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To Activate, Expose and Elicit thinking – a three step journey

Teacher actions and student thinking in 18 Swedish mathematics classrooms in upper secondary.

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Title in Swedish: Att aktivera, exponera och stimulera tänkande – en resa i tre steg.
Title in English: To Activate, Expose and Elicit thinking – a three level journey
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Abstract

Today's research, as well as current Swedish governance documents, stress the high importance of developing students' abstract, individual and critical thinking in mathematics education. The needs for such quality thinking however stand in stark contrast to the 'traditional mathematics education', which cannot be expected to develop such thinking. This study suggests an evolutionary instead of revolutionary approach in leaving the old paradigm of 'traditional education' behind, since project support for such a transformation project is claimed to be lacking on central level while the feasibility of such a paradigm shift on individual (teacher or even school) level is claimed to be very low in today's time-pressured reality. By investigating teacher actions which generate *occurrences of activated, exposed and elicited student thinking*, this study purports to suggest alternative ways forward.

A series of semi-structured classroom observations, complemented with questionnaires and interviews, has therefore been carried out of 18 mathematics teachers in upper secondary level in Sweden. The study is based on empowerment theory (as well as an adaptation from the industry of an applied empowerment model), a triangulation mixed-method design was adopted and a thematic analysis, underpinned by a latent theoretical approach from an epistemologically constructionist perspective as described by Braun and Clarke (2006), was used. The study identified several occurrences of the aforementioned student thinking, as well as their corresponding teacher actions and proposes a way for organizing the actions as well as their outcome into an overarching model.

Keywords: mathematical thinking, classroom observations, thinking progression, higher order thinking skills, HOTS, mathematics education, upper secondary.

Sammanfattning

Dagens forskning och samtliga svenska styrdokument betonar den stora vikten av att inom matematikundervisningen utveckla det abstrakta, individuella och kritiska tänkandet hos eleverna. Behoven av sådant ”kvalitetstänkande” står dock i kraftig kontrast till ”traditionell matematikundervisning”, som inte kan förväntas utveckla sådant tänkande. Denna studie föreslår ett evolutionärt, istället för revolutionärt, angreppssätt för att lämna det gamla paradigmet, eftersom projektstöd för ett sådant transformationsprojekt saknas på central nivå och genomförbarheten för ett paradigmskifte på individuell (lärar- eller t.o.m. skol-) nivå hävdas vara låg i dagens tidspressade verklighet. Genom att studera lärarhandlingar som genererar förekomster av *aktiverat*, *exponerat och stimulerat elevtänkande*, menar denna studie att hitta alternativa vägar framåt.

En serie semistrukturerade klassrumsobservationer, kompletterade med enkäter och intervjuer, har därför genomförts hos 18 matematiklärare på gymnasiet i Sverige. Studien är baserad på *empowerment*-teori (tillsammans med en industriell tillämpning av en *empowerment*-modell), en blandad metoddesign (*triangulation mixed-method*) samt en tematisk analys kännetecknad av ett latent-teoretiskt angreppssätt och av ett epistemologiskt konstruktionistiskt perspektiv enligt Braun och Clarke (2006). Studien identifierade flera förekomster av ovannämnda elevtänkande, likväl som motsvarande lärarhandlingar, och föreslår ett sätt att strukturera både handlingar och deras resultat i en övergripande modell.

Nyckelord: matematiskt tänkande, klassrumsobservationer, högre ordningens tänkande, tänkande-progression, HOTS, matematikundervisning, gymnasium.

Foreword

This study presents the results of a degree project in the Bridging Teacher Education Programme at Kungliga Tekniska Högskolan (KTH) in Stockholm. The project was initiated during autumn 2021 and finished during the spring term 2022. I'd like to thank my supervisor Ernest Ampadu for his knowledge, wise remarks and, above all, for the humoristic as well as intellectually stimulating work atmosphere during all our meetings. It has been a very interesting and inspiring work period which has taught me a lot. I would also like to thank my teacher in mathematics education Petter Norrthon, whose 'teacher actions' inspired many of my thoughts in this study. Petter's endless patience with my many, many questions must also be mentioned.

