Image modified from: $[49,51]$
the segmented detection plane of 4608 pads equaly divided into 18 ring and each pad has an area about $4 \mathrm{~mm}^{2}$ [55]. The TPC volume divided by mesh to two region: drift and amplification regions. The knockout and scattered protons leave the reaction chamber and pass through the TPC, ionizing the atoms of the gas in the TPC. The ionized electrons inside the TPC hits the mesh, they create the avalanche of the electrons in the amplification region and they are collected on the pads. The micro-mesh placed $128 \mu \mathrm{~m}$ from the anode pads and a strong electric field ( $40-70 \mathrm{kV} / \mathrm{cm}$ ) is applied in between mesh and pads to have short signal rise time for the electrons. The 4608 segmented anode allows the reconstruction of the proton trajectory in the xy plane. The $3^{t h}$ dimension $z_{\text {pad }}$ information is calculated by the drift time of electrons in the TPC by using the following formula:

$$
\begin{equation*}
z_{p a d}=\left(t_{p a d}-t_{0}\right) v_{d r i f t} \tag{3.4}
\end{equation*}
$$

here, $t_{0}$ is the time offset to take into account for trigger time, $t_{p a d}$ is the measured time in the pad, and $v_{\text {drift }}$ is the drift velocity of the electron. Eq. 3.4 can be considered as a MINOS calibration equation, where $v_{d r i f t}$ and $t_{0}$ were determined experimentally for each run in Sec.4.1.5.

### 3.3.2 DALI2/DALI2+

Detector Array for Low Intensity radiation 2 (DALI2) is an array composed of inorganic crystal scintillator, $\mathrm{NaI}(\mathrm{Tl})$, where thallium is used as an impurity activator to increase the efficiency. DALI2 is a $\gamma$-ray detector made of $186 \mathrm{NaI}(\mathrm{Tl})$ scintillators which was used in SEASTAR-2 campaign in 2015. The more developed version with 226 scintillators was used during the 3 rd campaign in 2017. It was
designed for the in-beam $\gamma$-ray spectroscopy experiments with high-velocity beams ( $\beta \approx 0.6 \mathrm{c}$ ).

Fig 3.8 shows an illustration of the DALI2 standard configuration. DALI2 is composed of three type of crystals. First two types of crystals, manufactured by SAINT-GOBAIN and SCIONIX, have the dimensions $45 \times 80 \times 160 \mathrm{~mm}^{3}$ and 40 $\mathrm{x} 80 \times 160 \mathrm{~mm}^{3}$ respectively. The third type is $60 \times 60 \times 120 \mathrm{~mm}^{3}$, produced for DALI by BICRON. Each crystal is coupled with PMTs and encapsulated with 1 mm thick aluminum housing. [56] The typical energy resolution of DALI2 is around


Figure 3.8: DALI2 detector array configuration from side(a-c) and back(b-d). (c-d) The GEANT4 simulation drawings from right and back, the red cylinder in the middle is liquid hydrogen target. Photos (a-b) are taken from [31]
$9 \%$ for the 662 keV line in a standard Cs source. The crystals are arranged in 11 cylindrical layers and one wall in the forward angle. Each layer consist of 6-14 detectors that is mounted in 5 mm thick aluminium frame. The last layer or wall consists of 64 crystal to cover the smallest angle in forward direction. The full array covers a range of polar angles in the laboratory frame between 15-160 deg. For the SEASTAR campaigns the original DALI2 geometry was changed to accommodate
the TPC of the MINOS system, one layer is removed and the angular coverage is changed to $12-118^{\circ}$ for the 2015 campaign. This geometry is chosen to maximize the detection efficiency, especially for forward angles, and the segmentation allows to reduce the angular resolution to $7^{\circ}$ FWHM.

### 3.4 Simulation of DALI2 response functions

For the DALI2, the complete Geant4 [57] simulation is designed in the most realistic way. This means that physical properties of the design are same as an experiment. The purpose of the simulation is to recreate experimental data and construct the response functions at different energies for different reactions. And these response functions are used to extract $\gamma$-ray detection efficiency and the energy resolution of the chosen detector geometry for realistic conditions. It also takes into account several effects, like the lifetimes of the excited states and line shape of the $\gamma$-ray due to the thick $\mathrm{LH}_{2}$ target. More information on the simulation package for DALI2 setup can be found in [58]. The simulation is vital to take care of the anisotropy of $\gamma$-rays emitted by moving nuclei. The anisotropy appears because of the Lorentz boost since the relevant nuclei are moving at relativistic speed ( $0.6 \mathrm{c} \sim 0.4 \mathrm{c}$ ), and therefore the Lorentz transformation has to be calculated for gamma-rays. The simulation code which works in GEANT4 is divided into three parts, and run separately independent of each other.

The first step is the EventGenerator, here we simulate the radioactive secondary ion beam interactions with 102 mm LH 2 target. It calculates the energy loss inside the target, which means before and after the nucleon knockout. Basically, it creates the source of radiation for the experiment. To run this part of the simulation we have to define several inputs. The main inputs are the beam, the target and the level scheme. For the beam; properties like A, Z, Q, energy, position and angle, and for the target; thickness, type, mass change inside the target and energy loss of projectile are used as an input. At the end, the level scheme with transition energies, lifetimes, decay probabilities are used as an input. In this step the atomic background from the beam can also be included.

The second part of the simulation is the EventBuilder which determines the interactions of radiation with the detectors. Here we define the DALI2 geometry in agreement with the experiment. The experimental resolution of each crystal which is found with calibration source in the rest frame used as input in the simulation. In the experiment residue isotopes after the second interaction with $\mathrm{LH}_{2}$ target, continue to move on with around 0.4 c velocity. In the simulation however we do not include that part, since we are only simulating the $\gamma$-rays. As an output, the simulation code creates ROOT files that contain all events and their relative information, such as the detector ID, the scattering angle etc. Finally, we analyse the simulated events in the Reconstruction part.

The last step of the simulation is the Reconstruction of the events detected by our detection system. The main aim of this reconstruction is the Doppler correc-
tion, the add-back procedure and to remove broken detectors from the system. The output of this step stored in ROOT tree, and can be compared with the experimentaly observed spectra. With this comparison we can determine the efficiency of the array and the number of de-excitation $\gamma$-rays that occurred in the reaction.


Figure 3.9: DALI2 response functions for relativisticly moving beam with add-back for $\gamma$ transitions between 0.25 to 2.5 MeV with 0.25 MeV steps.

The shape of the response functions as a function of energy can be seen in Fig 3.9 for the relativistically moving source. From the figure, one can understand that the photo-peak progressively merges with the Compton edge because of the low resolution above the 2 MeV . An add-back procedure is applied for distances below 15 cm . In the add-back analysis, our main aim is to reconstruct the Compton scattering events. Signals from a group of two or three neighbouring detectors are assumed to be caused by Compton scattered $\gamma$-rays and the deposited energies added to find the real initial energy of the photon. The detector with the highest deposited energy is assumed to be the first interaction point, and the Doppler shift is calculated with respect to the position of that detector.

### 3.5 Data Acqusition System (DAQ)

For all campaigns collection of the events are done using the RIBF DAQ data acquisition system [59]. In order to optimize the usage of disk space and dead time , data acquisition should focus on the interesting events only. For the SEASTAR-2 campaign this selection is done with using the combination of three different trigger signals.
F7 DS The first trigger signal gathered by the F7 plastic scintillator in BigRIPS, when the nucleus pass through the detector. It doesn't matter if this nucleus reaches
to the end of the ZeroDegree spectrometer or not the event is validated in DAQ. DS means "downscaled" by a factor of x50 and it is used to avoid recording too many events. This single beam trigger is used for cross section and transmission calculations described in the next chapter.
F7xF11 The second combination of the signal is created by the coincidence with F7 and F11 from ZeroDegree spectrometer. The nucleus that creates the signal in F7 should reach the end of the beam-line where the last plastic detector is placed at F11.
F7xF11x $\gamma$ Both F7 and F11 triggered and at least one $\gamma$ ray was detected in DALI2, this trigger is used for the $\gamma$-ray analysis. This acqusition system works with a common dead time, that is the longest deadtime in all beamline and in beam detection systems. For DALI2 and MINOS the dead time is around $100 \mu \mathrm{~s}$, and for beam line detectors the dead time is around $200 \mu s$, therefore the common dead time of the DAQ system is $200 \mu s$. The MINOS detector is not using the same DAQ as the rest of the detectors. The MINOS DAQ works as a slave to the general data acquisition system.

## Part 2 AGATA NEDA DIAMANT Campaign

The experiment reported in Paper I was performed at the Grand AccÃ@lÃ@rateur National dâIons Lourds(GANIL), located in Caen, France. The main nucleus $\left({ }^{88} \mathrm{Ru}\right)$ was populated via the heavy-ion fusion-evaporation reaction. The ${ }^{36} \mathrm{Ar}^{+18}$ ion beam was accelerated up to 115 MeV by the seperated sector cyclotron, CSS1, and directed to bombard in isotopically enriched ${ }^{54} \mathrm{Fe}$ thick target foil, which had a $6 \mathrm{mg} / \mathrm{cm}^{2}$ areal density, thick enough to stop the fusion products. The argon ions were produced in the Nanogan3 Electron Cyclotron Resonance Ion Source (ECRIS). The first acceleration was done in compact accelerators C01 and C02 with energies up to $1 \mathrm{MeV} / \mathrm{A}$. Following the initial acceleration in C01 \& C02 the ${ }^{36} \mathrm{Ar}^{+18}$ ions were sent to the separator selector cyclotron1 CSS1. The intensity range of the argon beam was $5-10 \mathrm{pnA}$ its energy was 115 MeV and it was used to irradiate the target for 12 days.

The Advance Gamma Tracking Array (AGATA) detector set-up for $\gamma$-ray detection coupled with the neutron detector array ( NEDA+NWall) and charge particle (DIAMANT) detector to select correct evoporation channel from the fusion reaction. The AGATA and ancillary detector setup is illustrated in Fig. 3.10

### 3.6 AGATA

AGATA (Advanced Gamma Ray Tracking Array) is a common European project which aims at building a $4 \pi \gamma$-ray tracking detector array [61]. The complete AGATA array will have 180 hexagonally shaped Hp-Ge crystals that are electronicaly segmented to 36 slices. Three different sized Agata crystals are grouped as a triple cluster. The geometrical properties are shown in Fig.3.11. In the current


Figure 3.10: The 3D conceptual drawing of AGATA (yellow colored,right hemisphere detectors), NEDA (purpel colored left hemisphere) and NWall (blue colored, at $90^{\circ}$ ) detector setup, with the beam is directed from right to left. The DIAMANT ancillary detector is not visible in the closed configuration in GANIL. the figure is taken from [60]
campaign only 14 triple clusters was available with 42 hexagonally shaped $\mathrm{Hp}-\mathrm{Ge}$ crystals.

Each crystal in a triple cluster detector shared one cryostat which is used for the cooling with liquide nitrogen. The cooling is important to avoid the leakage currents from the low energy gap of the germanium crystal. From the Monte Carlo simulations the photo-peak efficiency at the 121 keV line from the ${ }^{153} \mathrm{Eu}$ source is calcualted as $26.3 \%$ without target chamber and with a tracking algorithm for the AGATA $1 \pi$ array in the compact configuration [60], the same simulation gives 22.4 \% efficiency with the target chamber. For the high energy region in the same conditions, simulations gave $13.5 \%$ (without target) and $10.7 \%$ (with target chamber) detector efficiencies for a 2.6 MeV gamma transition. The $\gamma$-ray detection efficiencies are also effected by the ancillary detectors, and surrounding matterials. The graph in Fig.3.12 shows the final efficiencies for the AGATA+NEDA+DIAMANT setup for $1 \pi$ solid angle coverage of AGATA. Since we use the DIAMANT detectors for charge particles, the red dots in graph are related to our analysis.

In Fig.3.12 the peak-to-backgound ratio is also given, which is important to know. One of the main component that effect the peak-to-background ratio is the


Figure 3.11: (a)The three different AGATA crystal geometries that combined in the triple cluster. The side view shows the segmentation positions in mm . The colour code is preserved for figure (c).(b):The segmentation system of the AGATA HPGe capsules. Along the crystal axis the external contact is subdivided into six rings labelled 1â6. Each ring is subdivided into six sectors labelled aâf, in total 36 segment per crystal. (c): Computer aided full AGATA array design image of the 180 crystal configuration, the cryostats and electronics are not shown. (d) Photograph of the setup with five AGATA triple cluster detectors. The figure is modified from Ref. [61]
tracking algorithm. An electronically segmented AGATA crystal has 36 output signals from the segments and 1 output signal from the so-called core. The core signal comes from the central electronics of the crystal. This means that for each triple cluster we have 111 output signals, that has to be analysed. The tracking algorithm groups the signal with respect to 4-dimensional space ( $x, y, z$ and $t$ ), and use the Energy deposition in the segment to find the initial interaction point. Due to the segmentation of the detectors and the pulse shape analysis technique, one


Figure 3.12: a) Simulated photo-peak efficiency of the AGATA $1 \pi$ array in the compact configuration as a function of the $\gamma$-ray energy for the NWALL/DIAMANT set-up using the calorimetric energy (no velocity). The red (black) symbol corresponds to the results with (without) the DIAMANT array at the centre, respectively. b) Similar results for the peak-to-total ratio. The figure is taken from: [60]
can track the $\gamma$-rays in AGATA and determine the correct energies of the Compton scattered $\gamma$-rays and the first interaction points. The core signal is used here as a cross-check of the tracking algorithm. Finally, the core signal deposited energy should be equal to the tracked energy for a single $\gamma$-ray interacting with the AGATA crystal. A more detailed explanation for the tracking algorithm can be found in Ref. $[62,63]$

### 3.7 NEDA

The NEutron Detector Array (NEDA) [64] and the Neutron Wall(NWall) consist of organic liquid scintillator detectors constructed for the fast neutron detection. In the experiment the detectors are used to distriminate between fast neutrons and $\gamma$-rays using the Pulse Shape Discrimination (PSD) method. The total array is located in the forward hemisphere directly opposite the AGATA detectors in a closed configuration. The array covers approximately $1.6 \pi$ solid angle, the cconfiguration in GANIL is shown in Fig.3.13.


Figure 3.13: The NEDA array positioned in the left, AGATA clusters are in the right, target chamber with the DIAMANT inside is in the center. The beam comes from right to left. The Image is taken from [65]

The NEDA detector array is constructed by 54 hexagonal prism shaped sctintillators. The hexagonal shape was designed to give a high efficiency, and to minimise the cross-talk in between the detectors. The other possible designs and their efficiency was simulated and the optimum configuration was selected $[66,67]$ The technical details of the NEDA array is explained in Ref. [64]. For the experiment these neutron detectors were coupled with the 14 NWall hexagonal detectors. NWall detectors were placed in the outer ring of the sphere as can be seen in Fig 3.10. In the present experiment, the neutron- $\gamma$ discrimination plays a vital role to cleanly select the corresponding $\gamma$-rays. As a liquid scintillator the NEDA array has a fast timing response, therefore it can be used as time reference in the trigger system, also that makes NEDA signal a part of the master system in the DAQ. The total efficiency of the NEDA+NWall system was calculated as $20 \%$ for a singleneutron events. The NEDA and NWall detectors worked together with NUMEX02 digital cards, used to apply a PSD algorithm, as explained in the next chapter. With the digitalization of the output signals for NEDA+NWall and DIAMANT set up, we were able to write more (approximately x20) events with respect to the old system.


Figure 3.14: An illustration of the DIAMANT detectors in the forward hemisphere. The beam direction is shown as red arrows.

### 3.8 DIAMANT

Evaporation of charge particles protons and alphas from the fusion-evaporation reactions were detected by the DIAMANT detector array [68, 69], consisting of 60 $\operatorname{CsI}(\mathrm{Tl})$ scintillator detectors, and covering nearly $2 \pi$ solid angle inside the target chamber. The corresponding light pulse in the scintillator gives different pulse shapes with respect to energy deposition of the different charged particles. Therefore it is possible to separate different charged particles from each other. There are energy and time output of the DIAMANT detector, with the particle identification (PID) output which based on the slow/fast light input. The DIAMANT detector is placed inside the target chamber covering the forward half hemisphere to open a space for the incoming beam. For the ${ }^{88} \mathrm{Ru}$ analysis, the DIAMANT detector was used as a veto detector since we are interested in 2 neutron channel of the evaporation reaction. Also, DIAMANT was not a part of the master trigger system, therefore it worked in a slave mode with respect to the AGATA and NEDA systems.

## Chapter 4

## Data Analysis

## Part 1 SEASTAR Campaign

In the previous chapter, the experimental setups used for the SEASTAR campaigns from 2015 and 2017 with different detector systems were presented. This chapter describes the different stages of data analysis and procedure. The data analysis follows three main steps, the first step is to calibrate all detectors and remove the major background events, the second step is to make the Doppler correction for the $\gamma$-rays using MINOS vertex information, and the third and final step is analyzing Doppler corrected spectra by using the simulated response functions.

The second SEASTAR campaign was run for 5 different BigRIPS and ZeroDegree physical settings, each of them is focused on the different mass regions. The overview of the settings from the SEASTAR-2 campaign is listed in table 4.1.

Table 4.1: Characteristics of the three settings applied to collect the data for SEASTAR-2.

| Parameter | Setting 1 | Setting 2 | Setting 5 |
| :--- | :---: | :---: | :---: |
| Isotope studied in this thesis | ${ }^{85} \mathrm{Ge}$ | ${ }^{87} \mathrm{Ge}$ | ${ }^{111} \mathrm{Mo}$ |
| Isotope centered in BigRIPS | ${ }^{85} \mathrm{Ga}$ | ${ }^{89} \mathrm{As}$ | ${ }^{111} \mathrm{Nb}$ |
| Isotope centered in ZeroDegree | ${ }^{84} \mathrm{Zn}$ | ${ }^{88} \mathrm{Ge}$ | ${ }^{110} \mathrm{Zr}$ |
| ${ }^{238} \mathrm{U}$ beam current at the ${ }^{9} \mathrm{Be}$ target $(\mathrm{pnA})$ | 35 | 30 | 33 |
| Rate at F7 $\left(\mathrm{s}^{-1}\right)$ | 5500 | 5000 | 2500 |
| Rate at F11 $\left(\mathrm{s}^{-1}\right)$ | 700 | 800 | 1000 |
| Energy in front of the $\mathrm{LH}_{2}$ target $(\mathrm{MeV} / \mathrm{c})$ | 280 | 260 | 270 |
| Measurement period $(\mathrm{h})$ | 22 | 10.5 | 58 |

### 4.1 Calibrations and Selections

### 4.1.1 Calibration of the PID

A general method for the $\mathrm{A} / \mathrm{Q}$ and Z determination and the particle identification was explained in the previous chapter. Since the PID is the most important information for the reaction selection, an extra correction is needed on PID elements; A/Q and Z. Therefore the parameters for the optical matrix must be calibrated for the known nuclides [70].

For the BigRIPS, the ion optical corrections were done during the experiment by the accelerator team. Therefore for the settings in SEASTAR-2, BigRIPS PID is good enough to handle the analysis or need only small modifications. The similar correction procedure is used for BigRIPS and ZeroDegree spectrometers. However, ZeroDegree corrections had to be done after each settings.

For ZeroDegree, the online analysis matrix elements are used in the offline analysis. To refine the A/Q corrections, beamline elements (position and angle) are plotted in function of A/Q for each focal plane (F9 and F11) as shown in Fig.4.1(a)(d) and fitted with a second-order polynomial to determine the correct $A / O_{\text {cor }}$ ratio,

$$
\begin{align*}
A / Q_{c o r}=A / Q & +C_{F 9 X} \times F 9 X+C_{F 9 X^{2}} \times F 9 X^{2} \\
& +C_{F 9 A} \times F 9 A+C_{F 9 A^{2}} \times F 9 A^{2} \\
& +C_{F 11 X} \times F 11 X+C_{F 11 X^{2}} \times F 11 X^{2}  \tag{4.1}\\
& +C_{F 11 A} \times F 11 A+C_{F 11 A^{2}} \times F 11 A^{2}
\end{align*}
$$

where $C_{F 9 X}, C_{F 9 A}, C_{F 9 X^{2}}, C_{F 9 A^{2}}$ are the correction parameters for the secondorder polynomial for position and angle on the F9 focal point, while F9X/A are the positions / angles measured by the PPAC system in the F9 focal point. The corresponding values for focal point F11 is named as $C_{F 11 X / A}$ and F11X/A. The results of the A/Q corrections as $A / Q_{\text {cor }}$ are also shown in Fig. 4.1(e-h).

Fig. 4.2 shows the total impact of the correction on $\mathrm{A} / \mathrm{Q}$ where the blue line is initial distribution and red line is the corrected final $\mathrm{A} / \mathrm{Q}$ distribution. For the 3 settings this procedure is performed separately, since the ZeroDegree is centered to different ions. The velocity dependence was also checked to see its effect on the PID elements. The Z value can be corrected with respect to the velocity dependency, as shown in Fig. 4.3. The final ZeroDegree PID spectra are shown in Fig. 4.4 with and without the corrections of the $A / Q$ and $Z$ values.

### 4.1.2 Cleaning the BigRIPS and ZeroDegree PID

All the focal plane detectors in BigRIPS and Zerodegree can also be used to clean up the PID spectra from noise [70]. The PID parameters A/Q and Z depend on two variables, the magnetic rigidity $B \rho$ and velocity $\beta$ of the ions, as it is explained in the previous chapter in Eq. 3.1. To measure the velocity, the time


Figure 4.1: Effect of the A/Q corrections at the focal plane F9 and F11 for Setting 1. In panel a) and b) uncorrected A/Q plot against the angle and position at focal plane F11, while F9 is plotted in c) and d). The corrected correlations are shown in panel e) and f) for focal plane F11, and g) and h) for focal plane F9. Only one run is used for the plots.


Figure 4.2: Comparison of the $\mathrm{A} / \mathrm{Q}$ distribution before(blue) and after(red) the A/Q correction procedure mentioned in the text.


Figure 4.3: Impact of the Z corrections as a function of $\beta$ from F9 to F11 focal points.left figure is before the correction, and right figure $\left(Z_{c o r}\right)$ is after the correction


Figure 4.4: Zero Degree PID plot before(left) and after(right) corrections.
of flight information is needed and it is measured by plastic scintillators. Two photomultiplier tubes (PMTs) are placed on the two sides of the plastic scintillators. The timing information of the pulse and the total charge readout are collected from the PMT signals.

A properly operated PM tube should give the same position information which are obtained by using the t and q parameters from left and right PM tubes. From the Ref. [70], the following equation can be obtained :

$$
\begin{array}{r}
x=-\frac{V}{2}\left(t_{r}-t_{l}\right), \\
x=-\frac{\lambda}{2} \ln \left(\frac{q_{l}}{q_{r}}\right) \\
\therefore \ln \left(\frac{q_{l}}{q_{r}}\right) \propto\left(t_{r}-t_{l}\right), \tag{4.4}
\end{array}
$$

where $\lambda$ is the attenuation length of charge, and V is the propagation speed of light in the scintillating material. The information from Eq. 4.4 can be plotted for each plastic detector. The real events should lie on the $\mathrm{X}=\mathrm{Y}$ line while the background scattered around randomly; by applying the cuts on each plastics detector, these background events are omitted, as shown in Fig. 4.5, for 4 focal plane plastics in the BigRIPS and ZeroDegree.

Another contamination is coming from the different charge states of the isotopes in the Zerodegree spectrometer. As mentioned in the previous chapter, the $B \rho-$ $\Delta E-T O F$ method is used to create the PID. The $\mathrm{A} / \mathrm{Q}$ value is assumed to be constant in this method, and the ion is fully stripped. However, in reality, ions capture electrons in Zerodegree detectors after the target, or ions pick up the electrons inside the target and loose them in the Zerodegree detectors. This process causes a wrong velocity $\beta$ calculation and leads to incorrect $\mathrm{A} / \mathrm{Q}$ identifications. Such kind of events can be cleaned by comparing the $B \rho$ between the two focal plane intervals. If the ion is not stripping or picking up the electrons, then $B \rho$ is unchanged in between two focal planes since the charge state ratio is inversely proportional to the $B \rho$ ratio. Therefore in the ideal case, $B \rho[F 8-F 9] / B \rho[F 9-F 11]$ $=1$, and the spots to the left and right of 1 corresponding to electron pickup and stripping, respectively. As an example, we see the charge states of ${ }^{111} \mathrm{Mo}$ in $5^{t h}$ setting from SEASTAR-2 at $42 / 43=0.9767$ (for pickup) and $43 / 42=1.0238$ (for stripping) in Figure 4.6.

The Final cleaned and calibrated BigRIPS and ZeroDegree PIDs for each setting in SEASTAR-2 are shown in Fig. 4.7, left column is BigRIPS PID, while the right one is the ZeroDegree PID.

### 4.1.3 Third Campaign, Hodoscope calibration and alignment

As it is mentioned before, during the $3^{\text {rd }}$ campaign, instead of ZeroDegree spectrometer, the SAMURAI spectrometer was used. Three different detectors had to


Figure 4.5: Consistency check for the plastic scintillators at focal planes 3 to 11. A correlation that in Eq. 4.4 can be observed, and cleaning cuts are drawn in the red lines to remove the background. The figure is from Setting1-SEASTAR-2


Figure 4.6: Charge state correlation histogram. Y axis is $\mathrm{B} \rho$ values from focal plane 8 to 9 , while, the X axis is the ratio between focal plane $8-9$ to $9-11$. The real no charge exchange events are centered in 1 , the cut that used to clean the histogram is shown in red. The figure corresponds to Setting5


Figure 4.7: Particle identification plots for the three settings. The left plots shows the BigRIPS identification plot and right column is ZeroDegree PID. The red circles shows the cuts used for each one of the isotopes of interests.
be calibrated and corrected in this setup.
The Hodoscope consists of 24 plastic bar scinttilator detectors, that detects heavy ions after the reaction. Each bar was centered at different isotopic regions in order to cover a wider mass range and they were calibrated in energy and time. Two step correction was applied on the charge and time values in hodoscope bars. First, the bar's time spectrum was checked thought the experiment run by run, in order to detect possible shifts. Shifts were noted for each bar seperately, and corrected with respect to one reference run, as shown in Fig. 4.8. Afterwards, the 24 bars were compared to each other and their time peaks were aligned with respect to one reference time. The same correction is also applied for the hodoscope charge signal over the runs and the bars.


Figure 4.8: Hodoscope time correction with respect to run numbers for bar number 13,14 and 15 . The first row is uncorrected and the second row is corrected.

Other detectors that needed to be checked were the beam drift chambers, BDC1 and BDC 2 . BDCs were placed before the target chamber, and since the position information of the incoming ions are obtained from these detectors, it was important to align them with each other and with the MINOS TPC.

To get the final PID in SAMURAI, the velocity of the ions should be known as it was explained in the previous chapter. To calculate the velocity, the ToF information for the SAMURAI was measured in between hodoscope time and F13 focal plane plastic scintillator time. The flight length is not measurable since the ions are not following the straight line. In order to determine this, experiment simulations were used for the nucleus of interest with spesific mass and charge values. The simulation results gave us the parameters of a polynomial function, that is used to calculate flight length and $B \rho$ values using the measured FDC1 and FDC2 positions and the angle informations. As a result, using the flight length and


Figure 4.9: The beam is centered in BDC 1 , with respect to position and angle.


Figure 4.10: The SAMURAI PID after the corrrections for one run. The titanium isotopes from $\mathrm{A}=57$ to $\mathrm{A}=60$ are marked.
$B \rho$ values, the $\mathrm{A} / \mathrm{Q}$ values were calculated. For the Z measurements, the BetheBlock formula is used with the hodoscope charge information. The corrected PID spectra for the SAMURAI are presented in Fig. 4.10 for one example run.

### 4.1.4 DALI2 Calibration

DALI2 was calibrated before and after each setting with standard $\gamma$-ray sources, that are covering an energy range from 121 keV to 1.8 MeV . For the DALI2+ array, since we only have one physics setting, calibration data was taken for every second day to measure the ADC shift for the crystals. For the SEASTAR campaigns,
the sources are placed in the downstream end of the TPC. The calibration is done by fitting a Gaussian function combined with an exponential background. The centroid of the Gaussian and sigma (in ADC channel) are plotted with respect to known energies of the calibration source and fitted with a first-order polynomial to get gain and offset for the NaI detector setup. For some detectors, the 121.77 keV peak from the ${ }^{152} \mathrm{Eu}$ source is not visible or not resolvable because of the threshold and the low resolution in the low energy region. And for some detectors, two close-lying transitions from ${ }^{133} \mathrm{Ba}$ source are indistinguishable and can not be used for the calibration. As a result, each individual crystal is calibrated with a different number of calibration points which vary from 3 to 6 , depending on the crystal response and resolution in the low energy region. The energy differences of the measured and the tabulated energy for each detector for the 661,898 , and 1173 keV energies are shown in Figure 4.11.


Figure 4.11: The energy difference between known transitions and measured transitions from the DALI2 setup.

From the fits performed for the energy calibration, the detector energy resolution is also obtained. The results for 6 transitions belonging to four sources used in the third setting are shown in Fig. 4.12. The same analysis is performed for the other settings and third campaign data sets. The width of the photopeak is proportional to the square root of the transition energy for a scintillaton counter and is shown in Fig. 4.12. This procedure is performed for all the detectors individually. The extracted fit function parameters for each DALI2 crystal are used as an input for the simulation described in Sec. 3.4. The same resolution parameters are used for all three settings.


Figure 4.12: Obtained energy resolution of DALI2 detector with ID 165. The fit function is $\sigma=0.517 E^{0.594}$.

