Formulation of Design Space Exploration Problems by Composable Design Space Identification

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Abstract—Design space exploration (DSE) is a key activity in embedded system design methodologies and can be supported by well-defined models of computation (MoCs) and predictable platform architectures. The original design model, covering the application models, platform models and design constraints needs to be converted into a form analyzable by computer-aided decision procedures such as mathematical programming or genetic algorithms. This conversion is the process of design space identification (DSI), which becomes very challenging if the design domain comprises several MoCs and platforms. For a systematic solution to this problem, separation of concerns between the design domain and decision domain is of key importance. We propose in this paper a systematic DSI scheme that is (a) composable, as it enables the stepwise and simultaneous extension of both design and decision domain, and (b) tuneable, because it also enables different DSE solving techniques given the same design model. We exemplify this DSI scheme by an illustrative example that demonstrates the mechanisms for composition and tuning. Additionally, we show how different compositions can lead to the same decision model as an important property of this DSI scheme.

I. Introduction

The design space exploration (DSE) activity [1] is an integral element of embedded system design methodologies, such as HepSYCode [2], SDF3 [3], Daedalus [4], and AutoFOCUS3 [5]. Its main purpose in such methodologies is to aid the designer in refining a specification into a more concrete form through exploration of viable refinement alternatives. For instance, DSE in said examples is used to find software-to-core mappings, execution and communication schedules for the target hardware platform, memory allocation etc.

In these methodologies, the designer specifies the embedded system with formal models that define applications, platforms and extra-functional properties. We refer to these models as design models. To enable a precise definition and clean semantics of the design models, we advocate the usage of models of computation (MoCs) [6] for the applications, models of architecture (MoAs) [7] for the platforms, and object constraint language (OCL) [8] for extra-functional constraints that need to be satisfied. Additionally, there are objectives such as minimization of power consumption. DSE requires transforming these models into other representations; such as genome encoding for genetic algorithms (GAs), constraint programs for constraint programming (CP), or linear mathematical formulas for mixed integer linear programming (MILP). We refer to such DSE-ready models as decision models, and to the set of all design and decision models as design and decision domains, respectively. Finally, we refer to the activity of converting a design model to a decision model as design space identification (DSI).

Fig. 1 illustrates the role of DSI within the design process as described.

A tuneable but systematic DSI is key to achieve separation-of-concerns between decision domain and design domain, as DSI is the link between system modeling and automatic decision making. An appropriate DSI scheme should be:

- *composable*, in the sense that extensions to the design domain can be stepwise converted to the decision domain as effortlessly as possible;
- *tuneable*, in the sense that the same design model can lead to different decision models, e.g. a MILP and a GA encoding for the same design model.

Our contribution in this paper is a DSI scheme fulfilling these two properties, with the requirement that any design model can be translated to a graph, which for practical design methodologies does not imply any limitations.
II. IDENTIFICATION SCHEME

This DSI scheme is built around the concept of *composable rules*. These rules take a design model and a list of decision models in order to produce a new decision model. In this way, we can compose rules stepwise by appending the result of one rule to the input list of others repetitively.

The rules produce decision models stepwise by converting types of elements in a design model, for example, by converting all synchronous dataflow (SDF) actors in a design model. Therefore, given a design domain and a ruleset with rules for every type of element, any design model in this design domain can be converted to a decision model. If later we directly extend the design domain to accommodate new design models, e.g. an application model described by a MoC not covered yet, then we define new rules for the new type that is built over the existing ruleset, as illustrated in Fig. 2, indirectly extending the decision domain. The definition of new rules is always backwards-compatible: no previously converted decision model is lost. Therefore, this extension process can be repeated indefinitely.

We assume that all design models are given as a tuple \( M = (G, E_D, J, C) \) as follows. The graph \( G = (V, E) \) has nodes \( V \) and edges \( E \) representing elements from MoCs and MoAs including hardware and software elements such as schedulers. Additionally, \( E \) also contains *decision edges* such as “partitions” or “mappings” between nodes. The set \( E_D \) of edges represents all the possible decision edges that can further be added between nodes in \( V \) as a result of DSE. The design objectives \( J \) and the constraints \( C \) are given in a language like OCL, using references to \( G \).

Given a design model \( M \), we must find decision edges in \( E_D \) to be added in \( G \) satisfying the constraints \( C \) and fulfilling the objectives \( J \). Accordingly, we must find a decision model \( X \) where a DSE solving technique returns a solution that is equivalent to finding the decision edges in \( E_D \) satisfying constraints \( C \) and fulfilling objectives \( J \). Thus, a decision model \( X \) identifies \( M \) if there is a function \( dse \) that returns \( G' = dse(X) \), where the edges of \( G' \) are the same of \( G \) plus any subset of \( E_D \). In other words, all possible decisions can be converted to \( M \) from the solutions of \( X \) via \( dse \). Likewise, a decision model \( X \) partially identifies \( M \) if any solution via \( dse \) contains a strict subset of \( E_D \), that is, only a subset of the decisions are captured by \( X \).