Denna studie presenterar resultatet av ett examensarbete från Kompletterande Pedagogisk Utbildning (KPU) vid Kungliga Tekniska Högskolan i Stockholm. Arbetet påbörjades under hösten 2021 och slutfördes under vårterminen 2022. Jag vill tacka min handledare Ernest Ampadu för all kunskap, kloka inspel och framförallt för ett både humoristiskt och intellektuellt stimulerande samarbetsklimat under våra möten. Det har varit en för mig mycket både intressant och inspirerande arbetsperiod då jag lärt mig mycket. Jag vill även tacka min lärare i matematikdidaktik, Petter Norrthon, vars lärarhandlingar gav upphov till många av mina tankar i denna studie. Även Petters oändliga tålamod med mina många, många frågor måste nämnas.

Stockholm, summer 2022

Carina Bark

Contents

1 Introduction

The introduction aims to give the reader an overview, while the main thoughts presented are developed and substantiated in the corresponding subsections in the next chapter (2. Background).

I entered the educational world after a long career within start-up and transformation projects; (re)building companies in Latin America, Sub-Saharan Africa, Northern Africa, the Arabian Peninsula and Asia, which are highly complex and event-driven contexts. In order to make *informed real time decisions*, a leader needs very good multidimensional models for both the building phase and the operational phase (structural leadership). In addition, staff must be truly *empowered* (cultural leadership) in both phases, where one of the fundamental components of empowerment is ‘quality thinking’, for example, individual thinking which is critical, independent, flexible, and solution-oriented. Such thinking is seen as a *skill* – and can hence be developed through training and practice. A leader’s responsibility includes to develop such skills among her/his co-workers, else the projects will fail.

The level of similarities to the teacher situation amazed me: The classroom is a complex, event-driven context which of a teacher requires strong structural and cultural leadership in order to make informed real-time decisions. Empowerment of students is essential on many levels - of which a fundamental one is such critical and independent thinking.

The underlying reasons for the increased focus on such ‘quality thinking’ – on world-wide level - are for example, technology development, complexity of problems faced, interconnectivity of a globalized society and the speed of change. Most nations have seen the need for ‘quality-thinking’ citizens and hence included those requirements in school curricula - in most subjects. The need for the individual learner’s competence in abstract, critical and reasoning thinking in mathematics learning at all levels is well documented in the literature. Similarly in Swedish educational governance documents and reports, the requirement is recurring (e.g. Skolverket 2003, 2010; Skolinspektionen, 2009, 2016).

Today’s demands for quality thinking stand in stark contrast to the transmission pedagogy of older days, where a student ‘receives’ packaged knowledge (the what/how of a phenomenon) and thereafter is expected to acquire skills by pure repetition practice. Such traditional education does not in general teach the skill of quality thinking. In contrast a more contemporary view of teaching places more emphasis on the individual role of the student. Most modern perspectives share a common, first prerequisite: that the student’s own thinking should be activated[1].

Mathematics education is however still, today, fairly ‘traditional’ in many countries. Also in Sweden, according to reports from Swedish national entities Skolverket and Skolinspektionen[2]. Such education may in practice reduce students to relatively passive receivers of pre-packaged parcels of knowledge transmitted by the teacher. Activation of the students’ individual thinking receives limited attention.

In the aforementioned reports this deficiency is often linked to the low level of problem solving and/or exploratory activities[3]. There is also a large amount of research on problem solving, often with the same purpose/link; seeing this approach as the evident solution to the lack of

[1] Transmission pedagogy versus the importance of quality thinking is further developed below in section 2.1.

[2] The current situation in Sweden, as described by Skolverket and Skolinspektionen, is described in more detail in section 2.2.

[3] More on problems and problem solving, as well as some of the complexity they represent, is found in section 2.3.

development of quality thinking and the same link is made in international assessment reports such as TIMSS and PISA[4].

Meanwhile, in addition to this 'traditional' way of teaching, the teaching of mathematics is very textbook driven in Sweden and the teaching materials are generally not designed from a problem-solving perspective, neither as to content nor form. Problem solving and/or exploratory activities may then become *additional* activities to the prevailing paradigm. Mathematics teachers have – in the same reports - expressed concerns that there is not enough time for such additions. One may ask, given the well-established 'tradition' within mathematics education, whether the lack of transition to a problem-solving approach is caused by the transition difficulty: that such a transition would require a *more substantial transformation project* – a paradigm shift – which can be very hard to implement while ensuring day-to-day activities in a stressful reality.