### 4.1.5 MINOS Calibration

The MINOS system, as explained in the previous section, was used to reconstruct each reaction position inside the target to improve the Doppler-correction. The MINOS calibration on the drift velocity was done by measuring the time corresponding to electrons in the Micromegas. The drift velocity is affected by the water and oxygen impurities in the gas since the molecules can capture electrons, which slows down the electron transport through the gas in between the Micromega planes.

The time spectra of Micromegas is shown in Figure 4.13. The first peak in the time spectrum ( $\mathrm{t}_{\text {start }}$ ) corresponds to the time of electrons from the ionization of the gas right next to the Micromega mesh. This signal is not affected by the gas impurities, which means that it is nearly constant for each run. However, if the electron is ionized near the cathode, it travels the entire TPC before arriving at the mesh, so it is affected by all the gas properties. That electron creates the last tail in the time spectra ( $\mathrm{t}_{\text {stop }}$ ). Therefore drift velocity depends on the gas condition inside the TPC which is changing from run to run for both SEASTAR campaigns. Fitting the Fermi function left and right-hand side of the time spectrum, we obtained $\mathrm{t}_{\text {start }}$ and $\mathrm{t}_{\text {stop }}$ respectively. Since we know the TPC length $\left(\mathrm{L}_{T P C}=300 \mathrm{~mm}\right)$ we can easily calculate the drift velocity from $v_{\text {drift }}=\frac{L_{T P C}}{t_{\text {stop }}-t_{\text {start }}}$ formula. The obtained drift velocity in the TPC from SEASTAR-2 setting 1 is shown in Figure 4.14 and used within the tracking algorithm. The same method is applied for the SEASTAR3 campaign.


Figure 4.13: Drift time distribution in the MINOS TPC for one run, $t_{\text {start }}$ and $t_{\text {stop }}$ were marked.

Drift velocity for setting1


Figure 4.14: Evolution of drift velocity over runs during the experiment. each run contains 1 hour data.

### 4.1.5.1 MINOS Tracking algorithm \& vertex reconstruction

The MINOS tracking algorithm was developed by C. Santamaria (CEA,Saclay, 2015) and is extensively explained in her thesis Ref. [71]. A brief summary is given in this section. The algorithm to construct proton vertex has 4 main steps, identification of xy plane, calculation of z-coordinate, track filtering and interaction vertex reconstruction. From the individual Micromegas pads, time and charge signal is collected, with the pad-position location. For the first step the the pad position ( $\mathrm{x}, \mathrm{y}$ coordinates ) information is collected, if the pad is triggered with a signal above the threshold. The algorithm based on the modified Hough transform is applied to all points, and it selects the events where the hit pads form a straight line in the xy plane. Each of these paths should be measured by at least 10 different pads, with a condition that at least two of the pads should be in the first four inner rings. As a second step the timing information is retained to calculate drift distance by using Eq. 3.4. The 3D reconstruction can be made with the track projection along the $z$-axis. In the third step standart Hough filter, same procedure that was used in the first step for 2 D construction, is applied in the xy , yz , and xz planes. Each identified tracks with a Houge filter, rejected if less than 15 out of 18 rings are triggered. In the last step of the tracking algorithm, the filtered tracks are fitted in 3 dimensions to make a straight line with minimization in each of the $\mathrm{x}, \mathrm{y}, \mathrm{z}$ points. If only one proton track is reconstructed in TPC, the interaction vertex is calculated between track and the beam line trajectory measured by the PPAC detectors before and after the target. For two reconstructed tracks, the minimal distance between the two tracks are determined, and the vertex position is calculated as a midpoint of these two.

### 4.1.6 Doppler Corrections

The ions produced in the experiment are traveling with relativistic velocities $\beta \approx$ 0.6 with a kinetic energy higher than $250 \mathrm{MeV} / \mathrm{u}$. Thus the emitted $\gamma$-rays from the nuclei will be Doppler shifted. While the nuclei are passing through the $\mathrm{LH}_{2}$ target, their kinetic energies are reduced by $60-70 \mathrm{MeV} / \mathrm{u}$ and they are slowed down to $\beta \approx 0.45 \mathrm{c}$. Therefore the emitted $\gamma$-rays from the nuclei need a Doppler shift correction. The Doppler corrected energy $\mathrm{E}_{\text {Dopp }}$, can be calculated by using the following formula:

$$
\begin{equation*}
E_{D o p p}=E_{\gamma}(1-\beta \cos \theta) \gamma \tag{4.5}
\end{equation*}
$$

where $\mathrm{E}_{\gamma}$ is the measured $\gamma$-ray energy by the DALI2 array in the laboratory frame, $\beta$ is the velocity (in fractions of c ) of the nuclei in the vertex position, and $\theta$ is the angle between the $\gamma$-ray emission and the direction of the emitting nucleus. The angle is calculated event by event between reaction vertex and DALI2 crystal hit point by the $\gamma$-ray. As explained in Section 3.3.1 MINOS TPC is used to improve the quality of the Doppler correction in cases where the proton had enough energy to leave the $\mathrm{LH}_{2}$ target, such as in the (p,2p), (p,pn), (p,3pn), (p,2pn) reactions. If we consider the direct beam channel $\mathrm{X}\left(\mathrm{p}, \mathrm{p}^{\prime}\right) \mathrm{X}$ reaction, MINOS information will
not be useful, since, the nucleus transfer too little energy to the proton to leave the target. In such cases, we consider that the reaction happens in the middle of the target and doppler correction is done accordingly. To calculate the angle $\theta$ correctly the interaction point inside the DALI2 is needed. The average interaction point for DALI2 is simulated by a Monte Carlo simulation code by taking into account the DALI2 geometry and the Compton effects inside the detector materials. To calculate the correct $\beta$ value, the ion velocities before and after the target are measured in the BigRIPS and ZeroDegree spectrometers. As it is mentioned in Sec. 3.2 before the target, the velocity $\beta$ is calculated in between the F5 and F7 focal planes, known as $\beta_{57}$. However after the F5 focal plane there are several materials in the beam line, that causes an energy loss of the ions before they enter the $\mathrm{LH}_{2}$ target. Therefore, a recalculation is needed to obtain the correct value $\beta_{b e f o r e}$ before the target. The LISE ++ [72] program is used to calculate the energy loss in between F5 position and the target enterance. This program used all the beam line elements and the mean velocity distribution $\beta_{57}$ as inputs and calculated the entarence energy in the Kapton window of the target. Also in order to calculate the exit velocity from the target $\beta_{a f t e r}$, the LISE ++ program was used with an input $\beta_{89}$, measured in between F8 and F9 focal planes of ZeroDegree. To calculate the energy loss in between F5 to target and target to F9, the LISE++ calculation is performed from 0.65 c to 0.5 c before the target and 0.5 c to 0.4 c after the target with steps of 0.05 c . The graph of the calculated velocity change before and after the target is shown in Fig 4.15. With the first order polynomial fit to the corresponding graphs, the correct values for $\beta_{b e f o r e}$ and $\beta_{a f t e r}$ are calculated event by event. The corrected and uncorrected $\beta$ distribution with ${ }^{89}$ As ions selected in BigRIPS and ${ }^{87} \mathrm{Ge}$ ions selected in ZeroDegree are shown in Fig 4.16. To check if the energy loss is correctly calculated, the $\beta_{\text {middle }}$ is also calculated with LISE++ using the half of the $\mathrm{LH}_{2}$ target with both $\beta_{89}$ and $\beta_{57}$ as inputs. The half target corrections parameters to $\beta_{89}$ and $\beta_{57}$ are shown in Fig. 4.15 as a black line, and the results of this correction is shown in Fig. 4.17. This figure is also used as a cross-check.

To find the vertex velocity $\beta_{\text {vertex }}$ inside the $\mathrm{LH}_{2}$ target, $\beta_{\text {after }}$ and $\beta_{\text {before }}$ values are used together with the reaction vertex, $z_{\text {vertex }}$ and target length, $L_{\text {target }}$;

$$
\begin{equation*}
\beta_{\text {vertex }}=\beta_{\text {after }}-\frac{z_{\text {vertex }}}{L_{\text {target }}}\left(\beta_{\text {before }}-\beta_{\text {after }}\right) \tag{4.6}
\end{equation*}
$$

The $\beta_{\text {vertex }}$ values are calculated for each event for the ${ }^{89} \mathrm{As}(\mathrm{p}, 2 \mathrm{pn}){ }^{87} \mathrm{Ge}$ reaction are shown in Fig. 4.16 with a green line. For the cases where MINOS vertex cannot be used, $\beta_{\text {middle }}$ is taken as an average of $\beta_{a f t e r}$ and $\beta_{b e f o r e}$. Another important factor for the correct $\beta_{\text {vertex }}$ calculation is the relative positions of the DALI2 and the MINOS target to each other, which effect the angle $\theta$ in the Eq. 4.5. The DALI2 and MINOS offsets for the actual positions have been determined by Sidong Chen (RIKEN) for the SEASTAR-2 and 3 campaign. Finally the Doppler-corrected $\gamma$-ray energies were used for the following analysis.

### 4.2 Procedure for the Analysis and Building Level Scheme

In this part of the chapter, the steps of the $\gamma$-ray analysis from the SEASTAR data are described. These steps were applied to the ${ }^{85,87} \mathrm{Ge}$ and the ${ }^{111} \mathrm{Mo}$ experiments. The spectra that are used in this analysis were treated in the same way. A background function which could be a double exponential, or a double Landau function (both commonly used to describe the DALI2 background spectra) was used and simulated response functions for different transition energies were utilized for DALI2. Each response function had two degrees of freedom: intensity and energy. For the energies, each response function is allowed to move within the range of one $\sigma_{E}$, in order to give a better fit to the data. This was not effecting the fit, since the energy resolution of DALI2 does not change rapidly, the response function of E and $\mathrm{E} \pm \sigma_{E}$ have similar shapes in the same region. For the low energy region $\left(E_{\gamma} \lesssim 250 \mathrm{keV}\right)$ the shift range was kept lower than $\sigma_{E}$ in order to control the backscattered peak. The intensity of each transition is also determined by using the response functions. The scaling factor of each response function is multiplied by the number of simulated events to give the intensity of a certain transition. Another method for the intensity calculation is determining the area under the response funtion by taking the integral. The maximum likelihood principle is used in the fitting procedure.

Since the resolution of the spectra is not good, we need a procedure to identify the additional peaks, which are hard to dissolve in the histograms. The improvement of the $\chi^{2}$ value while adding extra response functions is one way of identifying the unresolved transitions. Also the intensity of the suggested peak should be at least two times greater than the statistical uncertinities of the corresponding fit.

Energy loss corrections before and after the target


Figure 4.15: Left - Energy loss correction parameters for ${ }^{89}$ As ions in BigRIPS (red), right - Energy loss correction parameters for ${ }^{87} \mathrm{Ge}$ in ZeroDegree (blue). Black fits are to compare the energy loss calculation until and from the middle of target.


Figure 4.16: Red solid (dashed) distribution is corrected (uncorrected) $\beta$ distribution before the target. Blue soldi (dashed) distribution is corrected (uncorrected) beta after the target. green distribution is $\beta_{\text {vertex }}$ calculated with Eq. 4.6. For the distributions ${ }^{89} \mathrm{As}(\mathrm{p}, 2 \mathrm{pn}){ }^{87} \mathrm{Ge}$ reaction was choosed.

The procedure that is followed in this analysis is based extracting as much information as possible for all possible transitions, and making a coincidence analysis and estimating the intensities. As a results of this informations we can build or develop the level schemes. The steps of the analysis are explained by using ${ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p})^{85} \mathrm{Ge}$ reaction as an example, the final results are shown in Paper II and in chapter 5. The first step in the analysis is to identify the visible transitions in the Doppler corrected $\gamma$-ray histogram. The addback is also applied in the $\gamma$-ray spectra shown in Fig.4.18.

Three peaks are clearly visible in Fig.4.18 at energies of 250, 600 and 800 keV . Also there is an accumulation of data in high energy region between 1000-2500 keV . Before we perform the $\gamma-\gamma$ coincidence analysis we fit the spectrum with the gaussian functions to identify the transition energies. The sigma parameter of the function is limited with respect to DALI2 resolution measured with calibration sources. The ${ }^{85} \mathrm{Ge}$ isotope was studied earlier with two different reaction with $\beta$ and $\beta$ delayed neutron decay [73-75]. In these studies $10 \gamma$-ray transitions at energies 107, 250, 365, 472, 596, 703, 773, 789, 1589, and 2241 keV were identified. Since the reaction mechanisms are totaly different, we are not expecting to populate exactly the same states, however it is still usefull as a starting point. The first fit with known transitions are shown in Fig. 4.19 up to 1000 keV , the 773 and 789 keV transitions are having the same response functions, and they are not resolvable.

For the next step, the information from the gaussian fit to identify the peaks


Figure 4.17: The corrected $\beta$ correlation in between $\beta_{\text {beforemidtarget }}$ calculated from $\beta_{57}$ from F5 to middle of the target and $\beta_{\text {aftermidtarget }}$ calculated from $\beta_{89}$ from middle of the target to F9. For the histogram ${ }^{89} \mathrm{As}(\mathrm{p}, 2 \mathrm{pn}){ }^{87} \mathrm{Ge}$ reaction was choosed.


Figure 4.18: The Doppler corrected $\gamma$-ray spectrum of ${ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p})^{85} \mathrm{Ge}$. The inset figure is the same spectrum in a logarithmic scale and increased binning.


Figure 4.19: The ${ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p}){ }^{85}$ Ge reaction 1 dimensional $\gamma$-ray energy spectra. For fitting previously known transitions are used.


Figure 4.20: The $773 / 789 \mathrm{keV}$ gated spectra, fitted with 1700 keV response function


Figure 4.21: The 773 keV gated and background subtracted spectra is fitted with 1700 keV response function.
between $1000-2500 \mathrm{keV}$ is used. Two peaks are identified at 1200 and 1700 keV . As a first step, the $\gamma$-ray spectra is fitted with this extra transitions to see the improvement in $\chi^{2}$ value. The next step is to scan all data to see $\gamma-\gamma$ coincidences, and confirm the new transitions. With the gate on $773 / 789 \mathrm{keV}$ shown in Fig. 4.20, we identify one transition at $1700(30) \mathrm{keV}$ that is visible also in the 1 D spectrum and it obeys the $2 \sigma$ rule. With a gate around 595 keV we found another transition at $1200(30) \mathrm{keV}$. The self coincidence is possible in such analysis without background subtraction. The Compton continuum of higher energy peaks may also induces false coincidences and auto coincidence is also possible. The reverse gating is important to confirm the coincidence analysis. The results are presented in chapter 5.

The background subtraction is applied for the spectrum which was obtained by putting a gate at 789 keV . The background is choosen from the right side of the peak with energy (900-1000). The background subtracted histogram is shown in Fig. 4.21. In order to identify transitions directly going into the ground state we use a special multiplicity condition, $\mathrm{M}=1$. As a results of such a strict selection, we can identify the transitions that are not in coincidence with any other low lying transitions. The final fitted spectrum shown in Fig. 4.22.

### 4.3 Momentum calculations

Momentum is a conserved quantity in nuclear reactions, therefore in the knock-out reaction the initial momentum carried by the ion before the secondary target, should


Figure 4.22: Final $\gamma$-ray histogram for ${ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p})^{87} \mathrm{Ge}$ reaction. The lowest $\chi^{2}=1.34$ value reached with 8 response function fits
be equal to sum of the fragment and knocked-out nucleon momenta. By calculating the momentum of the knock out nucleon, we can determine the orbit of the knocked out valence nucleon in the nucleus. The main uncertanity comes from the reaction vertex position, and it is overcomed by using the event by event $\beta$ measurements, and putting a requirement for the vertex position. The effect of reaction vertex position on uncertanities becomes negligible in the momentum resolution. The momentum distribution of the fragments from one nucleon knockout reaction in the laboratory frame can be calculated as [76]:

$$
\begin{equation*}
P_{f}^{l a b}=\left(1+\frac{x_{11}}{D}\right) Z_{f} B \rho \tag{4.7}
\end{equation*}
$$

here $x_{11}$ is the position distribution in F11 which is corrected to restore achromatic condition and $D$ is the dispersion relation from F8 to F11 in the ZeroDegree detector. $z_{f}$ is the atomic number of the fragment, and $B \rho$ is the magnetic rigidity, that is measured in spectrometers. Since the reaction we are dealing with is in flight, we need to transform the laboratory frame to projectile rest frame with Lorentz transformation

$$
\begin{equation*}
P_{\|}=\gamma_{b}\left(P_{f}^{l a b}-\beta_{b} E_{f}^{l a b}\right) \tag{4.8}
\end{equation*}
$$

We measure the velocity of the beam $\left(\beta_{b}\right)$ or incoming ions and the energy of the fragment in laboratory frame is calculated from the relativistic mass energy conversion formula $E_{f}^{l a b}=\sqrt{P_{f}^{l a b^{2}}+(m c)^{2}}$, and $\gamma_{b}$ is the relativistic correction. The


Figure 4.23: The measured longitudinal momentum distributions for two different reaction leading to ${ }^{85} \mathrm{Ge}$
momentum resolution of the system can be measured by calculating the momentum distribution of the direct beam channel $\mathrm{X}\left(\mathrm{p}, \mathrm{p}^{\prime}\right) \mathrm{X}$. For the current work, momentum calculations are performed only for the ${ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p})$ and ${ }^{86} \mathrm{Ge}(\mathrm{p}, \mathrm{pn})$ single nucleon knockout-reactions which populated the ${ }^{85} \mathrm{Ge}$ isotope. The sigma value of the distributions are related with the angular momenta carried out by the knockout nucleons. For ${ }^{86} \mathrm{As} \sigma=107 \mathrm{MeV} / \mathrm{c}$ for ${ }^{86} \mathrm{Ge} \sigma=55 \mathrm{MeV} / \mathrm{c}$, and for direct beam the momentum resolution is around $25 \mathrm{MeV} / \mathrm{c}$. The inclusive momentum distributions are shown in Fig.4.23 for a neutron and proton knockout reaction.

### 4.4 Treatment of reaction channels

In this section inclusive and exclusive cross section analysis is discussed for different knockout reactions. Cross sections are one of the observables of direct reactions, and they can be used in the DWIA calculations to derive Spectroscopic factors, see Sec. 2.4.

### 4.4.1 Reaction Cross sections

The inclusive cross section ( $\sigma_{\text {reaction }}$ ) can be derived from the equation:

$$
\begin{equation*}
N_{\text {out }}=\sigma_{\text {reaction }} \cdot n_{t} \cdot N_{\text {inc }}, \tag{4.9}
\end{equation*}
$$

$N_{\text {inc }}$ and $N_{\text {out }}$ are the number of incident ions impinging on a target and the number of outgoing particles. Number of scattering centers per area in the target are given
by $n_{t}$. For example, for the ${ }^{86} A s(p, 2 p)^{85} G e$ reaction, $N_{\text {out }}$ is equal to the number of ${ }^{85} \mathrm{Ge}$ nuclei produced in the target, and $N_{\text {inc }}$ is equal to number of ${ }^{86} \mathrm{As}$ nuclei entering to the MINOS target, for the SEASTAR-2 campaign $n_{t}$ is calculated as :

$$
\begin{equation*}
n_{t}=\rho_{L H_{2}} \cdot L_{L H_{2}} \cdot \frac{N_{A}}{M_{H}} \tag{4.10}
\end{equation*}
$$

where $\rho_{L H_{2}}$ is density of the target $=0.07322\left(\mathrm{~g} / \mathrm{cm}^{3}\right), L_{L H_{2}}$ is the length of the target $=102(\mathrm{~mm}), N_{A}$ is the Avogadro number $=6.022 * 10^{23}\left(\mathrm{~mol}^{-1}\right)$ and $M_{H}=$ $1.008 \mathrm{~g} / \mathrm{mol}$ is the molar mass of the hydrogen, As a result of the Eq. 4.10 one obtain $n_{t}=4.329(0.044) 10^{23}\left(\mathrm{~cm}^{-2}\right)$.

For a specific reaction, we use the ZeroDegree and BigRIPS dedectors for selecting incoming and outgoing particles. However, for calculating the cross sections, we first need to know the beam line efficiency, since not all the ions produced in the beginning of the BigRIPS fragment seperator reach the ZeroDegree spectrometer. This is due to the scattering along the beam line, and efficiencies of the beam line detectors. Therefore while determining the reaction cross sections one has to consider also the transimission probability T , and be aware of the fact that not all the ions are reaching to the ZeroDegree detector. The final inclusive reaction cross section is then:

$$
\begin{equation*}
\sigma_{\text {reaction }}=\frac{N_{\text {out }}^{\prime}}{N_{\text {inc }}} \cdot \frac{1}{n T} \tag{4.11}
\end{equation*}
$$

The total transmission probability $T$ is:

$$
\begin{equation*}
T=\epsilon_{\text {line }} \cdot \epsilon_{\text {target }} \cdot \epsilon_{Z D} \tag{4.12}
\end{equation*}
$$

where $\epsilon_{\text {line }}$ is the corresponding efficiency for the beam line, $\epsilon_{\text {target }}$ is the loss inside the MINOS target due to the scattering , $\epsilon_{Z D}$ describes the undetected particles based on the acceptance of ZeroDegree or the transmission efficiency due to $B \rho$ selection in the dipole magnets. Each of these efficiencies can be extracted from the experimental data.

In order to calculate the beam line efficiency $\left(\epsilon_{\text {line }}\right)$, we use an empty MINOS target measurement which is performed at the beginning of each setting in SEASTAR-2. The MINOS and ZD efficiencies are not effecting the measurements. The centered nuclei in ZeroDegree during this runs were : ${ }^{85} \mathrm{Ga}$ for setting $1,{ }^{111} \mathrm{Nb}$ for setting 5 , and ${ }^{95} \mathrm{Br}$ for setting 2 . The beam line efficiency is calculated by:

$$
\begin{equation*}
\epsilon_{\text {beamline }}=\frac{N_{Z D}}{N_{B R}} \tag{4.13}
\end{equation*}
$$

This efficiency is independent of the reaction channel since the target is empty, and it is assumed to be constant within the settings. The results for three settings are listed in table 4.2.

Target efficiency is determined by using the run with MINOS, while the ZeroDegree and BigRIPS detectors centered at the same nuclei. These kinds of measurements are done only before setting 2 . For setting 1 and $5, \epsilon_{\text {target }} . \epsilon_{\text {beamline }}$ information is extracted from the experimental runs. Therefore the regular formula :

Table 4.2: The value of $\epsilon_{\text {line }}$ for the three different settings from SEASTAR-2

|  | Setting 1 | Setting 2 | Setting 5 |
| :---: | :---: | :---: | :---: |
| $\epsilon_{\text {line }}$ | $0.901(11)$ | $0.87(3)$ | $0.80(2)$ |

$\epsilon_{\text {line }} \cdot \epsilon_{\text {target }}=\frac{N_{Z D}}{N_{B R}}$ cannot be applied, since the ZeroDegree and BigRIPS are centered at different ions. The beam line detector at Foci 5 x distribution F5X is used to extract $\epsilon_{\text {line }} \cdot \epsilon_{\text {target }}$. The scaling factor in between only BigRIPS gated distribution and BigRIPS and ZeroDegree gated histogram gives the beam line and target efficiency. The special region is choosed to calculate efficiency so that the acceptance of ZeroDegree is not effective. The obtained scaling factor takes into account the missing ions due to the beamline efficiency and the scattering inside the target, $\left(\epsilon_{\text {target }} \cdot \epsilon_{\text {beamline }}\right)$. This method is used for setting 1 and 5 and conducted by selecting different pairs of nuclei. Finally for further analysis, the average value for each setting is shown in table4.3.

Table 4.3: Average combined efficiencies for $\epsilon_{\text {beamline }} \epsilon_{\text {target }}$ three settings

|  | Setting 1 | Setting 2 | Setting 5 |
| :--- | :---: | :---: | :---: |
| $\epsilon_{\text {line }} \epsilon_{\text {target }}$ | $0.618(11)$ | $0.570(38)$ | $0.630(23)$ |

In order to calculate the final transmission, we should include the ZeroDegree efficiencies to the total calculation. Until now, the efficiencies do not change much for different isotopes in the same setting. Now we focus on the reaction of interests and the core of the cross section calculations the isotope reaction ratios, which also includes the ZeroDegree efficiency effects for specific focal planes. To include the ZeroDegree efficiencies we use the F5X fit method for each reaction. This is done by examining the position distribution in the BigRIPS dispersive focal plane F5. An example showing F5X distribution can be seen in Fig. 4.24(a) for ${ }^{86} G e(p, p n)^{85} G e$ reaction. The blue line shows the F5X distribution for ${ }^{86} \mathrm{Ge}$ in BigRIPS and red line is the part of this distribution with the ${ }^{85} \mathrm{Ge}$ selection in ZeroDegree. The red distribution has a cut of in the low F5X, the reason of this cut off can be explained by ZeroDegree optimization. The ZeroDegree is optimized for the proton knockout reactions, therefore the neutron channels are cut by the slits at F9X dispersive focal plane in ZeroDegree.

To overcome this neutron knockout reaction acceptance problem, we take the isotope ratio between the distributions where the acceptance of ZeroDegree is maximum ( $\mathrm{F} 5 \mathrm{X}=50-100 \mathrm{~mm}$ ). In Fig.4.24(b) the ratio plot for the above reaction is shown with a specific trigger condition downscale beam trigger and ZeroDegree trigger (fbit==3). The cut in low x (because of large F9X) is visible and the flat region can be fitted to extract the $\frac{N_{o u t}^{\prime}}{N_{\text {inc }}}$ ratio which is used in Eq. 4.11. The final inclusive cross sections with the corresponding uncertanities are shown in table 4.4


Figure 4.24: (a) The x distributions at F 5 for ${ }^{86} \mathrm{Ge}$ ions detected shown by the blue histogram and ${ }^{86} \mathrm{Ge}$ selected in BigRIPS with ${ }^{85} \mathrm{Ge}$ ions detected in ZeroDegree shown in red histogram. with trigger condition fbit $==3$. (b) The ratio between these two distributions. the red line shows the fit into the flat region and it is used to determine $\frac{N_{o u t}^{\prime}}{N_{i n c}}$.
for the reactions of interest.
Table 4.4: Reaction cross sections for the SEASTAR-2 data sets that were used in this work

| Reaction | $\sigma_{\text {inclusive }}(\mathrm{mb})$ |
| :--- | :---: |
| ${ }^{87} \mathrm{As}(p, 2 p n)^{85} \mathrm{Ge}$ | $9.07(2.2)$ |
| ${ }^{86} \mathrm{As}(p, 2 p)^{85} \mathrm{Ge}$ | $7.65(1.92)$ |
| ${ }^{86} \mathrm{Ge}(p, p n)^{85} \mathrm{Ge}$ | $38.62(2.98)$ |
| ${ }^{89} \mathrm{As}(p, 2 p n)^{87} \mathrm{Ge}$ | $10.34(0.87)$ |
| ${ }^{9} \mathrm{Se}(p, 3 p n)^{87} \mathrm{Ge}$ | $1.47(0.12)$ |
| ${ }^{113} \mathrm{Tc}(p, 2 p n)^{111} \mathrm{Mo}$ | $10.3(5)$ |
| ${ }^{112} \mathrm{Mo}(p, p n)^{111} \mathrm{Mo}$ | $61(3)$ |

### 4.4.2 Excitation cross sections for particular states

In order to find the exclusive cross sections for particular states, one has to determine how often the state is populated and how many ions undergo an interaction with a target. The number of ions going into the target is reported in the previous section $\left(N_{B R}\right)$, but not all the ions emitting a $\gamma$-ray are triggered by the MINOS system, as a result MINOS efficiency has to cooperated to the exclusive cross sections. The MINOS efficiency ( $\epsilon_{\text {Minos }}$ ) is found by taking the ratio of total reactions occuring versus tracked vertices. For a single proton knockout reaction Minos has an efficiency of $89 \%$, for the neutron knockout reaction this efficiency is $54 \%$, and for proton and neutron knockout reactions it is $88 \%$. Another detector that is important for the cross section determination for a particular state is DALI2.

The efficiency of DALI2 is already considered in the simulated lineshapes, and the effect of DALI2 efficiency is already considered. However, during the SEASTAR campaign in 2015, there was a problem in DALI2 trigger, therefore additional correction factor is needed for the cross sections. For some events, DALI2 trigger signal is not send to the DAQ while the $\gamma$-ray is already detected. By considering the different trigger conditions, DALI2 trigger efficiency $\left(\epsilon_{D A L I 2_{t r}}=\frac{N(\text { fbit }==3)}{N(\text { fbit }==3| | f b i t==7)}\right)$ is calculated to be $0.513(40)$ in average. By using the calculated efficiencies from section 4.4.1, the cross section for the excitation of a particular state is given by

$$
\begin{equation*}
\sigma_{\text {state }}=\frac{A \cdot N_{\text {simulated }}}{N_{B R}} \cdot \frac{1}{\epsilon_{D A L I 2_{t r}} \cdot \epsilon_{\text {Minos }}} \cdot \frac{1}{n T} \tag{4.14}
\end{equation*}
$$

where $\mathrm{N}_{\text {simulated }}$ is the number of simulated ions and $A$ is the amplitude of the fit of the response function to the data. The efficiency parameters for DALI2 is already included in the simulation, therefore by fitting the response function, we are already considering the DALI2 detector efficiency. For the neutron knockout reaction, the exclusive cross section is obtained for 250 keV state. And it is used for the normalization of the DWIA calculation.

