Risk of fragment hit on aircraft after releasing a bomb

Risk för splitterträff på eget flygplan i samband med bombfällning

JOAKIM HEDBERG
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Erik Joakim Robert Hedberg

supervisors
Matilda Ågren
Mats Hartmann

examiner
Raffaello Mariani

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Abstract

An aircraft is exposed for a certain risk during warhead deployment. If the distance is too short between the aircraft and warhead during burst, fragments may hit the aircraft with fatal consequences. The warhead consists not only of fragments created during burst but also parts to attach it to the aircraft such as mounting plates and fastening loops. These parts have a significantly larger mass than natural fragments and may travel far during trajectory.

The problem for a potential hit of fragment on a fighter jet after its warhead has detonated has been present since several decades. It is of interest to analyse the problem to effectively reduce the warheads arming time and the aircraft’s altitude during warhead deployment. The complexity consists of how the mounting plates and fastening loops behave during trajectory, which may affect the travelled distance if they rotate or tumble. Attempts to solve this problem for the Gripen fighter jet has been made by Staffan Harling at FOI which this thesis is a subsequent work.

This thesis treats a risk perspective analyse of the distance between the aircraft and warhead named range safety distance. Travelled distances for fragments are calculated with variation in velocity, drag coefficient and ejection angle to analyse the problem to a wider extent.

The conclusion states that the time from warhead deployed until it burst should be at least seven seconds. Generic data has been used in this master thesis due to classified information concerning real cases. Focus has been on the method and to develop a Matlab code that hopefully can be used to estimate range safety distances from a appropriate risk perspective.
Sammanfattning

Vid bombfällning är ett flygplan särskilt utsatt för risk. Om avståndet mellan flygplan och fälld stridsdel är relativt sett för kort vid bombens brisad, finns risk att splitter träffar flygplanet med potentiella förödande konsekvenser. Stridsdelen består förutom splitter som skapas vid brisad också utav en fästanordning bestående av fästplatta samt lytöglag i dubbel uppsättning. Dessa har en signifikant större massa vilket troligtvis gör att de flyger längre vid utkastning efter bombbrisad.

Problemet med vådabekämpning av eget flygplan har kvarstått under flera decennier och utgör en frågeställning intressant att besvär i övningssyfte och under skarp insats. Syftet är att kunna minimera armeringstiden och därigenom även flygplanets flyghöjd vid fällning. Komplexiteten utgörs av att lytöglag och fästplattan har svåranalyserat beteende under sin färdade bana, då de både kan rotera och tumla.

Denna avhandling behandlar ett riskperspektiv som analyserar avståndet mellan flygplan och stridsdel. Den färdade sträckan för splitter är beräknat med variation i hastighet, luftmotståndskoefficient samt utkastningsvinklar för att undersöka problemet i ett större perspektiv.

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Chapter 1

Introduction

An aircraft during training or combat situations intervene targets on land and sea which needs to be precise and effective, including minimise the risk of detection and therefore enemy counteractions. Even if the main risk for the aircraft is the obvious enemy in war times, there is also the risk of hit from fragments after a warhead has been deployed from the aircraft and thereafter detonated. Even if it is unusual that an aircraft been hit by fragments from its own deployed warhead, incidents have occurred during both peace- and wartime. If an aircraft is hit by a fragment, the consequences may be fatal and resulting in a crash.

For exercises during peacetime the probability for a fragment hit must be deemed sufficiently low such that the risk for the aircraft and pilot is acceptable. This must be true both for warheads reaching their target on the ground and in the case of an accidental initiation of the warhead in its trajectory after the set fuse arming time.

This thesis treats the potential risk of an aircraft getting hit by fragments from the warhead it has deployed which thereafter detonates. Earlier work in this subject has been performed by Staffan Harling at FOI and is described in the following reports [1] and [2]. In this thesis a Matlab program has been developed that allows the user to analyse arbitrary flight trajectories and vary different input parameters. Any suitable aircraft trajectory can be read and the time when the warhead is released as well as detonated can be varied. Velocities and rotations for the warhead may be set before detonation to simulate different initial conditions for fragments and other parts of the warhead for trajectory calculations. Different types of warheads can be read if given in the correct format, including trajectory for the warhead if a guided warhead would be used.

1.1 Background

A typical fighter jet is capable of transporting and dropping multiple types of bombs for different purposes. The commonly used GPB (General Purpose Bomb) Mk82, Mk83 and Mk84 with weights of 500, 1000 and 2000 pounds are three examples of fragmenting warheads [1]. The amount of explosives and fragments may vary depending on type of warhead which will affect the safe separation distance between the aircraft and warhead. This might force the pilot to take manoeuvres to avoid getting hit, depending on how the warhead has been deployed.

![Mk82 warhead](image)

Figure 1.1: Mk82 warhead

Figure 1.1 displays a Mk82 warhead [3]. On the top of the warhead in figure 1.1, two fastening loops is located with their respective mounting plates. These are used to attach the warhead to the weapon pylon under the aircraft’s wing. These will have a relative high mass in comparison to natural fragments resulting in a potential
higher risk for the aircraft. The fastening loops are reasonably defined as cubic shape, giving them an advantageous aerodynamic shape according to Staffan Harling’s report [1].

1.2 Problem

A warhead generates a considerable amount of fragments after detonation. Fragments with low mass will be in majority with a decrease in number of fragments in relation to increasing mass. This holds for natural fragments. Fragments may have a high initial velocity after the warhead detonates, around 2000 meters per second. In comparison, an aircraft may fly up to around 700 meters per second and therefore fragments may catch up with the aircraft and hit it. The time from warhead release until warhead burst creates a distance from aircraft to warhead. After the warhead has been deployed, a certain amount of time passes until the warhead is fully armed. At this time and after is a possibility that the warhead unintentionally bursts.

It is of interest to analyse that the risk for the aircraft is sufficiently low at all times after the fuse arming time. The following problem statement summarise the problem:

- How can the safe separation distance be determined for an arbitrary aircraft deploying a warhead?

The safe separation distance is between the aircraft and warhead just before burst. Given data in order to do the analysis are aircraft trajectory, bomb trajectory and bomb fragmentation characteristics.

1.3 Objectives

The objectives of this thesis is to develop a method to evaluate and estimate the potential risk for an aircraft of getting hit by fragments from a deployed warhead.

This has been performed through creating and implementing a model that evaluates the risk of parts of the warhead hitting the aircraft. The model process data in Split-X format for fragments and excel format to read trajectories for aircraft and warhead.

The developed program in Matlab should be able to implement data from fragmentation of a warhead model and perform calculation for fragments projected trajectories. The calculation needs to regard the projected trajectory and velocity for the warhead, aircraft and fragments.

1.4 Research Methodology

The general methodology consists of the initial literature studies to gather sufficient information and understanding of fragmenting warheads, fragments and risk. Complete data with fragments for a stationary warhead can be simulated with Split-X software. In order to estimate safe separation distances related to the safe separation time, a Matlab code is developed to calculate trajectories. The first part of coding consist of read in data from Split-X and develop a segment for trajectory calculations. A ballistic code is used to calculate trajectories for fragments and other parts of the warhead. The fragment’s position is stored together with the global time, in order to calculate the distance between the projectile and aircraft depending on time. The fragment’s position can thereafter be compared to the aircraft in a global coordinate system to decide the distances. The distances are presented in a plot where the fragment with the shortest distance to the aircraft is plotted in every time step, resulting in a curve. An overall view of the distances of all analysed fragments for the calculations is included. Travelled distances for large fragments and other parts of the warhead are analysed separately. Rotations for the warhead before point of burst is also analysed to see variation in trajectories and fragments positions. These results combined for the distances gives an estimation of risk for fragments hit on aircraft in the context of safety distances.

1.5 Limitations

For fragments during trajectory calculations, tumbling is represented with the so called Cauchy area, the average exposed area for a fragment during trajectory. No consideration is taken to air stream and turbulence. Correct values of fragmentation data is usually classified, generic data is used as an example in this master’s thesis. No
evaluation of damage for the aircraft is concerned, only a risk perspective focused on safety distances. To evaluate
damage, in-depth knowledge is required considered the aircraft’s structure and material. Not only is it a difficult
task to carry out the assessment, but the information required is most likely classified.

Velocities for the mounting plates and fastening loops are set to respective around 500 and 600 meters per second
only for the section where initial rotation is varied for the warhead before burst. In the case of analysing the
travelled distance the velocities are increased to around 1600 meters per second. Velocities are set accordingly to
provide a suitable perspective dependent on the analysis.

Lift force is disregarded for trajectory calculations for fragments, mounting plates and fastening loops. Mount-
ing plates may possibly act as a rotating frisbee and therefore generate lift, but theory for adequate results with an
adoption to use in calculations is difficult to find.
Chapter 2

Background

The overall layout for analysing the problem is first described containing sections for aircraft, warhead and fragments trajectories. To work with each individual section, basic concepts such as fragment types, air drag, drag coefficient, gravity and Cauchy area is described. A section for mass distribution explains a suitable way to group fragments. Initial angles and velocities are described for fragments. Parts of the warhead that are of interest for risk evaluation such as mounting plates and fastening loops are described. Some models for evaluating safety distances and risk are presented.

2.1 Aircraft, warhead and fragment trajectories

The aircraft will travel on a defined trajectory with the warhead attached underneath. At a certain point, the warhead will be released and begin to travel on its own trajectory towards its target. After the warhead has detonated, fragments will spread in different directions on individual trajectories. All this results in the need to describe the overall problem into three main parts, aircraft, warhead, fragments and the initial values for one trajectory will be given from a previous one.

2.1.1 Aircraft

The aircraft follows a trajectory and at a point the warhead is released. The initial values for position and velocities in X-, Y- and Z-directions may be sent from a code section treating the aircraft trajectory further to the code section for the warhead. Two trajectories are considered where the initial angles (yaw, pitch, roll motion) may be varied at the point of the warhead being released.

Figure 2.1 describes aeronautical terms for rotations [4], The definitions for respective angle yaw, pitch and roll are held fix to prevent miss conception.
2.1.2 Coordinate transformation

To perform any sort of evaluation and comparison for velocities, positions and time, each system included needs to be expressed in a global coordinate system. The aircraft may be regarded as acting in a global coordinate system but the warhead and fragmentation process requires transformation from respective local to global coordinate system.

When transforming local to global coordinates, Euler angles are advantageous due to their definition and are widely used in the aeronautical industry. Each coordinate axis can be rotated in a process with directions for yaw, pitch and roll with three predefined rotation matrices. The angles between respective pairs of x-, y- and z-axis for local and global coordinate systems have to be decided for the typical situation. Otherwise, the transformation process is applied with the rotation matrices.

![Figure 2.2: Directions for Euler angles against global coordinate system, figure from reference: [5].](image)

Euler angles are shown in figure 2.2 in a global coordinate system [5]. The angles $\alpha$, $\beta$ and $\gamma$ in figure 2.2 represent the angular definitions used in this thesis and Matlab code. Both local and global coordinate systems are included with this notation.

2.1.3 Warhead

The warhead most likely follows a trajectory depending on initial parameters when deployed. The initial values that may be given from the code section for the aircraft may be used for the warhead. Further, the same type of values for position and velocities in X-, Y- and Z-directions may be sent to the code section treating warhead, fragments and their ballistics.

2.1.4 Fragments

To estimate trajectories for fragments it may be suitable to investigate the equations concerning ballistics. Data for each fragment may be extracted from a file generated by Split-X for a typical warhead like the Mk-80 series. Initial velocities, mass and rotations in three dimensions may be given for individual fragments that will be used in a trajectory calculation. The fragment in its trajectory can be seen as a point mass that is exposed for air drag in opposite direction from the resultant velocity component and gravitational force pointing to the centre of earth. The resultant velocity component can be divided in components in X- and Y-direction as given in the formulas in this section, originally from the report [6]. To transform calculations from three to two dimensions, the resulting velocity in the XY-plane may be denoted $v_{xy}$ that acts orthogonal to the velocity in Z-direction $v_z$. The formulas may then be rewritten to following:

$$M \cdot \frac{d(v_{xy})}{dt} = M \cdot \frac{d(v \cdot \cos \varphi)}{dt} = -F_D \cdot \cos \varphi$$  \hspace{1cm} (2.1)
\[ M \cdot \frac{d(v_z)}{dt} = M \cdot \frac{d(v \cdot \sin\varphi)}{dt} = -F_D \cdot \sin\varphi - M \cdot g \]  

Where \( M \) is the fragment’s mass that will be moved to the right side and gives the following respective equations:

\[ \frac{d(v_{xy})}{dt} = -\frac{F_D \cdot \cos\varphi}{M} \]  

\[ \frac{d(v_z)}{dt} = -\frac{F_D \cdot \sin\varphi}{M} - g \]

The angle \( \varphi \) locates from the horizontal plane referred to as XY-plane, against the air drag force component. The differential equations 2.3 and 2.4 should be solved in order to calculate the fragments trajectory and velocity. These two equations are not analytically solvable due to large variations in the air drag coefficient and a numerical approach is needed. These large variation occurs due to tumbling and rotation of the fragment during trajectory that is dependent on velocity and surrounding conditions. The accuracy during trajectory calculations is mainly dependent on the complexity of determining the drag coefficient explained in section 2.5.

### 2.2 Warhead model

There are several factors that need to be considered for a warhead from a risk perspective. Fragments from the burst of the warhead but also parts constituting the lifting attachments such as mounting plates and fastening loops. These parts are described in this section. A section is included treating the Split-X model and the mass distribution for the fragments which the model consist of.

