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Technical Optimization of a Long Span Beam made of Glulam Elements

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Moelven Töreboda AB. Testings on a glulam truss, factory of the company.

Abstract

Timber constructions have during the past two or three decades become more and more common, mostly due to the easy prefabrication and the cheap, sustainable material. For long-span applications it is vital to find design solutions that optimize the structure from the point of view of material consumption, number of the connections, structural reliability etc.

This thesis focuses on the structural optimization of a long span timber structure earlier developed by a Swedish glulam company for industry buildings and sport hall applications. The main goal of this study has been the minimization of the volume of wood required to built the structure, given a set of geometrical restrictions and the assigned loads.

The optimal shape of the structure and the arrangement of the different elements has been investigated by means of theoretical analyses taking into account the principal directions of stress which would occurr in similar structures with massive cross sections. The results of this investigations give some guidelines to design new types of structures, where both mechanical efficiency and manufacturing issues have been taken into consideration.

Comparisons of the structural models proposed in this thesis with the original proposal provided by the Swedish glulam company conclude the work, suggesting some possible improvements.

Keywords

Glulam structures, long span ceilings, optimal arrangement, technical considerations, self weight minimum.

Sammanfattning

Träkonstruktioner har under de senaste 2-3 decennierna blivit allt vanligare, främst på grund av trä är ett billigt och hållbart material och det möjliggör enkel prefabricering. När det är frågan om stora spännvidder är det viktigt att hitta designlösningar som optimerar strukturen både ur materialförbrukning, antal knutpunkter, strukturell tillförlitlighet etc.

Detta examensarbete fokuserar på den strukturella optimeringen av en större takstol, lämpat för industri eller sporthallsbyggnad, som tidigare utvecklats av ett svenskt limträföretag. Huvudsyftet har varit att minimera takstolens trävolym som erfordras för att bära tilldelade laster.

Den optimala formen på takstolen samt positioneringen och lutningen av takstolens olika element har undersökts med hjälp av teoretiska analyser med beaktande av de huvudspänningslinjer som skulle uppstå i liknande konstruktioner med massiva tvärsnitt. Resultaten av dessa undersökningar ger några riktlinjer för att kunna utforma nya typer av takstolar, där både mekanisk effektivitet och tillverkningsmässiga frågor har beaktats.

Jämförelser av de takstolsmodeller som föreslås i denna avhandling med det ursprungliga förslaget från det svenska limträföretaget samt förslag på några möjliga förbättringar avslutar arbetet.

Nyckelord

Limträ strukturer, långa spantak, optimalt arrangemang, tekniska överväganden, egenvikt minimum.

Acknowledgements

This master thesis has been developed at the department of Civil and Architectural Engineering of KTH, Stockholm.
The work has concerned the activities of Moelven Töreboda AB in the field of glulam structures for large industrial ceilings.

The author wish to thank the following people for their contribution:

Supervisor Prof. Roberto Crocetti for the proposal of this interesting topic, the availability and the guidance provided. He has aided me to understand the issues of the company and to solve the most complicated ones.
His suggestions have been important to keep the right way along the whole path; it has been a long experience but it has worth it, now the matter is much clearer.

Examiner Prof. Magnus Wålinder, who have allowed me to deliver the work through a prompt assistance; even if he hasn't been involved in the topic, his help have been fundamental to achieve the degree.

Prof. Bert Norlin, who has followed the thesis from the beginning, providing useful material and clever remarks. His deep experience in the field of the structural analysis has always been a secure reference for all the doubts.

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1. Introduction

1.1 State of the art

1.1.1: Historical background

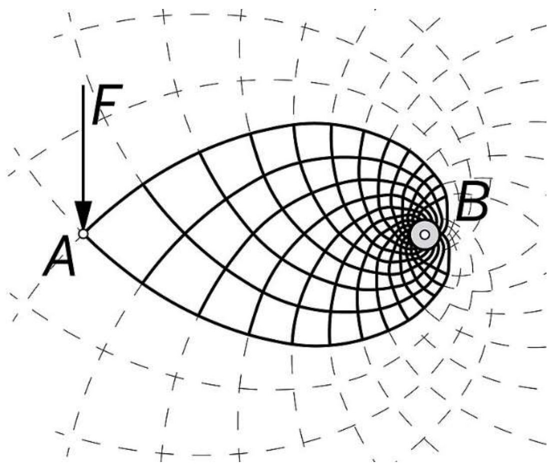
A truss structure is the last stage of the optimization of a beam: the height is so tall that the web is no more full but substitued with diagonals, so that the material is concentrated where is really needed, into the two chords. All the bars are subjected to pure axial force, that is the stress state with the highest mechanical efficiency.
Almost all the beams used for large structures are trusses, because the longer is the span the more important is to pursue the optimality of the system.

Figure 1: "Punt da Fèer", Sesto Calende (Italy). Truss bridge on Ticino for trains and cars.

Between all the possible configurations of the elements in a truss it has always been tried to find the most suitable layout to carry the loads with the minimum construction material.

The earlier works belong to Rankine (1858) and its reciprocal graphical representations for trusses. Afterwards, James Clerk Maxwell proposed some methods to find the minimum volume of material required for a truss, giving the loads and the boundary conditions. In his book "*On reciprocal figures, frames and diagrams of forces*" (1870) the formulation is based on the fact that the section of a bar is proportional to the load applied, thus the whole volume is related to the transferred load.

Maxwell searched the optimality condition through the principle of virtual work, however his studies were generalized by Anthony George Maldon Michell. His publication "*The limits of economy of material in frame-structures*" (1904) is considered to be the starting point of the structural optimization theories.
Given the domain where the truss can develop, Michell drew the optimal structural solution for several problems (see Fig. 2). An important property is that those configurations lead not only to the lowest volume of material, but also to the highest stiffness.



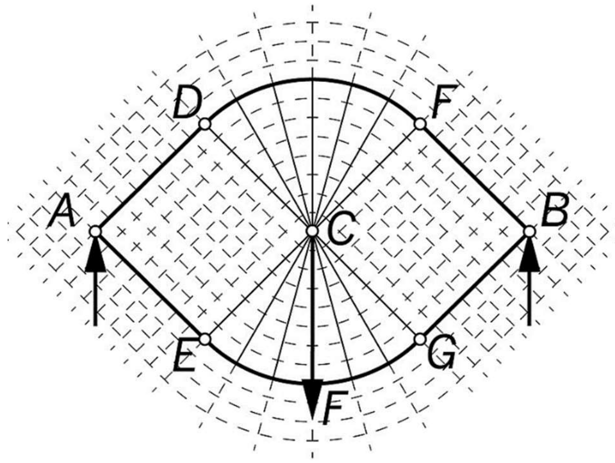


Figure 2: Optimal Michell trusses for different loads and boundary conditions.

The same results can be also found by drawing the principal directions of stress in a solid continuum. Following those trajectories the shear is always zero and the bars are placed exactely along the tension and compression paths. An example is the Lanificio Gatti, Rome, 1953, by Pier Luigi Nervi (Fig. 3).

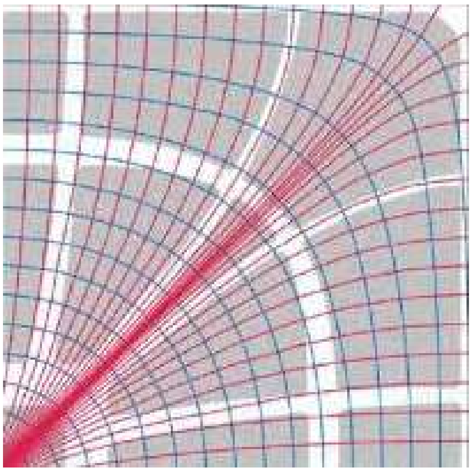



Figure 3: Principal directions of stress for a floor system and their structural application.

1.1.2: Current tools

Nowadays, the studies about the optimality of a truss can be done with specific softwares, confirming and deepening the results of the previous century. Due to their power and simplicity, the numerical techniques have by now substituted the analytical ones.

The most famous FE procedures start considering the allowed space for the truss as a solid continuum. After a preliminary structural analysis, the material is removed from the regions with few stresses such that it remains only where it's more needed. The domain is carved until the configuration of the optimal truss turns out to be defined.

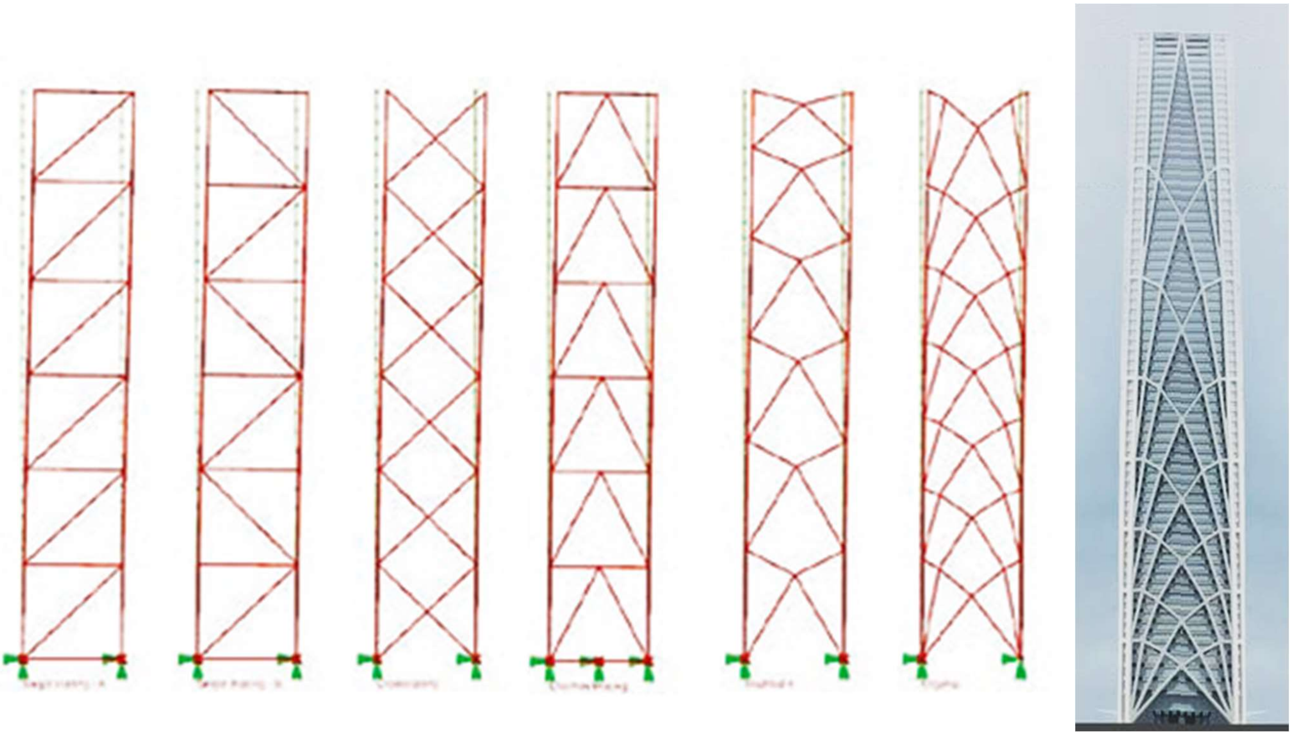


Figure 4: Bracing systems with increasing optimality and a real skyscraper, Shenzhen.

In Fig. 4 several truss solutions for a bracing system are shown. They are ordered from the less efficient (on the left) to the optimal one on the right. For the same forces to bear, the material used varies up to the 20%.
The last red configuration of Fig. 4 is faithfully reproduced in a real case, the CITIC Financial Center Tower 1, Shenzhen, China.

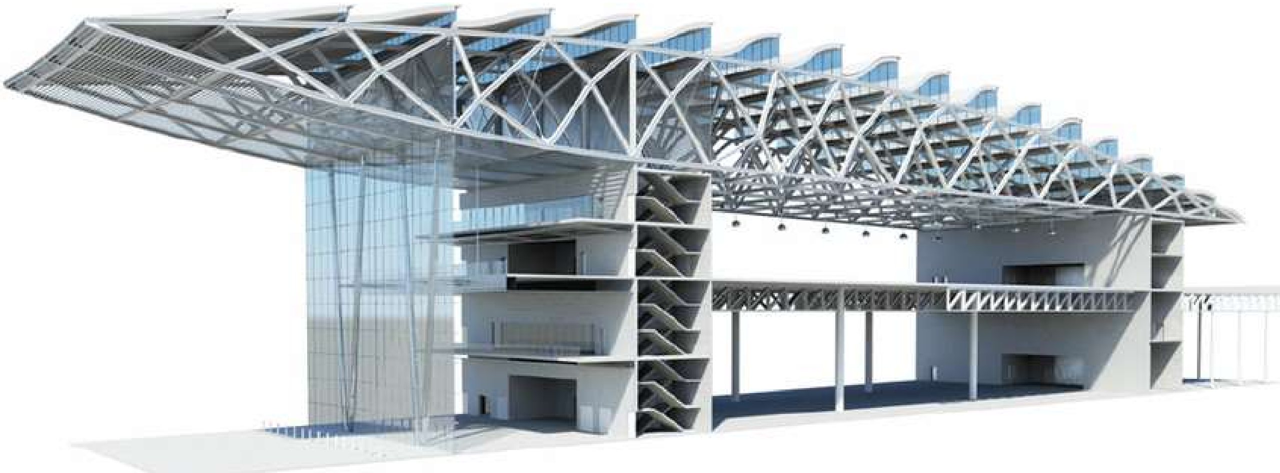


Figure 5: Rendering of a roof truss designed with topology optimization.

1.1.3: Generalities on truss beams and their optimization

The trusses developed in this thesis are related to a simply supported roof system subjected to a uniform load distribution. In Fig. 6 the principal directions of stress of the problem and the consequent optimal layout are shown.

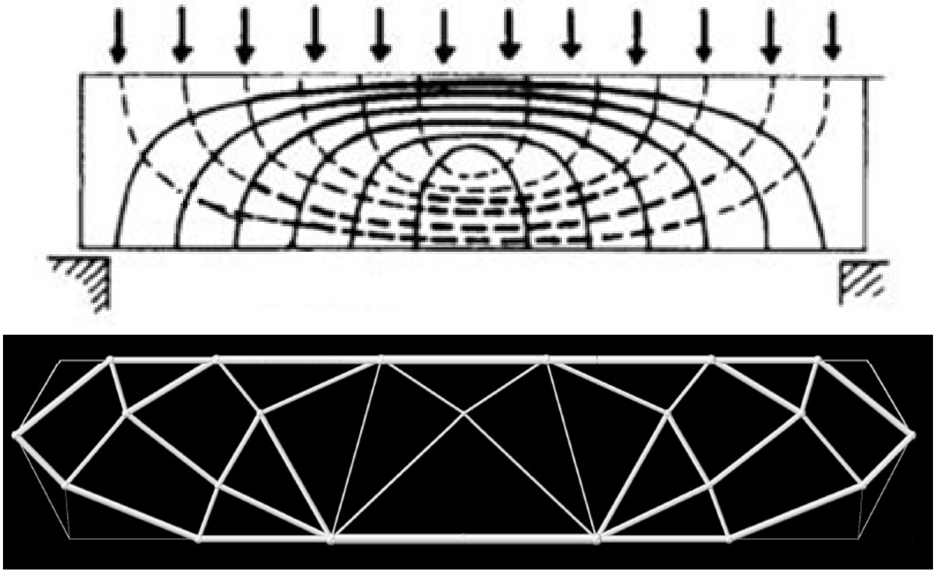


Figure 6: Isostatic lines of the problem and optimal layout.

Since for a roof truss the space allowed is not enormous, configurations like the rendering of Fig. 5 are quite rare due to their practical difficulties regarding the manufacturing. The structures designed with a topology optimization are generally reserved to the bracing of the skyscrapers.

Truss beams are normally designed with a more traditional layout of the elements. Even if they are not optimal, the classical schemes need a brief presentation because they cope with the technical issues that are also important in the real construction. In Fig. 7 there are some examples.

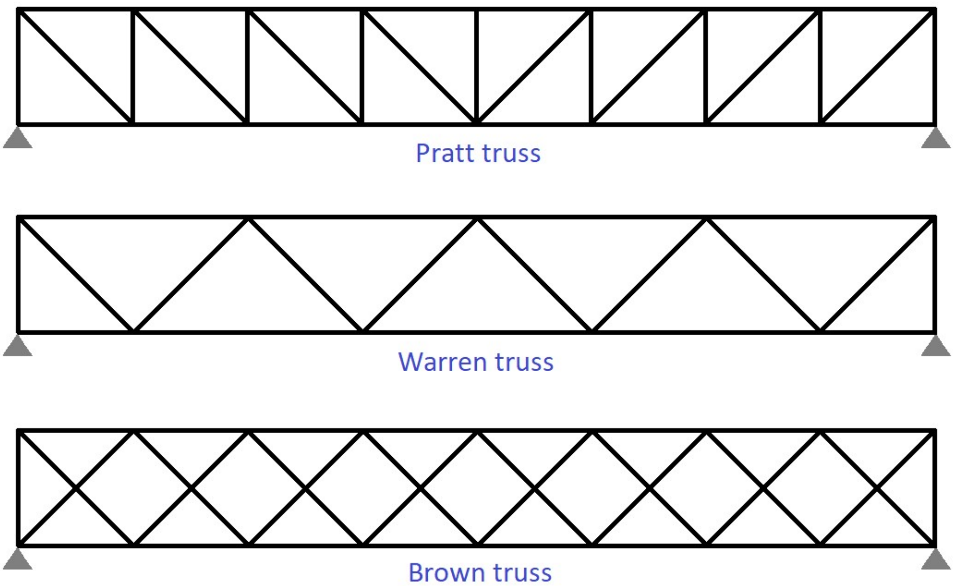


Figure 7: Different kind of trusses.

The Pratt scheme is one of the most common. The compressed diagonals are only the vertical ones, therefore the buckling problems are minimized. If the inclination of the diagonals is inverted, the Pratt layout becomes a Howe's one, better under uneven load.
As a general remark, an optimal truss doesn't have vertical diagonals because the principal directions of shear are inclined at 45° in the web of the beam. For this reason, the Warren scheme is structurally more efficient and sometimes preferred. It can be stiffened doubling the diagonals and getting the Brown truss, that is mechanically very performing. However, the joints in the web lead to other complications.

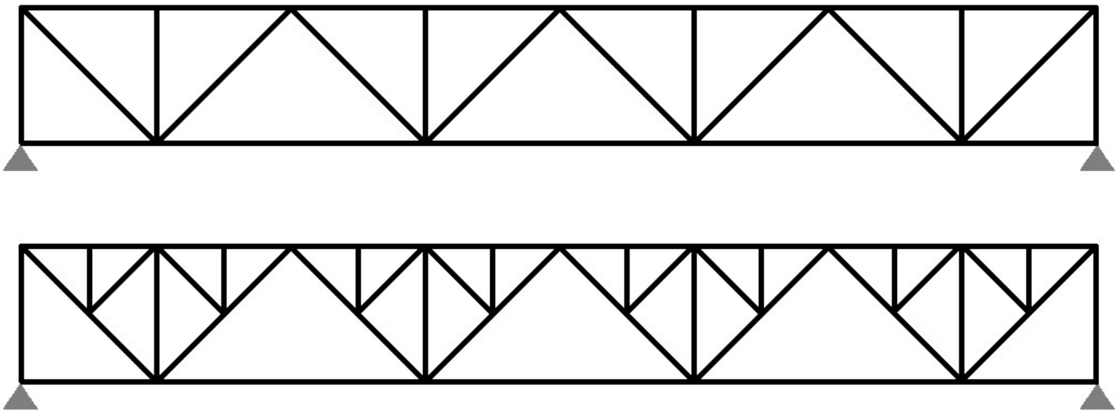


Figure 8: Other layouts for trusses.

The problems arising are of course much more numerous than what briefly presented. Even if not optimal, the vertical diagonals are often very useful to control the buckling of the upper chord, especially when it is directly loaded by a distributed pressure. In this case (which is the case of study in this thesis), the spacing of the nodes in the compressed chord is fundamental for the local bending moment of the upper beam.
It's clear that the issue of the spacing of the nodes is coupled with the inclination of the diagonals and a solution that is good for both must be defined. In Fig. 8 there are examples on how the vertical diagonals, which sometimes are non truss bars, have been introduced to cope with the flexural behavior.

For a sake of curiosity, examples of those things are available under our eyes, inside the university: the roof of the stands of the "Giuriati" sports field is a steel structure, where the main beams are Pratt trusses. All these beams are equal, even if two of them are more loaded since they support a larger area of the ceiling.
In the previous years Politecnico di Milano made a renovation of the whole arena and those two trusses were strengthened. Additional diagonals were added to the existing ones, in order to cope with the buckling of the upper chord. The improved scheme reflects basically what shown in the second layout of Fig. 8.

A further issue that has not been mentioned is the scale of the drawing: the same problem with the same truss scheme can behave differently for different dimensions.
Generally, the more the structure is large the more the self weight is important and therefore the bars are not simply proportional, they become much more thick. Moreover, enlarging the picture the problems of instability of the compressed members increase and the layout tends to become more complicated, to shorten the buckling lengths.

As said, truss beams are normally designed with a traditional layout of the elements because the optimal ones are impracticable in the reality. Between the configurations that are feasible to be built is then possible to perform some little adjustments, to search for some improvements.

In Fig. 9 it is represented a common type of tower crane. The configuration of the elements is ordinary as announced, however some optimization has been searched.

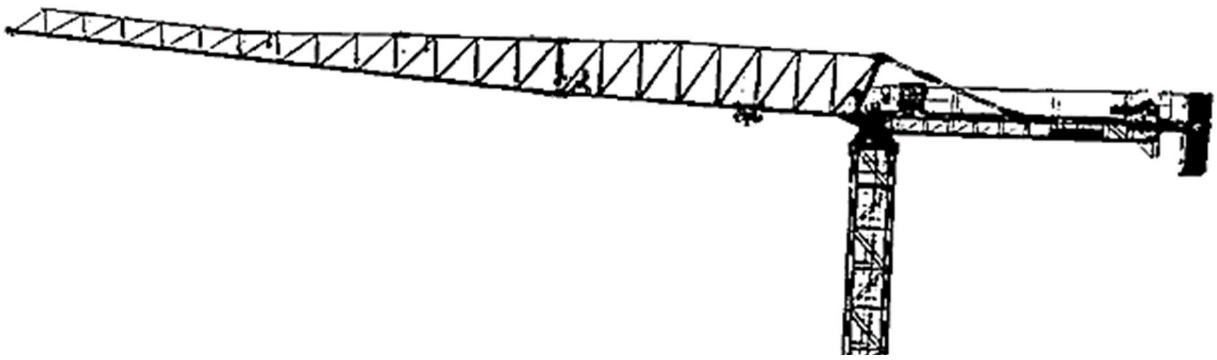


Figure 9: Truss structure of a tower crane, from a photography.

Firstly, it's evident the varying height of the truss, according to the structural behavior. Unfortunatelly it's not possible to appreciate also the varying thickness of the bars, that cooperate with the geometry for the mechanical efficiency.
Secondly, we notice that the diagonals are not uniform along the lenght: they are more dense and inclined near the tower, while their slope decreases going far from it. This design choice of optimality is related to the relations between the shear and the bending moment applied on the truss.

The structure under analysis in this work is made of glulam elements. Even if what just said it's traditionally related to the steel trusses, few novelties arise by changing the material. The main difference is that the timber members are thicker and therefore there are less problems of buckling.
The dowelled connections in use are a weak point, almost like the bolts for metals. Since the weldings don't exist for wood, the problem of the joints can be avoided only with the continuity of the member. Attention will be paid to this issue, that enters into the technical efficiency of the whole structure.

In this thesis all the matters just presented will be considered in order to find improvements of a real truss, commissioned by a company. The study will analyze both theoretical and technical issues, to achieve the optimization required.

1.2 Presentation of the thesis

1.2.1: <u>The working team and the organization</u>

This thesis belongs to an "Abroad thesis program" settled between Kungliga Tekniska Högskolan (Sweden) and Politecnico di Milano (Italy).
The work has been promoted and largely developed in Stockholm, following the swedish needs. The activities have been approved also by the italian reference person: Antonio Capsoni, a structural engineer of Politecnico di Milano, researcher and teacher in different courses.

The topic regards the application field of Moelven Töreboda AB, a company leader in the construction of glulam structures for civil use.
The relations with the firm have been filtered by one of its consultants: Roberto Crocetti, a swedish citizen born in Italy. He is a freelance engineer in the whole Europe and one of the leading experts of timber structures. He is also a teacher at the departments of structural engineering of KTH (Stockholm) and LTH (Lund).

While Crocetti is the Supervisor of the thesis, the Examiner is Magnus Wålinder, a professor of the department of building materials; he mainly dealt with the administrative procedures.

The working team has been completed by Bert Norlin, structural engineer with long experience at KTH and teacher in several courses related to timber and steel structures. He has been the theoretical counterpart with respect to Crocetti, useful to add academical suggestions to the technical matters of Moelven.

Figure 10: Logos of the two universities and the company involved.

Roberto Crocetti has ideated a new and promising concept for large ceilings of industrial buildings; he is intentioned to develop this thesis in order to verify the structural effectiveness of his proposal and bring the model to the company, to improve the constructions they are building.
Since the work will have some technical implications with Moelven, it has also been activated an internship with Crocetti: in this way, the practical issues related to the real applicability and optimality for the firm will be focused specifically.

The first part of the work has been performed in Stockholm, guided by the presenter of the matter. His main focus has been to investigate the eventuality of little improvements of his concept and to analyze other structural details.
Following the italian suggestions, the adjustment of the proposed configuration has been broaded up to a complete technical optimization: comparisons between different structural solutions have been performed, in order to get subsequent betterments.
Part of this further analyses has been completed in Italy.

1.2.2: <u>The swedish concept</u>

The proposed concept of Crocetti is the main glulam beam of a roof system, whose doubly braced assemblage is outlined in yellow in Fig. 11. The span is 50 m, maximum height 6,25 m. The slope of the pitch is constant and equal to the 5%, as fixed for the water drainage.

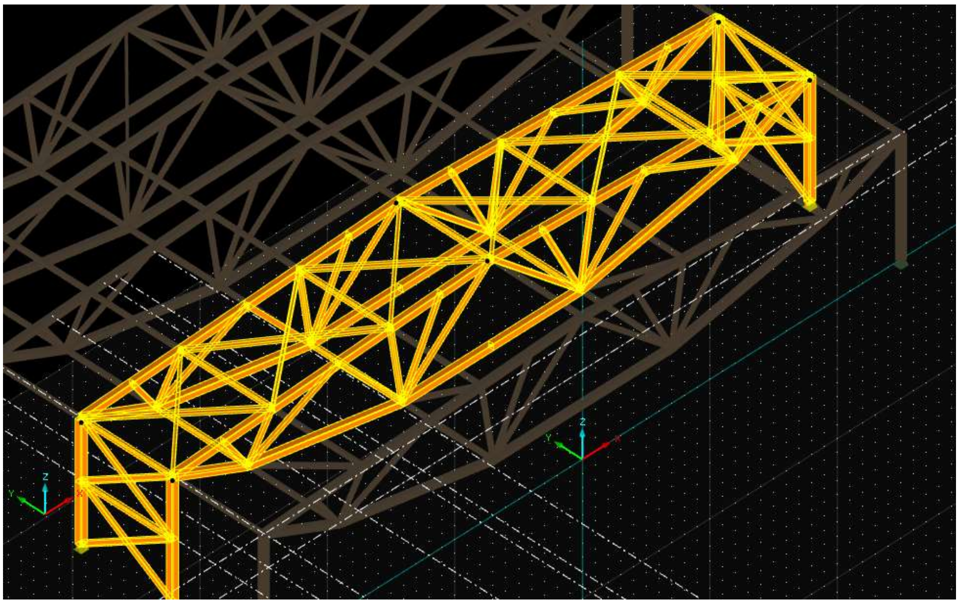


Figure 11: The swedish concept proposed.

The load is uniformly distributed from the roof through a corrugated steel plate and it's mainly provoked by the self weight and the snow load. Secondary joists are not used because practically it's very handy to have a prefabricated plate which satisfies at the same time the load bearing capacity between the main beams, the thermal insulation and the impermeability of the roof.
From what said, the main beams are loaded by a distributed pressure. They are made of glulam since wood is sustainable, largely available in Sweden and competitive in the industrial production.

It is not exact to speak about a truss system because the structural behavior is slightly different. Firstly, it's evident that a truss can be only subjected to concentrated forces on the nodes.
Secondly and more important, the diagonals (Fig. 11) don't respect a truss layout: most of them intersect alone the compressed flange and this imply that their axial force is balanced with shear in the upper element.

The ideator didn't conceive a truss at all. He has thought the upper chord as a continuous beam resting on a catenary through the diagonals. Those bars act like compressed springs and interact with the flexural behavior of the upper member, similarly to a cable stayed bridge.

The lower chord has been drawn with a funicular shape mainly because it should provide stiffness to the structure with its curvature and not with its truss level arm.
It's not possible to speak about a suspended structure because there isn't horizontal constrain at the supports and the tension of the lower elements becomes compression in the upper ones.

Eventually, we could define the structure a truss-flexural beam. Roberto Crocetti presented this concept as a configuration close to the mechanical optimality, because the shape of the lower chord follows the diagram of the bending moment. He promoted the thesis to verify the structural efficiency of his proposal.

1.2.3: <u>The development of the thesis</u>

Section 2. begins describing the assigned Moelven's beam, that is the geometry, the loads and the structural analysis performed.
The initial proposal had steel tendons in place of some glulam members. It has been found that the structure isn't stable under uneven load, because one cable becomes compressed and everything fails. The problem has been to having applied an inadequate uneven load into the structural analysis.

After having stabilized the proposal, the design has been performed with solely glulam members and the required volume of material has been measured, to have an index of the general expense.
The assigned configuration didn't allow for significative improvements by simply adjusting the existing elements.

The last part of Section 2. (Chapters 2.7, 2.8 and 2.9) contains insights on the structure proposed by Crocetti and related to the internship developed with him. There are analyses on how the results are affected by the bending stiffness of the upper chord and by the rigidity of the joints (rotational and longitudinal).

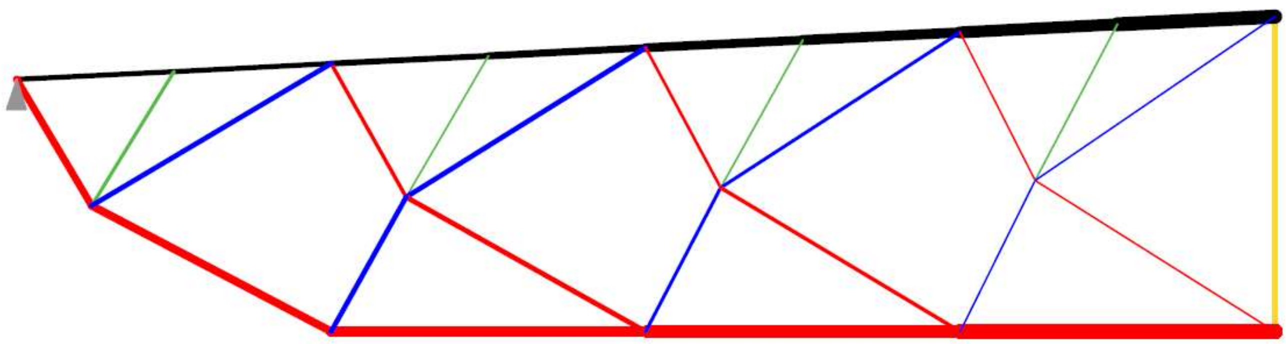


Figure 12: First theoretical truss developed to search for a better alternative.