## Part 2 AGATA NEDA DIAMANT Campaign

In this section, the AGATA data structure and the data reconstruction will be explained as a part of the ${ }^{88} \mathrm{Ru}$ analysis. The AGATA campaign in GANIL in 2018 was run with digital electronics for all the detectors. In this part of the work, the data flow management of the GANIL system will be explained briefly. The data chain steps are also mentioned here divided into by Local and Global levels. Finally, the Offline Data Analysis steps are discussed.

### 4.5 General Data Flow of AGATA setup

The synchronization and system coordinator of the detector system is called Global Trigger and Synchronization (GTS), which is developed for the AGATA experiments and used for all detectors [77]. All detectors in the setup shares the global time reference supplied by the GTS and operated on the same 100 MHz clock. The reference timing is important to tag the events with the timestamp, which is used later for the time correlations.The GTS alignment for all AGATA signals are shown in Fig.4.25. The GTS is also responsible for the hardware trigger process, and it can process 40 trigger requests at the same time. The architecture of the system is divided into two-levels, the Local and the Global levels.

At the Local level, individual detectors don't know each other; therefore, signal from the detector is processed individually. Data processing in this level is the same for all AGATA crystals, and it goes in parallel. For NEDA and NWall detectors local level analysis is also done on crsytal by crystal basis. The produced data for the AGATA, NEDA and DIAMANT detectros in that level is tagged with a timestamp which is giving the absolute time with a 10 ns uncertainty.

At the Global level of the system, detectors communicate with each other; therefore, they are not independent anymore. In this step, the interesting events are selected in order to decrease the data rate. Event Builder and merger are also processed in this level with the AGATA and ancillary detectors. The final step in Global replay is the tracking algorithm for the AGATA detectors, and then the data is ready for the $\gamma$-ray analysis.

### 4.6 Local level Process

At the Local level replay of AGATA detectors, all crystals pass through the same steps. There are several stages for the detectors: Crystal Producer, PreprocessingFilter, Pulse Shape Analysis (PSA) Filter, and PostPSAFilter. Before we go through the steps, the definition of the some concepts will be helpful to follow this topic. Producers are taking the data from the DAQ main system and it send the data to the data flow process. Filters are read and process data from the data flow, and send it back to data flow. Consumers read data from the flow and write it to


Figure 4.25: time alignment of all AGATA signals after the GTS reboot, for a single run from experiment, the TkT program is used for visualization.
the disk, withoout sending back the data to data flow. Therefore Consumers are the final actors in the data process [78].

The first step is to produce the data from the raw traces (waveforms from the digitizer) using CrystalProducer Program. For the current experimental setup, AGATA detector raw traces were not saved because the data rate was too high, and it took a lot of disc spaces to save the traces for 36 crystals with 38 output signals. On the other hand, NEDA and NWall are using the NEDA Producer which saved the traces of the signal to be replayed offline with specific parameters for increasing the neutron detection efficiency.

For the AGATA signals, digitizer gains could be arranged to give as low as 20 MeV or as high as 5 MeV full range. Since we were not interested in energies above 5 MeV for this experiment, the energy range for the segments were limited to 5 MeV , but for the core signal, we saved both the high and the low gain data. The number of output signals from one crystal is 38 ( 36 segment +2 core). With the help of CrystalProducer, we generated the raw spectra and send them to the data flow.

The second step of the AGATA analysis is the PreprocessingFilter where we performed the energy and time calibrations of the AGATA segments from the ADC channels. The calibration for the energy was done only in the first order polynomial, and time alignment of the segments done with respect to the core signal. Since the AGATA crystals are electronically segmented and the crosstalk between the segments is highly possible which can create a problem since the tracking algorithm uses the segment energies to calculate the $\gamma$-ray energies. The effect of crosstalk on the sum of the segment energies is shown in Fig.4.26. After the crosstalk corrections, if there are broken or missing segments, the missing energy is calculated by comparing the core signal to the sum of the segment signal. The system is forced to compensate the missing energy from the broken segment to have a energy value equal to the core energy.

After the energy - time corrections and calibrations with the help of the PreProcessingFilter, the data is stored in the units of keV for the energy, time is sampled in 10 ns intervals, and the position of the interactions inside the segments are saved in the units of mm . Another filter that has been applied in the local processing step is Pulse Shape Analysis Filter for the AGATA and the NEDA detector systems. For the NEDA analysis the NEDAProducer sends the data directly to PSA filter without any calibration. It is particularly important to get the correct parameters for the AGATA system because a mistake in this step is irreversible since the traces are not stored in the in-beam data.

PSAFilter is applied for the signal decomposition to find the closest position of the interaction inside the crystal. The algorithm is known as the Grid Search. The details can be found in ref [79]. The idea is to compare measured and generated pulses with the lowest $\chi^{2}$ criterion in order to determine the interaction points. Pulse shape analysis takes most of the CPU time in the data flow. The PSAFilterGridSearch uses 2 mm grids for the germanium crystals, making approximately 1500 points in one segment to analyze. The PSA parameters calculated in this step
are essential for the PostPSA analysis, where the final energy and time calibration are done, and neutron damage in the HpGe crystal is corrected.

The final step for the Local-level analysis on AGATA is the Post-PSA filter. For this step, we took data for one night-long ${ }^{60}$ Co source to get enough statistics for each segment. Also, it is convenient to use a ${ }^{60}$ Co source, with high energy transition at 1.3 MeV which can trigger the final layer segments. The electron-hole mobility in the segments is calculated, which is the first parameter for trapping. For a brand new detector (in a perfect case), the crystal does not have the neutron damage and the trapping should be zero. However, the AGATA crystals are already exposed to radiation. Therefore there is neutron damage in the crystal, which affects the germanium band gaps. This damage decreases the bandgap; therefore, the effective energy is measured lower than the real photon energy. This effect is corrected with a neutron correction, an artificial correction on the asymmetric distribution. After the correction is applied to the replayed data, all segments have to be calibrated again to fine-tune for the correct energy. The effect of the PostPSA Filter on the neutron damage correction can be seen in the Fig.4.27. With the final Filter, the local level processing is done for the AGATA detectors, and data is sent to the Global level processing.

For the NEDA/NWall detectors, Local level processing is also important since the Pulse Shape Analysis has to be done before the Global level processing. In the current experiments at GANIL, the NEDA traces/waveforms were saved in contradistinction to the AGATA array. Local-level NEDA Replay had two steps. Signal preprocessing optimization and signal processing. In the preprocessing op-


Figure 4.26: Croos-talk effect for the AGATA segments, the figure shows the different folds of the sum of segment energies, the shift in 1332 keV energy from ${ }^{60} \mathrm{Co}$ calibration source is visible in the higher folds.


Figure 4.27: $1172 \mathrm{keV} \gamma$-ray from ${ }^{60} \mathrm{Co}$ calibration source. The data is taken for the neutron damage correction before the physics runs starts. The 36 segment signal from the one AGATA crystal with 2 core signals and 2 sum of the segments signals are shown in the figure. The blue spectrum respressent the energy before neutron damage correction, the black spectrum show the final spectrum after neutron damage correction.
timization, the main aim is to optimize the three parameters Delay, Fraction, and Crossing line. First, the NEDA producer read raw data from NUMEX02 boards and send it to data flow to process in the PSA algorithm. One of the main reasons to save NEDA traces was to get a better time resolution of the Time to Digital Converter (TDC) values after the experiment.

After the NEDA producer PSA Filter is applied to NEDA/NWall data. Three types of Pulse Shape Discrimination (PSD) algorithms were tested in Ref. [64,80,81] with Charge Comparison (CC), Integrate Rise Time (IRT), and Neural Network (NN) techniques. Charge Comparison Pulse Shape Analysis (CCPSA) method is based on the signal shape differences between the $\gamma \mathrm{s}$ and the neutrons in the organic scintillator. In the CCPSA method different integration gates are chosen as short and long time gates. The short time gate is sensitive to the particle nature, while the long one is used to calculate the total charge in the signal. The ratio between
the slow and fast time gates resulted in two different distributions for neutrons and for gammas. As a Filter in the preprocess, signal optimization CCPSA algorithm is used to convert raw NEDA data to compressed NEDA data. After finding the optimum parameters for the CCPSA as the last step, the signal processing stage is started, and the CCPSA algorithm is applied as a filter to the raw data. The final TDC after the post CCPSA filter will be around 3 ns for the current experimental setup. The processed neda data without the trace information are sent to the Global Replay process to merge with other detectors.

DIAMANT detectors were not following the same steps with NEDA or AGATA. In a way, there is no filters applied to DIAMANT signals. The regular Trapezoid filter is used for raw DIAMANT signal from the digitizer to convert it to Energy and Time. In the local level analysis DIAMANT signals are time sorted and sent to the global analysis level

### 4.7 Global Level

In the Global level processing, the data is already saved in the binary format, so in this level, detectors start seeing each other, and the event building process starts to create the final correlated events. GANPRO (GANil PROcessing) and AGAPRO (AGAta PROcessing) frameworks is used at different levels to reach the final data structure. GANPRO is used for the compressed NEDA data and Diamant data, while AGAPRO has been used for the AGATA array and event merging steps. The first step is to sort the compressed NEDA frames in time order, and the sorted data is sent to the Event merger. The data structure and event time windows are shown in Fig.4.28 for the current experiment.

The first step is the event building which groups the events in a specific event time window. Building events are done for each type of detector seperately. Therefore AGATA, DIAMANT, and NEDA detectors build their events independently. The EventBuilder actor is responsible for building the events in AGATA detectors based on the time stamp information and constructing the events coming from different crystals. With the EventBuilder, the events are also filtered according to a real-time trigger condition to reduce the data rate. The coincidence time window between two $\gamma$-rays is determined in this level. For this experiment, the window is selected as 8 time stamp units. For the NEDA and DIAMANT detector setup, the BasicATSB actor is used to build events in a given time window. The BasicATSB system is simpler than the EventBuilder due to the fact that, it requires a single data flow and stors the data without building the events [83]. The data structure for the AGATA and the ancillary detectors are shown in the data flow Fig. 4.28.

The second step is Eventmerger that merges the events from different detectors, which is called global events. In this step, the user decides which detectors and crystals will be merged within the provided time window length. Also, the mandatory key is chosen to open global event window. In the current experiment, the mandatory key is the AGATA event. Only if an AGATA PSA event exists the new


Figure 4.28: Data structure and data flow in the Local and Global level process. The figure is taken and modified from Ref [82]
global event is merged. For this experiment, only AGATA is used for validation. Therefore, even only an AGATA event can create a global event. The structure of Global events for the experiment shown in Fig 4.28.

The third step is the TrackingFilter that applies the tracking algorithm to the AGATA data. When the experiment was ongoing, one type of tracking algorithm is available: the Orsay Forward Tracking (OFT) algorithm. New tracked events were added to the global event, with keeping the information before the tracking algorithm. For the tracking, each event is listed with deposited energies and positions from the segments of the AGATA detectors. The tracking aims to reconstruct the $\gamma$-rays, which deposit their full energy in several interactions within the segments. For that, the first, second, etc. interaction points are listed within a given time and position conditions to build the path of the $\gamma$-ray. The OFT algorithm requires several inputs; The first input comes from PSA hits, energy, and 3 dimensional ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) hit positions within a prompt coincidence window defined in the event builder step. It also requires the nominal position of the source. For the current algorithm, which is explained widely in Ref. [62, 84] for the energy; MeV and for the distance; cm is used as a unit. There are also three tracking parameters that are used to optimize the tracking. The main idea is to find the most probable sequence of the interaction points for each cluster and which sequence satisfies Compton scattering formula.

The OFT algorithm roughly follow these steps $[62,84]$ and the input parameters used for this experiment underlined in the following text:

- The algorithm first cluster the interaction points based on the polar and azimuthal angles. It calculates the distance between two interaction points in Ge and between these points and the source. The maximum angular separation has been calculated between interaction points and the source inside the same cluster. Two interaction point and angular correlation between them is calculated several times iteratively in the loop to identify different clusters.
- After the cluster search, clusters are evaluated with Compton scattering formula, and Figure Of Merit (FOM) is calculated for each possible track in the clusters, searching the best possible sequence.
- The average position uncertainties are calculated between the geometrical angles and energy angles from the tracks. The first input parameter in the OFT is the limit of the position uncertainty. For this data set, the average position uncertainty should be lower than 0.8 cm to be a valid cluster and pass the tracking filter.
- The clusters are sorted according to FOM and the clusters were marked if they have better FOM while they are sharing the same interaction points with other clusters. The sorted clusters are validated if the FOM value is lower than 0.05 , which is the second input parameter to the OFT tracking filter.
- As the last step, if there is only a single interaction point in a cluster, the Compton interaction probability and the depth of the interaction are calcu-
lated. The validation of such an event depends on the square root of the production of the probability, and depth should be bigger than 0.02 , which is the last third input parameter in the tracking filter.

After the tracking filter, The root [85] base treebuilder is used to build root trees which include the correlated branches from the tracking filter and event builder results of AGATA, NEDA, and DIAMANT detectors.

The final data output from the replay process is a root tree, which contains correlated NEDA, DIAMANT, and AGATA data. To reach nucleus ${ }^{88} \mathrm{Ru}$, the compound nucleus ${ }^{90} \mathrm{Ru}$ evaporates 2 neutrons, and these neutrons are captured in NEDA and NWall detectors. Since in the fusion evaporation reactions, several nuclei are populated with different combinations of nucleon evaporations, cleaning the data set is a critical step for the analysis. In order to make a clean selection on 2 neutron channels, the DIAMANT detector is used as a veto detector, which means if there is Diamant information in the global event frame, that event is rejected. In that way, only the neutron evaporation channels are boosted. More information on the ${ }^{88} \mathrm{Ru}$ selection and $\gamma$-ray analysis on this field can be found in Ref. [86].

## Chapter 5

## Results and Discussion

### 5.1 First observation of $\gamma$-ray transitions in ${ }^{111} \mathrm{Mo}$

The neutron rich nuclide ${ }^{111}$ Mo was studied previously in Ref. [87-90]. In the most recent study, the ground state half-life was determined to be 196(5) ms [90]. A beta-decaying isomeric state with a half-life of $200(10) \mathrm{ms}$ has been reported [89]. The spin and parity of the observed states were tentatively assigned to be $7 / 2-$ or $9 / 2$ - for the isomeric state and $1 / 2+$ or $3 / 2+$ for the ground state based on the systematics in this region.

In the present work, $\gamma$-ray transitions in ${ }^{111}$ Mo were observed for the first time. Two different reactions populated the ${ }^{111} \mathrm{Mo}$ nuclei and the inclusive cross sections for the reaction ${ }^{113} \mathrm{Tc}(\mathrm{p}, 2 \mathrm{pn})$ is $\sigma_{\text {inc }}=10.3(6) \mathrm{mb}$ and the ${ }^{112} \mathrm{Mo}(\mathrm{p}, \mathrm{pn})$ reaction is $\sigma_{\text {inc }}=61(3) \mathrm{mb}$.

The first step of the analysis was started with the verification of the methods, where we tested the previously known nucleus ${ }^{109} \mathrm{Mo}$, which was populated in the same experiment. The energy of the transition between the $(7 / 2+) \rightarrow(5 / 2+)$ states was measured to be $140(10) \mathrm{keV}$, in agreement with the earlier measurement which reported $144.0(3) \mathrm{keV}[26,91]$. Furthermore, the $\gamma-\gamma$ coincidence analysis method was also tested on the ${ }^{111} \mathrm{Mo}(\mathrm{p}, \mathrm{p} 2 \mathrm{n}){ }^{109} \mathrm{Mo}$ reaction with an applied gate on the $110.8(3) \mathrm{keV}$ transition. The results confirm the method by showing correct coincidences with the $138.9(3), 222.2(3)$ and $397.3(5) \mathrm{keV}$ transitions in agrement with the previously constructed level scheme [26], see Fig. 5.2.

The different knockout reactions populate the ${ }^{111}$ Mo isotope with different average excitation energies, the strongest $\gamma$-ray transitions (130(10) $\mathrm{keV}, 176(13) \mathrm{keV}$, $205(16) \mathrm{keV}, 235(19)$ and $290(28) \mathrm{keV}$ ) are observed in both reactions as seen in Fig 5.1. Two additional transitions at energies $380(22) \mathrm{keV}$ and $750(45) \mathrm{keV}$ were clearly observed in the ${ }^{112} \mathrm{Mo}(\mathrm{p}, \mathrm{pn})$ reaction. As mentioned above the isomeric states have been reported in neutron rich Mo nuclei, however our experiment is not suitable for studies of states with long half-lifes.

To investigate the lifetime effect on our study, we simulate different half-lives


Figure 5.1: (Color online) Doppler-corrected $\gamma$-ray spectra for ${ }^{111}$ Mo measured in the forward angles of DALI2 generated from (a) (p,pn) and (b) (p,2pn) reactions, where they contain the total spectrum (black solid line), normalized bremsstrahlung component obtained from (p,p') reactions (red striped area), and the backgroundsubtracted spectrum (open diamonds). The inset figure in (a) shows the spectrum in the higher energy range selected with a strict multiplicity condition $\mathrm{M}=1$ for all detectors. The total fit to the subtracted spectrum is shown by the thick blue line, the single $\gamma$-ray response functions of DALI2 are indicated by thick red line, and the dashed blue line shows the double exponential background fits. The figure is from Paper II.


Figure 5.2: Total $\gamma$-ray energy spectrum and energy-gated $\gamma$-ray coincidence spectrum from the ${ }^{111} \mathrm{Mo}(\mathrm{p}, \mathrm{p} 2 \mathrm{n}){ }^{109} \mathrm{Mo}$ reaction. The background-subtracted spectra shown as open diamonds. The coincidence cut in the $100-120 \mathrm{keV}$ range for 110 keV transition with background subtraction $170-190 \mathrm{keV}$ in the ( $\mathrm{p}, \mathrm{p} 2 \mathrm{n}$ ) reaction and only forward angle detector was used to optimize peak background ratio. The inset is the partial level scheme of ${ }^{109}$ Mo from Ref $[15,26]$.
from 0-100 ps for all transitions and tested to reach the lowest $\chi^{2}$ value in our fits, which was obtained for 60 ps . A half-life of 60 ps can affect the centroid of a 290 keV transition by about 10 keV , and such effects are included in the uncertainties of the energy.

In order to investigate the photopeak-photopeak coincidences in the data, background subtraction was applied to the gated histograms. The coincidence analysis for ${ }^{111} \mathrm{Mo}$ was done separately for each reaction since the different knockout reactions may not populate the same excited states. The data analysis method which is explained in Chapter 4, Sec. 4.2 was applied to identify the coincidences. The strongest mutual coincidence relationship was found between the 130 and 176 keV transitions in both reactions. Furthermore, the 176 keV gamma-ray is also in coincidence with the 205 keV transition, which was visible in both reactions. The coincidence spectra for the 130 and 176 keV transitions are shown in Fig 5.3.


Figure 5.3: (a) $\gamma$-ray spectrum obtained using a gate centered around 130 (115145) keV , with Compton background in the $150-170 \mathrm{keV}$ range, in the ${ }^{113} \mathrm{Tc}(\mathrm{p}, 2 \mathrm{pn})$ reaction. (b) $\gamma$-ray spectrum obtained using a gate centered around 176 (160200) keV , with Compton background in the $200-240 \mathrm{keV}$ range in the ${ }^{113} \mathrm{Tc}(\mathrm{p}, 2 \mathrm{pn})$ reaction.

The 205 and 235 keV transitions also revealed clear coincidence relationships, especially in the ${ }^{113} \mathrm{Tc}(\mathrm{p}, 2 \mathrm{pn})$ reaction, while the 205 keV line showed coincidence with the 176 keV transition. The $290-\mathrm{keV}$ transition, which was one of the strongests ones was observed in both reactions. was neither in coincidence with the 130 keV transition nor with the transition at 176 keV . On the other hand, it was observed to be in coincidence with the 380 keV transition. Additionally, the $750-\mathrm{keV}$ transition was found in the $\gamma-\gamma$ coincidence analysis in the single-neutron knockout reaction, and it was found to be in coincidence with the 290 keV transition as well as with the 130 keV and 176 keV gamma rays. The coincidence results are summarized in Table 5.1.

Due to the high density of peaks in the low energy region and the insufficient energy resolution of the DALI2 detectors, it was not possible to construct a firmlevel scheme for ${ }^{111} \mathrm{Mo}$. Considering the relative intensities and the coincidence analysis, the 130 keV and 176 keV transitions are clearly in cascade. We can tentatively assign the 130 keV transition to be placed on top of the 176 keV transition and the 290 keV transition as a crossover since their energies sum up within the experimental uncertainties.

For the odd-even ${ }^{109} \mathrm{Mo}$ and ${ }^{111} \mathrm{Mo}$ isotopes, the TRS calculations, which are explained in Chapter 2, were performed by blocking the lowest neutron configuration of different parity and signature in a self-consistent manner. The results for ${ }^{111}$ Mo at zero rotational frequency are shown in Fig. 5.4. The entire PES is quite gamma-soft for quadrupole deformations in the range $\beta_{2}=0.2-0.3$.

For the lowest negative-parity configuration, the TRS calculation produces coexisting oblate and triaxial minima, Fig 5.4(a). The lowest-lying oblate configuration is the predicted ground-state configuration and has deformation parameters $\left(\beta_{2}, \gamma\right) \approx\left(0.25,-60^{\circ}\right)$. The lowest positive-parity configuration lies approximately 600 keV above the predicted negative-parity ground state and exhibits a triaxial gamma-soft minimum, at $\left(\beta_{2}, \gamma\right) \approx\left(0.30,-25^{\circ}\right)$ that stays relatively unchanged with rotation.

In order to investigate the lowest active configurations in ${ }^{111} \mathrm{Mo}, \mathrm{PPR}$ calculations were performed by using the TRS deformation parameters. In the PPR calculation negative and positive parity states were calcualted separately. Their relative placement was obtained from the energy differences derived from the TRS minima.

According to the PPR calculations, the ground state in ${ }^{111} \mathrm{Mo}$ is predicted to have spin-parity $I^{\pi}=9 / 2^{-}$for the oblate minimum of the negative parity configuration, which is a structure of predominantly $1 h_{11 / 2}$ parentage. While the $7 / 2_{1}^{-}$ is predicted to be situated only 29 keV above the ground state, the $11 / 2_{1}^{-}$state is calculated to be $\approx 136 \mathrm{keV}$ higher in energy. The predicted ground-state oblate structure is dominated by the $9 / 2[514](77 \%)$ and $11 / 2[505]$ ( $16 \%$ ) Nilsson configurations. The effect of changing the shape to the somewhat less favored triaxial

Table 5.1: Gamma-ray transitions in ${ }^{111}$ Mo observed in this work. The relative intensities of the transitions are normalized with respect to the most intense 290 keV transition for the ${ }^{112} \mathrm{Mo}(\mathrm{p}, \mathrm{pn})$ and ${ }^{113} \mathrm{Tc}(\mathrm{p}, 2 \mathrm{pn})$ reactions. The coincidence analysis results are also shown in the right coloum of the table.

| $E_{\gamma}(\mathrm{keV})$ | $I_{\gamma}$ <br> ${ }^{112} \mathrm{Mo}(\mathrm{p}, \mathrm{pn})$ | $I_{\gamma}$ <br> ${ }^{113} \mathrm{Tc}(\mathrm{p}, 2 \mathrm{pn})$ | Coincident $\gamma$-lines |
| :--- | :---: | :---: | :--- |
| $130(10)$ | $84(6)$ | $52(9)$ | 176 |
| $176(13)$ | $83(7)$ | $59(11)$ | 130,205 |
| $205(16)$ | $44(8)$ | $59(12)$ | 176,235 |
| $290(28)$ | $100(7)$ | $100(12)$ | $290,380,750$ |
| $235(19)$ | $44(8)$ | $19(14)$ | 205 |
| $380(22)$ | $32(8)$ | $18(13)$ | 176,290 |
| $750(45)$ | $11(9)$ | $9(14)$ | $130,176,290$ |

shape is a rather dramatic rearrangement of the lowest-lying negative-parity levels. The $I^{\pi}=7 / 2^{-}$state then becomes the ground state while the $11 / 2_{1}^{-}, 9 / 2_{1}^{-}$, and $3 / 2_{1}^{-}$states become almost degenerate around $80-90 \mathrm{keV}$ above the ground state. The migration of the $3 / 2_{1}^{-}$state by more than 700 keV is remarkable and would have profound effects on the structure of ${ }^{111} \mathrm{Mo}$ as discussed below. For the similarly less favored prolate shape, the $I^{\pi}=7 / 2^{-}$state is also the ground state. The excited positive-parity structure is based on Nilsson configurations with mixed $1 g_{7 / 2}$ and $2 d_{5 / 2}$ parentage. Its excitation energy is around $600-700 \mathrm{keV}$ depending on the deformation. At an oblate shape, the band head is predicted to be $I^{\pi}=3 / 2^{+}$with a close-lying $I^{\pi}=1 / 2^{+}$state only 23 keV higher in excitation energy. The wave function of the $3 / 2^{+}$state is a mixture of mainly the [411] $3 / 2$ and [420]1/2 Nilsson configurations. At the favored triaxial shape, this is reversed with the $1 / 2^{+}$being 60 keV and 170 keV lower than the $3 / 2^{+}$and $5 / 2^{+}$states, respectively. Therefore the positive parity, $3 / 2^{+}$, and $1 / 2^{+}$were predicted to be very close in energy, almost degenerate. To study the further effect of shape coexistence on the level scheme, we also performed PPR calculations for a prolate ( $\gamma=0^{\circ}$ ) configuration. The lowest positive parity state is then changed to $5 / 2^{+}$with a more simple configuration dominated by the 5/2[402] Nilsson state with an admixture of $75 \%$.

There are no obvious long-lived isomeric states in the predicted theoretical level schemes in Fig. 5.5. However, in our calculation the relative position of the negative


Figure 5.4: Total Routhian surfaces in the $\beta_{2} \sin \left(\gamma+30^{\circ}\right)-\beta_{2} \cos \left(\gamma+30^{\circ}\right)$ plane for the lowest signature and parity $(\pi, \alpha)$ configurations in ${ }^{111} \mathrm{Mo}$ at $\hbar \omega=0.0 \mathrm{MeV}$. The blue diamond indicates the position of the collective minimum for each potential energy surface. Equipotential energy lines are separated by 0.2 MeV . (a) blocking of the lowest ( $\pi=-, \alpha=-1 / 2$ ) configuration. (b) blocking of the lowest ( $\pi=$ $+, \alpha=+1 / 2)$ configuration.


Figure 5.5: The theoretical level scheme results of the PPR calculations. The first lowest states of a given spin are drawn for both positive and negative parity. The energy difference between the first positive-parity state and the negative-parity ground state is taken from the TRS calculation at zero rotational frequency at each shape. The transitions are labeled with their corresponding reduced E2 transition probabilities in Weisskopf units as predicted by the PPR calculation. The dominant Nilsson component of the wave function is indicated at the bottom of the band.
and positive parity band heads are more uncertain than the relative positions of the states within the same band. In this work, we propose a possible scenario that could explain the existence of the long-lived isomer. Considering the oblate -triaxial shape coexistence predicted by the TRS results, if the energy of the triaxial $1 / 2^{+}$ band is shifted down to around 250 keV , it will come close to the $5 / 2^{-}$state which is built on the oblate ground state structure. Such a scenario which is within the estimated uncertainties of the theoretical predictions, could hence lead to a spintrap isomer with a sufficiently long half-life. Another scenario might be to reverse the parity as predicted by the TRS. In this case the triaxial positive parity structure and oblate negative parity structure, can become close in energy and this can also lead to a spin-trap isomer.

Using an oblate-triaxial shape coexistence, the PPR model can hence predict a low lying spin trap isomer. From the population of states in neutron knockout and proton-neutron knockout reactions, and based on the observation of negative parity states in ${ }^{109} \mathrm{Mo}$, it is most likely that the negative parity states are emanating from spherical $1 h_{11 / 2}$ subshell, while the positive parity states from $1 g_{7 / 2}$ and $2 \mathrm{~d}_{3 / 2}$ are not observed. We therefore conclude that the observed $\gamma$ rays are due to transitions between states within the negative-parity structure and the additional $\gamma$ rays observed in the proton-neutron knockout reaction are due to the population of higher spin states.

### 5.2 Results for Single Particle Structure in ${ }^{85,87} \mathbf{G e}$

### 5.2.1 Studies of ${ }^{85} \mathrm{Ge}$

The isotope ${ }^{85} \mathrm{Ge}$ has previously been investigated via $\beta$ decay $[74,75]$, and $\beta$ delayed neutron decay [73] reactions. In these studies, nine $\gamma$-ray transitions at energies; 107.7, $365.4,472.6,595.8,703,773.2,788.5,1589.4$, and 2240.5 keV from $\beta$ decay of ${ }^{85} \mathrm{Ga}$ and one additional $\gamma$-ray transition at 250 keV from $\beta$-delayed neutron decay of ${ }^{86} \mathrm{Ga}$ have been observed.

In the present in-beam study, the nucleus ${ }^{85} \mathrm{Ge}$ was populated via three different knockout reactions ${ }^{86} \mathrm{Ge}(\mathrm{p}, \mathrm{pn}),{ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p})$, and ${ }^{87} \mathrm{As}(\mathrm{p}, 2 \mathrm{pn})$ with corresponding inclusive cross-sections, $\sigma_{\text {inc }}=74.4(32) \mathrm{mb}, 12.6(44) \mathrm{mb}$ and $15.6(39) \mathrm{mb}$. Since each reaction has a different average excitation energy, they may populate different levels with different transitions in the observed spectra as a result. The transitions observed for each reaction are listed in table 5.2. Doppler-corrected $\gamma$-ray energy spectra are shown in Fig.5.6.

