Weekly planning of hydropower in systems with large volumes of varying power generation

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Abstract

Hydropower is the world’s largest source of renewable electricity generation. Hydropower plants with reservoirs provide flexibility to the power systems. Efficient planning techniques improve the flexibility of the power systems and reduce carbon emissions, which is needed in power systems experiencing a rapid change in balance between power production and consumption. This is due to increasing amount of renewable energy sources, such as wind and solar power. Hydropower plants have low operating costs and are used as base power. This thesis focuses on weekly planning of hydropower in systems with large volumes and varying power generation and a literature review and a maintenance scheduling method are presented.

The topic of hydropower planning is well investigated and various research questions have been studied under many years in different countries. Some of the works are summarized and discussed in literature reviews, which are presented in this thesis. First, some reviews are presented, which covers several aspects of hydropower planning. Literature reviews for long term, mid term and short term planning, respectively, are described.

Maintenance scheduling in power systems consists of preventive and corrective maintenance. Preventive maintenance is performed at predetermined intervals according to a prescribed criteria. This type of maintenance is important for power producers to avoid loss in electricity production and loss in income. The maintenance scheduling for hydropower plants prevent these phenomena since spill in the reservoirs and wear on the turbines can be avoided. Usually, the maintenance in hydropower plants is performed on the turbines or at the reservoir intake. A deterministic and a stochastic method to solve a mid term maintenance scheduling problem formulated as a Mixed Integer Linear Programming using dynamic programming is presented. The deterministic method works well in terms of computational time and accuracy. The
stochastic method compared to the deterministic method yields a slightly better result at the cost of a need for larger computational resources.

**Keywords**

Hydropower, Literature review, Weekly planning, Low carbon operation, Maintenance scheduling, Preventive maintenance, Mixed Integer Linear Programming, Dynamic programming, Cascaded river systems, value of stored water.
Sammanfattning


Ämnet vattenkraftplanering är väl undersökt och varierande forskningsfrågor har studerats under många år i olika länder. En del av arbetena sammanfattas och diskuteras i litteraturstudier, vilka presenteras i den här avhandlingen. Först presenteras några litteraturstudier, som täcker flera aspekter av vattenkraftplanering. Litteraturstudier, för långtids-, medeltidsplanering, respektive korttidsplanering beskrivs.

deterministiska metoden ger ett något bättre resultat dock till priset av ett behov av större datorresurser.

**Nyckelord**

Vattenkraft, Litteraturstudie, Veckoplanering, Låg koldioxid drift, Underhållsplanering,
Förebyggande underhåll, Blandad Heltalsprogrammering, Dynamisk programmering,
Kaskadälvsystem, vattenvärden.
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Charlotta Ahlfors
Acronyms

DP Dynamic programming
MILP Mixed Integer Linear Programming
SDP Stochastic dynamic programming
SDDiP Stochastic dual dynamic integer programming
SDDP Stochastic dual dynamic programming
UC Unit Commitment
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Chapter 1
Introduction

1.1 Background

Sustainability and environment is a global concern. The global established sustainability goals [1] aim to ensure access to affordable, reliable, sustainable and modern energy for all (goal number 7) and to take urgent action to combat climate change and its impacts (goal number 13).

In 2018 the amount of total world energy production originating from fossil fuel energy sources (oil, coal and gas) was 81% and only around 14% was from renewable energy sources [2]. To reach the goal of the Paris agreement of at most 2 degrees increase in global average temperature from pre-industrial levels the CO$_2$ emissions need to be reduced to net-zero emissions by the middle of the 21st century [3]. Many countries have also introduced legislation to achieve net-zero emissions. This puts a strong incentive to increase the amount of renewable energy sources.

Among the renewable energy sources, hydropower is responsible for around half of the power production [2]. Hydropower production uses the potential and kinetic energy of water to produce power using reservoirs and hydropower turbines. Besides being renewable, hydropower is also a flexible energy source. Other renewable energy sources are intermittent (mainly solar and wind), which means that their production fluctuates over short time intervals due to changing weather conditions. To compensate for the power fluctuations of these other power sources and increasing fluctuations in consumption the hydropower production can fulfil the important role to stabilise the power system, as there must always be a balance between power...
CHAPTER 1. INTRODUCTION

production and consumption. Therefore, hydropower plays an important role in the power system and will so even more in the future since renewable energy sources increase and other non-renewable flexible power sources are phased out.

However, there are some drawbacks with hydropower that must be kept in mind related to sustainability. A hydropower unit has a large local environmental effect disrupting the fauna and wildlife surrounding the unit and displacing nearby residents in towns and villages. There is also an effect on the whole ecosystem of the river where the unit is located as it changes water ways and increases the risks of drying up parts of the river during the dry season. The hydropower unit blocks fishes from swimming past the reservoir and diminishes the population of important species. On a global scale most of the suitable river systems already have hydropower installed and this leaves small room for increasing the hydropower production in the future. This means that hydropower must be utilised in an efficient manner and create sustainable planning methods [4].

An important part when running a hydropower unit is the planning of the power production. In this planning different time horizons must be considered. In the long-term planning, stochastic seasonal variations in precipitation need to be considered as well as even variations over longer time periods of multiple years (e.g., changes due to climate change). These plans concentrates on the utilization of the largest reservoirs which are usually located at the top of the river system. In the short term planning, with a time horizon of just a few days, more immediate factors influence the discharge plans, such as the current electricity price. Somewhere between these perspectives weekly planning of hydropower is relevant.

The planning period of a few weeks serves as an important bridge between the long term planning and short term planning. Weekly planning can use more detailed information about reservoir levels, electricity prices, power consumption and maintenance of turbines not considered in the long term planning. In contrast to the short term planning, weekly planning spans over a longer time period and it is therefore possible to account for changes within the planning that have a duration of a few days, which is not possible in the short term planning. Moreover, since the forecasts for the relevant information is more uncertain in 1-3 weeks horizon than in the next 24 hours it is important to consider stochastic behaviour in the weekly planning.
1.2 Research question

In weekly planning it is of significance to retain a high level of details in the problem formulation. This enables the study of various important questions, such as the influence of reservoir sizes and delay time of water running between reservoirs, on the ability of hydropower to function as a regulating energy source in the presence of a large amount of renewable energy sources. If stochastic behaviors are modelled, such as reservoir inflow, power production from other renewable energy sources (wind and solar) and electricity prices, this bring further significance to the model. However, the challenge with such detailed models that span over time periods of weeks is that the computational burden to solve the planning problem increases dramatically compared to solving the same problem for a short time period of a few hours to days. Therefore various approximations need to be included in the model.

The research question in this thesis is to study how detailed models can be used for weekly planning of hydropower. Especially, modeling turbines and keeping a high time resolution in the model, are parts of the research question.

1.3 Objectives

Earlier research has paid attention to long term planning and short term planning of hydropower. This project aims at developing efficient methods to plan the coordination between wind power and hydropower for weekly planning of hydropower. Weekly planning is needed when planning should be performed for one or two weeks ahead in order for the power producer to make an early plan before bidding to the day ahead market or to make specific maintenance plans. This type of planning makes it possible to reduce loss in income, avoid failure in turbines and unplanned disruptions. Stochastic behaviors can also be considered, such as inflow to the reservoirs and electricity prices. The following specific research objectives have been the foundation for the work:

- Develop efficient planning methods for power systems with a large share of renewable power sources, such as wind power. The first model is a deterministic maintenance scheduling model and the second is a stochastic maintenance scheduling model. This is due to the fact that this has been identified as a research gap and is relevant for the power producers. Relevant time resolution to be used
for these models should be considered. The models should be tested on two case studies, where two subsets of a cascaded river are used as test systems.

- Study the connection between short term, mid term and long term planning of hydropower.

## 1.4 Contribution

This section gives an account of the contributions in this thesis.

- The first contribution is a method based on dynamic programming that is used to solve detailed hydropower models including several hydropower stations, several turbines and detailed time resolution.

- The second contribution is to show that the method can be used to assess the costs of turbine maintenance.

- The third contribution is an application of the dynamic programming method to stochastic planning problems.

The contributions are presented in one conference paper to Power Tech 2021 and one Journal paper.

- Weekly planning of hydropower in systems with large volumes and varying power generation: A literature review [5],

- Preventive maintenance scheduling of hydropower plants using dynamic programming [To be published].

## 1.5 Overview of the thesis

The thesis consists of six chapters. The main chapters are the literature review, preventive maintenance scheduling and the stochastic model, which include scientific contributions. A summary of each chapter is found below:

- **Chapter 1**: Provides an introductory to the thesis, explains the research question, objectives, contribution and overview of the thesis.

- **Chapter 2**: Explains hydropower planning and related topics; different methods, mathematical optimization, the nordic electricity market, hydropower
production and renewable electricity production.

- **Chapter 3**: Presents a literature review that has been carried out including previous reviews on hydropower planning in different time perspectives and maintenance scheduling.

- **Chapter 4**: Describes preventive maintenance scheduling and contribution 1 and 2.

- **Chapter 5**: Describes the stochastic model and contribution 3.

- **Chapter 6**: Gives a summary of the content and suggestions for future work.
Chapter 2

Hydropower planning

2.1 Planning methods for power systems

Planning is aimed to utilize resources in the power system as efficiently as possible. It requires considering both technical and economical aspects. In the shortest time frame, seconds and minutes, the technical aspects are most important. It is important to keep a stable frequency and keep the balance between production and consumption at all times. In a time frame of hours or days, maintaining the function of the system is important but it also becomes important to consider economical decisions. When the time perspective is weeks or months, economical decisions play a major role. The available energy that can be utilized must be planned for the coming period. For hydropower plants this means that the producer must plan on how to utilize the water that is stored in the reservoirs. But also maintenance planning is important for this time frame. In the longest time frame, for months and years, investment decisions are crucial. Furthermore, in the case of hydropower plants, decisions are also taken on how to utilize the water that is stored in the largest reservoirs for the coming year or months [6].

For hydropower planning, specifically, the planning is divided in plans for the coming years or seasons (long term planning), months (mid term planning) and plans for short time frame of the next few days (short term planning). This can be seen in figure 2.1.1.
2.2 Mathematical optimization

The planning problems studied in this thesis are formulated as mathematical optimization problems and what follows in this section is a short description of the theory of optimization problems and a few examples of the most common types of optimization problems.

A general formulation of an optimization problem can be written in just two lines

\[
\begin{align*}
\min & \quad f(x), \\
\text{s.t.} & \quad x \in U.
\end{align*}
\]

The variables of the problem are collected in a vector \( x \in \mathbb{R}^n \) and under the constraint that \( x \in U \) the objective is to find the minimal value of the function \( f(x) \) [7].

Probably the most common type of optimization problem are linear programming problems (LP) and they can be formulated as

\[
\begin{align*}
\min & \quad c^T x \\
\text{s.t.} & \quad Ax \geq b, \\
& \quad x \geq 0,
\end{align*}
\]

where \( A \) is an \( m \times n \) matrix, \( b \in \mathbb{R}^m \) and \( c, x \in \mathbb{R}^n \). In these types of problems the objective function \( f(x) = c^T x \) is a linear function and the set \( U \) is defined in terms of linear inequalities. The mathematical theory for these types of problems is rich and an extensive set of methods have been developed to solve them. Two prominent methods are the Simplex Method and various interior point methods [7]. If some of the variables are assumed to be integers then we have a mixed integer linear programming problem (MILP). Some of the problems in this these are formulated as MILP problems.

For every linear programming problem there is a corresponding problem called the
dual problem. For every variable in the original problem, denoted the primal problem, there is a constraint in the dual problem and for every constraint in the primal problem there is a variable in the dual. The dual problem has the form

\[
\begin{align*}
\text{max } & b^T y \\
\text{s.t. } & A^T y \leq c, \\
& y \geq 0,
\end{align*}
\]

where \( A^T \) is the transpose of \( A \) and \( y \in \mathbb{R}^m \). An interpretation of the values of the dual variables \( y \) is that the component \( y_i \) measures how the optimal value of the primal problem changes if constraint \( i \) is relaxed. The dual variables are therefore often called shadow prices. This formulation is not directly used in the thesis but is important in the methods of the solver that has been used.

