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Event-based High Resolution X-ray Imaging using Compton Coincidence Detection

Händelsebaserad högupplöst röntgenavbildning med hjälp av Compton-sammanfallsdetektering

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Abstract

Research on photon counting detectors (PCDs) is focused on semiconductor materials, and silicon is a strong candidate to use in PCDs for photon counting computer tomography (CT). In a silicon detector, a significant portion of the counts is due to Compton scattering events. Since only part of the incident photon energy is deposited in a Compton interaction, Compton interactions lead to a loss of spectral information. By using Compton coincidence detection, i.e., combining information from multiple Compton events caused by the same incident photon, it is possible to obtain more spectral information from Compton scattered photons, increasing the energy resolution of the detector. The goal of this thesis is to develop and evaluate a method for Compton coincidence detection for photon counting CT.

In this thesis, a method for Compton coincidence detection based on Compton kinematics and a χ^2 test is presented and compared to a previously developed method based on maximum likelihood estimation. The χ^2 method utilised the connection between the energy before vs after a Compton interaction, and the scattering angle. The possible scattering angles due to deposited energy in each interaction were called the energy angles. The spatial angles between the interaction positions in the detector were calculated and compared to the energy angles through a χ^2 test in order to find the correct order of interaction and the incident photon energy. The χ^2 method correctly identified the interaction order of 85.8% of simulated interaction chains ending in photoelectric effect and 64.1% of simulated interaction chains containing only Compton interactions. The energy estimation was 100% correct for all chains ending in photoelectric effect, since all of the incident energy was deposited in the detector. For chains of only Compton interactions, the energy was estimated with an RMS error of 21.2 keV. Combining the results from chains ending in a photoelectric interaction and chains of only Compton interactions, the total RMS error of the energy estimation was 11.5 keV.

Keywords

Photon counting CT, Compton scattering, Compton kinematics, coincidence detection, χ^2 test

Sammanfattning

Datortomografi (CT) är en viktig del av dagens sjukvård, och fotonräknande detektorer för CT är på väg från forskning till klinisk användning. Forskningen inom fotonräknande detektorer fokuserar på att använda halvledande material, och kisel är en stark kandidat till att användas för fotonräknande detektorer. I en kiseldetektor interagerar en betydande andel av fotonerna genom Compton-spridning. Då endast en del av fotonenergin deponeras i detektorn när en Compton-interaktion sker leder det till en förlust av spektral information. Genom att kombinera information från flera Compton-interaktioner som orsakats av samma infallande foton, så kallad sammanfallsdetektering, är det möjligt att erhålla en ökad mängd spektral information från Compton-spridna fotoner. Målet med detta examensarbete är att utveckla och utvärdera en metod för sammanfallsdetektering för att erhålla spektral information från Compton-spridda fotoner i en detektor till fotonräknande CT.

I detta arbete presenteras en metod baserad på kinematiken bakom en Compton-interaktion och ett χ^2 test. Metoden jämförs sedan med en tidigare utvecklad metod baserad på maximum likelihood-uppskattning. χ^2-metoden utnyttjade sambandet mellan deponerad energi i en Compton-interaktion och möjliga spridningsvinklar, här kallade energivinklar. De spatiella vinklarna mellan interaktionerna i detektorn mättes och jämfördes genom ett χ^2-test för att hitta interaktionsordningen och den infallande energin. χ^2-metoden identifierade interaktionsordningen korrekt för 85.5% av alla simulerade interaktionskedjor som slutade i fotoelektrisk effekt och 64.1% av alla simulerade interaktionskedjor som endast innehöll Compton-interaktioner. Uppskattningen av infallande energi var 100% korrekt för alla interaktionskedjor som slutade med en fotoelektrisk interaktion, eftersom all infallande energi deponerats i detektorn. För kejdor som endast bestod av Compton-interaktioner uppskattades den infallande energin med ett RMS-fel på 21.2 keV. Genom att kombinera resultaten från kedjor som slutade med en fotoelektrisk interaktion och resultaten från kejdor som endast bestod av Compton-interaktioner blev det totala RMS-felet för energi-uppskattningen 11.5 keV.

Nyckelord

Fotonräknande CT, Compton-spridning, Compton-kinematik, sammanfallsdetektering, χ^2-test

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Contents

1 Introduction

Imaging of the human body is an essential part of today's healthcare. Computer tomography (CT) can be used to obtain cross-sectional and three-dimensional x-ray images. Conventionally, CTs are made with energy-integrating detectors (EIDs) that integrate the signal from multiple photons interacting in the detector during a set time span. Energy-integrating CT is, however, associated with multiple limitations [1], such as inadequate contrast between different soft tissues and relatively high required dose.

Instead of using EIDs it is possible to use photon counting detectors (PCDs) to avoid these limitations. CT with PCDs is currently progressing from research labs into clinical evaluations. With a PCD it is possible to count each photon individually as well as measure the energy of each photon, which cannot be done with an EID. The main advantage of this is that the energy information can be used for improved spectral imaging. The research on PCDs mainly revolves around semiconductor detectors, thus associating them with less noise [1] and higher spatial resolution with the same dose, alternatively the same spatial resolution with lower dose, compared to conventional CT with EIDs [2].

The silicon strip detector, invented by the Physics of Medical Imaging group at KTH Royal Institute of Technology [3] is one of the most promising technical solutions for photon counting spectral CT. Here the PCD is based on a silicon semiconductor. The photon can interact in the detector either through photoelectric effect or Compton scattering. In a Compton interaction only a fraction of the photon energy is deposited in the detector. This causes a loss of spectral information, thus decreasing the energy resolution of the detector. By using Compton coincidence detection and combining information from multiple Compton interactions induced by the same x-ray, both spatial resolution and energy resolution can be improved, which in turn will increase the image quality and decrease the radiation dose [4].

A current framework for estimating incident photon energy from Compton interactions has been developed by the same research group using maximum likelihood estimation [5]. The method is based on identifying the correct order of interactions caused by an incident photon. The maximum likelihood method utilises an expression for the total likelihood of a chain of N interaction events. Based on the measured interaction events in the detector, the likelihood function is calculated for all possible interaction orders and the order with the largest maximum of the likelihood function is estimated as the correct order [5].