The optimization analysis starts being developed more properly in Section 3, where the mechanical improvements have been investigated through deep reorganizations of the structural system and of the layout of the members.
A truss model (see Fig. 12) has been introduced for the same Moelven's problem. The configuration has been chosen according to a chart of the principal directions of stress, in order to be mechanically very competitive.

The comparisons have shown that the truss solution based on academical knowledge is lighter than the original proposal of the consultant of the company.
The research hasn't been interrupted: observing the results obtained and following inspiring ideas, different structural analyses have been experimented in order to search for possible improvements of the theoretical configuration.

In the middle of Section 3. the firm has asked to reduce the maximum height of the structure from 6,25 m to 4,50 m, due to logistical issues.
The concept of Moelven has been thus lowered in height and designed again, finding that it starts working quite poorly with a limited arm. The academical truss scheme has been also lowered and the optimization analysis has proceeded on the height of 4,50 m, with better skills and awareness.

Section 3. ends with the study of the last theoretical configuration, which benefits from all the improvements found and the knowledge acquired. There are considerations about the spacing of the nodes in the upper member, the shape of the lower chord and the orientation of the diagonals (with details on the support and at the midspan).
With respect to the initial proposal, the academical solution has achieved a reduction of the 25% of the required volume of glulam.

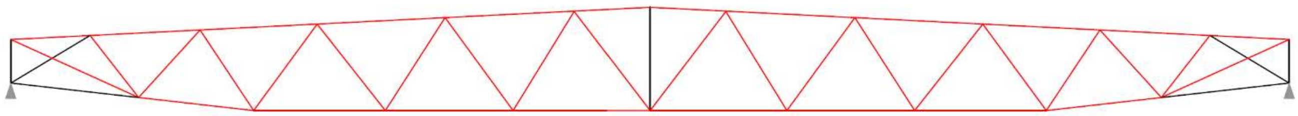


Figure 13: Final optimal configuration developed.

Even if structurally optimal, the configuration developed in principal directions of stress has been considered too academical; it has been rejected due to architectural reasons and due to some technical difficulties in the real production.
Section 4. has been therefore realized to find a configuration that is feasible to be really built and, at the same time, it takes advantage of the positive aspects theoretically found to be more efficient than the proposal of the company.

For the technical solution of Section 4 it has been chosen a shape for the lower chord that is in between the academical geometry and the Moelven's one. The member turns out to be aesthetically acceptable but also sufficiently performing from the mechanical point of view.
The most tricky details and connections have been abandoned, for an easier manufacturing. A region that has required specific attention is the arrangement near the supports, where the elements are a lot. Anyway, the layout has been designed to be structurally efficient and practically feasible.

A first proposal for the arrangement along the span has been a Warren scheme enriched with secondary non truss diagonals. It has proved to be well performing.
However, the flexural behavior of the upper chord has shown to allow for some small improvements. For this reason, the configuration has been adjusted a little, removing the non truss diagonals but keeping the same number of elements.

The final purely truss system proposed (Fig. 13) has shown to be only the 5% worse than the academical configuration in principal directions of stress. Moreover, it allows to save over the 20% of glulam with respect to the original concept assigned (i.e. more than 7 m^3 of timber per beam).

The last chapter of Section 4 is dedicated to the conclusions of the work. The most important technical aspects found in the thesis are recalled. It is also reminded how they contribute to the optimality of the solution developed, making it better with respect to the initial proposal.

After the description of the activities conducted, there is the bibliography and, in Section 6, the Appendix. It is a large portion of the thesis because it contains all the data regarding the studies performed. There are tables with the complete geometry of all the configurations, the cross sections of the members, the material properties and the loads applied. The Appendix contains also the static and kinematic results obtained for all the schemes, plus some safety assessments.
The information reported in the Appendix allows the reproduction and verification of the entire work.

2. The reference Moelven structure

2.1 Ceiling over a span of 50 m

The optimization performed in this work starts with the analysis of an existing structure and the suggestion of possible improvements.

In Fig. 14 there is a part of a technical drawing of a beam designed by Moelven Töreboda AB for a large span roof; for a better visualization only the left half has been shown in Fig. 14, the right one is symmetric. Further details are in the Appendix.

The hybrid solution combines glulam members and steel tendons; the total length is 50 m.

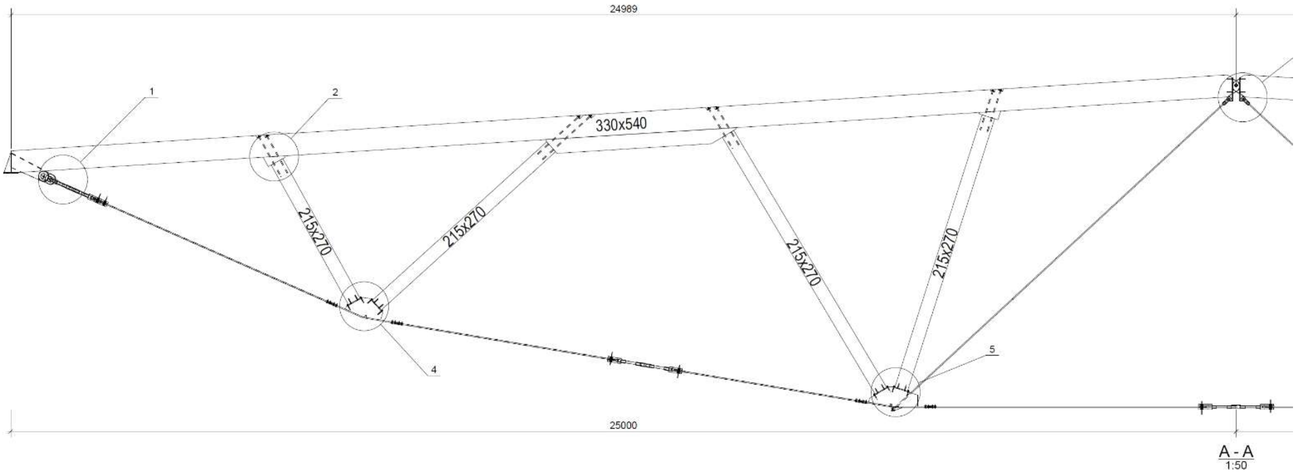


Figure 14: Left half of a roof structure of Moelven.

The slope Theta of the roof pitch is fixed at the 5% as in the swedish standards. The height at midspan has been kept the maximum possible, that is 6,25 m as in the existing design. The length of the half beam is of course imposed too, 25 m.

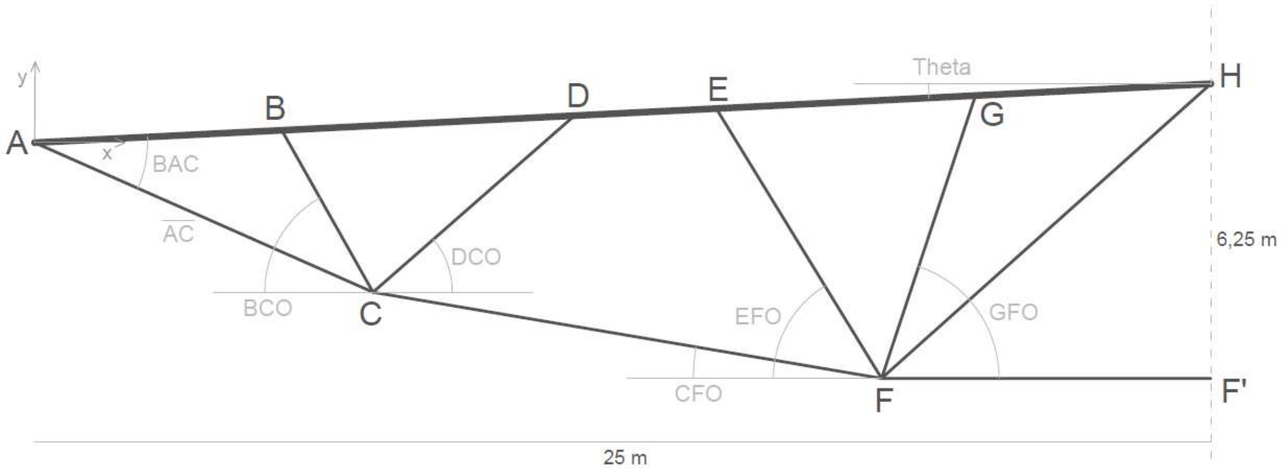


Figure 15: Arbitary geometrical parameters of the structure.

The structure has been analyzed and then some improvements have been searched, respecting the constraints just listed. Initially, improvements of the existing configuration have been proposed, then, new and better schemes have been experimented.

In the assigned structure there are only 7 arbitrary parameters that define the geometrical configuration of all the elements; we have chosen 6 independent angles and one length (AC), as indicated in Fig. 15.
The values of those arbitrary parameters taken from the technical drawings of Moelven are tabulated into the Appendix. There are also the coordinates of all the nodes with respect to a cartesian reference system centered in point A.

It's clear that those 7 quantities control all the current problem, therefore they are the degrees of freedom we need to manipulate in order to search for a better configuration of the existing elements.

2.2 Modeling of the problem

On the upper member AH, the load of the roof is uniformly distributed through a corrugated steel plate. For this reason, the bending behaviour of the beam AH is very important and must be considered properly.
The structure has been analyzed with hand calculations and the displacement method: according to the mechanical behavior, a Bernoulli beam model has been used for the upper member, while truss elements with hinges have been used for the other parts of the structure.

The support in A has been considered a roller, since for a roof is almost impossible to have perfect horizontal constraint at the supports (that is hinges at both the sides). Of course this choice must be verified since a horizontal obstruction to the second order shortening of the beam will lead to lower deflections and higher stresses. Generally, a final verification with the hypothesis of perfect hinges is good and enough.

In Fig. 16 it is shown the mechanical model developed for the structure, with 14 unknowns of translation and 4 unknowns of node rotation.
Initially, for a sake of simplicity, only half of the beam has been modelled, considering uniform load and perfect symmetry. Afterwards, for completeness, the full model has been studied, to be able to analyze also the non uniform load distribution on the two halves.

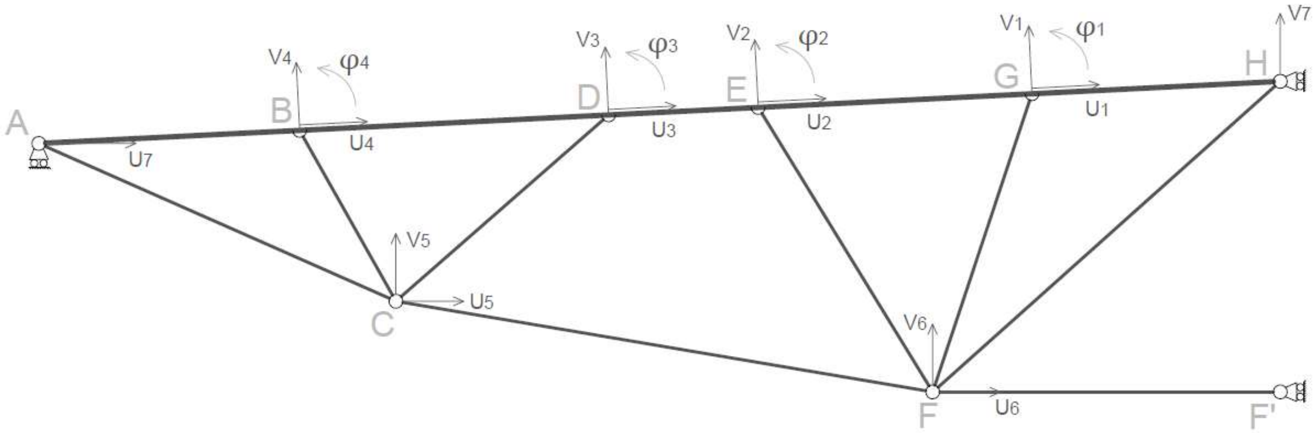


Figure 16: Kinematic unknowns of the Displacement Method.

In order to double the model and study the whole beam the unknowns in Fig. 16 have been mirrored specularly, so that the stiffness matrix will be the same (only the loads are non symmetric). The exception is the displacement U7, that doesn't have the twin since there is the hinge on the other side. One new unknown of displacement needs to be added, that is the horizontal translation of the point H, that is no more equal to zero.

The mechanical properties of the sections, taken from the Moelven's technical report, are ordered in the Appendix. They have been used for the structural analysis without any modification.

It can be seen that the structure is not properly a truss since if we have perfect hinges for all the nodes is no more stable. More clear is the fact that there is a uniform distributed load that produces bending in the upper member, aspect that is not allowed for perfect trusses.
In general we can see that the upper member behaves like a continuous beam on multiple supports, resting on a chain. In this perspective it's important to understand the bending behavior of the beam AH and the distribution of its stresses due to the spacing between the nodes and the effectiveness of the diagonals in carrying the load.
A comparison with a real truss behavior will be addressed afterwards.

2.3 Loads

2.3.1: The loads of Eurocode 1 and the Swedish National Annex

The load has been considered in the ULS combination. It is uniformly distributed as said and comes from the self weight of the roof and from the snow.

The self weight is of course equal everywhere and it leads to a perfectly symmetrical problem.
On the other hand, the snow can be non uniform on the roof and it requires the complete study of the structure since it breaks the symmetry.

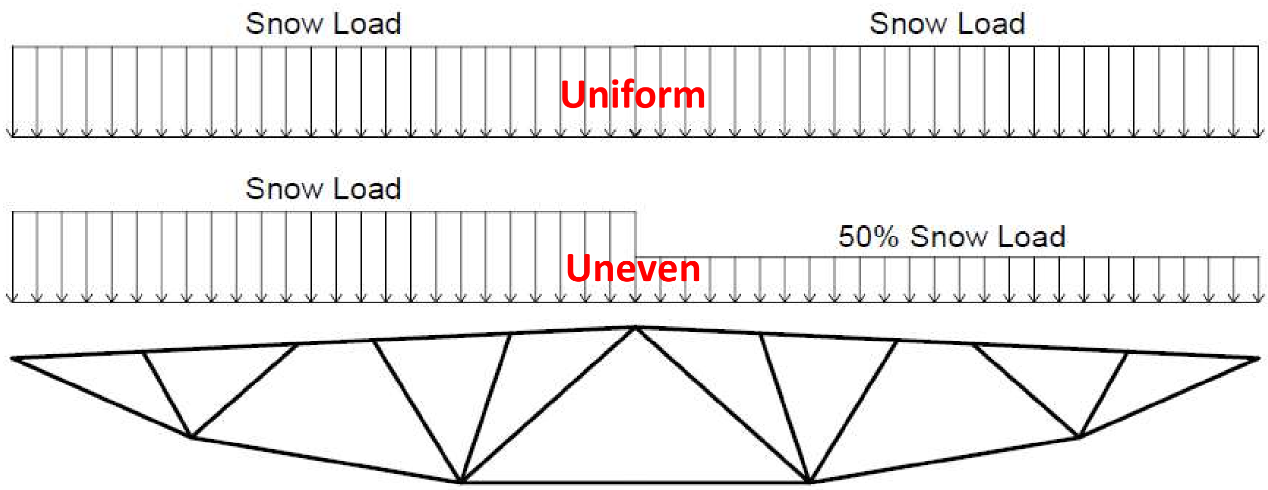


Figure 17: Uniform and non uniform snow load distribution (EC 1).

The snow load distribution can be considered differently in the Eurocode 1 and in the Swedish National Annex, because they follow a different idea.

Regarding the homogeneous snow load everywhere, both the codes agree in having it at the maximum amplitude (with the necessary safety factors).

The difference is related to the uneven distribution: Eurocode 1 takes into account the snow removal on one half, while the Swedish National Annex considers a non uniform snow deposition on the two pitches due to little wind during the falling.
As a consequence, Eurocode 1 takes away the 50% of the snow load from one pitch, leaving the maximum on the other half (as it is shown in Fig. 17). On the other hand, the Swedish National Annex increases a little the uniform load on one pitch with respect to the homogeneous distribution.

The evaluation of the snow load according to the Swedish National Annex is not easy and depends also on the inclination of the roof. The design values used by the consultant of Moelven have been copied without any modification. All the values are reported in the Appendix.

2.3.2: Rational choice of the load and detection of the lability

The values of the load used in the swedish design for the two pitches are quite similar (they differ of about the 3%), therefore they will lead to very similar stresses in the two halves of the beam.
It can be argued that they are useless to investigate the non-symmetrical behavior of the structure since this kind of load distribution is not really "uneven".

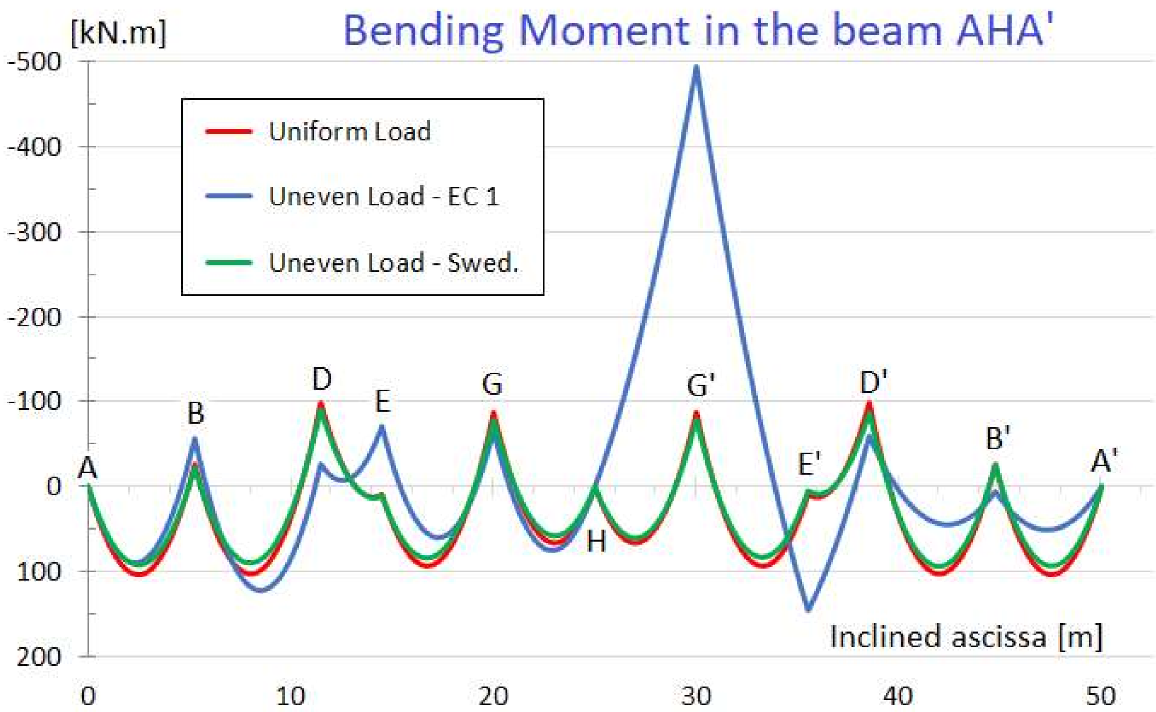


Figure 18: Bending moment in the upper beam under different load cases.

The structural model previously described has been written by hand as indicated and the system of equations has been solved in Excel. Both the loads of the Swedish National Annex and Eurocode 1 have been applied. The results in terms of bending moment in the upper chord are shown in Fig. 18.

As we expected, since the swedish uneven load is very similar to the symmetrical case, the red and the green curves are very close. The behavior is in line with the charts of the report of Crocetti.

The blue curve is the bending moment under the uneven load of EC 1. It seems wrong but it's correct.

Since we are analyzing a structure with wires, we need to check that they are in tension, otherwise we must remove them from the analysis since they can't carry any compression.

The diagonal FH is a steel wire, therefore it can work only under positive load. The uniform load distribution and the uneven case according to the Swedish National Annex produce little tension in the cables FH and F'H.
On the other hand, the uneven load of Eurocode 1 generates compression in F'H due to the high non-symmetry. Basically, the left side of the structure starts being supported from the right side that has few load, so that F'H becomes compressed.

The wire F'H (and not FH, that is in tension) must be removed and the structural analysis needs to be performed again. The left side of the beam can be supported only by the flexure of the upper member of the right side. For this reason, a huge peak of negative bending moment arises in G' and provokes the collapse of that member.

From this analysis it's clear that the initial structure proposed is not stable under the uneven load of EC 1, because it's not able to carry the shear which arises between the two halves. Due to the anti-symmetry of shear, in the couple of the diagonals FH and F'H one will be in tension and the other will be in compression, thus it's clear that it's impossible to use a couple of wires for this job.

2.3.3: Insights of the lability; its resolution

When considering the homogeneous load distribution nothing appears because, due to the symmetry, in the hinge H the force between the two halves is horizontal and it goes directly in compression in the upper member, therefore the diagonals FH and F'H are almost useless.

Looking at the technical report of the consultant of the company, there is a FE solution with RFEM using the load of the Swedish National Annex. It's clear that the software has not been able to detect the lability of the structure because the inadequate load has been applied: being it almost homogeneous, both the wires are in little tension and the stability seems satisfied.

The uneven load of the Swedish National Annex has been abandoned since it is not significant. As engineers, we always need to guarantee that our structures are safe according to our experience about the reality, prior than the rules written into the code.
For the whole thesis the uniform and uneven loads according to EN 1991-1-3 have been considered. In the Appendix there are all the details.

Regarding the structure under analysis, that is not safe, the elements FH and F'H have been considered to be in glulam, with the same section like the other diagonals. In this way, those bars can work as strut and tie for the shear between the two halves under uneven load, so that the bending in the upper member remains limited. This is enough to prevent the collapse of the assigned beam.

2.4 Little improvements of the existing configuration

Once the structural analysis has been done and verified, possible improvements of the mechanical behavior have been searched. The goal is now simply to enhance the structural performance using the same elements. At this first stage it has been considered only the homogeneous load case.

The existing configuration has been modified by adjusting the 7 arbitrary geometrical variables with a parametric analysis. The purpose has been the reduction of the midspan deflection and of the peaks of the bending moment in the upper element.

In general, it has been found that the angle BAC is the most important parameter governing the problem. Increasing it a little seems to be good in all the cases, because the arm for the global bending stiffness grows and the vertical settlements are usually lower.

The length of the tie AC is important to determine the position of the nodes C and F (and indirectly also of the diagonals), thus it influences the stiffness of those points against the vertical loads.

The orientation of the diagonals affects the position of the supporting points of the upper beam AH, therefore their role is more local and related mainly to the distribution of the bending moment in that member.

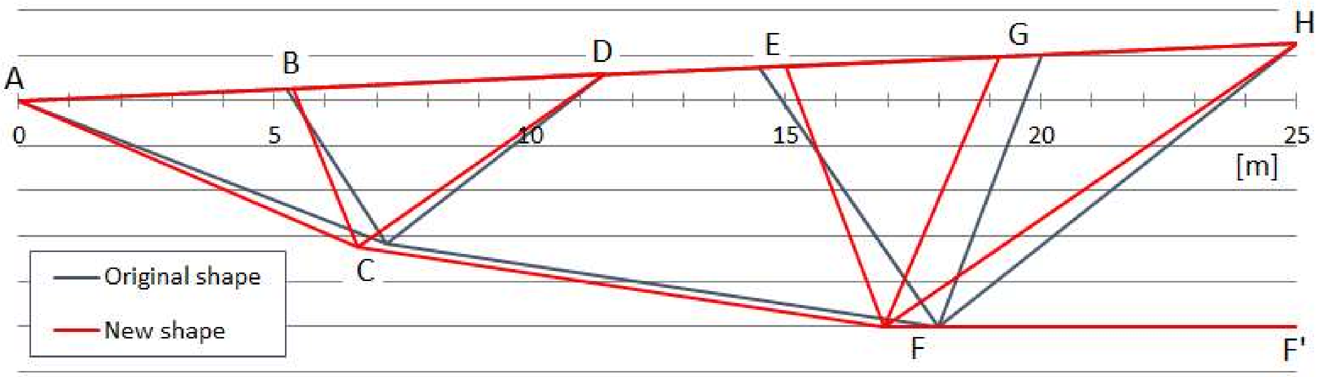


Figure 19: Comparison between the original configuration and the proposed one.

Having learnt those general phoenomena and with a deep optimization between the 7 parameters, it has been found a little improvement in the general configuration. In Fig. 19 there is a graphical representation of the original geometry of Moelven and the new one proposed with this study. Further details are in the Appendix.

The comparison in term of mechanical performance is resumed in Table 1 here below, other diagrams of the internal actions are in the Appendix.

		Original solution		New solution	
Midspan Deflection		153	mm	147	mm
Bending Moment	Min (AH)	-99	kN.m	-101	kN.m
	Max (AH)	104	kN.m	101	kN.m

Table 1: Mechanical comparison between the two solutions.

By increasing a little the angle BAC the reduction of the settlement is achieved (see Table 1). Then, it seems more difficult to control the bending moment in the upper beam AH, but the accurate optimization has found a good solution, where the maximum and the minimum values along AH are the same in modulus and interest several sections.

The improvements found by simply adjusting the configuration of the assigned elements of the project of Moelven are, unfortunately, very limited and not interesting. Possible enhancements must be searched through large changes, rethinking the structure from scratch.

2.5 Assessment of the structural solution of Moelven

In the previous chapter it has been searched a mechanical improvement of the structure by rearranging the geometrical configuration of the existing elements. The conclusion found is that the beam, as it is with 10 diagonals converging on 4 nodes on the lower chord, is optimal enough: better solutions are possible only with a deeper change of the elements and of the whole structural system.

In this chapter it is performed an assessment against the failure of the existing solution proposed by Moelven, that is the first optimal one we have. Moreover, the amount of material needed is evaluated since this is an interesting parameter to judge the optimality of a structure that is able to carry the loads. In this way, in the next chapters it will be possible to compare this design with the new optimal solutions that will be experimented.

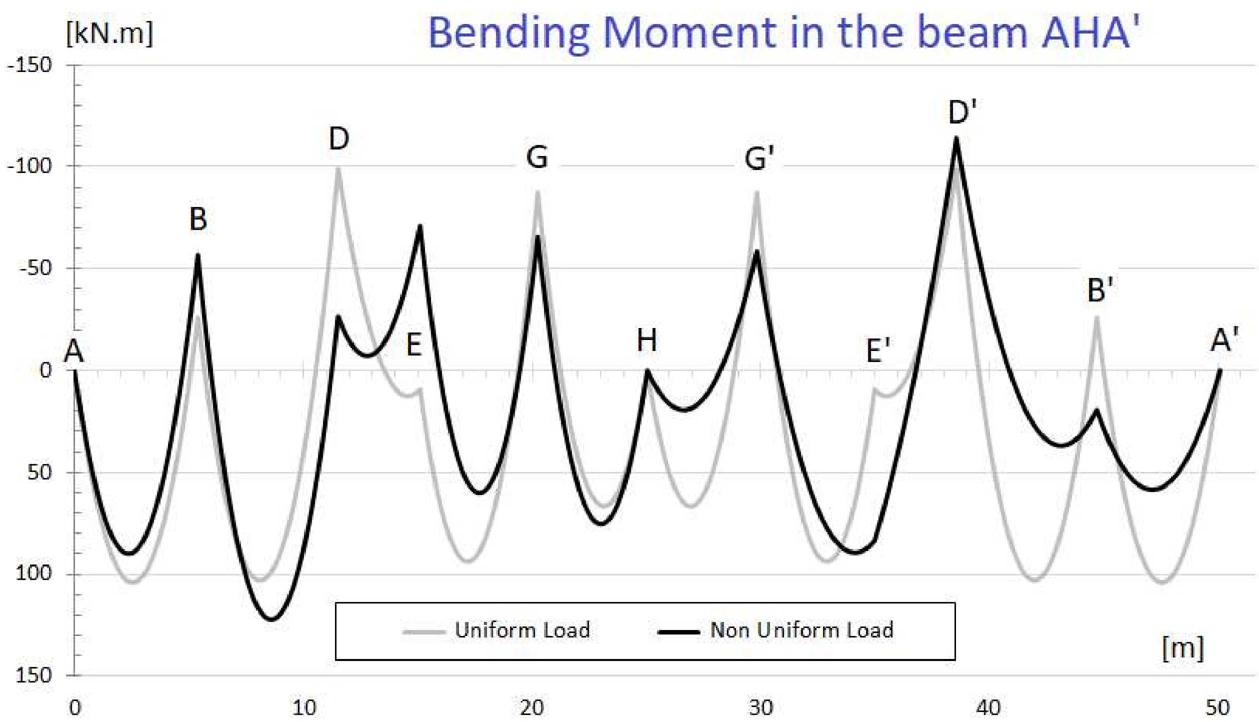


Figure 20: Bending moment diagram in the upper member, Moelven's structure.

The solution proposed by Moelven (and previously described) has been analyzed and the internal actions have been plotted for the two loading conditions previously seen. In Fig. 20 the diagram of the bending moment in the upper beam is represented. The other charts are in the Appendix.

The most important issue that need to be checked is the buckling of the compressed diagonals and of the upper beam, that is subjected to compression and bending. This phenomenon has been studied only at a local level for each element. Possible global modes of instability are not addressed here.

The eulerian buckling of timber members is described into the Eurocode 5, where it is normed through some empirical coefficients. The design formulas are shown in the next page, all the related data and the results are in the Appendix.

Assessment against compression and bending:

Slenderness: $$\lambda = L_0 \sqrt{\frac{A}{J}}$$

Relative slenderness: $$\lambda_{rel} = \frac{\lambda}{\pi} \sqrt{\frac{f_{c,0,k}}{E_{0,05}}}$$

If $\lambda_{rel} > 0,3$ the buckling risk need to be studied, as follow:

$$k = 0,5[1 + 0,1(\lambda_{rel} - 0,3) + \lambda_{rel}^2] \qquad k_c = \frac{1}{k + \sqrt{k^2 - \lambda_{rel}^2}}$$

$$\frac{\sigma_{c,0,d}}{k_c\, f_{c,0,d}} + \frac{\sigma_{m,d}}{f_{m,d}} \leq 1$$

The formulas of the Eurocode 5 are here resumed. The diagonals will naturally buckle along the weak axis. The roof prevents the upper beam AHA' from the buckling out of the plane of the structure, so that AHA' is forced to work with the strong axis of bending.
The buckling length has always been considered as the distance between the nodes, on the safe side.

From the calculations, the most solicited part is the upper beam AHA', that is near to failure but still safe, according to the Eurocode. The worst loading condition for that member is the homogeneous distributed load along all the length.