Seven transitions with peak centroid energies of 250(11), 365(22), 472(26), $595(25), 665(37), 703(33)$, and $803(36) \mathrm{keV}$ were fitted with the corresponding response function for all reactions. Eight additional transitions with peak centroids at 878(39), 960(45), 1200(70), 1452(75),1589(77), 1700(80), 2241(105), and 2500(124) keV were observed in the 1 -dimensional $\gamma$-ray spectra for the different reactions. It is likely that the 773 keV and 788 keV transitions reported in Refs. [74,75] and the

Table 5.2: Gamma-ray transitions observed in ${ }^{85} \mathrm{Ge}$. The relative intensities of the transitions observed in the different knockout reactions are normalized to the most intense $789+773 \mathrm{keV}$ peak. The coincidence analysis results are shown in the right colum of the table.

| ${ }^{85} \mathrm{Ge}$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
| $E_{\gamma}(\mathrm{keV})$ | $I_{\gamma}$ <br> ${ }^{86} \mathrm{Ge}(\mathrm{p}, \mathrm{pn})$ | ${ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p})$ | $I_{\gamma}$ <br> ${ }^{87} \mathrm{As}(\mathrm{p}, 2 \mathrm{pn})$ | Coincidence <br> results |  |
| $250(11)^{*}$ | $45.7(6.3)$ | $23.4(6.3)$ | $12.1(2.1)$ | 2500 |  |
| $365(22)^{*}$ | $30.6(5.4)$ | $27.3(6.8)$ | $11.9(2.3)$ | - |  |
| $472(26)^{*}$ | $20.4(5.2)$ | $26.2(7.2)$ | $6.6(2.3)$ | - |  |
| $595(25)^{*}$ | $33.6(7.5)$ | $49.86(5.7)$ | $26.4(5.2)$ | 1200 |  |
| $665(37)$ | $21.4(8.4)$ |  | $30.4(5.9)$ | $107,789,960$ |  |
| $703(33)^{*}$ | $17.5(6.4)$ | $24.6(9.3)$ | $18.2(3.9)$ | 1200 |  |
| $789,773(36)^{*}$ | $100.0(12.3)$ | $100.0(16.8)$ | $100.0(7.3)$ | $665,878,960$ |  |
| $878(39)$ | $60.9(10.8)$ | - | $20.0(3.8)$ | 789,107 |  |
| $960(45)$ | $37.4(7.5)$ | - | - | $789,665,107$ |  |
| $1200(70)$ | - | $27.9(10.3)$ | - | 596,703 |  |
| $1452(75)$ | $34.8(7.6)$ | - | - |  |  |
| $1589(77)^{*}$ |  |  | $16.6(3.8)$ |  |  |
| $1700(80)$ | - | $76.7(14.5)$ | - |  |  |
| $2241(105)^{*}$ | - | $61.9(12.1)$ | - | $7.8) 2.9)$ |  |
| $2500(124)$ | $41.59(7.19)$ | - | 250 |  |  |

* Transitions observed in previous studies.

796 keV transition reported in [74] merge into the 803 keV peak in the present work due to the low energy resolution of the NaI spectrum.

As explained in Chapter 4, each fit is controlled with a $\chi^{2}$ test, and each transition obeys the $2 \sigma$ error rule. To optimize the peak-to- background ratio, we include the low-multiplicity events involving the low-threshold detectors in the forward direction for the high statistics cases. Two dimensional $E_{\gamma}-E_{\gamma}$ correlation matrices were used to study coincidence relations. The low-energy 107 keV transition which was observed previously in several studies, is at the limit of the DALI2 threshold for some of the detectors, and it is masked by the abundance of low-energy bremsstrahlung photons in the range up to around 200 keV . Therefore this transition is not fitted in the 1-dimensional spectra shown in Fig 5.6. However, in some cases, the 107 keV transition is clearly observed, such as in the coincidence analysis, therefore it is used for building the level scheme.

The 365 and 472 keV transitions decay from the same energy level at 472 keV $[73,74]$. The previously known intensity ratio between these transitions should be the same for all reactions within the uncertainties. The previously measured $I_{472} / I_{364}$ ratio was $1.57(87)$ in Ref.[73] and 1.80(57) in Ref. [74]. And this ratio is in an agreement with our results shown in table 5.2 which is used as a verification


Figure 5.6: Doppler corrected $\gamma$-ray energy spectra measured by DALI2 obtained from three different nucleon knockout reactions as indicated in the top left corner of each panel. A $\gamma$-ray multiplicity cutoff of $\mathrm{M} \leqslant 4$ was chosen to optimize the spectra with respect to the low-energy atomic bremsstrahlung background. The experimental data are shown as open black diamonds with error bars, and the solid dark blue line is a fit to the entire spectrum. The background was fitted with a double Landau function and is given by the blue dashed line. The Monte Carlo simulated response functions for each transition are shown as a red line. The inset figures are highlight the high energy region with a logarithmic intensity scale, the figure is taken from Paper III.
step for this study.
The 250 keV transition is one of the particularly important transitions in this analysis since it is predicted to decay from the $1 / 2^{+}$level to the ground state. Until now, it has only been populated in $\beta$-delayed neutron decay while it was not observed in the higher statistics direct $\beta$-decay studies. In this study, the
relative transition intensity of the 250 keV line was determined to be different in each reaction. The neutron knockout reaction populates the 250 keV transition with approximately twice as high relative intensity as compared with the proton knockout reaction. In the coincidence result obtained by putting a gate on the 250 keV transition, we observe that it is in coincidence with the 2500 keV transition which is only observed in the neutron knockout reactions (p,pn) and (p,2pn). The spectra in coincidence with the 250 and 2500 keV transitions are shown in Fig.5.7(a, b). Similarly to the 2500 keV transition the 1452 keV transition is also populated only in the ( $\mathrm{p}, \mathrm{pn}$ ) and ( $\mathrm{p}, 2 \mathrm{pn}$ ) reactions. The coincidence results for the 1452 keV transition show one peak around 789 keV . However, for this coincidence analysis, it was not possible to distinguish the 789 and 773 keV transitions. One of the possible placements of the 1452 keV transition is to decay from the previously known 2348 keV state to the known 896 keV state, in agreement with the energy balance. On the other hand, if the current reaction populates the 2348 keV state, then the 2241 keV transition should also be visible in the same $\gamma$-ray spectrum, so the 1452 keV transtion is not placed in the level scheme, although it is confirmed that it belongs to ${ }^{85} \mathrm{Ge}$.

Both the 960 and 878 keV transitions are visible in the neutron knockout reaction and in coincidence with the 789 and 107 keV transitions, see Fig 5.7(c,d). We place them as directly feeding the 896 keV state in the level scheme. The 960 keV transition is also in coincidence with the 665 keV transition. The reverse coincidence analysis obtained by putting a gate on the 665 keV transiton, confirms the mutual coincidence between the 960 and 665 keV and the 665 and 789 keV transitions. Therefore, the 665 keV transition is tentatively placed as a feeding the newly observed 1856 keV state. The ordering of the observed transitions is done according to the relative intensities in the neutron knockout reaction. Based on these observations in neutron knockout reaction, the level scheme is extended. New transitions and levels are shown in red color in Fig 5.8.

The single proton knockout reaction ${ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p}){ }^{85} \mathrm{Ge}$ gave rise to two additional peaks at 1200 and 1700 keV in the single spectra in Fig. 5.6. For the coincidence analysis of the 1200 keV transition, we used a gate between 1150 and 1250 keV and selected a suitable background. The corresponding coincidence spectrum includes two peaks at 596 and 703 keV . This gives a clear hint for the placement of the 1200 keV transition, suggesting that it is connecting the newly observed 1903 keV excited state with the 703 keV excited state. The 703 keV transition has previously been found to decay directly to the ground state [74], while it was not observed in a later study [75] with lower statistics. The 1700 keV peak is clearly visible in this work in the Doppler corrected $\gamma$-ray energy spectrum from the ${ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p})^{85} \mathrm{Ge}$ reaction shown in Fig 5.6. The coincidence analysis does not provide a firm placement for this transition, which is in coincidence with the 789 or 773 keV transition. As a result, it is not placed in the extended level scheme. The additional transitions from the proton knockout reaction are shown in blue color in Fig.5.8.


Figure 5.7: Gated spectrum for the ${ }^{86} \mathrm{Ge}(\mathrm{p}, \mathrm{pn})^{85} \mathrm{Ge}$ reaction, The spectra have been produced with a forward-angle detector condition to improve peak-to- background ratio.(a) Coincidence cut in the $2200-2800 \mathrm{keV}$ range for the 2500 (124) keV transition with background subtraction gated in the energy range (2900-3700). A multiplicity cut $\mathrm{M} \leq 4$ has been applied to decrease low energy background. (b) Coincidence cut in the 220-280 keV range for $250(11) \mathrm{keV}$ transition with background subtraction (1000-1100). (c) Coincidence cut in the 910-990 keV range for 960(45) keV transition with background subtraction (1030-1170 \& 870-900). (d) Coincidence cut in the $860-920 \mathrm{keV}$ range for $878(39) \mathrm{keV}$ transition with background subtraction (1070-1110). This figure is adapted from Paper III.

### 5.2.2 Studies of ${ }^{87} \mathrm{Ge}$

The ground state of ${ }^{87} \mathrm{Ge}$ is known from earlier work although no gamma-ray transitions and excited states were reported [93]. In this study, ${ }^{87} \mathrm{Ge}$ is populated by two different reactions: ${ }^{89} \mathrm{As}(\mathrm{p}, 2 \mathrm{pn})$ (with a cross section $\sigma=10.34(0.87)$ ) and ${ }^{90} \mathrm{Se}(\mathrm{p}, 3 \mathrm{pn})(\sigma=1.47(0.12))$ with enough statistics for limited $\gamma$-ray spectroscopy. Furthermore, ${ }^{87} \mathrm{Ge}$ is also populated by a single-neutron knockout reaction with low statistics. The Doppler-corrected $\gamma$-ray spectra for each reaction are shown in Fig.5.9. Three peaks at energies $250(12), 510(37)$, and $630(31)$ are observed in the singles gamma-ray spectra. The $630(31) \mathrm{keV}$ transition is only visible in the ${ }^{90} \mathrm{Se}(\mathrm{p}, 3 \mathrm{pn})$ reaction. However it cannot be verified because of the large uncertainty in the intensity and could not be firmly assigned to the ${ }^{87} \mathrm{Ge}$ isotope. The 250 and 510 keV transitions are clearly visible in both the ${ }^{89} \mathrm{As}(\mathrm{p}, 2 \mathrm{pn})$ and the ${ }^{88} \mathrm{Ge}(\mathrm{p}, \mathrm{pn})$ reactions. For all reactions, the most intensely observed $\gamma$-ray transition is the 250 keV line, which is assigned to decay from the first excited state to the ground state. The coincidence analysis, shows that these two transitions are not in coincidence with each other, even with the limited statistics. As a result, the 510 keV transition is also expected to feed into the ground state. The energies and relative intensities
$\mathrm{Sn}-3046 \mathrm{keV}$


Figure 5.8: Measured transition energies and proposed level scheme for ${ }^{85} \mathrm{Ge}$. Symbols ${ }^{*}$, denote transition energies taken from the literature[75]. Coloured arrows and horizontal coloured lines indicate measured transitions and levels deduced for the first time in the present work. Red color represents the results of ${ }^{86} \mathrm{Ge}(\mathrm{p}, \mathrm{pn}){ }^{85} \mathrm{Ge}$ reaction, while the blue color is for ${ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p}){ }^{85} \mathrm{Ge}$ results. The dashed arrow indicates tentative placement. The calculated single neutron separation energy from Ref. [92] is also indicated in the figure.
of the observed $\gamma$-ray transitions in ${ }^{87} \mathrm{Ge}$ are shown in the table inside Fig.5.9 for the two reactions with highest statistics.

### 5.2.3 Theoretical calculations

For the ${ }^{83,85,87} \mathrm{Ge}$ isotopes, large-scale shell model calculations have been performed with parameters explained in Sec. 2.1.1. The detailed LSSM calculations were


Figure 5.9: DALI2 Doppler corrected $\gamma$-ray spectra for ${ }^{87} \mathrm{Ge}$ measured for the ${ }^{89} \mathrm{As}(\mathrm{p}, 2 \mathrm{pn})$ and ${ }^{90} \mathrm{Se}(\mathrm{p}, 3 \mathrm{pn})$ and ${ }^{88} \mathrm{Ge}(\mathrm{p}, \mathrm{pn})$ reactions. For all reactions, the forward-angle detectors relative to the beam were used in order to reduce the background in the low-energy region. Blue diamonds with error bars mark the experimental data, the continuous black line is the fit of the whole spectrum, and the corresponding background (blue dashed line) is fitted with a double landau function. The simulated response function for each transition is shown as a red dotted line. In the right bottom corner, the table summarises the intensities of the transitions for the different reactions, with the suggested decay scheme.
carried out in the proton $\pi\left(1 p_{1 / 2}, p_{3 / 2}, f_{5 / 2}, g_{9 / 2}\right)$ (denoted as fpg) and neutron $\nu\left(g_{7 / 2}, d_{3 / 2,5 / 2}, s_{1 / 2}, h_{11 / 2}\right)$ (denoted as gdsh) model space with respect to the ${ }_{28}^{78} \mathrm{Ni}_{50}$ core. In the LSSM calculation one of the uncertanities comes from the relative positions of the $d_{5 / 2}$ and $g_{7 / 2}$ orbitals. These orbitals are proven to be degenerate in nuclei around ${ }^{100} \mathrm{Sn}$. Therefore in order to investigate the effect of ordering in-between this two orbitals, we have tested the results with the energy of the $\nu$ $7 / 2^{+}$s.p. level raised by 1.5 MeV . In figures $5.10,5.11$ and 5.12 SM2 denotes this modified shell model calculation, while SM1 is for the original single- particle levels in the $\pi \mathrm{fpg}$ - $\nu \mathrm{gdsh}$ model space.

The first calculation is performed for ${ }^{83} \mathrm{Ge}$ in order to compare the previously known experimental level scheme with the theoretical calculations. For the unmodified single-particle energy set, the ground state of ${ }^{83} \mathrm{Ge}$ is calculated to be $7 / 2^{+}$ (with dominant $g_{7 / 2}$ parentage) whereas the lowest $5 / 2^{+}$state (with dominant $d_{5 / 2}$ parentage) is calculated to be nearly 400 keV higher. As expected, the raising of the $7 / 2^{+}$s.p.level by 1.5 MeV results in, the $7 / 2^{+}$state being nearly 1 MeV above the $5 / 2^{+}$ground state in the second calculation see Fig. 5.10. The experimental results are closer to the modified shell model calculations. If we consider the ${ }^{85,87} \mathrm{Ge}$ wave functions, the influence of such a raise will be more interesting. According to the calculations, we note that low-lying states are mostly dominated by the coupling of protons in the $p_{3 / 2}, f_{5 / 2}$ subshells, and neutrons in the $g_{7 / 2}, d_{5 / 2}$ subshells. With the odd valence neutrons as one of three or five neutrons in the mixed $g_{7 / 2}, d_{5 / 2}$


Figure 5.10: Comparison between observed excitation energies and large-scale shell model calculations for ${ }^{83} \mathrm{Ge}$. The experimental level energies and spin-parity assignments were taken from Ref. [94]
configuration, there will be a competition between the seniority-one (with only the odd un-paired particle) states and seniority-three (one broken pair) states for spinparity values $5 / 2^{+}$and $7 / 2^{+}$. In the case of ${ }^{85} \mathrm{Ge}$, the ground state is calculated to become $7 / 2^{+}$instead of $5 / 2^{+}$after raising the $g_{7 / 2}$ orbital energy. This favors a seniority-one configuration for that state, see Fig. 5.11.

The calculations for ${ }^{87} \mathrm{Ge}$ show slightly different ordering of the states, the spin-parity of the ground state and first excited states are calculated to be $3 / 2^{+}$ and $5 / 2^{+}$, respectively, see Fig. 5.12. Also it seems that the modification of the neutron $g_{7 / 2}$ orbital energy hasmuch less influence compared with the ${ }^{83,85} \mathrm{Ge}$. The ${ }^{87} \mathrm{Ge}$ ground state is predicted to be remarkably dominated by the coupling of seniority-three neutron states and proton particle-hole excitations. To see the collectivity in the complete picture of germanium isotopes, the LSSM calculation is


Figure 5.11: Comparison between observed excitation energies and large-scale shell model calculations for ${ }^{85} \mathrm{Ge}$. The tentative spin-parity assignments were taken from Ref. [74].
also performed for even-even germanium isotopes. The $2^{+}$state energy results are calculated slightly higher than the known experimental values. This indicates an increased collectivity and therefore a larger model space is needed to describe this region correctly. This is currently beyond the reach of the LSSM calculations due to the associated computational limits.

The previous studies on even-even isotopes showed a new triaxial deformed region in the neutron rich part of the Ge isotopic chain [95]. To investigate the shape evolution more in the Ge chain especially we have performed TRS calculations. For the ${ }^{84-88}$ Ge chain the TRS shows a clear trend of increasing quadrupole deformation in the predicted near-triaxial shapes as a function of increasing neutron number starting already at $N=52$, which is in agrement with previous calculations.

The ${ }^{85,87} \mathrm{Ge}$ isotopes show a similar trend as the even-even isotopes with increasing neutron number. One of the differences in the odd-even isotopes is that, they have much more pronounced softness in the triaxial $\gamma$-degree of freedom for the lowest negative parity configuration, which can be seen in the PES belonging


Figure 5.12: Comparison between observed excitation energies and large-scale shellmodel calculations for ${ }^{87} \mathrm{Ge}$. Experimental level energies are taken from the present work.
to ${ }^{85,87} \mathrm{Ge}$ isotopes in Fig.5.13. Since the effect is only visible in the negative parity configurations, it is likely that the shape-driving properties of the $1 / 2[550]$ Nilsson intruder configuration near the Fermi surface is causing this softness. The lowest negative parity $1 / 2[550]$ Nilsson orbit is an intruder orbit emanating from the $h_{11 / 2}$ subshell with a strong- shape driving configuration. This trend is also effective in the LSSM calculation, the negative-parity states of $h_{11 / 2}$ parentage move down rapidly with increasing neutron number from $\mathrm{N}=51-55$. However, the experimental data do not shed any light on whether this orbital is populated near the Fermi level in the neutron-rich germanium isotopes up to mass $A=88$ or not.

Another single-particle orbital, which is of major interest in the shell evolution studies in neutron-rich nuclei is the $3 s_{1 / 2}$ orbital. Above $N=50$, the neutron $3 s_{1 / 2}$ single-particle state is observed to come down in energy as a function of increasing $N / Z$ ratio relative to the $2 d_{5 / 2}$ level for proton numbers $Z \leq 40$ [96]. The momentum distributions observed in single-nucleon knockout reactions leading to ${ }^{85} \mathrm{Ge}$ and ${ }^{87} \mathrm{Ge}$ may shed light on the ordering of the neutron levels. The effect of


Figure 5.13: Total Routhian surfaces in the $\beta_{2} \sin \left(\gamma+30^{\circ}\right)-\beta_{2} \cos \left(\gamma+30^{\circ}\right)$ plane for the lowest signature and parity $(\pi, \alpha)$ configurations in ${ }^{85} \mathrm{Ge}$ and ${ }^{87} \mathrm{Ge}$. The red dot indicates the position of the lowest-lying minimum for each potential energy surface. Equipotential energy lines are separated by 0.2 MeV . (a)and (c) blocking of the lowest ( $\pi=-, \alpha=-1 / 2$ ) configuration. (b)and (d) blocking of the lowest ( $\pi=+, \alpha=+1 / 2$ ) configuration.
the $3 s_{1 / 2}$ single-particle state is visible in the neutron knockout reaction. However for ${ }^{87} \mathrm{Ge}$, the statistics of the ( $\mathrm{p}, \mathrm{pn}$ ) reaction is not enough to make a conclusive analysis of momentum distributions. The ${ }^{86} \mathrm{Ge}(\mathrm{p}, \mathrm{pn})^{85} \mathrm{Ge}$ reaction is therefore the only testing ground in the present data. The momentum distributions are calculated by using the measured observables as explained in Sec. 4.3. The exclusive momentum distribution for the 250 keV excited state with $I^{\pi}=1 / 2^{+}$is of special interest.

The momentum distribution was extracted by fitting the $\gamma$-ray spectra in coincidence with the $40 \mathrm{MeV} / \mathrm{c}$ sections of the inclusive momentum distribution instead of directly selecting the momentum distributions from $\gamma$-ray cuts in the singles spectra. In this way we are aiming to decrease the contribution from the higher energetic transitions on the distribution. By comparing the experimental distribution and calculated DWIA predictions, the orbit from which the valence neutron is removed from in ${ }^{86} \mathrm{Ge}$ can be predicted. In Fig 5.14 the parallel momentum distributions of the 250 keV state predicted by the DWIA calculations assuming the valence neutron is removed from the $3 \mathrm{~s}_{1 / 2}, \mathrm{~d}_{5 / 2}$ and $\mathrm{d}_{3 / 2}$ states, i.e. carrying $l=0$ or $l=2$ units of angular momentum are shown. While exclusive momentum distributions selected by the 250 keV transition agrees well with the $l=0$ prediction, the orbitals with $l=2$ are less probable to be occupied by the removed valence neutron. Therefore, the spin and parity of the 250 keV state is assigned to be $1 / 2^{+}$. These PMD results also confirm the previously observed reordering in the neutron orbitals for above $\mathrm{N}=50$ region [96].


Figure 5.14: Individual parallel momentum distributions of the ${ }^{85} \mathrm{Ge}$ residues from one-neutron knockout reaction. The exclusive momentum distribution for 250 keV level is shown in the graph, and compared with calculated DWIA distributions assuming 1 n removal from d (blue line from $\mathrm{d}_{5 / 2}$, pink dashed line from $\mathrm{d}_{3 / 2}$ ) and s (dashed red line from $\mathrm{s}_{1 / 2}$ ) orbitals.

As a result, there is evidence for a rapid onset of deformation in Ge isotopes beyond $N=50$, both in theoretical and experimental studies. Our TRS Strutinskytype mic-mac calculations of PES, as well as LSSM calculations, confirm this picture. The TRS calculations also predict that with increasing the neutron number, quadropole deformation is also increasing. Also with an analysis on the momentum distribution, it is clear that $3 \mathrm{~s}_{1 / 2}$ neutron orbit is also influencing in the positiveparity configuration in addition to the $g_{7 / 2}, d_{5 / 2}$ orbitals.

### 5.3 Results for the self-conjugate nucleus ${ }^{88} \mathrm{Ru}$

The $\mathrm{N}=\mathrm{Z}$ nucleus ${ }^{88} \mathrm{Ru}$ was investigated earlier by using the reaction ${ }^{58} \mathrm{Ni}\left({ }^{32} \mathrm{~S}, 2 \mathrm{n}\right)^{88} \mathrm{Ru}$. Four $\gamma$-rays at energies $616.2,799.8,964.3$, and 1100.5 keV were found and assigned to the ground state band up to a tentative spin of $8+[97]$. In the present work, the ${ }^{88} \mathrm{Ru}$ isotope was populated via the ${ }^{54} \mathrm{Fe}\left({ }^{36} \mathrm{Ar}, 2 \mathrm{n}\right)^{88} \mathrm{Ru}$ reaction. Its reaction cross section was extremely low compared to the other observed reaction channels. To identify the 2 n - evaporation channel from the compound nucleus $\left({ }^{90} \mathrm{Ru}\right)$ NEDA+NWall detectors, which cover $1.6 \pi$ solid angle, were coupled with the AGATA array (having around $1 \pi$ solid angle coverage). The experiment was run with the hard one neutron- gamma - gamma coincidence trigger condition to suppress the high cross-section channels. One difficulty in the experiment was the target contamination namely oxygen, which reacts with the beam resulting in the production of several different isotopes. The 2 n condition was used for the neutron detectors with special cuts to reject scattering neutrons in order to reduce the background and contaminations from other channels. The DIAMANT detector was used for the background subtraction and as charged-particle veto detector.

To identify the transitions belonging to ${ }^{88} \mathrm{Ru}$, a $2 \mathrm{D} \gamma-\gamma$ coincidence matrix was constructed with the 2 n and no charge particle condition. The known energies are used for the gates to observe transitions above the $8+$. The Compton background subtraction was applied for each gated histogram, although it was not enough to prevent leakage from the strong proton, alpha channels. To increase the statistics, the gated spectra were summed up to boost the new transitions. The resulting gated histogram is shown with red color in Fig.5.15. A background spectrum, which was shown in Fig.5.15 as a black solid line, was produced by using the same $\gamma$-ray energy gates requiring a coincidence with two neutrons and charged particles. With this double background subtraction, the three new transitions at 1063, 1153, and 1253 keV were found. The final background subtracted spectrum is shown in Fig.5.16, with the transitions belonging to ${ }^{88} \mathrm{Ru}$ marked with their energies in keV .

The new transitions were assigned to the cascade of the ground state band and ordering is determined with respect to their intensities. The 1063 keV transition is assigned as decay $10^{+} \rightarrow 8^{+}, 1153 \mathrm{keV} 12^{+} \rightarrow 10^{+}$, and the 1253 keV transition a decay from the tentative $\left(14^{+}\right)$state to the $12^{+}$state.

In order to understand the isospin properties of the deformed rotational ${ }^{88} \mathrm{Ru}$ nucleus, the kinematical moment of inertia $J$ is calculated from the angular mo-
mentum $I$ over the rotational frequency $w(w=d E / d I)$. Fig. 5.17 shows the kinematical moment of inertia as a function of rotational frequency for ${ }^{88} \mathrm{Ru}$ and the neighboring isotones ${ }^{86} \mathrm{Mo}$ and ${ }^{84} \mathrm{Zr}$. The ground -state bands in the $\mathrm{N}>\mathrm{Z}$ isotones show the characteristics of normal paired band crossings in rotating deformed nuclei of the isovector $\mathrm{T}=1$ type.

The band crossing frequency for the neighboring isotones is around 0.47 MeV which is predicted by standart mean field calculations to be normal for the regular $T=1$ isovector band crossing frequency. On the other hand, for the $N=Z{ }^{88} \mathrm{Ru}$ nucleus the band crossing frequency is around 0.54 MeV which is significantly higher than the regular $\mathrm{T}=1$ isovector value mentioned before.

Theoretical cranked shell model calculations predicted two-quasiparticle $\pi g_{9 / 2}$ alignment to occur at 0.45 MeV rotational frequency closely followed by the neutron alignment of $\nu g_{9 / 2}$ orbit [101, 102], which is in good agreement for neighboring isotone band crossing frequencies. However, the delay in band crossing frequency in ${ }^{88} \mathrm{Ru}$ cannot be explained in this standard mean-field model. Large Scale Shell Model calculations with isospin conserving Hamiltonians can explain the delay in frequency for $\mathrm{N}=\mathrm{Z}$ nuclei as resulting from the isoscalar pairing interaction effect on the rotational behaviours. According to the calculation in Ref. [103], significant delay in-band frequency for $\mathrm{N}=\mathrm{Z}$ and particularly for ${ }^{88} \mathrm{Ru}$ sharp irregularity at a rotational frequency $\hbar \omega_{c} \approx 0.65 \mathrm{MeV}$ is predicted as a result of the isoscalar pair correlations.


Figure 5.15: Gamma-ray energy spectrum detected in coincidence with previously known transitions ( $616,800,964$ and 1100 keV ). The additional condition on the red histogram is 2 n and no charge particle in coincidence, while the black solid histogram is the background spectrum created with the same $\gamma$-ray energy gates with 2 n and at least one charge particle in coincidence.


Figure 5.16: Gamma-ray energy spectrum detected in coincidence with the 616 , 800,964 , and $1100 \mathrm{keV} \gamma$ rays, with the additional requirement that two neutrons and no charged particles were detected in coincidence are a two neutron and charge particle background subtraction. The figure is taken from [98]

### 5.4 Preliminary results from SEASTAR-3 campaign for titanium isotopes $\mathrm{N}=34$-39

For nuclei far from stability, with a significant imbalance of neutron and proton numbers, the emergence of new magic numbers is a topic of large interest. Two new magic numbers at $\mathrm{N}=32$ and $\mathrm{N}=34$ were recently proven in calcium isotopes in the vicinity of $\mathrm{Z}=20$ [104].