Assuming a low count rate of one incident photon per time frame, the maximum likelihood method correctly identifies the interaction order for 98.3% of interaction chains of a length ≥ 2, and estimating the incident energy for interaction chains of any length with an RMS error of 2.54 keV for an ideal detector. (Note that in the original publication of reference [5] the reported RMS errors were incorrect. The numbers presented here correspond to the corrected RMS errors as given by the authors.) This shows the potential of using Compton coincidence methods to increase the amount of spectral information gained from Compton scattered photons. However, the maximum likelihood framework is a time-consuming solution, therefore this project aims to test a different statistical method to estimate photon energy from information extracted from Compton interactions in a silicon detector.

The aim of this thesis is to investigate if a method based on a χ^2-test combined with kinematic constraints can be used to extract information about the incident x-ray spectrum based on a set of registered events. The method will be evaluated on how well it can estimate incident photon energy based on a chain of interactions caused by an incident photon as well as how well it can identify the order of interactions in the chain. These results will then be compared to results from the maximum likelihood estimation.

To achieve the aim, the work is divided into two sub-tasks:

- Task 1: 1 photon per time frame interacts in the detector. The interaction chain caused by the incident photon ends in photoelectric interaction. All of the incident energy is deposited in the detector. Goal: estimate interaction order and incident energy.
- Task 2: 1 photon per time frame interacts in the detector. The interaction chain caused by the incident photon only consists of Compton interactions. Only part of the incident energy is deposited in the detector. Goal: estimate interaction order and incident energy.

2 Method

The method is based on the Compton Kinematic Discrimination technique, developed by Boggs & Jean (2000) [6] and further developed by Ramey (2019) [7]. The method section is dived into 4 parts: the theory, the simulation of the input data, the implementation for task 1, and the implementation for task 2. The work was performed using MATLAB (The Mathworks Inc., Natick, Massachusetts, USA), both for the simulation of input data and for the calculations of the χ^2 value.

2.1 Theory

An incoming photon can interact in the detector through Compton scattering. In this process the photon collides with an electron, deposits some of its energy to the electron and then continues in a different direction and with a different wavelength compared to the initial photon. This is shown in figure 1. The photon energy after the interaction can be described as

$$E' = \frac{E_0}{1 + \frac{E_0}{m_e c^2}(1 - \cos\theta)} \qquad \text{Compton kinematics} \qquad (1)$$

where E_0 is the energy before the interaction, E' is the energy after the collision, m_e is the electron mass, c^2 is the speed of light, and θ is the scattering angle. Equation 1 explains the kinematics of a Compton scattering event.

From equation 1 it can be observed that the scattering angle is connected to the difference in energy before and after the Compton interaction. The detector can measure the energy deposited in each interaction and if the photon energy before and after the interaction is known, the scattering angle can be calculated. In this thesis this is called the energy angle. If the direction of the photon after the interaction is known, the energy angle can be used to limit the possible directions of the photon before the interaction.

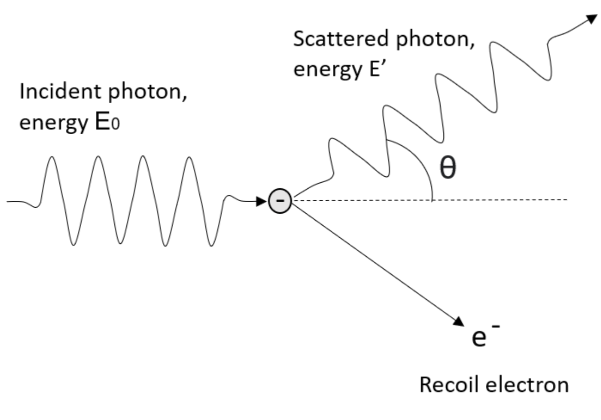

Figure 1: Illustration of Compton scattering

Each interaction in the detector is registered with information of the deposited energy as well as the interaction position. It is therefore possible to calculate the angles between the interactions spatially. In this thesis these are called the spatial angles. By comparing the spatial angles with the energy angles the most likely order of the interactions can be determined.

The interactions caused by an incoming photon can be described using the following equations, adapted from Boggs & Jean (2000) [6]

$$W_i = \frac{1}{m_e c^2} \sum_{j=i+1}^{N} E_j \qquad \text{Unitless energy} \tag{2}$$

$$\eta_i' = \cos\theta' = 1 + \frac{1}{W_{i-1}} - \frac{1}{W_1} \qquad \text{Energy angle} \tag{3}$$

$$\eta = \cos\theta = \hat{r}_i \cdot \hat{r}_{i+1} \qquad \text{Spatial angle} \tag{4}$$

$$\hat{r}_i = \frac{\bar{x}_i - \bar{x}_{i-1}}{|\bar{x}_i - \bar{x}_{i-1}|} \qquad \text{Direction before hit i} \tag{5}$$

where N is the total number of interactions, W_i is the unitless energy after interaction i, θ' is the energy scattering angle in interaction i, and θ is the spatial angle in interaction i. W_0 is the incident photon energy. $\hat{r}_i$ is the direction before hit i, $\hat{r}_{i+1}$ is the direction immediately after, and $\bar{x}_i$ is the position of interaction i. Equation 3 is a rewritten form of equation 1. To save computation time the angles are kept in the cosine form, η' and η and these values are then compared instead of θ' and θ.