A useful and interesting class of optimization problems is stochastic optimizations problems. In these problems a subset of the variables are stochastic and the objective function is to optimize the expected value of an expression. In the linear case such a problem takes the form

\[
\begin{align*}
\text{min } & c^T x + \mathbb{E} (d_s^T y_s) \\
\text{s.t. } & Ax \geq b, \\
& T_s x + W y_s = h_s \\
& x, y_s \geq 0,
\end{align*}
\]

where \( s \) is a stochastic variable, and \( d_s, T_s \) and \( h_s \) are stochastic functions dependent on \( s \) and \( y_s \) is a variable in the problem that can depend on \( s \). This type of problem is used in this thesis when the planning problem is studied with multiple price scenarios. However, due to the scenario based formulation these problems tends to be large and needs solution algorithms such as dynamic programming to be solved, which is also used in this thesis to handle this problem. Another option is to solve these type of problems by sampling the outcomes and apply Monte Carlo simulation [8].

One way to handle certain large linear optimization problems is by applying dynamic programming. For example linear programming problems including integer variables, so called Mixed Integer Linear Programming problems, can in favourable situations be solved by dynamic programming. A description of dynamic programming is given
in chapter 4. Dynamic programming can also be applied to stochastic optimization problems and is then called stochastic dynamic programming.

2.3 The Nordic electricity market

Usually, in everyday language, the market that is referred to as the electricity market is the day-ahead market. In this market the power producers trade power for a number of trading periods in advance. The Nordic electricity market Elspot, is a spot market and day-ahead market where hourly contracts for the coming 24 hours are traded. Bids for the next day can be submitted until 12:00 the day before and at 15:00 prices and quantities for the different hours in the next 24 hours are released. The market price is computed as a price cross between demand and supply curves for the different hours [9].

When the day ahead market has closed, in the hours before the power should be delivered, adjustments to the trading plan may have to be performed. This trading is handled in a separate market. The Nordic intraday market is called Elbas and actors can place or accept bids for a specific hour after the release of prices on Elspot, until one hour before the delivery hour [9].

The system operator has the responsibility to keep power balance between production and consumption at all times and this is done at balancing markets. In Europe the nominal value for the frequency is 50 Hz and a portfolio of frequency reserves are used to keep this value. The Swedish system operator uses the reserves FFR, FCR-N, FCR-D up, FCR-D down, aFRR and mFRR [10]. The reserves are called support services and operate either automatically or manually and at different frequencies. This can be seen in table 2.3.1.

2.4 Hydropower production

Hydropower is a carbon emission free energy source that can be used to keep the balance in the power system when use of intermittent renewable energy sources, e.g. photo voltaic generation and wind power, is rapidly increasing. In Figure 2.4.1 a typical hydropower plant is shown. Power is generated using the difference in potential energy between two water surfaces. Water is released from the reservoir, through the
Table 2.3.1: Support services for the power system used by the Swedish system operator [10].

<table>
<thead>
<tr>
<th>Type</th>
<th>Activation</th>
<th>Activation time</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFR</td>
<td>Automatic at frequency deviations at low levels of inertia &amp; Automatic at 49.9-50.10 Hz</td>
<td>Three options for 100%: 0.7 s at 49.5 Hz, 1.0 s at 49.6 Hz and 1.3 s at 49.7 Hz</td>
</tr>
<tr>
<td>FCR-N</td>
<td>Automatic at 49.9-50.10 Hz</td>
<td>63% within 60 s and 100% within 3 min</td>
</tr>
<tr>
<td>FCR-D (up)</td>
<td>Automatic linear activation at 49.9-49.50 Hz</td>
<td>50% within 5 s and 100% within 30 s</td>
</tr>
<tr>
<td>FCR-D (down)</td>
<td>Automatic linear activation at 50.1-50.5 Hz</td>
<td>50% within 5 s and 100% within 30 s</td>
</tr>
<tr>
<td>aFRR</td>
<td>Automatic activation at deviations from 50.00 Hz</td>
<td>100% within 120 s</td>
</tr>
<tr>
<td>mFRR</td>
<td>Manually activated</td>
<td>100% within 15 min</td>
</tr>
</tbody>
</table>

intake into the penstock. After passing the penstock water passes a turbine, which is connected to a generator, to the outlet and the river. The amount of power that can be generated depends on discharge, head height and efficiency of the turbine. The relation between discharge and power production in a power plant is a non-linear, non-concave function. This causes problems when models are constructed, since a global maximum cannot be guaranteed when a solution is reached and other assumptions are required [8]. In section 4.1.7 this non-linear relation is modeled by a piecewise linear function.

2.4.1 The Nordic river systems

Sweden has several regulated rivers. Most of the rivers are controlled by Vattenregleringsföretagen, which has the responsibility to make sure that several hydropower owners can operate in the same river [12]. Vattenregleringsföretagen decides how much each hydropower owner can produce each our and keep track of the production to guarantee a fair system. Typically the Swedish rivers have low heads, i.e. less than 100m. The rivers are cascaded, which means that water that is released in the upper reservoirs can be used in hydropower plants downstream. The most commonly used turbines in Swedish hydropower plants are Kaplan and Francis. A map of the Swedish hydropower plants can be seen in figure 2.5.1. The Norwegian rivers have larger head, are less cascaded and Pelton turbines are commonly used.
2.5 Renewable electricity production

Renewable energy is continuously recycled from natural resources. The use of renewable energy leads to reduced carbon emission and less global ecological footprint. Examples of renewable energy sources are wind power, solar power and hydropower. Wind and solar power are intermittent power sources that cannot be used as regulating power since they are weather dependent and their stochastic behaviours create fluctuations in the power production. These fluctuations can be compensated by introducing energy storage in the power system. Another option is to combine wind and solar power with hydropower plants that can balance the fluctuations. This also means that the system operator has to account for larger frequency deviations when the amount of wind power and solar power is increased in the power system [8]. These are challenges that must be handled since the electricity demand is globally increasing and there is a limited possibility to construct new hydropower plants.

The scope in this thesis is the Nordic river system and the case studies in chapter 4 and 5 are performed in one single owned river.
Figure 2.5.1: Map of the Swedish hydropower plants and their sizes [13].
Chapter 3

Literature review

3.1 Previous reviews on hydropower planning

The topic of hydropower planning is well investigated and various research questions have been studied under many years in different countries. Some of the works are summarized and discussed in literature reviews, which are presented in this chapter. First, some reviews are presented, which cover several aspects of hydropower planning. Literature reviews for long term, mid term and short term planning, respectively, are then described. This chapter is based on the Power Tech 2021 conference paper Weekly planning of hydropower in systems with large volumes and varying power generation: A literature review.

In [14], a review of scheduling under uncertainty is presented. This study spans over many fields of science and the different techniques used to model uncertainties. Hydropower planning is one of the discussed topics. Some of these techniques are two-stage stochastic programming, parametric programming, fuzzy programming, chance constraint programming, robust optimization, conditional value-at-risk and other risk reducing techniques. In [15], a review of operation of hydropower plants is presented. It is concluded that the main parameters affecting the operation of a hydropower plant are head, discharge, turbine and the generators. In [16], a review on hydropower plant models and control is performed. It summarizes the research work from 2005 and earlier of hydropower plant model development and its controller design.

In [17], from 2020, a comprehensive review is performed which spans over different time frames of hydropower scheduling. The different time spans for planning are
defined as follows: short term 2-14 days, mid term 3-18 months and long term 1-5 years. An important conclusion and suggestion for future work is that scheduling of hydropower systems would be more valuable by considering other sustainable energy sources, such as wind and solar power, included in the optimization. Some models take this into account but it can be further developed.

A review of stochastic dual dynamic programming (SDDP) for long term hydropower planning is presented in [18]. It is concluded that the main strength of the method is to obtain reliable water values for all reservoirs and part of the computations can be done using parallel processing, due to the dual formulation of the optimization problem. Its weakness is the long computational times. In [19], a review is made of the application of the SDDP method applied to long term planning in the Nordic countries. For example, head variations and price modelling are discussed.

Several other methods for long term planning is reviewed in [20]. The methods are categorized in four main classes: Implicit stochastic optimization, explicit stochastic optimization, real-time control with forecasting and heuristic programming.

Recent hydropower optimization research and development using metaheuristic approaches are discussed in [21]. For example, physics based algorithms, gravitational search algorithms, and swarm based algorithms, such as particle swarm optimization, are reviewed and suitable for long term planning.

A review of short term planning in power systems with a large amount of wind power in the system is presented in [22]. The paper discusses deterministic model approaches as well as stochastic models. The conclusion in this paper is mainly that introducing several types of uncertainties lead to more realistic models but also models that are more difficult to study and analyse.

Earlier literature reviews have paid attention to the focus areas which have been mentioned. However, new research related to hydropower planning is of importance and weekly planning has not been in focus in earlier reviews. The scope of this chapter is to summarize the reviews until today within hydropower planning, present selected representative studies within the different time frames of hydropower planning and present papers focusing in particular on weekly planning.
3.2 Long term planning

The aim of the long term planning is to make plans for one to several years in advance, where many parameters are uncertain, such as inflow and electricity prices. There are many examples of models where the stochastic behaviour is taken into account. The output from the long term planning is often water values or a weekly discharge plan of power generation. The output is used in mid term or short term planning directly.

In [23], water value methods that dates back to the 1960’s are used. In water value methods aggregation of the reservoirs is done to prevent the curse of dimensionality and make the computations efficient. In this article only one reservoir is considered to overcome this problem. Stochastic dynamic programming (SDP) is used and compared to results from deterministic dynamic programming (DP). A conclusion is that the hydropower plants is operated more efficiently with DP rather than with SDP, since the DDP method results in higher reservoir levels and higher efficiency. However, more water spillage can be a consequence. The conclusions are drawn based on a case study in Brazil.

Another paper where the SDP method is used is [24] where the costs are reduced compared to when a standard Markovian model is used. Inflows are determined in a stochastic process as Markov-chains.

SDDP was introduced in [25], which allows for modeling of multi-reservoir systems without aggregation and leads to more realistic and detailed models.

One major factor affecting the outcome of the SDDP method is a correct statistical model of the inflow and this is studied in [26] and [27]. In [26] a new level of detail is presented with respect to the hydro reservoirs. The approach is tested as a large scale market model. Typical SDDP models tend to consider smaller systems. The case study is performed on the Norwegian power system. The method is not recommended to involve individual decision variables for each reservoir.

Recent focus of the research on SDDP is on convergence properties, strategic bidding, risk aversion, emission reduction and integration of wind power and pumped-storage as outlined in [28]. A hybrid SDP/SDDP aproach is used. The model is tested on a 7 reservoir system in Norway with corresponding power stations with a scheduling horizon of two years and weekly decision stages. Traditionally long/mid term models only consider the energy market but a conclusion of the work is that a market for a
reserve capacity can be introduced as an extension.

Another example of a significant paper for long term planning is [29], which compares two models for hydro-thermal scheduling. Both models takes into account interconnected power markets and using detailed hydro modelling. The reference model is the EMPS model and the other model is the SOVN-model. The SOVN-model uses formal optimization while the EMPS-model uses a heuristic approach. The EMPS model uses the objective to minimize the expected cost in the whole system subject to all constraints. The simulated system in the EMPS model can be the Nordic system or Northern Europe. The planning horizon is up to 25 years and the time step is one week.

In [30] a stochastic dynamic programming and successive two-stage optimization problem is solved. The proposed method is tested in the hydropower system of Yuanhe River in Hunan province China. The results show that the method has good performance to optimal scheduling. In [31] flexible robust optimization dispatch for hybrid wind/photovoltaic/hydro/thermal power system is used. The model and the algorithm performs well. In terms of reliability and economy the results of the optimization dispatch are good.

3.3 Mid term planning

Mid term planning models do not consider multi-year horizons but limit their attention to months. The models for mid term planning include more details and output such as water values.

In [32], a model for the weekly scheduling of a hydropower system is developed, where bidding to both the day-ahead market and the reserve market are included. The model is constructed to protect against the risk of shortage of water and shortage of storage. It is shown that hydropower producers who participate in both markets have a significantly higher expected income than those only taking part in the day-ahead market. This method has the strength to account for both markets but at the same time it requires a proper scenario tree representation of the stochastic variables.

The model in [33] adds forward contracts as an ingredient in the operation of a hydropower plant, as well as pumped storage. It uses multistage stochastic quadratic programming. The results are water values and optimal forward bids to support the
daily operation. A case study is performed, which shows that the forward trading contributes to a 20% increase in profit. Its main strength is that the model is both realistic and tractable. On the other hand, it assumes a linear dependence between the amount of commodities and market price and is also limited to a horizon of one year.

In [34], a mid term and long term electricity contract decomposition method is proposed that considers the cascade water quantity matching constraints. Mid term and long term power quantity contracts are decomposed by calculating the matching constraint of cascade water quantity. The method is tested in the Yunnan power spot market in China. The results show that the model is applicable for the Yunnan power spot market.