The sub-shell closure at $\mathrm{N}=34$ was first observed in the ${ }^{54} \mathrm{Ca}$ isotope [104], and it is explained with the tensor force concept proposed by Otsuka et al. [105]. The tensor force acts between valence neutrons and protons depending on the spinorbit coupling in the single particle level i.e. $j_{<}=l-1 / 2$ or $j_{>}=l+1 / 2$. The interaction between the $\pi j_{<}$and $\nu j_{>}$is attractive, whereas if the valence neutrons and protons occupy the same type of orbit like both $j_{<}$or both $j_{>}$, the tensor force between these nucleons is repulsive [105]. In the case of a sub-shell closure at $\mathrm{N}=34$, the tensor force affects single-particle energies causing a gap in the level structure. The standard level structure and configuration for protons $(\pi)$ and neutrons $(\nu)$ is shown in the first two columns of Fig. 5.18 for the ${ }^{56} \mathrm{Ti}$ isotope. To preserve the $\mathrm{N}=34$ gap, the $\mathrm{f}_{5 / 2}$ neutron orbital should stay above the $\mathrm{p}_{1 / 2}$ state with increasing the neutron number, like the top row of the Fig. 5.18. However, the residual tensor force, produces an increasingly attractive interaction between the proton $\mathrm{f}_{7 / 2}\left(j_{>}=3+1 / 2\right)$ and neutron $\mathrm{f}_{5 / 2}\left(j_{<}=3-1 / 2\right)$ levels when adding more neutrons, as shown in the second row of Fig. 5.18. Therefore, with 22 protons the


Figure 5.17: Experimental values for the kinematical moment of inertia (J) for the low-lying yrast bands of the $\mathrm{N}=44$ isotones, the results for ${ }^{86} \mathrm{Mo}$ [99], and ${ }^{84} \mathrm{Zr}$ [100] are taken from the literute, while ${ }^{88} \mathrm{Ru}$ data is from this work. The black dashed vertical line indicates the approximate rotational frequency of the first isovector-paired band crossing due to $g_{9 / 2}$ protons as predicted by standard cranked shell model calculations $[101,102]$. The red- dotted vertical line indicates the band crossing frequency for the ground-state bandin ${ }^{88} \mathrm{Ru}$. The level scheme to the right of the graph is a result of current analysis. The figure is modified from Ref. [98]
titanium chain is a good testing ground for the effects of the tensor force. Also, one can find answers to the questions; how strong is the tensor force? For which isotope does the $f_{5 / 2}$ orbit return to its "normal" place like for near stable isotopes. The firm spin-parity assignments of excited states in the titanium isotopes may shed light on such questions.

Experimental data for the titanium chain from $\mathrm{N}=34$ - 39 were collected during the SEASTAR 3 campaign. Each isotope is populated via different knockout reactions. The BigRIPS PID for the incoming beam selection and the SAMURAI PID for outgoing ions after the $\mathrm{LH}_{2}$ target are shown in Fig. 5.19 and 5.20, respectively. The reaction channels with observed $\gamma$-ray transitions are listed in table 5.3. Transitions observed for the first time in the current work are marked with a "*".

The preliminary results of the experiment for the titanium isotopes, ${ }^{56-61} \mathrm{Ti}$ are


Figure 5.18: Schematic illustration highlighting the attractive interaction between the proton $\pi f 7 / 2$ and neutron $\nu \mathrm{f} 5 / 2$ single-particle orbitals for $\mathrm{Z}=22$ isotopes. Topordinary ordering of the $\mathrm{f} 5 / 2$ and $\mathrm{p} 1 / 2$ orbitals which creates the $\mathrm{N}=34$ shell gap, bottom- tensor force effective ordering of the neutron shell orbits
shown below for each different isotope in Fig.5.21, 5.22, 5.23, 5.24, 5.25, and 5.26. The known transitions are confirmed in the preliminary results for the even-even isotopes; there is also a possibility to find new transitions on even-even isotopes with more precise analysis.

For the odd-even Ti isotopes, the analysis is more complex. The theoretical calculations and previous studies reveal systematic long-lived excited states. With 22 protons, the protons mostly occupy the $2 \mathrm{p}_{1} / 2$ and $1 \mathrm{f}_{7 / 2}$ orbitals, while with $\mathrm{N}=35-39$, the valence neutron is in the $2 \mathrm{~d}_{5 / 2}$ or $1 \mathrm{~g}_{9 / 2}$ orbitals. Due to the large angular momentum differences between neutron and proton orbitals around the Fermi surface in $\mathrm{N} \sim 40$ nuclei, long-lived isomeric states become possible. Such isomeric states were observed in the Ni and Fe isotopic chains [106-108].

In our experimental setup, the transitions that are detected by the DALI2+ setup cannot be due to a decay from states with an effective (taking into account feeding) lifetime of a few ns or larger, then they would have decayed outside the field of the DALI2 + detection system and not be observed. Therefore, the isomeric decays in the odd- even ${ }^{59} \mathrm{Ti}$ nucleus with energy and half-life 113 keV ( $600 \mathrm{ns)}$ ) and 699 keV ( 70 ns ) [113-115] and in the ${ }^{61} \mathrm{Ti}$ nucleus with 125 keV (200ns) and 575 keV (314 ns) [113] could not be observed in the current analysis.

Several shell model calculations with different effective interactions were performed in this region. The results for the hybrid effective potential LNPS are


Figure 5.19: Preliminary BigRIPS particle identification plot for SEASTAR-3 experiment. The titanium isotopes are marked with dashed red circles.
shown in 5.27, the detailed information about the calculations can be found in Ref. [116]. The shell model calculations based on the LNPS effective interactions shown in Fig. 5.27 also predict a possible positive parity spin-trap isomeric state with spin-parity $9 / 2^{+}$, since the $9 / 2^{+}$state is calculated lower than the $5 / 2^{+}$state in the ${ }^{59} \mathrm{Ti}$ and ${ }^{61} \mathrm{Ti}$ isotopes. For the LSNP shell model calculations the full $p f$ shell


Figure 5.20: The preliminary SAMURAI PID plot from SEASTAR-3. The titanium chain is marked with dashed red circles.


Figure 5.21: The Doppler corrected $\gamma$-ray spectrum from the ${ }^{58} \mathrm{Ti}(\mathrm{p}, \mathrm{p} 2 \mathrm{n})^{56} \mathrm{Ti}$ reaction. The observed transition energies with the initial and final spin values [109,110] are marked in the figure.


Figure 5.22: The Doppler corrected $\gamma$-ray spectrum from the ${ }^{58} \mathrm{Ti}(\mathrm{p}, \mathrm{pn}){ }^{57} \mathrm{Ti}$ and ${ }^{59} \mathrm{Ti}(\mathrm{p}, \mathrm{p} 2 \mathrm{n})^{57} \mathrm{Ti}$ reactions. The first-time observed transitions are marked, the previously known transitions 364 keV and 980 keV are also observed in Ref. [111].


Figure 5.23: The Doppler corrected $\gamma$-ray spectrum from the ${ }^{59} \mathrm{Ti}(\mathrm{p}, \mathrm{pn})^{58} \mathrm{Ti}$ reaction. The populated transition energies with the initial and final spin values [112] are marked in the figure.


Figure 5.24: Doppler corrected $\gamma$-ray spectrum from the ${ }^{60} \mathrm{Ti}(\mathrm{p}, \mathrm{pn}){ }^{59} \mathrm{Ti}$ reaction. The transitions are observed for the first time in this work. The previously known isomeric decay at 109 keV [113] is not observed.


Figure 5.25: The Doppler corrected $\gamma$-ray spectrum from the ${ }^{61} \mathrm{Ti}(\mathrm{p}, \mathrm{pn}){ }^{60} \mathrm{Ti}$ reaction. The populated transition energies with the initial and final spin values are marked in the figure. The spin assignment are taken from the literature [112].


Figure 5.26: Doppler corrected $\gamma$-ray spectrum from the ${ }^{62} \mathrm{~V}(\mathrm{p}, 2 \mathrm{p})^{61} \mathrm{Ti}$ reaction.One new transition at 340 keV is visible. The previously known isomertic 125 and 575 keV [113] decays are not observed.

### 5.4. PRELIMINARY RESULTS FROM SEASTAR-3 CAMPAIGN FOR

 TITANIUM ISOTOPES $N=34-39$Table 5.3: Summary of the reactions used in the current analysis and the observed $\gamma$-rays. First-time observed transitions from the current analysis are marked with "*". For the previously known transitions, the corresponding references are listed in the last colum. Some of the known transitions are not listed in this table, since they are not observed in this work.

| Ti nuclei | Reaction | $\gamma$-ray | Ref. |
| :---: | :---: | :---: | :---: |
| ${ }^{56} \mathrm{Ti}$ | $\begin{aligned} & { }^{57} \mathrm{Ti}(\mathrm{p}, \mathrm{pn}) \\ & { }^{58} \mathrm{Ti}(\mathrm{p}, \mathrm{p} 2 \mathrm{n}) \end{aligned}$ | 1129, 1161, 690, | $\begin{gathered} {[109]} \\ ,[110] \end{gathered}$ |
| ${ }^{57} \mathrm{Ti}$ | ${ }^{58}$ Ti $(\mathrm{p}, \mathrm{pn})$ | $\begin{aligned} & 320^{*}, \quad 364, \quad 590^{*}, \quad 740^{*}, \quad 980 \\ & 1100^{*}, 1450^{*} \end{aligned}$ | [111] |
| ${ }^{58} \mathrm{Ti}$ | $\begin{aligned} & { }^{59} T i(\mathrm{p}, \mathrm{pn}) \\ & { }^{59} V(\mathrm{p}, 2 \mathrm{p}) \end{aligned}$ | 1047, 991, 619, 1835 | [112] |
| ${ }^{59} \mathrm{Ti}$ | $\begin{aligned} & { }^{60} \mathrm{Ti}(\mathrm{p}, \mathrm{pn}) \\ & { }^{60} \mathrm{~V}(\mathrm{p}, 2 \mathrm{p}) \\ & { }^{61} \mathrm{~V}(\mathrm{p}, 2 \mathrm{pn}) \\ & { }^{61} \mathrm{Ti}(\mathrm{p}, \mathrm{p} 2 \mathrm{n}) \end{aligned}$ | 860*, 1350*, 1800*, 450*, 330* |  |
| ${ }^{60} \mathrm{Ti}$ | $\left.{ }^{{ }^{61} T i} \mathrm{pi}, \mathrm{pn}\right)$ | 850, 866 | [112] |
| ${ }^{61} \mathrm{Ti}$ | ${ }^{62} V(\mathrm{p}, 2 \mathrm{p})$ | 340* |  |



Figure 5.27: Level schemes of ${ }^{57-61} \mathrm{Ti}$ in comparison with results from large-scale shell model calculations using LNPS effective interaction. The experimentally observed isomeric states are indicated with thicker lines. LNPS results for odd-mass isotopes are taken from Ref. [113], and for even isotopes from Ref.[112]. The experimental values are taken from Refs [111-115]
for protons and the $1 f_{5 / 2}, 2 p_{3 / 2}, 2 p_{1 / 2}, 1 g_{9 / 2}$, and $2 d_{5 / 2}$ neutron orbitals with ${ }^{48} \mathrm{Ca}$ core was used. In this level of the analysis only preliminary placements can be done for some isotopes. In the ${ }^{61} \mathrm{Ti}$ isotope, the 340 keV transition is the only transition that we can observe from the proton knockout reaction. Assuming it depopulates the first excited state to the $1 / 2^{-}$ground state predicted by the LNPS calculation we can make Weisskopf estimates of the half-life of the corresponding 340 keV level, for different multipolarities. Based on such Weisskopf estimates, if the transition is of E2 type the half life should be around 13 ns , which is out of our detection range. While if the transition is of M1 type than the 340 decay is quite fast with of the order of ps half-life. Therefore we can assign the 340 keV transition to be of M1 type from the $3 / 2^{-}$state to the ground state guided by the LNPS calculation (Fig. 5.27). The observed $3 / 2^{-}$state energy is also comparable with the theoretical prediction as shown in Fig. 5.27. For the lighter odd-even Ti isotopes, the analysis will be more complicated, since we observe more than one transition in each reaction. With a more comprehensive coincidence and intensity analysis for the different reactions we expect to assign level schemes for the Ti isotopes.

At this level of the analysis, it is not possible to conclude the placement of the all studied transitions. The differences in proton and neutron knockout reactions could bring extra informationas as well as momentum distributions. Since the analysis is preliminary, with the more detailed analysis on the knockout mechanism, one can get more detailed results on the odd-even ${ }^{57,59,61} \mathrm{Ti}$ isotopes in the continuing analysis.

## Chapter 6

## Summary of Papers

### 6.1 Paper I

The ${ }^{88} \mathrm{Ru}$ nuclei are populated via a fusion evaporation reaction using a ${ }^{36} \mathrm{Ar}$ beam impinging on a ${ }^{54} \mathrm{Fe}$ target. The compound nucleus ${ }^{90} \mathrm{Ru}$ evaporates two neutrons through the reaction, the neutrons are measured and identified by NEDA/NWall liquid scintillator detectors, while the charge particles evaporated from the compound nucleus are detected by the DIAMANT detector. The corresponding $\gamma$-rays from excited ${ }^{88} \mathrm{Ru}$ isotope were measured with the AGATA Hpge detector array, which was coupled with the previously mentioned detectors. The experiment was performed in GANIL for 13 days of irradiation time. The collected data from the experiment, was optimized and resorted using the agapro and ganpro programs, specially developed for AGATA analysis. The final $\gamma$-ray spectra were created and analyzed using the ROOT data analysis framework[85].

For the self-conjugate ${ }^{88} \mathrm{Ru}$ isotope, three new $\gamma$-ray transitions have been identified at energies 1063,1153 and 1253 keV . The ground state band is extended up to the $14^{+}$spin state at energy 6949 keV . The observed level scheme is consistent with a deformed rotational system. The rotational frequency of the alignment of the valence nucleons has a significantly higher value than for neighboring $\mathrm{N}>\mathrm{Z}$ isotopes and what is predicted by theoretical calculations performed without isoscalar neutron-proton pairing. The delayed rotational alignment is suggested to be a signature for the presence of isoscalar $\mathrm{T}=0$ neutron-proton pairing. By including isoscalar pairing in a theoretical calculation, an agreement is obtained with the experimentally observed delayed rotational alignment.

### 6.2 Paper II

Neutron-rich molybdenum isotope ${ }^{111}$ Mo was populated via knockout reactions ${ }^{112} \mathrm{Mo}(\mathrm{p}, \mathrm{pn})$ and ${ }^{113} \mathrm{Tc}(\mathrm{p}, 2 \mathrm{pn})$. The experiment was performed at RIKEN, using the fast-fission reaction from ${ }^{238} \mathrm{U}$ beam with ${ }^{9} \mathrm{Be}$. The fission products were iden-
tified and measured through the BigRIPS spectrometer. The secondary target was located inside the MINOS TPC, at the center of the DALI2 NaI detector array. Knockout reactions occur inside the $\mathrm{LH}_{2}$ target, and recoiling protons are tracked by the MINOS TPC. The $\gamma$-rays from the excited ${ }^{111}$ Mo isotope (which is in flight) are captured by the 186 NaI detectors covers nearly $4 \pi$ solid angle. The ${ }^{111} \mathrm{Mo}$ ions were identified after the MINOS+DALI2 setup by the ZeroDegree spectrometer.

Seven $\gamma$-ray transitions were observed for the first time in ${ }^{111} \mathrm{Mo}$ at energies $130,176,205,235,290,380$, and 750 keV . A coincidence analysis was performed and the 130 keV and 176 keV transitions were found in coincidence with each other, with several other results. TRS calculations have been performed for the nuclei in the molybdenum isotopic chain $A=108-112$. The calculations predict a shape transition from triaxial to oblate axially-symmetric shape with increasing neutron number in the isotopic chain. For ${ }^{111} \mathrm{Mo}$, shape coexistence was predicted for the low-lying states between a triaxial positive-parity configuration based on mixed $1 g_{7 / 2}$ and $2 d_{5 / 2}$ parentage and an oblate ground-state configuration with $1 h_{11 / 2}$ parentage. PPR calculations of the rotational structures built on these states were carried out. Following the results of the calculations, theoretical level schemes are proposed for positive and negative parity states and compared with the experimental findings.

### 6.3 Paper III

The experiment was a part of SEASTAR2 campaign as for the ${ }^{111}$ Mo analysis. A $\gamma$-spectroscopic study of the neutron-rich germanium isotopes ${ }^{85,87} \mathrm{Ge}$ has been performed. The excited states in ${ }^{85,87} \mathrm{Ge}$ were populated in nucleon knockout reactions following fast-fission reactions of ${ }^{238} \mathrm{U}$ at the Radioactive Isotope Beam Factory (RIBF). Two $\gamma$-ray transitions have been observed for the first time in ${ }^{87} \mathrm{Ge}$, both suggested to be decays to the ground state from 510 keV and 250 keV levels. In ${ }^{85} \mathrm{Ge}, 13 \gamma$-ray transitions have been observed, 7 of them for the first time in this study. Based on the analysis of intensities and systematics in neighboring Ge isotopes, a level scheme with two excited states is proposed for ${ }^{87} \mathrm{Ge}$, and the previously reported level scheme extended for ${ }^{85} \mathrm{Ge}$. Exclusive parallel momentum distributions obtained for the 250 keV state in ${ }^{85} \mathrm{Ge}$ was in agreement with previous assignments of spin-parity $\left(1 / 2^{+}\right)$. This confirms the predictions of a low-lying neutron $3 s_{1 / 2}$ single-particle state in this neutron-rich nucleus. LSSM and TRS calculations have been performed for the nuclei in the germanium isotopic chain between $A=83$ and $A=87$. The LSSM calculations indicate a close competition between the $\nu 7 / 2^{+}$and $\nu 5 / 2^{+}$seniority- 1 configurations for the ground states of ${ }^{83} \mathrm{Ge}$ and ${ }^{85} \mathrm{Ge}$, whereas for ${ }^{87} \mathrm{Ge}$, the ground state is predicted to be $3 / 2^{+}$based on the coupling of different seniority- $3 \nu 7 / 2^{+}, \nu 5 / 2^{+}$configurations, and proton particle-hole excitations. The TRS calculations also predicted consistent triaxial shapes for the lowest-lying configurations with a significant $\gamma$-softness for the negative-parity configurations.

## Division of work between authors

A short summary of the authors contributions to the articles which are the basis of this thesis is found below. The articles contain both theoretical and experimental work and the main contribution of the uthor is on the experimental side.

## Paper I

The author of this thesis took part in the preparation of the experimental set-up, participated in the experiment, performed the online data analysis and optimization of the detector setup, and was the third author of the Paper.

## Paper II

The author of this thesis performed the offline data analysis. She is the principal author of the paper.

## Paper III

The author of this thesis performed the offline data analysis. She is the principal author of the paper.

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# Isospin Properties of Nuclear Pair Correlations from the Level Structure of the Self-Conjugate Nucleus ${ }^{88}$ Ru 

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#### Abstract

The low-lying energy spectrum of the extremely neutron-deficient self-conjugate ( $N=Z$ ) nuclide ${ }_{44}^{88} \mathrm{Ru}_{44}$ has been measured using the combination of the Advanced Gamma Tracking Array (AGATA) spectrometer, the NEDA and Neutron Wall neutron detector arrays, and the DIAMANT charged particle detector array. Excited states in ${ }^{88} \mathrm{Ru}$ were populated via the ${ }^{54} \mathrm{Fe}\left({ }^{36} \mathrm{Ar}, 2 n \gamma\right){ }^{88} \mathrm{Ru}$ * fusion-evaporation reaction at the Grand Accélérateur National d'Ions Lourds (GANIL) accelerator complex. The observed $\gamma$ ray cascade is assigned to ${ }^{88} \mathrm{Ru}$ using clean prompt $\gamma-\gamma$-2-neutron coincidences in anticoincidence with the detection of charged particles, confirming and extending the previously assigned sequence of low-lying excited states. It is consistent with a moderately deformed rotating system exhibiting a band crossing at a rotational frequency that is significantly higher than standard theoretical predictions with isovector pairing, as well as observations in neighboring $N>Z$ nuclides. The direct observation of such a "delayed" rotational alignment in a deformed $N=Z$ nucleus is in agreement with theoretical predictions related to the presence of strong isoscalar neutron-proton pair correlations.


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Introduction.-Nucleonic pair correlations play an important role for the structure of atomic nuclei as well as for their masses. Some of the most well-known manifestations of the pairing effect in nuclei, which has strong similarities with superconductivity and superfluidity in condensed matter physics [Bardeen-Cooper-Schrieffer (BCS) theory [1,2] ], are the odd-even staggering of nuclear masses [3], seniority symmetry [4-6] in the low-lying spectra of spherical even-even nuclei, and the reduced moments of inertia and backbending effect $[7,8]$ in rotating deformed nuclei. Atomic nuclei, which are formed by the unique coexistence of two distinct fermionic systems (neutrons and protons), may also exhibit additional pairing phenomena not found elsewhere in nature. In nuclei with equal neutron and proton numbers $(N=Z)$ enhanced correlations arise between neutrons and protons that occupy orbitals with the same quantum numbers. Such correlations have been predicted to favor a new type of nuclear superfluidity, termed isoscalar neutron-proton ( $n p$ ) pairing [9-12]. In addition to the normal isovector $(T=1)$ pairing mode based on like-particle neutron-neutron (nn) and proton-proton ( $p p$ ) Cooper pairs that have their spin vectors antialigned and occupy time-reversed orbits, neutrons and protons may here also form $n p T=1, I=0$ pairs. Of special interest is the long-standing question of the possible presence of a $n p$ pairing condensate [9-15] predicted to be built primarily from isoscalar $T=0$, $I>0 n p$ pair correlations that still eludes experimental verification. The occurrence of a significant component of $T=0$ correlated $n p$ pairs in the nuclear wave function is also likely to have other interesting implications, e.g., the proposed "isoscalar spin-aligned $n p$ coupling scheme" in the heaviest, spherical, $N=Z$ nuclei [16].

Despite vigorous activity over the last decade or so, the fundamental questions concerning the basic building blocks and fingerprints of $n p$ pairing are still a matter of considerable debate. Even though until now there has been no substantial evidence for the need to include isoscalar, $T=0, n p$ pairing to explain the known properties of
low- or high-spin states in even-even $N=Z$ nuclei the available data for the heavier $N=Z$ nuclei are very limited due to experimental difficulties: No accurate information on masses for $N=Z$ nuclei above $A \approx 80$ is currently known, shape coexistence effects have muddled the analysis of rotational patterns of deformed $N=Z$ nuclei in the mass $A \sim 70$ region, and $n p$ transfer reaction studies on the lighter $N=Z$ nuclei are suffering from the complexity in the interpretation of the experimental results. Furthermore, correlations of this type are enhanced in heavier nuclei where more particles in high- $j$ shells can participate. Many theoretical calculations suggest that the best place to look for evidence of an isoscalar pairing condensate is in nuclei with $A>80$; for a recent review, see Ref. [17]. Calculations using isospin-generalized BCS equations and the Hartree-Fock-Boguliubov (HFB) equation including $p p, n n, n p$ ( $T=1$ ), and $n p(T=0)$ Cooper pairs indicated that there may exist a second-order quantum phase transition in the ground states of $N=Z$ nuclei from $T=1$ pairing below mass 80 to a predominantly $T=0$ pairing phase above mass 90 , with the intermediate mass $80-90$ region showing a coexistence of $T=0$ and $T=1$ pairing modes [18]. There are even predictions for a dominantly $T=0$ groundstate pairing condensate in $N \sim Z$ nuclei around mass 130 [19] (although such exotic nuclei are currently not experimentally accessible).

The interplay between rotation and the like-particle pairing interaction has been studied in great detail in deformed nuclei where, normally, the neutron and proton Fermi levels are situated in different (sub-) shells; and hence the neutrons and protons can be considered to form separate Fermi liquids dominated by $T=1$ pair correlations. However, the isoscalar, $T=0, n p$ coupling has the interesting property of being less affected by the Coriolis interaction in a rotating system, which tends to break the time-reversed pairs with $T=1$. Therefore, the presence of a $n p$ pairing condensate may reveal itself in the rotational states of deformed $N=Z$ nuclei where one might expect that the $T=0$ pairing correlations are active while
the normal isovector pairing mode is suppressed by the Coriolis antipairing effect [20]. Calculations within the isospin-generalized HFB framework indeed also suggested such a mixed $T=1 / T=0$ pairing phase with a transition from $T=1$ to $T=0$ dominance as a function of increasing angular momentum [21]. Hence, medium- to high-spin states of rotating $N=Z$ nuclei appear to be among the best places to search for the presence of $T=0 n p$ pairing, and it is important to reach the heaviest possible $N=Z$ nuclei where, however, the experimental conditions are most challenging. One of the key signatures proposed for isoscalar pairing is a significant "delay" in band crossing frequency in deformed $N=Z$ isotopes compared with their $N>Z$ neighbors, which necessitates the study of such nuclei up to angular momentum around $I=10 \hbar$ or higher [17]. Such delays have previously been observed in the deformed $N=Z$ nuclei ${ }_{36}^{72} \mathrm{Kr}_{36},{ }_{38}^{76} \mathrm{Sr}_{38}$, and ${ }_{40}^{80} \mathrm{Zr}_{40}$ but were not considered as conclusive evidence for isoscalar $n p$ pairing effects due to the possible influence of shape coexistence on the alignment frequencies [22-24]. The nuclei ${ }_{42}^{84} \mathrm{Mo}_{42}$ and ${ }_{44}^{88} \mathrm{Ru}_{44}$ also have indications of delays in the rotational alignments; however in these cases the experimental data did not reach the required rotational frequency in order to draw firm conclusions [25,26]. The nucleus ${ }^{88} \mathrm{Ru}$ is here of particular interest, as it is predicted to be the last deformed self-conjugate nuclear system before the $N=Z=50$ closed shells [27]. The structure of its intermediate-to-high-spin states constitutes one of the most promising cases for discovering effects of a BCS-type of isoscalar pairing condensate. However, due to the large experimental difficulties in producing and selecting such exotic nuclei in sufficient quantities excited states in ${ }^{88} \mathrm{Ru}$ were previously known only up to the $I^{\pi}=8^{+}$state [25], just where normal (isovector) paired band crossings are expected to appear in the absence of strong isoscalar pairing. In the present work the level scheme of ${ }^{88} \mathrm{Ru}$ has been extended to higher angular momentum states in the ground-state band, leading to a conclusive measurement of the rotational alignment frequency. The experimental difficulties have been overcome through the use of a highly efficient, state-of-the-art detector system and a prolonged experimental running period.

Experimental details.-Excited states in ${ }^{88} \mathrm{Ru}$ were populated in fusion-evaporation reactions induced by a ${ }^{36} \mathrm{Ar}$ beam produced by the CIME cyclotron at the Grand Accélérateur National d'Ions Lourds (GANIL), Caen, France. The ${ }^{36} \mathrm{Ar}$ ions were accelerated to an energy of 115 MeV and used to bombard target foils consisting of $99.9 \%$ isotopically enriched ${ }^{54} \mathrm{Fe}$ with areal density of $6 \mathrm{mg} / \mathrm{cm}^{2}$, which was sufficient to stop the fusion products of interest. The beam intensity varied between 5 and 10 pnA with an average of 7 pnA during 13 days of irradiation time. Prompt $\gamma$ rays emitted in the reactions were detected by the Advanced Gamma Tracking Array (AGATA) spectrometer [28] in its early phase 1 implementation [29], consisting of 11 triple
clusters of segmented HPGe detectors. Emission of light charged particles and neutrons was detected in prompt coincidence with the $\gamma$ rays by the nearly $4 \pi$ solid angle charged particle detector array DIAMANT [30,31], consisting of $64 \mathrm{CsI}(\mathrm{Tl})$ scintillators, and the neutron wall [32] and NEDA $[33,34]$ neutron detector arrays consisting of 42 and 54 organic liquid-scintillator detectors, respectively. The trigger condition for recording events for subsequent offline analysis was that at least two of the high-purity germanium crystal core signals from the AGATA triplecluster detectors were registered in fast coincidence with at least one neutronlike event recorded in the liquid scintillator detectors. The condition for the neutronlike events was determined by pulse-shape discrimination (PSD) via a firmware threshold set for the so-called charge comparison (CC) ratio between the charge integrated over the tail part of each liquid scintillator pulse and its total integrated charge. Similar PSD criteria made it possible to discriminate between different types of charged particles detected in the $\mathrm{CsI}(\mathrm{Tl})$ scintillators. The final discrimination between neutrons and $\gamma$ rays was performed off line by setting twodimensional gates on the neutron time of flight vs the CC ratio. The rare two-neutron evaporation events were separated from events where a neutron scattered between detectors by applying simultaneous cuts on the deposited energy and time of flight as a function of the distance between detectors that fired. For the off-line charged particle selection, individual two-dimensional gates on the particle identification and energy parameters of the DIAMANT detectors enabled the identification of $\gamma$ rays as belonging to specific charged particle evaporation channels. A 50 ns wide time gate was applied to the time-aligned Ge detector timing signals in order to select prompt $\gamma$-ray emission. The $\gamma$-ray energy measurements with AGATA rely on tracking algorithms [35-39] that reconstruct trajectories of incident $\gamma$ ray photons in order to determine their energy and direction. This is achieved by disentangling the interaction points and corresponding interaction energies in the germanium crystals that are identified using pulse shape analysis of the detector signals and thereafter establishing the proper sequences of interaction points using the characteristic features of the interaction mechanisms (primarily the photoelectric effect, Compton scattering, and pair production). The energy calibration of the germanium detectors was performed using standard radioactive sources $\left({ }^{60} \mathrm{Co}\right.$ and $\left.{ }^{152} \mathrm{Eu}\right)$. Figure 1 shows projected spectra from the $2 n$-selected $E_{\gamma}-E_{\gamma}$ coincidence matrix obtained requiring anticoincidence with detection of any charged particle in the DIAMANT $\mathrm{CsI}(\mathrm{Tl})$ detector array. The spectrum in Fig. 1(a) was produced for events where $\gamma$ rays coincident with the $616,800,964$, and 1100 keV transitions assigned to ${ }^{88} \mathrm{Ru}$ were selected. The background spectrum was produced by using identical energy cuts on a selection of the data requiring coincidence with two neutrons and a charged particle summed with the background spectrum obtained by shifting the energy cuts a


FIG. 1. (a) Gamma-ray energy spectrum detected in coincidence with the $616,800,964$, and $1100 \mathrm{keV} \gamma$ rays, with the additional requirement that two neutrons and no charged particles were detected in coincidence. (b) Expanded part of the unsubtracted gated spectrum around the new $\gamma$-ray transitions at $1063 \mathrm{keV}\left(10^{+} \rightarrow 8^{+}\right), \quad 1153 \mathrm{keV} \quad\left(12^{+} \rightarrow 10^{+}\right)$, and $1253 \mathrm{keV}\left[\left(14^{+}\right) \rightarrow 12^{+}\right]$is drawn in red together with the background spectrum (black) used to produce the spectrum shown in (a). Gamma-ray peaks due to contaminant reactions on oxygen leading to the population of excited states in ${ }^{49,50} \mathrm{Cr}$ and ${ }^{49} \mathrm{Mn}$ are indicated. (c) Level scheme of ${ }^{88} \mathrm{Ru}$ deduced from the present work. Relative intensities are proportional to the widths of the arrows.
constant offset of +20 keV in the two-neutron gated data requiring anticoincidence with the detection of charged particles. These transitions were previously identified as belonging to ${ }^{88} \mathrm{Ru}$ in a study involving a different reaction: ${ }^{58} \mathrm{Ni}\left({ }^{32} \mathrm{~S}, 2 n \gamma\right)^{88} \mathrm{Ru}^{*}$ [25]. All $\gamma$ rays observed in prompt coincidence and assigned to the ground-state band of ${ }^{88} \mathrm{Ru}$ in this work are indicated with their energies in keV .