In this work, the energy angle and the spatial angle were compared using a χ^2 test, see equation 6. The χ^2 test is typically used as a measure of the deviation of observed values from the expected outcome [8]. Here the χ^2 test is used to compare the differences between spatial angles and energy angles.

$$\chi^2 = \frac{1}{N-1} \sum_{i=1}^{N-1} \frac{(\eta - \eta')^2}{\delta\eta_i^2 + \delta\eta_i'^2} \qquad \chi^2 \text{ test} \tag{6}$$

where $\delta\eta$ and $\delta\eta'$ are the uncertainties in the spatial and energy angles respectively. $\delta\eta^2$

and $\delta\eta'^2$ were as follows

$$\delta\eta_i^2 = \delta\theta_{i,r}^2 \sin^2\theta_i \quad \text{Spatial angle uncertainty} \tag{7}$$

$$\delta\theta_{i,r} = \sqrt{\delta\theta_{i,x}^2 + \delta\theta_{i,y}^2 + \delta\theta_{i,z}^2} \quad \text{Spatial uncertainty} \tag{8}$$

$$\delta\theta_{i,x} \approx \tan\delta\theta_{i,x} = \frac{\delta x_i'}{r_i'} \cdot \sqrt{1 - (\hat{r}_i \cdot \hat{x})^2} \quad \text{Angle uncertainty in x} \tag{9}$$

$$\delta x_i' = \sqrt{\delta x_i^2 + \delta x_{i-1}^2} \quad \text{Uncertainty in x position} \tag{10}$$

$$\delta\eta_i'^2 = \frac{\delta W_{i-1}^2}{\delta W_{i-1}^4} + \delta W_i^2 \left(\left(\frac{1}{\delta W_i^2} - \frac{1}{\delta W_{i-1}^2} \right)^2 - \frac{1}{\delta W_{i-1}^4} \right) \quad \text{Energy angle uncertainty} \tag{11}$$

$$\delta W_i = \frac{1}{m_e c^2} \sqrt{\sum_{j=i+1}^{N} \delta E_j^2} \quad \text{Unitless energy uncertainty} \tag{12}$$

The uncertainties in y and z, $\delta\theta_{i,y}$, $\delta\theta_{i,z}$ and $\delta y_i'$, $\delta z_i'$, were similarly calculated according to equations 9 and 10 respectively. δx_i, δy_i, δz_i is the uncertainty in the measurement of the interaction position. These uncertainties were assumed to be $\delta x_i = 10\ \mu m$, $\delta y_i = 500\ \mu m$ µm, and $\delta z_i = 500\ \mu m$ µm for all i. δE is the uncertainty in the measured deposited energy and was assumed to be $\delta E = 0.5$ keV for all interactions.

Equation 6 shows that the smaller the difference is between the spatial and energy angles, the lower the χ^2 value will be. The most likely order of the interactions was therefore determined as the order with the lowest χ^2 value. Since the method is based on comparing angles between interactions at least two interactions per interaction chain are required in order, thus this method did not work for photons interacting only once in the detector.

2.2 Input data/interactions in detector

The input data to the χ^2 function consisted of simulated photon interactions in a silicon detector. It contained information of the deposited energy in each interaction, the position of each interaction, and if the interaction is photoelectric or Compton scattering. The input data was created through a Monte Carlo simulation of how photons interact in a silicon detector. The same simulation method as is thoroughly described in reference [5] was used, with a few alterations.

For each of N_γ number of incident photons, the incident photon energy E_γ was first sampled. Using the Beer-Lambert law, it was then determined whether the incident photon interacts and in which position. The interaction type (photoelectric or Compton scattering) was determined using the interaction cross sections. In the case of a Compton interaction, the scattered photon energy and the scattering angle were sampled, as well as if the scattered photon interacts or not. If the scattered photon interacts, the interaction position and interaction type are determined and the process continues until there are no more interactions or until a photoelectric event occurs. The detector was assumed to be infinite in the xy-plane and have a detector depth of $L_d = 8$ cm in the z-direction, which corresponds to the incident photon direction. All incident photons were sampled within a 1×1 cm^2 area, meaning all primary interactions were located within this area.

In this work, an energy discretisation of 0.25 keV was used. For the first task the incident energies were sampled from a spectrum of energies from an x-ray source operated at 120 kVp with 30 cm soft tissue filtration between the source and the detector. For the second task photons were simulated with set energies of 30 keV, 40 keV, 50 keV, 60 keV, 70 keV, 80 keV, 90 keV, 100 keV, 110 keV and 120 keV. Compared to the simulation in reference [5], a different approximation of the Compton scattering distribution was used in this work which allows for a larger interval of possible scattering angles for each combination of incident energy and scattered energy. The reason for using a different approximation of the distribution was due to an error found in the approximation used in [5] that resulted in a narrower span of possible scattering angles.

2.3 Task 1: 1 incident photon per time frame, ends in photoelectric effect

The first task was to create a code that performed the χ^2 calculation for 1 photon interacting per time frame, where the interaction chain ends with a photoelectric interaction. A total of 5000 photons were simulated for the input data, which consisted of all sorts of interaction chains. In this case only the ones with a length ≥ 2 and ending in a photoelectric interaction were considered. The steps of the calculation are described in algorithm 1.

Algorithm 1: 1 photon, ends in photoelectric effect

Input: *all_photons* - cell array with interaction positions and deposited energy in each interaction, from each incident photon.

The total number of incident photons, *#incident photons*, is the number of elements in the *all_photons* cell array;

for *j=1:#incident photons* **do**
- Choose current incident photon;
- Find number of interactions in interaction chain N;
- Reset correct order;
- Reset minimum χ^2 to infinity: $\chi^2_{min} = \infty$;
- **if** *chain is longer than 1 & ends in photoelectric effect* **then**
  - Permute number of interactions to find all possible orders;
  - **for** *each order combination* **do**
    - Choose current order to evaluate;
    - Choose energy deposited in final interaction as current total energy, recalculate it to unitless energy;
    - Reset total χ^2 value;
    - **for** i = *second to last interaction:first interaction* **do**
      - Calculate spatial direction $\hat{r}$ before and after interaction;
      - Calculate cosine of spatial angle η;
      - Calculate spatial angle uncertainty $\delta\eta^2$;
      - Calculate unitless energy before and after interaction, where energy after is the same as current total energy and energy before is energy after + energy deposited in the interaction;
      - Calculate cosine of energy angle η';
      - Calculate energy angle uncertainty $\delta\eta'^2$;
      - Calculate χ^2 value for interaction i and add to total χ^2 value;
      - Update current total energy to energy before;
    - **end**
    - **if** $\chi^2 < \chi^2_{min}$ **then**
      - Update minimum χ^2: $\chi^2_{min} = \chi^2$;
      - Note current order as estimated correct order;
    - **end**
    - Incident photon energy W_0 is set as the total energy;
    - Recalculate unitless energy to keV: $W_0 \rightarrow E_0$;
  - **end**
- **end**

end

Algorithm 1 calculates $\hat{r}$ using equation 5, η using equation 4, η' using equation 3, spatial angle uncertainty $\delta\eta^2$ using equation 11, and χ^2 using equation 6. The interaction order with the lowest χ^2 value was assumed to be the correct order of the interaction chain. Since each chain contained a photoelectric interaction, all of the incident photon energy was deposited in the detector. The estimated incident energy W_0 was therefore calculated as the sum of the deposited energies in all interactions, and did not change depending on the ordering of the interactions. The unitless incident energy W_0 was then recalculated to energy with unit keV, E_0.