In [35], stochastic linear and non linear programming is used. The objective is to maximize the expected revenue of a producer, and the decision variables are generation and forward contracts in each period for each scenario. The parameters are limited by constraints on reservoir balance of water, upper and lower limits of power generation, bilateral contract and reservoir spillage.

In [36], a model is suggested of combined SDP and SDDP, where variables and functional relationships are linear. The main purpose is to fulfil the hydropower units operators’ demands to get steady operation for the power grid. A case study is performed in Lysebotn in Norway. The study has resulted in a quantitative valuation including sales of capacity. However, the results from the paper may be case specific.

An extended version of the SDDP method is used in [37], stochastic dual dynamic integer programming (SDDiP). The method is applied to mid term planning. A test is performed on how to incorporate stagewise-dependent stochastic variables on the right-hand side of the objective function. The method has proven convergence in the nonconvex case. A case study is performed for a Norwegian hydropower producer. The case study shows that it is possible to solve the mid-term hydropower scheduling problem but time consuming to solve it to the optimal value.

In [38], the SDDiP method is applied in another Norwegian case study. The paper highlights that the method can be used to include nonconvex environmental constraints in mid term and long term planning. A case study is performed on a multireservoir system, with three reservoirs and corresponding hydropower plants. A
particular nonconvex environmental constraint is modelled and a scheduling horizon of one year and weekly decision stages was applied.

3.4 Short term planning

The short term planning is performed for the nearest future and input parameters depend on the planning in the longer time horizons. The outcome of the planning is a production plan, usually with hourly resolution, of the amount of water that should be used in each power plant of the planning period and submitted to the day-ahead trading. Since the purpose is to find a detailed generation schedule, the models used for this planning include detailed information of the hydropower plants, turbines, and the topology in the river. Furthermore, the models include data for efficiency curves, discharge, inflow, spillage and reservoirs sizes.

In [39], a mixed integer stochastic model is developed that optimizes the bidding in a day-ahead market. The model treats the inflow and market price as stochastic. Through simulation, the results of the model are compared to results from a deterministic practice-based model, similar to what is used in the industry. The conclusion of the simulations is that the stochastic model performs better by 0.7%. This model has a number of positive features. For example, it is fast, can be used on a daily basis and is designed for the scheduling hierarchy used by producers today. The framework can be used to study other approaches to short term planning. The study is, however, limited to one system and needs further work to reach a reliable conclusion.

In [40], a stochastic short term planning model is proposed where day-ahead electricity prices and water inflows are modeled as scenarios based parameters on time series analysis. The model is constructed for a river with two cascaded hydropower plants. The schedule is determined for 7 days.

A non-linear dynamic programming model of a hydropower plant is presented in [41], where head and efficiency curves are treated as non-linear. The revenue of a producer that sells electricity in a pool-based electricity market is maximized. The plan is made for 1 day to 1 week with hourly resolution and the outcome is both the number of units in operation and the generation dispatch. A case study is made where the model is tested and an optimal weekly operation schedule has been obtained. The results of
the model have been compared to results from a nonlinear programming model in 15 different daily cases. An increase in revenue is found with the dynamic programming model, ranging from 0.9% to 3.3%.

In [42], the non-linearity effects of head and efficiency curves are included in a stochastic model, where inflow and market price are considered uncertain. The non-linearity is handled using successive linear programming. The model is tested in three fictitious case studies and performs well. The paper investigates which factors influence the result the most but reaches no general conclusion.

Non-linearity is studied in [43]. A deterministic non-linear mixed integer optimization model is used to solve short term planning problems. The optimization problem is solved using Lagrangian relaxation leading to separable dual subproblems that are solved by the bundle method. A small hydropower system is used as a case study. The conclusion is that this approach is feasible but needs to be further studied on larger scale.

In [44], a mathematical model is proposed to simulate day-ahead markets of large-scale multi energy systems with high penetration of renewable energy towards 2050. Unit commitment (UC) is added to the model and the results are analysed. Including unit commitment constraints with integer variables lead to a more realistic behavior of the units at the cost of increasing the computational time. Relaxing integer variables significantly reduces the computational time without compromising the accuracy of the results. Hydro reservoirs were not modelled with UC in order to reduce the complexity of the problem.

The novelty of [44] involves to develop a method for integration of renewable energy sources, planned maintenance scheduling, influence of unit commitment, multi-energy markets, full year hourly results, annual storage scheduling, several years of scenarios and a large scale system. The optimisations and simulations, except for variable renewable energy, are performed with the energy system model Balmorel (an open source, energy system tool, deterministic with bottom-up approach). The variable renewable energy hourly resolution time series data is simulated in CorRES and used as input to Balmorel. Data is aggregated into the regions that are used in Balmorel. A case study of the North Sea offshore Grid Denmark project was performed. Countries involved are Germany, Great Britain, Norway, Denmark, Belgium and Netherlands. The outcome of the tested scenario of electricity and heat show an
increase in \( CO_2 \)-price: 6.06, 76.70 and 130.38 €/ton in 2020, 2030 and 2050, respectively. The aggregated planned energy content along the year of hydro reservoirs with seasonal inflow of the Norwegian hydro storage reservoirs is investigated. Results from scenarios for 2020, 2030 and 2050 strengthen the importance of performing full year optimisation towards 2050. A conclusion from the study is that the case study is valid to other regions than the North Sea region, where high penetration of variable renewable energy is expected. The penetration of wind and solar is likely to challenge the need for balance in the system and the profitability of thermal units.

In [45], the integration of wind power into a power system with limits on transmission capacity is studied. Stochastic linear programming is used and includes trading on the spot and regulating market. It is concluded from the study that coordination of hydropower production with wind power is beneficial for the utility of both power sources, great reduction in wind power curtailment and a more efficient usage of the transmission lines. Some model limitations are mentioned, such that the outcome of stochastic variables are assumed to be known in planning for the next day. Future work is suggested, for example, that a longer case study needs to be performed and the impact of coordination of start-ups for hydropower plants needs to be investigated.

In [46], a short term model, formulated as a mixed integer linear programming (MILP) problem, is presented. Power output and the number of units committed at each hydropower plant and hour of the day or week ahead are modeled. The aim of the planning is to maximise the hydropower efficiency while reducing start up or shut down costs and produce sufficient power to cover the load in the system. The method is to relax the hydraulic constraints, simulate the solution and reintroduce constraints that are violated in an iterative way. This leads to a computational efficient method that can be used in a large hydropower system.

In [47], a non-linear model for hydropower planning and scheduling of maintenance is proposed. The thermal generation complement is minimized and the future value of stored water is maximized. The maintenance scheduling is modeled as a continuous variable. In [48], an algorithm is presented for the unit commitment problem that includes head and efficiency curves. The optimization problem is formulated as MILP problem. The optimization results in an accurate planning with reasonable computational time, since aggregation is used in the model.
A stochastic mixed integer model is developed in [49], that includes bidding to the day-ahead and balancing market and with start/stop decisions. Three implementations of the start/stop decisions (using none, real and binary variables) are tested. The conclusion is that there is only a small additional value in participating in the balancing market and there is a weak dependence of start/stop costs. The case study is performed on a small hydropower system in Norway and the power prices do not include the effect of a larger amount of intermittent power sources.

There are many other notable works on short term planning and a few are mentioned in this paragraph. The commitment problem in power systems with intermittent and fluctuating characteristics is studied in [50]. In [51], an overview is presented of formulations and optimization methods for unit-based short term hydro scheduling. A short term planning method is presented in [52] where Benders decomposition is used to solve a hydropower maintenance scheduling problem. Moreover in [53] a stochastic short-term hydropower optimization method which emphasizes inflow scenario tree is presented. Results show that there is a gain in using a stochastic model for the short-term hydropower optimization model with multiple scenarios compared to a median scenario since it offers a more robust solution.

### 3.5 Maintenance scheduling

Maintenance scheduling in power systems consists of preventive and corrective maintenance. Preventive maintenance is performed at predetermined intervals according to a prescribed criteria. This type of maintenance is important for power producers to prevent wear on turbines and to repair malfunctioning equipment, and thus to avoid loss in electricity production and loss in income. Usually the maintenance in hydropower plants is performed on the turbines or at the reservoir intake. The maintenance periods usually last for a few hours to weeks and in a few cases for months. During the maintenance a part of the power plant is out of service and in some cases the whole power plant is affected and offline.

Many studies have been carried out on maintenance scheduling in different areas of electrical applications. A literature review is done in [54], of maintenance scheduling in regulated and deregulated electricity markets, and MILP models are discussed as possible solution methods. For example, [55] is mentioned where outage planning for thermal power plants is studied. The model presented is a stochastic MILP model.
and the maintenance scheduling is based on hourly price based unit commitment. The random variables are modeled with scenarios and the problem is solved with the Monte Carlo method. The maintenance is fixed within a time window for each thermal unit and the duration of the maintenance is also specified. The objective function is the best cost effective solution of the maintenance, thus the maintenance is a decision variable. Furthermore, there are several constraints on the maintenance. A case study is performed with 29 units for one year, time steps of one hour and the maintenance duration for each unit is one day.

A model of maintenance for long term hydropower planning is presented in [56]. The objective function is quadratic, i.e. the model is nonlinear. The duration of each maintenance is fixed but the start time of the maintenance are decision variables. The model is deterministic and the time steps are days. The objective is to decide the most cost effective way to schedule the maintenance for each turbine in the power plants.

In [57], a linear stochastic model is developed for long term planning of 1 year for different types of power plants. Benders’ decomposition technique is used to solve the resulting model. The maintenance periods for the hydropower plants are variable. The objective of the planning is to minimize start-up costs of generators, production costs of generators and maintenance cost to put a generator into maintenance. The conclusion is that the method has interesting convergence characteristics. Several turbines in each power plant are not considered nor the hydrological balance is not considered.

In [58], two models are developed. The first model is deterministic and formulated as a MILP problem. The second model extends this approach to include the effect of uncertain water inflows by formulating a two stage stochastic linear problem. This problem is solved by using Benders decomposition in combination with acceleration techniques. The conclusion is that the second model is superior to the first model. The aim of the planning is to maximize the value of electricity production minus maintenance costs under a number of constraints. The planning period is 30 days with 18 maintenance tasks. Maintenance is a decision variable. A case study is performed on both models where four hydropower plants of Rio Tinto in Canada are used as test system. Turbines are not modeled separately.

The importance of integrating the maintenance scheduling in the detailed hydropower scheduling is highlighted in [59]. A method to solve these problems as one is developed
by the use of Benders decomposition. The maintenance scheduling problem is solved as a MILP problem in the first step providing a trial maintenance scheduling to be considered in the hydropower scheduling. The hydropower scheduling is evaluated using multi-stage Benders decomposition. Furthermore, the hydropower scheduling part of the problem is evaluated using stochastic dynamic programming and stochastic dual dynamic programming. Inflow to reservoirs, prices for energy and reserve capacity are treated as stochastic variables. Maintenance is a decision variable and the maintenance duration is weeks. Hydropower units are considered but turbines are not modeled separately. The method has been tested in a case study on a Norwegian watercourse with 7 hydropower reservoirs and associated power plants and the conclusion is that the model works well on the tested system.

### 3.5.1 Tables

Three tables summarizing the review papers are presented in appendix A; Table A.0.1, Table A.0.2 and Table A.0.3.

Figure 3.5.1 shows countries (indicated by grey dots) where case studies from papers in this chapter are performed. Several studies are performed in Europe, but also in Canada, Brazil, New Zealand and China.
CHAPTER 3. LITERATURE REVIEW

3.6 Summary

Different reviews have paid attention to, for example, review of reservoir based as well as run of river hydropower plants, hydropower plants model and control, different algorithms for different time frames, methods for long-term planning models and short term planning models. Moreover, some reviews have paid attention to SDDP for long term hydropower planning and its application in the Nordic countries.

Models for long term planning are performed in e.g. the Nordic countries, Brazil, New Zealand and China. Mid term planning models have been developed in e.g. Spain, Switzerland, China and Norway. Models for short term planning has been performed in the Nordic countries, Spain, Brazil and Canada.

The long term planning models are aimed for planning for years or several months. They cover both deterministic and stochastic models. The SDDP method is commonly used in different models.

Mid term planning models do not consider multi-year horizons but limit their attention to months. The models for mid term planning include more details and output such as values of stored water. Earlier studies have paid attention to decomposition, stochastic linear and nonlinear programming, combined SDP and SDDP and stochastic dual dynamic programming. Short term planning is aimed for a week or days. Many studies have been performed within this time frame and they include methods such as mixed integer stochastic model, stochastic model handling non-linearities with time series data, successive linear programming, non-linear Mixed Integer optimisation model, unit commitment model and mixed integer linear programming model.