Discussion.-Figure 2 shows values of the kinematical moment of inertia $\left(J^{(1)}\right)$ for the low-lying yrast level energy bands in the $N=44$ isotones ${ }_{44}^{88} \mathrm{Ru}_{44}$ (this work), ${ }_{22}^{86} \mathrm{Mo}_{44}$ [40,41], and ${ }_{40}^{84} \mathrm{Zr}_{44}$ [42]. The ground-state bands in the even- $Z, N>Z$ isotones ${ }_{42}^{86} \mathrm{Mo}_{44}$ and ${ }_{40}^{84} \mathrm{Zr}_{44}$ exhibit a variation of $J^{(1)}$ (defined as the angular momentum, $I$, divided by the rotational frequency, $\omega=d E / d I$ ) as a function of rotational frequency that is characteristic of a normal paired band crossing in a rotating deformed nucleus of the isovector $(T=1)$ type. The band crossing frequency is $\hbar \omega_{c} \approx 0.47 \mathrm{MeV}$ in both cases (indicated by the black vertical dashed line in Fig. 2). For the $N=Z$ nucleus ${ }_{44}^{88} \mathrm{Ru}_{44}$ the increase in $J^{(1)}$ also resembles a paired band crossing, albeit at a significantly higher rotational frequency, $\hbar \omega_{c} \approx 0.54 \mathrm{MeV}$, indicated by the red vertical dotted line in Fig. 2.


FIG. 2. Experimental values for the kinematical moment of inertia ( $J^{1}$ ) for the low-lying yrast bands of the $N=44$ isotones ${ }_{44}^{88} \mathrm{Ru}_{44}$ (this work), ${ }_{42}^{86} \mathrm{Mo}_{44}$ [40,41], and ${ }_{40}^{84} \mathrm{Zr}_{44}$ [42]. The black dashed vertical line indicates the approximate rotational frequency of the first isovector-paired band crossing due to $g_{9 / 2}$ protons as predicted by standard cranked shell model calculations [43,44]. The red dotted vertical line indicates the band crossing frequency for the ground-state band in ${ }_{44}^{88} \mathrm{Ru}_{44}$ observed in this work.

Theoretical predictions of the rotational response of excited states and the associated spin alignment can be provided by cranked shell model calculations [45], which predict the first proton two-quasiparticle $\left(\pi g_{9 / 2}\right)^{2}$ alignment to occur at $\hbar \omega_{c} \approx 0.45 \mathrm{MeV}$ followed closely by a neutron $\nu\left(\mathrm{g}_{9 / 2}\right)^{2}$ alignment [43,44]. Mountford et al. have demonstrated that the first alignment in ${ }^{84} \mathrm{Zr}$ is due to $g_{9 / 2}$ protons by means of a transient-field $g$-factor measurement [46]. The slopes of the $J^{(1)}$ curves around the crossing point also exhibit an expected variation, reflecting the change in interaction strength between the ground-state band and the broken-pair $S$ band as the proton Fermi level changes within the $g_{9 / 2}$ subshell. The large delay in band crossing frequency for ${ }_{44}^{88} \mathrm{Ru}_{44}$ compared with its closest $N=44$ isotones can not readily be explained using standard mean field models.

Developments of computational methods in recent years enable shell model calculations to be performed with large model spaces, providing nuclear structure predictions for medium-mass nuclei away from closed shells. Large-scale shell-model (LSSM) calculations with an isospinconserving Hamiltonian are also the method of choice

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for theoretical investigations of the isospin dependence of nucleonic pair correlations [17]. In Ref. [26], projected shell model calculations following the approach of Ref. [47] predicted a delay in the band crossing frequency in the $N=Z$ nuclei ${ }_{42}^{84} \mathrm{Mo}_{42}$ and ${ }_{44}^{88} \mathrm{Ru}_{44}$ as an effect of enhanced neutron-proton interactions. Kaneko et al. [48] employed LSSM calculations using a "pairing-plus-multipole" Hamiltonian [49] in the $\left(1 p_{1 / 2}, p_{3 / 2}, f_{5 / 2}, g_{9 / 2}, d_{5 / 2}\right)$ (often denoted as $f p g d$ ) model space for studying ${ }_{44}^{88} \mathrm{Ru}_{44},{ }_{44}^{90} \mathrm{Ru}_{46}$, and ${ }_{44}^{92} \mathrm{Ru}_{48}$ and concluded that $T=0 \mathrm{np}$ pairing is responsible for the distinct difference in rotational behavior between the $N=Z$ and $N>Z$ nuclei. These calculations also predicted a significant delay in the band crossing frequency for $N=Z$ and their prediction for the $J^{(1)}$ moment of inertia of ${ }_{44}^{88} \mathrm{Ru}_{44}$ revealed a sharp irregularity at a rotational frequency $\hbar \omega_{c} \approx 0.65 \mathrm{MeV}$ [48]. We therefore conclude that the delayed alignment of $g_{9 / 2}$ protons observed in the ground-state band of ${ }^{88} \mathrm{Ru}$ in the present work is likely not to be in agreement with the response of a deformed rotating nucleus in the presence of a normal isovector pairing field and that isoscalar pairing components may be active in this self-conjugate nucleus.

Summary.-In summary, new $\gamma$-ray transitions in the selfconjugate nuclide ${ }_{44}^{88} \mathrm{Ru}_{44}$ have been identified, extending the previously reported level structure. The observed groundstate band exhibits a band crossing that is significantly delayed compared with the expected behavior of a rotating deformed nucleus in the presence of a normal isovector ( $T=1$ ) pairing field. The observation is in agreement with theoretical predictions for the presence of isoscalar neutronproton pairing in the low-lying structure of ${ }^{88} \mathrm{Ru}$.

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# First observation of $\gamma$ - ray transitions in ${ }^{111} \mathrm{Mo}$ 

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#### Abstract

Excited states in the extremely neutron-rich nuclei ${ }^{109} \mathrm{Mo}$ and ${ }^{111}$ Mo have been studied following nucleon knock-out reactions. Seven $\gamma$-ray transitions, some of them in prompt mutual coincidence, have been identified for the first time in ${ }^{111}$ Mo using the DALI2 and MINOS detector systems at the BigRIPS and ZeroDegree electromagnetic fragments separator at the RIBF, RIKEN, Japan. Total Routhian surface (TRS) and Particle- Plus Rotor calculations have been performed to investigate the predicted shape coexistence and its effect on the structure of nuclei in this region of the nuclear chart. Following the results of the calculations, theoretical level schemes are proposed for positive and negative parity states and compared with the experimental findings.


## I. INTRODUCTION

The evolution of nuclear shell structure as a function of neutron and proton numbers has revealed new phenomena that were not observed in the valley of stability. Neutron-rich nuclei in the $A \sim 110$ region are predicted to exhibit rapidly changing equilibrium groundstate shapes and shape coexistence with competing prolate, oblate, triaxial, and spherical shapes $[1-3]$. They, therefore, constitute an important testing ground for nuclear structure models.

[^0]These nuclei are also situated on the path of the astrophysical rapid neutron-capture process (r-process) [4]. Knowledge of their properties are essential for a better understanding of isotopic abundances in the solar system, astronomical observations, and the production of the heaviest elements in the Universe. For example, in the $100 \leq \mathrm{A} \leq 115$ mass region, below the $\mathrm{A}=130$ r-process abundance peak, the predictions of state-of-the-art model calculations underestimate by up to orders of magnitude or more the observed isotopic solar abundances $[5,6]$.

The neutron-rich nuclide ${ }^{111}$ Mo was identified for the first time by Bernas et al. following in-flight fission of a ${ }^{238} \mathrm{U}$ beam accelerated to around $750 \mathrm{MeV} / \mathrm{u}$ and identified in the FRS fragment separator at the GSI accelerator facility [7]. Pereira et al. measured its half-life for
the first time to be $200(10)_{-35}^{+40} \mathrm{~ms}$ at the National Superconducting Cyclotron Laboratory at Michigan State University following fragmentation of a ${ }^{136} \mathrm{Xe}$ beam at a beam energy of $120 \mathrm{MeV} / \mathrm{u}$ [5]. In an experiment at the JYFL Accelerator laboratory, Kurpeta et al. used deuteron-induced fission of natural uranium to produce ${ }^{111}$ Mo nuclei, which were separated using the IGISOL3 mass separator [8]. Isobaric purification was achieved by the JYFLTRAP Penning trap system resulting in a monoisotopic beam of ${ }^{111}$ Mo. Kurpeta et al. determined the $Q_{\beta}$ for the first time to be $9085(5) \mathrm{keV}$ and suggested the presence of a beta-decaying isomeric state with spin-parity (7/2-) (or possibly (9/2-)) proposed from systematics, close above a low-spin (tentatively $(1 / 2+$ ) or $(3 / 2+))$ ground state. The half-life of the ground state was determined to be $186(9) \mathrm{ms}$ in agreement with the previous measurement [5], while the beta decaying isomeric level was determined to have a similar half-life of around 200 ms . Later, ${ }^{111}$ Mo nuclei have also been produced at the RIKEN RIBF facility using in-flight fission of a ${ }^{238} \mathrm{U}$ beam accelerated to $345 \mathrm{MeV} / \mathrm{u}$, and the halflife was determined to be $196(5) \mathrm{ms}$ [9].

## II. EXPERIMENTAL SET-UP

The experiment was performed at the Radioactive Isotope Beam Factory (RIBF), operated by the RIKEN Nishina Center for Accelerator-Based Science and the Center for Nuclear Study of the University of Tokyo, Wakô Japan. A ${ }^{238} \mathrm{U}$ beam with an intensity of 30 pnA was accelerated to $345 \mathrm{MeV} / \mathrm{u}$ and impinged on a $3-\mathrm{mm}$ thick Be target to create rare-isotope beams via in-flight fission. The primary Be target was placed at the entrance of the fragment separator BigRIPS [10]. From the cocktail of isotopes produced in the fission reactions, the isotopes of interest were selected by the $\mathrm{B} \rho-\triangle \mathrm{E}-\mathrm{B} \rho$ method. Data were collected during 58 hours at an average $183 \mathrm{~s}^{-1}$ event rate of the full cocktail beam, which had average incident energy of $270 \mathrm{MeV} /$ nucleon. The selected isotope beam was incident on a 99 mm thick liquid hydrogen $\left(\mathrm{LH}_{2}\right)$ target in the MINOS time projection chamber (TPC) [11]. The individual intensities for the secondary beams of interests, ${ }^{113} \mathrm{Tc}$ for ( $\mathrm{p}, 2 \mathrm{pn}$ ) and ${ }^{112} \mathrm{Mo}$ for ( $\mathrm{p}, \mathrm{pn}$ ), were 23 and 35 particles/s, respectively.

The ${ }^{111}$ Mo nuclei were created via neutron or proton and neutron knockout reactions in the $\mathrm{LH}_{2}$ target. After passing through the $\mathrm{LH}_{2}$ target, the selected nuclei reduce their kinetic energy to around $134 \mathrm{MeV} / \mathrm{u}$. Products from the secondary reactions in the $\mathrm{LH}_{2}$ target were identified by the ZeroDegree spectrometer, using the $\mathrm{B} \rho-\triangle \mathrm{E}-\mathrm{B} \rho$ method [12]. Each reaction vertex was reconstructed using proton tracks detected in 300-mm-long TPC, which was surrounding the $\mathrm{LH}_{2}$ target in MINOS. The vertex reconstruction precision was around 5 mm FWHM with a tracking efficiency of $64 \%$ for the (p,2pn) and $65.5 \%$ for the (p,pn) reactions. Following
the vertex reconstruction, precise Doppler correction of the $\gamma$-rays was performed event by event as described in Ref [13]. The $\gamma$-rays emitted in the reactions were detected with the $186 \mathrm{NaI}(\mathrm{Tl})$ scintillator array DALI2 [14], covering $12^{\circ}$ to $118^{\circ}$ polar angle range with respect to the central beam axis and the center of the $\mathrm{LH}_{2}$ target inside MINOS. DALI2 was calibrated with peaks from 121 keV to 1332 keV using ${ }^{152} \mathrm{Eu},{ }^{60} \mathrm{Co},{ }^{137} \mathrm{Cs}$ and ${ }^{133} \mathrm{Ba}$ radioactive sources and an energy resolution of 37 keV full width at half maximum (FWHM) was obtained for the 344 keV peak of ${ }^{152} \mathrm{Eu}$. Monte Carlo simulations using the GEANT4 tool kit [15] predict the full energy peak detection efficiency of DALI2 to be $35 \%$ at 500 keV , and a 0.6 c source velocity $[14,16]$. The DALI2 detection threshold was set around $100-150 \mathrm{keV}$ (in the center of -momentum frame) with higher values for the backward detectors due to the kinematic boost of the $\gamma$-rays in the forward direction. Analysis of the data in the present work only considered the forward angle $\left(13^{\circ}-30^{\circ}\right)$ detectors to increase the peak-to-background ratio in the low energy region.

## III. DATA ANALYSIS AND RESULTS

The off-line analysis of selected nuclei and relevant spectra was performed using the ROOT and Anaroot software packages [17]. The inclusive cross section for the ${ }^{113} \mathrm{Tc}(\mathrm{p}, 2 \mathrm{pn})$ reaction, for which the obtained $\gamma$-ray energy spectrum is shown in Fig. 1b, was determined to be $\sigma_{\text {inc }}=10.3(6) \mathrm{mb}$. The $\gamma$-ray energy spectrum obtained for the ${ }^{112} \mathrm{Mo}(\mathrm{p}, \mathrm{pn})$ reaction $\left(\sigma_{\text {inc }}=61(3) \mathrm{mb}\right)$ is shown in Fig. 1a. The spectra's main background component is due to the unresolved transitions, and Bremsstrahlung that was created by fast ion beam interactions with the hydrogen target, mainly affecting the low-energy region [18]. There's also background coming from the protons hitting the surronding material. This atomic background contribution was determined by measuring the DALI2 spectrum observed in prompt coincidence with the unreacted beam particles from the ${ }^{111} \mathrm{Mo}(\mathrm{p}, \mathrm{p}$ ') reaction (red stripe shaded histogram in Fig 1). The background was normalized with respect to the number of events associated with ${ }^{111}$ Mo nuclei in the reacted beam before it was subtracted from the experimental data.

The background subtraction method was tested on the reaction channels leading to ${ }^{112} \mathrm{Mo}$ from the same experiment, and the $2^{+} \rightarrow 0^{+}$transition was found at 235 keV, well separated from the bremsstrahlung bump at low energy [18]. In this way, it was verified that this subtraction method was not creating artificial peaks. In addition to ${ }^{112} \mathrm{Mo},{ }^{109} \mathrm{Mo}$ was analyzed to verify the procedure, and the $\left(7 / 2^{+}\right) \rightarrow\left(5 / 2^{+}\right)$transition was measured to be 140 (10) keV, in agreement with an earlier measurement of 144 keV [19, 20]. The recently found isomeric state at 69 keV with 180 ns half-life [21] could not be observed in this study because of the current experimental setup is not capable to measure such long life


FIG. 1. (Color online) Doppler corrected $\gamma$-ray spectra measured in the forward angles of DALI2 detectors for ${ }^{111} \mathrm{Mo}$ generated from (a) (p,pn) and (b) (p,2pn) reactions, where they contain the total spectrum (black solid line), normalized bremsstrahlung component obtained from ( $p, p^{\prime}$ ) reactions (red striped histogram), and the background-subtracted spectrum (open diamonds). The inset figure in (a) is strict multiplicity selection $\mathrm{M}=1$ in all detector. The total fit to the subtracted spectrum is shown by the thick blue line, the single $\gamma-$ ray response functions of DALI2 are indicated by the thick red line, and the dashed blue line shows the double exponential background fit.
times. Furthermore, the background-subtracted spectra (open diamonds in Fig. 1) were fitted using the response functions obtained from Monte Carlo simulations using the GEANT4 software package $[15,16]$. In the formation of the response functions for the DALI2 detectors, the individual DALI2 detector element thresholds and resolutions were employed and measured during the experiment, used as an input. In these simulations, absorption of $\gamma$-rays in different materials around the target was also included. To define the remaining background, originating from unresolved high energy transitions and particle induced background, which has unknown shape in low energy, the double explonantial function was used, which is also preferred for the other isotope analysis in the same experiment [22, 23]. The fitting parameters were determined by fitting the background function and the response functions simultaneously. $\chi^{2}$ tests were performed to choose the best possible combinations for the transition energies. In order to obtain the upper limit of intensities for each transition, the singles $\gamma$-rays spectra were fitted with the response functions (red lines in Fig.1) and double exponential background functions


FIG. 2. (a) Total $\gamma$-ray energy spectrum with multiplicty less than 4 selection spectrum from the ${ }^{111} \mathrm{Mo}(\mathrm{p}, \mathrm{p} 2 \mathrm{n}){ }^{109}$ Mo reaction. The inset is the partial level scheme of ${ }^{109} \mathrm{Mo}$ from Ref [20, 21]. (b) 111 keV energy-gated $\gamma$-ray coincidence spectrum for forward angle detectors to optimiza peak background ratio. For the gate $100-120 \mathrm{keV}$ range and for the background subtraction $170-190 \mathrm{keV}$ range is used.
(blue dashed line in Fig. 1)as explained above, and the results are shown in Table I.

The different knockout reactions leading to ${ }^{111}$ Mo result in different average excitation energies, the strongest $\gamma$-ray transitions (130(10) keV, 176(13) keV, 205(16) $\mathrm{keV}, 235(19)$ and $290(28) \mathrm{keV}$ ) are observed in both reactions as seen in Fig. 1. Two additional transitions at energies 380 (22) and 750 (45) keV were more clearly observed in the neutron knockout reaction from the ${ }^{112} \mathrm{Mo}$ isotope. Energy uncertainties are dominated by the fitting error and energy calibration. No information about the half-lives of the levels was available in the current analysis; therefore, we simulate different half-lives from $0-100 \mathrm{ps}$ for all transitions and tested to reach the lowest $\chi^{2}$ value, which was obtained in 60 ps . A half-life of 60 ps can affect the centroid of a 290 keV transition by about 10 keV . This lifetime values was adopted with an error of $100 \%$, resulting a final uncertanities.

The known systematics of excited-state level energies for odd-even Mo and Ru isotopes in this region of the nuclear chart [19, 24-26] reveals high-level densities with several low-energy transitions at low excitation energy. With the limited DALI2 energy resolution, there is always a possibility of unresolved close-lying transitions in the one-dimensional gamma-ray energy spectrum.


FIG. 3. (Color online) Selected energy-gated $\gamma$-ray coincidence spectra from the ${ }^{113} \mathrm{Tc}(\mathrm{p}, 2 \mathrm{pn}) \&{ }^{112} \mathrm{Mo}(\mathrm{p}, \mathrm{pn})$ reactions. The experimental data are shown as open black diamonds with error bars, and the continus dark-blue line is a fit to the entire spectrum. The background was fitted with a double exponential function and is given by the blue dashed line. The simulated response function for each transition is shown as a red line.
(a) Spectrum obtained using a gate centered around 130 (115-145) keV, with Compton background in the $150-170 \mathrm{keV}$ range, in the ( $\mathrm{p}, 2 \mathrm{pn}$ ) reaction.
(b) Spectrum obtained using a gate centered around 176 (160-200) keV, with Compton background in the 200-240 keV range in the ( $\mathrm{p}, 2 \mathrm{pn}$ ) reaction.
(c) Spectrum obtained using a gate centered around $205(190-215) \mathrm{keV}$, with Compton background in the $400-450 \mathrm{keV}$ range and requiring a $\gamma$-ray multiplicity less than 4 , in ( $\mathrm{p}, 2 \mathrm{pn}$ ) reaction.
(d) Spectrum obtained using a gate centered around $235(210-250) \mathrm{keV}$, with Compton background in the 400-500 keV range and requiring a $\gamma$-ray multiplicity less than 4 , in ( $\mathrm{p}, 2 \mathrm{pn}$ ) reaction.
(e) Spectrum obtained using a gate centered around $750(700-800) \mathrm{keV}$, with Compton background in the $800-860 \mathrm{keV}$ range, in ( $\mathrm{p}, \mathrm{pn}$ ) reaction. The inset figure shows the relative intensity of 290 keV the coincidence transition as a function of the centroid energy of several 100 keV - wide gates in the range $600-900 \mathrm{keV}$.
(f) Spectrum obtained using a gate centered around $290(260-310) \mathrm{keV}$, with Compton background in the $470-530 \mathrm{keV}$ range,in (p,pn) reaction.

A $\gamma-\gamma$ coincidence analysis helped to resolve additional transitions. To identify the photopeak-photopeak coincidences in the data, background subtraction was applied to the gated histograms. First, to reduce the contribution due to the Compton distribution from $\gamma$ rays with higher energy than the peak of interest, a background gate was chosen close to the $\gamma$-ray peak of interest from the same reaction and subtracted from the gated histogram (results shown with a blue diamond in Fig.3). The coincidence analysis was first carried out for ${ }^{109}$ Mo events from ${ }^{111} \mathrm{Mo}(\mathrm{p}, \mathrm{p} 2 \mathrm{n})$ reactions to verify the procedure. As an example, a gate was applied to the $100-120 \mathrm{keV}$ range for the known 111 keV transition [19-21], with a Comp-
ton background gate applied in the range from 170 to 190 keV . The resulting coincidence spectrum (Fig. 2) shows three peaks at $139 \mathrm{keV}, 223 \mathrm{keV}$, and 397 keV in the agreement with the level scheme (inset figure) from Ref [21].

The coincidence analysis for ${ }^{111} \mathrm{Mo}$ was done separately for each reaction since the different knockout reactions may not only populate the same excited states. The strongest mutual coincidence relationship was found between the 130 and 176 keV transitions in both reactions. Furthermore, the 176 keV gamma-ray is also in coincidence with a 205 keV transition, which is visible in both reactions. The 205 and 235 keV transitions also showed

TABLE I. Gamma-ray transitions in ${ }^{111}$ Mo observed in this work. The relative intensities of the transitions as observed in different reactions are normalized with respect to the most intense 290 keV transition for the ${ }^{112} \mathrm{Mo}(\mathrm{p}, \mathrm{pn})$ and ${ }^{113} \mathrm{Tc}$ ( $\mathrm{p}, 2 \mathrm{pn}$ ) reactions. The coincidence analysis results are also shown in the right coloum of the table.

| $E_{\gamma}(\mathrm{keV})$ | $I_{\gamma}$ <br>  <br>  <br> ${ }^{112} \mathrm{Mo}(\mathrm{p}, \mathrm{pn})$ | $I_{\gamma}$ <br> ${ }^{113} \mathrm{Tc}(\mathrm{p}, 2 \mathrm{pn})$ | Coincident $\gamma$-lines |
| :--- | :---: | :---: | :--- |
| $130(10)$ | $84(6)$ | $52(9)$ | 176 |
| $176(13)$ | $83(7)$ | $59(11)$ | 130,205 |
| $205(16)$ | $44(8)$ | $59(12)$ | 176,235 |
| $290(28)$ | $100(7)$ | $100(12)$ | $290,380,750$ |
| $235(19)$ | $44(8)$ | $19(14)$ | 205 |
| $380(22)$ | $32(8)$ | $18(13)$ | 176,290 |
| $750(45)$ | $11(9)$ | $9(14)$ | $130,176,290$ |

a clear coincidence relationship in ${ }^{113} \mathrm{Tc}(\mathrm{p}, 2 \mathrm{pn})^{111}$ Mo reaction, while 205 keV shows extra coincidence with the 176 keV transition. The $290-\mathrm{keV}$ transition, which is one of the strongest transition observed in both reactions, is neither in coincidence with the 130 keV transition nor the transition at 176 keV . On the other hand, it is in coincidence with the 380 keV transition. Additionally, the $750-\mathrm{keV}$ transition was found in the $\gamma-\gamma$ coincidence analysis in the single-neutron knockout reaction, and it is in coincidence with the 290 keV transition as well as with the 130 keV and 176 keV gamma rays. The inset graph in figure 3 (e) show the intensity change of 290 keV transition in different gates in between 600 to 900 keV to prove that 750 keV transition exist and in coincidence with 290 keV . The 750 keV transition is not clearly visible in the $\gamma$ - ray spectra in Fig. 1, and the calculated number of count is around 900 , which is close to what is deduced from the 290 keV gated histogram ( 606 counts) in Fig. 3 assuming the 750 keV and 290 keV are exclusively coincidental. The coincidence results are summarized in Table I.

Due to the apparent complexity of the data and the insufficient energy resolution of the DALI2 detectors, it was not possible to construct a firm-level scheme for ${ }^{111} \mathrm{Mo}$.

In the previous $\beta$ decay study of ${ }^{111} \mathrm{Mo}$ [8], the ground state spin parity was tentatively assigned as $(1 / 2+)$ or $(3 / 2+)$, and evidence for a low-lying $\beta$ decaying isomeric state with a similar half-life was deduced. The possible isomeric state was tentatively assigned spin-parity (7/2-) (or possibly (9/2-)) proposed based on systematics. However, there is no experimental data that gives precise information about the ground state properties of ${ }^{111} \mathrm{Mo}$.

## IV. THEORETICAL CALCULATIONS

Neutron rich nuclei with mass $\mathrm{A}=100-112$ reveal interesting phenomena such as shape coexistence $[1,8,18]$ and triaxiality $[18,24,26,27]$. Previous studies have also predicted a prolate to oblate shape transition for the
ground states of the Mo isotopes for neutron numbers in the range $\mathrm{N}=68-72[2,28]$. There is considerable variation in these predictions, in particular with respect to the role of single-particle orbits for polarizing the nuclear shape.

To investigate further the shape evolution in neutronrich molybdenum, we have performed extended Total Routhian Surface (TRS) and Particle Plus Rotor (PPR) calculations. The TRS calculations of equilibrium deformations are based on a Woods-Saxon's potential with parameters from [29] and the cranked Strutinsky formalism [30]. In the calculations, pairing correlations were taken into account by means of seniority and double stretched quadrupole pairing force [30] and the LipkinNogami method [31] as an approximate particle number projection. The TRS results are shown in the $X=\beta_{2} \cos \left(\gamma+30^{\circ}\right), Y=\beta_{2} \sin \left(\gamma+30^{\circ}\right)$ plane, where the $\beta_{2}$ is quadrupole deformation parameter, and $\gamma$ is a shape degree of fredoom measured in degrees $\left(\gamma=0^{\circ}\right.$ prolate, $\gamma=30^{\circ}$-oblate, $\gamma=60^{\circ}$ - triaxial ) in quadrupole deformation space. The results are shown in Figs. 4, and 5.

For the even-even ${ }^{108,110,112}$ Mo isotopes (Fig. 4) the TRS calculations were performed for the vacuum groundstate configuration. The potential energy surface (PES) of ${ }^{108}$ Mo exhibits an energy minimum with the shape elongation parameter, $\beta_{2} \approx 0.33$ and the triaxial deformation parameter $\gamma \approx-18^{\circ}$. In one of the first theoretical studies of nuclear shapes carried out in the neutronrich mass $\mathrm{A}=100$ region ${ }^{108} \mathrm{Mo}$ was previously studied using a different WS potential by Skalski et al. [2] with very similar results. Also, recent results using the calculations predict a shape evolution towards oblate shape as a function of increasing neutron number [32]. For mass number $A=110$ the predicted ground-state deformation is $\left(\beta_{2} \approx 0.30, \gamma \approx-30^{\circ}\right)$ while for $A=112$ the predicted ground-state deformation as ( $\beta_{2} \approx 0.23, \gamma \approx$ $-60^{\circ}$ ). Hence, according to our calculations, there is a pronounced shape change between neutron numbers 66 and 70 . This is in accordance with earlier calculations $[1,2,32]$. The shape change from prolate like structures to oblate with increasing neutron number is accompanied by a reduction in $\beta_{2}$ deformation.

For the odd-even ${ }^{109}$ Mo and ${ }^{111}$ Mo isotopes, the TRS calculations were performed by blocking the lowest neutron configuration of different parity and signature in a self-consistent manner. For more details of the method, we refer to ref. [30]. For ${ }^{109} \mathrm{Mo}$, the lowest negative and positive parity configuration gives triaxial gamma soft minimum at $\beta_{2}=0.34$ and $\gamma=-20$, which is also consistent with the precious odd-even Molibdenium studies [21]. The results for ${ }^{111} \mathrm{Mo}$ at zero and $0.40 \mathrm{MeV} / \hbar$ rotational frequency are shown in Fig. 5. The entire PES is quite gamma-soft for values of the quadrupole deformation in the range $\beta_{2}=0.2-0.3$.