Our proposed DSI scheme has three steps. First, we define a ruleset \( R = \{r_1, \ldots, r_n\} \), where each rule \( r_i \) takes a design model \( M \), a set of decision models \( X = \{X_1, \ldots\} \), and outputs a decision model \( X = r(M, X) \). Second, we define the mechanism that uses \( R \) to produce, stepwise, composed decision models out of \( M \):

\[
X_{i+1} = \begin{cases} \{r(M, X_i) \mid r \in R\} \cup X_i, & i \geq 0 \\ \{X_0\} \end{cases}
\]

where \( X_0 \) is a baseline “empty” decision model. The recursion stops at the fixpoint \( \hat{X} \), when \( \hat{X} = X_{i+1} = X_i \). This mechanism relies on the fact that we formally require every rule \( r \in R \) to also take a set of decision models, so that \( r \) can partially identify more elements of \( M \) by taking previously partially identified \( X \in \hat{X} \). Third, we choose one decision model from \( \hat{X} \) that identifies \( M \), as there may be several \( X \) that identify \( M \), and perform a \( dse \) technique on it.

In summary, the scheme consists in a) choosing an initial design domain and defining rules \( R \) that compositionally identify any design model \( M \) in it by partial identification, then b) applying the mechanism of Equation (1) to construct these decision models \( \hat{X} \), and lastly c) pick a decision model \( X \in \hat{X} \) that identifies \( M \) and feed it to a \( dse \) technique, formalized as a function. Extending the supported design domain involves adding new rules to \( R \) as done for the initial design domain.

Two remarks should be made for this rule-based DSI scheme. First, as all decision models are always kept in Equation (1), the order of evaluation for the ruleset \( R \) is irrelevant as long as \( X_i \) is computed before \( X_{i+1} \). Second, Equation (1) explains how the scheme is backwards-compatible: if a ruleset \( R \) can construct one \( X \) that identifies \( M \), then additional rules in \( R \) cannot remove \( X \) from the existing decision models.

III. ILLUSTRATIVE EXAMPLE

The design model is based on the use case found in [9]. It consists of a Sobel filter modeled as a SDF application, a predictable time-division multiplexing (TDM) bus-based multi-processor system-on-chip (MPSoC) model, where each tile consists of a processor, memory and a network interface, and abstract cyclic order elements. The order elements describe an arbitrary number of slots, each of arbitrary size. They can represent non-preemptive cyclic execution of programs on a processor, or time slices in a TDM bus that are processed in cyclic order. They also indicate that the Sobel SDF actor firing and communication must be mapped and scheduled order-based in the MPSoC platform. The objective is to optimize the throughput of the Sobel filter. A visual representation of this design model can be seen in Figs. 3 to 5.

A. Ruleset definition and identification mechanism

There are mainly three different types of elements in the design model, i.e. SDF graph, order elements, and platform. We define a ruleset \( R_{aop} = \{r_{a}, r_{ao}, r_{aop}\} \), discussed in order of rule composition, and shown in Fig. 3.
First, \( r_{ao} \) takes the empty decision model \( X_0 \), and partially identifies the Sobel SDF actors and channels to give \( X_a \), following the semantics of SDF [10]. The new decision variable \( x_t \) is the SDF firing vector at index \( t \), \( b_t \) is the token vector at \( t \), \( \mathbf{1} \) the topology matrix and \( q \) is the repetition vector.

Next, \( r_{ao} \) takes \( X_a \) and partially identifies the ordering elements to give \( X_{ao} \), introducing the semantics of static cyclic execution for the SDF actors. A new decision variable \( y_{t,i,j} \) is introduced for communication that represents the number of tokens sent from tile \( i \) to \( j \) at \( t \), and \( \delta_t \) is an indicator function that is zero if \( t < 0 \) and \( 1 \) otherwise. The new \( i \) index for \( x \) represents each schedule slot \( i \) where an actor can fire.

Afterwards, \( r_{ao} \) takes \( X_{ao} \) and partially identifies the MPSoC to give \( X_{aop} \), introducing execution and communication times as the platform provides these details. Since \( X_{aop} \) has the Sobel SDF semantics and platform information, it is an identification. The new index \( l \) for \( y \) describes which slot \( l \) of the TDM bus is being used at \( t \), and \( M \) is a matrix so that if TDM slot \( l \) is used to communicate from \( i \) to \( j \) at \( t \), then it must be used by \( i \) to \( j \) on all \( t' > t \). The variable \( \tau_{t,i} \) is the starting time at \( t \) for tile \( i \). The constants \( W_i \) is the worst case execution time (WCET) for every actor in tile \( i \), \( C_{ij} \) is the worst case communication time (WCCT) for a token from tile \( i \) to \( j \). The function \( \tau_{t,i,j} \) gives the throughput for tile \( i \) based on \( x, y, \tau, WCET \) and WCCT. In this example, we assume that WCET and WCCT are given as design constraints, and that \( \tau_{t,i,j} \) has a linear representation.