2.4 Task 2: 1 incident photon per time frame, ends in Compton scattering

The second task was to create a code that performed the χ^2 calculation for 1 photon interacting per time frame, where the interaction chain ends with a Compton interaction and the final scattered photon escapes the detector. A total of 5000 photons (500 for each energy level mentioned in section 2.2) were simulated for the input data, which consisted of all sorts of interaction chains. In this case only the ones with a length ≥ 2 and ending in a Compton interaction were considered. The steps of the calculation were similar to to those for task 1 and are described in algorithm 2.

Algorithm 2: 1 photon, ends in Compton scattering

Input: *all_photons* - cell array with interaction positions and deposited energy in each interaction, from each incident photon.

The total number of incident photons, *#incident photons*, is the number of elements in the *all_photons* cell array;

```
for j=1:#incident photons do
    Choose current incident photon;
    Find number of interactions in interaction chain N;
    Reset correct order;
    Reset minimum χ^2 to infinity: χ^2_min = ∞;
    if chain is longer than 1 & ends in Compton scattering then
        Permute number of interactions to find all possible orders;
        for each order combination do
            Choose current order to evaluate;
            for all possible escape energies do
                Choose current escape energy to evaluate;
                Choose energy deposited in final interaction + current escape energy
                  as current total energy, recalculate it to unitless energy;
                Reset total χ^2 value;
                for i = second to last interaction:first interaction do
                    Calculate spatial direction r̂ before and after interaction;
                    Calculate cosine of spatial angle η;
                    Calculate spatial angle uncertainty δη^2;
                    Calculate unitless energy before and after interaction, where
                      energy after is the same as current total energy and energy
                      before is energy after + energy deposited in the interaction;
                    Calculate cosine of energy angle η';
                    Calculate energy angle uncertainty δη^2;
                    Calculate χ^2 value for interaction i and add to total χ^2 value;
                    Update current total energy to energy before;
                end
                if χ^2 < χ^2_min then
                    Update minimum χ^2: χ^2_min = χ^2;
                    Note current order as estimated correct order;
                    Update incident photon energy W_0 to total energy with current
                      estimated escape energy ;
                    Recalculate unitless energy to keV: W_0 → E_0;
                end
            end
        end
    end
end
```

Algorithm 2 calculates $\hat{r}$ using equation 5, η using equation 4, η' using equation 3, spatial angle uncertainty $\delta\eta^2$ using equation 11, and χ^2 using equation 6. Since the chain ends with a Compton interaction, not all of the incident photon energy is deposited in the detector. To find the total energy all possible, escape energies are looped through for each interaction order. The interaction order and escape energy with the lowest χ^2 value was assumed to be the correct energy and order of the interaction chain. The estimated incident energy W_0 was calculated as the sum of the deposited energies in all interactions + the escape energy used for the lowest χ^2 value. The unitless incident energy W_0 was then recalculated to energy with unit keV, E_0.

To evaluate the results of the energy estimation the RMS error was calculated both separately for correctly ordered and incorrectly ordered interaction chains, and for all Compton interaction chains combined. Finally also a total RMS error for the energy estimation in both task 1 and task 2 combined was calculated.

3 Results

The results are split into the results from task 1 and task 2. Results from task 1 are found in table 1. Results from task 2 are found in table 2 and 3, and figure 2.

Table 1: Results for the identification of interaction order for interaction chains consisting of Compton interactions and 1 photoelectric interaction. The far-right column shows the proportion of chains of each length with a length ≥ 2, ending in a photoelectric interaction, out of a total of 5000 simulated photons. Note that for the longest types of chains there is very little data and the statistics are not representable.

Interaction chain	**Correctly identified interaction order [%]**	**Proportion of each chain length out of all simulated chains [%]**
1 Compton + 1 photoelectric	95	22
2 Compton + 1 photoelectric	88	14
3 Compton + 1 photoelectric	79	8.9
4 Compton + 1 photoelectric	77	4.2
5 Compton + 1 photoelectric	51	3.0
6 Compton + 1 photoelectric	78	1.2
7 Compton + 1 photoelectric	20	0.35
8 Compton + 1 photoelectric	75	0.28
Total:	86	54

For task 1, the energy estimation was correct 100% of the time due to all of the incident photon energy being deposited in the detector when the interaction chain ends with a photoelectric interaction.

Table 2: Results for the identification of interaction order for interaction chains consisting only of Compton interactions. The far-right column shows the proportion of chains of each length with a length ≥ 2, consisting only of Compton interactions, out of a total of 5000 simulated photons. Note that for the longest types of chains there is very little data and the statistics are not representable.