The diagonals have a larger safety margin, especially the member BC due to its short length. The fact of having the same type of element for all of them and not with a square cross section is not so optimal, however the choice is probably related to the easier manufacturing.

The lower chain is verified against the tension failure, therefore all the elements are generally safe.

The assessment performed doesn't include the integrity of the connections, that are considered to be ok. They are not so interesting because they are uncoupled with respect to the optimization of the configuration of the members.

The evaluation of the necessary volume of timber (to have an index of the mechanical efficiency of the structure) is postponed to the next chapter.
It will be seen soon that the company has decided to substitute all the steel tendons with glulam pieces. We think it's better to re-design from the beginning the structure according to those new Moelven's directives and to calculate the volume of the required material only afterwards, finding a value that is significative for the future comparisons.

2.6 Structure with solely glulam elements

2.6.1: Development of the new suggestions from Moelven

Since the lower chord is a steel tendon, it reults to be very thin and it is also very weak against fire. The use of particular layers to protect the cable from the flames has been seemed not to be efficient; for this reason that steel tie has been substituted with a timber member. If the same element is made of wood it will be much more thick and it will resist longer under fire.

Using timber instead of steel for the lower chord is good also for another reason, that is the resistance against negative loads. If the wind provokes a suction on the roof, the lower chain could be compressed and it would buckle for sure if it is a steel tendon.

Due to all this, a new concept has been proposed for the ceiling, where all the elements are in glulam. It has been also decided to add a couple of diagonals to the scheme of the previous chapters, so that the configuration is what is shown in Fig. 21.

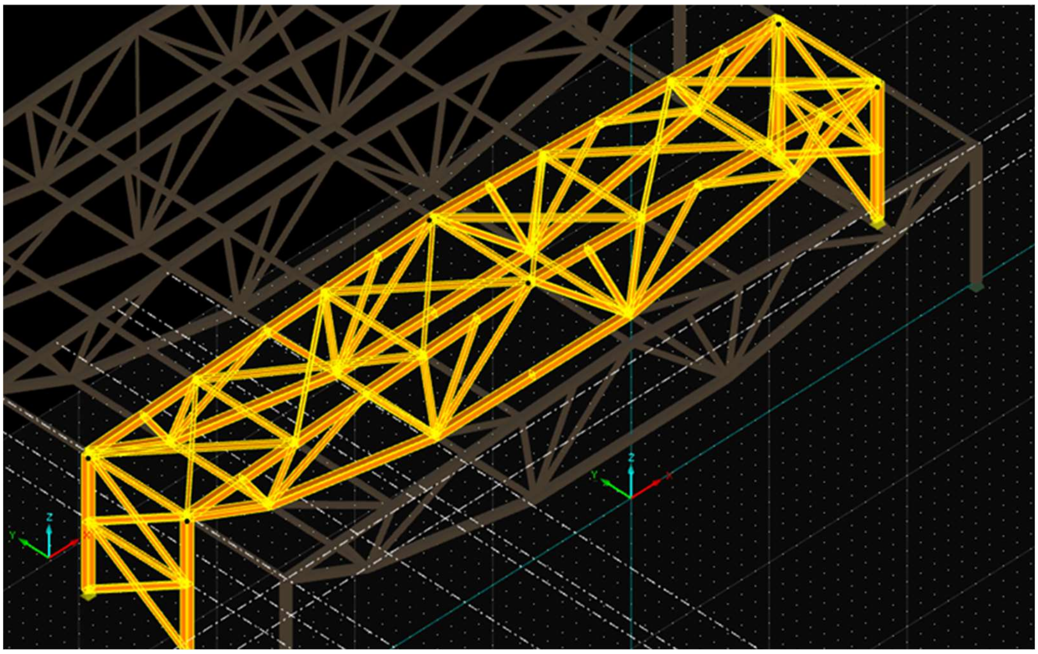


Figure 21: Concept of the structure proposed by Moelven with solely glulam members.

From the point of view of the structural analysis there is nothing new with respect to what has been seen so far. The beam has the same structural behaviour.
The same model used previously has been adjusted, changing the mechanical properties of the lower chord and adding the two new diagonals.

The sketch of Fig. 21 is only conceptual and there are not precise geometrical data about. For this reason, the boundaries of the structure have been kept as before and, regarding the configuration of all the elements, their optimal position has been searched directly with a parametric analysis on Excel.

Like before, the goal of the parametric research is to find a scheme of the members that distributes the forces in a good way, using the geometry appropriately. The mechanical behavior is felt and guided with the positioning of the elements. The charts of the internal actions are an important help to detect and correct anomalies, concentration of stresses or useless parts.

It must be noticed that the geometrical degrees of freedom that define the configuration are no more 7 but 8, due to the addition of the diagonal FL. Between all those parameters it has been performed the optimization of the structure desired by Moelven.

In Fig. 22 it is shown the best solution found, superposed to the previous hybrid model. The two new diagonals EC and FL are perfectly visible.
The coordinates of all the nodes of the beam and the values of the arbitrary parameters are in the Appendix, in order to allow a perfect reproduction of the work.

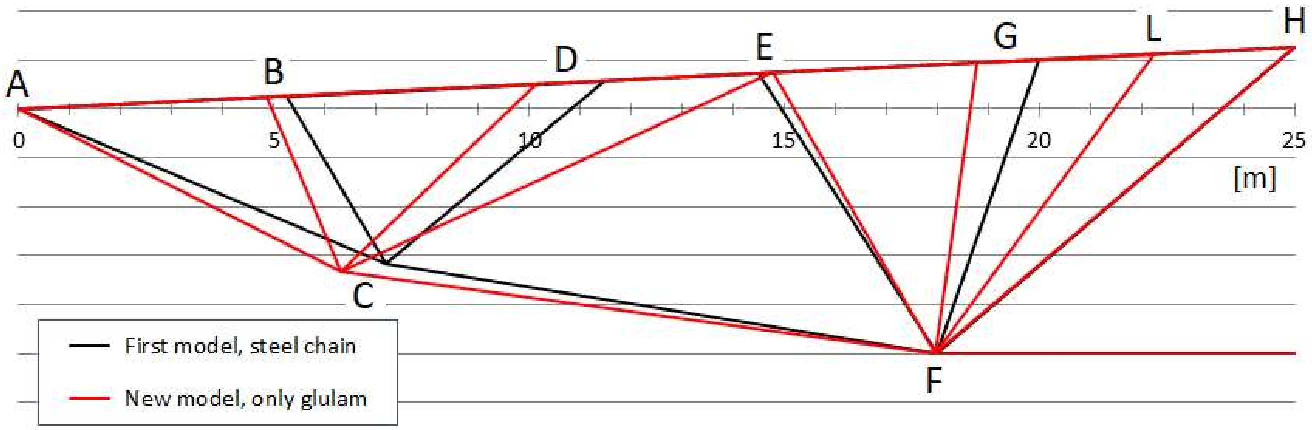


Figure 22: Optimal configuration with two new diagonals, compared to the old one.

2.6.2: Design of the structure

Like in the previous chapter, the structure has been subjected both to uniform and to non uniform distribued load at the ULS. Then it has been designed according to Eurocode 5, as before.
In Fig. 23 the commercial dimensions of the lamellas used in the design are shown. In the Appendix there are the sections of all the elements.

Figure 23: Dimensions of the commercial lamellas [mm] used for the design.

The assessment against local buckling of the diagonals and of the upper member has been performed, as previously done.

Regarding the lower chord under tension, the resistance is most likely imposed by the connections, that are the weakest part of that element. The design of the joints is something to do at the last stage of the process, while our focus is on the preliminary study of the most suitable configuration. An approximate estimation of those details is enough.

The lower chord has been roughly defined keeping an extra safety factor of about the 30% on the tension resistance, in order to implicitly consider that the joints will lower the load bearing capacity.

In compression this issue is not so important since the buckling takes over, therefore no extra reduction has been considered under negative load.

In the Appendix there are all the data regarding the safety of the structure against the collapse. In the upper member AH this value is very near to 1 everywhere, while in the diagonals the optimization is less pronounced. As a general concept, it is more difficult to approach the ideal section with commercial lamellas when the dimensions are limited.

2.6.3: Analysis of the results and comparisons

In the Appendix there are the data about the internal actions of all the elements, under uniform and non uniform load. In general, the charts are similar to what has been seen in the previous chapters and they don't require special comments.

An interesting detail that can be observed is that the amplitude of the bending moment is not uniform along the beam AHA', but it's lower near the midspan, exactly where the axial force is higher. The optimization procedure has determined this result.

Since the buckling of the upper beam is induced both by the compression and the bending, the diagonals have been oriented in order to leave the largest bending moment where the axial force is low, and vice versa. In this way the section will be used at its maximum everywhere.
The safety factor close to 1 along all the beam AHA' is also a consequence of this aspect.

		Old Solut. - Steel chain		New Solut. - Only glulam	
Midspan Deflection		153	mm	106	mm
Bending Moment	Min (AH)	-99	kN.m	-59	kN.m
	Max (AH)	104	kN.m	79	kN.m

Table 2: Midspan deflection and maximum bending moment for the two solutions.

Let's come to the more interesting comparison with the previous, hybrid solution with the steel chain. In Table 2 are indicated, for the two structures, the midspan deflection under the ULS uniform load and the maximum bending moment arising in the upper beam AH.

From the comparison it's clear that the new solution with solely glulam elements performs better than the first one.
The reason is not immediate, however more detailed analyses have shown that the merit is in the two new diagonals that have been added. Thanks to them, the web of the structure is stiffer and this leads to lower deflections. Moreover, the distance between the nodes on the upper beam can be reduced and therefore the bending moment arising can be lower, due to the shorter span.

2.6.4: Optimality of the solution, volume of timber required

Of course, the more diagonals are used the better the load can be carried, but the solution will be also more expensive. The degree of optimality of this configuration is not known.

Since the structure is mechanically equivalent to what has been seen so far, we consider it as the starting point of the optimization procedure.
This is the Moelven's proposal that will be compared with the next solutions that will be developed with deeper changes of the elements.

At this stage it has sense to compute the total volume of timber needed. This datum will be compared afterwards with the future proposals. The amount for each part of the structure is reported in Table 3.

Volume of timber				
Upper beam	6,89	m^3	31,6	%
Lower pieces	10,37	m^3	47,5	%
Diagonals	4,58	m^3	21,0	%
Total	21,8	m^3	100	%

Table 3: Volume of timber needed for the structure requested by Moelven.

Looking at the table it can be immediately seen that the lower chord is the element that requires the highest amount of wood.

It could be expected that the upper beam asks the largest section instead of the lower chord, since under compression there is the problem of buckling.
On the other hand, we should not forget that the design resistance in tension is smaller than in compression due to the different influence of the defects. Moreover, we have used only the 70% of the tensile strength in order to implicitly include the effect of the connections.

All these aspects lead to the fact that the biggest element is the lower chord.

The rough design of the lower tie is not accurate and the error deeply propagates to the total volume, that can't be precise now. However, it's clear that it will be extremely important to search a way to minimize the stresses in the lower pieces and the weakening provoked by the connections.

Further studies focused on the lower chord will probably be useful to find other possibilities, where the stresses are carried in a better way in that member.

<u>Note</u>:

In the next chapters the improvement of the configuration is temporarily abandoned: the research of possible solutions with a better mechanical performance than the structure just designed is suspended. In the pages below there are technical insights about this beam of Moelven, more closely related to the internship developed at the company.

The reader who is interested solely in the topic of the optimization can directly proceed to the Section 3. of this thesis.

2.7 Influence of the stiffness of the upper member

2.7.1: Description of the mechanical phenomenon

Since the upper member AH is not hinged on each node but is a continuous beam, we are dealing with a redundant structure. As a consequence, the stiffness of the elements will affect the stresses.

The beam AH is subjected to bending by the distributed load applied on the roof. Since the analysis of optimization is general, the cross section has been chosen to be good in bending but whitout searching an extreme solution: the originary beam, working with the strong axis of inertia, has been adjusted but without reaching a very slender web, avoiding all the problems of sectional instability that are not the main focus now. The chosen dimensions are 255x540 mm.

In this chapter the influence of the stiffness of the upper beam on the results is analysed.

If the section of the compressed chord becomes very tall it will be very stiff on the strong axis and, once the lateral instability is prevented, it will be optimal in bending. Unfortunately, it will also attract higher stresses due to the redundancies of the system. In particular, the bending moment in AH will increase because its flexural behavior starts covering multiple sub-spans. Some diagonals will reduce their compression because the load of the roof will be trasferred in bending by the upper member using few of them.
Conversely, if the upper chord AH is flexible, it will deform more and it will load all the diagonals along his length, taking fewer bending moment.

The mechanical behavior is similar to a beam resting on discrete elastic supports with different stiffnesses: increasing the bending stiffness of the beam, the smaller deflections will make it work mainly on the stiffest springs, unloading the others.
It's important to recall what said at the beginning of this paragraph: without the redundancies nothing would happen. If the beam is not continuous but hinged at each spring, its bending behavior is always local for every sub-span and the stresses can't change, they are statically determined.

The reader can correctly argue that what has been described is not properly the behavior of a truss. This is correct, in fact this was exactly the general idea of the designers of Moelven, and their unusual arrangement of the diagonals doesn't follow a truss action, but the concept of a elastically supported continuos beam.
In the structure proposed by Crocetti the upper member is similar to a Winkler beam where the axial compression related to the truss system is added.

The two mechanisms of bending of the beam AH and compression in the diagonals cooperate in parallel against the load; the dominant one depends on how the stiffness is distributed between the elements of the structure.

The "bending behavior" with the stiff upper beam AH is a priori not optimal, because we are using the flexural performance of that beam instead of the the system action of the diagonals and of the whole structure.

2.7.2: Operative procedure

We are not so worried to have a problem of "excess of stiffness" in the beam AH because the diagram of the bending moment is very nice and it shows a countinuos beam on multiple supports, where all of them do their job. However, further studies have been performed in order to better describe the phenomenon and see when it becomes an issue.

It has been redone the calculations several times, changing solely the dimensions of the cross section of the upper beam AH. The area of the section of that beam has been kept fixed as in the design previously done, that was b x h = 255x540 mm (that means a sectional area of 1377 cm^2).

It has been tried to reduce the height h enlarging the base b with the same ratio, and vice versa.
The height has been modified stepwise, changing of an integer quantity the number of the lamellas; they have a commercial thicknes equal to 45 mm each. The dimension of the base can't be commercial like the height since the area is imposed to 1377 cm^2, therefore the base b is simply a rational number that follows the steps of the height h.

The area of the section of AH has been kept constant in order to keep constant the volume of that member and therefore the amount of timber used in the structure. It must be remembered that the length of all the elements doesn't change because we are changing only the cross section of the upper beam.
In this way, all the possibilities with different sections for AH will have roughly the same cost.

Summing up, we have kept as a costant the amount of material of the design and tried to change the stiffness of the upper beam, looking at what happens.

In must be underlined that, keeping the same area of the cross section and increasing the height h, the inertia grows in a parabolic way. For a rectangular section we have:

$$\mathrm{I} = \left(\frac{1}{12}\right)\left(\frac{b}{n}\right)(\mathrm{nh})^3 = n^2\left(\frac{1}{12}\right)bh^3$$

Reformulating the problem, we increase the stiffness of AH in a parabolic way at constant volume; we will have higher strength of the beam AH but also higher bending moment and we want to quantify the consequences.

2.7.3: Analysis of the results obtained

The results of the analyses have been studied in terms of safety of the beam AH against the local buckling induced by the bending and compression.
Here below it is recalled the equation of the Eurocode 5 that describes this interaction:

Safety:

$$\frac{\sigma_{c,0,d}}{k_c\, f_{c,0,d}} + \frac{\sigma_{m,d}}{f_{m,d}} \leq 1 \qquad [\ ]$$

The load applied has been always the uniformly distributed at the ULS, for simplicity and since it has been seen to be the critical condition during the design.
The results obtained are shown in Fig. 24 and 25.

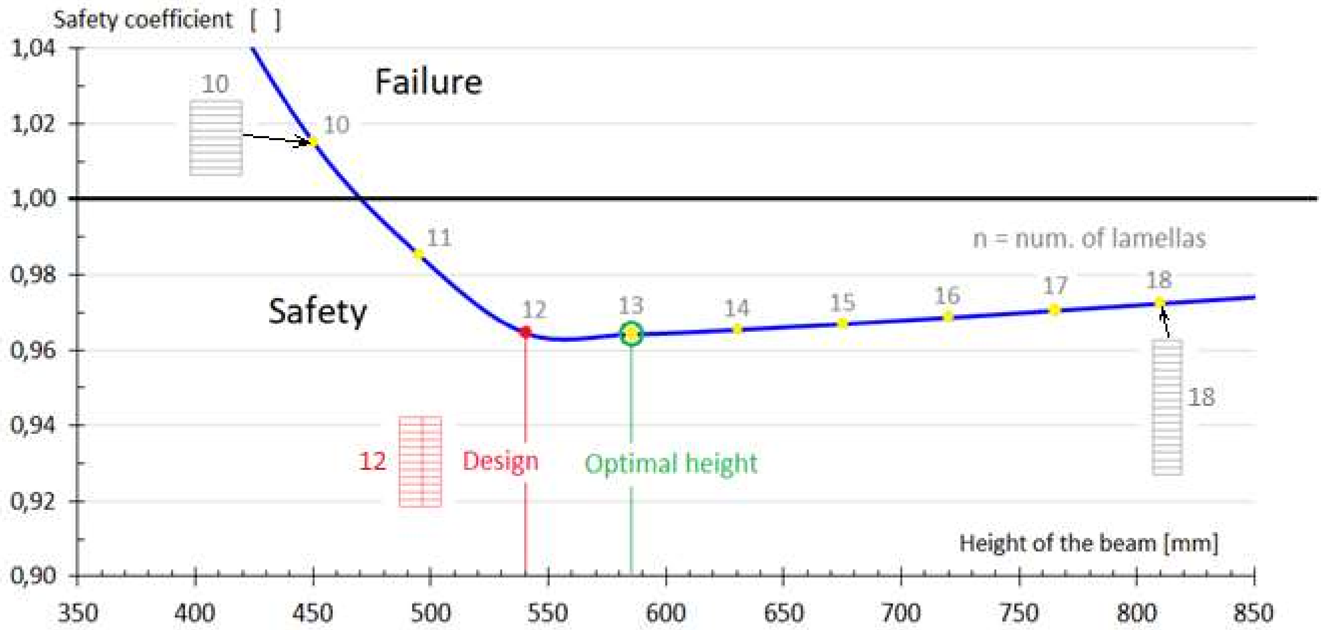


Figure 24: Safety against failure changing the section of the beam AH, at constant area.

The red dot in Fig. 24 is the design cross section, with the commercial dimensions 255x540 mm. The other yellow dots are obtained increasing or decreasing the number n of the lamellas and keeping the same area. There is also a qualitative sketch of some sections, in scale.

The horizontal black line separates, according to the Eurocode, the solutions allowed from the forbidden ones.

It is clear that, decreasing the number of the lamellas, the structure will quickly collapse because the reduction of the strength is not counteracted by a sensible reduction of the bending moment. In Fig. 25 this aspect is even clearer.
This fact means that the upper beam AH of the design is flexible enough and further redistribution of stresses by reducing its stiffness are not significant.

The green circle in Fig. 24 indicates the number of lamellas that leads to the lowest safety factor (keeping the area of the design). The value is so close to the design's one that the difference is not detectable in the chart and it's completely neglible.

It's interesting to notice that, increasing the height h of the beam AH, the safety factor doesn't reduce, but starts a very little growth, up to the failure for super tall and slender webs (the last data of Fig. 25 are not buildable in reality but they are shown for completeness).

This slow rise of the safety factor is very important, because it means that, also for a little increasing in the number of lamellas, the height becomes tall enough so that the bending stiffness of the beam AH is significant and the diagonals start loosing importance; the bad flexural behavior of the upper member is appearing.

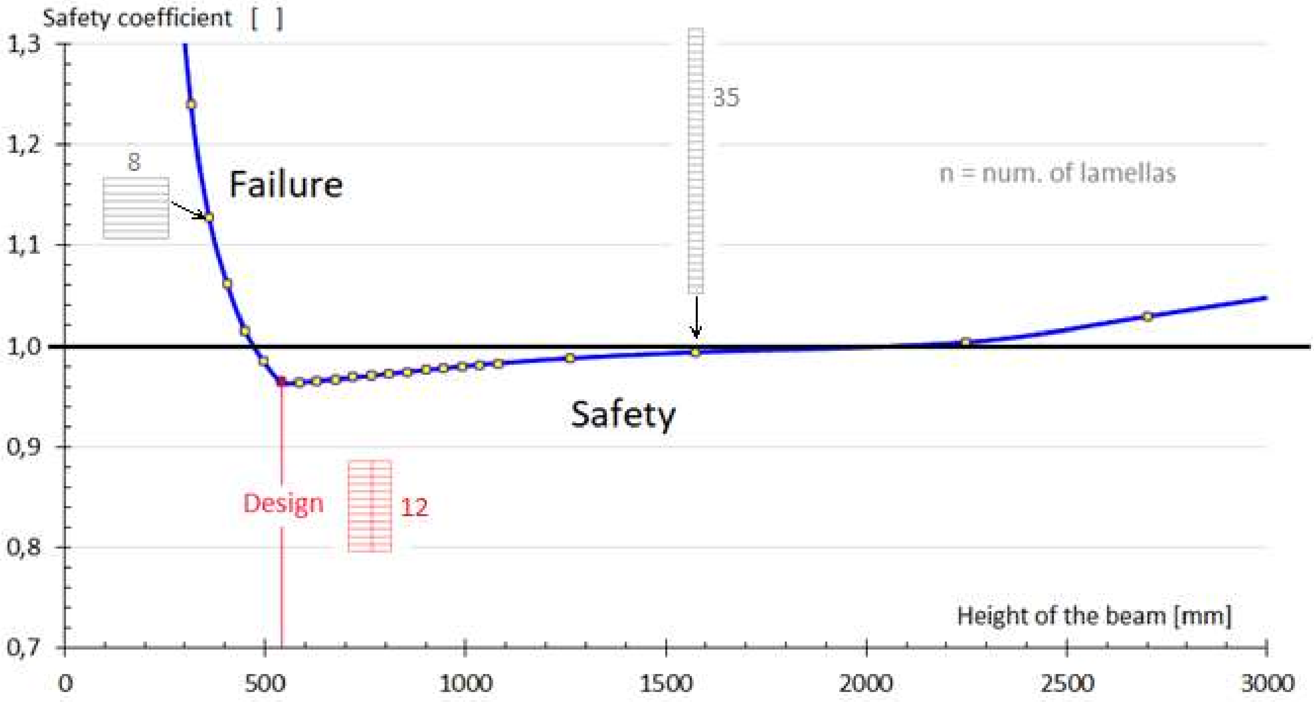


Figure 25: Safety against failure for large changes of the dimensions of the section of AH.

The diagram of Fig. 25 also shows that is very difficult to defeat the strenght of a tall beam by increasing the bending moment due to its own stiffness, because the inertia goes with the square of the height.

However, it is also true that it's not good to search for a very slender web (with all the related problems), to make it optimal in flexure, because the increasing of the bending moment will vanish the advantages of the tall cross section.

2.7.4: Insights on the flexural behavior of the upper beam

Since the topic is very interesting, we think it's appropriate also to draw the diagrams of the main internal actions in those cases that we are analyzing, with varying cross section at constant area.

The charts of the bending moment in the upper beam AH and of the axial forces in the diagonals are shown in the Fig. 26 and 27. The load is again uniform everywhere at the ULS, therefore only half of the beam is enough, due to the symmetry. Further details are in the Appendix.

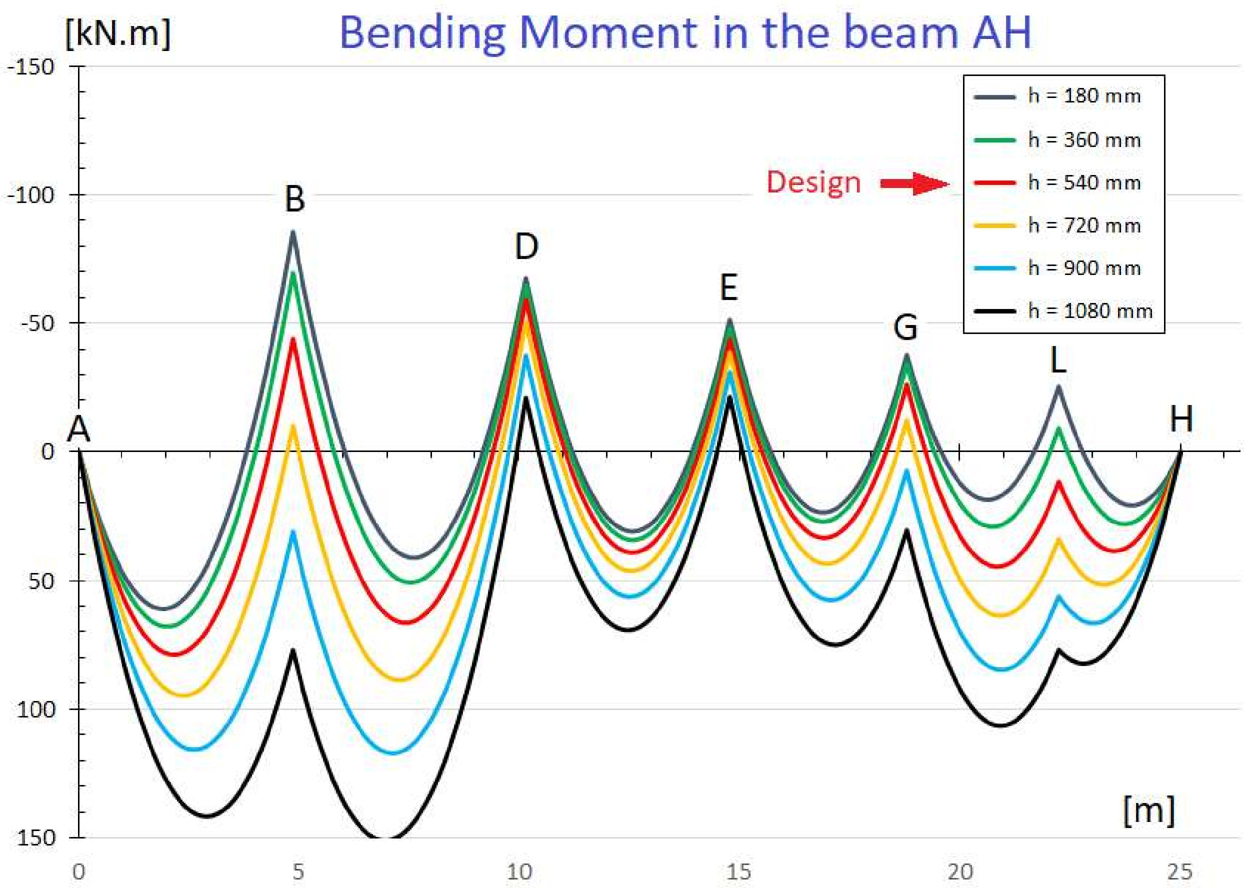


Figure 26: Bending moment in AH for differents shape ratio h/b.

Looking at Fig. 26 it's clear that the positive bending moment becomes always bigger by stiffening the beam AH. This is the reason why the safety coefficient doesn't reduce increasing the height; the responsible is the bad global flexural behavior of the upper beam, exactly as described before.

The diagrams of Fig. 26 confirm also that, increasing the stiffness of the upper member, some diagonals become less effective than the others in supporting it: the curves tend to become less affected in the nodes B and L, while the influence of the diagonals in D and E is much larger.

The concept just recalled is shown also in Fig. 27; the lines are almost horizontal, however the little changes in the values match perfectly to the mechanical behavior of the upper beam. The weakest diagonals are BC and FL and they are clearly unloaded when the beam AH becomes stiffer.
The load subtracted by the soft springs is redistributed by the beam to the stiffest ones: the elements CD and CE are the more rigid and they increase their force by stiffening the upper member.

The bad flexural redistribution of stresses provoked by the upper beam means growing of inequalities.

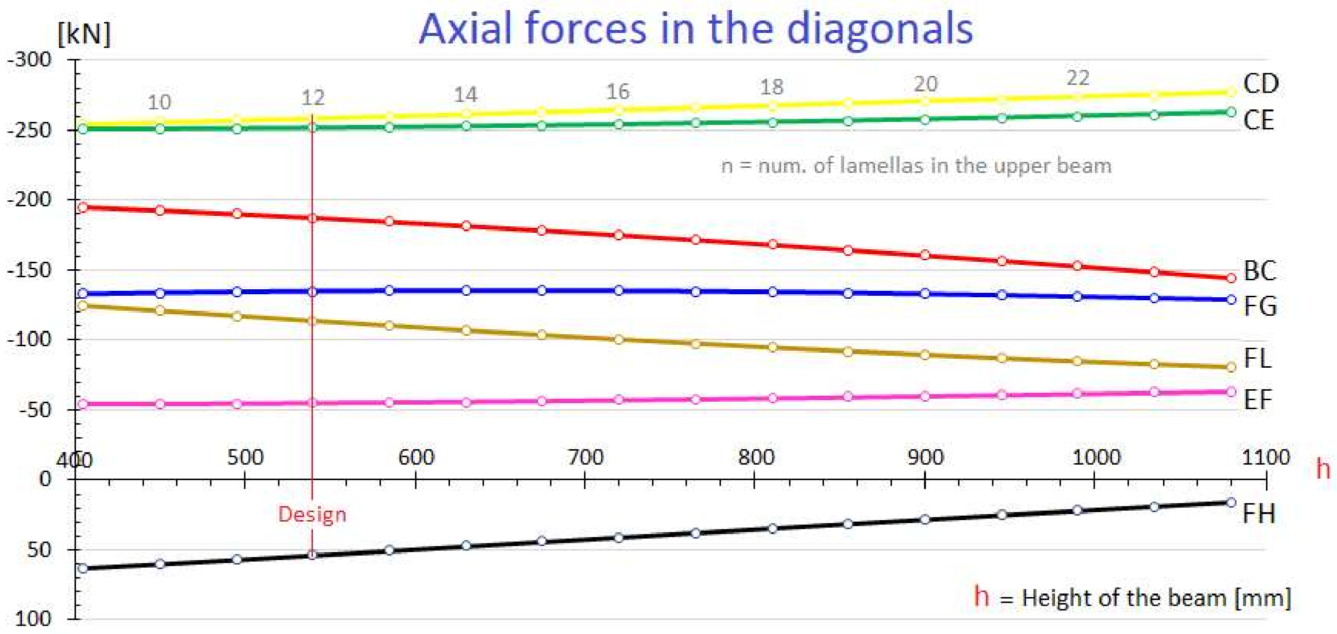


Figure 27: Axial forces in the diagonals varying the shape ratio h/b of the upper beam.

A doubt that arise is why a beam of ~ 0,5 m of thickness can manifest a global flexural behaviour in a structure that is 50 m long. Intuitively, this phenomenon should arise for much more tall beams: considering the dimension of the problem we would expect heights of the order of some meters.