The question which then surges is – given the fundamental importance of activating and developing students' abstract and critical independent thinking in mathematics education – whether *other ways than such a complete transformation* exist. Considering the existing reality, perhaps **evolution** (a series of smaller changes) has a higher degree of *feasibility* than **revolution** (a larger, simultaneous transformation). Using empowerment theory (as well as an adaptation from the industry of an applied empowerment model), a series of semi-structured classroom observations have been carried out of 18 mathematics teachers in upper secondary level in Sweden with the primary purpose of detecting the *occurrences of activated, exposed and elicited student thinking*, in order to hopefully identify potential alternative ways forward; ways that already have been proven in practice.

2 Background

2.1 The need for Quality thinking vs. Traditional education

Why should transition take place - An important need and counterproductive ways of working?

Today, with the progress of technology, any task that can be performed using a set of predefined rules can be automated and therefore performed by computers, robots and devices. Even the art of defining those rules – a task previously belonging to programmers and engineers - can today be delegated via machine learning/artificial intelligence; rules are defined and redefined during operations by the machine itself; not beforehand. With communication improvement, so-called intelligent devices can communicate among themselves, drawing upon clustered intelligence. The trend is strong and growing fast in several dimensions; e.g. as to application areas; as to the level of delegation possible, as to the complexity of needs addressed etc. Due to previous project experience in the industry, I have been part of implementation as well as strategy development within this field, at national and international levels. The conclusion is that the requirements for human thinking have changed fundamentally – and continue to change at an increasing rate.

At the same time, world problem complexity and urgency increase in all areas of society, but perhaps especially as to sustainability demands; be they environmental, social or of any other kind. A globalized world means that most issues are interconnected and hence problem-complexity grows even further. That the seriousness of problems on world-wide level also increases at an alarming speed, cannot be doubted. The requirement for individually empowered critical thinking grows by the minute.

In Swedish educational governance documents, this situation is described as e.g.:

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[4] TIMSS; Trends in International Mathematics and Science Study *and* PISA; Programme for International Student Assessment.

> ”Changes in working life, digitalisation and technical development, internationalisation and the complexity of environmental issues put new requirements on people’s knowledge and ways of working. The school must stimulate students’ creativity, curiosity and self-confidence as well as their will to test and implement ideas and solve problems” (SKOLFS 2010:144, p.4, my translation)[i].

A few examples of how the needs for quality thinking are expressed (from the Swedish educational governance documents and reports) can be found below[5]:

- On legislation level for primary and lower secondary levels, the introductory chapters do not focus on mathematical knowledge, but on mathematical *thinking* as a tool for future life: ”The school is responsible for that each student after finished education … can use mathematical thinking for further studies and in daily life” (SKOLFS 2010:37, p.7, my translation)[ii] and
  “Students shall practice critical thinking” (SKOLFS 2010:37, p.4, my translation) [iii]
- For upper secondary, the introductory chapters are more detailed:
  ”… the school’s responsibility that each student… can use their knowledge as a tool to
  - formulate, analyse and test hypotheses and solve problems,
  - reflect on own experiences and own ways of learning,
  - critically assess norms, claims and conditions, and
  - solve practical problems and tasks, …(SKOLFS 2010:144, p.6, my translation)[iv]
- The math course plan for primary and lower secondary states that “Mathematical activity is intrinsically a creative, reflecting and problem-solving activity” (SKOLFS 2010:37, p.29, my translation)[v] and uses wording such as: “formulate and solve problems… reflect on and value chosen strategies, models and results… argue logically and reason mathematically…. communicate about mathematics… see the context and relevance of mathematics…” (SKOLFS 2010:37, p.30, my translation)[vi]
- The math course plan for upper secondary states that ”Ultimately mathematics is about detecting patterns and formulate general statements” (SKOLFS 2010:261, p.108, my translation)[vii] and that the education shall…(inter alia):
  - “challenge the students and provide experiences of the logics of mathematics, its generalizability, creative qualities and multi-facetted character”…
  - “strengthen the students’ self-confidence in using mathematics in various situations and provide opportunities for problem-solving as a means and as an end in itself” (SKOLFS 2010:261, p.108, my translation)[viii]
- “The mathematical capabilities largely revolve around being able to identify connections and patterns, to assess and evaluate, to estimate reasonability and to reason one’s way to a solution etc” (Skolinspektionen 2016, p.18, my translation)[ix].