#### 2.2.1 Fastening loops and mounting plate

The MK80 GPB is attached to the aircraft with two fastening loops and two mounting plates. These parts are large objects consisting of solid steel and will most likely not be fragmented when the warhead detonates. Data for the mounting plates and fastening loops is compiled in table 2.1. The large mass of these parts needs to be considered in ballistic calculations due to the long ranged trajectories they will follow [1]. The warhead has two plate shaped reinforcements intended to distribute forces when the warhead is attached to the aircraft. These mounting plates have a relatively large mass in comparison to fragments and it is possible for it to act as a unit after the warhead has detonated. The warhead hangs in two fastening loops as shown in figure 2.3. These fastening loops have in earlier work by Staffan Harling been modelled as a cube [7] during ballistic calculations. The cube shape has a favourable aerodynamic in comparison to natural fragments with equal mass. The average cross section area for the fastening loop matches the shape of a square rather than a circle when considering three viewing angles. The shape is therefore more like a cuboid than a sphere. The fastening loops are compact and have a high mass and has the potential to travel a considerable distance in comparison to fragments in the rest of the core casing.

![Fastening loop](image)

Figure 2.3: Fastening loop.

Figure 2.3 shows a fastening loop retrieved from the report [1].
<table>
<thead>
<tr>
<th>Part</th>
<th>Type</th>
<th>Mk82</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mounting plate</td>
<td>Plate</td>
<td>3000</td>
</tr>
<tr>
<td>Fastening loop</td>
<td>Cube</td>
<td>350</td>
</tr>
</tbody>
</table>

Table 2.1: Data for mounting plate and fastening loops for Mk82 warhead.

The mounting plate can be described either as a plate or natural fragment. Depending on how it will detach from the core casing during warhead burst, the two cases may be possible.

### 2.2.2 Split-X model

A model of a generic 500lb warhead may be suitable to use for calculations. With a developed Matlab code, a picture of the warhead may be obtained when plotting all fragments initial location, revealing its symmetric shape. The real Mk80 series shape is not as symmetric due to guiding fins and a long narrow structured shape that is more suitable from a aerodynamic perspective.

![MK80 form created from Split-X data](image)

Figure 2.4: All fragments for the 500lb model plotted with respective initial positions before trajectory calculations. Local coordinate system included.

The plot in figure 2.4 is almost looking solid due to the large amount of fragments it consists of, the total amount is 43502. Axes for the local coordinate system shows how the warhead is defined. The majority of the fragments for the 500lb warhead has a mass under ten grams. To better see the distributions, the fragments are divided into four classes containing a span for respective distribution. Fragments with a mass of less than one gram is excluded.
Figure 2.5: Fragments from 500lb Split-X model grouped with respect to mass. Fragments with a mass less than one gram are neglected.

The majority of fragments are under ten grams and regarding figure 2.5, it is observed that the amount of fragments decrease drastically from ten grams and up to 100 grams.

<table>
<thead>
<tr>
<th>mass class (grams)</th>
<th>1 - 2</th>
<th>2 - 5</th>
<th>5 - 10</th>
<th>10 - 100</th>
<th>&gt; 100</th>
</tr>
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<tbody>
<tr>
<td>amount</td>
<td>3781</td>
<td>6164</td>
<td>5340</td>
<td>14254</td>
<td>2367</td>
</tr>
</tbody>
</table>

Table 2.2: Data for mass distribution for 500lb Split-X model.

Fragments presented in figure 2.5 are summed regarding all stacks for each category in table 2.2. The amount of fragments is reduced from 43502 to 31906 when fragments with mass less than one gram is neglected. This is the sum of all the categories in table 2.2.
From a risk perspective, it is mainly fragments with a relative large mass that may be ejected in the direction of the aircraft. To further categorise and reduce the number of fragments to be considered in the analysis, a condition for velocities in global X- and Z-direction may be implemented to evaluate those fragments ejected in the first positive quadrant where the aircraft may be located. The total amount of fragments are reduced to 13292 and when excluding fragments with a mass less than one gram, the amount is further reduced to 9990.
Figure 2.7: Distribution of fragments with a mass over 1 gram from 500lb Split-X model. Velocities are positive in X- and Z-direction.

The reduced amount of fragments due to restrictions in mass and velocities are displayed for mass distributions in figure 2.7.

<table>
<thead>
<tr>
<th>mass class (grams)</th>
<th>1 - 2</th>
<th>2 - 5</th>
<th>5 - 10</th>
<th>10 - 100</th>
<th>&gt; 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>amount</td>
<td>1192</td>
<td>1947</td>
<td>1747</td>
<td>4813</td>
<td>858</td>
</tr>
</tbody>
</table>

Table 2.3: Data for mass distribution for 500lb Split-X model. Only velocities in positive X- and Z-direction allowed.

The amount of fragments is reduced for each category in table 2.3 in comparison to table 2.2.
Figure 2.8: Distribution over fragments with a mass over 0.1 kilogram from 500lb Split-X model. Restrictions with velocities in positive X- and Z-direction allowed.

The number of fragments in figure 2.8 is reduced in comparison to the unrestricted case in figure 2.6. Only fragments with a velocities in positive X- and Z-direction is allowed.

2.3 Fragments

Fragmentation depends on how a warhead is composed regarding type of explosives, geometry and material [6]. When treating geometry, parts of a warhead composed of a higher percent of explosives generates smaller fragments when the warhead burst. Which in opposite yields that for regions with a lower percentage explosives, larger fragments during warhead burst are generated [8]. This occurs due to the variation of acceleration of mass in the local region and how the shell expands before burst.

Fragmentation in general is categorised in three major sections depending on how they have occurred. Every type of fragment has crucial differences that affect the aerodynamic impact during trajectory calculations.

2.3.1 Natural fragmentation

No methods are used to control the fragments produced under fragmentation. The fragment distribution will depend on the type of explosives, thickness and size of the shell, material properties and the overall geometry [6].

2.3.2 Preformed fragmentation

Fragments that are shaped during manufacture depending on the type of target are called preformed fragmentation. It is a composition of fragments that can be held together by a shell [9]. It is often cubes or spherical metal parts that are packed in the shell to control the size and amount of desirable fragments. The drawback of a warhead with preformed fragments is reduced initial velocity and a considerably higher economic cost than for a warhead which gives rise to natural fragments. Preformed fragmentation is suitable when weight is an issue, as for military troops carrying a mine. An example is the American M18 Claymore mine with the Swedish name "försvarsładning 21".
2.3.3 Controlled fragmentation

During the manufacturing of a warhead, the amount of explosives can be varied in the warhead. This may give a variation of acceleration of the shell during burst, resulting in a pressure wave producing the desirable type and amount of fragments. Another type of controlled fragmentation is an intentional structural weakness of the shell, shaped to maximise the amount of desirable fragments. There are different ways to construct controlled fragmentation, a weakened structure could be moulded inside the shell or machined grooves could be notched [10].

To characterise fragmenting warheads, it is needed to determine the fragments mass, initial velocity, initial direction and typical shape of fragments for different classes chosen suitable for trajectory calculations[9].

2.3.4 Mass distribution

To estimate the mass distribution for fragments of a warhead after burst, formulas for estimation are given in this section. For natural fragmentation warheads, a software may be used to estimate the mass distribution such as Split-X. The mass distribution can also be estimated with Mott Linfoot’s relation or with Hörner’s equation for smaller warheads. A more time consuming approach would be to gather the information regarding mass distribution through experiments.

An early model is Hörner’s fragmentation formula that can be used for warheads with a diameter less than 200 millimetre. The total mass \( M \) of all fragments over a certain weight \( m \) is given by [8]. The total number of fragments that exceeds a certain mass is estimated with [8]:

\[
N(m_1 < m < \infty) = \gamma M_0 \int_{m_1}^{\infty} \frac{e^{-\gamma m}}{m} \, dm
\]  

(2.5)

Where \( M_0 \) is the fragment mass before warhead burst and \( \gamma \) is the fragmentation number given as [6]:

\[
\gamma = \gamma_0 e^{-(0.264d_y + 15.8 \frac{d}{d_y})}
\]  

(2.6)

Where \( \gamma_0 \) is an explosive and shell thickness dependent factor independent of geometry [9]. It is therefore unique for every combination of explosive and shell material [11]. The value \( d_y \) is the outer diameter of the shell and \( d \) the thickness of the shell. This model is only valid for bombs with a maximum diameter of 200 millimetres and thus not suitable for calculations and predictions for the Mk bomb series. Instead, Mott-Linfoot’s relation should be used for these larger bombs because it is valid for warheads with a diameter greater than 200 millimetres [11]:

\[
N(m) = \frac{M_0}{2M_k} e^{-\frac{m}{M_k}}
\]  

(2.7)

The term \( M_k \) is a relation corresponding to the fragmentation number \( \gamma \) and is determined with an empirical developed constant \( B_2 \), the inner diameter of the shell \( d_i \) and the thickness of the shell \( t \).

The experimental method to collect fragments contains a closed or semi open capsule containing sawdust or water, surrounding the bomb. A finite volume around the warhead is left with air to let the explosive shell expand undisturbed. After a burst, all fragments are collected and weighted to determine the number of fragments and their individual mass [9].

2.3.5 Initial angle and velocity

The Gurney equation predicts how fast an explosive will accelerate a surrounding layer of materials and therefore gives the initial velocity for a fragment as [9]:

\[
v_0 = \sqrt{2E} \left( \frac{M}{m} \right)^{\frac{1}{2}}
\]  

(2.8)

Where \( v_0 \) is the initial velocity, \( \sqrt{2E} \) the Gurney velocity for the chosen explosive, \( M \) the mass of the explosive and \( m \) the mass the core casing. The Gurney model is commonly used when fragments or projectiles are driven by explosives. The model does not consider effects of shock mechanics during burst and relies on the following assumptions [10]:

\[
\frac{M}{m} \gg 1
\]
• The gases produced during burst have a one-dimensional velocity profile in the spatial coordinate system with a uniform density.

• The released amount of energy during detonation is fixed for the given explosive.

The initial angle is approximated with Taylor’s pitch formula for an arbitrary fragment as:

\[ \sin \alpha = \frac{v_0}{2U_D} \sin \beta \]  (2.9)

Where \( v_0 \) is the initial velocity of the fragment, \( U_D \) velocity for detonation that directly depends on what explosive is used and diameter of the warhead. The angle \( \alpha \) is the angle between the normal to the boundary line and the resultant velocity component \( v_0 \). The angle \( \beta \) is the angle between the detonation initial direction (normal to the boundary line) and the normal line to the shells enclosed area [9]. The values \( \alpha, \beta \) and \( v_0 \) in equation 2.9 described in figure 2.9.

![Figure 2.9: The relation between quantities that describe the Taylor angle \( \sin \alpha \).](image)

2.4 Ballistic calculations

The ballistic trajectories of the fragments depend on a range of parameters of which the most important are described in this chapter.

2.4.1 Air drag

Air drag is the main reason for fragment retardation and is derived through the momentum equation. The drag is dependent on a projectile's velocity and exposed area against flow direction during trajectory. It is here given as a differential equation on the following form [12]:

\[ m_s \ddot{x} = -F_D = -\frac{1}{2} C_D \rho_{air}(h) A \dot{x}^2 \]  (2.10)

Where \( m_s \) is the projectile or fragment mass and \( \dot{x} \) the acceleration derivative in two steps from position. Where \( F_D \) is the air drag force, \( C_D \) is the air drag coefficient, \( \rho_{air}(h) \) the density of the surrounding air on the height \( h \) over the sea and \( A \) the fragment’s effective area in equation (2.10). Where \( C_D \) is dependent on the fragment...
geometrical shape and is seen as a constant for air velocities much lower or larger than the velocity of sound. This results in a span with given data points for the drag coefficient corresponding to mach numbers that may be used for calculations. The drag coefficient makes the momentum equation (2.10) difficult to solve due to the dependence for \( C_D \) to the trajectory \( x \). With the approximation that \( C_D \) is a constant, it becomes independent of \( x \) and can be rewritten as:

\[
\dot{x} = \text{constant} \cdot e^{\frac{C_D A \rho_{air} x}{2m}}
\]  

(2.11)

The initial conditions in equation 2.11 are set as \( v_0 \) at \( t = 0 \). This originates from \( x(t = 0) = 0 = v_0 \). Thus, the constant in 2.11 is equal to the initial velocity for the fragment, i.e. \( \text{const.} = v_0 \). Equation 2.11 can now be rewritten as following [13] [12]:

\[
v = v_0 e^{\left(-\frac{C_D A \rho_{air} x}{2m}\right)} = v_0 \cdot e^{-k_0 x}
\]

(2.12)

Where \( v \) is the fragment velocity, \( v_0 \) initial velocity and \( k_0 \) a constant defined in equation 2.13.

\[
k_0 = \frac{C_D}{2m} \rho_{air} A
\]

(2.13)

A fragment on a trajectory in air is affected by the downward gravitational force and air drag in opposite direction from the fragments velocity [6]. The air drag is given as the following formula rewritten for better understanding from equation (2.10):

\[
F_D = \frac{1}{2} \cdot \rho_{air} (\dot{h}) \cdot C_D \cdot A \cdot v^2
\]

(2.14)

In equation (2.14), \( v \) the resultant velocity component through the trajectory. The air drag coefficient \( C_D \) is e.g. dependent on the Mach number \( Ma \) given as:

\[
M = \frac{v}{v_s}
\]

(2.15)

In equation (2.15), \( v \) is the object’s velocity in relation to the surrounding air and \( v_s \) is the velocity for sound under the given circumstances. Conditions can vary such as the altitude that directly affect the temperature, air density and pressure. Therefore, the velocity of sound \( v_s \) will vary and is given by equation (2.16).

\[
v_s = \sqrt{\kappa RT}
\]

(2.16)

Where \( \kappa \) in equation (2.16) is dependent on the fluids atomic composition and usually given as the value 1.4 for air that is a diatomic gas. The gas constant \( R \) for air is 287 J/(kg·K) and the temperature \( T \) is measured in Kelvin. The value for the drag equation (2.14) is therefore dependent on the atmospheric conditions but also on the exposed area for a fragment during trajectory.