Last, we could evaluate \( milp(X_{aop}) \) or \( cp(X_{aop}) \) (a MILP or a CP solver) to obtain a solution containing order, slot and mapping edges missing in the input design model, as exemplified in Fig. 4. In it, Orders 1 to 4 are used to schedule the SDF actors, and are then mapped to individual tiles. Order 5 is used to schedule the communication and is mapped to individual slots of the TDM bus.

B. Equivalent ruleset definitions

Alternatively, we define a ruleset \( R_{poa} = \{ r_p, r_{po}, r_{poa} \} \) that starts the scheme from the MPSoC to identify \( X_{aop} \), as seen in Fig. 5. The decision model that partially identifies the platform \( X_p \) has no \( dse \) function, and serves as a stepping stone for \( X_{po} \). The decision model \( X_{po} \) partially identifies the order elements and platform. It has abstract execution and communication slots, \( u \) and \( v \), which means that a process is executed or communicated non-preemptively in order, akin to \( x \) and \( y \) in \( X_{aop} \). The semantics of the SDF Sobel is introduced as \( r_{poa} \) takes \( X_{po} \) and partially identifies Sobel to give \( X_{aop} \).

Note that both \( R_{aop} \) and \( R_{poa} \) could be unified, leading to the same decision model \( X_{aop} \). If \( R_{both} = R_{aop} \cup R_{poa} = \{ r_a, r_{ao}, r_{poa}, r_p, r_{po}, r_{poa} \} \), then the final decision model set would be \( X = \{ X_0, X_a, X_{ao}, X_{aop}, X_p, X_{po} \} \) via \( R_{both} \).

C. Definition of new solving functions for tuning

The decision models identified until now are given in a mathematical programming format, amenable to MILP or CP techniques, which may require significant computation time to yield a solution. Without adding a new rule to \( R_{both} \) to transform \( X_{aop} \), we can also define other solution techniques over \( X_{aop} \). For example, let us define a \( greedy(X_{aop}) \) function that is a list-scheduling algorithm, which greedily allocates SDF actors in any available tile (greedily choosing \( x_{t,i} \), and...
allocates TDM slots as necessary whenever communication must occur (greedily choosing \(r_{t,i,j,l}\)). This would give the designer a choice between an exact solution and a fast solution for this input design model, or any other which is part of the design domain covered by \(R_{both}\).

If instead a GA-based solution is desired, we have to define \(R_{ga} = R_{both} \cup \{r_{topa}\}\), with the rule \(r_{topa}\) that takes \(X_{aop}\) and gives \(X_{ga}\) in genome-encoded format. Then \(ga(X_{ga})\) is evaluated via GA algorithms.

**IV. Related Work**

In [11], the authors surveyed a body of DSE methods and identified the need for DSI to increase flexibility and reusability of embedded systems design flows. Based on this survey, they proposed a DSI meta-framework with base models that can be extended to specific design domains. These extensions can be translated to new DSE instances. Likewise, in [12], the authors proposed a meta-model that cover distinct embedded system design instances. If extensions to these models satisfy certain meta-model interfaces, the framework generates CPs instances of the DSE problem based on clock constraints.

Two alternatives to a systematic DSI are to narrow the design domain, for instance, only SDF graphs on MPSoCs, as in SDF3 [3], or, to use simulation-based methods such as in COMPLEX [13]. In the latter, the possible design model decisions can be explored directly via meta-heuristics that use simulators and/or performance estimators to test every solution candidate. These methods do not compete with the scheme proposed here, but can be described in it by choosing a descriptive ruleset. For example, in [12], for every type of application and platform element, we can define rules that partially identify the elements to clock constraints and variables in CP format, which another rule later uses to produce the overall CP formulation.

**V. Conclusion**

We presented a systematic design space identification (DSI) scheme that is composable and tunable for efficient design space exploration (DSE). The DSI scheme is rule-based and enables the systematic extension of both design and decision domains in tandem, can be tuned to employ different DSE solving techniques, allows arriving at the same decision model by different rule compositions, and is backwards-compatible with respect to rule addition. We demonstrated these properties and the potential of this novel DSI scheme using an illustrative example, where the mechanism of the scheme and different rules were stepwise applied. While this paper provided the basic ideas and mechanisms for the DSI scheme, future work aims to demonstrate how the scheme can cope with heterogeneous design models in form of different underlying models of computation (MoCs) and complex platforms, and to integrate it with existing DSE methods.

**References**