Interaction chain	**Correctly identified interaction order [%]**	**Proportion of each chain length out of all simulated chains [%]**
2 Compton	61	3.5
3 Compton	68	1.6
4 Compton	74	1.0
5 Compton	63	0.38
6 Compton	44	0.18
Total:	64	6.7

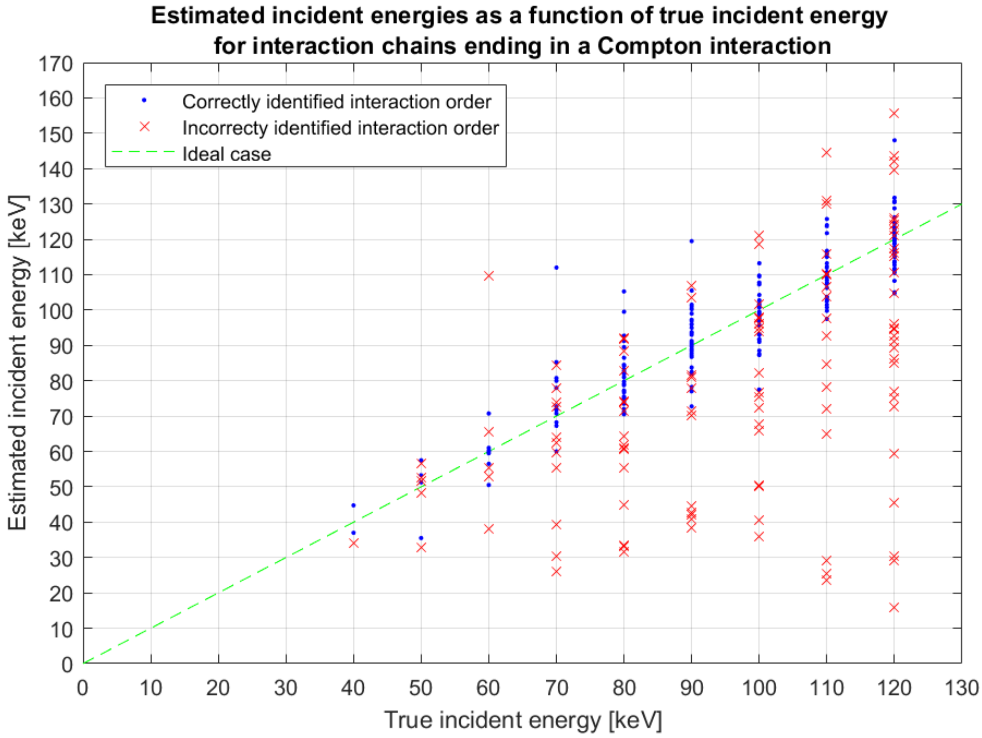

Figure 2: Estimated energies from the χ^2 test as a function of the true energies for interaction chains consisting only of Compton interactions. Blue dots the are results for interaction chains with correctly identified interaction order, red x:s are results for interaction chains with incorrectly identified interaction order, and the dashed line represents ideal results.

Table 3: RMS error for energy estimation. Row 1-3 shows results for interaction chains consisting only of Compton interactions and row 4 shows RMS error for both interaction chains of only Compton interactions and chains ending in photoelectric effect.

RMS error for Compton chains with correctly identified interaction order [keV]	8
RMS error for Compton chains with incorrectly identified interaction order [keV]	33
Total RMS error for both correctly and incorrectly ordered Compton chains [keV]	21
Total RMS error for both chains of only Compton interactions and chains ending in photoelectric effect [keV]	11

4 Discussion

Table 1 shows that we get a higher percentage of correctly identified interaction orders for shorter interaction chains. This is mainly because we have more possible combinations for longer chains, meaning more possibilities to having similar χ^2 values for both correct and incorrect orders. For the shortest chains, there is only one event of each type (photoelectric effect and Compton scattering), and it is most of the time possible to distinguish these and place the photoelectric one last. The longest chains only make up for a very small fraction of all chains, meaning there is very little data and the statistics therefore are not fully representable. The energy estimation for task 1 was done by summing the deposited energy in each interaction in the chain. It was correct 100% of the time since all of the incident energy was deposited in the detector when the interaction chain contained a photoelectric interaction.

Table 2 shows that for task 2 the identification of the interaction order was less affected by the length of the interaction chains, compared to for task 1. In total, the results for the order identification for task 2 are lower than for task 1. This is due to it being more unknown variables when the escape energy must be estimated compared to when all energy is deposited in the detector. Looping through all possible escape energies results in more possibilities to test, thus enabling more possible errors to be made. Different estimated escape energies can cause different orders to seem most likely, therefore a larger fraction of interaction chains become incorrectly ordered.

With this method it is currently not possible to analyse photons that interact only once in the detector, thus only chains with a length ≥ 2 were considered. We see in table 2 that chains with at least two interactions containing only Compton interactions are rare, only 6.7% of 5000 simulated photons met the criteria. Long chains of only Compton interactions are very rare, therefore the data is insufficient to give fully representable statistics in this case.

For the first task incident energies were sampled from a spectrum, while for the second task set energy levels were used for the incident energies. The reason for this was to to know the true incident energy in order be able to compare the estimated incident energy to the true incident energy. In task 1, all of the incident energy was deposited in the detector, thus the incident energy was known by summing all energy depositions. In task 2, only part of the incident energy was deposited in the detector so set energy levels were used to know the true incident energy.

Figure 2 shows the results of the energy estimation in task 2. Here it is shown that the energy estimation is generally significantly better for chains with correctly identified interaction order than for incorrectly ordered chains. This is also clear from the RMS errors found in table 3. The total RMS error for both correctly and incorrectly identified orders is therefore in between the two separate RMS errors for the correctly ordered and incorrectly ordered chains. The reason for the significantly worse energy estimation is that when the algorithm incorrectly identifies the order, it has misinterpreted what happened, and therefore it is more difficult to correctly determine the incident energy. The RMS error for task 1 and 2 combined is also calculated in order to be able to make a better comparison to the maximum likelihood method (see below). However, it must be noted that task 1 and task 2 are two different cases with different conditions.

Comparing the results of the χ^2 method in this thesis to the results of the similar work in [5] where the Compton coincidence detection is done with a maximum likelihood estimation (see Appendix A, section A.4), it shows that the maximum likelihood gets generally better results. It manages to correctly identify the interaction in 98% of the cases compared to 85%/64% with the χ^2 method, as well as achieves energy estimation with an RMS error of 2.5 keV compared to 11 keV for the χ^2 method. There are multiple reasons that contributes to this. The main reason is that the maximum likelihood method takes more different physical effects in the interactions into account compared to the χ^2 method. The χ^2 method only looks at the relationship between spatial angles between interaction and possible scattering angles due to the deposited energy while the maximum likelihood method includes factors such as the probability distribution of different scattering angles, the probability to interact in a certain position, the probability for photoelectric and Compton interaction, the probability of escape etc. This results in the maximum likelihood method utilizing more information to identify the interaction order, thus making it more reliable.