We must not forget that, also if the beam AH was infinitely stiff, it would not be able to carry the load alone, transferring it directly at the supports in bending and forgetting all the lower structure. The reason is the presence of the hinge in H, that makes this configuration unstable. Consequently, the lower structure is always necessary for the equilibrium.

It has been also seen that not all the diagonals are equal in transfering the forces from the upper chord to the lower one: in the nodes D and E there are the most important ones.

It is possible now to understand why the bad flexural behaviour of the beam AH appears sooner than what was expected, for a section that is small compared to the whole structure. The reason is that there are internal nodes that will work always, so that the global bending behavior arises when the beam AH is stiff enough to work between those nodes forgetting the secondary ones, largely before than when its height is significant for the whole structure.

The large stiffness of the upper beam is therefore related to the spacing between the nodes and not to the dimensions of the problem. Much earlier than what expected, the unequal work of the diagonals manifests and determines a non clever bending of the beam itself.

The analysis of the flexural influence of the upper beam is not straightforward and requires a specific analysis for each particular configuration of the structure.
For simplicity, a rough conclusion that can be done in this chapter is that a ordinary shape ratio for the upper beam is generally suitable. The gain by using a slender cross section seems to be negligible for the bending resistance. Moreover, the risks related to the slender web increase and the problem of the global flexural behavior is around the corner.

2.8 Influence of the rotational nodes stiffness

2.8.1: Description of the mechanical phenomenon

In all the analyses performed so far it has been used the assumption that all the nodes of the structures are hinges. This means that all the connections between the diagonals and the longitudinals members are not able to transfer any bending moment.

The model described is accurate enough for a lot of cases of trusses. For bolted structures, the interstice between the bolt and the steel plate leads to the fact that, for small rotations of the element, the moment transferred is almost zero.

In our case the joints will be quite large because we are using a weak material like timber, and a huge number of screws will be necessary. In this way, the rotation in the node can be hampered and some bending moment can arise.

What we are describing is more complicated than a phenomenon of the connection alone, it depends on the whole structure. It's not only a matter of the rotational stiffness of the joints, it also depends on how much the elements will try to rotate and what is their bending stiffness against this rotation.

If a bar deforms due to bending from transversal loads, the node rotation can be important, hence the bending transferred by clamping it will not be negligible. Conversely, if the element is flexible and without transversal loads, it will adapt to the displacements of the extremities with minimal stresses.

In a perfect truss the loads are applied only at the nodes, thus they don't induce external bending moment or rotation. The structure works with the axial forces and elongation of the bars, which interact mainly with the displacements of the nodes and minimally with their rotation.
Generally, no matter what is the real rotational stiffness of the joints, the effect is negligible and the hypothesis of hinged nodes is very appropriate.

It's also true that in this case there is the uniform distributed load that provokes bending moment and rotations in the upper member, plus a non-perfect truss arrangement of the diagonals.

In this chapter the influence of the nodes rotational stiffness on the solution is analyzed.

2.8.2: Operative procedure

The structure has been modelled with perfect internal clamps at all the nodes, For sure the reality must be in between, stiffer than the hinge model but more flexible than the clamp model.
The real rotational stiffness of the nodes is not known but it won't be interesting if the two models will provide a very similar response.

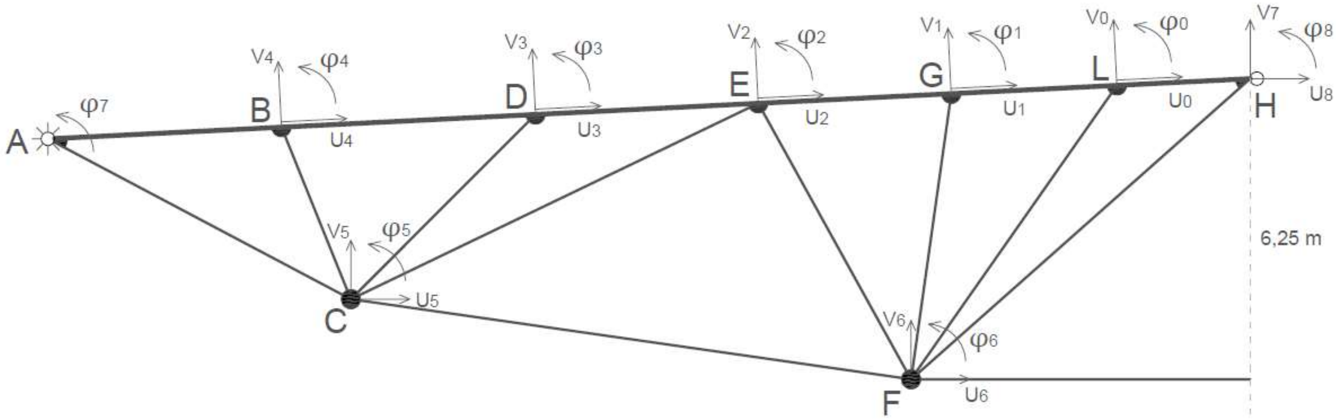


Figure 28: Structural model of the structure, left half of the beam.

In Fig. 28 it is shown the model with the clamps at all the nodes. Four new rotations (on the nodes A, C, F and H), have been added to the unknowns, in order to study the bending moment transferred.
For a better visibility of the picture only the left part of the beam is shown in Fig. 28; the right side is symmetrical except for the roller at the right support in A', with his horizontal degree of freedom. Of course also V7, U8 and φ8 in the middle don't have their specular.
In the point H the hinge has been left because in the technical drawings it is represented as a real hinge of the design, with a single steel pin.

The structural analysis has been performed with the geometry and the sections defined in the optimal design of the previous chapters for the beam with solely timber elements.
An important fact that need to be underlined (and that was totally useless before) is related to the diagonals: it has been considered that they work in the plane of the problem with the strong axis of bending. In this way, their bending stiffness against rotation is maximum and this model with all clamps is really an upper limit of the reality.

2.8.3: Analysis of the static and kinematic results obtained

The first result shown is a very general comparison between the two models with hinges or clamps nodes, see Table 4.
The data confirm the theory: the model with all clamps is stiffer and therefore it leads to lower deformations. As expected, the difference is very little since the rigidity we have added by clamping the nodes is almost negligible compared to the whole structural stiffness.

		All hinges		Only clamps	
Midspan Deflection		106,1	mm	105,8	mm
Bending Moment	Min (AH)	-59	kN.m	-63	kN.m
	Max (AH)	79	kN.m	70	kN.m

Table 4: Midspan deflection and maximum bending moment for the two models.

The internal actions are more interesting because they are related to the stresses in the elements and to the failure. All the diagrams for homogeneous and uneven load in the hypothesis of hinged or clamped connections are in the Appendix.
A chart of the bending is in Fig. 29; since the load is uniform everywhere only half beam is enough.

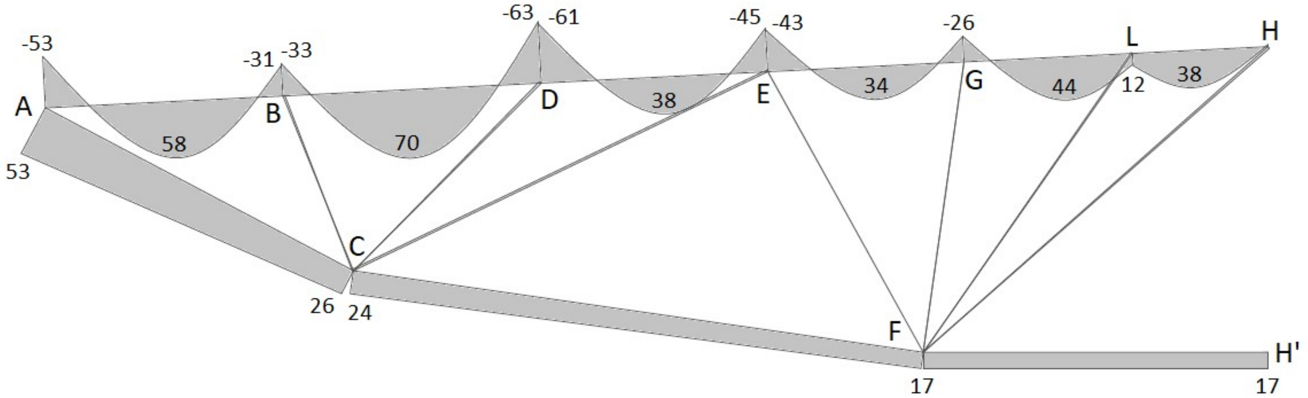


Figure 29: Diagram of the bending moment [kN.m] under uniform load, clamped joints.

The diagram of the bending moment of Fig. 29 is in scale and the stresses in the diagonals are so small that are almost not visible.

An important conclusion can be immediately drawn: the node rotation between the diagonals is so limited that, since they are slender and no external bending is applied on them, the bending moment they take from the joints is negligible compared to the other structural elements.

The interesting novelty is the bending moment that arises all along the lower chord (Fig. 29) and that was null before, by definition. It's probably related to the global settlement of the structure.

It can also be noticed that the connection in A brings a lot of bending moment between the upper and the lower chord. The element AB rotates in A under the external bending and both AB and AC are very stiff in flexure (also if AC was designed only for tension).
Due to all this, a significant bending moment is transferred locally in the point A from the upper beam to the bar AC. Basically, the clamp in A rigidly links AB and AC, that work in parallel in bending against the distributed load of the roof.

We will evaluate later if the new bending moment in the lower chord and in the node A is an issue, for sure this fact would be avoided if a steel wire is used instead of glulam pieces.

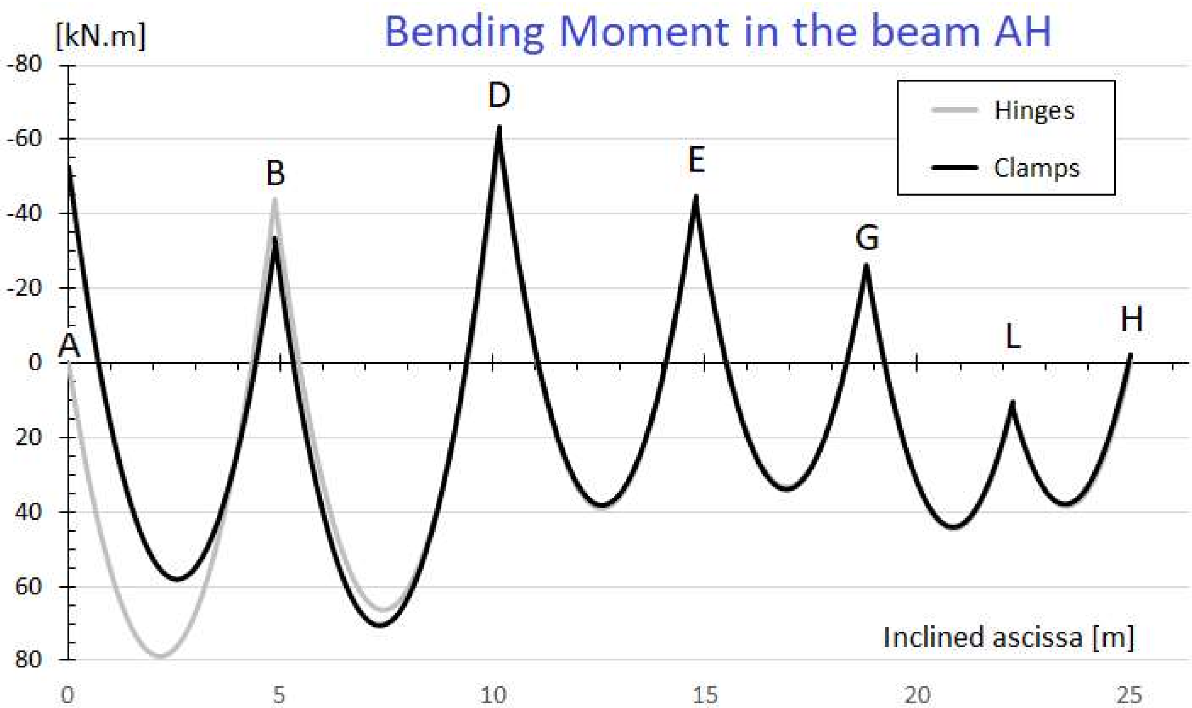


Figure 30: Diagram of the bending moment in the beam AH for the two models.

The comparison is full when the stresses are different from zero in both the models with hinged or clamped nodes. The bending moment diagrams of the upper beam AH for the two studies are overlapped in Fig. 30, uniform load at ULS.

It can be seen the peak of negative bending moment imposed by the clamp in A and shared with the lower chord. However, the effect of the added constraint on the support vanishes in some meters and the other nodes don't feel to be clamped on the diagonals instead of being hinged.

We can say that in the upper beam there is a non negligible difference between the two models, anyway it doesn't seem to be significant for the global stability of that member.

The other charts of the internal actions in the upper member are not very interesting: all of them confirm that the biggest difference between the two models is localized around the support as we have just seen, for both the two load distributions. In any case, they are reported in the Appendix.

2.8.4: Analysis of the results obtained: safety factors

The complete comparison between the models with hinged and clamped joints requires to analyze also the safety against failure. In the Appendix there are all the diagrams for all the structural elements and the load cases. Here the most important considerations are reported.

Regarding the failure of the upper beam under compression and bending almost nothing changes: it's true that on the support in A some negative bending moment is shared with the lower chord, however the value is smaller in modulus than the positive bending moment induced by the external loads, therefore the safety is not affected.

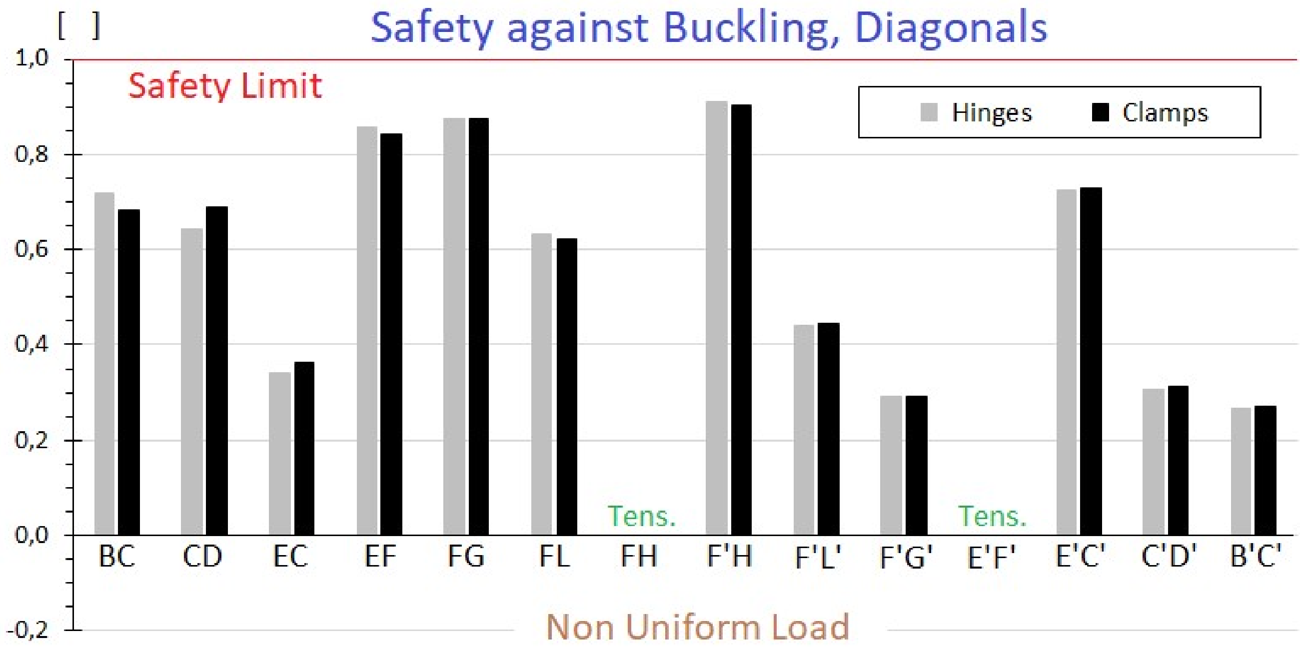


Figure 31: Safety against buckling of the diagonals, for both the axes of inertia.

In the "hinge" model previously seen there was no bending into the diagonals, therefore the buckling had to happen for sure under pure compression, along the weak axis of inertia.

The diagonals are now bent in the plane of the structure, where they work with the strong axis of inertia. Regarding their safety, there is a new important fact that needs to be underlined. Unlike before, two different kinds of buckling can occurr: along the weak axis as previously, out of plane under pure compression; otherwise in the plane of the beam, along the strong axis, due to compression and bending.
The interaction between the two modes is automatically considered into the Eurocode.

For the “clamp” model both the two buckling failures have been studied, in order to find the critical one that imposes the design. For simplicity, the black column in the diagrams of the safety of the diagonals represents directly the worst between these two cases.

It has been seen that the bending moment arising into the inclined members is quite small. The two buckling modes have almost the same preference, the winner one is not known a priori.
Anyway, the final value of the safety of the diagonals is not modified so much with respect to the hinge model.

The practical interesting information is therefore that, at the end of the analysis, the models with hinges and clamps provide very similar results for the diagonals; the chart in Fig. 31 shows that the differences are very limited.

Summarizing the analyses about the diagonals we know that the physical problem changes significantly at the theoretical level, because there is the new buckling mode promoted by the bending that didn’t exist before.
However, from the practical point of view of the design, the changes in the final assessment of the safety of the diagonals are almost negligible, therefore the new phenomenon is not noteworthy.

For the studies of all the compressed members done so far the differences shown are related to the different stresses provided by the two models with “hinges” and “clamps” nodes.
The buckling length of the elements has always been considered equal to the length between the two nodes, on the safe side.
If the connection is perfectly rigid the buckling length will decrease but it won’t become the half, like for a real clamp. The other elements will provide some rotational stiffness working in bending and the real buckling length can be calculated with a second order analysis of the whole structure; however, this goes outside this general evaluation of the influence of the nodes stiffness.

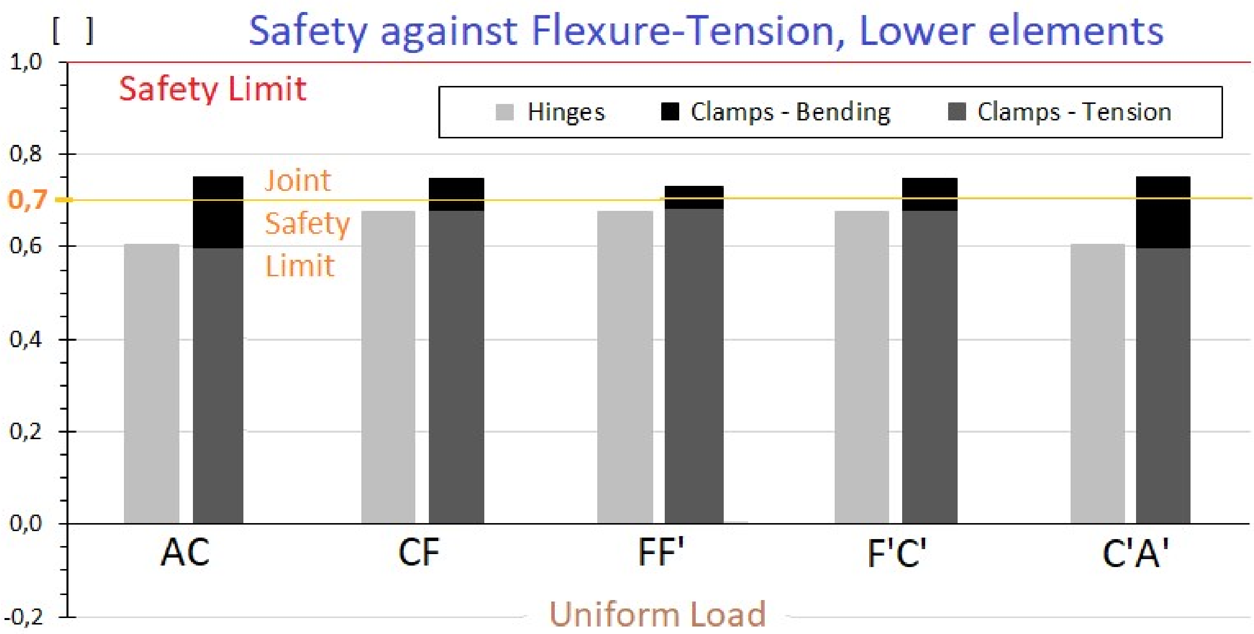


Figure 32: Safety against tension and bending of the lower elements.

The last element that requires the study of the failure is the lower chord. Here, the longitudinal connections are for sure the biggest of the structure and their rotational stiffness will be the highest in the reality.

The design performed has always kept an extra-safety of the 30% on the tension resistance, in order to implicitly consider the weakening provoked by the joints.

Having done the analysis under the “clamps” assumptions, it’s necessary to assess now the same section with the new stresses.

We remember that, in this new model, there is a significant bending moment affecting the lower members that didn’t exist before, when they worked as truss elements under pure tension.

The assessment against the failure due to combined tension and flexure has been performed according to the Eurocode 5. The interaction formula is the following:

Safety: $$\frac{\sigma_{t,0,d}}{f_{t,0,d}} + \frac{\sigma_{m,d}}{f_{m,d}} \leq 1 \qquad [\ ]$$

In Fig. 32 the coefficient that describes the collapse of the lower chord is represented in the two models with hinges or clamps. The column representing the failure in the “clamp” model is divided between the two addenda of the interaction formula just seen: tension and bending.

It is clear that the tension stress is almost the same as before, because it mainly depends on the global system of the elements and very marginally on the rotational constraints between them.

The additional bending moment that arises due to the fix joints provokes a little increase of the risk of failure, that is quantified in Fig. 32. The growing is not dramatic also if the bending moment applied is not small. It is evident, however, that the difference is not negligible and it’s enough to no longer be able to satisfy the 30% of extra-safety we imposed.

The fictitious strength reduction of the 30% is just a very rough estimation of the weakening of the connections. More detailed analyses could show that the increasing of the stresses doesn’t provoke the collapse of the structure, especially considering that the bending moment doesn’t apply uniform stresses into the section (and into the joints).

One thing is sure: the lower elements are the ones where the issue of the rotational stiffness of the joints is more important. This is mainly due to the high rigidity of the connections, the large bending moment arising and the quantity of material involved.
If design problems are found, specific analyses for the reduction of the real stiffness of the connections could be developed in order to restrict the phenomenon.

2.8.5: <u>Rotational stiffness of the joint in A</u>

The last analysis we perform in this chapter regards the rotational stiffness of the joint in A. Since we have seen that the clamps in the lower chord provoke a non negligible bending moment in those pieces, it is interesting to deepen the knowledge about those connections.

The investigation has been done with the focus on the node A, that is where the bending moment is trasmitted from the upper member in AC through the clamp; the same study could be extended also to the other nodes.

The rotation of the elements AB and AC converging on the support has been divided in two different unknowns; between them it has been placed a rotational spring with stiffness equal to K, in order to contrast the relative rotation.
For the value $K = 0$ the two bars can rotate independently and the bond is exactly like an hinge. If K becomes equal to infinite, the spring behaves like a rigid rotational link, therefore the two elements must share the same rotation and the connection is a clamp.

The structural analysis has been performed several times with different values of the rotational stiffness K of the spring in A. The results have been plotted in a diagram, so that it is possible to know the value of the bending moment arising once it is known the real rigidity of the connection (for example through empirical measurements).

In Fig. 33 it is shown the bending moment in A for all the possible values of the stiffness of that joint, from a theoretical hinge to a perfect clamp.
In has been applied the uniform distributed load at the ULS since this is the worst condition for the lower chord. The non uniform load distribution is interesting almost only for the stresses in the diagonals.

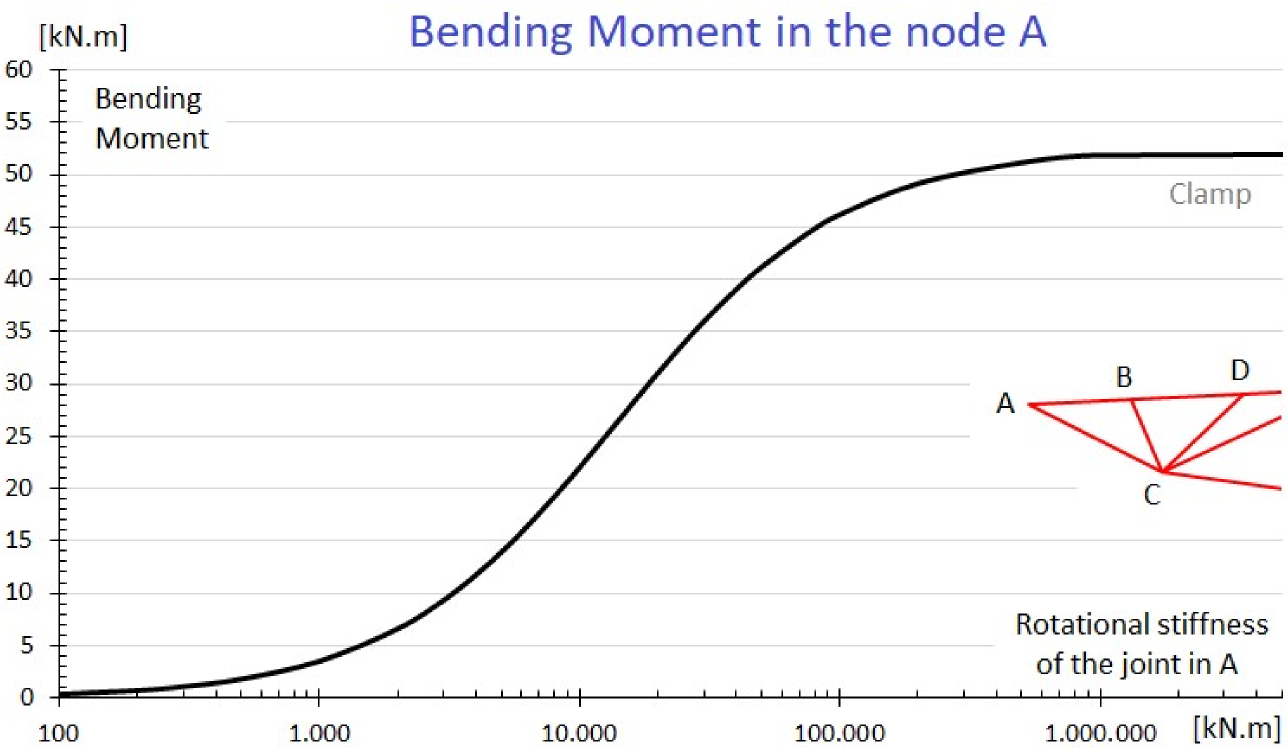


Figure 33: Bending moment in the node A for different stiffnesses of the joint.

The connection behaves as a hinge for $K < \sim 200$ kN.m and like a clamp for $K > 10^6$ kN.m.
It must be noticed that the abscissa of the chart is in logaritmic scale, therefore it is not necessary to have a very precise empirical datum, it’s important only to know the order of magnitude of the rotational stiffness that the connection is able to provide.

For completeness we report also the diagram of the bending moment along the lower pieces as the spring stiffness *K* changes, see Fig. 34.

It can be seen that the effect of the stiffness of the joint in A is quite local and it affects in a minimum way the bars beyond the element AC.

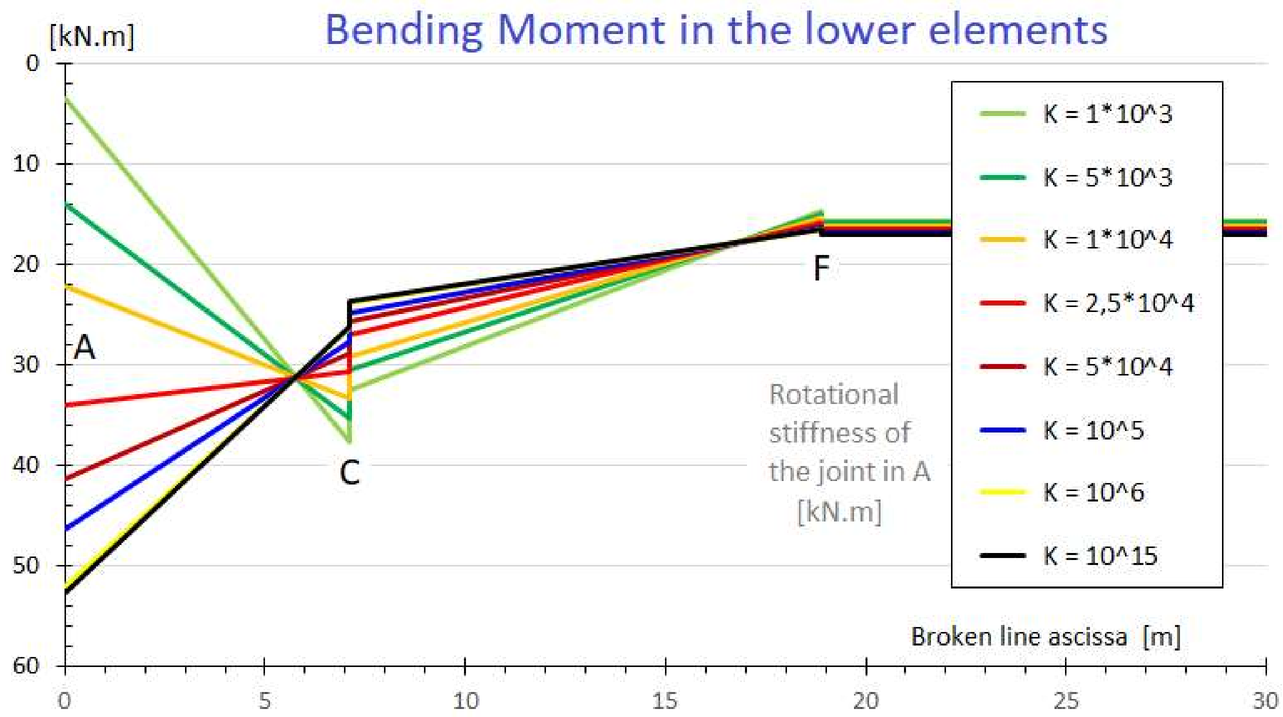


Figure 34: Bending moment in the lower member for different values of K.

Summarizing all the considerations of this chapter, it has been seen that the rotational stiffness of the joints doesn't affect so much the deformations, the stresses and the failure of the structure.
The only relevant difference is related to the bending moment arising in the lower chord, that brings the elements closer to the collapse but with values that are manageable. This effect can be also deepened through a specific analysis of the rotational stiffness of the longitudinal connections in the lower member.

2.9 Influence of the slip of the joints

2.9.1: Description of the mechanical phenomenon

In the previous chapter it has been analyzed how the rotational stiffness of the joints affects the solution. Even if the structure is not a real truss it has been confirmed that the influence on the settlements is very limited. We remind that they are determined almost solely by the axial stiffness of the elements and minimally by the rotational properties.