### 2.4.2 Cauchy area

Due to the inconsistency in stability for natural fragments during trajectory movement, the frontal area is not as easy to determine as for e.g. an artillery projectile that has a relatively stable trajectory with a rotation around its longitudinal axis. The natural fragment will most likely tumble. To determine an area, named effective area, for the fragment it is needed to decide a time averaged exposed frontal area.

The average displayed area for a fragment on a trajectory can be approximated with its Cauchy area [12]. For every body with an non concave boundary surface [14], the mean value of its projected cross section for all projection angles is equal to a fourth of the body’s total surface. For natural fragments, the Cauchy area is:

\[
A = \pi \cdot r^2 = \frac{\pi}{4} \cdot d^2
\]

(2.17)

Where \( r \) in equation (2.17) is the mean valued radius, \( d \) the mean valued diameter and \( A \) mean valued surface (Cauchy area) for the fragment. For fragments with a cubic shape, the Cauchy area will be given from the following formula:

\[
A = 3 \cdot \frac{a^2}{2}
\]

(2.18)

Where \( a \) in equation (2.18) is the side length of the cubic fragment. In cases where the fragment is a cuboid (shape of a plate), the Cauchy area can be calculated from the following equation [14]:

\[
A = \frac{ab + ac + bc}{2}
\]

(2.19)
Where \( a \) in (2.19) is the thickness, \( b \) the width and \( c \) the height of the plate.

### 2.4.3 Gravity

The gravitational constant will vary with the location on earth. Usually, an approximation is used and the gravitational constant \( g \) is set to 9.81 \( m/s^2 \). In calculations and estimations with higher fidelity, the gravitational constant can be calculated with the international gravitational model [15]:

\[
g = 9.78049 (1 + 0.0052844 \sin^2 \phi - 0.0000059 \sin^2 (2\phi))
\]

\( \phi \) in equation (2.20) is the latitude. The small numerical values in equation (2.20) do affect the gravitational constant in a way that \( g \) is 9.83 \( m/s^2 \) at the north and south poles and 9.78 \( m/s^2 \) at the equator. The equation is dependent on the distance from point of measurement to the centre of earth and not true altitude.

### 2.5 Drag coefficient

The drag in equation 2.14 is dependent on the Reynolds number related to velocity, projected area and surface smoothness for the fragment. Surface smoothness may be difficult to measure for fragments but affect the fragments drag coefficient and thereby its velocity. The drag coefficient is a considerable factor for a fragment’s travelled distance. Depending on shape of the projectile, the drag coefficient may vary depending on it’s velocity. This relation is usually experimentally produced and compiled in tables.

#### 2.5.1 \( C_D \) for Mk warheads

The drag coefficient for the Mk82 warhead will vary with velocities according to values given in a graph from a DSTO report [16]. The drag coefficient for this type of warhead is given in figure 2.10:

![Drag with corresponding velocities for Mk warhead](image)

Figure 2.10: Values for \( C_d \) from DSTO report [16] plotted against corresponding Mach number.

The drag coefficient in figure 2.10 increases drastically during transonic flow. It is relative low in comparison to natural and preformed fragments that will be shown in the following sections.
2.5.2 $C_D$ for natural fragments

An arbitrary fragment in vertical equilibrium in a trajectory is effected by the drag force $F_D$ and fragment weight $W_e$ given in [17]:

$$F_D = W_e = \frac{C_D \rho A v^2}{2} \quad (2.21)$$

Equation (2.22) gives equation (2.21) rewritten to clarify $C_D$:

$$C_D = \frac{2W_e}{\rho A v^2} \quad (2.22)$$

The drag coefficient for fragments in equation (2.22) demonstrates the dependence to the fragment weight and projected area. This relation may be used when determine the drag for a natural fragment. The method is unfortunately only valid up to a drag coefficient of 0.1 due to the equilibrium occurred in the air stream for this experiment in a vertical wind tunnel. It serves as an example to understand the theory and especially how the drag coefficient may be determined without interference from assembly stands and other instruments used to hold the fragment in place as if it were tested in a horizontal wind tunnel.

Values for the drag coefficient used for fragments trajectory calculations plotted against corresponding Mach number are given in figure 2.11. These values are achieved from extensive horizontal wind tunnel experiments combined with estimations for rotations and tumbling described by Frank McCleskey [17]. The data for drag coefficient of natural fragments is given from Split-X data and is published in the article [18].

![Figure 2.11: Values for $C_d$ from Split-X data plotted against corresponding Mach number.](image)

The drag coefficient for natural fragments in figure 2.11 is substantially larger than for the warhead in figure 2.10.

2.5.3 $C_D$ for mounting plates and fastening loops

The drag coefficient for mounting plates and fastening may be approximated as preformed fragments.
When comparing drag coefficient for preformed fragments in figure 2.12 against natural fragments in figure 2.11, the plotted curve shows similar values for respective Mach number. The data for drag coefficient of sphere and cube fragments is given from Split-X data and is published in Martijn van der Voort [18].

### 2.5.4 $C_D$ variation depending on tumbling or rotation

The drag coefficient may be difficult to approximate and especially for objects that are unstable during trajectory. The result is a variation in exposed area that may seem to vary arbitrarily. It may be of interest to analyse how the exposed area may vary and therefore affect the drag coefficient, especially for the thin plate shaped mounting plates and cubic shaped fastening loops.

This complete section 2.5.4 with theory regarding tumbling and rotation is based on the reasoning in the report Moxnes et.al "Projected area and drag coefficient of high velocity irregular fragments that rotate or tumble", Defence Technology 13 (2017) 269-280 [14].

Consider a fragment shaped as a parallelepiped. During supersonic velocities, a bow shock is formed in front of the parallelepiped that generates a restoring force, so the parallelepiped is statically stable [14]. The dynamic pressure (kinetic energy per unit volume of a fluid) is relatively much lower at transonic and subsonic velocities resulting in the restoring force which is not sufficient to stabilise the parallelepiped and tumbling is much more likely to occur. This may be interesting for fragments with a high velocity that act as parallelepipedes, namely the fastening loops.

If initial conditions for an air stream are held constant such as air density and velocity, the air drag equation (2.14) will be directly dependent of $C_D$. The drag coefficient is dependent on the fragments projected area described in the section 2.4.2. The dependence to area and mach number is as follows:

$$C_D = C_D \left( \frac{A_{\text{max}}}{A_C}, M \right)$$

(2.23)

In equation (2.23), $M$ is the Mach number, $A_{\text{max}}$ the maximum projected area of the fragment and $A_C$ the Cauchy area. The Cauchy area may be denoted as the projected area $A_P$ during calculations. Unfortunately the constant $A$ used in section 2.4.2 will here be given the name $A_C$ and is defined as fourth times the surface area $A_S$ of the fragment [14]. For simplicity, $M$ that denotes the Mach number dependency is now suppressed. The projected
area $A_P$ varies for fragments during flight because of tumbling and rotation. An infinitesimal thin plate is used for analysing tumbling and rotations since it has dimensions that are suitable for calculation in comparison to point masses. The relation between the projected tumbling area $A_T$, total area $A_{TOT}$ and Cauchy area for a thin plate is given by equation (2.24):

$$A_T = A_C = \frac{1}{4} A_{TOT} = \frac{1}{4} (A + A) = \frac{A}{2} \quad (2.24)$$

The infinitesimal thin plate in equation (2.24) is convex with area of each side of the plate $A$. For the projected area $A_P(\theta)$ the following relation holds:

$$A_P = A \sin(\theta), 0 \leq \theta \leq \frac{\pi}{2} \quad (2.25)$$

In equation (2.25), $\theta$ is the angle of attack with corresponding variation. To define the tumbling and rotation, approximations for probability densities for the angle of attack give:

$$Tumbling: \rho_T^T = \cos(\theta), Rotation: \rho_R^R = \frac{1}{\pi/2} \quad (2.26)$$

The fragments are rotating around an axis normal to the velocity vector and normal to the normal vector of the plate. Expectations for the projected area during tumbling are:

$$E_T(A_P) = \int_0^{\pi/2} A \sin(\theta) \cos(\theta) \, d\theta = \left[ \frac{1}{2} A \sin^2(\theta) \right]_0^{\pi/2} = \frac{1}{2} A \quad (2.27)$$

For the rotational case:

$$E_R(A_P) = \frac{1}{\pi/2} \int_0^{\pi/2} A \sin(\theta) \, d\theta = \frac{2A}{\pi} \quad (2.28)$$

So if the thin plate tumbles, the projected area is half from the maximal projected area $A$ in equation (2.27) and if it rotates the projected area is 30 percent larger in equation (2.28).

In the case for a parallelepiped that actually is a thin plate with height, rotation and tumbling appear to be slightly more advanced. The area of sides of the parallelepiped is denoted $A$, $B$ and $C$ with projected area as:

$$A_P = A \cos(\theta) + B \sin(\theta) \sin(\phi) + C \sin(\theta) \cos(\phi), \theta \leq \frac{\pi}{2}, \phi \leq \frac{\pi}{2} \quad (2.29)$$

The boundaries defined in equation (2.29) is limited to the first quadrant due to symmetry. The projected area for tumbling becomes:

$$E_T(A_P) = \frac{2}{\pi} \int_0^{\pi/2} \int_0^{\pi/2} (A \cos(\theta) + B \sin(\theta) \sin(\phi) + C \sin(\theta) \cos(\phi)) \sin(\theta) \, d\phi \, d\theta = \frac{A}{2} + \frac{B}{2} + \frac{C}{2} = A_C \quad (2.30)$$

The expected area in equation (2.30) is equal to the Cauchy area in equation (2.19). The parallelepiped is assumed to rotate uniformly with the axis of rotation normal to the velocity component. For a rotational motion, one side of the parallelepiped will not interact with the air flow and in this case the side $S$ is chosen. The parallelepiped therefore is approximated as two parallel plates merged together with area $A$ and $B$ that rotate. The projected area then becomes:

$$A_P = A \cos(\theta) + B \sin(\theta) \quad (2.31)$$

Where $\theta$ in equation (2.31) is the rotation angle. The projected area during rotation is estimated as:

$$E_R(A_P) = \frac{1}{\pi/2} \int_0^{\pi/2} (A \sin(\theta) + B \cos(\theta)) \, d\theta = \frac{2}{\pi} (A + B) \approx 0.64 (A + B) \quad (2.32)$$

Which gives a projected area in equation (2.32) around 30 percent larger than for tumbling.

### 2.5.5 Drag dependence on shape

The drag force is dependent on the shape of the object travelling in an air stream. A flat plate with the largest area facing the air flow, have a drag coefficient of 1.28 [19]. For comparison, a sphere can have a drag coefficient as low as 0.07 up to 0.5. Shape has a very large effect on drag in summary.
2.6 Risk

The definition of risk may vary depending on the actual case but usually consists of three main areas [20].

- The probability for a certain non desirable event to occur.
- An expression for the negative consequence of a certain event.
- An expression for the total estimate of probability and consequences.

To achieve a meaningful reasoning including the ability to discuss risk and actions for risk reduction, both the probability and consequences are concerned. Keep in mind that the risk may not always be measured in a numeric value as a probability from zero to one. Risk may be quantified as the concept to evaluate shortest distances between aircraft and fragments. It can be a difficult task to define how the risk should be measured and presented as a numerical value and therefore the approach of comparing least distances is a way of more easily grasp the concept.

The acceptable risk depends on the conditions, the risk for operations during war may vary from peace time. It is usually not possible to eliminate the risk completely, resulting in accepted values for a actual case. Different models can be used to calculate risk that includes the amount of explosives, distances to personnel and directions.

A risk perspective for military applications considering the distance between aim point and friendly troops is one way of measure risk when the probability of hit is regarded as low. This will be denoted as risk estimate distance. During training, the risk acceptance is regarded as low in comparison to military applications and named the minimum safe distance. This may be set based on the maximum allowed horizontal distance that a fragment may travel or by the ballistic limit for skin penetration [21].

How to approach the risk analysis depends on how vulnerable the aircraft is expected to be. In this work no consideration is taken of vulnerability of the aircraft. The risk of hit on an aircraft is directly dependent on the amount of fragments being generated during detonation. It is fair to suppose that the risk of being hit will decrease with distance from centre of burst. The risk will decrease until it can be regarded as zero, for relatively long distances.

2.6.1 Risk during war - risk estimate distance

The distance between an aim point (desired ground zero) and own personnel on the ground is usually described as risk estimate distance, only valid for combat situations. In other words, the distance in meters from the intended centre of impact at which a specific degree of risk and vulnerability will not be exceeded. The risk is usually expressed as the probability of incapacitation (PI), defined as the probability that a soldier will suffer an incapacitating injury. Air-to-surface risk estimate distances can be found in [22] for Mk82. If the probability of incapacitation is set to 0.1, the distance for Mk82 500lb warhead is 300 meters for airburst. If the (PI) is increased to ten percent, the corresponding distance is 135 meters.

Danger close is a term including that friendly forces are within close proximity of the target. It is used for close air support. Regulations for the Swedish armed forces compiled in [23] treats established information concerning close air support for personnel on ground. Depending on the actual conditions, the publication sets values for maximal accepted safety distance considering the allowed risk for personnel to be hit, altitude of aircraft when releasing warhead and templates of risk for calculating distances in height and sideways. Danger close is not the same as minimum safe distance or risk estimate distance. It allows the observer to inform the field artillery team of the close proximity to friendly forces.
The grey box in figure 2.13 indicates the restricted flight zone for aircraft in close proximity to friendly artillery.

### 2.6.2 Risk during peace - minimum safe distance

Minimum safe distance are the equivalent to risk estimate distance but for peace time. For a Mk82 500lb warhead, the minimum safe distance is set to 1200 meters. Values for minimum safe distance are acquired from the report [22] and are presented in the same way as risk estimate distances. Unfortunately, equations to calculate the minimum safe distance are left out in the report [22].