3.7 Conclusion

Power systems are rapidly changing into a large share of renewable power sources. This requires development of hydropower models, which are efficient and suitable for these power systems. The planning models used until today are denoted as long term planning, mid term planning and short term planning. From this study it can be concluded that the definitions of the different planning time frames are ambiguous and vary. Moreover, within the studied works on maintenance scheduling no work was found which focuses on weekly planning, where turbines are modeled.
separately and where dynamic programming is used as solution method. A possible explanation to why this area has not been studied thoroughly is that weekly planning is in between short term and mid term planning. This means that the methods used in short term planning are difficult to extend to the longer time frame used in weekly planning. At the same time, the approximate methods used in mid term planning are not sufficiently accurate. A motivation for studying the planning problem within the weekly perspective is that even a slight improvement of the outcome of a percentage or a fraction of a percentage leads to a substantial increase in revenue in the order of millions of euro for the power plant owner.
Chapter 4

Preventive maintenance scheduling

Preventive maintenance is used to keep hydropower plants in good condition and avoid failures. The preventive maintenance usually lasts for one hour up to several days and is performed on the hydropower turbines to prevent water spill, loss in electricity production and wear on the turbines. Plans that take disruptions into account are required to control the maintenance. These plans can be utilized by the power producer in order to schedule the maintenance of turbines for a few hours up to the next fourteen days. The net result of a successful maintenance planning can be a substantial financial gain for the power producer.

In Sweden, there are rivers with several cascade connected power plants and the exact configuration of the power plants varies between the different rivers. Each power plant usually have several turbines and commonly a few turbines are disconnected at the same time during the maintenance. A power plant is seldom shut down for maintenance of all turbines at the same time. Therefore, the maintenance plans require that the possible combinations to switch the turbines on and off during a planning period are considered. However, the complexity of the problem is to avoid a combinatorial explosion, i.e. a rapid growth of the complexity of a problem. The combinatorial explosion may occur due to how the combinatorics of the problem is affected by the input, constraints and bounds of the problem. A method that avoids the combinatorial explosion is required to solve a maintenance scheduling problem. Standard techniques, such as Mixed Integer Linear Planning (MILP), are not adequate due to the size of the problem.

This chapter consists of the following parts. First the problem is formulated as a MILP...
problem. Then a relaxation of the problem using value of stored water is performed. The relaxed problem is solved by dynamic programming. The MILP problem is solved by using the relaxed solution. To test the planning method a case study is performed where a Swedish river with cascade coupled power plants has been used. Finally, a stochastic method is presented, with three different setups for electricity price scenarios, which is compared to the deterministic method.

4.1 Problem formulation

A method to solve preventive maintenance scheduling problems is developed in this chapter, where planned outages in specific turbines are assumed. The purpose of this chapter relates to the research objective: develop efficient planning methods for power systems with a large share of renewable power sources, such as wind power. The problem that should be solved is a scheduling problem for a mid term planning period of two to three weeks. This problem is solved in two situations, in the first situation the problem is deterministic and the second situation stochastic electricity prices are taken into account. The first situation is handled in this chapter. This leads to a large number of variables because of the number of hydropower units, reservoirs, segments and time steps. Therefore, a solving strategy must be applied in several steps.

The problem is formulated as a Mixed Integer Linear Programming and solved by dynamic programming. The final value of stored water, i.e. the total value of stored water in all reservoirs, is used as target at the end of the planning period. The problem is investigated from an economic perspective of a power producer with sole ownership of the river in the context of a competitive market using time steps of one hour.

4.1.1 Model limitations

The model has some limitations. It is assumed that the power producer knows when maintenance should be performed and on which turbine. The model is aimed to compute the loss in income if the planned maintenance is executed. Moreover, the model is constructed to be utilized by a single owner of the river system. Nonlinear dependence of the power production in turbines is modeled as concave piecewise linear functions. The value of stored water assumes discrete values and delay time of water is not modeled.
4.1.2 Indices

We introduce the following indices,

\[ i = \text{unit}, \]
\[ j = \text{reservoir}, \]
\[ k = \text{segment}, \]
\[ t = \text{time}. \]

4.1.3 Parameters

We introduce the following parameters,

\[ A_{unit_{i,j}} = \begin{cases} 
-1, & \text{if reservoir } j \text{ releases water directly to unit } i, \\
+1, & \text{if unit } i \text{ releases water to reservoir } j, 
\end{cases} \]

\[ A_{spill_{j,j'}} = \begin{cases} 
-1, & \text{if reservoir } j \text{ spills water to } j', \\
+1, & \text{if reservoir } j' \text{ spills water to } j, 
\end{cases} \]

\[ \beta = \frac{\text{(value of stored water at the end of the planning period)}}{\text{(value of stored water at the start of the planning period)}}, \]

\[ \text{cost}_i = \text{Start up cost in unit } i, \]

\[ P_{\text{max}_i} = \text{Maximal power output in unit } i, \]

\[ P_{\text{min}_i} = \text{Minimum power output in unit } i, \]

\[ price_t = \text{Electricity price hour } t, \]

\[ Q_{\text{max}_{i,k}} = \text{Allowed maximum discharge level in unit } i \text{ and segment } k, \]

\[ Q_{\text{min}_{i,k}} = \text{Allowed minimum discharge level in unit } i \text{ and segment } k, \]

\[ \varrho_{i,k} = \text{Slopes in efficiency curves unit } i \text{ and segment } k, \]

\[ \theta_i = \text{Constant term in the efficiency curve in unit } i, \]
\( V_{max_j} = \text{Maximum volume in reservoir } j, \)
\( V_{min_j} = \text{Minimum volume in reservoir } j, \)
\( V_{0_j} = \text{Start volume in reservoir } j, \)
\( w_{j,t} = \text{Local inflow to reservoir } j \text{ hour } t. \)

### 4.1.4 Variables

We introduce the following variables,

\[
\begin{align*}
    p_{i,t} &= \text{Power output in unit } i \text{ hour } t, \\
    q_{i,t,k} &= \text{Discharge in unit } i \text{ segment } k \text{ hour } t, \\
    s_{j,t} &= \text{Spillage in reservoir } j \text{ hour } t, \\
    u_{i,t} &= \text{Binary variable, unit } i \text{ is on or off hour } t, \\
    v_{j,t} &= \text{Volume in reservoir } j \text{ hour } t, \\
    y_{i,t} &= \text{Binary variable, unit } i \text{ is switched on or off hour } t.
\end{align*}
\]

### 4.1.5 Dynamic programming

The full maintenance scheduling problem is formulated as a MILP problem and solved with a dynamic programming strategy.

Dynamic programming is a method developed in the 1950’s by Richard Bellman to solve large optimization problems that can be formulated in terms of smaller subproblems in the same form.

An optimization problem that can be formulated as

\[
\begin{align*}
    \min_{u_k} \quad & \phi(x_N) + \sum_{k=0}^{N-1} f_0(k, x_k, u_k) \\
    \text{subject to} \quad & x_{k+1} = f(k, x_k, u_k), \\
    \quad & x_0 \text{ given}, \\
    \quad & u_k \in U(k, x_k), \\
\end{align*}
\]

for \( k = 0, \ldots, N − 1 \) and where
• $u_k$ are optimization variables, also called control variables,

• $x_k$ are state variables controlled by the dynamics $x_{k+1} = f(k, x_k, u_k)$,

• $\phi$ and $f$ are functions, and

• $U(k, x)$ are feasible sets for the control variables $u_k$.

can then be solved by Bellman’s equation

$$J(n, x) = \min_{u \in U(n, x)} \left[ f_0(n, x, u) + J(n + 1, f(n, x, u)) \right]$$

working backwards from $n = N$, when $J(N, x) = \phi(x)$, down to $n = 0$ arriving at the solution of the full problem. For further discussion of dynamic programming, see [60].

### 4.1.6 Value of stored water

An important aspect of mid term planning is the value of stored water. The value of stored water represents the total energy of the water stored in the reservoirs of the river system. Water in a reservoir upstream has a higher energy value than water in a downstream reservoir since the water can be used in several turbines downstream. The value of stored water at time $t$ can be written as

$$\alpha_t = \sum_j b_j v_{j,t}$$

where the coefficients $b_j$ are constants that are determined by how the reservoirs are connected. The variables $v_{j,t}$ are the volumes of the reservoirs at time $t$.

Since the reservoirs have a known volume at the start of the planning period the value of stored water is known in the beginning of the planning period.

An example of this can be illustrated with a three hydropower reservoir system as in Figure 4.1.1. If the units (turbines) connected to the reservoirs are equal in performance then the value of stored water in this case is $\alpha_t = 3v_{1,t} + 2v_{2,t} + v_{3,t}$. Water in reservoir 1 can flow through all downstream units of the three reservoirs, water in reservoir 2 can flow through the units of the second and third reservoirs, while water in reservoir 3 can only flow through the units connected to this reservoir.
4.1.7 Efficiency curves for the power production

The power production has highly nonlinear dependence between discharge and electricity production. In this chapter the power production in each unit is modeled as a piecewise linear concave function with two segments as illustrated in Figure 4.1.2.

This dependence is formulated as

$$ p_{i,t} = \sum_k \rho_{i,k} q_{i,t,k} + \theta_i u_{i,t} $$

(4.1)

where $\rho_{i,k}$ is the slope of each linear part and $\theta_i$ is a constant term. The discharge is written as a sum of parts $\sum_k q_{i,t,k}$, where each term is the amount of discharge in the corresponding segment of the piecewise linear curve.

Figure 4.1.2: Example of a piecewise linear function of the power production. The horizontal axis shows the discharge through the turbine and the vertical axis shows the power output. Note that in this example there is a small amount of discharge through the turbine before any power is produced.
4.1.8 MILP formulation of the full problem

The full problem is formulated as a MILP problem. The formulation is

$$\text{max} \quad \text{The income of sold electricity}$$
$$\quad - \text{Start up costs for the turbines}$$

subject to  

Hydrological constraints
- Final value of stored water (Energy)
- Power generation
- Unit commitment
- Variable limits
- Maintenance periods

The objective function is expressed as

$$\text{max} \quad \sum_{i,t} (\text{price}_t \ p_{i,t} - \text{cost}_i \ y_{i,t}). \quad (4.2)$$

The hydrological balance in the river system is formulated as the constraint

$$v_{j,t+1} = v_{j,t} + w_{j,t+1} + \sum_{i,k} A_{\text{unit}}_{i,j} q_{i,t+1,k}$$
$$+ \sum_{j'} A_{\text{spill}}_{j',j} s_{j',t+1}. \quad (4.3)$$

At the start of the planning period the reservoirs are assumed to contain a known volume of water,

$$v_{j,0} = V_{O_j} \quad (4.4)$$

and at the end of the planning period the total value of the reservoir water should have a predetermined percentage $\beta$ of its value at the start of the planning period,

$$\sum_j b_j v_{j,T} = \beta \sum_j b_j v_{j,0}. \quad (4.5)$$

The reservoir levels are only allowed to vary between prescribed fixed values,

$$V_{\text{min}_j} \leq v_{j,t} \leq V_{\text{max}_j}. \quad (4.6)$$
CHAPTER 4. PREVENTIVE MAINTENANCE SCHEDULING

The discharge from a unit \(i\) in segment \(k\) is limited by \(Q_{\text{min},i,k}\) and \(Q_{\text{max},i,k}\) unless the unit is switched off,

\[
Q_{\text{min},i,k} u_{i,t} \leq q_{i,t,k} \leq Q_{\text{max},i,k} u_{i,t}. \tag{4.7}
\]

Note that \(Q_{\text{min},i,k} = 0\) if \(k > 1\) since the discharge in segment \(k\) is equal to 0 as long as segment \(k-1\) is not fully utilised. As already discussed the dependence between power production and discharge is formulated as

\[
p_{i,t} = \sum_k \varrho_{i,k} q_{i,t,k} + \theta_i u_{i,t}. \tag{4.8}
\]

Limits for minimal and maximal power production are formulated as,

\[
P_{\text{min},i} u_{i,t} \leq p_{i,t} \leq P_{\text{max},i} u_{i,t}. \tag{4.9}
\]

If unit \(i\) is on during hour \(t\) then \(u_{i,t} = 1\) and if it is off then \(u_{i,t} = 0\). If unit \(i\) is switched on hour \(t\), i.e. it is off hour \(t - 1\) and on hour \(t\), then \(y_{i,t} = 1\), otherwise \(y_{i,t} = 0\). The connection between \(u_{i,t}\) and \(y_{i,t}\) is

\[
u_{i,t+1} - u_{i,t} \leq y_{i,t+1}. \tag{4.10}
\]

The maintenance of a turbine \(i\) during a time period \([t_1, t_2]\) is specified by introducing the constraints

\[
u_{i,t} = 0 \quad \text{for} \quad t_1 \leq t \leq t_2. \tag{4.11}
\]

4.1.9 Subproblem and relaxation

The full problem has been formulated in the previous subsection as an optimization problem. This MILP problem cannot be solved for any longer periods of time than just a few hours for a large system (such as for example in the test of a full river system described in section 4.3). As the number of binary variables increase, it leads to an exponential increase in time to solve the problem. To counter act this phenomena a relaxation of the problem is performed before the problem is solved by dynamic programming.