In this chapter we are going to analyze another phenomenon related to the joints: the axial slip. When the connection transfers the force of the bar, there is some local deformation that arises in the link due to the concentration of the stresses around the connectors.
The consequence is an additional longitudinal deformation of the element with respect to the Young strain considered so far. The total axial stiffness of the member is reduced and all the displacements of the structure must increase.

The intrinsic characteristics of the joint affect the magnitude of its slip, however in general this is not negligible since a connection is flexible compared to the longitudinal rigidity of the bar.

Regarding the rotational joint stiffness, the theory says that all the results are affected for every type of structure; practically it has been shown that the effect is normally unimportant.
For the joint slip, conversely, the settlements can be really large, but the internal actions can't change in an ideal truss without redundancies, because they depend solely on the global equilibrium with the external forces; the slip properties of the connections are not involved.

In our case the beam is not statically determined, hence not only the strains, but also the stresses are theoretically influenced by the joint slip.

The purpose of this chapter, thus, is firstly to study how much the deformations of the connections lead to an increase of the global settlements; secondly, which forces and how much they are affected by the redundancies of the system.

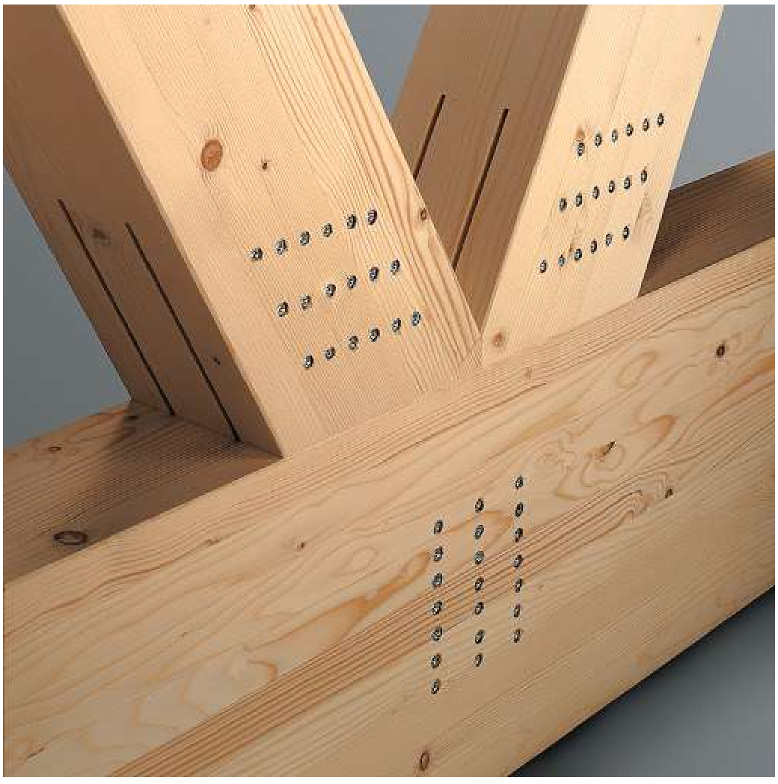


Figure 35: Dowelled joint.

2.9.2: Design of the connections

In order to study the joint slip the first thing to know is the joint itself, in all the details.
In Fig. 36 it is represented a dowelled connection very common in Sweden. All the thicknesses of the timber and the steel parts are standardized, the dowel is of class S355 with d=12 mm and the glulam is GL30c.

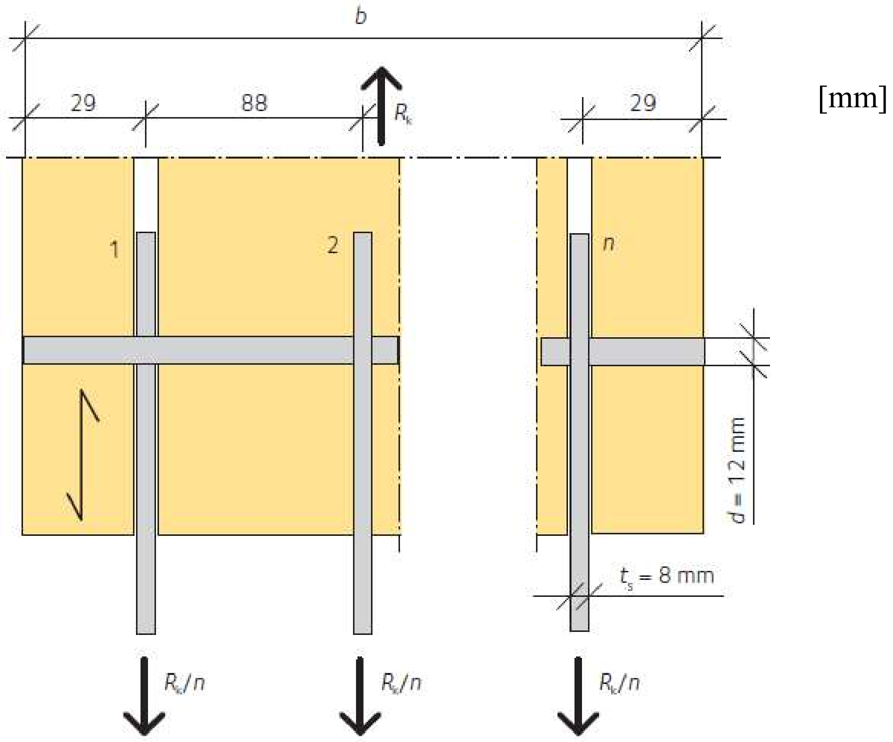


Figure 36: Typical dowelled connection used in Sweden.

There are books with specific empirical design rules for the joint of Fig. 36 widely used in Sweden, therefore all the connections have been dimensioned according to this literature.
In the Appendix there are all the information regarding the design of the joints, according to *Limträhandbok – Del 3.* Here the main aspects are described.

Since the thickness of the diagonals is known, the number of steel plates allowed for each member can be found in the charts. Then, the number of dowels needed to carry the axial force is found through experimental formulas.

Since each bar trasfers the same axial force at the two extremities, the two joints of every element are twin in the design. For this reason, the connections have always been associated to the name of the member they link, not to their node. In this point the plates from the different elements will converge, each with its dowels.

All the diagonals have been thought to work with two steel plates. The longitudinal connections of the lower chord, instead, have four steel plates since the forces to carry are much more high and the section is larger. Of course, the more plates are used, the less dowels are necessary.
Practically, the joints in point C and F will have two longitudinal plates plus two plates that involve also the diagonals.

The upper member is continuous and so it doesn't have connections and axial slip.

2.9.3: Joint slip modulus and structural stiffness reduction

Once the connections have been designed from the forces of the initial structural analysis, it's possible to calculate the longitudinal displacement they provoke.

The slip of the joint is defined into the Eurocode 5. The local deformation of the timber loaded from the dowel is considered to be elastic and it's described with a Hooke relation.
The stiffness K_{ser} of the link is empirically related to the diameter of the connector and to the density of the timber; the whole connection has a stiffness K_{ser_joint} that is proportional to the number of shear planes that share the total load. The details are in the Appendix.

Figure 37: Elastic model of a jointed bar: three springs in series.

Having found the stiffness K_{ser_joint} of each connection it is necessary to insert it into the mechanical model of the whole structure.

Fig. 37 shows that a member jointed at the extremities is a system of three springs in series, constituted by the two connections and the element itself.

The three pieces of Fig. 37 are exactly like a bar without joints and with an equivalent axial stiffness EA*/L. This is obtained by reducing the area from A to A* in order to account for the flexibility added by the connections, so that the total structural stiffness it's the same.

In the Appendix there are the passages to find the equation to calculate the fictitious reduced area A*, hence to obtain the equivalent axial stiffness.

The structural analysis performed so far doesn't account for the joints. However, if we replace A with the reduced A*, all the bars will deform as if they were jointed. It's clear that we can use the old model with the new A* of the elements and the results will automatically consider the connections.

2.9.4: Implementation and kinematic results

The same structural analysis as before has been redone using A* instead of A for all the bars (starting from the cross sections previously designed). Hinged nodes like in the first study performed.

The load at the ULS has been kept, in order to have results aligned to those already found.
Very often, the displacements are studied at the SLS, not at the ULS. Anyway, the solution is linear with respect to the forces applied, thus, conceptually, is not important the value of the load.
Furthermore, only the uniform load distribution has been considered since we are now focusing on the effect of the axial slip, that is the same for every load case.

To get the stresses in post processing it would be necessary to depurate from the joint slip the displacements found in the resolution; however it's easier and equivalent to keep using the reduced area A* to obtain the internal actions.
All the diagrams are in the Appendix. The charts refer to "No Joint Slip" model indicating the old analysis with perfect hinges at the nodes. "With Joint Slip" is the new model that adds the longitudinal stiffness of the connections to the previous one.

		No Joint Slip		With Joint Slip	
Midspan Deflection		106	mm	115	mm
Bending Moment	Min (AH)	-59	kN.m	-63	kN.m
	Max (AH)	79	kN.m	82	kN.m

Table 5: General results neglecting and considering the joint slip, ULS uniform load.

Let's start with the most interesting outcome: the midspan deflection. Table 5 says that the axial strains in the joints provoke an increasing of the total settlement of the 9%. The value is not negligible, as we expected: the joint slip directly affects the axial deformation of the bars, that is the responsible of the midspan deflection.

The value is, however, not too high. Dowels are stiff in transferring the forces since they are a lot and very tight. It would be enough a clearance between the connectors and the holes (like for bolts) to have much more larger displacements. Also a stronger class of resistance of the steel would lead to higher deflections.

In general, we can be satisfied about the kinematic results just obtained.

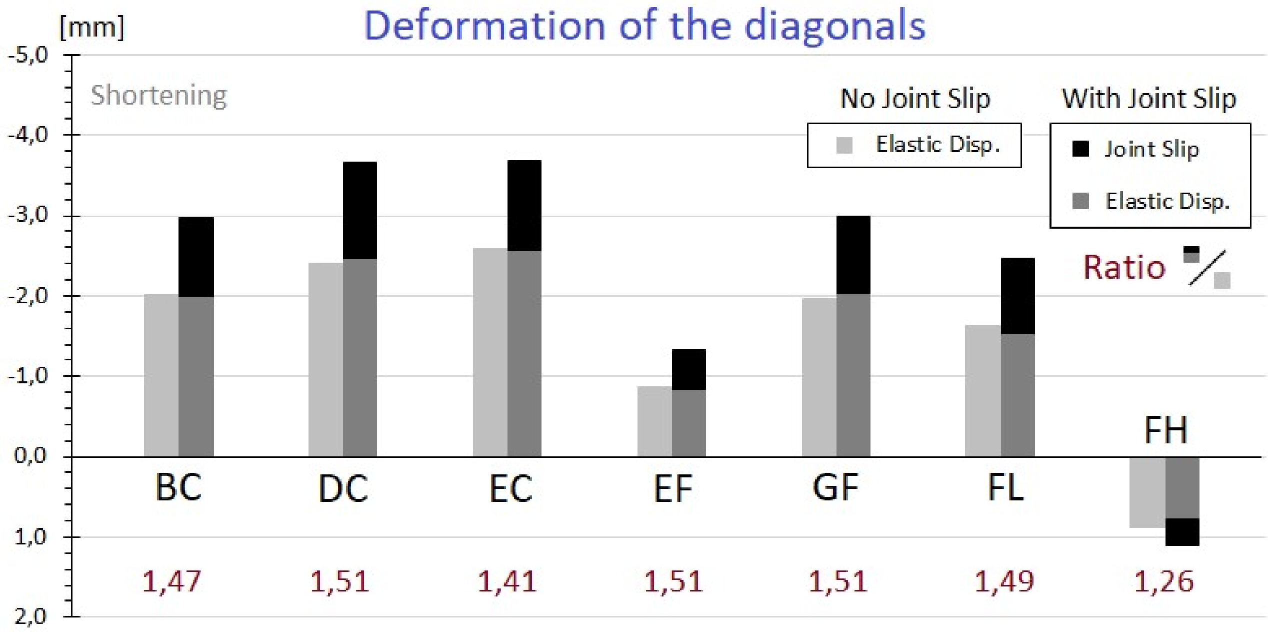


Figure 38: Axial deformation of the diagonals, ULS uniform load.

At the level of the single bar, the total deformation found through the analysis with A* can be divided into the slip and the Young components as shown in Fig. 38, since the problem is local.
The connections are only the two at the extremities and, knowing the force from the structural analysis, the joint slip is found. The same force provides also the elastic displacement of the element.

The percentage of increasing of the displacements is quite large in the diagonals, where the single element with its connections feels the local slip much more than the whole beam.
This fact is clearly shown in Fig. 38, with ratios of the order of 1,4 - 1,5.

2.9.5: Considerations on the internal actions obtained

As previously said, in a statically determinate structure the stresses don't depend on the stiffness of the members. If this was the case of our beam, the forces in the diagonals and in the whole structure couldn't change in the two cases, whatever is the joint slip.

The real structure has little redundancies due to the continuity of the upper chord, therefore, neglecting the joint slip, some errors affect also the stresses, not only the total displacements.

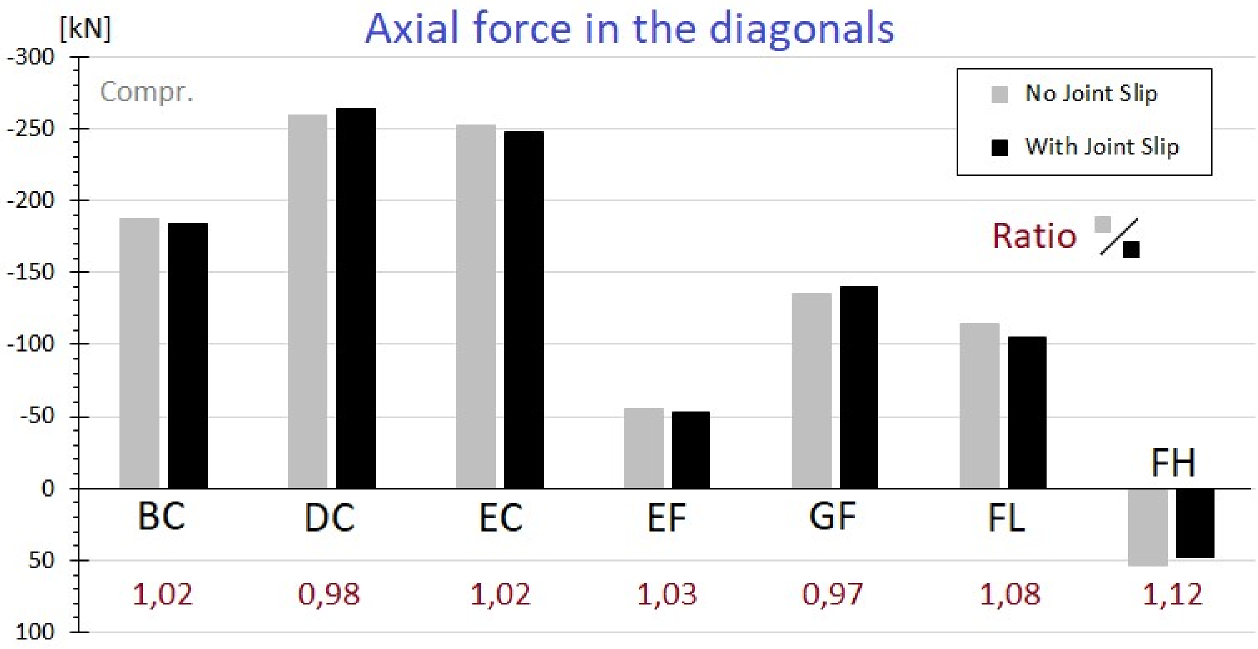


Figure 39: Axial forces in the diagonals, ULS uniform load.

The static differences are not so clear in Fig. 38, because the ratio is only in total and not also between the elastic components. Fig. 39 is better from this point of view, because it reports the ratio between the forces (that is the same as between the Young strains of Fig. 38).

In the Appendix there is also the diagram of the lower pieces, where it can be seen that the element FF' has a tension (and thus a elastic strain) that doesn't change adding the joint slip: it's the only one element whose force is statically determined by solely equilibrium from the external load.

Looking at Fig. 39 it can be seen that the change in the stresses is minimal, because the redundancies are not very influential in the system.
However, it can also be noticed which bars are more affected by the redundancies and which are less. It's clear that the elements FL and FH are the most involved; they are more inclined, less effective as diagonals and the flexural behaviour of the upper beam is more pronounced.

The last evaluation we need to do is if the changes in the internal actions are significant with respect to the safety of the structure.

We remember that the joint slip provoked the failure of the Sandö arch bridge in Sweden, during the casting of the concrete in the formwork (1939).
In this structure we don't have significant second order effects like in that case, the stresses are slightly affected at the first order by the redundancies, not by the displacements.
In any case, a further check is never useless.

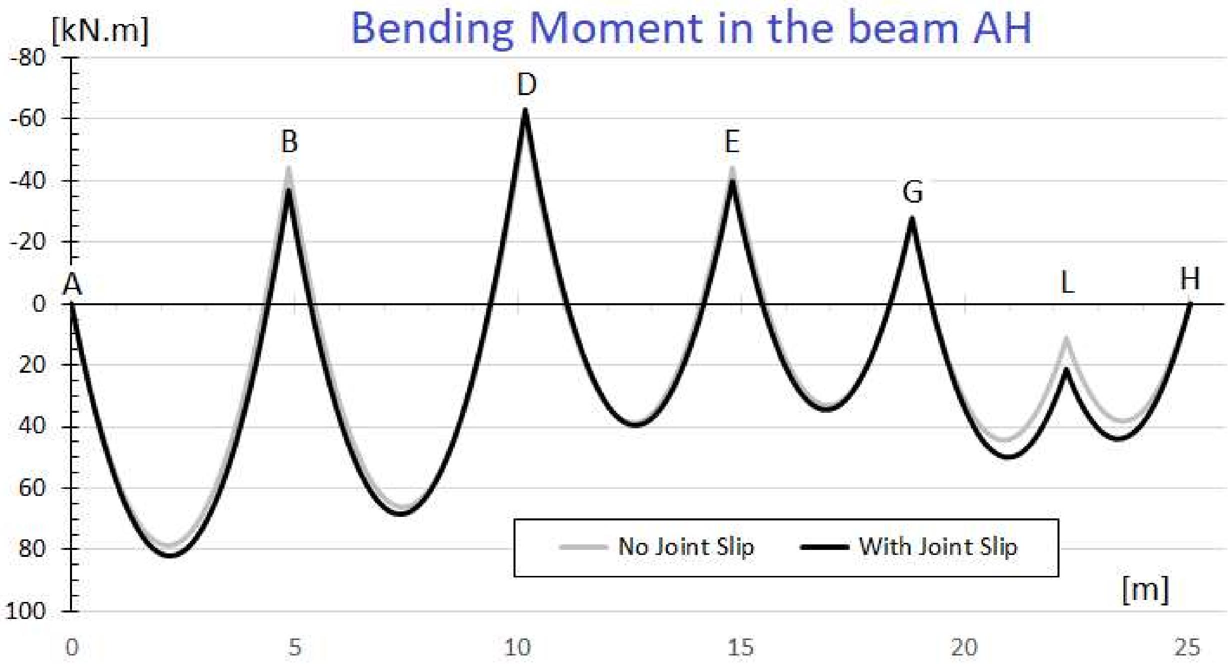


Figure 40: Bending moment in the upper beam, ULS uniform load.

In Fig. 40 it is shown the diagram of the bending moment in the upper beam, neglecting or considering the joint slip. As it has been seen for the diagonals, the differences are very limited and almost negligible in the design.

The diagonals lose stiffness adding the joints, therefore the flexural behavior of the upper beam rises up. The effect is similar to what seen in Fig. 26 but is related to the opposite phenomenon.

The element FL, that was not effective in supporting the upper member, worsens its performance since it's more flexible. The bending moment in the beam AH grows a little and its safety reduces.
All those aspects are very limited according to what is shown in the last pictures, however it's good to keep them in mind and avoid a design that is extremely close to the collapse.

If a good accuracy is searched, the buckling risk of the upper member should be evaluated through a complete analysis at the second order instead of the local studies. The change of the bending due to the joint slip is probably less important than the other approximations of this model.

3. Alternative theoretical solutions

Having studied the structure proposed by the consultant of Moelven, we move now to search for widely different configurations of the timber elements, in order to improve the mechanical performance of the beam. In this Section 3. this goal is going to be pursued by developing different theoretical concepts, their analysis and adjustment through subsequent investigations.

3.1 Configuration that follows the principal directions of stress

3.1.1: <u>From the theoretical curves to a truss scheme</u>

The idea that we introduce in this chapter in order to improve the mechanical performance is to place the bars according to the stress paths in a solid continuum.

In Fig. 41 are drawn the principal directions of stress in a simply supported beam subjected to uniformly distributed load.

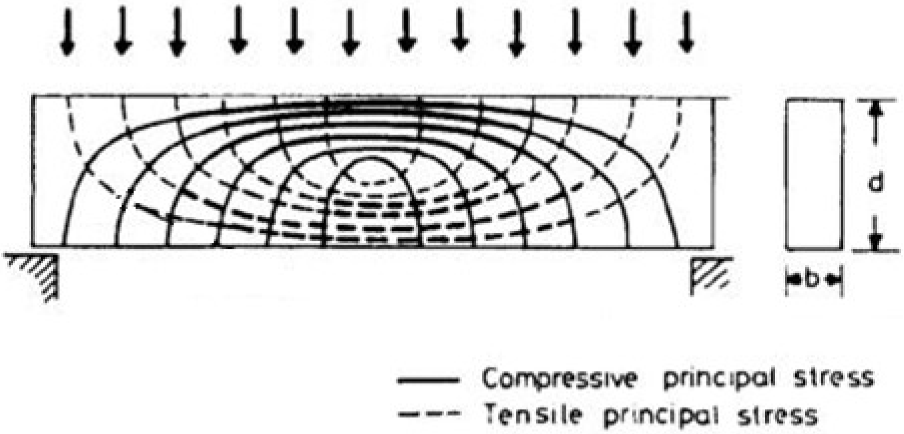


Figure 41: Principal directions of stress in a simply supported beam under uniform load.

We can read through those curves the internal actions that we know: at the midspan there is pure bending moment and therefore all the lines are horizontal, tension in the bottom and compression at the top. On the supports there is pure shear and the lines are inclined at 45°.

The problem of Fig. 41 is very similar to the our, same constraints and loading conditions. Moreover, we can think at the height of the full web of the beam as the area where our elements are placed.

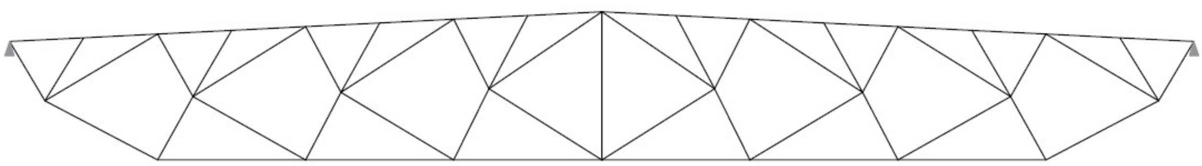


Figure 42: New proposal for the arrangement of the elements.

Going from a full wall beam to a truss it is useful to remove the material where it is not fundamental, according to Fig. 41; in technical words it means to arrange the diagonals so that they follow the principal directions of stress in the web. If they are aligned with those lines, their force works more directly in the structure and the material is used in an optimal way.

In Fig. 42 it is represented the new proposed consifiguration, developed applying the theoretical considerations just done. A more detailed picture of the left half of the beam is shown in Fig. 43.

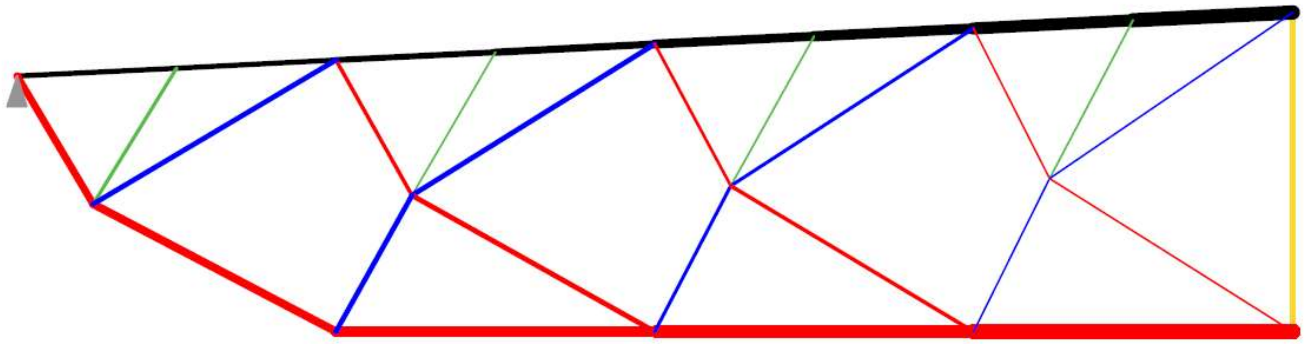


Figure 43: Left half of the new proposed configuration.

In Fig. 43 the elements used to follow the principal directions of the tension stress are drawn with red color. They are descending chains that converge into the lower chord, which becomes more loaded going far from the support. This increasing force is related to the condensing of the dashed lines of Fig. 41 near the midspan and it's marked with the thickening of the red line in Fig. 43.

The blue lines are specular with respect to the red ones and refer to the elements placed to follow the principal directions of the compression stress. The negative force grows along the black upper chord up to the midspan, where it reaches its maximum, similarly to the tension path.

The green diagonals are an addition to the basic structural scheme and they are non-truss elements, because their axial force is balanced by shear in the upper member. Those elements have been placed to counteract the uniform distributed load on the roof, that is carried in bending from the black continuous beam.
In the previous schemes proposed by Crocetti there where issues related to large bending moment in the upper member, with values of the order of 100 kN.m. The green diagonals allow to reduce the spacing between the nodes from more than 5 m to ~3,1 m and this seems to be a priori very good.

As said, we have transformed the theoretical layout of Fig. 41 into the technical scheme of Fig. 43, trying to follow the curves with the bars of the truss.
It must be noticed, however, that the upper chord can't be horizontal like the upper theoretical profile, it needs to be inclined of the 5% in order to allow for the water drainage from the pitch. The consequence is that in the middle the flow of the forces is distorted due the imposed roof slope, therefore we need to modify the arrangement of the elements theoretically suggested.

At the midspan the two symmetrical black beams converge with a certain angle and this implies that their compression has an upward resultant. This force must be carried by the diagonals, but the more they are inclined the more is difficult for them to do this job.

The yellow vertical element has been thought to take in tension the vertical resultant of the upper beams; it's an additional bar with respect to the theoretical scheme of Fig. 41, where it wasn't necessary since there was no upward resultant due to the horizontal convergence of the compressed arches.

The yellow member is intended also to create a stiff link between the two chords at the midspan, where the diagonals are too weak and inclined. This is another problem that arised in the original solution and that we are trying to solve in this way.

3.1.2: Structural analysis; considerations on the static results obtained

The complete structural analysis of the introduced truss has been done, considering hinges in all the joints (except the upper chord that is a continuous beam, like before). Both the uniform and non uniform load distribution at the ULS have been applied.

The configuration has now 18 independent geometrical parameters, thus it's quite complicated to optimize all their values. It has been considered that the proposed configuration is intrinsically good and only a few minor adjustments have been made to improve the distribution of the forces.

The design has been performed for all the elements with local assessment of buckling and tension failure of the bars, as before. This procedure seems enough for a preliminary estimation of the efficiency of the structure.
All the coordinates of the nodes, the design cross sections and the internal actions are in the Appendix, here the most important results are commented.

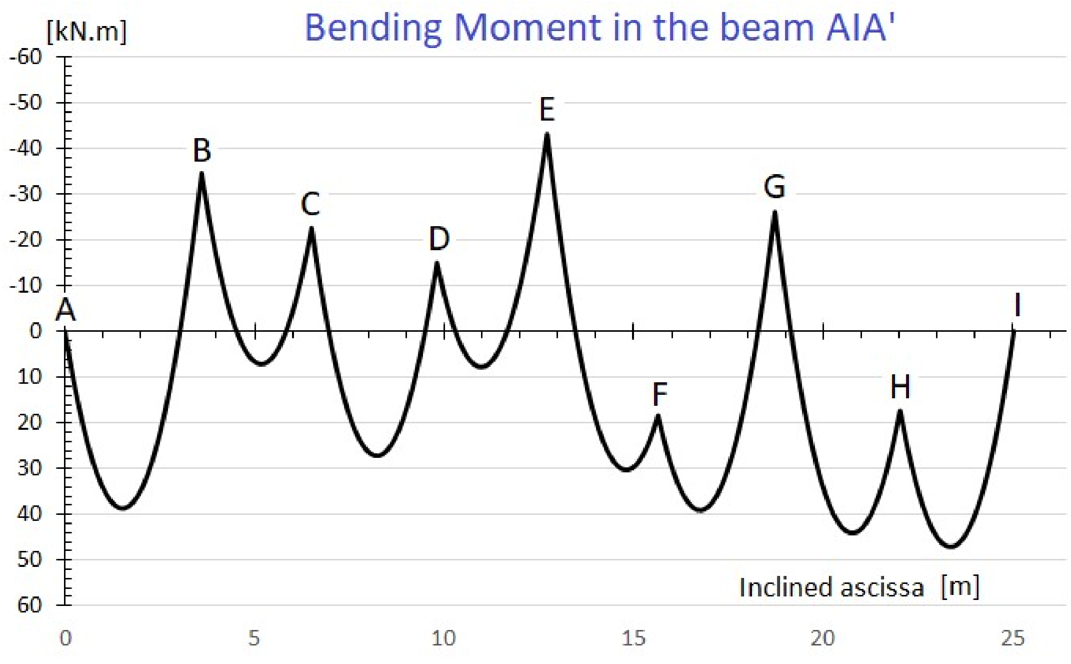


Figure 44: Bending moment diagram in the upper beam, uniform load at the ULS.

In Fig. 44 it is shown the diagram of the bending moment in the beam AI under the uniform distributed load at the ULS. We can notice that the choice to increase the number of nodes in the upper member is beneficial since the peaks are considerably reduced and now they are around 40 kN.m.
The distribution of the bending is also quite nice since there are several elements with similar stresses.

The lower chord becomes more loaded going towards the midspan (Fig. 45), as expected. The path A-L-Q has some peculiarities and it will be discussed further in the next chapter.
The diagonal IT works with good tension as wanted and seems a right choice.