### 2.6.3 Danger zone

The danger zone is categorised into three main areas, laser surface, ground to ground and ground to air. The air to ground case is also called weapons danger zone including the minimum land and air requirement, to include terrain mitigation, needed to safely employ a given weapon [24].

The danger zone on ground level with corresponding ricochet and fragmentation zones in figure 2.14. The fragmentation zone contain fragments after warhead burst. It is an area that may fluctuate in size depending on the
global position for the fragments at a certain time. This means that for a longer time, fragments have travelled longer on their trajectory and the corresponding fragmentation zone have expanded. The fragments may be categorised by mass and initial velocities to effectively reduce the amount of fragments for calculations. Important parameters to define the danger zone are also initial velocities for aircraft and warhead, height of dropping and height of detonation.

2.6.4 Range safety distance and vertical danger area

The model for range safety distance can be used to estimate danger areas for ammunition destruction by open detonation on ground level [25]. The model used would be for bare exposed explosive only and is defined as a distance outside which no more than one fragment would be expected to fly and therefore not absolutely safe.

\[ D = K \cdot AUW^{1/6} \]  

(2.33)

The distance \( D \) in equation (2.33) given from \( AUW \) meaning 'All Up Weight of Ammunition or Bare Explosives' measured in kilogram. The equation treats single ammunition. For the case of public access concerning civilians, the constant \( K \) is set to 634. For controlled access, meaning no civilians or animals in the area, \( K \) is set to 444.

The vertical danger area limits an area to warn air traffic of demolitions on ground level. No ballistic parabola needs to be taken account of. The vertical danger area calculated with the following formula [26]:

\[ D = 314 \cdot AUW^{1/4} \]  

(2.34)

No consideration is taken to if the aircraft is civilian or military in equation (2.34).

2.6.5 Risk from a kinetic energy perspective

A major reason for categorise fragments as distributions according to mass is simplicity that gives an comprehensible view. To motivate and value the risk from different mass distributions it is necessary to analyse the impact energy for a potential hit on target. The energy is calculated from following formula:

\[ E = \frac{1}{2} M_{\text{fragment}} \cdot v^2 \]  

(2.35)

Where the energy at impact in equation (2.35) is dependent on the fragment’s mass \( M_{\text{fragment}} \) but also on the velocity \( v \) for the fragment at impact. The velocity range is quite narrow and considered as around 1800 meters per second during burst. The velocity for the fragment is reduced due to air drag and therefore also the kinetic energy. A fragment with a relative large mass have a large kinetic energy in comparison to a fragment with less mass travelling at the same velocity. From a risk perspective considering the kinetic energy, fragments with large mass will expose the aircraft for a risk of hit during a longer time after warhead burst with a devastating consequence if hit.
Chapter 3

Method

The structure of the method that has been developed has been divided into four main parts. They are with respect of order: Split-X generated fragment data including drag coefficient for mach numbers in the flow regime. A given aircraft trajectory. A given guided or unguided bomb trajectory and last the fragment calculation. A Matlab code has been developed to perform the analysis. The following steps are with respect of order, used in the program:

- Read aircraft trajectory data which will be in a global coordinate system.
- Read or calculate warhead trajectory. Calculations begin at the time when the aircraft released the warhead. This will be the release time and location.
- Calculate warhead burst location and orientation after the point of release.
- Read fragment data from Split-X and convert it to SI-units. The Cauchy area is also calculated.
- Transform fragment local positions to global coordinate system using warhead orientation and location at time of burst.
- Calculate fragment ballistic trajectories, keep data in global coordinates.
- Calculate distance between aircraft and each fragment at each evaluation time. Find shortest distance to the closest fragment at each evaluation time to conclude if fragments from the warhead are at a safe distance from the aircraft at all times.

Each part is adjusted to begin with values for the global time. This to facilitate the extraction of data from scripts when the global time is the parameter.

3.1 Read data files

The processed data file given by Split-X is in .pit format, a specific format for the Split-X program. A Matlab script is used to extract information to 26 columns presented in appendix in the table A.1. Each row describes individual fragment velocities, mass, position and Cauchy area. The data file is converted to SI units and thereafter transformed with Euler angles to global coordinates. Trajectories for aircraft and warhead may be read from excel sheets in .xlsx format. Only position and time is needed as in data to recreate corresponding velocities and therefore sufficient information for calculations.

3.2 Trajectories

One row from the extracted data including all predicted information for a fragment is sent from the main script in Matlab to a function that first divide the row into values and name them as suitable variables. A matrix for drag coefficient \( C_D \) is created from given data. Cauchy area, velocities in X- Y- and Z-direction, absolute velocity, the mass of the fragment and the angle between XY-plane and absolute velocity is obtained. The thermodynamic constant \( \kappa \) and gravitational constant \( g \) is set. The positions in X-, Y- and Z-direction are given in global coordinates for the trajectory of the warhead. The variables are now sent to scripts for calculating the fragment’s trajectories after burst.
A code for trajectory calculations originated from artillery shell calculations where first developed for an overall understanding and can be found in the appendix B.1. As mentioned earlier, the preferred method for trajectory calculations was chosen to be the Runge–Kutta model due to the significantly higher accuracy for fragments with relative low mass. The two models trajectories with corresponding absolute velocities are verified against data for an artillery shell that may be read more about in the appendix B.2. The correctness for the models is also confirmed with comparisons for drag coefficient calculations presented in the appendix B.2.1.

3.2.1 Runge-Kutta

The method used for trajectory calculations is Runge-Kutta. The problem is transferred from three dimensions to two dimensions meaning that velocities in X- and Y-direction will be joined to a velocity in the plane. This velocity in the XY-plane is perpendicular to velocity component in Z-direction. After calculations, transformation back to three dimensions occur.

Values for the drag coefficient, velocities, position, time step, global time, gravitational constant and load factor for the fragment are sent to the Runge-Kutta calculator. The current altitude $h$ is calculated as:

$$h = z + v_{z1s} \cdot dt$$

(3.1)

Where $v_{z1s}$ is the initial velocity in Z-direction in equation (3.1). The altitude is used to get the air density $\rho$ used to calculate the interpolation constant:

$$C_0 = \frac{1}{2} \cdot \rho \cdot A_m$$

(3.2)

The interpolation constant $C_0$ is used in the equation for velocity change during iterations. Equation (3.1) and (3.2) are held constant during the four iteration that Runge-Kutta consists of that will be described as following parts:

- Determine an absolute velocity and movement in XY-plane and Z-direction at current altitude. The movement is decided from the initial velocities multiplied with the time step.

$$v_0 = \sqrt{v_{xy1s}^2 + v_{z1s}^2}$$

(3.3)

The absolute velocity $v_0$ in equation (3.3) is in global coordinates, meaning that $v_{xy1s}$ is the velocity in XY-plane and $v_{z1s}$ is the velocity in Z-direction.

$$k_1 = v_{x1s} \cdot dt \quad l_1 = v_{z1s} \cdot dt$$

(3.4)

The movement $k_1$ and $l_1$ in respective XY-plane and Z-direction is given in equation (3.4).

- Linear interpolate is performed in each direction over the drag coefficient matrix with the the current mach number. The interpolation should be multiplied with the negative interpolation constant $C_0$, the absolute velocity $v_0$ and respective movement $k_1$ and $l_1$ in both directions. There should also be the negative gravitational constant and the time step multiplied to the interpolation in Z-direction. The result of the interpolation is the velocity change in XY-plane $m_1$ and in Z-direction $n_1$.

- The velocities in XY-plane and Z-direction needs to be updated. This is done by multiplying the current velocities with a mean valued change, with the newly interpolated value in respective direction.

$$v_{x2s} = v_{xy1s} + \frac{1}{2} \cdot m_1$$

(3.5)

$$v_{z2s} = v_{z1s} + \frac{1}{2} \cdot n_1$$

(3.6)

The variable $m_1$ in equation (3.5) is the interpolated value in XY-plane and $l_1$ in equation (3.6) the interpolated value in Z-direction.

The first cycle is now complete. The updated velocities will be input data for the same procedure in three more identical cycles with a total of four cycles. After the four cycles complete, the results for position and velocities are given as weighted mean value of the four interpolation steps for each variable.

26
3.2.2 Aircraft

The aircraft trajectory is read from an excel document, with initial values for positions and time that is used to calculate velocities. The aircraft’s coordinate system is concerned as the global system. All following actions for local coordinate systems will be transformed using Euler coordinates to norm it correctly. The aircraft trajectory used in the analysis and calculation is an example and all data is compared against it.

3.2.3 Warhead

The warhead is considered unguided and therefore treated as a descending projectile. The trajectory is calculated with the Runge–Kutta method for initial conditions given from the interpolated point of release from aircraft. It is possible to read warhead trajectory data according to the same procedures as for the aircraft trajectory.

3.2.4 Fragments

A warhead will produce a large amount of fragments during burst. The generic warhead used which is described in section 2.2.2 consists of a total of 43502 fragments. To reduce the number of fragments, restrictions for velocities can be implemented to only consider fragments with a positive velocity in X- and Z-direction resulting in a reduction to around a fourth of the total amount of fragments. The velocity restriction in X-direction depends on the orientation of the warhead in the global coordinate system and how they travel on their trajectory. The fragments direction therefore is pointing in both the trajectory direction for the aircraft and warhead that naturally should expose the aircraft for risk.

3.2.5 Mounting plate and fastening loop

Calculations for the mounting plates and fastening loops are performed with the same code as used for fragments. The difference is the initial values given in a 26 column row matrix in the same sequence that the code transforming .pit files generates. The main difference is the ability to set an initial angle $\alpha$, $\beta$ and $\gamma$ that will simulate a motion corresponding to pitch, yaw and roll. The code calculates trajectories for the projectiles for a given span of angles reaching from negative to positive values to simulate a motion around the neutral line. The steps for the rotational sequences is set by the user for ability to calculate a desirable range.

3.3 Risk

The aircraft has an exposed vulnerable area, typical the bottom because the aircraft will most likely not fly under the warhead it has deployed. Fragments will travel on their trajectories out from the point where the warhead detonated in an expanding cloud. The frontal area on these trajectories will expand and greatly increase but contain the same amount of fragments which reduces the ”density” of fragments per volume.

The risk of fragments hitting the aircraft will be evaluated from a least distance approach. Depending on the size of fragments the travelled distance will vary with mass, but the amount of fragments of higher mass classes will drastically decrease the larger the mass is. To estimate a risk area after detonation that may be a suitable approach for a risk comparison approach, a large fragment (dependent on the mass distribution) is used in calculations for fragments trajectory. The area is a convex hull enclosing all fragment’s trajectory points at a certain evaluation time. So naturally, the convex hull will likely expand with increased time after warhead detonation.

3.3.1 Safety distances

The minimum safety distance will be a parameter for the user to type in for the Matlab code. It gives a understandable visualisation if the aircraft is sufficiently far away from fragments or if burst time need to be changed. To perform comparisons with existing risk models, the perspective with risk area is considered and calculated.

The estimations for the risk estimate distance and range safety distance consider burst of warhead on ground level. To compare the tabulated values, an approximation for the risk area of the aircraft to warhead needs to be considered. Independent on where the warhead burst after deployment, fragments with a positive velocity in X- and Z-direction are considered resulting in a half sphere expanding in the positive Z-direction.
3.4 Fragment data reduction

A warhead model usually generates a large amount of fragments. Due to the large amount of fragments, calculations took considerable time and reduction was needed. A limit for the lowest mass was set to adjust the calculations only concerning fragments expected to travel a certain length. Fragments with positive velocity components in X- and Z-direction is a potential threat for the aircraft on its trajectory and therefore a suitable reduction. This will summarise to a sector in the first positive quadrant defining a fourth of the warhead. The sector may vary depending on the symmetry of the model, an authentic model surely wouldn’t be modelled as absolute symmetric and resulting in another division.

3.5 Condition for termination

Trajectory calculations for fragments, mounting plates and fastening loops have different condition for termination. Calculations for fragments with relative small mass are terminated when the velocity in Z-direction is zero and therefore the fragment has reached its highest point on trajectory. For fragments with larger mass, the travelled length during trajectory may be of a distance that the fragment is ahead of the aircraft and therefore exposes it for a risk of hit when the fragment falls free from the highest point on its trajectory. This also holds for the mounting plates and fastening loops, mainly due to the large mass in comparison to fragments. The condition to cancel the trajectory calculations is therefore set to when the projectile’s height (Z-direction) is equal to the lowest global altitude of the aircraft trajectory. The unguided warhead trajectory is calculated until the warhead reaches ground level.

3.6 Coordinate transformation

The aircraft, warhead and fragments are treated in separate coordinate systems but needs to be put in a global coordinate system for interconnection. The coordinate axles are transformed with Euler angles and verified with controlled rotations for chosen fragments of the warhead.

3.6.1 Euler angles, Tait-Bryan notation

The rotation of coordinate axes in local system is transformed individually with the respective rotation matrix to be correctly described in a global system. Each rotation is described with the following three matrices:

\[ R_z(\gamma) = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \] (3.7)

\[ R_y(\beta) = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix} \] (3.8)

\[ R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix} \] (3.9)

A yaw motion is described by matrix 3.7, a pitch motion by matrix 3.8 and an roll motion by matrix 3.9. The angles \( \alpha \), \( \beta \) and \( \gamma \) are defined according to figure 2.2.

\[ R_{\text{total}} = R_z(\gamma) \cdot R_y(\beta) \cdot R_x(\alpha) \] (3.10)

The full transformation for a extrinsic rotation is described by the total rotational matrix 3.10.