To start, the volumes of the reservoirs can be plotted over time as shown in Figure 4.1.3. The planning period \([0, T]\) is divided into \(n\) intervals \([0, \Delta t], [\Delta t, 2\Delta t], ..., [(n - 1)\Delta t, T]\), where \(\Delta t = T/n\). Now, the volumes of the reservoirs are studied at the end points
of these intervals, \( t = 0, t = \Delta t, t = 2\Delta t \) and so on, as done in Figure 4.1.4. In the Figure 4.1.4 two value of stored water scale bars are shown for each of these times. The left scale bar at time \( t = \Delta t \) corresponds to the time \( t = \Delta t^- \), i.e. the instant before \( t = \Delta t \), and the right corresponds to the time \( t = \Delta t^+ \), i.e. the instant after \( t = \Delta t \). The values of stored water are obviously equal at those two times. The full problem is now relaxed by allowing the volumes of the reservoirs instantly before and after each of the times \( t = \Delta t, t = 2\Delta t, \ldots, t = (n-1)\Delta t \) to be unequal as shown in Figure 4.1.5. However, the values of stored water \( \alpha_t \) of the system are required to be equal at the times instantly before and after. Also the variables \( u_{i,t} \) and \( y_{i,t} \) are decoupled at the times \( t = \Delta t, t = 2\Delta t, \ldots, t = (n-1)\Delta t \). This relaxation splits the full problem into subproblems over the time intervals \([0, \Delta t], [\Delta t, 2\Delta t], \ldots, [(n-1)\Delta t, T]\). Each subproblem is connected to the next by the fact that the value of stored water at the end of the subproblem is equal to the value of stored water at the start of the next. This is illustrated with an example in Figure 4.1.6.
4.1.10 Solution by dynamic programming

Introduce the following functions

$\Delta F(k, \alpha, \alpha') = $ The optimal objective function value for the subproblem from $t = (k-1)\Delta t$ to $t = k\Delta t$ with $\alpha_{(k-1)\Delta t} = \alpha$ and $\alpha_{k\Delta t} = \alpha'$,

$F(k, \alpha, \alpha') = $ The optimal objective function value for the optimization problem from $t = (k-1)\Delta t$ to $t = T$ with $\alpha_{(k-1)\Delta t} = \alpha$ and $\alpha_T = \alpha'$. 

Figure 4.1.5: Example of a three reservoir system where the water volumes are allowed to be different instantly before and after the times $\Delta t, 2\Delta t, ...$. The values of stored water instantly before and after these times are equal.

Figure 4.1.6: The full problem is split into subproblems connected by the values of stored water.
According to Bellman’s equation the function $F$ satisfies the recurrence equation

$$F(k, \alpha, \alpha') = \max_{\alpha''} \left[ \Delta F(k, \alpha, \alpha'') + F(k+1, \alpha'', \alpha) \right]$$  \hspace{1cm} (4.12)

for $k = 1, \ldots, n - 2$. This equation is illustrated in Figure 4.1.7 and can be expressed as: The optimal value of going from value of stored water $\alpha$ at time $t = (k - 1)\Delta t$ to value of stored water $\alpha'$ at time $t = T$ is equal to the optimal value of going one time step to value of stored water $\alpha''$ added to the optimal value of going the rest of the time steps from value of stored water $\alpha''$ to the value of stored water $\alpha'$ at time $t = T$ when $\alpha''$ is chosen to maximize the expression.

![Figure 4.1.7: Illustration of Bellman’s equation.](image)

The recurrence relation can be used to calculate the values of $F$ by working backwards,

$$F(n - 1, \alpha, \alpha') = \Delta F(n - 1, \alpha, \alpha'),$$
$$F(n - 2, \alpha, \alpha') = \max_{\alpha''} \left[ \Delta F(n - 2, \alpha, \alpha'') + F(n - 1, \alpha'', \alpha') \right],$$
$$\vdots$$
$$F(1, \alpha, \alpha') = \max_{\alpha''} \left[ \Delta F(1, \alpha, \alpha'') + F(2, \alpha'', \alpha') \right].$$  \hspace{1cm} (4.13)

To make it feasible to solve this recurrence relations it is also assumed that all values of stored water assume a discrete set of values and values of $\Delta F$ are calculated for all transitions between these levels as illustrated in figure 4.1.8. The value of $F(1, \alpha_0, \alpha_T)$, where $\alpha_0$ is the value of stored water at the start of the planning period $t = 0$ and $\alpha_T$ is the value of stored water at the end of the planning period, is then the optimal value.
of the relaxed full planning problem. This value is calculated using the recurrence relations (4.13) and can be seen as finding the optimal path from the start value of stored water $\alpha_0$ to the final value of stored water $\alpha_T$ in the graph of transitions as illustrated in Figure 4.1.9.

4.1.11 A solution to the full problem

The solution derived for the relaxed problem determines the values of stored water at the intermediate times $\Delta t$, $2\Delta t$, $\ldots$, $(n-1)\Delta t$. These values of stored water are used in order to find a solution close to the optimal solution to the non-relaxed problem by solving the full MILP problem stepwise:

- From $t = 0$ to $t = \Delta t$ with the constraint that the value of stored water is $\alpha_{\Delta t}$ at $t = \Delta t$.
- From $t = \Delta t$ to $t = 2\Delta t$ with the constraint that the value of stored water is $\alpha_{2\Delta t}$ at $t = 2\Delta t$.

Figure 4.1.8: The value of $\Delta F$ is calculated for all transitions between the discrete set of value of stored water levels.

Figure 4.1.9: Illustration of solution to the full problem by determining values of stored water at the intermediate times $t = \Delta t$, $t = 2\Delta t$ and so on.
• From \( t = (n - 1)\Delta t \) to \( t = T \) with the constraint that the value of stored water is \( \alpha_T \) at \( t = T \).

The time interval length \( \Delta t \) is short enough to make each of the subproblem in the list above computationally tractable.

### 4.2 Validation of the method

The objective of the empirical studies is to reduce the system so that the dynamic programming approach can be compared to a direct solution of the MILP-problem. This is done by studying two subsets, A and B, of a full river system. The planning problem is solved twice per case study, first using a MILP-solver in Gurobi and then with the dynamic programming method explained in this chapter.

#### 4.2.1 Two solution methods: MILP-solver and dynamic programming

The two solution methods, using a MILP-solver and using dynamic programming, are studied in this section. This is done in a case study that uses the river A subset of the full river system. This subset of the river consists of 3 reservoirs and 4 turbines. The objective of this case study is to evaluate how the dynamic programming method performs by comparing the results with the optimal solution obtained by solving the MILP problem.

In table 4.2.1 results from the case study is presented when the planning problem is solved in Gurobi for one day with the two methods. The number of intermediate steps in the dynamic programming were 4 and each step was 6 hours. The optimal values for different target of the value of stored water at the end of the planning period is presented as well as the running time.

When using the dynamic programming method the optimal values are slightly smaller compared to solve the problem directly. This originates from the fact that in the dynamic programming method first a relaxed problem is solved and then a constrained problem is finally solved. However, since the difference is fairly small this shows that this relaxation does not affect the original problem in any major way.
What is perhaps not readable from table 4.2.1 is that the DP method is sensitive to how fine grained the value of stored water levels are chosen. A too coarse choice of levels will lead to a solution that deviates from the true optimal solution and an optimal values that is significantly worse. On the other hand, a too fine grained set of value of stored water levels will increase the runtime. In this case study 10 levels are used. This means that $10 \times 10 = 100$ subproblems need to be solved in each step of the dynamic programming. By increasing this to 20 levels a total of $20 \times 20 = 400$ subproblems need to be solved for each DP step. To find a suitable number of levels an iterative approach needs to be considered.

To overcome this the DP method is implemented as one full optimization problem in Gurobi where all the subproblems are grouped together and the intermediate values of stored water joining the subproblems are introduced as additional optimization variables. This means that the intermediate values of stored water can attain real values and not just values from discrete set of values of stored water and are determined by solving the optimization problem. This yields a fully satisfactory solution. The running time of the DP method compared to solving the problem directly is roughly equal. This approach is only used for this case study in contrast to the full river system where this is not computationally tractable.

Table 4.2.1: Optimal values from empirical study subset of a river A. The first column shows percentage of the final value of stored water compared to the initial value of stored water. The second and fourth columns are the profits during the planning period.

<table>
<thead>
<tr>
<th>Final v.w.</th>
<th>MILP problem</th>
<th>Dynamic programming</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.96%</td>
<td>276.801</td>
<td>2.0</td>
</tr>
<tr>
<td>99.97%</td>
<td>276.791</td>
<td>1.7</td>
</tr>
<tr>
<td>99.98%</td>
<td>276.780</td>
<td>2.2</td>
</tr>
<tr>
<td>99.99%</td>
<td>276.769</td>
<td>2.5</td>
</tr>
<tr>
<td>100.00%</td>
<td>276.759</td>
<td>2.3</td>
</tr>
<tr>
<td>100.01%</td>
<td>276.748</td>
<td>2.0</td>
</tr>
<tr>
<td>100.02%</td>
<td>276.738</td>
<td>2.6</td>
</tr>
<tr>
<td>100.03%</td>
<td>276.727</td>
<td>2.3</td>
</tr>
<tr>
<td>100.04%</td>
<td>276.716</td>
<td>2.2</td>
</tr>
<tr>
<td>100.05%</td>
<td>276.706</td>
<td>1.7</td>
</tr>
<tr>
<td>100.06%</td>
<td>276.695</td>
<td>1.7</td>
</tr>
</tbody>
</table>

In figure 4.2.1 the different values of stored water that are assumed during the planning period are shown both when solving the MILP problem directly and by using dynamic...
programming. It can be seen that the values of stored water are similar for the two methods.

![Image of graph showing stored water values over time for dynamic programming and MILP problem.]

Figure 4.2.1: The values of stored water of the system during the planning period when the problem is solved with and without dynamic programming, and no maintenance. The black curve is with dynamic programming and the blue curve is the MILP-problem.

A conclusion from this is that the dynamic programming method performs well compared to solving the MILP problem directly since the optimal values only differ 1–2%. The intermediate values of stored water are comparable in the two solution methods.

### 4.2.2 Case study with maintenance

The case study is repeated when maintenance of turbine 1 is required and it is forced out of commission during 5 hours. In table 4.2.2 the corresponding results are reported. The objective in this part of the case study is to evaluate how the introduction of maintenance affects the dynamic programming method compared to the optimal solution.

In figure 4.2.2 the discharge from a turbine with and without maintenance is plotted when the dynamic programming method is applied. It can be seen from the figure that when the maintenance period (hour 4 to 7) has stopped the two curves follow almost the same pattern.

In figure 4.2.3 the discharge from a turbine when the dynamic programming method is applied and the electricity prices is plotted. As expected the discharge is high when the electricity prices are high during the planning period.

Both graphs in figure 4.2.2 and 4.2.3 show patterns which are expected compared to the
Chapter 4. Preventive Maintenance Scheduling

Table 4.2.2: Optimal values from empirical study subset of a river A when a turbine is under maintenance. The first column shows percentage of the final value of stored water compared to the initial value of stored water. The second and fourth columns are the profits during the planning period.

<table>
<thead>
<tr>
<th>Final v.w</th>
<th>MILP problem</th>
<th>Dynamic programming</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.96%</td>
<td>272.001</td>
<td>1.4</td>
</tr>
<tr>
<td>99.97%</td>
<td>271.994</td>
<td>1.8</td>
</tr>
<tr>
<td>99.98%</td>
<td>271.987</td>
<td>1.4</td>
</tr>
<tr>
<td>99.99%</td>
<td>271.980</td>
<td>1.2</td>
</tr>
<tr>
<td>100.00%</td>
<td>271.972</td>
<td>1.4</td>
</tr>
<tr>
<td>100.01%</td>
<td>271.965</td>
<td>1.4</td>
</tr>
<tr>
<td>100.02%</td>
<td>271.958</td>
<td>1.4</td>
</tr>
<tr>
<td>100.03%</td>
<td>271.950</td>
<td>1.3</td>
</tr>
<tr>
<td>100.04%</td>
<td>271.943</td>
<td>2.3</td>
</tr>
<tr>
<td>100.05%</td>
<td>271.936</td>
<td>1.4</td>
</tr>
<tr>
<td>100.06%</td>
<td>271.929</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Figure 4.2.2: The discharge from a turbine in case study river A with and without maintenance. Blue curve shows no maintenance and black curve with maintenance.

discharge and electricity prices during the planning period. When the electricity prices are higher the values of stored water are decreasing compared to when the electricity prices are lower.