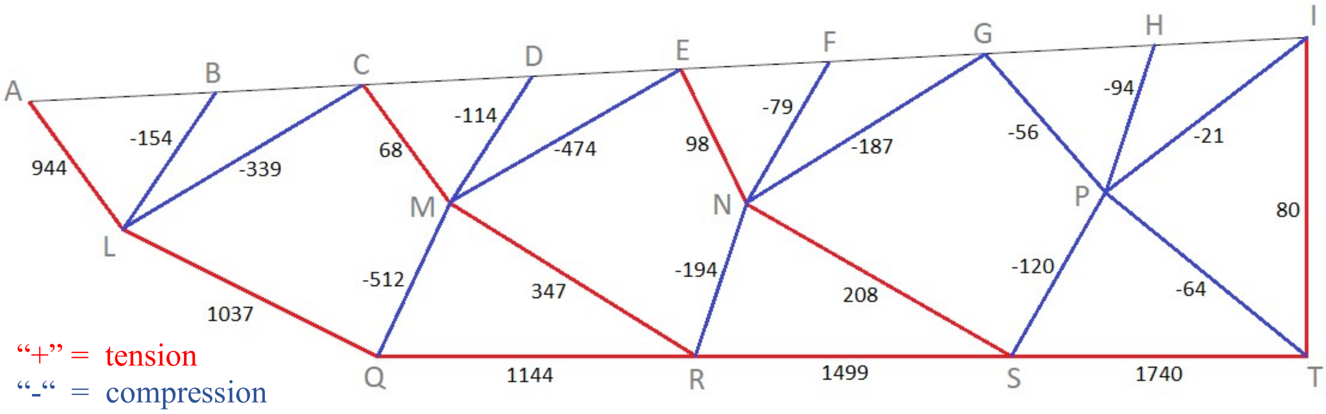


Figure 45: Axial forces [kN] in the truss bars, ULS uniform load.

The forces in the diagonals are almost as figured in Fig. 43, with descendent chains and rising arches in compression; the values get lower far from the support, as the shear they are taking. It can be also seen, however, that the structural scheme imagined is broken around the node P, where all the bars have small compression under uniform load.
Specific considerations about those diagonals near the midspan will be done subsequently.

3.1.3: Comparison with the previous configuration of Moelven

In Table 6 it is reported the volume of timber needed for this new theoretical structure.
On the left side it's also recalled the Table 3, with the analogous data related to the previous solution of Moelven with solely timber elements, deeply studied so far.

Volume of timber - Previous solut.		
Upper beam	6,89 m^3	31,6 %
Lower pieces	10,37 m^3	47,5 %
Diagonals	4,58 m^3	21,0 %
Total	21,85 m^3	100 %

Volume of timber - Stress Princ. Dir.		
Upper beam	5,58 m^3	30,7 %
Lower pieces	8,17 m^3	45,0 %
Diagonals	4,41 m^3	24,3 %
Total	18,15 m^3	100 %

Table 6: Material needed for the previous solution by Moelven and the new one.

It is rewarding to see that this new configuration allows to reduce the construction material: the theoretical concepts have found a verification also in the reality.
The difference is not enormous but it's larger than the 15% and it's interesting.

Reading the table it's clear that both the two chords are thinner than before.
The upper beam (now with fewer bending) is again continuous but it has no more a constant cross section: it has been designed with increasing number of lamellas approaching the midspan, in order to follow the increasing of the compression brought by the diagonals.

Also the lower chord has a varying thickness. The lever arm against the bending is now large along all the span, hence it is possible to follow the diagram of the bending moment with a thinning of the lower member near the support, not with a reduction of the height of the beam.
The drawing of Fig. 46 clearly shows that the available geometry is much more used in the red scheme, thus the mechanical efficiency is surely higher for the theoretical configuration.

The diagonals require now more timber in percentage, since they are a lot and very used; however, their volume is again lower than before, probably because the forces are better distributed and the buckling lengths are shorter.

Those technical considerations are the reflection of the theoretical ones.

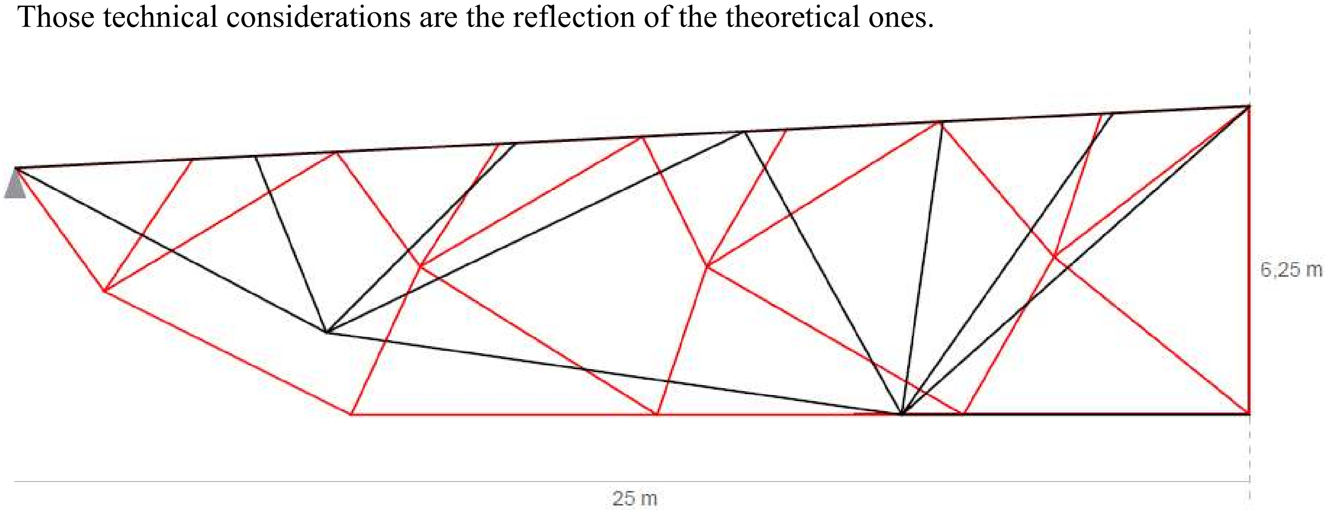


Figure 46: Qualitative superposition of the previous solution (black) and the new one (red).

We can finally have an order of magnitude of the problems that affect the solution previously studied, with great bending behaviour of the upper chord, concentration of forces on the lower one and global wrong truss mechanism. They worsen the performance for more than the 15%.

The improvements found are not absolute: the reality is much more complicated than the data shown and the best choice remains difficult to be defined. Afterwards it will be necessary to compare the advantages of a better structural system with the logistical complications that could arise for a more sophisticated layout of the elements.
Even from the structural point of view, the theoretical proposal just found will have additional troubles that will require more detailed analyses.

Anyhow, before than addressing the new structural issues or the practical ones, we think it’s better to further develop this theoretical configuration and improve it again. This will be the topic of the next chapter.

3.2 Mechanical improvement by lowering the support

3.2.1: Inspiring ideas and general considerations

Having done the structural analysis of the theoretical framework just seen we have now a clearer idea about how it behaves, with some good aspects and others that can be improved.

Looking in the Appendix at the diagrams of the axial forces it's evident that the high tension of the lower chord largely travels along Q-L-A, up the support. The elements AL and LQ visually seems diagonals, but they have been considered as the bottom chord togheter with Q-R-S-T, because they have the same role.

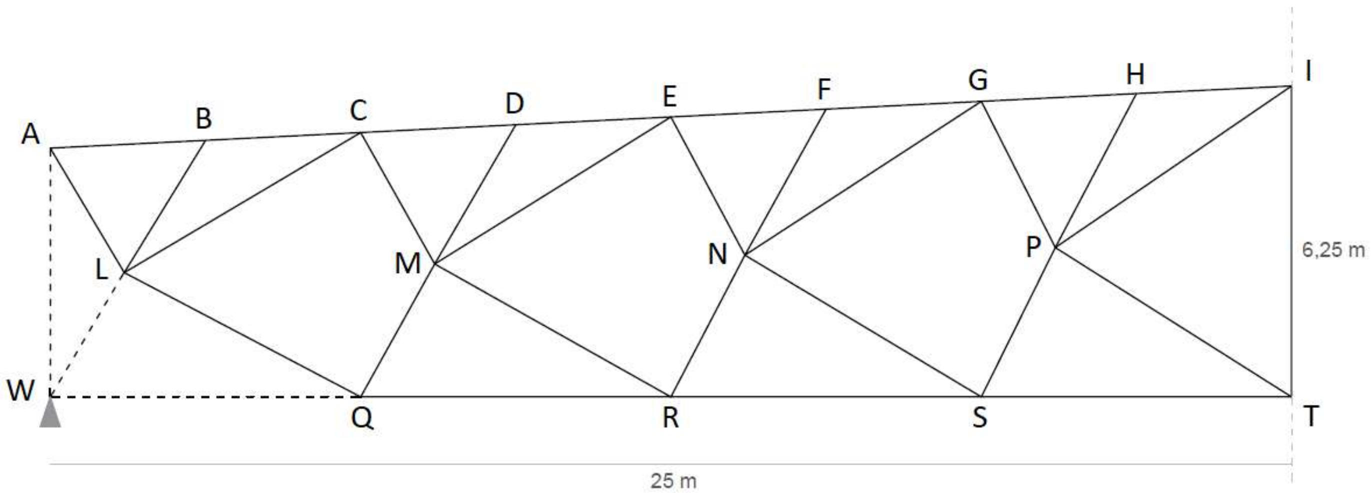


Figure 47: Theoretical configuration and its improvement.

We think that the existing theoretical configuration could be improved if we move the support from point A to point W, adding the three dashed elements shown in Fig. 47.

The choice of having the support in A was simply adopted keeping the design of the old Moelven's solution, where the two chords converge on the abutment. However, this aspect doesn't seem mandatory and from the engineering point of view there is no problem in this suggested shifting.

The structural changing we are performing has a priori several promising factors.
The first advantage is more academical: from the theoretical chart of the principal directions of stress we notice that the uniform load applied flows through compressed arches and tight chains up to the support. If this point is in W, the path of the forces is shorter because the load goes down more directly, instead of descending through the web and then rising up to A.
The general shape of the whole beam will be also more similar to the theoretical sketch of Fig. 41.

Secondly, about the observation on Q-L-A, after the shifting we think that the elements AL and LQ will act more like diagonals than like a lower chord, therefore their forces will be lower and the static scheme will be more regular.
It's better to confine the large tension force in the lower horizontal member, because we will be able to manifacture it with a continuous straight beam, with few longitudinal joints and minimal weakening provoked by them. Conversely, the inclined bars are connected at each extremity, hence it's not good to carry high tension with the diagonals.

Finally, the column supporting the structure will be 5 m shorter, it will have few problems of buckling and probably it will be also thinner.

3.2.2: Technical peculiarities and structural analysis

Trustful in the suggestion of lowering the support from A to W, it has been performed the structural analysis of the beam of Fig. 48.

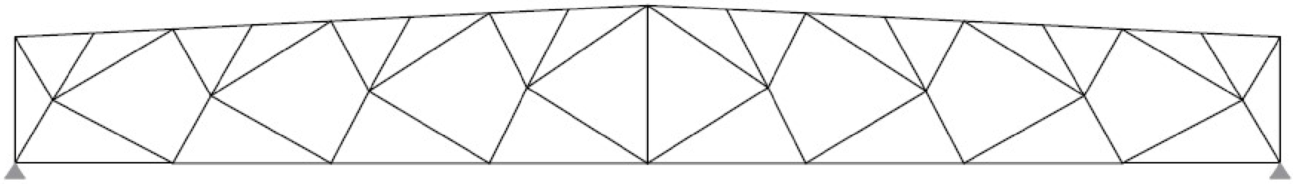


Figure 48: Theoretical configuration with horizontal lower chord.

As a consequence of the support reaction arising through the diagonals, the compression in the upper member has a strong jump in the node C, like if that chord starts in C and no more in A (Fig. 47). Due to this fact, it's good to have two completely different cross sections for the elements A-C and C-I, not simply an increasing in the number of lamellas like before.
Again, we can imagine the greater efficiency introduced with this model. Structurally we are shortening the upper chord since the flow of compression doesn't stream until A but, once arrived in C, it descends through the diagonals. The material needed will probably be lower.

The element A-C will be narrow, with the local role of carrying the bending moment applied by the distributed load. The beam C-I will be instead much more wide due to the high compression transferred.
It must be underlined that, since in the point C the upper member will be no more continuous, the structural analysis has been adjusted, putting a rotational spring that simulates the longitudinal connection. The stiffness of that spring can be set at each value, however the effect is very local.

The element AW (like IT previously treated) is not drawn in the theoretical scheme of Fig. 41. It has been placed to help the transmission of the shear arising from the support through the web.
The arch W-L-C (Fig. 47) will be highly compressed by bringing the flow of forces from W to the upper chord and therefore it's useful to have also the bar AW carrying some negative force.

The structural analysis has been done as described, with the lower chord W-T as a continuous, straight beam with increasing thickness approaching the midspan. An hinge in T has been placed since in this point there will be a structural pin like in point I, for the assembling on site of the prefabricated pieces.

Same loads as always, at the ULS, same assessment against the failure as before.
It can be argued that the local assessments of buckling are not enough for the compressed arches, because the nodes L, M, N and P can have an out of plane buckling mode due to the couple of converging compressed diagonals on them. This type of instability has not been studied due to the complexity and the uncertanties of the system resistance that governs the problem.
Some transversal bracing between the trusses at middle web will probably be necessary and the efficiency of the structure will decrease a little. Anyway, in these pages we prefer to focus on the other aspects introduced so far.

All the results in terms of internal actions and cross sections of the elements are in the Appendix, where it is possible to find also the coordinates of all the nodes; they have been obtained with little adjustments of the configuration of Fig. 48, in order to optimize the design. The most important aspects are commented in the next paragraph.

3.2.3: Results obtained and comparisons

The shifting of the support hasn't affected significantly the diagram of the bending moment of the beam AI, that is still nice. The jump of the compression of the upper chord in point C is perfectly interpreted with the change of cross section, exactly as anticipated before.

It's very satisfying to see that the axial forces in the diagonals have also followed the expectations: the tension in the chain A-L-Q is now considerably lower with respect of having the support in A.

The element AW is working properly in carrying some of the reaction force arising from the support; it has been verified that without him the compression in W-L-C is much more high and the buckling problem becomes a real trouble.

Volume of timber - Support in A				
Upper beam	5,58	m^3	30,7	%
Diagonals	4,41	m^3	24,3	%
Lower chord	8,17	m^3	45,0	%
Total	18,15	m^3	100	%

Volume of timber - Support in W				
Upper beam	5,29	m^3	29,6	%
Diagonals	6,45	m^3	36,1	%
Lower chord	6,14	m^3	34,3	%
Total	17,89	m^3	100	%

Table 7: Material needed for the two theoretical configurations.

In Table 7 it is reported the volume of timber needed for the two theoretical configurations, with the support in A and in W. It is intresting only the total value, the intermediate ones are not very significant since some bars have been shifted from the group of the lower chord to the diagonals' one. We can see that the material needed for the last proposal with the support in W is slightly lower. The promising considerations behind this solution have been rewarded by the technical design.

The advantage gained lowering the support from A to W isn't so high, but it's however enough to have an improvement of the 18% with respect to the old structure assigned.
This estimation of the reduction of volume is probably not so accurate: if it's true that we are not considering the shortening of the supporting column, we are also forgetting the extra bracing required at middle web. Maybe, the total amount of timber will be slightly larger than what expected.

The schemes we are developing are promising; hopefully, the ~4 m^3 of material we are saving are enough to cover all the new issues and the additional difficulties in the production.
In any case, even if those theoretical structures are too complicated for the real manufacturing, they will be able to provide suggetions on how to improve the existing one, looking at the good structural performances arising here.

For a complete analysis of the theoretical configuration it's important to consider not only the volume of timber needed, but also the displacements. In Table 8 there is a general comparison between the last configuration with the support in W and the one of Crocetti previously studied. The midspan deflection is calculated under the ULS uniform load everywhere, that is the highest weight.

		Moelven, glulam elem.		Theoret, support in W	
Midspan Deflection		106	mm	109	mm
Bending Moment	Min (AI)	-59	kN.m	-48	kN.m
	Max (AI)	79	kN.m	47	kN.m

Table 8: General results about the studied Moelven's model and the last theoretical one.

From the table we can see that the elastic settlement is very similar. If it is thought that the theoretical solution with the support in A provide a value of only 104 mm (see the Appendix), it's clear how the assigned structure and the academical ones have the same stiffness.

It must not be forgotten, moreover, that we are always doing a design for load bearing capacity, not for deformations. The truss solutions can easily become stiffer than the old proposal of the company if we add a little of the timber we are saving.
The improvements performed (principal directions of stress, truss scheme with full usage of the geometry, lowering of the support) are general and they better both the resistance and the settlements.

The result obtained is not surprising: a structure that follows the principal directions of stress is optimal both in strength and in stiffness, because in general the strain energy of the whole system tends to be minimal.

3.2.4: Summary of the improvements, criticism

At the end of this theoretical chapter it's useful to resume all the positive aspects found.

The choice of reducing the spacing of the nodes in the upper beam from 4-5 m to less than 3,5 m seems beneficial, the decrease of the bending moment arising it's visible also in Table 8.
In this way it is possible to save material keeping the flexural usage of the section of the upper chord around the 30% of the total; of course the lower is this value the better is, since it's not good to have large flexural behavior in a structure. This compromise seems reasonable.

The vertical elements IT and AW satisfy their purpose and therefore it will be useful to keep them also in the subsequent analyses.
The little improvement in lowering the support to point W has been verified from the structural point of view and it seems good to execute it in the design.

The truss behavior seems in general better than the chain-flexural behavior; the diagonals work in a more proper way with the lower chord and the technical advantage of having the latter straight and continuous seems sure.

The issues arising with those theoretical configurations are mainly divided in two branches.
The technical problems, of course, are related to the complications in the manufacturing, like the increasing number of elements and joints, the additional bracing at middle web or the complexity of the star connections.
Secondly, there are criticism regarding the aesthetics of the beam: from the architectural point of view a straight, horizontal lower chord is much more uglier than a funicular shape, hence it is not acceptable.

From what have been just said, the requirements for the real solutions to develop will be to avoid the connections in the web and to have the two chords converging on the support.

3.3 Transposition of the Moelven's structure on h = 4,50 m

3.3.1: The request of the company

Moelven has asked us to develop a design for a structure on the same span of 50 m but with a maximum height of the beam equal to 4,50 m.

The request, that of course is not optimal from the structural point of view, has been made because 4,50 m is the maximum transportable height on a truck in Sweden. If the beam isn't taller than this, the whole web can be assembled previously and brought already ready at the construction site. Exceptional transportations of 25 m in length are allowed on the swedish roads, therefore the structure would be prefabricated in two halves of 25 m in length and 4,50 m in height. A long truck would carry the two pieces on site, where only the joints at the midspan remain to be done.

It's clear that the production in the factory is much more fast and cheap with respect to the manufacturing at the construction site.
On the other hand, a structure with such a slenderness will require a lot of timber because there is few geometry to use, hence the cost of the material needed will be higher for sure.

In the next pages it will be analyzed if it's feasible to have a beam made of timber with a ratio L/h equal to 50/4,50 m = 11.1. Secondly, several designs will be performed and compared, in order to find a configuration that can compete commercially with the structures of 6,25 m studied before.

3.3.2: Operative procedure

In this Chapter 3.3 the configuration proposed by Moelven for h = 6,25 m has been designed again for 4,50 m. The scheme has been simply "compressed" vertically in order to fulfill the requirement, no changes in the number of the elements have been made.

For completeness, both the configurations with 10 and 14 diagonals in the web have been studied.
In Fig. 49 there is the optimized configuration on 6,25 m deeply studied before (in red), superposed with the same on 4,50 m (black) and the first proposal that lacked of the diagonals EC and FL (blue).

It can be seen in Fig. 49 that the noded are not in the same position in the three structures. During the design the geometrical parameters of configuration have been adjusted in order to have a good distribution of the forces for each scheme, as usual.

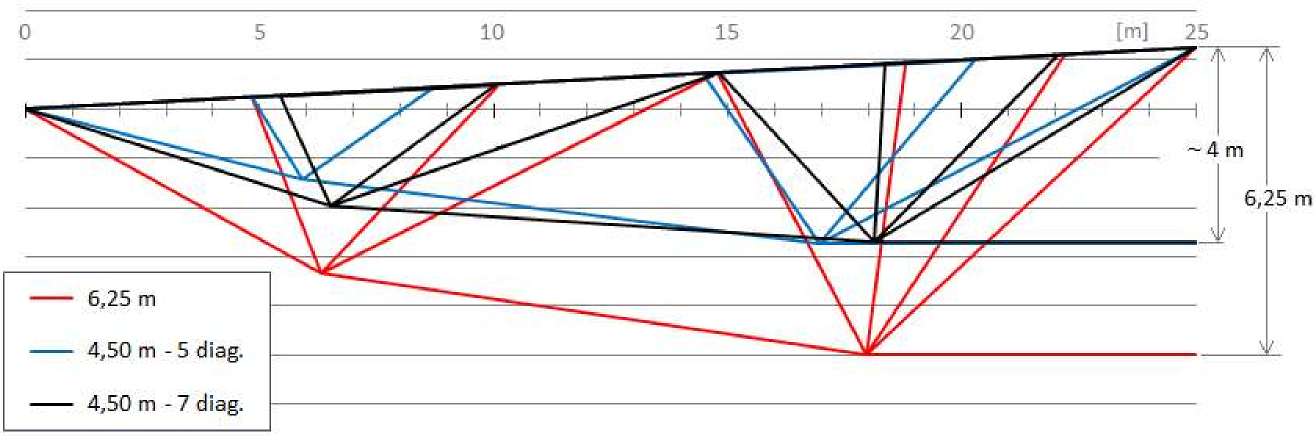


Figure 49: Moelven's configurations for different heights.

The analyses have been performed with the hypothesis of clamps everywhere. With this assumption some bending moment arises in the lower chord and we remember that it's not negligible in the design of that member under tension and flexure.
Since the longitudinal connections are quite big and will have some rotational stiffness, the clamp model is appropriate and on the safe side.

A technical aspect that now is important to underline is related to the height of the beam.
In the studies done so far, all the bars have been modelled as geometrical lines without any thickness, hence a distance of 6,25 m between the two chords is the distance between the center lines of those two elements. In the reality they will have a physical dimension that is not interesting for the engineering calculations.

Now, the real thickness of the two chords is fundamental: h = 4,50 m is no more the arm that we can use against the bending moment, is the maximum height of the piece that can be loaded on the truck. The total height of the structure can't be larger than 4,50 m, therefore the distance between the center lines of the two chords will be smaller due to the thickness of the timber members. In Fig. 49 the height is approximated to ~ 4 m since the sections have a depth around 0,5 m; in the Appendix there are the exact measures.
This fact has been now considered in the design of all the beams with imposed total height at 4,50 m.

Conceptually there is almost nothing new in the analyses performed in this chapter.
The only relevant fact is that the lower pieces have been thought to work with the weak axis of inertia against the bending spreaded by the joints, that means that the base of their cross section is larger than the height.

This fact has a double advantage. Firstly, a large base with little height for the lower chord allows to have great area for the tension resistance with a thin element, therefore the distance between the two members is maximized.
Secondly, it's not fundamental to work with the strong axis of inertia against the bending induced by the joints. If we use the stiff side of the lower chord we are stronger in flexure but we are also attracting higher bending moment since the element is much more rigid. At the countrary, a flexible lower chord can stand the rotations of the joints taking minimal bending moment.

For all those aspects the lower pieces have been designed with the base larger than the height.
The upper beam, conversely, is always working with the strong axis of inertia since it's directly loaded by the external weight. The h/b ratio, however, is not extreme since the main task must be the compression and not the bending.

3.3.3: Results obtained and comparisons

With the technical peculiarities mentioned in the previous paragraph the design has been performed as usual, uniform and non uniform load at the ULS; the geometrical parameters of configuration have been adjusted a little to improve the performance.

It has been seen that the scheme of Crocetti with 14 diagonals is better than the beam with 10 diagonals, both for the stresses arising and for the volume of timber needed. For this reason, we describe and comment directly the promising structure with 14 diagonals.

In the Appendix there is some basic information about the solution with 10 diagonals, just to show what has just been stated. The beam with 14 diagonals, instead, is more widely described, with diagrams of all the relevant internal actions. Here below the most important aspects are analyzed.

Volume of timber - 14 diag. - h = 6,25 m		
Upper beam	6,89 m^3	31,6 %
Lower pieces	10,37 m^3	47,5 %
Diagonals	4,58 m^3	21,0 %
Total	21,8 m^3	100 %

Volume of timber - 14 diag. - h = 4,50 m		
Upper beam	11,28 m^3	34,9 %
Lower pieces	17,67 m^3	54,6 %
Diagonals	3,40 m^3	10,5 %
Total	32,3 m^3	100 %

Table 9: Configuration with 14 diagonals: material needed for different heights.

In Table 9 it is reported the design volume of timber needed for the structural scheme of Moelven with 14 diagonals, working with h = 4,50 m (plus the old data for h = 6,25 m).

The first thing to say is that it has been possible to design the beam with only 4,50 m of height with feasible cross sections of the elements, hence the ratio L/h requested is not impossible. The volume of timber claimed of course increases, of about the 50%, but it doesn't become absurd.

It's clear that the growing of the material needed is provoked by the two chords, that are much thicker than before (their area rises up more than the 60%).
The arm of the structure for the bending moment is shorter and therefore the cross section of the upper and lower member must be larger. This was known a priori and it's the price we need to pay if we want a slimmer beam that can fit on a truck in one piece.

It will be charge of the company to judge whether the higher volume needed is acceptable for the production. For an economical perspective it's not the percentage that is interesting but it's the net value, 10 m^3 of construction timber. A cost analysis will evaluate if this quantity is paid by the cheaper manufacturing.

		6,25 m - 14 diag.	4,50 m - 14 diag.
Midspan Deflection		106 mm	159 mm
Bending Moment	Min (AH)	-59 kN.m	-58 kN.m
	Max (AH)	79 kN.m	137 kN.m

Table 10: General comparison of the same Moelven's scheme on different heights.

Going on with our engineering considerations we look now at the technical data. In Table 10 it is reported the vertical midspan settlement of the adjusted configuration of Moelven with 14 diagonals and h = 4,50 m (plus the old data of the same scheme with h = 6,25 m); ULS uniform load.

Both the designs are for load bearing capacity, hence the displacements are not directly controlled. The new structure is more slender than the previous one and of course it will be also more flexible, indeed the midspan deflection becomes the 50% larger.
The increasing is important, however we are analyzing a roof, where the deformations are not the main issue (compared to floors for example); besides, we haven't received any limitation about.

The deflection is still lower than L/300, a value that is not of high quality but it should be acceptable.

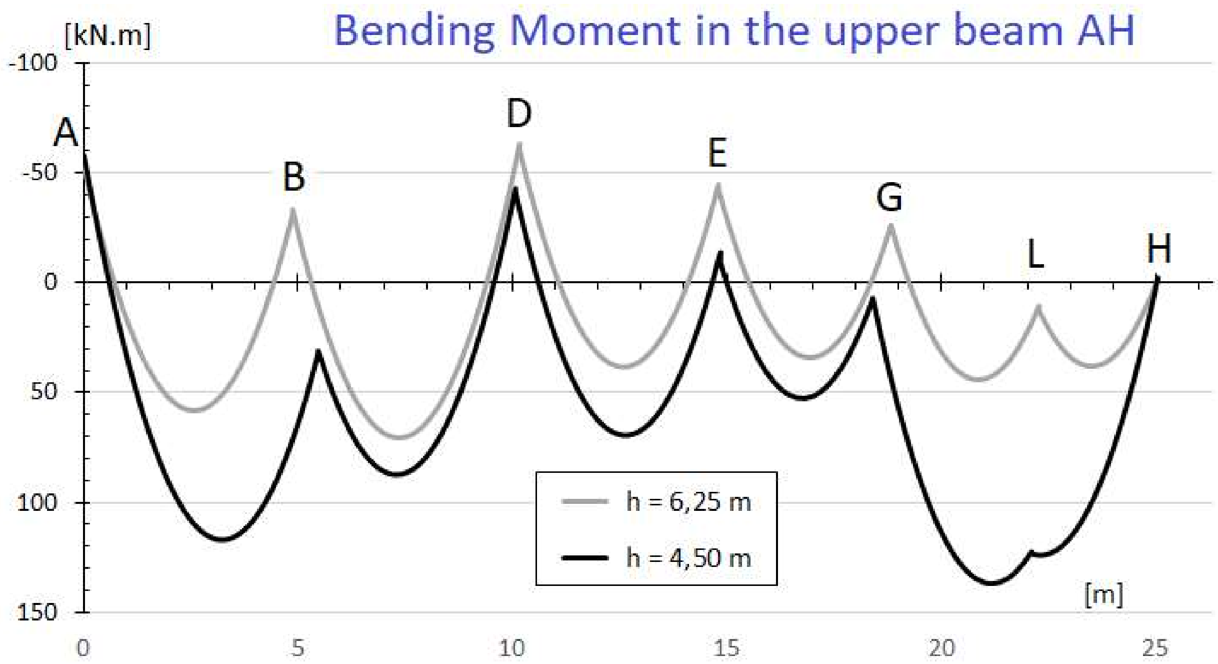


Figure 50: Bending moment in the beam AH for different heights, ULS uniform load.

Table 10 also shows another problem that arises reducing the height of the beam, that is even more clear in Fig. 50 here above. Lowering the height of the beam from 6,25 m to 4,50 m the diagram of the bending moment in the upper beam worsens.

The chart of Fig. 50 refers to the configuration of Moelven with 14 diagonals, deeply studied before for the height of 6,25 m and just redesigned now for h = 4,50 m.
It can be seen that with a shorter web the diagonals become less effective in transferring the load applied on the upper chord, likely due to their little inclination. The flexural behavior of that member rises up and the bending moment increases a lot.

It should not be forgotten that the orientation of the diagonals has been adjusted in both the cases in order to distribute the bending in a good way along the beam AH (for this reason the nodes of the two curves are not perfectly aligned). However, Fig. 50 shows the importance of the height of the web.

The diagonal FL, which was already the worst for h = 6,25 m, becomes really weak now; it’s clear that a shape ratio for the inclination of the diagonals should be always kept in the design.

A further problem can be seen looking at the diagram of the shear in the Appendix.
The uneven load is now much more serious than before. The force transmitted between the two halves in the hinge in H is very large, brought with a high axial force in the element FH.
Also in this case the poor inclination of the diagonals is probably the main responsible.

The only positive aspect that emerges from the diagrams of the internal actions is the flexural behavior of the lower chord. Thanks to the proper ratio h/b assigned at the cross section of the lower pieces, the bending moment arising is very similar to the values previously obtained for h = 6,25 m.

3.4 Transposition of the theoretical configuration on h = 4,50 m

3.4.1: Theoretical studies to learn technical improvements

It has been seen in the previous chapter that the configuration proposed by Crocetti, acceptable even if non optimal for h = 6,25 m, worsens a lot if h = 4,50 m is imposed.

It's clear that a technical improvement is necessary, especially in the arrangement of the diagonals.

Before searching for better practical solutions, it's preferable to have a theoretical scheme as a reference.
It has been said that the configurations that follow the principal directions of stress have logistical issues that lead to a questionable effectiveness in the real production. This does not imply that is useless to study them.
The ideal solutions get an idea about how far we are from them and how much we can earn with technical adjustments of the real structure. Moreover, they show the good mechanical behavior and suggest how to improve the existing one.