3.6.2 Implementation

The transformation matrix is placed in a separate function to easily be called upon to transform values in matrices. The data from Split-X is sent to this function with burst data to transform local coordinates to global. Interpolated values from the warhead trajectory for the time where the warhead detonates constitutes the burst data. The results are given in a correctly described global coordinate system ready to be sent to fragmentation calculations.
3.7 Warhead rotated with Euler angles

The warhead is able to rotate around its x-axis (in the trajectory direction) during trajectory that means that all fragments on the first half of the warhead has an equal probability of having a positive velocity in Z-direction and therefore may be directed towards the aircraft. This is valid if the warhead is positioned decently horizontal. If the warhead deviates substantially from its projected trajectory, more or less fragments than the typical fourth may be included in the simulation. The warhead may be rotated around its Z- and Y-axis for an arbitrary angular set to simulate a warhead wobbling as a yaw and pitch motion. This results in adjusted initial angles for fragmentation for warhead burst.

3.8 Distance between aircraft and fragment

The implementation of measurement for distance between fragments and aircraft is the difference in absolute distance with global time as parameter. Data from the flight trajectory needs to be adjusted and the corresponding matrix begin with values at burst time and following. Fragments are filtered depending on mass and velocity in X- and Z-direction. For matrices standardised by length, the difference in X-, Y- and Z-direction is calculated column wise and the absolute distance thereafter. The minimum distance is extracted from the distance matrix with a \textit{min} command in Matlab.

3.8.1 User input safety distance

The safety distance between aircraft and warhead may vary depending on the users preferences. An "user input safety distance" can be set by the user of the Matlab code to compare a reference value against minimum distance plots for a fast evaluation. The safety distance is set to 100 meters as a starting value.

3.9 Warhead model

The model used for calculations originates from Split-X and is initially described in section 2.2.2. The model is symmetric and an approximation of a real Mk82 warhead considering weight and measurements. Positions for the mounting plates and fastening loops on the Mk82 shell are estimated visually.

3.10 Mounting plate and fastening loop

Data achieved from Staffan Harling’s report [1] are used for the attachment plates and fastening loops for the Mk82 warhead and are used to calculate trajectories. The warhead is attached to the aircraft’s wing with two fastening loops connected to the warhead through respective mounting plate. The velocities for the mounting plates and fastening loops are set to 600 meters per second for an investigative purpose. The trajectories are calculated with the same method as for fragments, using the same values for drag coefficient with corresponding Mach numbers. Initial values concerning mass, Cauchy area and angle to warhead x-axis are given in Staffan Harling’s report [1].

The initial position for both fastening loops and mounting plates depends on the rotation in respective direction for roll, pitch and yaw. To evaluate the possible trajectories to calculate, a preset span for angles can be set with corresponding steps. The mounting plates and fastening loops are positioned in pairs in two locations on the surface of the warhead resulting in total of four objects.
Figure 3.1: Angle variation from the nominal direction for variation in roll, pitch and yaw for warhead. The nominal direction should be viewed equivalent for respective angle variation.

The nominal direction in figure 3.1 for the mounting plates and fastening loops are set from calculations for the warhead. The user defines a span for the angles named "roll", "pitch" and "yaw" that will be used in both positive and negative direction during calculations, e.g. ten degrees in figure 3.1 results in a total span of twenty degrees. The evaluation angle is also defined by the user, that will give fixed values for the span during trajectory calculations. So the "span for an angle" is divided in steps with the "evaluation angle". Trajectories are calculated for all evaluation angles and the end points for the trajectories, at the highest point on trajectory, constitutes the boundaries for a convex hull serving as a risk area.

3.11 Perturbation - sensitivity analysis

To estimate the robustness of the calculations and understand how variation of different variables will affect the results, a variation with a given percentage is desirable. The percentage is set to ten percent and will be applied to velocity and drag coefficient. The different distributions of mass for the warhead includes a sensitivity analysis in the process as variation in mass already occur due to the large amount of fragments with varying masses. The same also applies to initial angles where a variation in $\alpha$, $\beta$ and $\gamma$ gives rotation sequences for potential placement in global coordinate system during burst.

The theory presented in section 2.5.4 exposes the uncertainty when determine the drag coefficient for fragments under rotation and tumbling. According to the results in report [14], the drag coefficient may vary rigorously for a thin plate and a parallelepiped. This motivate a variation in drag coefficient to be analysed. The value is set to a variation of ten percent for the drag coefficient for both mounting plates and fastening loops. During rotation, the expected projected area for the plate is the Cauchy area with a 30 percent increase that will be used as variation in calculations for the mounting plates. The fastening loops will most likely not rotate according to theory in section 2.5.4 and are therefore not analysed with a higher expected projected area.
Chapter 4

Results

Presented results originates from calculations for the 500lb warhead model. Calculations for fragments trajectories for different mass intervals is presented. An approach where kinetic energy for fragments is calculated to motivate a reasonable safety distance between aircraft and fragments is also presented. Trajectory calculations for mounting plates and fastening loops are calculated for a case where these fragments following the global XZ-plane, meaning no variation sideways for trajectory. This to estimate the travelled distance for the fastening loops and mounting plates depending on front or back position. To further investigate how the trajectories vary for the mounting plates and fastening loops during a potential roll, pitch and yaw motion for the warhead during deployment, trajectories are calculated for a span of rotation in respective direction presented as a convex hull. The Mounting plates and fastening loops have the potential to rotate and tumble depending on the current flight regime. Trajectories are calculated for several cases.

4.1 Kinetic energy variation with time and fragment mass

Fragments with a mass less than 0.1 grams is neglected due to the minimal kinetic energy linked to the strong retardation due to air drag for relative short trajectories. 80 Joules is considered as sufficient to put a soldier in a non-combat condition and seen as a value to compare with. The initial velocity of around 1800 meters per seconds for fragments independently of mass, results in a fragment travelling approximately 1000 meters under the first second of trajectory. For fragments with a mass of 1 gram and above, comparisons is presented in the following table:

<table>
<thead>
<tr>
<th>[kg]</th>
<th>0.001</th>
<th>0.002</th>
<th>0.005</th>
<th>0.01</th>
<th>0.05</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>[J]</td>
<td>0.63</td>
<td>2.30</td>
<td>13.37</td>
<td>49.49</td>
<td>808.95</td>
<td>2498.08</td>
</tr>
</tbody>
</table>

Table 4.1: Kinetic energy one second after burst for fragments in respective distribution.

In table 4.1, trajectories are calculated up to one second after burst for fragments in each class for those fragments in the ±2% span. The initial velocity is taken from the mean value over all fragments in each category where the velocity in X- and Z-direction is positive. The total amount of fragments are therefore reduced to a fourth for the symmetric generic warhead that as been used. If the time for fragment calculation is increased to two seconds, kinetic energy is occurring to:

<table>
<thead>
<tr>
<th>[kg]</th>
<th>0.01</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[J]</td>
<td>12.76</td>
<td>228.27</td>
<td>734.84</td>
<td>2245.30</td>
</tr>
</tbody>
</table>

Table 4.2: Kinetic energy two seconds after burst for fragments in respective distribution.

The impact energy has drastically decreased for respective typical mass in table 4.2 in comparison to the case for one second. Fragments with a relative large mass has still a high impact energy and the same procedure will follow for a later time:
<table>
<thead>
<tr>
<th>[kg]</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>[J]</td>
<td>64.39</td>
<td>208.58</td>
<td>649.29</td>
<td>1514.23</td>
</tr>
</tbody>
</table>

Table 4.3: Kinetic energy four seconds after burst for fragments in respective distribution.

Fragments with a mass of around 0.35 kilograms has been added in table 4.3 that can be compared to the Mk82 fastening loop.

4.2 Distance between aircraft and fragments

Fragments from the first quadrant, meaning velocities in X- and Z-direction are considered. Due to the symmetrically shaped Split-X model, one quadrant represent approximately a fourth of the warhead if the distribution of fragments for mass and directions is fairly equal for the whole warhead.

The burst time will be set between two and four seconds for calculations. The burst time is defined as the time after the warhead has been deployed from the aircraft until it detonates. The shortest distance in the following sections is calculated as the shortest distance for all fragments at every time step, thereafter the smallest value is selected and plotted in the graph.

![Trajectory for aircraft and fragment](image)

Figure 4.1: A visualisation of the measured shortest distances (dashed lines) between aircraft (blue line) and a fragment (orange line) at four evaluation times. The blue dot is the start of the fragment's trajectory.

The blue line in figure 4.1 is a part of an aircraft trajectory. The orange line is a fragments trajectory after warhead burst. Burst occurs in the blue dot in the end point of the orange line. The dotted lines are the shortest distances between fragment and aircraft at evaluation times. At evaluation times, distances for all fragments in calculations is measured and the shortest is presented.

4.2.1 Fragment with median mass, 10.8 grams

When excluding fragments with a mass less than 1 gram, the total amount of fragments is reduced to 31906. The corresponding median is then 10.8 gram and in the span of 10.6 to 11.0 grams, 98 fragments remain that will serve as type fragments.
Figure 4.2: Release time is two seconds, bomb trajectory three seconds and burst time is five seconds. Distances calculated with time step 0.01 seconds.

The user input safety distance of 100 meters are not fulfilled for the burst time three seconds as illustrated in figure 4.2. There are fragments too close to the aircraft during a period of slightly over one second, the local minimum at a distance of 5.5 meters. The distance increases after 5.5 seconds without any exceptions and therefore the X-axis is adjusted to maximum of seven seconds in figure 4.2 for advantageous visualisation. Since arming time of three seconds is not enough, figure 4.3 gives the corresponding results with arming time of four seconds.

Figure 4.3: Release time is two seconds, bomb trajectory four seconds and burst time is six seconds. Distances calculated with time step 0.01 seconds.
The condition for safety distance of 100 meters is here fulfilled with a local minimum of 103 meters. For the burst time set to four seconds after release, the warhead has travelled 261 meters in negative Z-direction from point of release from aircraft.

### 4.2.2 Fragments with mass over 100 grams

The amount of fragments over 0.1 kilogram with positive velocities in X- and Z-direction is 858.

Figure 4.4: Release time is two seconds, bomb trajectory four seconds and burst time is six seconds. Distances calculated with time step 0.01 seconds.

The minimum distance in figure 4.4 is only eleven meters and well under the safety distance.

### 4.2.3 Mass between 1 and 2 grams

Fragments in the span of one to two grams of mass will travel the shortest distance in comparison to other mass intervals analysed. The burst time is set to four seconds. The amount of fragments in this mass interval is 1192.
The safety distances is well over the least accepted value of 100 meters that could be observed both in the graph and heat map in figure 4.5. The minimum distance is 180 meters at 6.45 seconds. The heat map presents the least distance to aircraft for every fragment in the given interval. The shortest distance plot shows the minimum distance of the fragments and the aircraft in every time step.

4.2.4 Mass between 2 and 5 grams

The amount of fragments for this distribution has increased from the lower one and is 1947.

4.2.5 Mass between 5 and 10 grams

The current interval of mass consists of 1747 fragments.
4.2.6 Mass between 10 and 100 grams

The current interval of mass consists of 4813 fragments.

Figure 4.7: Release time is two seconds, bomb trajectory four seconds and burst time is six seconds. Distances calculated with time step 0.01 seconds. The X-axis on the left plot is adjusted to a maximum of eight seconds for a suitable visualisation of the graph. The colour bar on the heat map is measured in meters.

The minimum distance is 111 meters at 6.6 seconds in the shortest distance plot in figure 4.7.

4.2.7 Mass over 100 grams

The current distribution of mass consists of 858 fragments.

Figure 4.8: Release time is two seconds, bomb trajectory four seconds and burst time is six seconds. Distances calculated with time step 0.01 seconds. The X-axis on the left plot is adjusted to a maximum of eight seconds for a suitable visualisation of the graph. The colour bar on the heat map is measured in meters.

There are a bunch of fragments with a short distance to the aircraft. The shortest distance plot in figure 4.8 indicates several direct hits with a minimum distance of three to seven meters.
Figure 4.9: Release time is two seconds, bomb trajectory four seconds and burst time is six seconds. Distances calculated with time step 0.01 seconds. The X-axis on the left plot is adjusted to a maximum of eight seconds for a suitable visualisation of the graph. The colour bar on the heat map is measured in meters.

The minimum distance is seven meters at 6.3 seconds in the shortest distance plot in figure 4.9.

4.3 Sensitivity analysis - velocities

The effect of a ten percent velocity increase is studied. The mass class with fragments over 100 grams is used as reference values.

Figure 4.10: Release time is two seconds, bomb trajectory four seconds and burst time is six seconds. Distances calculated with time step 0.01 seconds. The X-axis on the left plot is adjusted to a maximum of eight seconds for a suitable visualisation of the graph. The colour bar on the heat map is measured in meters. The dashed line in the left figure consists of calculations originated from original velocity. The solid line is calculated with an ten percent velocity increase.

The shortest distance up to 6.4 seconds seems to be unchanged for the two graphs in the left figure 4.11. The global minimum at 6.3 seconds have a reduced value to half from fourteen meters to seven meters. This holds for one of the fragments with the shortest difference to the aircraft. The heat map displays blue areas that reveal an not negligible amount of fragments with a distance to the aircraft of 100 meters or down to 80 meters. A comparison for velocity is also done for the distribution of mass in the span of ten to hundred grams as follows:
The shortest distances between aircraft and one of the fragments in the mass class at every time step in figure 4.11, is approximately the same for the two velocity regimes. The shortest distance is under twenty meters for a relative long time for this mass class containing 4813 fragments. The heat map indicates with blue regions that their exist a not negligible amount of fragments with a shortest distance of under hundred meters.