Figure 4.2.4 corresponds to figure 4.2.1 but with maintenance. The figure shows the values of stored water during the planning period.

A conclusion from introducing maintenance is that the dynamic programming method still performs well in comparison with the optimal strategy.
4.2.3 A second case study

In this section a second case study is performed and a different subset of the full river system is studied. This subset, river B, consists of three cascaded reservoirs and three units. The objective this time is to make sure that the conclusions from studying river A can be applied to a similar but different case.

In table 4.2.3 the optimal values from this case study is shown. The data that has been used in this case study is the same as for the first case study. The planning period is one day. The planning problem is solved as a MILP problem and with the dynamic programming method. In the dynamic programming method a time step of 6 hours is used.

The results in table 4.2.3 can be compared to the results obtained in table 4.2.4 where
CHAPTER 4. PREVENTIVE MAINTENANCE SCHEDULING

Table 4.2.3: Optimal values from empirical study subset of a river B. No maintenance is scheduled. The first column shows percentage of the final value of stored water compared to the initial value of stored water. The second and fourth columns are the profits during the planning period.

<table>
<thead>
<tr>
<th>Final v.w</th>
<th>MILP problem</th>
<th>Dynamic programming</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.96%</td>
<td>75.670</td>
<td>75.448</td>
</tr>
<tr>
<td>99.97%</td>
<td>75.668</td>
<td>75.445</td>
</tr>
<tr>
<td>99.98%</td>
<td>75.665</td>
<td>75.441</td>
</tr>
<tr>
<td>99.99%</td>
<td>75.662</td>
<td>75.437</td>
</tr>
<tr>
<td>100.00%</td>
<td>75.659</td>
<td>75.433</td>
</tr>
<tr>
<td>100.01%</td>
<td>75.656</td>
<td>75.429</td>
</tr>
<tr>
<td>100.02%</td>
<td>75.653</td>
<td>75.426</td>
</tr>
<tr>
<td>100.03%</td>
<td>75.651</td>
<td>75.422</td>
</tr>
<tr>
<td>100.04%</td>
<td>75.648</td>
<td>75.418</td>
</tr>
<tr>
<td>100.05%</td>
<td>75.645</td>
<td>75.414</td>
</tr>
<tr>
<td>100.06%</td>
<td>75.642</td>
<td>75.411</td>
</tr>
</tbody>
</table>

The problem is solved with maintenance scheduled for one unit during six hours. A comparison shows that the optimal values are somewhat smaller in the DP case which is explained by the fact that the dynamic programming approach can be seen as solving a more restricted problem.

Table 4.2.4: Optimal values from empirical study subset of a river B. Maintenance of turbine 2 is scheduled for hours 12 to 17. The first column shows percentage of the final value of stored water compared to the initial value of stored water. The second and fourth columns are the profits during the planning period.

<table>
<thead>
<tr>
<th>Final v.w</th>
<th>MILP problem</th>
<th>Dynamic programming</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.96%</td>
<td>74.495</td>
<td>72.911</td>
</tr>
<tr>
<td>99.97%</td>
<td>74.493</td>
<td>72.908</td>
</tr>
<tr>
<td>99.98%</td>
<td>74.490</td>
<td>72.905</td>
</tr>
<tr>
<td>99.99%</td>
<td>74.487</td>
<td>72.902</td>
</tr>
<tr>
<td>100.00%</td>
<td>74.484</td>
<td>72.899</td>
</tr>
<tr>
<td>100.01%</td>
<td>74.482</td>
<td>72.897</td>
</tr>
<tr>
<td>100.02%</td>
<td>74.479</td>
<td>72.894</td>
</tr>
<tr>
<td>100.03%</td>
<td>74.476</td>
<td>72.891</td>
</tr>
<tr>
<td>100.04%</td>
<td>74.473</td>
<td>72.888</td>
</tr>
<tr>
<td>100.05%</td>
<td>74.471</td>
<td>72.885</td>
</tr>
<tr>
<td>100.06%</td>
<td>74.468</td>
<td>72.882</td>
</tr>
</tbody>
</table>

In figure 4.2.5 the discharge from one turbine is plotted for the cases when maintenance is not required and when it is required and the dynamic programming method is applied. The maintenance period is between hour 12 to 17. There is a
significant difference in discharge between the two cases.

![Discharge vs Time](image1.png)

Figure 4.2.5: The discharge from a turbine in case study river B with and without maintenance. Blue curve shows no maintenance and black curve with maintenance.

In figure 4.2.6 the electricity price is plotted together with the discharge when the dynamic programming method is applied. The discharge is high during peak electricity prices.

![Price vs Time](image2.png)

Figure 4.2.6: The discharge from a turbine in case study river B with maintenance and the electricity prices. Blue curve shows electricity prices and black curve discharge with maintenance.

To conclude the comparison, the values of stored water of the system are plotted in figure 4.2.7 and 4.2.8. As can be seen the difference between solving the problem with and without dynamic programming is rather small in the no maintenance case. When maintenance is performed on the second turbine there is no discharge during hours 12-17 and this prevents the values of stored water to decrease as rapidly as in the optimal solution when there is no maintenance.

The conclusion that can be drawn from this second case study is that the dynamic programming method still performed in line with what was obtained from the first case study.
Figure 4.2.7: The values of stored water of the system during the planning period when the problem is solved with and without dynamic programming, and no maintenance. The black curve is with dynamic programming and the blue curve is without dynamic programming.

Figure 4.2.8: The values of stored water of the system during the planning period when the problem is solved with and without dynamic programming, and maintenance. The black curve is with dynamic programming and the blue curve is without dynamic programming.

4.3 Empirical study full river

The model has been implemented in Python and tested on a Swedish cascaded river system consisting of 15 hydropower plants, 34 turbines and 16 reservoirs. The objective of the study is to show that the model works for a full river of more than 10 power plants. The river consists of two branches with power plants and reservoirs which merge into the same river downstream. The reservoirs located in the north part of the river system are larger and can store water for long time periods while the reservoirs located downstream are smaller and can store water for short time periods of a few hours. The planning period is 11 days and based on historical data from public and nonpublic sources. The number of maintenance are 8 and ranging from 2 to 160 hours,
see Figure 4.3.1. In the case study the index sets of the indices $i, j, k$ and $t$ are $\mathcal{I} = \{1, 2, \ldots, 34\}, \mathcal{J} = \{1, 2 \ldots, 16\}, \mathcal{K} = \{1, 2\}$ and $\mathcal{T} = \{1, 2, \ldots, 264\}$, respectively.

![Figure 4.3.1: Diagram showing the planned maintenance (red color) of the 34 generators (vertical axis) during the planing period (horizontal axis).](image)

The electricity prices during the planning period are spot prices from Nord Pool and shown in Figure 4.3.2.

![Figure 4.3.2: Electricity spot price during the planning period in the case study.](image)

The local inflows are nonpublic data but are mostly constant for each reservoir during the planning period.
4.4 Results full river

A table of optimal values with and without maintenance for different values of the final value of stored water are shown in Table 4.4.1. An immediate observation is that the optimal value is lower in the case of maintenance compared to the case without maintenance. Typically the decrease is around 0.3%. The optimal value is lower when maintenance is performed since water cannot be fully utilized at peak electricity prices. Another aspect is that a larger final value of stored water target also means a lower optimal value since water is then stored in the reservoirs that can be utilized in the future.

The final values of stored water are chosen to deviate very little from the start value of stored water since the planning period is during a part of the year (November/December) when the water inflow to the reservoirs are low.

<table>
<thead>
<tr>
<th>Final v.w.</th>
<th>Without m. [MEUR]</th>
<th>With m. [MEUR]</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.996%</td>
<td>16.528756</td>
<td>16.248609</td>
</tr>
<tr>
<td>99.997%</td>
<td>16.528752</td>
<td>16.248604</td>
</tr>
<tr>
<td>99.998%</td>
<td>16.528747</td>
<td>16.248600</td>
</tr>
<tr>
<td>99.999%</td>
<td>16.528742</td>
<td>16.248595</td>
</tr>
<tr>
<td>100.000%</td>
<td>16.528737</td>
<td>16.248590</td>
</tr>
<tr>
<td>100.001%</td>
<td>16.528732</td>
<td>16.248585</td>
</tr>
<tr>
<td>100.002%</td>
<td>16.528728</td>
<td>16.248580</td>
</tr>
<tr>
<td>100.003%</td>
<td>16.528723</td>
<td>16.248575</td>
</tr>
<tr>
<td>100.004%</td>
<td>16.528718</td>
<td>16.248571</td>
</tr>
<tr>
<td>100.005%</td>
<td>16.528713</td>
<td>16.248565</td>
</tr>
</tbody>
</table>

The discharge in one particular turbine is plotted in Figure 4.4.1 under two situations. In the first situation (plotted in blue) there is no scheduled maintenance and in the second situation (plotted in black) the turbine is out of commission during the first 63 hours.

By combining Figure 4.3.2 and Figure 4.4.1 as done in Figure 4.4.2 it is observed that the times the turbine is switched on corresponds strongly to the times the electricity price is high.

The computational time of running the model in Python with and without maintenance, respectively, is 6-7 hours. The solver used in Python is Gurobi. The computer used an Intel Core i5-10400T CPU with a clock frequency of 2.00 GHz and
CHAPTER 4. PREVENTIVE MAINTENANCE SCHEDULING

Figure 4.4.1: The discharge from one turbine during the planning period with and without maintenance scheduled.

In table 4.4.2 results are presented from a study were the final value of stored water is chosen to differ up to two percent from the start value of stored water. The purpose with this study is to show that the method works for larger differences in start and final value of stored water than what has been presented earlier. This time the intermediate values of stored water were chosen to be five. This change decreased the computational time but the optimal values are different from if ten values would have been chosen. It can be seen in 4.4.2 that the optimal values for 100% of the value of stored water are higher compared to the optimal values at that level in 4.4.1. The reason to this is that there is a larger span of possible values of stored water in 4.4.2. However, this illustrates that the method also works for larger variations in start and final value of stored water.

a RAM-memory of 16.0 GB.
Table 4.4.2: Optimal values from empirical study full river 2% difference in start and final value of stored water.

<table>
<thead>
<tr>
<th>Final v.w</th>
<th>Optimal value Without m. [MEUR]</th>
<th>Optimal value With m. [MEUR]</th>
</tr>
</thead>
<tbody>
<tr>
<td>98%</td>
<td>16.750615</td>
<td>16.463418</td>
</tr>
<tr>
<td>98.5%</td>
<td>16.748299</td>
<td>16.461100</td>
</tr>
<tr>
<td>99.0%</td>
<td>16.743493</td>
<td>16.456244</td>
</tr>
<tr>
<td>99.5%</td>
<td>16.741017</td>
<td>16.453755</td>
</tr>
<tr>
<td>100.0%</td>
<td>16.738329</td>
<td>16.451115</td>
</tr>
</tbody>
</table>

4.5 Conclusion

A model for fixed preventive maintenance scheduling has been developed in this chapter, which can be used by the industry. The case studies, river A and B, show that the model works well. The computational time for the case studies is similar when using dynamic programming compared to solving the MILP problem except in one case. This case is when the MILP problem is solved for river B and there is no maintenance. A possible explanation to the longer computational time compared to when the problem is solved for river B with maintenance can be seen in figure 4.2.5. From the figure it is clear that the optimal solutions for the two cases differ significantly and this could explain why it might take different time to reach the solution even though the only difference between the cases is the applied maintenance. However, the solution strategy gives a solution close to the optimal one. This is established by comparing the value of the objective function and values of stored water for the obtained solution with and without using dynamic programming.
Chapter 5

Stochastic model

5.1 Problem formulation

In this part a stochastic model is presented and compared to the deterministic model discussed previously. The aim is to investigate if and to what extent a stochastic model for the electricity price performs better than the deterministic model. This is determined by comparing the outcomes of the stochastic model and the deterministic model by calculating the expected value of perfect information (EVPI) and the value of stochastic solution (VSS). The data that has been used is assumed to be representative of an average scenario for a year. The different scenarios are assumed to be equally probable.