Against this background of urgent needs for quality thinkers, a closer look at education seems appropriate: What kind of thinkers do our educational systems create? In mathematics, what does the aforementioned transmission pedagogy do to thinking? Swan (2005) provides an illustrative comparison of transmission and collaborative orientations[6]:

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[5] The requirements for quality thinking are prevalent in most subjects. The Swedish educational context furthermore has – by international comparison –a high focus on education in democratic values and competencies. It goes without saying that empowered and critical thinking is highly valuable also from this perspective. In this study however, the focus is mathematics education.

[6] The concept of *collaborative orientation* above– *or collaborative learning* - is in this study merely used as an illustrative contrast to the transmission orientation.

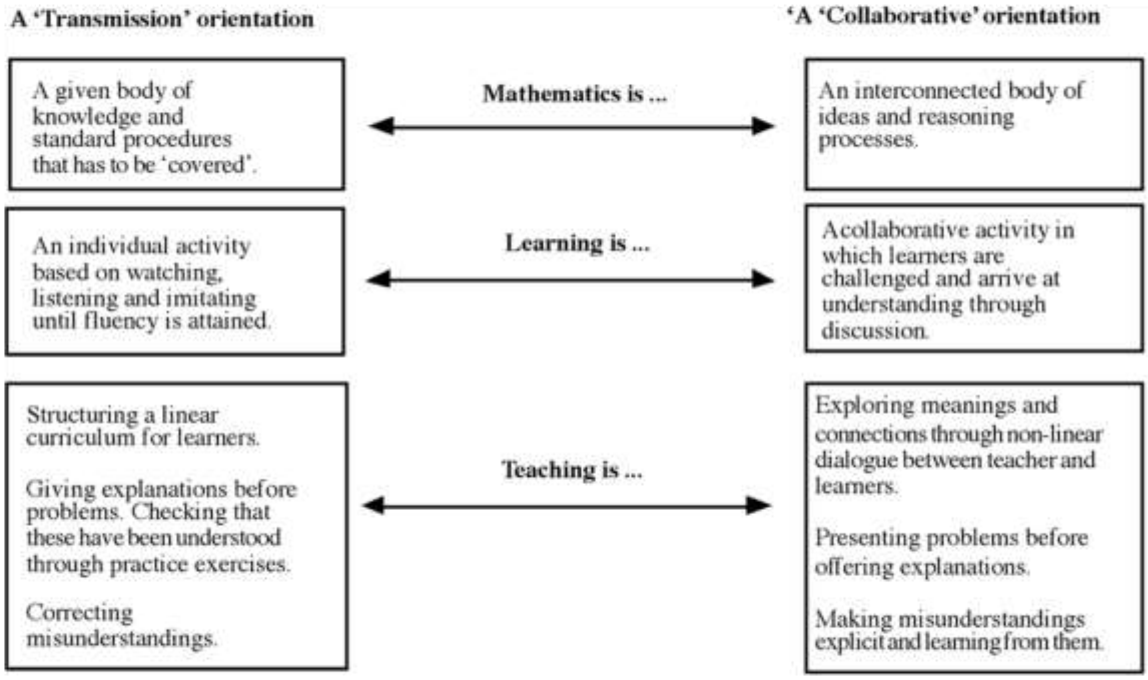

Figure 1: A 'Transmission' orientation vs. a 'collaborative' orientation (Swan, 2005)

From a learning theory perspective, transmission pedagogy can be seen as behaviouristic, with – inter alia - a strong *focus on the teacher as the active provider*, while the student is seen as a more or less passive receptacle. An important aspect is also the teacher as the owner of *truth*; correcting student errors. Education characterized by (albeit various degrees of) this transmission approach is in this study called *traditional education*.

A contrasting and more modern example is a constructivist view, where the individual student actively *builds* the knowledge (Piaget, 1969, 1970), extended into social constructivism (or even constructionism, e.g. Burr, 2003) where knowledge and meaning are *jointly constructed via participation* in a social context. Another example is a sociocultural perspective, conveying a view of the student as an *actively participating actor* in her/his own, individual learning process (e.g. Säljö, 2015). Although modern learning theories constantly evolve and diversify, their common denominator is that *the student's own active thinking* is key. By contrast, a transmission approach does not only focus on procedural knowledge, but also creates a more general thinking behaviour which is substantially limited; more passive and closed (in opposition to e.g. active and open/flexible).