### 4.4 Distance depending of fragment size

Trajectory calculations have been done for the three largest fragments with positive velocities in X- and Z-direction for the Split-X model. These fragments are by far the largest of the natural fragments. Other large projectiles are the mounting plates and fastening loops. The initial conditions for trajectory calculations are the point of burst for the warhead after four seconds.
4.5 Mounting plate and fastening loop

The trajectories for mounting plate and fastening loops are calculated. The initial absolute velocities are set to 1612 meters per second for both the mounting plate and fastening loop. The chosen velocity is set as a reference value compared with other velocities during calculations. The velocity in X-direction is set to 800 meters per second and 1400 meters per second in Z-direction.
The same values for drag coefficient during calculations is used as for natural fragments, this makes the travelled distance more dependent on the projectiles mass. This due to lack of data. The mounting plate and fastening loop are not rotating during trajectory in figure 4.13, but they are tumbling according to the theory for Cauchy area. The fastening loop reaches an altitude of 740 meters after 13.3 seconds. Ground level is reached after 31 seconds and the maximum travelled distance is then 685 meters. The mounting plate reaches an altitude of 1230 meters after 15.8 seconds. Ground level is reached after 38 seconds and the maximum travelled distance is 1186 meters.

Calculations for mounting plates and fastening loops with a drag coefficient data related to preformed fragmentation is displayed to evaluate any potential difference concerning drag.
Figure 4.14: Comparing trajectory for mounting plate and fastening loop. Drag coefficient data for preformed fragmentation has been used during calculations.

Trajectories in figure 4.14 for both the mounting plate and fastening loop are similar to the trajectories in figure 4.13. The fastening loop reaches a maximum of 748 meters in altitude and 693 meters travelled distance. The mounting plate reaches 1243 meters of maximum altitude and 1201 meters of travelled length.

### 4.6 Calculations of risk

For a burst time of three seconds, the absolute distance between aircraft and warhead is 676 meters and for a burst time of four seconds giving 904 meters. These distances would serve as the radius of the half sphere describing the risk area depending on the safe separation distance.

The vertical danger area corresponds to the distance between aircraft and warhead in Z-direction. For a burst time of three seconds, the distance is 32 meters and for a burst time of four seconds it is 5 meters.

<table>
<thead>
<tr>
<th>Mk82 warhead</th>
<th>Explosives [kg]</th>
<th>Range safety distance [m]</th>
<th>Vertical danger area [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>87</td>
<td>934</td>
<td>1391</td>
</tr>
</tbody>
</table>

Table 4.4: Safety distances for Mk82 warhead, K is set to 444 in calculations of vertical danger area.

The range safety distance given in table 4.4 is calculated for the case of controlled access.

### 4.7 Rotations for warhead after released

Trajectories for mounting plates and fastening loops are calculated for rotations with angles $\alpha$, $\beta$ and $\gamma$. The rotations is further described in section 3.10. Each part of the mounting plates and fastening loops will be presented.
individual, resulting in four cases. A segment from a typical aircraft trajectory has been used for comparison against the warhead trajectory. The distance is measured from the mounting plates or fastening loops point on its trajectories against the location of the aircraft at given time. This means that each rotational series gives a certain point in the convex hull as measurement point. Observe that the distance is not measured from the whole set of convex hull against the aircraft’s location. The mounting plate and fastening loop is assumed to separate during burst and be treated as individual object. The velocities for the mounting plates and fastening loops are set to 603 meters per second to give a reasonable distance between fragments and aircraft during the first seconds after warhead burst. The reason is rather to use the elaborated methodology rather than analyse numerical values.

![Trajectories for aircraft and warhead](image)

Figure 4.15: Trajectories for aircraft and warhead. The point when the aircraft deploy the warhead is marked with a yellow dot.

The aircraft trajectory used in this section is plotted in figure 4.15 together with the calculated warhead trajectory. The warhead trajectory begin when the warhead is deployed from the aircraft and marked with a yellow dot.

### 4.7.1 Fastening loop and mounting plate in front position

The angles $\alpha$, $\beta$ and $\gamma$ is set to ten degrees variation around untouched position each, meaning a total of 20 degrees for the span. The rotations are analysed in steps of two degrees. The time step is set to 0.1 seconds for both aircraft and warhead trajectory calculations. The initial velocity are the same for both the mounting plate and fastening loop and set to 603 meters per second.
A convex hull is produced for every sixth time step and shown in figure 4.16. The burst time is set to two seconds and the convex hull is spread above the aircraft’s positions already after 0.6 seconds. The green circles are the position of the aircraft on its trajectory and the blue circles are the corresponding warhead position for the same time if it had not detonated. The blue circles are left to compare velocities for the aircraft and warhead for a understanding of their relation linked to burst time. Each convex hull is corresponding to these positions to measure distances at the same time.

So the first green and blue circle are respective positions for aircraft and warhead at the same evaluation time. The smallest convex hull are the first produced and corresponds to the first circles meaning they are all valid at the same time for comparison. Each pair of circles corresponds to one convex hull for respective time that originates from fragments during burst.

The aircraft and warhead trajectories have limitations on all axis to better present the result from a visual perspective. This will make the figures hard to compare in another way than distances from convex hull to aircraft. The results in this section should be seen as an visual explanation. The idea is to compare distances between the convex hull and the aircraft at different times and from that evaluate a suitable burst time. Where if the burst time is increased to four seconds and the same case is converted to the following:
Figure 4.17: Risk area for front fastening loop with burst time of four seconds. The first blue circle are the position of warhead during burst. The first green circle are the position of the aircraft when warhead bursts.

The set of convex hulls is positioned lower when the burst time is set to four seconds in figure 4.17 than for the hulls for burst time of two seconds in figure 4.16. The set of convex hulls for corresponding position of the aircraft at given time, is always behind the aircraft. The axes are adjusted in figure 4.17 for a suitable visual presentation and to distinguish the convex hull to the aircraft’s evaluation positions.
Figure 4.18: Risk area for front fastening loop with burst time of three seconds. The first blue circle are the position of warhead during burst. The first green circle are the position of the aircraft when warhead bursts.

When the burst time is set to three seconds as in figure 4.18, the convex hull will be in front of the aircraft in the second point of measurement (1.2 seconds). After three seconds, the aircraft is again in front of the convex hull area.
Figure 4.19: Risk area for front fastening loop with burst time of three seconds. The convex hull plotted is valid for the time of three seconds after warhead burst.

The convex hull area will tangent the aircraft after three seconds, visualised in figure 4.19 for different points of view. Only the convex hull at the time of three seconds is plotted in the graphs of figure 4.19 to show the relation between the aircraft trajectory and the risk area. The idea is to present the size of the convex hull in comparison to the aircraft’s position marked as a red circle. The convex hull seems to cross the red circle. The yellow dot is the aircraft’s position when warhead burst.
The distance between the aircraft and front fastening loop for all rotational sequences is plotted in figure 4.20. The global time of burst is five seconds, including two seconds of time after beginning of the aircraft’s trajectory until time of warhead release. The shortest distance is 34 meters after 5.5 seconds and is the global minimum. The shortest distances concerning all rotational sequences results in the following plot:
Figure 4.21: Shortest distance between aircraft and front fastening loop with burst time of three seconds.

The shortest distance for every time step is extracted from all rotational sequences for the fastening loop and plotted in figure 4.21. The blue line is the lower bound for all plotted lines in figure 4.21. For the mounting plate with a significantly larger mass, the following result is obtained:
Figure 4.22: Distance between aircraft and front mounting plate with burst time of three seconds. Each curve represents the distance between the aircraft and mounting plate for one rotational sequence.

The distance between the aircraft and front mounting plate for all rotational sequences is plotted in figure 4.22. The global time of burst is five seconds, including two seconds of time after beginning of the aircraft’s trajectory until time of warhead release. The shortest distance is 52 meters after 5.4 seconds and is the global minimum. The shortest distances concerning all rotational sequences results in the following plot:
The shortest distance for every time step is extracted from all rotational sequences for the mounting plate and plotted in figure 4.23.

4.7.2 Fastening loop and mounting plate in back position

For the back position of the warhead, the same approach is applied as for the front section. Risk area for back fastening loop with burst time of three seconds. The initial velocity are the same for both the mounting plate and fastening loop and set to 602 meters per second.
Figure 4.24: Risk area for back fastening loop with burst time of three seconds. The first blue circle are the position of warhead during burst. The first green circle are the position of the aircraft when warhead bursts.

The back position for the mounting plates and fastening loops will have a slightly inclination upwards in comparison to the front position. This results in a relative larger positive angle in pitch direction and therefore in the set of convex hull directed more upright than for the same case in front position.

The limit for the axis is set to better visualise the aircraft and warhead position in comparison to the convex hull.
Figure 4.25: Distance between aircraft and back fastening loop with burst time of three seconds. Each curve represent the distance between the aircraft and mounting plate for one rotational sequence.

The distance between the aircraft and back mounting plate for all rotational sequences is plotted in figure 4.25. The global time of burst is five seconds, including two seconds of time after beginning of the aircraft’s trajectory until time of warhead release. The shortest distance is 20 meters after 5.4 seconds and is the global minimum. The shortest distances concerning all rotational sequences results in the following plot:
Figure 4.26: Shortest distance between aircraft and back fastening loop with burst time of three seconds.

The global minimum at 5.4 seconds in figure 4.26 is approximately 25 meters. Finally, the mounting plate for the back position is analysed.
Figure 4.27: Distance between aircraft and back mounting plate with burst time of three seconds. Each curve is represent the distance between the aircraft and mounting plate for one rotational sequence.

Figure 4.28: Shortest distance between aircraft and back mounting plate with burst time of three seconds.
The least distance is 38 meters and obtained after 5.4 seconds for least total distance in figure 4.28. Values for graph in figure 4.28 is gathered from figure 4.27.

4.8 Variation in drag coefficient and average exposed area for mounting plates and fastening loops

Calculations in this section is performed with the initial conditions for burst during warhead deployment with burst data from time of burst of two seconds. Initial velocities for warhead at point of burst is set to 240 meters per second in X-direction and negative 85 meters per second in Z-direction. Velocities for the mounting plate and fastening loop in the front position is set to 520 meters per second and for the back position 603 meters per second.

4.8.1 Mounting plates and fastening loops with increased drag coefficient

The drag coefficient is varied with an increase and decrease of ten percent for trajectory calculations in this section.

![Trajectories for mounting plates with drag variation](image)

Figure 4.29: Trajectories for mounting plates. Dashed lines are trajectories with a ten percent drag coefficient increase. Dashed dot line are trajectories with a ten percent decrease for drag coefficient. The first pair of trajectories that reaches highest are from the back position and the pair reaching the longest distance are from the front position.

Data for trajectories in figure 4.29 are gathered in table 4.5.

<table>
<thead>
<tr>
<th>Mounting plate</th>
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<th>Front</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drag coefficient correction factor</td>
<td>0.9</td>
<td>1.0</td>
</tr>
<tr>
<td>Z-direction [m]</td>
<td>1117</td>
<td>1029</td>
</tr>
<tr>
<td>X-direction [m]</td>
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<td>177</td>
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<tr>
<td>% difference Z-direction</td>
<td>8.6</td>
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</tr>
<tr>
<td>% difference X-direction</td>
<td>9.6</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.5: Values for mounting plate during variation in drag coefficient.
The variation in altitude and travelled distance for mounting plates differ from 6.9 percent up to 9.6 percent from calculations with values for the original drag coefficient in table 4.5. The same procedure is used for analysing the fastening loops in front and back position.

![Trajectories for fastening loops with drag variation](image)

Figure 4.30: Trajectories for fastening loops. Dashed lines are trajectories with a ten percent drag coefficient increase. Dashed dot line are trajectories with a ten percent decrease for drag coefficient. The first pair of trajectories that reaches highest are from the back position and the pair reaching the longest distance are from the front position.

Data for trajectories in figure 4.30 are gathered in table 4.6.

<table>
<thead>
<tr>
<th>Fastening loop</th>
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<th>Front</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drag coefficient correction factor</td>
<td>0.9</td>
<td>1.0</td>
</tr>
<tr>
<td>Z-direction [m]</td>
<td>700</td>
<td>644</td>
</tr>
<tr>
<td>X-direction [m]</td>
<td>115</td>
<td>105</td>
</tr>
<tr>
<td>% difference Z-direction</td>
<td>8.7</td>
<td>0</td>
</tr>
<tr>
<td>% difference X-direction</td>
<td>9.5</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.6: Values for fastening loop during variation in drag coefficient.

The variation in altitude and travelled distance for fastening loops differ from 7.3 percent up to 9.6 percent from calculations with values for the original drag coefficient presented in table 4.6.

### 4.8.2 Rotating plate

The mounting plates may rotate during trajectory for all flight regimes. The expected projected area during rotation is 30 percent larger than the Cauchy area used in calculations in other sections.
Figure 4.31: Trajectories for mounting plates. Dashed lines are trajectories with an 30 percent area increase. The first pair of trajectories that reaches highest are from the back position and the pair reaching the longest distance are from the front position.

The difference between trajectories for the increased area is presented in figure 4.31.

<table>
<thead>
<tr>
<th>Mounting plate</th>
<th>Back</th>
<th>Front</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area [m²]</td>
<td>0.0105</td>
<td>0.01365</td>
</tr>
<tr>
<td>Z-direction [m]</td>
<td>1030</td>
<td>838</td>
</tr>
<tr>
<td>X-direction [m]</td>
<td>177</td>
<td>140</td>
</tr>
<tr>
<td>% difference Z-direction</td>
<td>18.6</td>
<td>18.1</td>
</tr>
<tr>
<td>% difference X-direction</td>
<td>21</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 4.7: Values for mounting plate during rotation.

The variation in altitude and travelled distance for mounting plates differ from 18.1 percent up to 21 percent from calculations with values for the original drag coefficient presented in table 4.7.
Chapter 5

Discussion

Sections are discussed individually for an overall understanding. The individual chapters are later woven together for a comprehensive reasoning. It is important to point out that focus is to evaluate the method of solving the problem and objective, rather than present useful numeric values. This due to the use of generic data for the warhead and aircraft since realistic data is classified.