The purpose of the next pages is to develop a theoretical configuration that works properly for the fixed height h = 4,50 m. The previous performing ones are the starting point, however they need to be further adjusted during the lowering from h = 6,25 m to h = 4,50 m. Through this procedure, the theoretical knowledge already achieved could be even enriched with new aspects.

All this will provide new awareness and better skills for the real improvement of the existing Moelven's solution.

3.4.2: Operative procedure

What we do in this chapter is to take the last theoretical configuration developed for h = 6,25 m and compress the geometry into the new requirement of h = 4,50 m. The same elements are kept, as in the most performing scheme found.

Probably, new issues will arise since we are distorting the shape of the structure, however this study is fast and easy since we have the whole structural analysis already set. The problems will be good for a further improvement of the theoretical configuration.

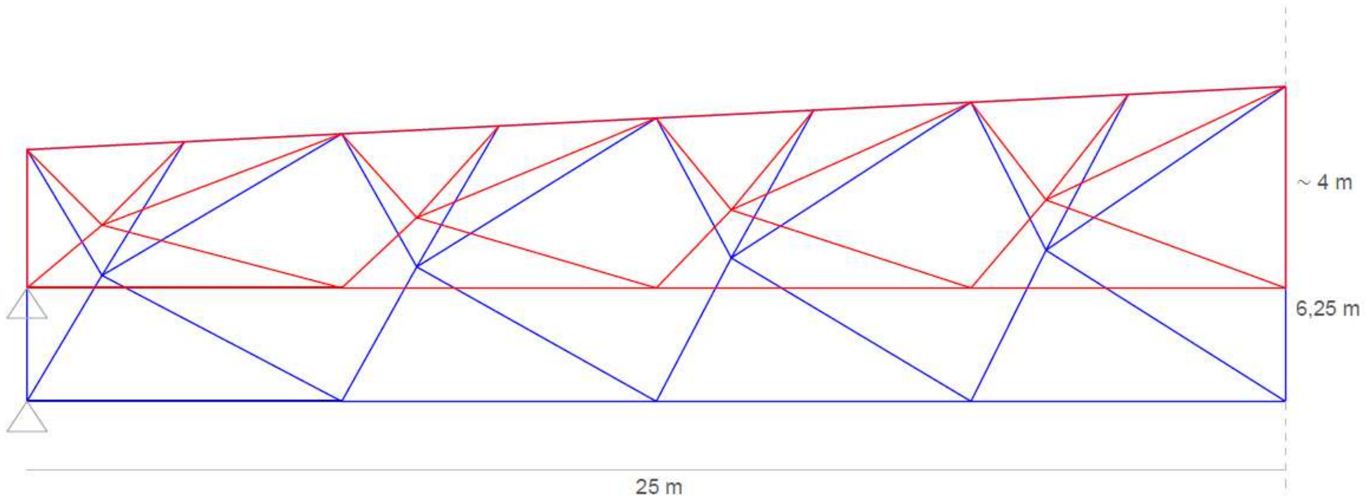


Figure 51: Lowering of the theoretical configuration.

In Fig. 51 it is shown the new theoretical structure with the lowered maximum height. All the promising aspects found before are kept: same scheme of diagonals (with IT and AW), same spacing between the nodes in the upper member, straight horizontal lower chord directly supported in W.

The lines in Fig. 51 represent the center axis of the elements. As in the previous technical chapter, the thicknesses of the two chords are ~ 0,5 m; the height of the beam in the drawing has been set at 4 m, to forecast the final height. In the Appendix there are the precise measures from the design.

From the technical solution of Moelven on h = 4,50 m it has been taken the choice of having the lower chord that bends with the weak axis of inertia (section with the base larger than the height). It has been seen that it's a good solution to avoid high bending moment in that member.

The configuration in red of Fig. 51 has been subjected to uniform and non uniform load at the ULS, as usual. During the design the position of the nodes has been adjusted and improved.
All the necessary information are in the Appendix, the main results are commented in the next paragraph.

3.4.3: Results obtained and comparisons

In Table 11 it is reported the volume of timber needed for this theoretical structure, compared with the technical solution of Moelven just designed in the previous chapter.

Volume of timber - Moelven - 14 diag.				
Upper beam	11,28	m^3	34,9	%
Lower pieces	17,67	m^3	54,6	%
Diagonals	3,40	m^3	10,5	%
Total	32,3	m^3	100	%

Volume of timber - Theoret. Compressed				
Upper beam	8,10	m^3	32,9	%
Lower chord	10,13	m^3	41,2	%
Diagonals	6,36	m^3	25,9	%
Total	24,6	m^3	100	%

Table 11: Volume of timber needed for different structures on h = 4,50 m.

It can be seen that the improvement brought by the theoretical configuration is now quite significant and much larger compared to the gain for the structure with h = 6,25 m. Keeping the same scheme as before and simply compressing it in 4,50 m we have been able to save the 24% of the material (and maybe even more if we use a more refined theoretical arrangement).
The extent of the achievable technical improvement of the solution of Crocetti is now very interesting.

The advantage obtained resides in the two chords, that now works better and can be thinner. When the height of the beam is reduced, it becomes more important to use the little geometry available in a good way. Due to this, the technical solution worsens a lot and the difference is now more relevant.

In order to evaluate the quality of the solution just developed it's necessary also to consider the stresses arising in the structure.

In Fig. 52 it is drawn the diagram of the bending moment in the upper member, under the ULS uniform load distribution.
The chart immediately shows the problem that arises lowering the height of the theoretical configuration: the flexural behaviour of the beam AI near the midspan becomes really large.
The diagonals have great difficulty in supporting the upper chord to limit the bending moment, the values grow to ~ 100 kN.m and become comparable to those of the technical Moelven's model.

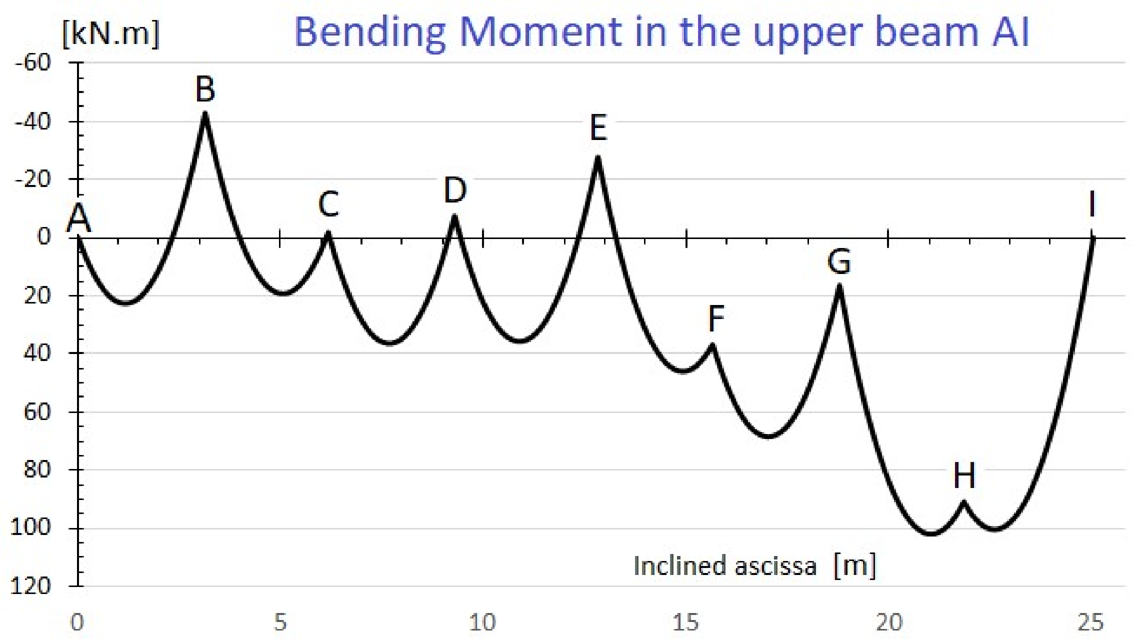


Figure 52: Bending moment in the upper beam, ULS uniform load.

The fault is not in the distance between the nodes, that is the same as for the beam with h = 6,25 m. Near the supports, indeed, the peaks are limited to ~ 40 kN.m as in that case.
Since the spacing between the nodes is quite small it's not acceptable such a large bending moment at the center: it means we are using a lot of diagonals for nothing.

The problem that eloquently arises affected also all the previous structures, less visibly.
The issue is now highlighted by the lowering of the previous scheme, that has produced a distorsion of the shape ratio of the diagonals in the web. They are now less inclined and they suffer the same problems as in the original configration, due to the little angle to the horizontal.

It could be seen also for the case of h = 6,25 m that in the theoretical configuration the diagonals near the midspan don't work properly. They are subjected to very few forces and they disturb themselves, breaking the scheme of tense and compressed elements.

The inclined members near the center, which come from the theoretical Fig. 41, interact with the vertical rod IT, that is extraneous with respect to those academical curves and it was added to cope with the inclination of the upper chords. Furthermore, the diagonals near the midspan need to work on the front line for the uneven load distribution, for which the principal directions of stress change. Due to all those reasons it's not a casuality that near the midspan the initial scheme is modified a lot with the adjustment that fix up the things during the design.

It will be necessary to deeply reorganize the arrangement of the diagonals at the center of the beam, thinking no more at the principal directions under the uniform load but at the uneven load, at the flexural behavior of the beam AI and the effects of the added rod IT.

Starting from these considerations and the knowledge achieved we are going to improve the theoretical configuration in the next chapter.

3.5 Improvement of the theoretical configuration on h = 4,50 m

3.5.1: Rearrangement of the configuration and considerations behind

In this chapter we modify the theoretical configuration developed for h = 6,25 m in order to make it suitable on h = 4,50 m. The knowledge achieved simply compressing that scheme in height is now the guideline for the adjustments to perform.

As a first thing we try to impose the optimal shape ratio of the diagonals.
The most suitable inclination of the elements in the web is not constant but depends on the shear applied to the structure. Where the shear is high (near the supports) the diagonals need to be more inclined and dense since they work a lot. Conversely, where the bending is dominant on the shear, like at the midspan, the slope of the diagonals should be reduced.
All those concepts could be seen also through the curves of the principal directions of stress (Fig. 41), now they have been clarified and highlighted for the bars of a truss.

As a general remark, the best inclination is around 45°, at least for a preliminary sketch of the structure. Of course it's also necessary to contextualize the theory with the geometrical constraints and the real dimensions and boundaries of the beam.

For the academical configuration with h = 6,25 m the two chords had the main nodes with a spacing equal to ~ 1,1 times the height of the web (following the theoretical charts). The tall truss had an average height of ~ 5,5 m, thus a preliminary spacing of 6,25 m (= L/8) had been selected for the lower chord.
The actual slender beam has an average height of ~ 4 m, therefore in this case we draw the initial scheme with a spacing of ~ 4,4 m between the nodes on the two chords, to keep the ratio of ~ 1,1. It will be investigated if a higher slope is necessary, especially near the supports.
The web has now more, smaller cells than for h = 6,25 m (let's see Fig. 53), but the inclination of the diagonals is the same. The important thing is the proportion of the cells, not their number.

Keeping the shape of the cells in the web, we have made closer the main nodes in the upper beam: the spacing has been reduced from 6,25 m to 4,50 m since it depends also on the dimension of the cell and not only on its shape. Due to this, we think it's no more necessary to add the secondary diagonals for the upper beam (marked in green in Fig. 43).

The span for the bending in the upper member passes from 3,1 m for the tall beam (h = 6,25 m) to 4,50 m in this case. A priori it's not good since the bending moment will probably increase. However, halve the spacing between the nodes to 2,2 m through the secondary diagonals seems excessive. Moreover, the upper chord will be thicker since its compression will be larger due to the reduced height. Probably it won't be a big issue to increase a little the bending by enlarging the spacing.

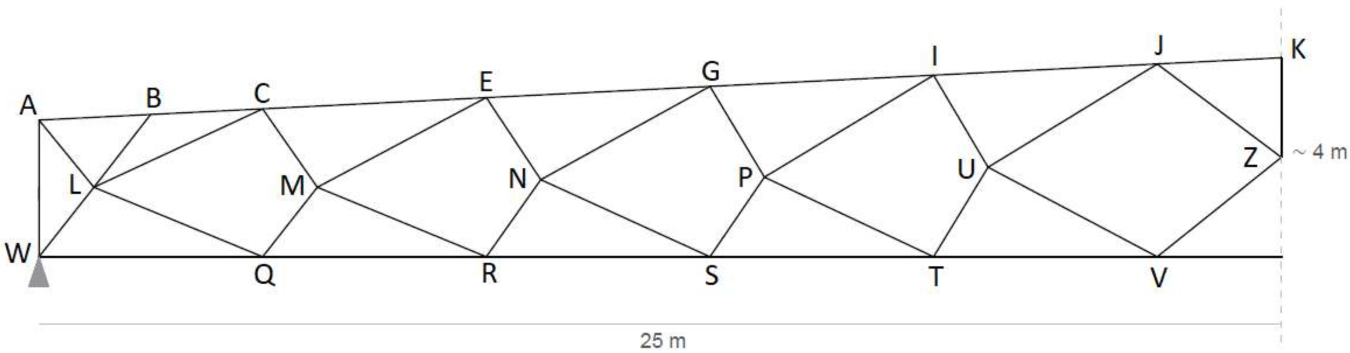


Figure 53: Proposed refinement of the theoretical configuration on h = 4,50 m.

The only secondary diagonal that we leave is the first one, BL. In that region a big reaction force arises from the support and must be transferred through the web, therefore it seems good to have also a secondary element that shares some load. As we said theoretically, in a region of high shear dense and inclined diagonals are preferable.
Furthermore, the compression in the upper beam A-B-C is very limited (because it arrives in C through W-L-C), thus the section would gladly be much smaller and designed mainly for bending, therefore a little span is better.

A region that need a deep reorganization of the elements is the web near the midspan. It has been seen in the previous chapter that the diagonals are too weak and aren't able to work properly.
Apart their slope, it can be argued that they are oriented in the wrong direction.

In all the structures previously developed they descend departing from the midspan, through points that settle with the beam.
It has been thought to put a diagonal (JZ) in reverse, pointing to the midspan. The symmetry axis can't slide under uniform load, therefore it provides infinite horizontal stiffness to sustain the inclined members that stabilize the upper chords.

With this intuition we hope to solve the problem related to the high bending moment that affects the upper beam near the midspan.

3.5.2: Results: material needed and the role of the diagonals

The structural analysis has been developed as in the previous cases: continuous straight lower chord with b > h, hinged at the diagonals (but with no more design hinge at the midspan); upper member longitudinally jointed in C. Uniform and non uniform load distribution at the ULS. Design of all the elements with little adjustments of the geometrical scheme of Fig. 53 and local assessment of failure. All the data of the structure and the results are in the Appendix, here the main ones are commented.

Volume of timber - Moelven - 14 diag.		
Upper beam	11,28 m^3	34,9 %
Lower pieces	17,67 m^3	54,6 %
Diagonals	3,40 m^3	10,5 %
Total	32,3 m^3	100 %

Volume of timber - Theor. Compressed		
Upper beam	8,10 m^3	32,9 %
Lower chord	10,13 m^3	41,2 %
Diagonals	6,36 m^3	25,9 %
Total	24,6 m^3	100 %

Volume of timber - Theoret. Improved		
Upper beam	8,10 m^3	34,3 %
Lower chord	10,48 m^3	44,3 %
Diagonals	5,06 m^3	21,4 %
Total	23,6 m^3	100 %

Table 12: Material needed for different structures on h = 4,50 m.

In Table 12 it is reported the material needed for these last two theoretical configurations, plus the technical solution of Moelven.

It's rewarding to see that another cubic meter of timber has been saved with respect to the previous truss scheme. The gaining is almost solely in the diagonals, that reduce their volume of the 20%. This means that the changes performed on their shape ratio were correct and very important, indeed the improvement is considerable.

The upper and the lower chord almost don't change their volume of timber in this improved structure. This is in line with the expectations since those members haven't been modified significantly and they continue working with the full geometry for the global bending of the beam.

The last theoretical gaining of timber is thus important but limited at the diagonals, that are not the main volume of the total. The global improvement with respect to the solution of Moelven, however, goes over the 25% and it's notable, much more than what could be saved having h = 6,25 m.
Now there are more than 8 m^3 of timber available to search for a technical solution that performs better than the existing one.
When the available geometry is limited it's more important to use it optimally…

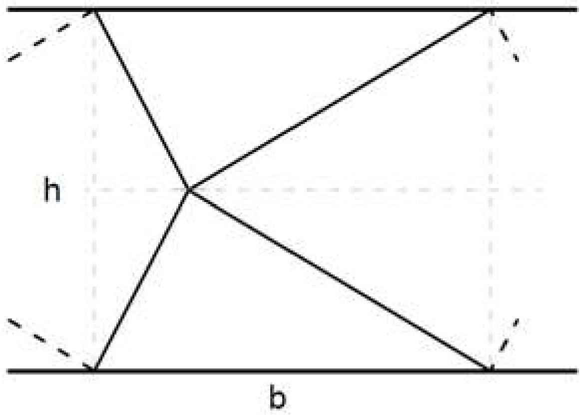


Figure 54: Geometrical scheme of one cell of the web.

We underline again that the original shape ratio proposed on h = 6,25 m is very good (it has the merit of the improvement of this chapter). It had been extrapolated from some schemes of trusses following the principal directions of stress and it's represented in Fig. 54.

The proportions of the cell and the ratio of the diagonals inside vary a little around the reported sketch, depending on the shear and the bending acting on the structure. It can be happily seen in the Appendix that, after the adjustment of the configuration, the members are less inclined at the midspan and more dense at the support.

The stroke with the knee is intended to follow the curve of the principal direction of stress. The bars are straight since the truss is discrete and the curve must be interpolated with a broken line path, to avoid the arising of bending under the curvature.

The approximation of Fig. 54 with just two elements per path seems rough but it should be quite good. The reality will be even poorer since we are not allowed to have star joints at middle web between the diagonals, therefore the stress curve will be followed with a straight line.
The web of a truss is generally too little to allow for very refined schemes of diagonals, which normally are reserved for larger structures, like the bracing of the skyscrapers. Anyway, we don't think the result will be too bad since the diagonals are the secondary elements of a truss, the main ones are the two chords.

The design will be a matter of finding the best inclination of the bars, playing around 45°. The game will be then complicated by dealing with the real global geometry of the structure, its dimensions and the spacing of the nodes.

3.5.3: Description of the static results obtained

From the Appendix, the bending moment arising in the lower member is quite low in magnitude: under uniform load it is substantially provoked by the global deflection of the lower chord. The uneven load makes the curve more irregular, but the values remain limited.
The choice of the weak axis of inertia for the lower chord is confirmed again to be good.

The descending chains and rising arches scheme has been abandoned around the midspan, as previously explained. Looking at the diagrams of the axial forces in the diagonals (see the Appendix) we can see that now they behave properly under both the two load cases.

In Fig. 55 it is drawn the diagram of the bending moment in the upper beam AK, under uniform distributed load at the ULS.

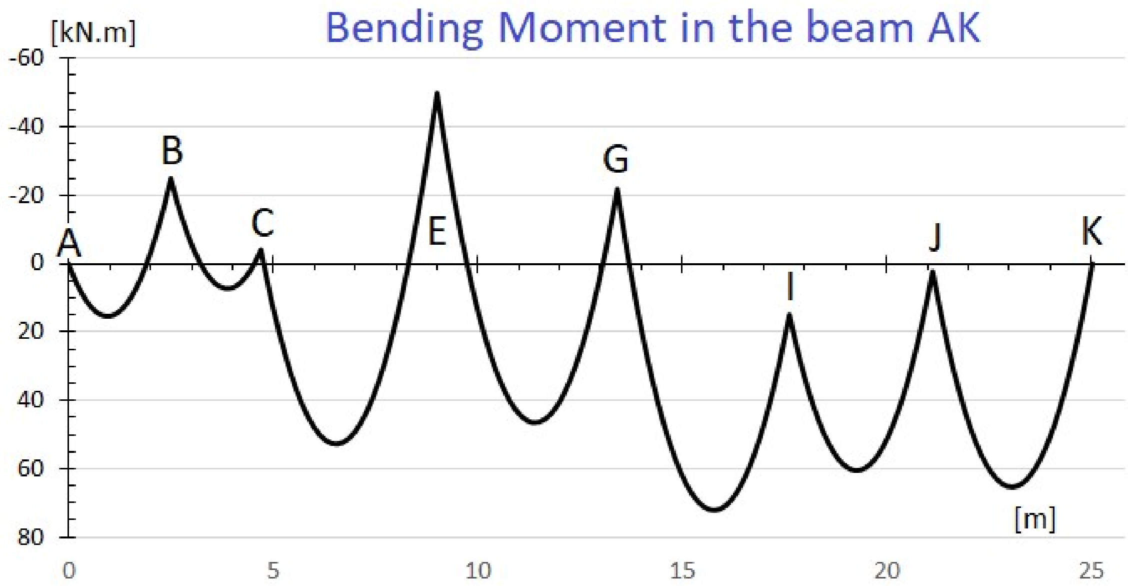


Figure 55: Bending moment in the upper beam, uniform ULS load.

It's immediate to see that the chart is much more better than what found in the theoretical configuration of the previous chapter. After the rearrangement of the bars the problem of high bending near the midspan is finally solved, the nodes are all effective in supporting the upper member.
We can proudly affirm that the choice of having the diagonal JZ set against the symmetry axis of the structure is very good in stiffening the central part of the beam. It will also be mantained in the future.

Of course, if the load is non uniform, the line KZ is not fixed horizontally and it will be pushed through the half with the lower weight. The bar JZ loses rigidity at its base and the mechanism described is altered. However, the load is also decreasing and it emerges that the safety level isn't reduced.

The secondary diagonal preserved near the support makes his job as hoped. The bar BL helps the elements AW and CL in transferring the reaction force of the support (see the charts in the Appendix). At the same time it halves the segment AC, so that the bending arising is only ~ 20 kN.m.

Looking at Fig. 55 it's also possible to see the consequence of having avoided the secondary diagonals.
In the previous theoretical solution those elements provided a spacing between the nodes in the upper member of ~ 3,1 m. In this last improved configuration their lack leads this value to ~ 4,5 m.

The peaks of the bending moment increase from ~ 40 kN.m (where the structural behavior was proper) to ~ 60 kN.m. The growing is not exaggerated, as we wanted.
The compression is still dominant in the upper chord, the bending takes less than the 20% of the resistance of its cross section (according to the interaction formula of the Eurocode 5). The addition of the secondary diagonals would probably be excessive and not effective.

Summing up, if the structure is slendered from h = 6,25 m to h = 4,50 m, the spacing between the nodes can be enlarged to ~ 4,5 m (thanks to the intrinsic thickening of the upper member due to the higher compression). This value must be saved and kept in mind for the next technical improvements.

3.5.4: Structural stiffness and general summary

The assessment of the displacements is always important, even if in this case it's not the main goal. In Table 13 there is a general comparison between the different solutions developed for h = 4,50 m: the technical and the two theoreticals. The load is uniform at the ULS.

h = 4,50 m		Moelven - 14 diag		Theoret. Compress.		Theoret. Improved	
Midspan Deflection		159	mm	174	mm	176	mm
Bending Moment	Min	-58	kN.m	-43	kN.m	-50	kN.m
	Max	137	kN.m	102	kN.m	72	kN.m

Table 13: General comparison between the technical and the theoretical configurations.

It can be seen that the proposal of Moelven is the stiffest, even if the difference is not so high, it's around the 10%.
We remember again that all those designs are for the load bearing capacity and not for the settlements, therefore it's substantially wrong to compare the displacements if different quantities of timber have been used. It has been verified that the last improved solution becomes the stiffest by simply thickening the two chords up to a total volume of 26,5 m^3, well below the 32,3 m^3 needed for the proposal of Crocetti.

It has been confirmed that an optimal distribution of the forces in the structure is beneficial both for the load bearing capacity and for the stiffness, the amount of material always verifies this.

At the end of this Chapter 3.5 we summarize the new positive aspects found with these theoretical configurations.
In general, it has been seen the importance of the right shape ratio of the elements in the web, where the diagonals should lay around 45° of inclination. Moreover, with the imposed h = 4,50 m, it seems that ~ 4-4,5 m is a good spacing between the nodes in the upper member (regarding its flexural behavior).
The bars converging into the symmetry axis at the midspan are probably the most important aspect found with these studies, fundamental for the stiffening of the structure at the center.

4. Conclusive technical proposals

In this last Section 4. we are going to search for a synthesis of the investigations done so far.
We have studied the proposal of the company and detected its issues. The subsequent analyses of the theoretical configurations have highlighted the improvements achievable and the positive aspects to be pursued, even if the practical construction turns out to be quite tricky.
It’s now time to apply all the knowledge acquired to formulate a conclusive structural solution that is suitable for the technical manufacturing and optimal enough from the mechanical point of view.

4.1 Configuration with “trident diagonals”

4.1.1: The shape of the lower chord: technical and aesthetical requirements

The first and most important decision that must be taken for the conclusive proposal is the shape of the lower chord.
In the previous Sections it has been said that, mechanically, the horizontal member is the best choice because it allows to use everywhere the full geometry available and because the straight element minimizes the longitudinal connections and their weakening. It has been also stated, however, that this solution isn’t aesthetically pleasant from the architectural point of view.

Structurally speaking, having the two chords converging on the support seems quite bad: apart the arm for the global bending, on the support there is a huge shear provoked by the reaction force and we need space to put strong diagonals in order to distribute it in the web. The chords alone with their angle seem weak for this task.

As a compromise, the horizontal lower chord has been waived but still keeping a minimum height of the web near the support. The aesthetic is saved and the structural system is not totally destroyed.

The scheme has been inspired by the roof trusses of the Allianz Stadium in Torino (Fig. 56).

Figure 56: Allianz Stadium, Torino (Italy).

The structural issue of the extremities of those beams is very similar to the our and it's the conciliation between the pleasant shape and the structural performance.
The solution adopted for the football stadium of Juventus is probably very clever: there is a strong link for the support reaction but it has been almost hidden by the architectural design. The funicular lower chord seems converging on the upper one and the effect is very beautiful.

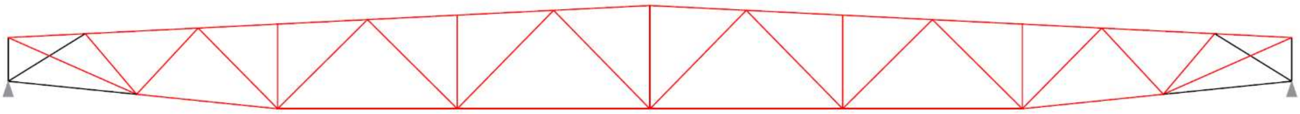


Figure 57: Concept of the proposed configuration with "trident diagonals".

The proposed concept for our beam is shown in Fig. 57. The lower chord is made with two straight lines, one horizontal and one rising on the support. Of course the scheme isn't as nice as the one of the Allianz Stadium, but the usage of the two buildings is not the same and the aesthetic requirement is also different.

In our truss, anyway, some bars can be painted with a different colour so that visually the shape seems much more tapered than what really is. The compressed rising diagonal is thus less visible and the whole scheme results to be much more pleasant, similarly to our inspiring example.

The proposal for the lower chord is architecturally approved and we can go on defining the other details of the arrangement.

4.1.2: <u>The reasons behind the chosen configuration</u>

The concept of the technical arrangement of this chapter has already been anticipated in Fig. 57, however it's necessary to explain how the positive aspects arised in the previous Section have been translated into the choices made. In Fig. 58 it is better shown the proposed configuration, focused on half beam.

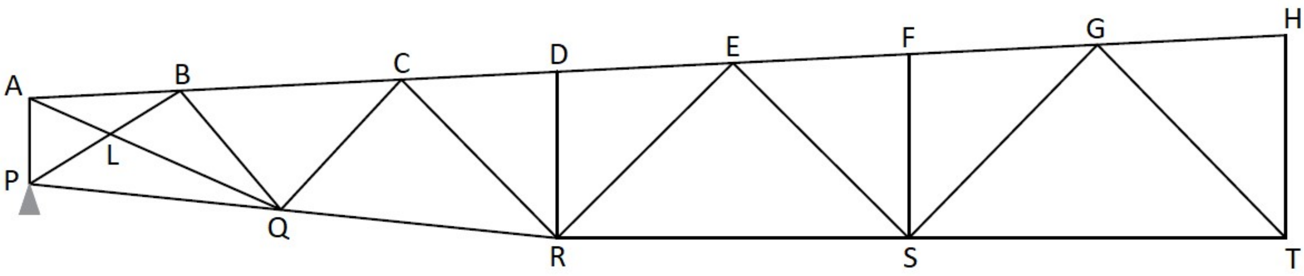


Figure 58: Proposed technical scheme with "trident diagonals", left half.

The arrangement at the supports needs further comments than the compromise between aesthetic and efficiency just seen: the reaction force is properly distributed only through a right design.

It has been seen with the academical studies that the diagonals need to be strong and dense in that region. The complex theoretical configuration has been simplified into the bars PA, PB and ALQ.
The elements PA and PB are thought to work in parallel to transfer the compression to the upper chord. The chain ALQ (together with BQ) is the complementary on the tension side, like before.

It has been stated that the joints in the web introduce additional manufacturing issues and the necessity of a lateral bracing to avoid the out of plane buckling of the compressed branches. Knowing this, the majority of those connections have been eliminated.
The node L is the most important between the ones in the web and it has been mantained. This joint is however different from those seen before in the academical configurations: the bar PB is straight and continuous, therefore it suffers the buckling like the other compressed diagonals. Since PB is not longitudinally jointed along its length, no additional out of plane bracing needs to be planned.
The fastenings for the elements AL and LQ are ordinary and the manufacturing issues are limited.

The proposed solution in L is thus technically fine and structurally very promising. Another time, this is a proper engineering compromise between the mechanical optimization and the practical demand.

A good choice that clearly emerges from Fig. 58 is the placing of the diagonals GT and TH. They directly derive from the studies of the last theoretical configuration.
The bar TH links the two chords at the midspan, stiffening the structure and providing the base for GT. This inclined element has proved to be very important for the control of the flexural behavior of the upper member near the center.

An important issue that was analyzed in the theoretical Section is the relation between the layout of the diagonals and the spacing of the nodes in the upper member.
In this technical chapter, the solution adopted consists of an ordinary Warren truss system that has been enriched with the non-truss diagonals DR and FS. Those two members have been thought to halve the span of the bending of the upper chord and reduce its stresses.

The average distance between the nodes turns out to be smaller than 4 m and this seems even better than the values previously used. It is believed that imposing the same spacing with a densing of the diagonals of the Warren truss scheme (avoiding DR and FS) would not be effective. The elements would probably be too numerous and too inclined.
For these reasons, the mixed layout has been selected instead of the pure Warren one.

From all our reasonings, the resulting configuration is what shown in Fig. 57 and 58. Even if the scheme has a clear truss behavior at the base, visually there are groups of three diagonals pointing on the lower chord, similarly to the old Moelven's proposal. Due to this, the arrangement is referred to be with "trident diagonals".