Another affecting factor could be that a different approximation of the Compton scattering distribution was used in reference [5] to simulate photon interactions. The reason for choosing a different approximation in this work was because an error was found in the approximation used in [5]. The error caused a smaller interval of possible scattering angles in [5], compared to what was used in this work. Different incident energies were also used in the simulation of photon interactions. In reference [5], 60 keV is used for all photons while a spectrum of energies up to 120 keV, filtered through 30 cm soft tissue is used for chains ending in a photoelectric interaction and different set energy levels for chains containing only Compton interactions are used in this work.

Regarding the correct order of interactions, a large difference between the maximum likelihood method and the χ^2 method presented in this work is that photoelectric interactions are typically placed at the end of the interaction chain with the maximum likelihood method. In the χ^2 method it is possible to place the photoelectric interaction anywhere in the interaction chain, depending on where the algorithm thinks the best fit is. With the maximum likelihood method, the errors in interaction order therefore occur only from incorrectly placed Compton interactions. With the χ^2 method, the errors can also occur from incorrectly placed photoelectric interactions.

Yet another factor affecting the comparison of the results is that with the Maximum likelihood method, only chains of length up to 4 interactions are included, while in this work chains of length up to 9 interactions are included. Since long chains are more difficult to correctly identify, including them can affect the results, compared to if they are excluded. However, since it is only a very small portion of interaction chains that have a length ≥ 5 the effect of this is small.

There are many possible improvements to this method. The main improvement would be to include some more physical effects in the calculation, e.g., the probability distribution of the scattering angles. However, it is not desired to include all the effects included in the maximum likelihood method, since the goal of this method is to be more simple and faster. Another improvement could be to put constraints for the angle calculations. Since the value of a cosine function is always between -1 and 1 it must be that $\eta, \eta' \in [-1, 1]$. It is thus possible to discard all interaction chains where this is not true.

Further steps for developing this method would be to analyse multiple photons interacting within the same time frame. When looking at multiple photons simultaneously it is desired to have a fast algorithm that quickly discards incorrect interaction chains. Though speed of the algorithm has not been previously discussed in this work, an implementation to improve the speed of the algorithm, would be to put constraints on the χ^2 value so that an interaction order is discarded as soon as the χ^2 value exceeds a certain threshold, or as soon as the χ^2 value exceeds the minimum χ^2 value. Another development would be to extend the method do include photons only interacting in the detector once.

A limitation to this method is that it does not take Rayleigh scattering into account. In reality photons can be scattered in the detector without depositing energy, and this could affect the results of the calculations since it assumes that the angles between the interactions are only caused by Compton scattering events.

5 Conclusion

In this work, a method for Compton coincidence detection based on Compton kinematics and a χ^2 test have been presented. The interaction order and the incident photon energy were estimated first for interaction chains ending in a photoelectric interaction, then for interaction chains only containing Compton interactions, assuming one incident photon per time frame for both cases. For the interaction chains ending in photoelectric effect, the interaction order was correctly identified for 86% of the interaction chains and the energy estimation was correct for 100% of the interaction chains ending in photoelectric effect. For the interaction chains containing only Compton interactions, the interaction order was correctly identified for 64% of the interaction chains. The incident photon energy was estimated with an RMS error of 21 keV for chains of only Compton interactions.

The results imply the potential of using a method based on Compton kinematics to increase the amount of spectral information gained from Compton scattered photons in a silicon detector. Further improvements could be made by taking additional physical factors into account in the estimation of interaction order and incident energy. Further steps are to test the method for multiple photons interacting within the same time frame.

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Appendices

A State of art

Computer tomography (CT) is an important part of diagnostics in today's healthcare. It can be used to acquire cross-sectional or three-dimensional images of a patient. Research on photon counting detectors shows they are about to be the next big improvement when replacing the energy integrating detectors conventionally used today. One ability the current photon counting detectors lack is the ability to utilise energy information from photons that interact in the detector through Compton scattering. This literature review describes in short the function of a photon counting detector, and a few methods of Compton coincidence detection that could be used to extract energy information from Compton interactions in the detector.

A.1 Semiconductor detectors

Photon counting detectors (PCDs) are based on semiconductor materials. The research on PCDs for CT is mainly focused on two materials: cadmium telluride/cadmium zinc telluride (CdTe/CZT) and silicon (Si) [1]. A photon that interacts in a semiconductor material deposits energy that causes a high energy electron to be released from one of the atoms in the material [2, 3], which in turn collides with other electrons leading to an avalanche of released electrons. For each negatively charged electron, a positively charged hole is left in the atom. To produce an electron-hole pair, a material specific energy is required, e.g., for silicon the energy needed to create one electron-hole pair 3.6 eV [3]. Thus, the number of electron-hole pairs produced is proportional to the energy deposited by the incident photon [2].

These electrons and holes form two charge clouds, one positively charged and one negatively charged charge, which drift towards electrodes on each side of the detector due to a voltage bias applied across the material [4]. The charge carriers are also affected by Coulomb repulsion and thermal diffusion effects, causing the charge clouds to increase in size while they drift through the semiconductor material [5].