For the lowest negative-parity configuration, the TRS calculations produce coexisting oblate and triaxial minima (Fig. 5(a)). The lowest-lying oblate configuration


FIG. 4. (Color online) Total Routhian surfaces in the $\beta_{2} \sin \left(\gamma+30^{\circ}\right)-\beta_{2} \cos \left(\gamma+30^{\circ}\right)$ plane for the vacuum groundstate configuration in ${ }^{108,110,112} \mathrm{Mo}$ at $\hbar \omega=0$ (first row). The blue diamonds indicate the positions of the ground - state minima for different shapes. Equipotential energy lines are separated by 0.2 MeV .
is the predicted ground-state configuration and has deformation parameters $\left(\beta_{2}, \gamma\right) \approx\left(0.25,-60^{\circ}\right)$. This minimum in the PES becomes more gamma-soft with increasing rotational frequency, as seen in Fig.5(c). The second triaxial minimum lies 220 keV above the oblate one with $\left(\beta_{2}, \gamma\right) \approx\left(0.30,-30^{\circ}\right)$ and is washed out by rotation. The lowest positive-parity configuration lies approximately 600 keV above the predicted negative-parity ground state and exhibits a triaxial gamma-soft minimum, $\left(\beta_{2}, \gamma\right) \approx\left(0.30,-25^{\circ}\right)$ that stays relatively unchanged with rotation.
In order to shed further light on the lowest active configurations in ${ }^{111} \mathrm{Mo}$, Particle-Plus-Rotor (PPR) calculations were performed. The PPR calculations employed the same Woods-Saxon potential as for the TRS calculations. Likewise, all deformation parameters of the PPR calculations were taken from the minima of the TRS calculations.
To test the agreement between the PPR calculations and experimental observables, we run the PPR calculations also for ${ }^{109} \mathrm{Mo}$, which has a relatively well-developed level scheme. The results are shown in Fig. 6. One can see the parallel pattern between experiment and theory. The PPR calculations also predict an isomeric $5 / 2^{+}$state with a half-life of 68 ns experimentally observable halflife of 180 ns. Since the PPR calculations are carried out separately for different parities, the theoretical $7 / 2^{-}$ state has been arbitrarily placed at the same excitation energy as the experimentally observable $7 / 2^{-}$state to facilitate the comparison.

The PPR calculations allow the determination of the ground state spins at the relevant and coexisting deformation values. In particular, they reveal an interesting relative migration between different levels as a function of the triaxial parameter $\gamma$, see Fig. 7. According to the PPR calculations, the ground state in ${ }^{111} \mathrm{Mo}$ is predicted to have $I^{\pi}=9 / 2^{-}$for the oblate minimum of the negative parity configuration, which is a structure of predominantly $1 h_{11 / 2}$ parentage. While the $7 / 2_{1}^{-}$is pre-


FIG. 5. (Color online) Total Routhian surfaces in the $\beta_{2} \sin \left(\gamma+30^{\circ}\right)-\beta_{2} \cos \left(\gamma+30^{\circ}\right)$ plane for the lowest signature and parity ( $\pi, \alpha$ ) configurations in ${ }^{111} \mathrm{Mo}$ at $\hbar \omega=0.0$ MeV and 0.40 MeV . The blue diamonds indicate the position of the collective minima for each potential energy surface. Equipotential energy lines are separated by 0.2 MeV . (a) blocking of the lowest ( $\pi=-, \alpha=-1 / 2$ ) configuration. (b) blocking of the lowest ( $\pi=+, \alpha=+1 / 2$ ) configuration. (c)blocking of the lowest ( $\pi=-, \alpha=-1 / 2$ ) configuration with 0.20 MeV rotational frequency. (d) blocking of the lowest ( $\pi=+, \alpha=+1 / 2$ ) configuration with 0.20 MeV rotational frequency.
dicted to be situated only 29 keV above the ground state, the $11 / 2_{1}^{-}$state is calculated to be $\approx 136 \mathrm{keV}$ higher in energy. The predicted ground-state oblate structure is dominated by the $9 / 2[514]$ ( $77 \%$ ) and $11 / 2[505]$ ( $16 \%$ ) Nilsson states. The corresponding Nilsson states for the following discussion can be seen in Fig. 8. The effect of changing the shape to the somewhat less favored triaxial shape is a rather dramatic rearrangement of the lowest-lying negative-parity levels. The $I^{\pi}=7 / 2^{-}$state then becomes the ground state while the $11 / 2_{1}^{-}, 9 / 2_{1}^{-}$, and $3 / 2_{1}^{-}$states become almost degenerate around $80-90$ keV above the ground state. The migration of the $3 / 2_{1}^{-}$ state by more than 700 keV is remarkable and would have profound effects on the structure of ${ }^{111} \mathrm{Mo}$ as discussed below. For the similarly less favored prolate shape, the $I^{\pi}=7 / 2^{-}$state is also the ground state.

The excited positive-parity structure is based on Nilsson configurations with mixed $1 g_{7 / 2}$ and $2 d_{5 / 2}$ parentage. Its excitation energy is around $600-700 \mathrm{keV}$ depending on the deformation. At an oblate shape, the band head is predicted to be $I^{\pi}=3 / 2^{+}$with a close-lying $I^{\pi}=1 / 2^{+}$state only 23 keV higher in excitation energy. The wave function of the $3 / 2^{+}$state is a mixture of


FIG. 6. Level schemes of ${ }^{109} \mathrm{Mo}$, the experimental data are compared with Particle Plus Rotor calculations using the same Woods-Saxon potential as for the TRS calculations. The experimental data are taken from [20,21]
mainly the [411]3/2 and [420] $1 / 2$ Nilsson configurations. At the favored triaxial shape, this is reversed with the $1 / 2^{+}$being 60 keV and 170 keV lower than the $3 / 2^{+}$and $5 / 2^{+}$states, respectively. Therefore the positive parity, $3 / 2^{+}$, and $1 / 2^{+}$were predicted to be very close in energy, almost degenerate. To study the further effect of shape coexistence on the level scheme, we also perform PPR calculations for a prolate ( $\gamma=-120^{\circ}$ ) configuration. The lowest positive parity state is then changed to $5 / 2^{+}$with a more simple configuration dominated by the $5 / 2[402]$ Nilsson state with $75 \%$ admixture.

It is noteworthy that the predicted excitation energy of the positive-parity low-spin band head is relatively speaking more uncertain than the predicted relative excitation energies within the positive-parity and negative-parity structures Fig. 7.

## v. DISCUSSION

Although the present experimental data does not allow a firm determination of the level structure in ${ }^{111} \mathrm{Mo}$, some important clues may be drawn from the different emissions of $\gamma$ rays in the two different knockout reactions, the observations from the previous $\beta$ decay study $[8]$, and from the theoretical calculations. The TRS calculations reveal equilibrium shapes that are strongly configuration
dependent at this particular neutron number. As can be seen from Figs. 4 and 5 the nucleus ${ }^{111}$ Mo appears to be at the tipping point between triaxial and oblate shapes. This might also be reflected in a shape coexistence between oblate and triaxial states in the structure of ${ }^{111} \mathrm{Mo}$ itself. We note that Kurpeta et al. concluded that there is evidence for two beta-decaying levels in ${ }^{111}$ Mo based on the observed wide spin range of levels populated in ${ }^{111} \mathrm{Tc}$ following the $\beta$ decay of ${ }^{111} \mathrm{Mo}$. This might be a reflection of this shape coexistence and its effect on the detailed level structure.

Considering the theoretical level schemes predicted by the PPR calculations for oblate, triaxial, and prolate shape (Fig. 7), there are no obvious candidates for a longlived isomeric state. However, note also that the relative energies of the negative-parity and positive-parity band heads are more uncertain than the relative positions of the states within each structure. We find that the oblatetriaxial shape coexistence predicted by the TRS calculations provides a possible scenario. If the energy of the triaxial $1 / 2^{+}$band head is shifted down only by around 250 keV , it would become similar or lower in energy than the $5 / 2_{1}^{-}$state built on the oblate ground-state structure and would hence become a spin-trap isomer with a sufficiently long half-life that it might well decay predominantly via $\beta$ decay. If the two lowest configurations are a positive-parity structure built on a triaxial shape and a close-lying negative-parity structure built on an oblate shape as predicted by the TRS calculations, such a spintrap isomer could also be produced if the band head energies are reversed. In other words, given such oblatetriaxial shape coexistence, the PPR prediction of a lowlying spin-trap isomer is rather robust. From the point of view of the population of states in the single-neutron and proton-neutron knockout reactions, it is most likely that negative-parity states emanating from the spherical $1 h_{11 / 2}$ subshell, and positive parity states from the $1 g_{7 / 2}$ and $2 d_{3 / 2}$ are observed.

For ${ }^{109} \mathrm{Mo}$, the observation of the 111 keV transition shows that negative-parity states can be populated from neutron knockout reactions while the possible population of positive-parity states is inconclusive since known transitions from positive-parity states have the same energies within the uncertainties as low-lying transitions connecting negative-parity states. We, therefore, assume that the observed $\gamma$ rays are due to transitions between states within the negative-parity structure and that the additional $\gamma$ rays observed in the proton-neutron knockout reaction are due to the population of higher spin states enabled in such reactions compared with single-nucleon knockout reactions.

## VI. SUMMARY

A $\gamma$-spectroscopic study of the neutron-rich molybdenum isotope ${ }^{111}$ Mo has been performed. Excited states in ${ }^{111}$ Mo were populated in nucleon knockout reactions


FIG. 7. The theoretical level scheme results of the PPR calculations. The first lowest states of a given spin are drawn for both positive and negative parity. The energy difference between the first positive-parity state and the negative-parity ground state is taken from the TRS calculation at zero rotational frequency at each shape. The transitions are labeled with their corresponding reduced E2 transition probabilities in Weisskopf units as predicted by the PPR calculation. The dominant Nilsson component of the wave function is indicated at the bottom of the band.


FIG. 8. The calculated single neutron levels for universal Woods-Saxon potential. Positive (negative) parity is indicated by solid (dashed) lines
following fast-fission of ${ }^{238} \mathrm{U}$ ions at the Radioactive Isotope Beam Factory (RIBF). Seven $\gamma$-ray transitions were observed for the first time in ${ }^{111} \mathrm{Mo}$, one of them only resolved after the $\gamma$ - $\gamma$-coincidence analysis. TRS calculations have been performed for the nuclei in the molybdenum isotopic chain $A=108-112$ (even- N isotopes) and for $A=111$. The calculations predict a shape transition from triaxial to oblate axially-symmetric shape accompanied by a somewhat decreasing quadrupole defor-
mation with increasing neutron number in the isotopic chain. For ${ }^{111} \mathrm{Mo}$, shape coexistence is predicted for the low-lying states between a triaxial positive-parity configuration based on mixed $1 g_{7 / 2}$ and $2 d_{5 / 2}$ parentage and an oblate ground-state configuration with $1 h_{11 / 2}$ parentage. PPR calculations of the rotational structures built on these states were carried out. The possibility of a $\beta$ decaying spin-trap isomer is predicted by the PPR calculations in this picture. This would be in agreement with previous observations related to the $\beta$ decay of ${ }^{111}$ Mo at the IGISOL facility.

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# Single-particle structures in ${ }^{85,87} \mathrm{Ge}$ 

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Gamma-ray transitions have been identified for the first time in the extremely neutron-rich $(N=$ $Z+25$ ) nucleus ${ }^{87} \mathrm{Ge}$ following nucleon knockout reactions studied at the RIBF, RIKEN, Japan. New $\gamma$-ray transitions from excited states in ${ }^{85} \mathrm{Ge}$ were also observed and placed in a tentative level scheme. The exclusive parallel momentum distribution was measured for the $1 / 2^{+}$state for the neutron knockout reaction leading to ${ }^{85} \mathrm{Ge}$ which is compared with calculated distorted wave impulse approximation (DWIA) distributions. The ${ }^{85,87} \mathrm{Ge}$ results are compared with large-scale shell-model calculations and potential energy surface calculations based on the total Routhian surface formalism.

## I. INTRODUCTION

Exotic, neutron-rich, atomic nuclei provide us with new phenomena not found near stability. A particularly striking example is the evolution of shell structure as a function of neutron or proton number, which has forced a revision of the concept of nuclear magic numbers $[1-3]$. The shifting of single-particle levels as more

[^1]neutrons are added in the isotopic chains can significantly influence the spin-orbit energy splitting, resulting in rearrangements of the pronounced shell gaps observed closer to stability see, e.g., refs. [4-8]. It might be one effect of the prominent tensor interaction component of the nucleon-nucleon force in addition to the isospin-dependent part of the spin-orbit interaction. The neutron-rich nuclide ${ }_{28}^{78} \mathrm{Ni}_{50}$ was recently found to exhibit structural features consistent with a doubly-magic "stronghold" against such effects while neighboring, more neutron-rich systems were predicted to be subject to a
breakdown of the $N=50$ and $Z=28$ magic numbers [9]. Here, the predicted rise in energy of the proton $f_{7 / 2}$ orbital may cause a reduction of the $Z=28$ shell gap. The rise in energy can also be understood from a simple mean-field perspective in relation to the fact that nucleons with large angular momentum are more sensitive to the change of nuclear potential inside the nucleus and lose their energies much faster than those with small $l$ values when the potential gets shallower approaching the drip-line $[10,11]$. Other orbitals that may be affected similarly are the neutron orbitals above $N=50$, where the $s_{1 / 2}$ (and $d_{5 / 2}$ ) orbital is lowered relative to the rising $g_{7 / 2}$ orbitals [9]. The relative lowering of the above low$l$ orbitals can drastically weaken the $Z=28$ and $N=50$ shell gaps. On the other hand, theoretical calculations tend to suggest that the weakening of the spin-orbit shell gaps may not be as significant as for the shell gaps predicted to emerge for harmonic oscillator magic numbers like $N=8$ and $20[11]$, which is still an open problem.

Therefore, the evolution of single-particle levels as a function of increasing neutron numbers in this region of the nuclear chart is an important testing ground for nuclear models. One consequence of the rapid migration and rearrangement of single-particle levels in the isotopic chains is the resulting changes in spin and parity for the ground-states of the odd-mass nuclides, which may also lead to significant variations in $\beta$-decay halflives. This region of nuclei overlaps the astrophysical r-process path in the nuclear chart and their structure can influence the rates at which heavier nuclei are produced in cosmic explosions, such as in the recently observed neutron star merger GW170817, which was characterized as an r-process site with parallel observations of gravitational waves and in different electromagnetic regions, from radio frequencies to $\gamma$ rays [12].

## II. EXPERIMENTAL SET-UP

The experiment was performed at the Radioactive Isotope Beam Factory (RIBF), Wakô, Japan, operated by the RIKEN Nishina Center for Accelerator-Based Science and the Center for Nuclear Study of the University of Tokyo. A ${ }^{238} \mathrm{U}$ beam with an intensity of approximately 30 pnA was accelerated to $345 \mathrm{MeV} / \mathrm{u}$ and impinged on a 3 -mm-thick Be target to create rare isotope beams via inflight fission. The Be target was placed at the entrance of the fragment separator BigRIPS [13, 14]. From the cocktail of isotopes produced in the fast-fission reactions, the isotopes of interest were selected by the $\mathrm{B} \rho-\Delta \mathrm{E}-\mathrm{B} \rho$ method and identified event-by-event by ToF-B $\rho-\Delta \mathrm{E}$ method [14]. Two different settings, magnetically centered on ${ }^{89} \mathrm{As}$ and ${ }^{85} \mathrm{Ge}$ were used to populate the ${ }^{87} \mathrm{Ge}$ and ${ }^{85} \mathrm{Ge}$ nuclide, respectively. Data were acquired for 10.5 hours and 22 hours for the ${ }^{89} \mathrm{As}$ and ${ }^{85} \mathrm{Ge}$ setting, with a full cocktail event rates of $140 \mathrm{~s}^{-1}$ and $730 \mathrm{~s}^{-1}$, respectively. The selected isotope beams were incident on a $99(1)$-mm-thick liquid hydrogen $\left(\mathrm{LH}_{2}\right)$ target at ki-
netic energy approximately $270 \mathrm{MeV} / \mathrm{u}$, which was reduced to around $70 \mathrm{MeV} / \mathrm{u}$ while passing through the $\mathrm{LH}_{2}$ target. The ${ }^{87} \mathrm{Ge}$ and ${ }^{85} \mathrm{Ge}$ nuclei were created via nucleon knockout reactions in the $\mathrm{LH}_{2}$ target and identified by the ZeroDegree spectrometer $[13,14]$ using ToF-B $\rho$ $\Delta \mathrm{E}$ selection. Each reaction vertex was reconstructed using the proton tracks detected in a 300 mm long time projection chamber (TPC), which was surrounding the $\mathrm{LH}_{2}$ target in a setup known as MINOS [15]. The vertex reconstruction precision was around 5 mm FWHM with efficiencies of $64 \%$ and $65.5 \%$ for the ( $\mathrm{p}, 2 \mathrm{pn}$ ) and ( $\mathrm{p}, \mathrm{pn}$ ) reactions, respectively. Gamma-rays emitted in the reactions were detected in the $186 \mathrm{NaI}(\mathrm{Tl})$ scintillator array DALI2 [16], covering the polar angle range $12^{\circ}-118^{\circ}$ with respect to the central beam axis and the center of the $\mathrm{LH}_{2}$ target inside MINOS. The reaction vertex reconstruction enabled precise Doppler correction of the $\gamma$ rays event-by-event as described in Ref. [17]. DALI2 was calibrated with peaks from 121 keV to 1332 keV with ${ }^{152} \mathrm{Eu},{ }^{60} \mathrm{Co},{ }^{137} \mathrm{Cs}$ and ${ }^{133} \mathrm{Ba}$ sources and an energy resolution of 37 keV (FWHM) were obtained for the 344 keV peak of ${ }^{152} \mathrm{Eu}$. Monte Carlo simulations using the GEANT4 tool kit [18] predict the full-energy peak detection efficiency of DALI2 to be $35 \%$ at 500 keV and a source velocity of 0.6 c [16]. The DALI2 detection threshold was set around $100-150 \mathrm{keV}$ (in the center-ofmomentum frame) with higher values for the backward detectors due to the kinematic boost of the $\gamma$ rays in the forward direction.

## III. DATA ANALYSIS AND RESULTS

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\text { A. }{ }^{85} \mathrm{Ge}
$$

The Doppler-corrected singles $\gamma$-ray energy spectra obtained for ${ }^{85} \mathrm{Ge}$ are shown in Fig. 1. The spectra are fitted with the response functions and line shapes obtained from the GEANT4 Monte Carlo simulations with a twocomponent Landau background function. The results of the fits were analyzed using the maximum likelihood and $\chi^{2}$ methods. The uncertainties in the fitted $\gamma$-ray energies and intensities include the the uncertainty from the energy calibration, and the statistical uncertainties from the fitting procedure.

The structure of ${ }^{85} \mathrm{Ge}$ has previously been studied following $\beta$ decay of ${ }^{85} \mathrm{Ga}[19,20]$ and $\beta$-delayed neutron decay of ${ }^{86} \mathrm{Ga}$ [21]. In the recent work of Miernik et al. eight $\gamma$-ray transitions belonging to ${ }^{85} \mathrm{Ge}$ with energies $107.7,365.4,472.6,595.8,773.2,788.5,1589.4$, and 2240.5 keV were identified and placed in a level scheme following $\beta$ decay of ${ }^{85} \mathrm{Ga}$ [20]. An additional transition at 250 keV was observed following $\beta$ delayed neutron emission [21]. The previous study of Korgul et al. reported two additional peaks at 703 and 793 keV energies [19]. In the present in-beam study, data for ${ }^{85} \mathrm{Ge}$ were obtained from the three different knockout reactions ${ }^{86} \mathrm{Ge}(\mathrm{p}, \mathrm{pn}),{ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p})$, and ${ }^{87} \mathrm{As}(\mathrm{p}, 2 \mathrm{pn})$ with corre-



FIG. 1. (Color online) Doppler corrected $\gamma$-ray energy spectra measured by DALI2 obtained from three different nucleon knockout reactions as indicated in the top left corner of each panel. A $\gamma$-ray multiplicity cutoff of $\mathrm{M} \leqslant 4$ was chosen to optimize the spectra with respect to the low-energy atomic bremsstrahlung background. The experimental data are shown as open black diamonds with error bars, and the solid dark blue line is a fit to the entire spectrum. The background was fitted with a double Landau function and is given by the blue dashed line. The Monte Carlo simulated response functions for each transition are shown as a red line. The inset figures are highlight the high energy region with a logarithmic intensity scale
sponding inclusive cross-sections, $\sigma_{\text {inc }}=38.62(2.98) \mathrm{mb}$, $7.65(1.92) \mathrm{mb}$ and $9.07(2.02) \mathrm{mb}$ respectively. Although each reaction has different average excitation energy they mainly produced the same transitions as shown in Fig. 1. Seven transitions with peak centroid energies of around $250(11), 365(22), 472(26), 595(25), 665(37), 703(33)$, and $790(36) \mathrm{keV}$ were fitted with the corresponding response function for all reactions. Seven additional transitions with peak centroids at $878(39), 960(45), 1200(70)$, $1452(75), 1589(77) 1700(80), 2241(105)$, and $2500(124)$ keV were observed in the 1 -dimensional $\gamma$-ray spectra for the different reactions. It is likely that 773 keV and 789 keV transitions reported in Ref. [19, 20] merge into the 790 keV peak, which is observed in the present work due to the lower energy resolution of the NaI spectrum.
$E_{\gamma}-E_{\gamma}$ coincidence relationships were investigated us-
ing a $2 \mathrm{D} E_{\gamma}-E_{\gamma}$ correlation matrix, which was created from the Doppler-corrected $\gamma$-ray data for each knockout reaction leading to ${ }^{85} \mathrm{Ge}$. Because of the abundance of low-energy bremsstrahlung photons in the range up to around 200 keV , it was difficult to identify the previously reported $107 \mathrm{keV} \gamma$-ray transition, which was assigned to connect the lowest excited state to the ground state in ${ }^{85} \mathrm{Ge}[19,20]$ in the singles spectra. However, it was possible to observe this transition by selecting proper $\gamma$-ray coincidence gates and selecting low-multiplicity events involving low-threshold detectors in the forward direction. The 773 and 788 keV transitions could also be separated in this way for some coincidence results.

The 250 keV transition, that is clearly visible for all knockout reactions, was also populated in the $\beta$-delayed neutron decay of ${ }^{86} \mathrm{Ga}$ [21] while it was not observed in the $\beta$ decay of ${ }^{85} \mathrm{Ga}[19,20]$. This indicates a spin-parity of the corresponding state at 250 keV that produces a $\beta$ decay hindrance in the $\beta$ decay from ${ }^{85} \mathrm{Ga}$, which is not present in the $\beta$-delayed neutron decay. In addition, the 250 keV relative transition intensities were different in each reaction. The neutron knockout reaction populates the 250 keV transition with approximately twice as high relative intensity as compared with the proton knockout reaction. Furthermore, in the $\gamma-\gamma$ coincidence analysis, we observe that the 250 keV peak is only in coincidence with the 2500 keV transition, which is only observed in the neutron knockout reactions (p,pn) and (p,2pn). The spectra in coincidence with the 250 and 2500 keV transitions are shown in Fig.2. Based on these observations, we place the 2500 keV transition to directly to feed the 250 keV state in the level scheme (Fig 5).

Similar to the 2500 keV transition the 1452 keV transition is populated only in the ( $\mathrm{p}, \mathrm{pn}$ ) and ( $\mathrm{p}, 2 \mathrm{pn}$ ) reactions. The 1452 keV transition energy matches the energy gap between the 2348 and 896 keV states, which were observed previously [20]. A possible explanation for that the 1452 keV transition was not observed in previous work may be that the 1455 keV transition, which belonged to ${ }^{84} \mathrm{As}$ populated by secondary $\beta$ decay was masking the 1452 keV transition in the spectrum. Although the coincidence analysis shows the coincidence with 789 keV , the placement of the 1452 keV is not certain. One of the reason for not placing the 1452 keV $\gamma$-ray decay from 2348 keV level is the unpopulated 2241 keV transition, which should decay from the same level. Another possibility of is the coincidence with 773 keV , however for this gated spectrum it is not possible to distinguish 773 and 789 keV transitions by coincidence with the 107 keV transition. Therefore the 1452 keV has not been placed in the level scheme. Both the 960 and the 878 keV transitions are visible in the neutron knockout reaction and in coincidence with the 789 and 107 keV transitions, see Fig 3. We place them as directly feeding the 896 keV state in the level scheme. In figure $3(\mathrm{a})$, the 960 keV transition is also in coincidence with the 665 keV transition. The 665 keV gated (in the range 630690) spectrum revealed four peaks at energies $789 / 773$,

960,1452 and 2241 keV . However the 1452 and 2241 keV transitions have lower than $2 \sigma$ significance, therefore only two real coincidence observed in the 665 keV gated histogram. The most probable placement of the 665 keV transition is in the cascade with 960 and 789 so, it is tentatively placed on top of the 1856 keV level in the newly constructed level scheme.

The single proton knockout reaction ${ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p}){ }^{85} \mathrm{Ge}$ gave rise to two additional peaks at 1200 and 1700 keV in the singles spectra. For the coincidence analysis of the 1200 keV transition, we have adopted the gate region $1150-1250 \mathrm{keV}$ with a suitable background gate. The resulting coincidence spectrum revealed peaks at 596 and 703 keV , see Fig.4. The 703 keV transition has previously been found to decay directly to the ground state from the 703 keV level [19], while it was not observed in a later study [20] with lower statistics. Since both the 703 keV and 595 keV transitions decay from the 703 keV level, the 1200 keV transition is placed as decaying into the 703 keV level from a new level at an excitation energy of 1903 keV . The 1700 keV peak is clearly visible in the Doppler corrected $\gamma$-ray energy spectrum from the ${ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p})^{85} \mathrm{Ge}$ reaction shown in Fig 1(b). The corresponding coincidence gate in the $\gamma-\gamma$ matrix in the range of $1650-1740 \mathrm{keV}$ shows that a peak in the $773 / 789 \mathrm{keV}$ region, however it was not possible to distinguish these transitions from each other, therefore it was not placed in the level scheme.
The coincidence analysis was also applied to previously known transitions in each reaction. For the ${ }^{86} \mathrm{Ge}(\mathrm{p}, \mathrm{pn}){ }^{85} \mathrm{Ge}$ reaction, a combined gate on $773 / 789$ keV shows coincidence with the 878,960 , and 665 keV transitions in the multiplicity less than 4 selection. The same coincidence gate $(773 / 789 \mathrm{keV})$ for the ${ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p}){ }^{85} \mathrm{Ge}$ reaction gives different results than the neutron knockout reaction and shows coincidence with the 1700 keV peak. Another known transition is 596 keV , visible in the singles spectra for all reactions. By performing a coincidence cut on the $E_{\gamma}-E_{\gamma}$ coincidence matrix in the range of $560-620 \mathrm{keV}$, a transition around 1200 keV is weakly visible in the ${ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p})^{85} \mathrm{Ge}$ reaction, while there is no clear evidence for such a peak in the ${ }^{86} \mathrm{Ge}(\mathrm{p}, \mathrm{pn}){ }^{85} \mathrm{Ge}$ reaction.
The coincidence analysis results from the protonneutron knockout reaction ${ }^{87} \mathrm{As}(\mathrm{p}, 2 \mathrm{pn}){ }^{85} \mathrm{Ge}$ was in agreement with the results from the single-neutron and single-proton knockout reactions discussed above. A summary of $\gamma$-rays assigned to ${ }^{85} \mathrm{Ge}$ is shown in Table I.