5.1 Mass classes

Increased mass for a fragment results in an increased travelled distance. The kinetic energy is directly dependent on the fragments mass and gives an estimate on the energy released during impact if an hit on the aircraft is registered. After four seconds of trajectory calculations for a natural fragment, the impact energy is around 1500 J for a fragment with mass of 0.35 kilograms. There are seven of these fragments in the positive quadrant of the warhead, meaning it consists of eleven fragments in total which may travel the longest trajectories and expose the aircraft for a risk of hit the longest time. Mounting plates and fastening loops are included.

The kinetic energy for fragments with a mass under one gram is 0.63 J after one second and approx 700 meters of travelled distance. This low value motivates the exclusion of the smallest fragments and constitutes the boundary for the first mass class. An upper bound is very inappropriate due to the relative enormously kinetic energies for fragments presented in table 4.1. Even if the evaluation of a hit is omitted, it would not be unreasonably to state that the largest fragments would severally damage the aircraft if being hit and therefore focus would be to study the higher mass classes combined with the fastening loops and mounting plates.

5.2 Distance between aircraft and fragments for different mass classes

The distance between aircraft and fragments will increase with reduced fragment mass and increased burst time. A suitable burst time was found to be around four seconds with a small amount of fragments on a close range to the aircraft for the higher mass distributions. Due to the results from heat map in figures 4.8 and 4.9, it should be reasonable to suppose that the amount of fragments hitting the aircraft are relative few. Otherwise they should indicate their existence with reserving a colour full region on the heat map and displayed in the colour bar for low valued distances. Since no hit on aircraft is accepted, the burst time should be increased. The compilation of calculated values in a shortest distance plot combined with a heat map for the same situation, complements each other for an overall better view. The short distance plot displays the critical points for a few fragments during a cropped time span. The heat map displays all distances during a larger time span for all fragments but at the cost of accuracy when interpret the graph. It should be assumed that the approach for presenting the results in this way is a preferable approach.

5.2.1 Fragment with median mass, 10.8 grams

The idea to use a type fragment outgoing from the mass based median for fragments with a mass greater than one gram, is to reduce the amount of fragments to be analysed but still represent an average mass for a fragment.
For the calculation with a burst time of three seconds presented in figure 4.2, a hit on the aircraft occurs almost immediately after burst at global time of 5.4 seconds. Only 98 fragments were used for the calculations which most likely indicates that an increase of fragments will result in more hits and the burst time is therefore increased to four seconds. The minimum distance between aircraft and one of the fragments is then increased to 103 meters presented in figure 4.3. The red dotted line named "user input safety distance" is arbitrarily set to 100 meters. This was done intuitively considering the size of a fighter jet and may be changed to whatever the user choose as appropriate according to the risk level.

5.2.2 Mass up to 10 grams
The shortest distance between aircraft and fragment in figures 4.5, 4.6 and 4.7 is reassuringly large when considering the user input safety distance of 100 meters. This indicates that fragments with a mass less than 10 grams have short trajectories and may not expose the aircraft for a severe risk. The total amount of fragments in these three divisions of mass distribution is 4886 fragments for the positive quadrant of the warhead, fragments with positive velocities in X- and Z-directions. Through calculations and assumption of the warheads direction during deployment, the amount of fragments exposing the aircraft of a potential hit at a burst time of four seconds is drastically decreased.

5.2.3 Mass between 10 and 100 grams
The shortest distance plot in figure 4.8 indicates several hits on the aircraft. This is the largest division of mass distribution containing 4813 fragments. The heat map reveals blue zones for fragments with a distance of around ten meters for times of six, seven and eight seconds. This short distance during the whole measured time span gives an undesirable result with a risk of hit during up to three seconds after burst.

5.2.4 Mass over 100 grams
Several hits are presented for the time of around 6.3 seconds in figure 4.9. The blue areas in the heat map also reveal fragments with a shortest distance to the aircraft of around 40 meters during the whole registered time interval.

5.2.5 Risk dependence on mass distribution
For the burst time of four seconds, fragments with a mass over ten grams exposes the aircraft for a potential risk of hit when view the shortest distances. It would be wise to increase the burst time further to reassure that the aircraft may not be hit. On the opposite side, the burst time has to be kept as short as possible to enable an effective deployment of the warhead during a reasonably low altitude of flight.

5.3 Sensitivity analysis velocities
An increase of ten percent is added to velocities for fragments to evaluate the effect of their trajectories and further the least distance to aircraft. The chosen burst time of four seconds gives a sufficient value of the safety distance for the lower mass distributions. In comparison for the heat maps in figures 4.9 and 4.10, The value of the blue area has decreased with a higher velocity. This implies that the amount of fragments with a least distance under the safety distance of hundred meters has increased, even though the shortest distance graph could be fooling with the marked global minimum points. The decrease of distance indicated in the colour bar for increase in time for the first hundred fragments indicates that the fragments "chasing" the aircraft after achieved their highest point on fragment trajectory. The large reduction for least distances revealed when comparing heat maps in the end of the time span could be avoided with the earlier mentioned manoeuvre. When viewing the largest distribution of 4813 fragments with masses from ten to hundred grams, the least distance is not decreasing as much. Whith this reasoning, the effect of a velocity increase on fragments from the distributions with lower mass will not be devastating due to the increased large retardation due to air drag. Therefore they will not be evaluated from this perspective.

5.4 Distance depending of fragment size
The three largest fragments in the section 4.4 have the longest calculated trajectories and will constitute the limit for how far fragments may travel. A fragment may travel up to around 1750 meters if tossed out in a disadvantageous
direction. Together with the mounting plates and fastening loops, these objects exposes the aircraft for the largest risk of hit if the burst time is set large enough. When analysing the largest natural fragments individual, it gives a comprehensive view of the most dangerous projectiles for the aircraft together with the mounting plates and fastening loops. These parts spans the risk area at a distance far beyond what fragments from the lower mass classes do.

5.5 Velocities for calculations

The velocities of up to 1800 meters per second for fragments is the maximum value used in calculations. For mounting plates and fastening loops rotation analyse, the velocities used are down to 520 meters per second and 603 meters per second depending on mounted in front or back position. The approach of choosing velocities states from what is suitable for a preferred method of analyse. Relative low velocities for the mounting plates and fastening loops results in an overall better visual understanding of the distances between aircraft and convex hull. It is the method for analysing the problem that is in focus due to the generic data that has been used. It is fully possible to use realistic data for velocities and do an analysis for the mounting plates and fastening loops with the developed Matlab code.

5.6 Mounting plates and fastening loops

Both the fastening loops and mounting plates travel a severe distance, given in figure 4.13. The Cauchy area is obtained from Staffan Harling’s report [2] and may unfortunately not be examined further. The travelled distance for the mounting plate with a 3.2 kilogram mass is only 1186 meters which in comparison to the trajectories for the largest analysed fragments in section 4.4 seems short, may depend on just the average projected area. The same reasoning is valid for the fastening loops with a mass of 0.35 kilogram and a travelled distance of 685 meters. Calculations with a drag coefficient for a preformed fragmentation gave similar results such as 1201 meters and 693 meters respective in figure 4.14. It is likely that the calculations are correct and that the mounting plates and fastening loops initial angles during burst is the main reason to the trajectories distance. Drag coefficient data for preformed fragmentation has been used for the calculations considering mounting plates and fastening loops.

5.6.1 Sensitivity analysis ejection direction

The warhead has no guiding fins and may therefore rotate around its x-axis (longitudinal axis) without any obstacle. The mounting position for the warhead where it is connected to the aircraft before deployment, rotates with the warhead after it has been dropped. The initial angles may therefore vary depending on how the warhead is positioned during burst. The warhead may even wobble in the other two directions during trajectory but not fully rotate. This is not as likely but motivates that the analysis for ejection direction concerns variation in all three directions.

The mounting plate and fastening loop at the back position will get a positive contribution in $\alpha$ direction due to the shape of the warhead leaning backwards. This directs the projectiles upwards resulting in potential collision in an earlier stage when the impact energy is significantly higher than during the later part of trajectory. The assumption that mounting plate and fastening loop is separated during burst resulting in four objects to concern. Two for the front position and two for the back position.

The convex hull is produced of the points in the trajectory for the specific rotation series for a certain time meaning that all points in the given area is not able to compare to the aircraft’s position. This problem can be relieved by looking at distance in z-direction and assume that for a given height, all points enclosed by the convex hull is regarded as risk for hit. The convex hull could be enlarged widely if $\alpha$, $\beta$ and $\gamma$, is increased. The chosen span of twenty degrees in each direction gives an advantageous visualisation to understand the growth of the risk area. For a more realistic case with a span of 40 degrees in pitch and yaw motion and 360 degrees in roll, the convex hull would be enormous and hard to grasp.

No concern has been taken to which rotational series trajectory represent the least distances with the motivation that the risk is severe for failure if the aircraft is hit, no mater if it is by the fastening loop or mounting plate. The burst time of three seconds is a trade off between the criteria for when the warhead can be armed and sufficient safety distance. For the case when the burst time was set to two seconds in figure 4.16, the convex hull
was positioned above the aircraft trajectory and increase risk of hit. When the burst time was set to four seconds in figure 4.17, the aircraft would be in front and above the set of convex hull and out of risk (if the rotational sequences not increased). A burst time of three seconds was chosen due to in between the cases discussed. The intention for the Matlab code is to let the user define variables such as, time of release and burst, span of angles, time step and what projectile to evaluate.

Velocities have been set to around 600 meters per second during calculations. If the velocities are increased to around 1800 meters per second as for many of the natural fragments, the burst time of four seconds would certainly not be enough. The burst time would need to be increased resulting in a higher altitude of warhead release so the warhead not would make ground contact to early.

The method for calculating the convex hull is a way of representing the risk in the form of area. Another way would be to evaluate the amount of fragments in a volume which has not been done but may possible give a value for the risk. The area approach is suitable to use with the method for ballistic calculations.

5.6.2 Front and back positions

The front and back position are angled slightly different resulting in a variation for initial velocities them between for mounting plates and fastening loops. The positions can be seen as small fastening loops in figure 1.1. This assumption that the position actually gives origin to different ejection angles gives a larger spread of risk area when fragments from both position is calculated. It is better to have a larger total risk area for safety reasons as the risk of the safe separation distance is chosen to short.

Presentation for the distance between aircraft and respective mounting plates and fastening loops for all rotational sequences gives a plot as in figure 4.20. The variation in respective \(\alpha\), \(\beta\) and \(\gamma\) angles for a set, span with fix interval results in curves following a similar pattern. The least total distance presented in figure 4.21 gathered from all rotational series will effectively present the range to deal with for the aircraft.

Least total distances for the front position are 50 meters and for the back position under 40 meters for both the mounting plates and fastening loops in section 4.7. The burst time of three seconds should be increased with regard to this short distances. The global minimum at 5.4 seconds indicates the shortest distance achieved already after 0.4 seconds after warhead burst. It is likely that at this time, the fragments may have caught up on the aircraft from its lead. When evaluate the position of the convex hull in any of the plots presenting them, it is noticed that the minimum in the least distance plot correlates with the position of the convex hull near the aircraft trajectory.

5.6.3 Variation in drag coefficient

Approximations regarding the drag coefficient set as values for preformed fragmentation for the mounting plates and fastening loops is arbitrary. It would be wise to analyse a variation in drag coefficient that is done in section 4.8.1. The difference for trajectories are up to 9.6 percent for a ten percent decrease of drag and has to be considered when estimate the safe separation distance. The variation for the dag coefficient may be larger or smaller depending on assumptions but the idea of analyse a variation should be considered to fully embrace a risk perspective.

5.6.4 Average exposed area

The case for a rotating plate with the assumption that the expected projected area are 30 percent larger in section 4.8.2 obviously results in shorter trajectories due to increased air drag. This may not contribute to the risk analysis but is important to understand how different fragments behave during ballistic calculations. The maximum 21 percent trajectory decrease in table 4.7 gives a reassuring contribution to the risk perspective that a rotating mounting plate is less of a risk for hitting the aircraft.

5.7 Time step

The time step for fragment trajectory calculation is set to 0.01 seconds which gave a descent accuracy and acceptable time for calculations linked to the division of the mass distribution. For calculations concerning a larger amount of fragments, a larger time step may be used and vice versa for a lower amount of fragments. This is strictly
connected to the time of calculation. Trajectory calculations for larger fragments may use a larger time step such as 0.1 seconds and still give descent results. For fragments with a mass less than ten grams, a time step of 0.01 second is needed or else the ballistic calculation will terminate. It reveals the trade of between accuracy of trajectory calculations and the analyse of the complete warhead with all fragments linked to the chosen calculation time. It is by this means motivated to divide the fragments into mass classes to effectively estimate the risk.

5.8 Calculations of risk

An overarching problem is that given theories for safety distances is not directly applicable on the given problem to compare results with. The calculation of range safety distance is primarily for detonation of ammunition stockpiles on ground level where the explosives and victim is fix in a coordinate system. It may be used for comparison due to the relative equal velocities between aircraft and warhead but inherent inaccuracies are likely to exist. If the volume around the detonating warhead is modelled as a half sphere, the range safety distance model should be arbitrarily suitable for comparison. The distance between aircraft and warhead is 904 meters at point of burst if the burst time is set to four seconds. In comparison for the range safety distance of 934 meters in section 4.6, the burst time should be set to 4.5 seconds to be on the safe side.