The drift of the charge carriers induces a current in the electrodes which is measured and compared to a set of energy thresholds. If the pulse height of the induced current is higher than the lowest threshold it is registered as a count [4]. The current $i(t)$ induced in the electrode can be calculated according to the Shockley-Ramo theorem [6]:

$$i(t) = -q\bar{v} \cdot \bar{E}_W \tag{A1}$$

where q is the charge, $\bar{v}$ is the charge carrier velocity, and $\bar{E}_W$ is the component of the electric field in the direction of $\bar{v}$ at the charge's instantaneous position [6, 7]. $\bar{v}$ is calculated as

$$v = \mu E \tag{A2}$$

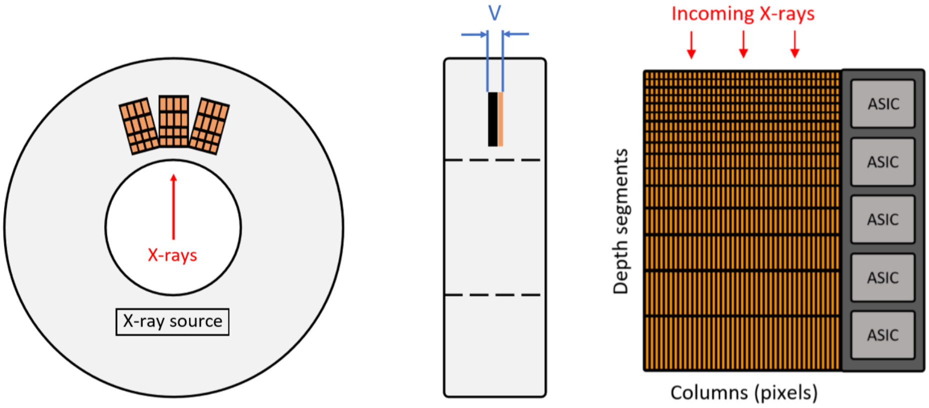
where μ is the carrier mobility specific to the material, and E is the electric field [7].

If two incident photons interact in the detector closely in time there is a risk of overlap between the two signals so that they cannot be separated, but are instead counted as coming from a single photon. This is called pulse pileup. Pulse pileup can result in a decrease in counts as two photons are counted as one, as well as detecting a higher incident energy than the true energy when the signals from the two photons are summed, causing a higher pulse height.

Photons can deposit energy in the detector through two types of interactions: photoelectric interactions or Compton interactions. In a photoelectric interaction the photon deposits all of its energy to the released electron while in a Compton interaction the photon is scattered by an electron and only deposits part of its energy to the electron. If the Compton scattered photon interacts again, multiple events can be registered from the same incident photon. A third type of interaction is Rayleigh scattering, but in this case the photon changes direction without depositing energy and this interaction can therefore not be detected. After a Rayleigh interaction, the photon can subsequently interact again through Compton scattering or photoelectric effect.

A.1.1 Silicon strip detector

The Physics of Medical Imaging group at KTH Royal Institute of Technology has developed a photon counting silicon strip detector [8] to be used for photon counting spectral CT. It consists of a number of silicon strip wafers placed in a CT gantry as showed schematically in figure A1a. A schematic illustration of a silicon wafer is shown in figure A1b. Silicon has several advantages [1, 4] as a material for an x-ray detector. It is easy to produce at low cost compared to other semiconductor materials, and has a high mobility for electrons and holes, resulting in a short collection time of the charge carriers caused by photon interactions. This makes the detector less sensitive to pulse pileup at high detection rates [1]. This detector can handle a count rate of up to 90 Mcs/mm^2, at which about 3% of the input counts are lost, and has an estimated dead time of $\tau_n = 20.2 \pm 5.2$ ns [9].



(a) (b)

Figure A1: (a) Schematic illustration of the placement of silicon strip wafers in a CT gantry. V is the applied bias voltage. (b) Schematic illustration of a segmented silicon strip wafer for spectral CT.

However, due to the low atomic number of silicon, a high fraction of the incident photons interact through Compton scattering for the energies used in CT. Due to the small size of the pixels (<1 mm [5]) in the detector, secondary interactions can cause pixel cross-talk, meaning that the same incident photon is registered by multiple pixels. Tungsten shielding between the pixels is typically used to prevent cross-talk. Thus each incident photon corresponds to only one registered count, and can contribute to image contrast and dose efficiency [1]. Nevertheless, Compton interactions lead to a loss of spectral information since not the entire incident photon energy is deposited and registered [6].

A.2 Compton coincidence detection

The performance of the silicon strip detector could be further improved by using coincidence detection methods to extract energy information from the Compton interactions. By removing the tungsten shielding it would be possible to obtain long interaction chains resulting from one incident photon [10]. The incident photon energy could be found by adding information from the events in the interaction chain. When detecting a set of interactions, it is desirable to identify interactions that come from the same incident photon in order to estimate the number of incident photons and the corresponding incident photon energies.

To be able to utilise information from Compton interactions it is crucial to have a detector with a high spatial resolution to identify the position of each interaction in the detector. The research group behind the silicon strip detector has previously presented a solution to achieve 1 µm resolution in their detector [11], using pixels of size 12×500 µm. The detector is divided into depth segments with multiple electrodes along the depth axis, thus enabling detecting multiple events caused by the same photon, alternatively interactions from the same incident photon occur in different pixels due to the small pixel size. It is thus possible to differentiate the interactions in an interaction chain despite the interactions occurring nearly simultaneously. However, the temporal resolution of a CT detector is usually not high enough to determine the temporal order of interactions and more advanced coincidence schemes are therefore required.

A.3 Monte Carlo simulation

Photon interactions in a detector can be simulated through Monte Carlo simulation. Monte Carlo simulation is a mathematical method used to predict the outcome of a process that includes random variables by estimating the probabilities of different outcomes [12]. The technique includes using multiple values an uncertain variable. This way, a model of multiple results is created, and then the average of all results are then used as the estimation.

A.4 Maximum likelihood estimation of incident photon energy

Maximum likelihood estimation is a statistical method to estimate parameters of a model or probability distribution, given some observations. This is done by maximising a likelihood function so that the observed data is the most probable outcome of the model.

The research group behind the silicon strip detector has developed a framework for maximum likelihood estimation of the incident photon energy of photons which have interacted through Compton interactions in the detector, utilising Compton coincidence [10]. The following is a summary of that method.