Given the initially described important needs, it becomes paramount to find ways of eliciting such active, open and flexible thinking skills and competencies. In theory, such quality thinking could possibly be stimulated without making it visible. However, if the teacher during the development is to be able to continuously adapt to a student's current level (cf. the zone of proximal development, Vygotskij 1978), it is here argued that making student thinking visible, to *expose* the thinking, is an absolute prerequisite for elicitation. Exposure is, in a wider sense, also required for formative assessment Black and Wiliam (2009).

The question surges whether the current education in Sweden can be said to expose and elicit such quality thinking or if it rather adheres to the traditional education described above. The current situation, as described by the Swedish national entities Skolverket and Skolinspektionen, is developed in the next section.

2.2 The starting point: Traditional mathematics education?

Is the status today – the starting point for any transition – (mostly) a traditional and textbook-driven teaching approach?

Concretising the term “traditional education” as described in the previous section, it could for mathematics education be described as: an instruction method where the teacher provides the “what” and the “how” by introducing a mathematical concept; showing examples of the “correct” solution methods/procedures to use and then instructing students to practice these procedures by providing a list of practice tasks (e.g. Chapko & Buchko, 2004).

The educational approach in Sweden is very similar to the above description according to reports from Swedish national entities Skolverket and Skolinspektionen: Mathematics education is all too often characterised by individual ’silent task solution’ (or even ‘silent calculation’) in the maths textbook, after an initial lecture-like activity (often referred to as a ‘go-through’ in Swedish) and a strong focus on procedural skills. This view – sometimes even *explicitly* labelled as ‘traditional’ *and* ‘firmly-rooted’ in the reports - is relatively consistent across several different reports, since quite a long time, both on primary and secondary levels[7] (e.g. Skolverket 2003, 2010; Skolinspektionen, 2009, 2016). A few examples are found below:

- “The by far most common form of working is that the students sit by themselves and work individually on the textbook tasks…. seldom is mathematics discussed on class level neither as a teacher-lead activity, nor among the students themselves” (Skolverket 2003, pp. 45-46, my translation)[x].
- “…more time is spent on individual forms of work while whole-class teaching gets less space” (Skolverket 2010, p.70, my translation)[xi].
- “…a majority of the lesson time in Swedish mathematics classrooms is spent on so-called silent calculation…" (Skolinspektionen 2016, p.18, my translation)[xii]
- “It can be observed that, with increasing age, the amount of mathematical reasoning decreases while focus on procedure management increases” (Skolinspektionen 2009, p.22, my translation)[xiii].

The same reports in general also convey the consequences; that such traditional education does not stimulate high-quality thinking (albeit in various different formulations). They then often directly (explicitly or implicitly) link this lack of high-quality thinking to the (still) low level of problem solving and/or exploratory activities. Implicit linking may for example occur when *directly* after a description of the traditional situation (sometimes along with its negative consequences), problem-solving is mentioned as a means to generate critical, abstract and independent thinking. For example, the Skolinspektionen (2016) report posits that “The education needs to provide support for exploring mathematical problems and stimulate abstract thinking” p.13, my translation)[xiv].

In the same way, problem-solving often is woven into the descriptions of ‘mathematical thinking’ in the Swedish governance documents (cf. the previous section) as well as in e.g. the PISA 2022 assessment framework, where problem solving is linked to the more general concept of mathematical literacy; “the capacity of individuals to reason mathematically and solve problems in a variety of 21st century contexts” (OECD, 2018, p.6). One may argue that these formulations refer to problem-solving as a *student skill*, but several of the teachers in this study interpreted them as the *teaching approach:* problem-solving as the answer to the need for development of quality thinking. There is also a large amount of research on problem solving, often with the same purpose/link; seeing this approach as the evident solution to the lack of development of quality thinking.

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[7] The total age span of primary and secondary levels coincides with the Swedish age span; however, the border between the two is different: in Sweden *grundskola* includes the age span 7-15 years and *gymnasium* comprises the age span 16 to 19 years of age.