In the stochastic model the electricity price is viewed as a stochastic variable. Scenarios for the electricity price are used to investigate a comparison of the stochastic model with a deterministic model. These scenarios are not necessarily realistic real scenarios but test scenarios. The prices from the last eight years (2013–2020) are used as the foundation to create scenarios in the following manner:

1. Let \( \text{price}^R_{y,t} \) be the electricity price year \( 2012 + y \) hour \( t \).

2. First, for a given year (fixed \( y \)), the time series \( \{\text{price}^R_{y,t}\} \) is normalized by defining a new series \( \text{price}^N_{y,t} = (\text{price}^R_{y,t} - m)/s \), where \( m \) is the average of the time series \( \{\text{price}^R_{y,t}\} \) and \( s \) is the standard deviation.

3. Then a scenario of the electricity price is defined as \( \text{price}^{\omega,t} = \sigma' \text{price}^N_{y,t} + z \), where \( z \) is the mean value and \( \sigma' \) is the standard deviation of the electricity price. The
mean value $z$ is chosen as the outcome of a normal distributed random variable with mean equal to the average value of the time series $price^R_{y,t}$ and variance $20$. The standard deviation $\sigma'$ will be allowed to vary to simulate different volatility of the electricity price.

In Figure 5.1.1 the electricity price time series $price^R_{y,t}$ are plotted. In Figure 5.1.2 the normalized electricity price time series $price^N_{y,t}$ are plotted. In Figure 5.1.3 the electricity price times series $price^\omega_{y,t}$ used in the stochastic model are plotted.

![Figure 5.1.1: Electricity price during the eleven days following the first Monday of the years from 2013 to 2020.](image1)

![Figure 5.1.2: Electricity price time series normalized to have mean 0 and standard deviation 1.](image2)

Another method is also used to construct the electricity price scenarios. These scenarios are $price^R_{y,t} + c_y$ where suitable constants $c_y$ are added to the historical electricity price to give the scenario a desirable mean value. The resulting scenarios are
Figure 5.1.3: Electricity price time series in the stochastic model ($\sigma' = 40$).

plotted in Figure 5.1.4. The difference from earlier constructed price scenarios is that the variance of the scenario is the same as in the corresponding historical year.

Figure 5.1.4: Electricity price time series in the stochastic model, variance is unchanged.

If we were to plan for each scenario then we would solve eight independent deterministic optimization problems as described in section 4.1.8, one for each scenario. In order to help to formulate the stochastic model these independent optimization problems can be combined into one large joint optimization problem with objective function

$$\max \sum_{\omega} \pi_{\omega} \left( \sum_{i,t} \text{price}_{\omega,t} p_{i,t,\omega} - \text{cost}_i y_{i,t,\omega} \right). \quad (5.1)$$
where the variables are made scenario dependent

\[ p_{i,t,\omega}, q_{i,t,k,\omega}, s_{j,t,\omega}, v_{j,t,\omega}, u_{i,t,\omega} \quad \text{and} \quad y_{i,t,\omega}. \]

The parameter \( \pi_\omega \) is introduced as the weight for the scenarios and the index \( \omega \) is used to keep track of the scenarios. The constraints and variable limits from the deterministic model are kept.

This can be concluded from the fact that in (5.1) the sum over \( \omega \) is made up of terms that are maximized independent of each other.

The stochastic model is a recourse problem where decisions are made in two stages. The first stage decision is made at the start of the whole planning period when one scheduling is done taking all eight scenarios into account. This planning is used during a first time period \( 0 \leq t \leq T \). At time \( t = T \) a schedule is made for the rest of the planning period taking only one scenario into account, i.e. at time \( t = T \) it is revealed which scenario occurs.

The stochastic planning model uses the large joint optimization problem with objective function (5.1) by adding the following constraints

\begin{align*}
\text{Stage 1 constraints (for } 0 \leq t \leq T) \\
p_{i,t,\omega} & \text{ are equal for all } \omega, \\
q_{i,t,k,\omega} & \text{ are equal for all } \omega, \\
s_{j,t,\omega} & \text{ are equal for all } \omega, \\
v_{j,t,\omega} & \text{ are equal for all } \omega, \\
u_{i,t,\omega} & \text{ are equal for all } \omega, \\
y_{i,t,\omega} & \text{ are equal for all } \omega. \\
\end{align*}

(5.2)

This makes the optimization variables equal during stage 1 irrespective of the scenario. (During stage 2 they can have different values depending on the scenario.) The stochastic model is solved with a similar dynamic programming approach used for the deterministic model and will be described now.

During stage 2, from time \( t = T \) to the end of the planning period \( t = T_{\text{end}} \), the scenarios are treated separately in the dynamic programming. If the following notation
is introduced

\[ F_{t \rightarrow T_{\text{end}}}^\omega (\alpha, \alpha_{\text{end}}) = \text{The optimal objective function value for the optimization problem from time } t \text{ to time } T_{\text{end}} \text{ in scenario } \omega \text{ with start value of stored water } \alpha \text{ and final value of stored water } \alpha_{\text{end}}, \]

\[ \Delta F_{t \rightarrow t + \Delta t}^\omega (\alpha, \alpha') = \text{The optimal objective function value for the subproblem from time } t \text{ to time } t + \Delta t \text{ in scenario } \omega \text{ with start value of stored water } \alpha \text{ and end value of stored water } \alpha'. \]

then Bellman’s equation can be formulated as

\[ F_{t \rightarrow T_{\text{end}}}^\omega (\alpha, \alpha_{\text{end}}) = \max_{\alpha'} \left[ \Delta F_{t \rightarrow t + \Delta t}^\omega (\alpha, \alpha') + F_{t + \Delta t \rightarrow T_{\text{end}}}^\omega (\alpha', \alpha_{\text{end}}) \right] \quad (5.3) \]

and this recurrence relation can be used to calculate the optimal objective function value for stage 2, i.e. from time \( T \) and onward for each scenario \( \omega \).

The dynamic programming steps in stage 1 starts by introducing the following quantity

\[ F_{T \rightarrow T_{\text{end}}}^\omega (\alpha, \alpha_{\text{end}}) = \sum_{\omega} \pi_\omega F_{T \rightarrow T_{\text{end}}}^\omega (\alpha, \alpha_{\text{end}}) \quad (5.4) \]

which is the optimal expected value of the objective function over all scenarios from time \( T \) to time \( T_{\text{end}} \) as expressed in (5.1). The calculation done in (5.4) is illustrated in figure 5.1.5. The optimal expected value of the objective function \( F_{t \rightarrow T_{\text{end}}} (\alpha, \alpha_{\text{end}}) \) is calculated by using the following version of Bellman’s equation

\[ F_{t \rightarrow T_{\text{end}}} (\alpha, \alpha_{\text{end}}) = \max_{\alpha'} \left[ \Delta F_{t \rightarrow t + \Delta t} (\alpha, \alpha') + F_{t + \Delta t \rightarrow T_{\text{end}}} (\alpha', \alpha_{\text{end}}) \right] \quad (5.5) \]

for \( t = T - 1 \) down to \( t = 0 \). The quantity \( \Delta F_{t \rightarrow t + \Delta t} \) is the optimal expected value of the subproblem from time \( t \) to time \( t + \Delta t \) over all scenarios. It is calculated by solving the large joint stochastic optimization problem over all scenarios as outlined earlier in this text, i.e. solving one eight times larger optimization problem than for the deterministic problem.

By relaxing the constraints (5.2) the wait-and-see solution is obtained. This corresponds to the situation where we have complete information and planning can
be performed optimally, i.e., we know beforehand which scenario will occur and can plan accordingly. From this the expected value of perfect information can be computed

$$\text{EVPI} = \text{WS} - \text{RP},$$

where WS is the expectation value of the optimal values when planning for each scenario and RP is the optimal value of the stochastic problem.

The value of the stochastic solution can be calculated as

$$\text{VSS} = \text{RP} - \text{DS},$$

where DS is the optimal value of the stochastic problem when the first-stage variables are fixed to values provided by a deterministic problem using the average value for electricity price during the stage 1 and scenario dependent electricity prices during stage 2.

### 5.2 Results of stochastic computations

The case study is performed on river A and the total time period is one week, where the first stage is 42 hours. To achieve reasonable computation time and memory consumption the time step is increased from 1 hour to 7 hours. Using the smaller time
step results in hitting the memory limit of the computer after about 12 hours and the computation is aborted. The electricity price scenarios used are those outlines above by using the price at the start of each 7 hour time step for the whole 7 hours.

The results from the computations are reported in table 5.2.1 and the values of EVPI and VSS are

\[
\begin{align*}
\sigma' = 40: & \quad \text{EVPI} = 11.726 - 11.545 = 0.181, \\
& \quad \text{VSS} = 11.545 - 11.539 = 0.006, \\
\sigma' = 80: & \quad \text{EVPI} = 11.913 - 11.366 = 0.548, \\
& \quad \text{VSS} = 11.366 - 11.339 = 0.026, \\
\text{Shifted:} & \quad \text{EVPI} = 11.712 - 11.365 = 0.347, \\
& \quad \text{VSS} = 11.365 - 11.337 = 0.028,
\end{align*}
\]

where the three different electricity price scenarios, as described at the start of this section, have been used. The standard deviation of the shifted scenarios ranges from 30 to 120 with a mean of 63.

<table>
<thead>
<tr>
<th>Method</th>
<th>Scenario [Mkr] (e' = 40)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect information</td>
<td>11.7</td>
<td>10.4</td>
<td>10.7</td>
<td>11.7</td>
<td>12.4</td>
<td>13.0</td>
<td>11.0</td>
<td>13.0</td>
<td></td>
</tr>
<tr>
<td>Average scenario</td>
<td>11.6</td>
<td>10.3</td>
<td>10.4</td>
<td>11.7</td>
<td>12.3</td>
<td>12.8</td>
<td>10.9</td>
<td>12.3</td>
<td></td>
</tr>
<tr>
<td>Stochastic</td>
<td>11.6</td>
<td>10.3</td>
<td>10.3</td>
<td>11.7</td>
<td>12.3</td>
<td>12.8</td>
<td>10.9</td>
<td>12.5</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Method</th>
<th>Scenario [Mkr] (e' = 80)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect information</td>
<td>11.3</td>
<td>9.9</td>
<td>11.0</td>
<td>11.7</td>
<td>12.9</td>
<td>13.6</td>
<td>11.2</td>
<td>13.7</td>
<td></td>
</tr>
<tr>
<td>Average scenario</td>
<td>10.9</td>
<td>9.7</td>
<td>9.9</td>
<td>11.6</td>
<td>12.5</td>
<td>12.7</td>
<td>10.9</td>
<td>12.4</td>
<td></td>
</tr>
<tr>
<td>Stochastic</td>
<td>10.7</td>
<td>9.7</td>
<td>9.9</td>
<td>11.6</td>
<td>12.3</td>
<td>12.9</td>
<td>11.0</td>
<td>12.7</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Method</th>
<th>Scenario [Mkr] (shifted)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect information</td>
<td>10.9</td>
<td>11.4</td>
<td>11.2</td>
<td>12.6</td>
<td>12.5</td>
<td>11.9</td>
<td>11.0</td>
<td>12.2</td>
<td></td>
</tr>
<tr>
<td>Average scenario</td>
<td>10.5</td>
<td>11.3</td>
<td>10.9</td>
<td>12.3</td>
<td>12.3</td>
<td>11.2</td>
<td>10.9</td>
<td>11.4</td>
<td></td>
</tr>
<tr>
<td>Stochastic</td>
<td>10.5</td>
<td>11.1</td>
<td>11.0</td>
<td>12.3</td>
<td>12.3</td>
<td>11.2</td>
<td>10.9</td>
<td>11.7</td>
<td></td>
</tr>
</tbody>
</table>

From the calculation of EVPI a conclusion is that with increasing standard deviation (e') the gap between the result from the stochastic method and solving the problem with perfect information increases. The VSS calculations show that the stochastic method performs better than a purely deterministic method where the
expected values of the electricity price is used. In this comparison the difference increases with increasing standard deviation ($\sigma'$). With the introduction of a larger share of intermittent power sources in the power systems the standard deviation is expected to increase and the result in this study indicates that stochastic models will perform better for planning purposes. Comparing EVPI and VSS show that the gap between the stochastic method and solving the problem with perfect information is at least an order of magnitude greater than the difference between the stochastic method and the deterministic method.