A.4.1 The likelihood expression

The incident photon is registered by the detector with the energy deposited by the photon and the interaction position. The number of interactions in the detector originating from the incident photon together with the position of each interaction and the energy deposited in each interaction can therefore be used to describe the incident photon. The likelihood function can be calculated as the probability of measuring a specific interaction chain coming from an incident photon of energy E_γ entering the detector in position $\bar{x}_\gamma = (x_\gamma, y_\gamma, z_\gamma)$. The likelihood can be described as $P(N = n, \{\bar{x}_1, ..., \bar{x}_N\}, \{E_1, ..., E_N\}|E_\gamma, \bar{x}_\gamma)$, where N is the number of resulting interactions, $\{\bar{x}_1, ..., \bar{x}_N\}$ is the position of each interaction, $\bar{x}_i = \{x_i, y_i, z_i\}$ describes the position of of the i:th interaction, and $\{E_1, ..., E_N\}$ describes the deposited energy in each interaction.

The likelihood expression consists of the probability to interact in a position, the probability of certain deposited energy and scattering angle in a Compton interaction, the probability of a Compton interaction, the probability of a photoelectric interaction, the probability of a certain deposited energy with an unknown scattering angle in a Compton interaction, and the probability of escape meaning the photon does not interact again and leaves the detector. A maximum likelihood estimate of E_γ and $\bar{x}_\gamma$ can then be achieved by measuring $\{\bar{x}_1, ..., \bar{x}_N\}$ and $\{E_1, ..., E_N\}$, and maximising $P(N = n, \{\bar{x}_1, ..., \bar{x}_N\}, \{E_1, ..., E_N\}|E_\gamma, \bar{x}_\gamma)$.

Since the detector cannot determine the order of the interactions, the likelihood function must be calculated for all possible orders of interaction. The order with the largest maximum of the likelihood function is then set as the estimated interaction order.

A.4.2 Likelihood framework implementation and results

In reference [10] the method described above was evaluated for both an ideal detector with perfect energy and spatial resolution and for a realistic detector with limited energy resolution (standard deviation of $\sigma_E = 0.5$ keV) as well as limited spatial resolution (standard deviation in each direction x, y, z, of $\sigma_x = 10$ μm, $\sigma_y = 500$ μm, and $\sigma_z = 500$ μm). For the ideal detector the interaction order was correctly identified for 98.3% of interaction chains of length ≥ 2. The incident energy was estimated with a root mean square (RMS) error of 2.54 keV for interaction chains of any length. (Note that in the original publication of reference [10] the reported RMS errors were incorrect. The numbers presented here correspond to the corrected RMS errors as given by the authors.) For the non-ideal detector the corresponding results were 94.2% of the interaction chains of length ≥ 2 were correctly ordered and the energy was estimated with an RMS error of 2.57 keV.

A.5 Kinematic constraints and kinematic fitting

An alternative approach to finding the incident photon energy could be to use kinematic constraints and kinematic fitting. Kinematic constraints are constraints that limits the movement possibilities for a particle. Kinematic fitting is a mathematical method to improve the measurements that characterise the procedure of a particle decaying or interacting by utilising the physical laws affecting the particle, decay, or interaction in question [13]. Compton kinematics has been used for event reconstruction in Compton telescopes for γ-ray astrophysics [14]. There, a method called Compton Kinematic Discrimination (CKD) is used for interaction ordering and background rejection. Similar methods could be used for finding incident photon energy from Compton coincidences in a silicon strip detector for CT. Compton kinematics has also been used for reconstruction of Compton events in PET. In reference [15] it is used in combination with deep learning to identify the true line of response (LOR) for intra-detector scatters (IRS) and inter-detector scatters (IDS) in a PET system based on a cadmium zinc telluride (CZT) detector.

A.6 Compton Kinematic Discrimination (CKD)

The following is a description of the Compton Kinematic Discrimination method developed by Boggs & Jean (2000) [14], described in references [14] and [16]. CKD is a method to find initial direction of an incoming photon in a γ-ray telescope. A γ-ray that interacts multiple times through Compton interactions and finally in a photoelectric interaction, forms a chain of interactions where all of the initial photon energy is deposited in the detector. However, the temporal resolution of the detector is not high enough to determine the order of the interactions. CKD utilises the connection between the energy before vs after a Compton interaction and the scattering angle to find the direction from where the photon was coming before the interaction. By comparing the angles measured spatially between the interactions and the possible scattering angle in each interaction, it is possible to backtrack the photon's path through the detector and find the interaction order and finally determine a cone of possible directions from where the incident photon came. This method can also help discriminate background radiation.

A.7 Compton kinematics and deep learning in PET

The following is a description of the method of using a deep neural network to identify the LOR, using Compton kinematics, in a PET system based on cross-strip CZT detectors proposed by Nasiri & Abbaszadeh (2021) [15]. In CZT detectors there is a significant fraction of multiple interaction photon events (MIPEs) due to Compton scattering in the detector, ending with a photoelectric interaction. Nasiri & Abbaszadeh used a dual-panel PET system, and analysed intra-detector scatter chains in two cases: 1 photoelectric interaction in one detector and 1 Compton interaction followed by 1 photoelectric interaction in the opposite detector; and 1 Compton interaction followed by 1 photoelectric interaction in each of the opposing detectors. A deep learning neural network was trained with simulated training data to identify the true LOR between the events based on Compton kinematics, then tested and evaluated with new simulated test data.

A.8 Method choice for this thesis

This thesis aims to test a simple method to utilise Compton kinematics for Compton coincidence detection in a silicon strip detector for CT. The maximum likelihood method is an estimator with good statistical qualities. However, it is a time consuming process since it requires integration over a number of variables (described in section A.4.1) to calculate the likelihood function. It is therefore desirable to find a more simple method to decrease computation time. By using a method based on Compton kinematics that only considers the connection between the scattering angle and the photon energy before and after an interaction the computation time could be reduced. However, since fewer affecting physical factors are taken into account the decrease in computation speed is at the expense of the accuracy.

The methods described in sections A.6 and A.7 are examples of methods based on Compton kinematics. However, both of them only utilises interaction chains ending with a photoelectric effect while in this thesis it is desired to also analyse interaction chains that consists of only Compton interactions. Due to the time frame of the project and the desire to create a simple but effective method, the CKD technique is chosen as a base for this work, since deep learning methods are more complicated and time consuming to train.

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