The value for vertical danger area of 1391 meters in table 4.4 will be difficult to fulfil with the calculated example. The altitude of the aircraft may therefore be increased to achieve a adequate distance equivalent to the desirable vertical danger area. Another way to increase the distance in Z-direction to that extent, would be for the aircraft to perform a manoeuvre to rapidly gain altitude which is not a part of this thesis. The condition for the vertical danger area may therefore not be achieved if the burst time is sufficiently long so the drop in altitude of the warhead is large enough to be equal the vertical danger area distance.

5.9 Safe separation distance

For the earlier reasoning concerning distance between aircraft and fragments, the burst time was set to four seconds. For mounting plates and fastening loops in rotational sequences, the burst time of three seconds where chosen. For the burst time of four seconds, the set of convex hull in figure 4.17 will be positioned after the aircraft at actual time and therefore not expose it for risk of hit. Considering the two cases, the over all burst time should be set to four seconds for this type of suitable aircraft trajectory. The amount of fragments in combination with the four projectiles (mounting plates and fastening loops), will expose the aircraft for a risk of hit but to prevent it totally, the burst time have to be increased with three seconds further. A burst time of at least seven should be enough.

The safe separation distance is the distance between aircraft and warhead that is required to reduce the risk of hit on the aircraft to the extent of no hit. For the point of warhead burst, the vertical distance in Z-direction is basically zero meaning that the aircraft and warhead is positioned on the same altitude. The condition for a vertical danger area of 1391 meters is therefore unrealistic to achieve. Values for the minimum safe distance during peace time is 1200 meters and the range safety distance is 1391 meters. These distances are on ground level and no consideration is taken to how the fragments trajectories vary in height. Because these are static values taken from a table which not concerns the dynamics of a system with relative velocities between aircraft, warhead and fragments, it is not possible to do a spotless comparison. The distance between aircraft and warhead at time of burst is 676 meters at burst time of three seconds and 904 meters at burst time of four seconds. These distances are not satisfying the range safety distance of 1391 meters nor the minimum safe distance at 1200 meters.

The method to evaluate the safe separation distance would be better if there existed models for the case of air detonation to compare calculations to. The models used are for burst on ground level and not directly applicable even if they can provide a good reference value. The idea to use several models for approximate cases helps to justify the method of estimate a safe separation distance.

An averaged valued distance from the models estimates and calculated values may give values to estimate a danger close distance for closed air support which would be the most similar of the given models. The danger zone model takes consideration to the risk of hit on ground level for friendly forces, for a deployed warhead. This would complement the CAS for the ground case and therefore help to give an overall estimate for risk of hit.
Chapter 6

Conclusion

The method of analysing the complete problem seems to work. The approach to compare distances should create a understandable overview for the uninitiated regarding a risk perspective. The model does not calculate any risk, which would be an measured value to compare.

Sensitivity analysis have been performed to see variations for the trajectory calculations. Values for drag coefficient and velocities may vary depending on initial velocities linked to a read aircraft trajectory. The use of generic data and limitations linked to classified information makes the calculated values less relevant and highlights the importance of the method in this thesis.

A continuing work should focus on determine the risk of being hit, maybe to estimate the amount of fragments in a defined volume at a certain distance from the aircraft. It would be wise to analyse manoeuvres performed by the aircraft to actively avoid fragments and the reasonableness linked to exerted g forces.
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Appendix A

Split-X

Split-X is a simulation tool to calculate fragments data for warheads. A program with direct visual content gives an understanding of the warhead concerning dimensions, material and displacement. The user can choose material properties from a predefined library to construct a warhead for a suitable purpose. A simulation results in datafiles preferable in the format of *.pit and *.zdata [10]. These files contain information for post processing in e.g MatLab.

A.1 Data to Matlab file

Data from Split-X analysed in a Matlab script and presented as a 26 column long row matrix for each fragment.

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Unique projectile reference number</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>Fragmentation category (1=preformed, 2=controlled, 3=natural)</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>Fragmentation sub type</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>Material number</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>Projectile length</td>
<td>[mm]</td>
</tr>
<tr>
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<td>Projectile width</td>
<td>[mm]</td>
</tr>
<tr>
<td>7</td>
<td>Projectile height</td>
<td>[mm]</td>
</tr>
<tr>
<td>8</td>
<td>X-coordinate of the projectile center in the design system</td>
<td>[mm]</td>
</tr>
<tr>
<td>9</td>
<td>Y-coordinate of the projectile center in the design system</td>
<td>[mm]</td>
</tr>
<tr>
<td>10</td>
<td>Z-coordinate of the projectile center in the design system</td>
<td>[mm]</td>
</tr>
<tr>
<td>11</td>
<td>Kardan rotation angle about the x-axis</td>
<td>[rad]</td>
</tr>
<tr>
<td>12</td>
<td>Kardan rotation angle about the y-axis</td>
<td>[rad]</td>
</tr>
<tr>
<td>13</td>
<td>Kardan rotation angle about the z-axis</td>
<td>[rad]</td>
</tr>
<tr>
<td>14</td>
<td>X-component of the final projectile velocity</td>
<td>[m/s]</td>
</tr>
<tr>
<td>15</td>
<td>Y-component of the final projectile velocity</td>
<td>[m/s]</td>
</tr>
<tr>
<td>16</td>
<td>Z-component of the final projectile velocity</td>
<td>[m/s]</td>
</tr>
<tr>
<td>17</td>
<td>X-component of the final angular velocity</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>18</td>
<td>Y-component of the final angular velocity</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>19</td>
<td>Z-component of the final angular velocity</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>20</td>
<td>Time at which acceleration of the projectile starts</td>
<td>[s]</td>
</tr>
<tr>
<td>21</td>
<td>Acceleration time</td>
<td>[s]</td>
</tr>
<tr>
<td>22</td>
<td>Sum of velocity components</td>
<td>-</td>
</tr>
<tr>
<td>23</td>
<td>Mass</td>
<td>[g]</td>
</tr>
<tr>
<td>24</td>
<td>Angle between velocity and x-axis</td>
<td>[deg]</td>
</tr>
<tr>
<td>25</td>
<td>Cauchy area</td>
<td>[mm²]</td>
</tr>
<tr>
<td>26</td>
<td>Reference diameter</td>
<td>[mm]</td>
</tr>
</tbody>
</table>

Table A.1: Data processed from Split-X in Matlab for each fragment.
Appendix B

Verification of ballistic calculations

B.1 Model

A second ballistic code was written and the results are as follows. The artillery procedure of determine the trajectory is divided into steps shown in this section. Two versions, two and three dimensional cases is written. The difference in the two dimensional case from the third described in this section, is that global Y-direction is replaced by global X-direction to gain the contribution from gravitation.

- A loop is created with a counter named \( t_{ctr} \) that will increase for each iteration and update the matrix including the results. The starting position is \( x, y \) and \( z \) in global coordinates. The algorithm starts by find values for velocity of sound \( a \), air pressure \( P \) and density \( \rho \) at the current true altitude (global Z-coordinate) with the function \textit{atmosisa}. The mach number is given from the current absolute velocity and velocity of sound in the following equation:

\[
M = \frac{fV_{Total}}{a} \quad (B.1)
\]

- To calculate the drag coefficient \( C_d \), the span with given values of \( C_d \) from Split-X is linearly interpolated for the current mach number with the function \textit{interp1}.

- Cartesian coordinates is used for correct description with the following equations:

\[
\alpha = \arctan\left(\frac{v_{y01}}{v_{x01}}\right) \quad (B.2)
\]

\[
r = \sqrt{v_{x01}^2 + v_{y01}^2 + v_{z01}^2} \quad (B.3)
\]

\[
\beta = \arccos\left(\frac{v_{z01}}{r}\right) \quad (B.4)
\]

Where \( \alpha \) in equation (B.2) is the angle between the velocity components in the XY plane including transformation from radians to degrees. The velocities \( v_{x01}, v_{y01}, v_{z01} \) is naturally in respective direction. The absolute velocity \( r \) is calculated with equation (B.3). The angle between the XY plane and Z plane is given as \( \beta \) in equation (B.4).

- The drag equation needs to be solved given as:

\[
F_v = \frac{1}{2} \kappa PM^2 A \frac{1}{m} \quad (B.5)
\]

The drag in equation (B.5) varying for different mach numbers \( M \), air pressure \( P \), the Cauchy area \( A \) and fragment mass \( m \). Known as before that \( \kappa \) is the thermodynamic constant.

- The differences in velocities is calculated as following:

\[
dv_{x1} = -F_v \cos(\alpha) \sin(\beta) \cdot dt \quad (B.6)
\]

\[
dv_{y1} = -F_v \sin(\alpha) \cos(\beta) \cdot dt \quad (B.7)
\]
\[ dv_z = -(F_\text{c}, \cos(\beta) + g) \cdot dt \]  
\[ (B.8) \]

The difference in velocity for the X-component given by equation (B.6), the Y-component from equation B.7 and Z-component from equation (B.8). The time step \( dt \) is decided depending on how fine the calculations need to be.

- The velocity components in respective directions as following:
  
  \[ v_{x1s} = v_{x01} + dv_{x1} \]  
  \[ (B.9) \]
  
  \[ v_{y1s} = v_{y01} + dv_{y1} \]  
  \[ (B.10) \]
  
  \[ v_{z1s} = v_{z01} + dv_{z1} \]  
  \[ (B.11) \]

The updated velocity component for X-component given by equation (B.9) the Y-component from equation (B.10) and Z-component from equation (B.11).

- The same procedure also applies to absolute velocity:

  \[ v_{1s} = \sqrt{v_{x1s}^2 + v_{y1s}^2 + v_{z1s}^2} \]  
  \[ (B.12) \]

  \[ dv_{1s} = \sqrt{dv_{x1}^2 + dv_{y1}^2 + dv_{z1}^2} \]  
  \[ (B.13) \]

The absolute velocity from equation (B.12) and its corresponding difference in absolute velocity from equation (B.13).

- In order to decide the current position for the fragment, the averaged velocity is calculated in following equations:

  \[ v_{xmed1} = \frac{v_{x01} + v_{x1s}}{2} \]  
  \[ (B.14) \]

  \[ v_{ymed1} = \frac{v_{y01} + v_{y1s}}{2} \]  
  \[ (B.15) \]

  \[ v_{zmed1} = \frac{v_{z01} + v_{z1s}}{2} \]  
  \[ (B.16) \]

The average velocity at current time step in X-direction is given from equation (B.14), in Y-direction from equation (B.15) and in Z-direction from equation (B.16).

- The position in the current step is decided as following:

  \[ x_{1s} = x + v_{xmed1} \cdot dt \]  
  \[ (B.17) \]

  \[ y_{1s} = y + v_{ymed1} \cdot dt \]  
  \[ (B.18) \]

  \[ z_{1s} = z + v_{zmed1} \cdot dt \]  
  \[ (B.19) \]

The new position given by the X coordinate from equation (B.17), Y coordinate from equation (B.18) and Z coordinate from equation (B.19).

- As the last step, variables is updated in preparation for the next iteration.

  \[ z = z_{1s} \]  
  \[ (B.20) \]

  \[ fV_{\text{Total}} = v_{1s} - d_{1s} \]  
  \[ (B.21) \]

  \[ v_{x01} = v_{x1s} \]  
  \[ (B.22) \]

  \[ v_{y01} = v_{y1s} \]  
  \[ (B.23) \]

  \[ v_{z01} = v_{z1s} \]  
  \[ (B.24) \]

  \[ v_{01} = v_{1s} \]  
  \[ (B.25) \]

  \[ t_{ctr} = t_{ctr} + 1 \]  
  \[ (B.26) \]

The calculated values is stored in a matrix that will be updated for each iteration and finally returned to the fragment script.

It is possible to run parts of the loop in individually iterations to increase precision. Those parts are difference in velocities, equation B.6, B.7 and B.8. The velocity components in equations B.9, B.10 and B.11. The averaged velocities in equations B.14, B.15 and B.16.
B.2 Verification of models with artillery table

To verify the models for fragment trajectory calculations, identical data has been used for a 15.5cm artillery shell with known trajectories and travelled distance from firing table [27] at page 600. The projectile has an initial absolute velocity of 559 meters per second, fired at an angle of 34.17 degrees with a mass of 43.17 kilograms. This gives an estimated travelled distance in global X-direction of at least 14000 meters and not exceeding 14100 meters. The altitude is 2996 meters in global Z-direction. The values from the firing table is calculated with atmospheric data given from "Försvarets normallufthav" and data at corresponding height for the ArtSS and Runge-Kutta methods using the Matlab function \textit{atmosis}. ArtSS2D and ArtSS3D indicates a method primarily for a two and three dimensional perspective that will be equal for a two dimensional case.

![Trajectory for 15.5cm projectile 77B](image)

**Figure B.1:** Comparing trajectory for a 15.5cm projectile with all three methods.

The trajectories are almost the same in figure B.1. The projectile travel 14155 meters for the ArtSS methods and 14124 meters for the Runge-Kutta method in X-direction. The total height in Z-direction is equal for both methods and estimated to 3003 meters.
The corresponding absolute velocity in figure B.2 to the trajectory in figure B.1 agrees well between the three used methods. The projectiles velocity naturally finds a minimum value when reaching the highest point on the trajectory of around 3003 meters. Thereafter it begins to accelerate until reaching ground level with an impact absolute velocity of around 316 meters per second at the travelled distance of around 14124 meters in X-direction. The results shows good agreement with firing table data.

B.2.1 Drag coefficient

The drag coefficient $C_d$ for fragments is given from the earlier equation (2.22) and will be given here again:

$$C_D = \frac{2W_e}{\rho A v^2} \quad \text{(B.27)}$$

During an iteration with both methods of ArtSS and Runge – Kutta, the drag coefficient is calculated for the absolute velocity at conditions for the given time step.
All three methods in figure B.3 align well for the drag coefficient, as the absolute velocity is the only variable that will be different between the methods depending on how it is rounded off. The density is gathered from the Matlab function `atmosis` and both fragment weight and Cauchy area is given from Split-X as constants.