Meanwhile, in addition to the aforementioned ‘traditional’ way of teaching, there is firstly a certain discrepancy between the governance documents and the teaching materials in Sweden, which are generally not designed from a problem solving perspective, neither as to content nor as to form[8]. Secondly, the subject of mathematics is very textbook driven in Sweden and the situation is firmly established. Examples:

- “The education is strongly textbook-driven. This has the consequence that the students have little or no possibilities to develop their problem-solving skill-set, their ability to use logical reasoning and their ability to contextualize mathematical problems” (Skolinspektionen 2009, p.9, my translation)[xv].
- “There is… a strong negative correlation between [the use of] textbooks and other [non-procedure-related] skills” (Skolinspektionen 2009, p.16, my translation)[xvi].

Problem solving and/or exploratory activities may then become *additional* activities to the prevailing paradigm and mathematics teachers have expressed concerns that there is not enough time for such additions: “Teachers do find that learning problem-solving is important, but state that it loses out since there is no time” (Skolinspektionen 2016, p.12, my translation)[xvii].

Even if there *were* time enough, we will however in the next section see that the transition to a problem-solving teaching approach may present a number of obstacles for the teacher.

2.3 Problems and problem solving – some transitional difficulties

Is it easy (or even possible) for most teachers to transition from their current position to the new paradigm?

As mentioned above, besides being a student skill, *problem solving* can also be seen as a teaching approach. For any teacher adhering to more traditional ways of teaching, there are quite a few hurdles to overcome, if undertaking the transformation to the new paradigm. These stumbling blocks may constitute contributions to that several Swedish educational reports firstly describe a low level of (implemented) problem solving (e.g. Skolverket 2003, 2010; Skolinspektionen, 2009, 2016). Secondly, when such activities *do* exist, the reports (e.g. Skolinspektionen, 2016, p.13) express a certain doubt as to whether the activities *labelled* as ’problem solving’ – as implemented in today’s practice – fulfil their intended purpose. Something is amiss – and a few possible causes are developed below.

Generally, the terms *problem* and *problem solving* have had many different (and sometimes conflicting) meanings over the years – a view which comes across relatively consistently in the literature (e.g. Schoenfeld 2013; Frejd & Lundberg 2015; McDonald & Smith, 2020).

In the Swedish curriculum and other top level governance documents, no definition can be found of the terms *problem* and *problem solving* (adding to potential confusion may be the fact that *problem solving* is considered both as a *capability* and as *central content*, without any further explanation). As mentioned in the previous section, Swedish maths textbooks are generally not oriented towards problem solving at all, while “[t]he textbook has a function as an interpreter of mathematics for teachers, pupils and parents” (Kongelf 2011, p.6)

Firstly, as a consequence of the above, confusion – or even misconception – of what “a problem” is on general level may exist. Some (albeit few) teachers in this study interpreted problem-solving as the solution of textual tasks. However, tasks – even in textual form - are not necessarily problems in the *problem-solving* sense. If a task can be identified as a standard task (e.g. a linear equation, a combinatorial problem, etc), there is often one (or a few)” best” *solution method(s)*. Knowledge of such methods is generally called *methods /procedural knowledge*. This implies (if

[8] There is no textbook review or approval on governmental level in Sweden since the closing of Statens institut för läromedel – SIL (National institute for learning materials) in 1991.

the knowledge is well-founded) that the steps of the method can be executed almost ‘automatically’. The previous knowledge of the problem solver thus defines whether an assignment is a standard task or a problem (Blum & Niss, 1991, p.37) and this may of course change over time, as a student learns new things. If such methods/procedural knowledge is lacking, the assignment belongs to the upper cognitive levels for *this particular student*; “what constitutes a problem is individual” (Larsson, 2013, p.2, my translation[xviii]). By extension perhaps even a creative (re)creation is required: “students create or re-create a reasoning sequence that, to some extent, is new to them” (Granberg & Olsson, 2015, p.50), even if *imitative reasoning* (in opposition to *creative reasoning*; Lithner, 2008) also is included.

How then can a teacher secondly (search for and) *identify* appropriate (true) problems for the students, when the above does not provide information of the intrinsic properties/character of *the task in itself*?