4.1.3: Technical peculiarities and structural analysis

The most important novelty, for which a specific description is required, regards the peculiarities of the elements at the support.
The bars AL and LQ are aligned and hinged on PB. Apart from the reduction of their tension resistance, they help to restrain the in plane instability of PB. There will be indeed two different buckling lengths for the compressed diagonal: PB for the out of plane mode and the longest between PL and LB for the in plane mode.
For a matter of sectional efficiency, the bar PB will work with the strong axis of inertia on the full lenght and with the weak axis of inertia on PL and LB.

Accordingly to what seen with the academical studies, the compression of the support should arise mainly through PB, therefore the compression in the upper chord will have a huge jump in point B. Two completely different cross sections have been proposed for AB and BH, longitudinally jointed as in the previous chapters. The height of the upper member increases approaching the midspan.

Specularly to the compression branch, the chain ALQ (together with BQ) works unloading the tension of the lower chord.
Regarding the lower members, it's better to limit the connections, therefore the element PR is continuous, with reduced section on PQ. The segment RT is also continuous and it's longitudinally jointed in R and in T. In the point T there is a design hinge like in H, while in point R a rotational spring like in B has been placed, to account for some rotational rigidity.

Except for the structural details just described, the other diagonals are trivially hinged at the chords.

The structural analysis has been performed like always, uniform and non uniform distributed load at the ULS. Design of the members as described at the beginning of the thesis.
The exact geometry of the proposed configuration is in the Appendix, together with the cross sections of all the elements and their stresses. In the next paragraphs the most important results are commented.

4.1.4: Static results

Having done the structural analysis we discuss now about the stresses obtained. In Fig. 59 are represented the axial forces arising in the truss members, uniform load at the ULS. Under the same load condition in Fig. 60 there is the diagram of the bending moment in the upper beam.

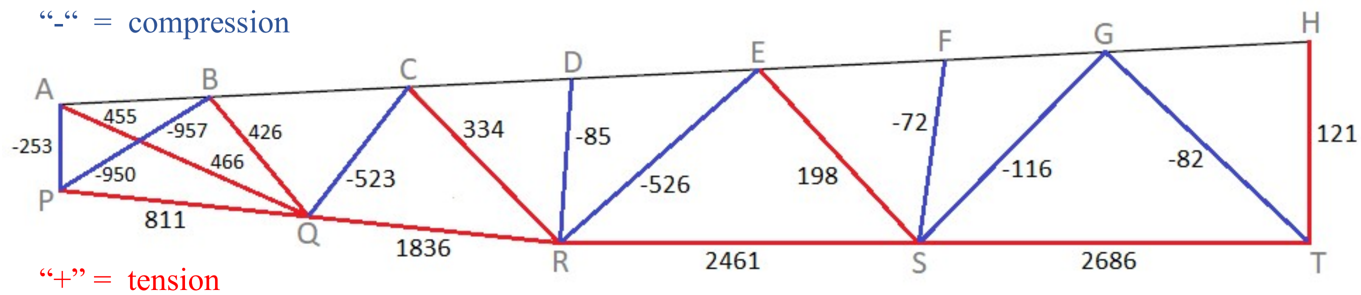


Figure 59: Axial forces [kN] in the bars, ULS uniform load.

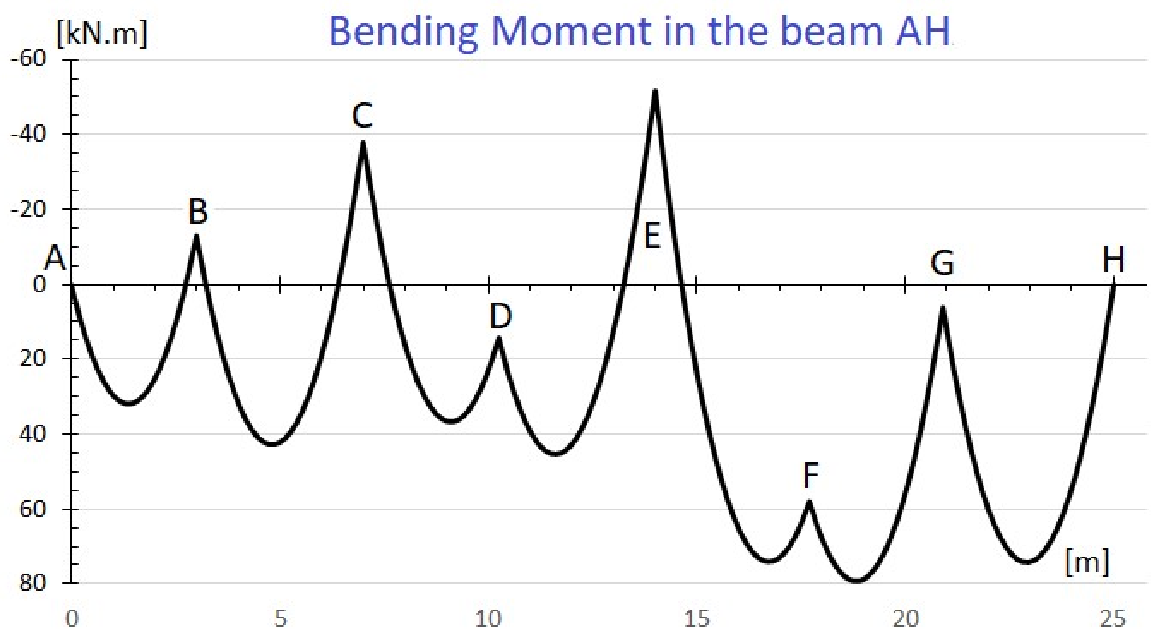


Figure 60: Bending moment diagram of the upper beam, ULS uniform load.

The values of the axial forces are very nice: the arrangement of the diagonals near the support works exactly as wanted and the tension path is distributed in point Q to counteract the shear of the beam. Specularly it happens in the compression branch, as clearly visible in the Appendix.

Regarding the diagram of the bending moment of Fig. 60 we find again similar things to what obtained with the academical configurations. For example we appreciate the good behavior of the element GT, for which specific comments are not needed.
Since the stresses arising are analogue to what found in the previous Section, it's clear that the passage from the theoretical schemes to this technical one has been done correctly.

We notice however that the additional non truss diagonals DR and FS are weaker than what expected and they are not very effective in controlling the flexural behavior of the upper chord. They had been placed in order to keep low the bending arising in the upper member, by reducing the distance between its nodes. The spacing adopted is quite small (lower than 4 m on average), but this little value is probably counteracted by the compliance of the two vertical bars.

Eventually, the stresses arising are acceptable and, regardless of these little problems, the bending moment on AH remains limited enough (well below the values of the initial assigned concept).
The proposed technical arrangement is thus suitable from the structural point of view.

4.1.5: <u>Volume of timber required</u>

The most important index to judge about the optimality of the technical solution developed is the volume of the timber required for the structure.
Table 14 is fundamental because it makes a transversal comparison between the whole thesis: it indicates the material needed for the Moelven's concept, for the theoretical configuration and for the technical solution just proposed.

Volume of timber - Moelven - 14 diag.			Volume of timber - Last Theoretical			Volume of timber - Technical proposal		
Upper beam	11,28 m^3	34,9 %	Upper beam	8,10 m^3	34,3 %	Upper beam	8,77 m^3	34,3 %
Diagonals	3,40 m^3	10,5 %	Diagonals	5,06 m^3	21,4 %	Diagonals	4,34 m^3	17,0 %
Lower chord	17,67 m^3	54,6 %	Lower chord	10,48 m^3	44,3 %	Lower chord	12,43 m^3	48,7 %
Total	32,3 m^3	100 %	Total	23,6 m^3	100 %	Total	25,5 m^3	100 %

Table 14: Material required for the theoretical, Moelven's and technical solution.

It's very pleasant to read the data reported in Table 14, they are the reward of the optimization research performed.
The reduction of the material required with respect to the initial proposal exceeds the 20% and it's remarkable. It's evident how the two chords are tinner now because they work better, with fewer bending moment and a more neat use of the elements and of the geometry.

With respect to the academical configuration the difference is acceptable and it's the price of a simpler layout of the bars.
Table 14 also says that the material used for the diagonals is now sligthly lower than the value of the theoretical arrangement. This is due to the fact that the configuration in principal directions of stress has a more complicated (and heavier) scheme of diagonals, but that allows an optimal usage of the two chords.

All those results are very promising for a real enhancement of the concept of the company.

4.2 Pure truss configuration

4.2.1: Improvement of the existing technical scheme

The technical scheme "with trident diagonals" just seen in the previous chapter is very good: it doesn't present particular technical difficulties and it allows to save more than the 20% of glulam with respect to the initial concept, going close to the theoretical results.

Regarding the arrangement of the diagonals near the support we are totally satisfied and, likely, further improvements are negligible. However, the structural behavior along the span can probably be enhanced a little bit. This investigation is the topic of this chapter.

It has been said that the non truss diagonals DR and FS aren't very effective in their task. For this reason, it has been decided to avoid them and to see if any betterment is achievable.
The general architectural concept of the new arrangement is shown in Fig. 61.

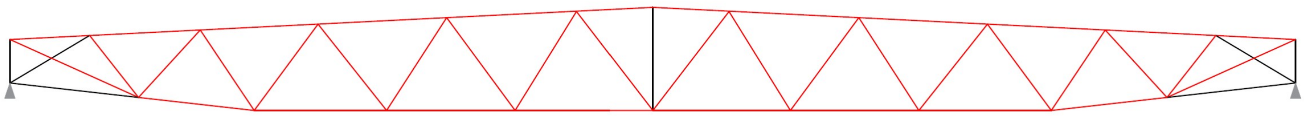


Figure 61: Concept of the proposed truss configuration.

Visually, the scheme is much more regular and ordered than before. It can be considered also more pleasant, even if the architectural judgement is not our task.
It's important to underline, however, that near the support the arrangement hasn't been modified and thus it's identical to the previous one. The same nice tapered effect is also kept.

The vertical bar at the center is the same as before but it has been drawn with a different colour: since it is the only vertical diagonal now, for aesthetical purposes it can be decided to hide it with a different painting or using a thin steel tendon. The architect will judge this aspect, but we strongly recommend not to eliminate the element because structurally is very useful.

For a better observation of the modifications introduced, the left half of the beam is reported in more detail in Fig. 62.

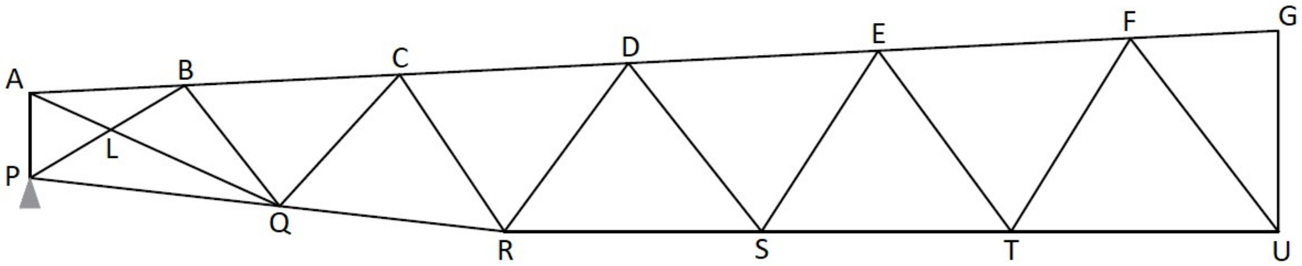


Figure 62: Proposed technical scheme of the pure truss configuration, left half.

The adjustments performed along the span aren't minimal but are not substantial as it could appear as a first impact. The total number of diagonals, for example, has been kept as before.
The sole significative difference is the elimination of the non truss bars, which have proven not to be very effective. For this reason, the configuration has now become a pure truss scheme.

The newly introduced choice implies that the distance between the nodes in the upper member and the density of the diagonals must be defined again.

Keeping the same number of diagonals as in the previous layout, we have now an average spacing on the upper member of ~4,4 m. This value is larger than before but it's still in agreement with the studies of the past theoretical Section.
Having a larger spacing means accepting larger bending moment arising in the compressed chord. However, if the diagonals are more efficient, the flexural behavior will be probably smaller and the stresses arising will remain limited.

The comparison is thus between a reduced spacing with weak elements and a larger spacing with a good layout. The number of members has been maintained in order to make the analysis more close and sure.
In the next paragraphs the results obtained are discussed.

4.2.2: Static results

The first result we analyze is of course related to the modifications performed.
In Fig. 63 there are the curves of the bending moment in the upper chord for this last pure truss configuration and the previous one, with "trident diagonals"; uniform load at the ULS.

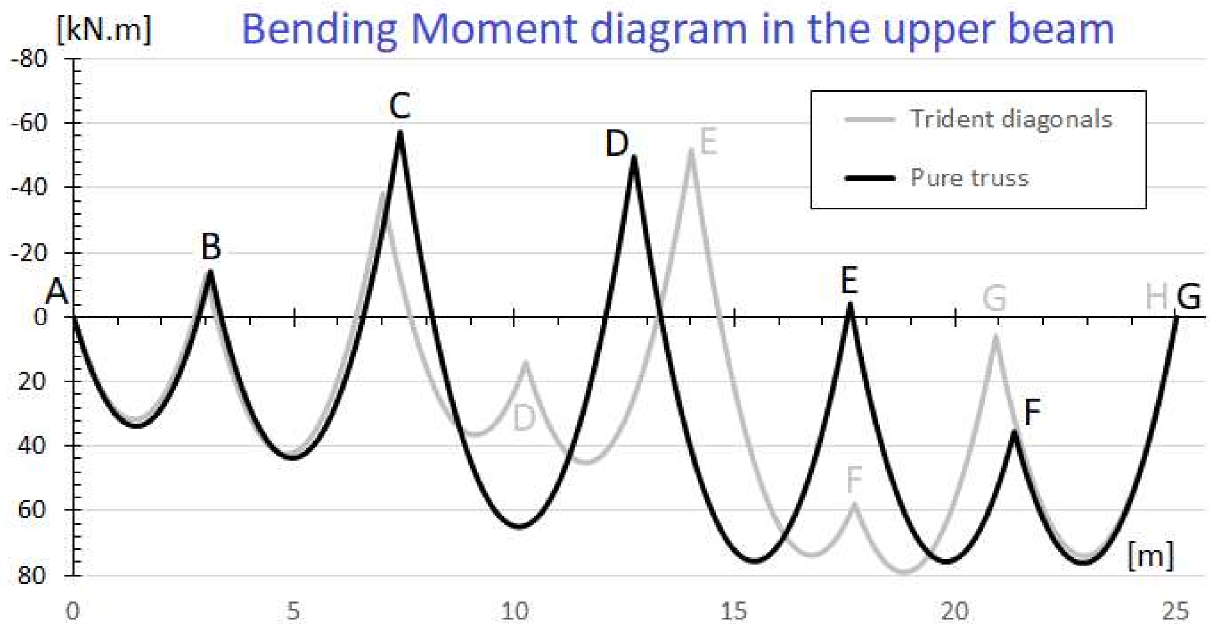


Figure 63: Bending moment in the upper chord for the two technical configurations.

The chart is very nice, the black curve is much more better than the the grey one.
With respect to the previous configuration we have lost one node, however the remaining ones are all very stiff and the bending moment arising doesn't exceed the previous values (indeed, it's slightly lower). The curve obtained is finally close to the last theoretical one, not only regarding the shape but also considering the magnitude; this is really very good.

We can proudly say that the intuition was correct: even if the spacing on the upper chord has been enlarged a little, the higher efficiency of the new layout of the diagonals is stronger and the resulting stresses are better.

The charts of the other internal actions are similar to those of the previous chapter, therefore no specific analysis is needed. For a sake of completeness, they are reported into the Appendix.

4.2.3: Material required and general comparisons

The interesting result awaited now is the verification that the improved structural performance really corresponds to a reduction of the material needed.
After the design with the same rules as always, the data have been collected. Table 15 compares the volume of timber required for these two technical configurations under analysis.

Volume of timber - "Trident diagonals"			Volume of timber - Pure truss		
Upper beam	8,77 m^3	34,3 %	Upper beam	8,92 m^3	35,8 %
Diagonals	4,34 m^3	17,0 %	Diagonals	3,91 m^3	15,7 %
Lower chord	12,43 m^3	48,7 %	Lower chord	12,06 m^3	48,5 %
Total	25,54 m^3	100 %	Total	24,88 m^3	100 %

Table 15: Volume of glulam needed for the two technical configurations.

The results obtained are very rewarding: thanks to the last enhancement we have been able to go under the 25 m^3 of glulam per beam.
The highest reduction of material is concentrated in the diagonals, where we focused the adjustments. There is also some improvement of the lower chord, probably related to the higher number of nodes, which leads to a better distribution of the stresses.
The upper member is now a little heavier because having less sub-spans implies having less steps of lamination number to follow the increasing of the compression. Anyway, those are minimal details, a lot of cross sections haven't been modified since the adjustments involved few diagonals only.

The further improvement just achieved can't be enormous, however the total gain with respect to the original concept has become very significative, since we are now saving more than 7 m^3 of material. This quantity is equivalent to the cost of many hours of manufacturing and it should cover the additional practical issues arising with this truss configuration.
The main logistical complication is simply related to the higher number of members and joints but it seems sustainable, especially considering that several identical beams will be needed and that the production of the elements can proceed in parallel.

The volume achieved now, 24,9 m^3, is quite close to the theoretical value previously obtained (equal to 23,6 m^3). The reason is probably related to the fact that in a truss the diagonals are not the major part, which instead resides in the two chords. As a consequence, the abandonment of the arches in the web has not dramatic consequences if the global layout remains clever.

We have found a technical solution which is just the 5% worse than the academical configuration. Generally, the optimized real arrangements are a few percentage lower than the theoretical schemes found through parametric analyses with FEMs.
We are in the presence of a double verification of quality, not only for our technical configuration but also regarding the academical one and it's really notable.

4.3 Conclusions

4.3.1: Summary of the work performed

This last chapter is dedicated to the synthesis of all the analyses performed and the obtained results.

The consultant of the company ideated his promising concept forging the shape of the lower chord, making it similar to the trend of the bending moment of a beam under uniform load.
The non truss mechanical behavior of the proposed structure has been immediately noticed. Actually, this was exactly the idea of the swedish designer, where the upper chord is supported (through the diagonals) by the curvature of the lower one, like in a catenary bridge.

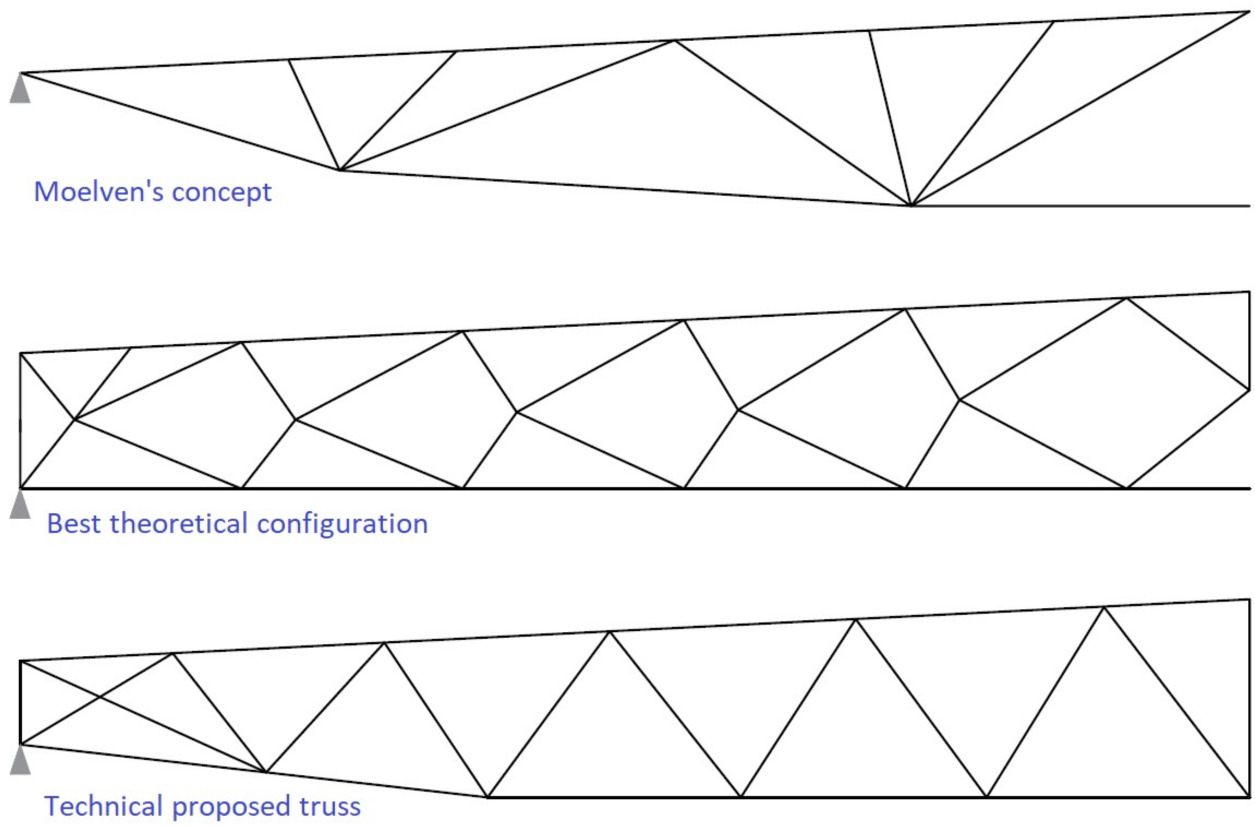


Figure 64: Main configurations analyzed (left half).

The request of technical improvements has been developed and the thesis has proceeded, firstly by studying the Crocetti's concept and then by comparing it with the new solutions developed.
The alternatives have initially followed the lines of the principal directions of stress, in order to aim to the maximum mechanical efficiency. Afterwards, technical trusses have been developed in order to conciliate the structural optimality with the manufacturing issues.

All the conclusions found are in the next paragraph.

4.3.2: Positive aspects arised and comparisons

The most general and important aspect highlighted in the thesis has been the primacy of the truss behavior on the flexural-chain one. Even if this sentence seems evident, the schemes can appear similar because they differ only for the arrangement of some diagonals. Those little differences, however, determine the structural behavior and are fundamental for the mechanical performance.

The arrangement of the web with arches and chains that interact with the two chords has been theoretically proven to be very powerful.
In the proposed solution most of the cross branches have been eliminated and the layout has been simplified, moving to a suitable Warren scheme practically accepted. Between the technical configurations this is one of the closest to the academical one.
The global performance is still good since the main elements are the two chords and not the diagonals.

The theoretical analyses have confirmed, as it could be expected, that the optimal shape for the lower member is the one that uses all the geometry available. The straight line is very good also because it minimizes the longitudinal connections.
The final truss developed with linearly varying depth (Fig. 64) has shown to be appropriate for both the structural purposes and the aesthetical requirements.

Improving the original structure, on the support it's good to leave some space between the chords, where the web is fundamental to carry the high shear provoked by the reaction force. In this region, the third sketch of Fig. 64 is theoretically inspired and structurally very good, but it's also practically feasible without additional transversal bracing, because the strut is continuous.

Since the upper members are imposed inclined for the water drainage, at the midspan their compression has an upward resultant. Due to this, it's very useful to have a vertical tie that carries this uplift force. This is another fundamental aspect that was not considered in the initial concept.

The element on the simmetry axis has also another advantage: it provides a stiff node on the lower chord to fasten the first diagonal. This inclined bar is therefore very stiff under uniform load and very effective in limiting the bending moment of the upper beam near the midspan.

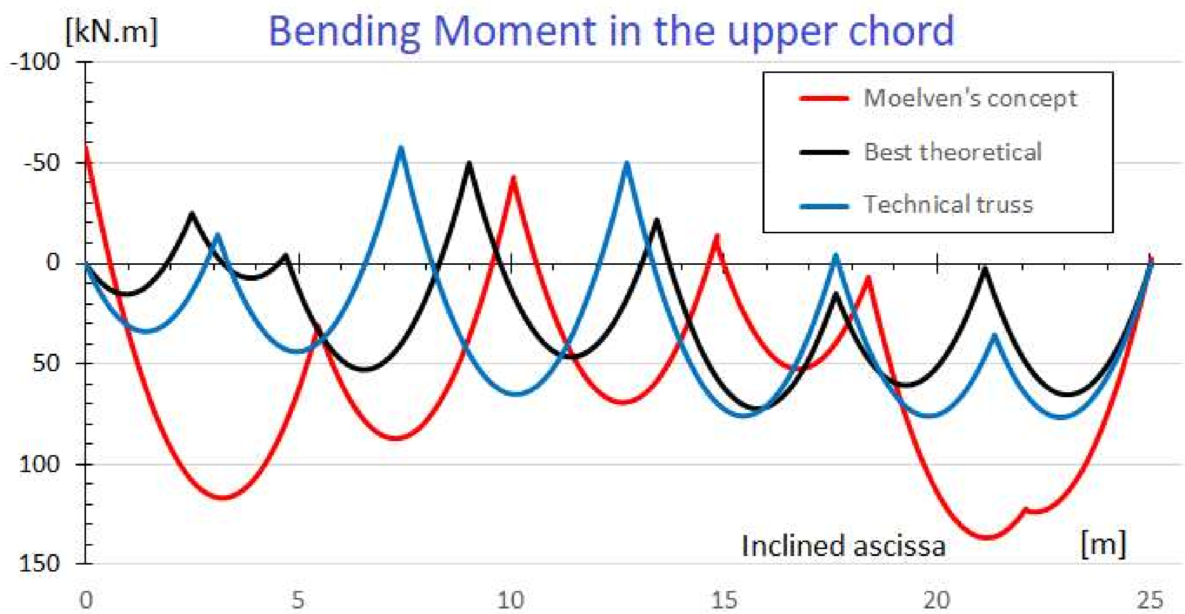


Figure 65: Bending moment in the upper chord for the main configurations, ULS uniform load.

Inside the general truss configuration, it has been experimented the possible advantage of having secondary non truss diagonals that help to reduce the flexural behavior of the upper chord, by shortening the spacing between its nodes.
The choice seems reasonable when the structure is tall and the allowed height is equal to h = 6,25 m. Conversely, when the imposed depth is reduced to h = 4,50 m, the secondary diagonals become ineffective because the cost of their addition overcomes their benefit.

Essentially, having h = 4,5 m, the distance between the principal nodes is enough to limit the flexural behavior of the upper beam and no additional rods are necessary. The good spacing with a pure truss system has been shown to be 4 - 4,5 m.

The consequence on the upper member of all the recalled facts is clear: the bending moment arising reduces from 130 kN.m to less than 80 kN.m (see Table 16); the proposed technical truss approaches a lot to the theoretical one.
It must be noticed that the number of nodes on the compressed chord has not been increased, there are 6 sub-spans both for the Moelven's proposal and for technical solution (7 segments in the academical scheme). It's clear that, with the same average spacing, the flexural behavior has improved a lot, thanks to the proper usage of the diagonals and of the whole structural system.

All the positive aspects just presented are also nicely synthesized in Fig. 65: the dark curves are much more regular than the red one, which is penalized by the bad mechanical behavior.

		Moelven's concept		Best theoretical		Technical truss	
Midspan Deflection		159	mm	176	mm	173	mm
Bending Mom. Upper Chord	Min	-58	kN.m	-50	kN.m	-57	kN.m
	Max	137	kN.m	72	kN.m	76	kN.m

Table 16: General comparison between the main configurations analyzed, ULS uniform load.

Regarding the stiffness, Table 16 reports the midspan settlement of the different structures studied. The two trusses are slightly more flexible than the concept of the company and this seems strange since the optimal arrangements should be also the stiffest ones.
The design has always been done for strength, therefore the aim has been to reduce the amount of material satisfying the safety. The latter two configurations are indeed lighter than the one of the firm. If their chords are thickened a little it can be demonstrated that they become also stiffer, keeping a lower volume of glulam.
It can be surely said that the proposed technical layout is better also considering the stiffness.

An interesting aspect that has been positively experimented has been the varying thickness of the two chords. It has been very important to improve the performances of the solution developed.

The upper member has been designed with a constant base and a stepwise height, obtained with a different number of lamellas for each sub-span. In this way, it is possible to follow the increasing of the compression along the length.
Note that the first segment of the upper beam has a different cross section, much smaller and slender since the axial force is trifling and the element is working for the local bending.

The flexural behavior of the upper chord arises earlier if the reduced depth of h = 4,50 m is imposed. This is due to the fact that, due to the increased compression, the member becomes thicker, therefore it's also stiffer.
In order to avoid bad global flexural performances, the height of the upper beam can't grow much further than 0,5 m, otherwise the high bending moment arising makes vain the slenderness of the section. The base results to be quite large and the ratio h/b is smaller than 2.
Summing up, the upper chord can't be very optimal in bending due to the global flexural behavior.

The lower member has also been designed with a different number of lamellas on each segment, in order to follow the entity of the applied force. Unlike the upper one, the height is constant and the varying dimension is the base, which grows up to becoming very large at the midspan.
Since the total depth of the structure is fixed, having a limited thickness for the tension member allows to increase the global arm of the truss, i.e. the distance between the centers of the two chords. Simultaneously, the weak axis of inertia for the lower elements prevents them from taking too much bending moment due to the global deflection of the beam.

All those good aspects regarding the two chords (the varying thickness, the limitation of the flexural behavior on both the members, the enmity to the longitudinal connections) are fundamental since these are the thickest elements of the structure.
Even if the axial force in the two chords is near to be statically determined, these technical measures have been able to save a lot of construction material. The data of Table 17 are very eloquent.

Moelven's concept		
Upper beam	11,28 m^3	34,9 %
Diagonals	3,40 m^3	10,5 %
Lower chord	17,67 m^3	54,6 %
Total	32,3 m^3	100 %

Best theoretical configuration		
Upper beam	8,10 m^3	34,3 %
Diagonals	5,06 m^3	21,4 %
Lower chord	10,48 m^3	44,3 %
Total	23,6 m^3	100 %

Technical proposed truss		
Upper beam	8,92 m^3	35,8 %
Diagonals	3,91 m^3	15,7 %
Lower chord	12,06 m^3	48,5 %
Total	24,9 m^3	100 %

Table 17: Volume of glulam required for the main configurations analyzed.

4.3.3: <u>Final results and future developments</u>

The amount of material needed for the different configurations is the arrival point of the thesis and the most interesting aspect of the analyses.

The result obtained is very rewarding: even if the technical configuration has abandoned the academical branches of the principal directions of stress, the arrangement is still good enough to limit the worsening to the 5%; this value is lovely for the engineering design.

The concept of the company has been improved of the 23%, that is more than 7 m^3 of structural glulam and it's remarkable in the real manufacturing.
The amount of saved timber is finally known; it's the main ingredient needed for a practical comparison with the increased production difficulties.

This thesis ends delivering a feasible optimal solution, the strategies that make it possible and the index of judgement.
It will be the task of Moelven to decide about the modality of application of these technical measures.

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