## B. ${ }^{87} \mathrm{Ge}$

No gamma-ray transitions have been previously reported for the nucleus ${ }^{87} \mathrm{Ge}$ and its half-life is unknown. In the present experiment, ${ }^{87} \mathrm{Ge}$ nuclei were produced by the two reactions ${ }^{89} \mathrm{As}(\mathrm{p}, 2 \mathrm{pn})$ and ${ }^{90} \mathrm{Se}(\mathrm{p}, 3 \mathrm{pn})$ corresponding to $72 \%$ and $28 \%$ of the total statistics ob-


FIG. 2. Gated spectrum for ${ }^{86} \mathrm{Ge}(\mathrm{p}, \mathrm{pn}){ }^{85} \mathrm{Ge}$ reaction, Spectra have been produced with forward angle detector condition to improve peak background ratio.(a) Coincidence cut in the $2200-2800 \mathrm{keV}$ range for the $2500(124) \mathrm{keV}$ transition with background subtraction (2900-3700) the extra multiplicity cut $\mathrm{M} \leq 4$ has been applied to decrease low energy background. (b) Coincidence cut in the $220-280 \mathrm{keV}$ range for $250(11) \mathrm{keV}$ transition with background subtraction (1000-1100)


FIG. 3. Gated spectrum for ${ }^{86} \mathrm{Ge}(\mathrm{p}, \mathrm{pn})^{85} \mathrm{Ge}$ reaction, Spectra have been produced with multiplicity $\mathrm{M} \leq 4$ and limited forward angle detector condition to improve peak background ratio.(a) Coincidence cut in the $910-990 \mathrm{keV}$ range for the $960(45) \mathrm{keV}$ transition with background subtraction (1030$1170 \& 870-900$ ). (b) Coincidence cut in the $860-920 \mathrm{keV}$ range for $878(39) \mathrm{keV}$ transition with background subtraction (1070-1110)

TABLE I. Gamma-ray transitions observed in this work for ${ }^{85} \mathrm{Ge}$. The relative intensities of the transitions as observed in the different knockout reactions are normalized with respect to the most intense $789+773 \mathrm{keV}$ peak. The coincidence analysis results are shown in the right colum of the table.

| $E_{\gamma}(\mathrm{keV})$ | $\begin{gathered} I_{\gamma} \\ { }^{86} \mathrm{Ge}(\mathrm{p}, \mathrm{pn}) \\ \hline \end{gathered}$ | $\begin{gathered} I_{\gamma} \\ { }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p}) \\ \hline \end{gathered}$ | $\begin{gathered} I_{\gamma} \\ { }^{87} \mathrm{As}(\mathrm{p}, 2 \mathrm{pn}) \\ \hline \end{gathered}$ | Coincident $\gamma$-ray transitions (keV) |
| :---: | :---: | :---: | :---: | :---: |
| 250(11)* | 45.7(6.3) | 23.4(6.3) | 12.1(2.1) | 2500 |
| 365(22)* | 30.6(5.4) | 27.3(6.8) | 11.9(2.3) | - |
| 472(26)* | 20.4(5.2) | 26.2(7.2) | 6.6(2.3) | - |
| 595(25)* | 33.6(7.5) | 49.86(5.7) | 26.4(5.2) | 1200 |
| 665(37) | 21.4(8.4) |  | 30.4(5.9) | 107, 789, 960 |
| 703(33)* | 17.5(6.4) | 24.6(9.3) | 18.2(3.9) | 1200 |
| 789,773(36)* | 100.0(12.3) | 100.0(16.8) | 100.0(7.3) | 665, 878, 960 |
| 878(39) | 60.9(10.8) | - | 20.0(3.8) | 789, 107 |
| 960(45) | 37.4(7.5) | - | - | 789, 665, 107 |
| 1200(70) | - | 27.9(10.3) | - | 596, 703 |
| 1452(75) | 34.8(7.6) | - | - |  |
| 1589(77)* |  |  | 16.6(3.8) |  |
| 1700(80) | - | 76.7(14.5) | - |  |
| 2241(105)* | - | 61.9(12.1) | - |  |
| 2500(124) | 41.59(7.19) | - | 7.8)2.9) | 250 |

* Transitions observed in previous studies.


FIG. 4. Gated spectrum for ${ }^{86} \mathrm{As}(\mathrm{p}, 2 \mathrm{p})^{85}$ Ge reaction, Spectra have been produced with forward angle detector condition to improve peak background ratio. Coincidence cut in the $1150-1250 \mathrm{keV}$ range for the $1200(70) \mathrm{keV}$ transition with background subtraction (2800-4000)
tained for this nuclide, respectively. The inclusive cross sections for the reactions are $10.34(0.87) \mathrm{mb}$ for the proton-neutron knockout reaction and $1.47(0.12) \mathrm{mb}$ for the 2 -proton-neutron knockout reaction. With a very low statistics, ${ }^{87} \mathrm{Ge}$ is also populated via single-neutron knockout from ${ }^{88} \mathrm{Ge}$ isotope. The statistics is not enough to calculate the cross sections, around 600 ions were detected by the BigRIPS and the ZeroDegree spectrometer.
The $\gamma$-ray energy spectra observed for each reaction are shown in Fig. 6. Three peaks at energies 250(12), $510(37)$, and $630(31)$ were observed in the single gammaray energy spectra. The 630 keV peak is visible only in the spectrum produced from the 2 proton-1neutron knockout reaction from ${ }^{90} \mathrm{Se}$. The intensity of such transition has a large uncertainty, and the existence of the transition is questionable with this statistics. The tran-

TABLE II. Gamma-ray transitions in ${ }^{87} \mathrm{Ge}$ observed in this work. The relative intensities of the transitions as observed in the different knockout reactions are normalized with respect to the most intense 250 keV transition.
${ }^{87} \mathrm{Ge}$

| $E_{\gamma}(\mathrm{keV})$ | $I_{\gamma}$ <br> ${ }^{89} \mathrm{As}(\mathrm{p}, 2 \mathrm{pn})$ | $I_{\gamma}$ <br> ${ }^{90} \mathrm{Se}(\mathrm{p}, 3 \mathrm{pn})$ |
| :--- | :--- | :--- |
| $250(12)$ | $100(19)$ | $100(30)$ |
| $510(37)$ | $54(20)$ | - |
| $630(31)$ | - | $52(46)$ |

sition is added to the fitting procedure, since it makes a differences in $\chi^{2}$ values, however with the large uncertanity, it is not proven that it belongs to ${ }^{87} \mathrm{Ge}$.

A tentative level scheme for ${ }^{87} \mathrm{Ge}$ is shown in the right side of the Fig 6 based on the following observations. For both reactions, the most intense observed $\gamma$-ray transition is 250 keV , which is assigned to decay from the second excited state to the ground state. Despite of the limited statistics the 250 keV and the 510 keV transitions appear not to be in coincident, and they were placed in a tentative level scheme as directly feeding the ground state. The energies and relative intensities of the observed $\gamma$-ray transitions in ${ }^{87} \mathrm{Ge}$ are shown in Table II for the two different reactions.

## IV. THEORETICAL CALCULATIONS

## A. Large space shell model calculations

We have performed detailed large-space shell model (LSSM) calculations in order to investigate the struc-


FIG. 5. Measured transition energies and proposed level scheme for ${ }^{85} \mathrm{Ge}$. Symbols ${ }^{*} *$, denote transition energies taken from the literature[20]. Solid red arrows and horizontal lines indicate transitions and levels deduced from the neutron knockout reaction, while the blue represent the proton knockout reaction results measured for the first time in the present work. The dashed arrow indicates tentative placement. The calculated single neutron separation energy from Ref [22] is also shown in the figure.
ture of highly neutron-rich Ge isotopes. The calculation were carried out in two different model spaces. First, the calculations were performed within the proton $\pi\left(1 p_{1 / 2}, p_{3 / 2}, f_{5 / 2}, g_{9 / 2}\right)$ (denoted as fpg) and neutron $\nu\left(g_{7 / 2}, d_{3 / 2,5 / 2}, s_{1 / 2}, h_{11 / 2}\right)$ (denoted as gdsh) model space with respect to the ${ }_{28}^{78} \mathrm{Ni}_{50}$ core. The proton-proton effective interaction, which was optimized for nuclei with both protons and neutrons within the fpg model space, was taken from Ref. [23]. The neutron-neutron interaction was taken from the monopole-optimized realistic CD Bonn nucleon-nucleon potential of Ref. [24], which is known to describe well nuclei just above $N, Z=50$. We used the same realistic nucleon-nucleon potential for the cross-shell neutron-proton interaction [25]. For comparison, calculations were also carried out in the model space $\pi \nu\left(1 p_{1 / 2}, p_{3 / 2}, f_{5 / 2}, g_{9 / 2}, g_{7 / 2}, d_{5 / 2}\right)$ (denoted as fpgd) in order to investigate the influence of the $N=50$ core breaking effect. That expanded model space includes all orbitals between the $N=Z=28$ and $N=Z=64$


FIG. 6. (Color online) DALI2 Doppler corrected $\gamma$-ray spectra for ${ }^{87} \mathrm{Ge}$ measured for the ${ }^{89} \mathrm{As}(\mathrm{p}, 2 \mathrm{pn})$ and ${ }^{90} \mathrm{Se}(\mathrm{p}, 3 \mathrm{pn})$ and ${ }^{88} \mathrm{Ge}(\mathrm{p}, \mathrm{pn})$ reactions. For all reactions, the forward-angle detectors relative to the beam were used in order to reduce the background in the low-energy region. Blue diamonds with error bars mark the experimental data, the continuous black line is the fit of the whole spectrum, and the corresponding background (blue dashed line) is fitted with a double landau function. The simulated response function for each transition is shown as a red dotted line. In the right bottom, proposed level scheme for ${ }^{87} \mathrm{Ge}$ is shown, the thickness of the transitions are relative to intensities from ${ }^{89} \mathrm{As}(\mathrm{p}, 2 \mathrm{pn})$ reaction.
subshells. The Hamiltonian was the same as that used in Refs. [26] and [27], including core excitation effects in nuclei around ${ }^{100} \mathrm{Sn}$. With that Hamiltonian, no significant contribution from the neutron excitation across the $N=50$ shell gap was found for ${ }^{83,85,87} \mathrm{Ge}$. The result is consistent with previous systematic potential energy surface calculations, which show that the nuclei ${ }^{83,85} \mathrm{Ge}$ are near-spherical, while ${ }^{87} \mathrm{Ge}$ may show a modest quadrupole deformation [28]. A significant contribution from the neutron excitation from the $g_{9 / 2}$ orbital to the $d_{5 / 2}$ orbital would have led to strong neutron-proton quadrupole interaction and thus a large deformation. A more noticeable weakening of the $N=50$ shell may be expected when the proton number is reduced to $Z=28$ (see, for example, ref $[11,29]$ ). In the following discussion, we focus on the first group of calculations (in $\pi \mathrm{fpg}$ $\nu$ gdsh space) without explicit core breaking effects.

A priori, it can be assumed that a relatively considerable uncertainty in the calculation may come from the relative positions of the $d_{5 / 2}$ and $g_{7 / 2}$ orbitals. These single-particle levels have proven to be nearly degenerate in nuclei around ${ }^{100} \mathrm{Sn}$ (i.e., signaling the onset of the so-called pseudospin symmetry). However, as the proton number decreases, the $d_{5 / 2}$ orbital is expected to become lower in energy than the $g_{7 / 2}$ orbital, in rela-
tion to the mean-field drifting effect as mentioned in the Introduction, even though little is known for the cases of neutron-rich nuclei when one approaches $Z=28$. In the $N=51$ isotones ${ }^{85} \mathrm{Se}$ and ${ }^{87} \mathrm{Kr}$, the ground states are tentatively assigned as $I^{\pi}=5 / 2^{+}$, whereas the $7 / 2^{+}$ state may be found at an excitation energy of around 1.5 MeV higher. The $s_{1 / 2}$ orbital is also expected to come lower as one approaches the neutron drip line (see, for example, $[11,30]$ ). By considering the above theoretical uncertainty, we have performed two calculations in the $\pi \mathrm{fpg}-\nu \mathrm{gdsh}$ space in order to study the effect of the relative energy shift between the $g_{7 / 2}$ and $d_{5 / 2}$ orbitals. In the first calculation, we used the original single-particle energies determined from the monopole optimization. In the second one, we raised the energy of the $\nu 7 / 2^{+}$s.p. level by 1.5 MeV to study the influence of its energy evolution on the wave function from a qualitative point of view.
With the original single-particle energy set, the ground state of ${ }^{83} \mathrm{Ge}$ is calculated to be $7 / 2^{+}$(with dominant $g_{7 / 2}$ parentage) whereas the lowest $5 / 2^{+}$state (with dominant $d_{5 / 2}$ parentage) is calculated to be nearly 400 keV higher. As expected, the $7 / 2^{+}$state is nearly 1 MeV above the $5 / 2^{+}$ground state in the second calculation. Consider how such a change influences the wave functions of ${ }^{85,87} \mathrm{Ge}$. We note that low-lying states are mostly dominated by the coupling of protons in the $p_{3 / 2}, f_{5 / 2}$ subshells, and neutrons in the $g_{7 / 2}, d_{5 / 2}$ subshells. For three (or five) neutrons in the mixed $g_{7 / 2}, d_{5 / 2}$ configuration, one can expect an interesting competition between the seniority-one (with only the odd un-paired particle) states and seniority-three (one broken pair) states for spin-parity values $5 / 2^{+}$and $7 / 2^{+}$(see, for example, Ref. [24]). In the case of ${ }^{85} \mathrm{Ge}$, the ground state is calculated to become $7 / 2^{+}$instead of $5 / 2^{+}$after raising the $g_{7 / 2}$ orbital energy. This favors a seniority-one configuration for that state.
The spin-parity of the ground state and first excited state in ${ }^{87} \mathrm{Ge}$ are calculated to be $3 / 2^{+}$and $5 / 2^{+}$, respectively. The excitation energies and wave functions of the low-lying states are much less influenced by the shift of the neutron $g_{7 / 2}$ orbital compared with ${ }^{83,85} \mathrm{Ge}$. Furthermore, the calculations do not predict any significant contribution from the $d_{3 / 2}$ orbital to the ground state configuration of ${ }^{87} \mathrm{Ge}$. The ${ }^{87} \mathrm{Ge}$ ground state is predicted to be remarkably dominated by the coupling of seniority-three neutron states and proton particle-hole excitations. Fig. 7, 8, 9 shows the calculated level energies compared with the tentative experimental level energies. We have also calculated the low-lying states in the even-even ${ }^{84,86} \mathrm{Ge}$ isotopes. The $2^{+}$state energies are calculated to be around 1.0 and 0.64 MeV for ${ }^{84,86} \mathrm{Ge}$, respectively. The corresponding experimental values are 624 keV [19] and 527 keV [21], respectively. This indicates an increased collectivity compared with the theoretical results suggesting that a larger model space might be needed to describe these states by means of LSSM calculations more accurately. Neither the energies nor the


FIG. 7. Comparison between observed excitation energies and large-scale shell model calculations for ${ }^{83} \mathrm{Ge}$. "SM" denotes calculations with standard parameters, and "SM2" denotes calculations for which the energy of the neutron $7 / 2^{+}$s.p. level is raised by 1.5 MeV in order to investigate the competition effect between $\nu 7 / 2^{+}$and $\nu 5 / 2^{+}$. The experimental level energies and spin-parity assignments were taken from Ref. [31]. $S_{n}$ value is taken from Ref. [22]
wave functions are found not to be sensitive to an energy shift of the neutron $g_{7 / 2}$ orbital.

## B. Total Routhian surface calculations

Previous studies of even-even Ge isotopes have indicated the existence of a new triaxial-deformed region in the neutron-rich part of the Ge isotopic chain. Lettman et al. [32] performed symmetry-conserving configuration mixing Gogny (SCCM) calculations for which the predicted potential energy surfaces revealed triaxial minima for both ${ }^{86} \mathrm{Ge}$ and ${ }^{88} \mathrm{Ge}$. The nucleus ${ }^{88} \mathrm{Ge}$, however, revealed a more significant $\beta$ deformation as well as an increased $\gamma$ softness. From the analysis of E2 matrix elements, a value of the quadrupole shape invariant, $K_{3}$, of 0.0027 was derived for both ${ }^{86,88} \mathrm{Ge}$, corresponding to an effective triaxial deformation $\gamma=29.5$, i.e., near maximum triaxiality. The fluctuations in $K_{3}$ indicated that ${ }^{86} \mathrm{Ge}$ has a larger degree of triaxial rigidity than ${ }^{88} \mathrm{Ge}$,


FIG. 8. Comparison between observed excitation energies and large-scale shell model calculations for ${ }^{85} \mathrm{Ge}$. "SM" denotes calculations with standard parameters, and "SM2" denotes calculations for which the energy of the neutron $7 / 2^{+}$s.p. level is raised by 1.5 MeV in order to investigate the competition effect between $\nu 7 / 2^{+}$and $\nu 5 / 2^{+}$. The tentative spinparity assignments were taken from Ref. [19]. $\mathrm{S}_{n}$ value is taken from Ref. [22]
and therefore ${ }^{86} \mathrm{Ge}$ was predicted to exhibit the most stable triaxial shape in this region of the nuclear chart. Also, as reported in Ref. [33], another study of non-axial nuclei beyond doubly-magic ${ }^{78} \mathrm{Ni}$ indicated that a maximum of triaxiality could appear in ${ }^{86} \mathrm{Ge}$.
In order to investigate further the shape evolution in the most neutron abundant germanium isotopes, we have performed Woods-Saxon total Routhian surface (TRS) calculations based on the cranked Strutinsky formalism [34]. Pairing correlations were taken into account by means of seniority and double stretched quadrupole pairing force [34] and the Lipkin-Nogami method [35]. The results are shown in Figs. 10, 11, and 12.
For the even-even ${ }^{84,86,88} \mathrm{Ge}$ isotopes (Fig. 10), the TRS calculations were performed for the vacuum groundstate configuration. The potential energy surface (PES) of ${ }^{84} \mathrm{Ge}$ exhibits an energy minimum as a function of the deformation parameters $\beta_{2}$ and $\gamma$ at a triaxial quadrupole-deformed shape with the shape elongation parameter, $\beta_{2} \approx 0.18$ and the triaxial deformation parameter $\gamma \approx 0^{\circ}$. The ground-state $\beta_{2}$-deformation increases with increasing neutron number, accompanied by an increase in both softness and deformation in the triaxial degree of freedom. For mass number $A=86$, the pre-


FIG. 9. Comparison between observed excitation energies and large-scale shell-model calculations for ${ }^{87} \mathrm{Ge}$. "SM" denotes calculations with standard parameters, and "SM2" denotes calculations for which the energy of the neutron $7 / 2^{+}$s.p. level is raised by 1.5 MeV in order to investigate the competition effect between $\nu 7 / 2^{+}$and $\nu 5 / 2^{+}$. Experimental level energies are taken from the present work. $S_{n}$ value is taken from Ref. [22]
dicted ground-state deformation is ( $\beta_{2} \approx 0.22, \gamma \approx 17^{\circ}$ ) while for $A=88$ the predicted ground-state deformation is $\left(\beta_{2} \approx 0.25, \gamma \approx 15^{\circ}\right)$. In all three cases, the potential energy surfaces reveal a competition from a near-degenerate gamma-soft prolate configuration, which is calculated to become less favored at non-zero rotational frequency. The predicted softness in the PES for these even- $N$ germanium isotopes in the $\gamma$-degree of freedom indicates susceptibility to shape-polarizing effects induced by the occupation of certain single-particle orbits. For example, the lowest negative-parity $1 / 2[550]$ Nilsson orbit is an intruder configuration emanating from the $h_{11 / 2}$ subshell with a strong shape-driving force towards positive $\gamma$ deformation [36]. For the odd-even ${ }^{85,87} \mathrm{Ge}$ isotopes, the TRS calculations were performed with blocking of one neutron orbital with different parity and signature configurations. The results are shown in Fig. 11 and Fig. 12. The PES of the odd-neutron germanium isotopes ${ }^{85,87} \mathrm{Ge}$ exhibit a similar evolution of shapes as for the even-even isotopes with increasing neutron number. A notable difference is, however, the much more pronounced softness in the triaxial $\gamma$ degree of freedom for the lowest negative-parity configurations. This effect is not present in the lowest PES


FIG. 10. Total Routhian surfaces in the $\beta_{2} \sin \left(\gamma+30^{\circ}\right)-$ $\beta_{2} \cos \left(\gamma+30^{\circ}\right)$ plane for the vacuum ground-state configuration in ${ }^{84,86,88} \mathrm{Ge}$. The red dot indicates the position of the ground- state minimum. Equipotential energy lines are separated by 0.2 MeV .


FIG. 11. Total Routhian surfaces in the $\beta_{2} \sin \left(\gamma+30^{\circ}\right)-$ $\beta_{2} \cos \left(\gamma+30^{\circ}\right)$ plane for the lowest signature and parity $(\pi, \alpha)$ configurations in ${ }^{85} \mathrm{Ge}$. The red dot indicates the position of the lowest-lying minimum for each potential energy surface. Equipotential energy lines are separated by 0.2 MeV . (a) blocking of the lowest ( $\pi=-, \alpha=-1 / 2$ ) configuration. (b) blocking of the lowest ( $\pi=+, \alpha=+1 / 2$ ) configuration.
for the positive-parity configurations. In the absence of any other negative-parity configurations than those mentioned above, $1 / 2[550]$ Nilsson intruder configuration near the Fermi surface, it is likely the shape-driving properties of this orbital that is causing the increased $\gamma$ softness in the negative-parity PES. This orbit is also strongly favored by an increasing $\beta_{2}$ deformation and hence comes closer to the Fermi surface with increasing neutron number. Interestingly, this trend also seen in the LSSM calculations for which the negative-parity states of $h_{11 / 2}$ parentage move down rapidly with neutron number between $N=51$ and $N=55$, see Figs. 7, 8, and 9.

## C. Parallel momentum distributions with DWIA calculations for ${ }^{85} \mathrm{Ge}$

It was possible to extract parallel momentum distributions of ${ }^{85} \mathrm{Ge}$ nuclei following the ( $\mathrm{p}, \mathrm{pn}$ ) reaction. These experimental results were compared with calculated mo-


FIG. 12. Total Routhian surfaces in the $\beta_{2} \sin \left(\gamma+30^{\circ}\right)-$ $\beta_{2} \cos \left(\gamma+30^{\circ}\right)$ plane for the lowest $(\pi, \alpha)$ configurations in ${ }^{87} \mathrm{Ge}$. The red dot indicates the position of the lowest-lying minimum for each potential energy surface. Equipotential energy lines are separated by 0.2 MeV . (a) blocking of the lowest ( $\pi=-, \alpha=-1 / 2$ ) configuration. (b) blocking of the lowest ( $\pi=+, \alpha=+1 / 2$ ) configuration.
mentum distributions of neutron removal from $s_{1 / 2}, d_{3 / 2}$ and $d_{5 / 2}$ orbitals populating final states in ${ }^{85} \mathrm{Ge}$ using the distorted wave impulse approximation (DWIA) model [37]. In the DWIA approach, the single particle wave function and the nuclear density of ${ }^{86} \mathrm{Ge}$ were calculated using the single particle potential of Bohr and Mottelson [38]. Optical potentials for the distorted waves in the states were constructed by the microscopic folding model [39], calculating the nuclear density and employing the Melbourne g-matrix NN interaction [40]. For the $n p$ interaction, the Franey-Love effective interaction is used [41]. The parallel momentum distributions (PMD) in the projectile frame were derived for neutron knockout, ${ }^{86} G e(\mathrm{p}, \mathrm{pn})$ reaction using the TOF and $B \rho$ information of the Zero-Degree spectrometer. The Lorentz transformation was applied using the measured velocity of ${ }^{86} G e$ ions in BigRIPS to correct the momentum spread from the incoming beam. The PMD is shown in Fig. 13. Parallel momentum resolutions of $\sigma=25 \mathrm{MeV} / \mathrm{c}$ were obtained for by measuring the unreacted ${ }^{86} \mathrm{Ge}$ ions. The inclusive momentum distribution for the ${ }^{86} \mathrm{Ge}(\mathrm{p}, \mathrm{pn})^{85} \mathrm{Ge}$ reaction has wider distribution with $\sigma=55 \mathrm{MeV}$. The momentum distribution for one excited state was extracted by fitting the $\gamma$-ray spectra in coincidence with the 40 $\mathrm{MeV} / \mathrm{c}$ small sections of the inclusive distribution. By subtracting the PMD of the feeding transition, one can reach the pure single particle state momentum distributions. By comparing the measured PMDs and DWIA predictions assuming the removal of a neutron or a proton from single- particle states and folded with the experimental momentum resolutions, the angular momentum assignments can be made. As shown in Fig 13, the PMDs of the 250 keV state in ${ }^{86} G e(\mathrm{p}, \mathrm{pn})$ reaction reproduced by the theoretical approximations. Assuming a neutron removed from the $\mathrm{s}_{1 / 2}$ state is agreed well with the experimental data, while $l=2$ transfer from the $\mathrm{d}_{3 / 2}$


FIG. 13. (Color online) Individual longitudinal momentum distributions of the ${ }^{85} G e$ residues from one-neutron knockout reaction. The exclusive momentum distribution for 250 keV level is shown in the graph, compared with calculated DWIA distributions assuming 1 n removal from d (blue line from $\mathrm{d}_{5 / 2}$, pink dashed line from $\mathrm{d}_{3 / 2}$ ) and s (dashed red line from $\mathrm{s}_{1 / 2}$ ) orbitals.
and $d_{5 / 2}$ levels would lead to wider momentum distributions. Therefore the spin and the parity of the 250 keV state is assigned as $1 / 2^{+}$. To assign the spin-parity for the ground state, we need to subtract the excited states PMDs from the inclusive momentum distributions. However in this data set with this method we can only reliably subtract eneries as low as 250 keV , as such the "ground state" distribution would be mixed with the 107keV PMD.

## V. DISCUSSION

There is evidence for a rapid onset of deformation in Ge isotopes beyond $N=50$, both in theoretical studies (e.g., [32]) and from experimental observations, such as from the observation of a 527 keV transition that is interpreted as the de-excitation of the first $I^{\pi}=2^{+}$state in ${ }^{86} \mathrm{Ge}$ following the $\beta$-decay of ${ }^{86} \mathrm{Ge}[21]$. Although it is difficult to draw firm conclusions from the present experimental observations on the collective nature of the observed states, our TRS Strutinsky-type microscopic-macroscopic calculations of PES, as well as LSSM calculations, confirm this picture. While the former shows a clear trend of increasing quadrupole deformation in the predicted near-triaxial shapes as a function of increasing neutron number starting already at $N=52$, the latter predicts a transition from seniority-1 ground-state configurations in ${ }^{83,85} \mathrm{Ge}$ to a ground-state configuration dominated by seniority-3 components in ${ }^{87} \mathrm{Ge}$. Although the LSSM calculations most likely are lacking the full model space needed to take into account well developed collectivity, this is a clear indication that such effects are in play. The theoretical calculations also highlight the importance of the negative-parity $1 / 2[550]$ intruder configuration emanating from the $h_{11 / 2}$ subshell with its strong shape-driving
force towards larger $\beta_{2}$ and positive $\gamma$ deformation [36]. However, the experimental data do not shed any light on whether this orbital is populated near the Fermi level in the neutron-rich germanium isotopes up to mass $A=88$. Another single-particle orbital which is of major interest in shell evolution studies in neutron-rich nuclei is $3 s_{1 / 2}$. Above $N=50$, the neutron $3 s_{1 / 2}$ single-particle state is observed to come down in energy as a function of increasing $N / Z$ ratio relative to the $2 d_{5 / 2}$ level for proton numbers $Z \leq 40[42]$. This opens for the interesting possibility of loosely bound halo-like $l=0$ states close to the ground state [42].

The momentum distributions observed in singlenucleon knockout reactions leading to ${ }^{85} \mathrm{Ge}$ may shed light on this question. The exclusive longitudinal momentum distributions for the ${ }^{86} \mathrm{Ge}(\mathrm{p}, \mathrm{pn})^{85} \mathrm{Ge}$ reactions deduced by selecting events associated with the detection of $\gamma$ rays of 250 keV , i.e., selecting the population of the 250 keV states, tentative assigned as $\left(1 / 2^{+}\right)$.
The 250 keV level was previously observed following $\beta$-delayed neutron decay of ${ }^{86} \mathrm{Ga}$ and proposed as a candidate for the first excited $I^{\pi}=1 / 2^{+}$state [21]. The structure of the first excited $1 / 2^{+}$state is predicted to be dominated by the odd neutron in the $3 s_{1 / 2}$ orbital coupled to the proton $0^{+}$state In order to relate these predictions to the information obtained from the exclusive momentum distributions from the ${ }^{86} \mathrm{Ge}(\mathrm{p}, \mathrm{pn})^{85} \mathrm{Ge}$ reactions shown in Fig.13,the data is compared with the calculated DWIA distributions.

According to the measured exclusive parallel momentum distributions, the $250 \mathrm{keV}\left(1 / 2^{+}\right)$state is associated with $l=0$ s-wave neutron knockout. A spin-parity assignment of $1 / 2^{+}$for the 250 keV state is therefore in agreement with $l=0$ neutron knockout from the ground state of ${ }^{86} \mathrm{Ge}$.

## VI. SUMMARY

A $\gamma$-spectroscopic study of the neutron-rich germanium isotopes ${ }^{85,87} \mathrm{Ge}$ has been performed. The excited states in ${ }^{85,87} \mathrm{Ge}$ were populated in nucleon knockout reactions following fast-fission reactions of ${ }^{238} \mathrm{U}$ at the Radioactive Isotope Beam Factory (RIBF). Two $\gamma$-ray transitions have been observed for the first time in ${ }^{87} \mathrm{Ge}$. In ${ }^{85} \mathrm{Ge}, 13 \gamma$-ray transitions have been observed, 7 of them for the first time in this study. Based on the analysis of intensities and systematics in neighboring Ge isotopes, a level scheme with two excited states is proposed for ${ }^{87} \mathrm{Ge}$, and the previously reported level scheme extended for ${ }^{85} \mathrm{Ge}$. Exclusive parallel momentum distributions obtained for the 250 keV state in ${ }^{85} \mathrm{Ge}$ are in agreement with previous assignments of spin-parity $\left(1 / 2^{+}\right)$. This confirms the predictions of a low-lying neutron $3 s_{1 / 2}$ single-particle state in this neutron-rich nucleus. LSSM and TRS calculations have been performed for the nuclei in the germanium isotopic chain between $A=83$ and $A=87$. The LSSM calculations indi-
cate a close competition between the $\nu 7 / 2^{+}$and $\nu 5 / 2^{+}$ seniority- 1 configurations for the ground states of ${ }^{83} \mathrm{Ge}$ and ${ }^{85} \mathrm{Ge}$, whereas for ${ }^{87} \mathrm{Ge}$, the ground state is predicted to be $3 / 2^{+}$based on the coupling of different seniority$3 \nu 7 / 2^{+}, \nu 5 / 2^{+}$configurations, and proton particle-hole excitations. The LSSM calculations also predict a rapid decrease in the $h_{11 / 2}$ level energies with increasing neutron number, which has been found to agree with the predictions of TRS calculations which indicate increasing quadrupole deformation with increasing neutron number in the isotopic chain. The TRS calculations also predicted consistent triaxial shapes for the lowest-lying configurations with a significant $\gamma$-softness for the negativeparity configurations.

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