The runtime of the calculations of EVPI and VSS are 10.6 minutes for the scenarios with $\sigma' = 40$, 70.7 minutes for the scenarios with $\sigma' = 80$ and 27.8 minutes for the scenarios that are only shifted (and not rescaled). In these calculations the optimality gap was set to 0.5% instead of 0.01% in order have reasonable runtimes and memory consumption, yet sufficient accuracy, when solving the subproblems in the dynamic programming method. With the smaller optimality gap (0.01%) the runtime could be up to 10–12 hours and in many cases the computation was aborted due to lack of memory. In cases when the computation was completed with a smaller optimality gap the results were not different from the results reported with the larger optimality gap in table 5.2.1. The runtimes also increase significantly with increasing time period (1 week) or shorter time resolution (7 hours).

All results in this study are affected by the model setup and data used. A conclusion from the study is that adequate electricity price forecasts are important but the runtimes of the stochastic models tend to be long and difficult to use. A general conclusion is that building a stochastic model is time consuming and requires large computer resources. The increase in the optimal value if fairly small when using the stochastic model compared to the deterministic model. However, this can translate to substantial increase in revenue over time. Some form of stochastic planning might therefore be useful in practise, but it is needed to always keep computational tractability in mind by, for example, using long time steps and sufficiently large optimality gap.
Chapter 6

Discussion

6.1 Summary

One goal of the project was to study the connection between short term, mid term and long term planning of hydropower. This is achieved in a literature review that has been carried out with the focus of weekly planning of hydropower. Some earlier reviews are presented followed by a presentation of works describing long term planning, mid term planning and short term planning. The long term planning models cover both deterministic and stochastic models. It could be seen that the SDDP method is commonly used. The mid term planning models cover decomposition, stochastic linear and nonlinear programming, combined SDP and SDDP and stochastic dual dynamic programming. The short term models pay attention to methods such as mixed integer stochastic model, stochastic model handling non-linearities with time series data, successive linear programming, non-linear Mixed Integer optimisation model, unit commitment model and mixed integer linear programming model.

Another goal of the project was to develop efficient planning methods for a power system with a large share of renewable power sources. This goal has been fulfilled since a model has been developed for weekly planning of hydropower where a relaxation is performed before the optimization problem is solved by dynamic programming. The model has been applied in an empirical study of a Swedish cascaded river system consisting of 15 hydropower plants, 34 turbines and 16 reservoirs. To test the model two case studies are performed where two subsets of the full river are used. The test models have been developed since the size of the problem makes it hard
to solve with standard techniques. The full optimization problem is to maximise the income of sold electricity minus start up costs for the turbines due to a set of constraints. The constraints are final value of stored water, power generation, unit commitment, variable limits and maintenance periods. The method works well in terms of computational time and monetary value, which are similar when dynamic programming is used compared to when the solver Gurobi is used in the case of the smaller test systems, river A and river B, except in one case. The solutions found with the dynamic programming method are close to the optimal solutions.

Furthermore a stochastic method is developed. The stochastic model is compared to the deterministic model. The stochastic model is a two stage model where the electricity price is a stochastic variable and data from eight years are used to create scenarios for the electricity price. The objective function is the expected value of profit. In general, the stochastic method performs better than the deterministic method and this is more evident when the price variations increase. However, accurate electricity price forecasts are important and the number of scenarios needs to be kept down in order to avoid long computational runtimes. In practice, it is not clear which of stochastic or deterministic planning models perform better.

To conclude, this project has shown that it is possible to solve a detailed hydropower planning problem with hourly resolution and a planning horizon of 2-3 weeks. The deterministic method can be used to solve large hydropower planning problems.

### 6.2 Future Work

Below some directions of future work are given.

- One suggestion is to combine the planning between different markets (such as the spot market, intraday trading and real time trading) and integrate maintenance scheduling, i.e. detailed models for mid term planning.

- It would be interesting to include stochastic inflows in the model. One way of approaching the problem could be with the SDDP method to circumvent the problem with exponential growth of the required resources to solve the problem.

- A possible extension of the maintenance scheduling study is to model the maintenance as a decision variable, which gives the power producer a larger
flexibility to schedule the maintenance.

- It would be interesting to include nonlinear dependence of the power production.

- As the electricity market is developing, it would be interesting to study shorter time steps in the model, e.g. 15 minutes instead of 1 hour. This could be formulated as a continuous time problem leading to a formulation as an optimal control problem and applying different numerical methods to solve the resulting differential equations.

- Delay time of water could be further studied.

- Further studies of electricity price forecasts and stochastic models which are adapted to planning periods of several weeks are needed.

- Use other structures and methods for stochastic planning where decision and information arrive at different times.

- Study approaches to circumvent the problem with limited computer resources in order to be able to solve larger problems by, for example, using more powerful computer resources.
Table A.0.1: This table summarizes the review papers.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Title</th>
<th>Literature reviews</th>
</tr>
</thead>
<tbody>
<tr>
<td>[14]</td>
<td>Planning and Scheduling under Uncertainty: A review Across Multiple Sectors</td>
<td>Review that spans over many fields of science and the different techniques used to model uncertainties.</td>
</tr>
<tr>
<td>[19]</td>
<td>Long-and medium-term operations planning and stochastic modelling in hydro-dominated power systems based on stochastic dynamic programming</td>
<td>Review of stochastic dual dynamic programming applied to hydropower scheduling in the Nordic countries.</td>
</tr>
</tbody>
</table>
Table A.0.2: This table summarizes the papers on long- and mid term planning.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Main characteristics</th>
<th>Case study</th>
<th>Conclusions</th>
</tr>
</thead>
<tbody>
<tr>
<td>[24]</td>
<td>SDP, Markovian model for inflow aggregated over time scales.</td>
<td>Brazil</td>
<td>Reduced costs compared to standard Markovian model.</td>
</tr>
<tr>
<td>[26]</td>
<td>SDDP, large system, model for inflow.</td>
<td>Norway</td>
<td>SDDP is preferred to aggregate-disaggregate models.</td>
</tr>
<tr>
<td>[27]</td>
<td>SDDP, IFS model for inflow compatible with SDDP.</td>
<td>New Zealand</td>
<td>IFS model is an accurate representation of the inflow in two aspects.</td>
</tr>
<tr>
<td>[28]</td>
<td>Hybrid SDP/SDDP, inflow, energy price and reserve capacity stochastic.</td>
<td>Norway</td>
<td>Linearization error is pronounced in sales of energy and reserve capacity.</td>
</tr>
<tr>
<td>[29]</td>
<td>Formal optimization model compared to aggregated heuristic model.</td>
<td>Fictitious</td>
<td>Both models give comparable results.</td>
</tr>
<tr>
<td>[30]</td>
<td>SDP, multi-stage problem is decomposed into two-stage optimization problems.</td>
<td>China</td>
<td>The method avoids the curse of dimensionality with good performance.</td>
</tr>
<tr>
<td>[31]</td>
<td>Flexible robust optimization dispatch for hybrid wind/solar/hydro/thermal power system.</td>
<td>China</td>
<td>Flexible robust optimization with adjustable uncertainty budget dispatch model built for a hybrid power system.</td>
</tr>
</tbody>
</table>

**Mid term planning**

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Main characteristics</th>
<th>Case study</th>
<th>Conclusions</th>
</tr>
</thead>
<tbody>
<tr>
<td>[32]</td>
<td>Stochastic MILP, day-ahead and reserve markets considered.</td>
<td>Spain</td>
<td>Higher expected income when participating in both day-ahead and reserve markets than just day-ahead.</td>
</tr>
<tr>
<td>[33]</td>
<td>Stochastic multi-stage quadratic model, pumped storage and forwards used</td>
<td>Switzerland</td>
<td>Trading with forwards increase profit.</td>
</tr>
<tr>
<td>[34]</td>
<td>Mid term and long term power quantity contracts are decomposed.</td>
<td>China</td>
<td>Tested model works well for the Yunnan power spot market.</td>
</tr>
<tr>
<td>[35]</td>
<td>Medium term power planning with bilateral contracts.</td>
<td>Norway</td>
<td>Stochastic linear and non-linear programming is used.</td>
</tr>
<tr>
<td>[36]</td>
<td>A case study on medium-term hydropower scheduling with sales of capacity.</td>
<td>Norway</td>
<td>SDP and SDDP are combined.</td>
</tr>
<tr>
<td>[37]</td>
<td>Nonconvex Medium-Term Hydropower Scheduling by Stochastic Dual Dynamic Integer Programming</td>
<td>Norway</td>
<td>The method works to solve to optimal value but is time consuming.</td>
</tr>
<tr>
<td>[38]</td>
<td>Nonconvex Environmental Constraints in Hydropower Scheduling</td>
<td>Norway</td>
<td>A particular nonconvex environmental constraint was modelled.</td>
</tr>
</tbody>
</table>
Table A.0.3: This table summarizes the papers on short term planning.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Main characteristics</th>
<th>Case study location</th>
<th>Conclusions</th>
</tr>
</thead>
<tbody>
<tr>
<td>[40]</td>
<td>Stochastic MILP, day-ahead price and inflow stochastic.</td>
<td>Norway</td>
<td>Profit and risk negatively correlated.</td>
</tr>
<tr>
<td>[41]</td>
<td>NLDP, head and efficiency curves non-linear.</td>
<td>Spain</td>
<td>Model considers many non-linearities and gives useful results.</td>
</tr>
<tr>
<td>[42]</td>
<td>Stochastic non-linear model solved with successive LP.</td>
<td>Fictious</td>
<td>The solution is close to optimal. Stochastic model performs better than deterministic.</td>
</tr>
<tr>
<td>[43]</td>
<td>Deterministic non-linear mixed integer optimization model solved with Lagrangian relaxation.</td>
<td>Brazil</td>
<td>The approach is feasible but needs to be further studied.</td>
</tr>
<tr>
<td>[44]</td>
<td>Mathematical model to simulate Day-ahead markets of large-scale multi-energy systems with large amount of renewable energy towards 2050.</td>
<td>North Sea region</td>
<td>The high penetration of wind and solar is likely to challenge the need for balancing in the system. Flexibility will be very important towards 2050.</td>
</tr>
<tr>
<td>[45]</td>
<td>Stochastic LP model with integrated wind power and trading.</td>
<td>Sweden</td>
<td>Coordinating hydro and wind power is beneficial.</td>
</tr>
<tr>
<td>[46]</td>
<td>MILP model with power output and unit commitment as variables.</td>
<td>Brazil</td>
<td>The model is efficient and generates a smooth generation schedule.</td>
</tr>
<tr>
<td>[47]</td>
<td>Non-linear model for power output and unit commitment.</td>
<td>Brazil</td>
<td>The model is compact and efficient but not convex.</td>
</tr>
<tr>
<td>[48]</td>
<td>MILP, unit commitment model.</td>
<td>Brazil</td>
<td>An accurate planning result within reasonable computational work.</td>
</tr>
<tr>
<td>[49]</td>
<td>Stochastic MIP commitment model with bids to day-ahead and balancing market.</td>
<td>Norway</td>
<td>Small additional revenue to participate in balancing market and weak dependence on start/stop costs.</td>
</tr>
<tr>
<td>[50]</td>
<td>MILP commitment model with large amount of wind power solved with robust optimization.</td>
<td>Fictious</td>
<td>A flexible model that ensures reliability and security.</td>
</tr>
<tr>
<td>[51]</td>
<td>Overview on formulations and optimization methods for the unit-based short-term hydro scheduling problem.</td>
<td>Norway</td>
<td>Fewer works summarize hydro scheduling works on single turbine models compared to aggregated models.</td>
</tr>
<tr>
<td>[52]</td>
<td>Acceleration of Benders decomposition for two-stage stochastic maintenance scheduling.</td>
<td>United Kingdom</td>
<td>The formulation approximates the three dimensional nonlinearity of hydropower production.</td>
</tr>
<tr>
<td>[53]</td>
<td>Stochastic two-stage NLP and MILP, inflow stochastic</td>
<td>Canada</td>
<td>Using multi-scenarios is more robust than using median scenarios.</td>
</tr>
<tr>
<td>[61]</td>
<td>The model is a combination of simulation and stochastic two-stage LP. Inflow and electricity price are stochastic.</td>
<td>Nordic Countries</td>
<td>The model is well suited to valuate the flexibility of a power system.</td>
</tr>
</tbody>
</table>
Bibliography


[50] Chen, Yue, Liu, Feng, Wei, Wei, Mei, Shengwei, and Chang, Naichao. “Robust unit commitment for large-scale wind generation and run-off-river hydropower”. In: CSEE journal of power and energy systems 2.4 (2016), pp. 66–75.