As a first criteria, the solution strategy must contain at least one step of pondering (non-automated thinking), also for a student with extensive methods /procedural knowledge - for the task to potentially count as a true problem. Kongelf (2011) expresses this as” a situation that requires a decision and/or an answer, no matter if the approach of solution is readily available or not to the problem solver” (p.11). The voluminous research on problem solving (please see section 4) extends this characteristic and provides various lists of criteria for a “good” problem. As an example, Taflin (2007) defines “rich problems” as having the following characteristics:

1. The problem should introduce important mathematical ideas or solution strategies.
2. The problem itself should be easy to understand and everyone should be able to work on it.
3. The problem should be perceived as a challenge, require effort and problem solving must be allowed to take time.
4. The problem should be solvable in a number of different ways, using different strategies and representations.
5. The problem should have the potential to serve as a base to initiate a mathematical discussion among the solvers; a discussion eliciting different strategies, representations and mathematical ideas.
6. The problem should have the potential to build bridges.
7. The problem should have the potential to result in students and teachers formulating new interesting problems.”
   (Taflin 2007, p.56, my translation[xix]).

Thirdly, if the teacher is fully able to distinguish between true problems and standard tasks and/or has found some sort of identification criteria that can be used, she/he may still find it difficult or too time consuming to *find* true problems – not to mention finding a sufficient *volume* of good problems over time - for a group of very different individual students. Teacher education may vary – while some universities put strong focus on the practical challenges of problem solving as a teaching approach, providing good sources of true problems, others may not.[9]

Fourthly, if “true” problems were indeed found, a teacher may perceive difficulties in the “how-to” design such a lesson (and/or assess the student performance, as mentioned by one of the teachers in this study) around such problems.

Fifthly, there are a number of cultural issues related to changing prevalent socio-mathematical norms (Yackel & Cobb, 1996) in the classroom. The traditional mathematics education approach of individual ’silent task solution’ in the maths textbook after an initial lecture is very different from a problem-solving set-up. The student familiarity with (and will to) work according to think-pair-share (Lyman, 1981) - regardless of problem solving - may vary greatly according to the

[9] Two lessons often used in teacher education in Sweden are the mathematical problems “The Tower” and “Tile Placement” (Swedish real classroom applications can be found on https://youtu.be/xTg5fYKM9zY and https://larportalen.skolverket.se/#/modul/1-matematik/Grundskola/435_problemlosning%20%C3%A5k7-9/4_urvalordningochsammankopplingavelevlosningar/flik/flikmeny/tabA respectively).

existing didactic contract (Brousseau, 1984). Time must also be spent *beforehand* on establishing students' dialogical skills and an open communication climate (leadership tasks) for the approach to be successful (Granberg & Olsson, 2015). Student values, attitudes and beliefs are also important (e.g. Schoenfeld 1985). A rather high change threshold may therefore exist, since cultural change is difficult and takes time.

Finally, and perhaps most importantly, transformation puts heavy change requirements on the teacher herself. One issue for more traditional teachers may be *time management* in a perhaps heterogeneous student group (including the management of personal stress levels), given that "problem solving must be allowed to take time" (Taflin 2007, p.56).

True teacher *behaviour change* is also required: The students must "be given the opportunity to formulate and test hypotheses and assumptions…" (Skott et al, 2010, p.17, my translation[xx]), which in turn implies that the teacher **must not tell** or explain the 'how to do it'. Teachers used to providing full explanations of facts and solution methods - upon which the students individually practice by repetition - may find themselves at a loss. If telling, the teacher would lower the cognitive level of the problem by "'*taking over'* the thinking and the reasoning and then tell students *how* to solve the problem" (Larsson, 2013, p.6, my translation[xxi]). Instead an a-didactical situation should be created (Brousseau, 1997). If less experienced in teaching problem solving, a teacher may not know *how* to guide the students forward (scaffolding) *without* explicitly telling them what to do. Explicit telling becomes piloting in the navigational sense; the teacher becoming the tug-boat trailing the student behind (the Topaze-effect, Brousseau, 1984) and the task at hand is emptied of its learning potential.

If insecure on *how* to achieve the new way of teaching, the teacher may avoid student questions (by moving on) or even yield to the temptation to express enthusiastic (but empty) encouragement such as "I am sure you'll think of a really good solution", as once observed during this study (very similar examples can be found in Lingefjärd & Meier, 2010).

Teacher behaviour change may however not be enough; a fundamental *new way of thinking* may be required. Helenius (2019) uses the expression *task discourse,* i.e. "you talk and think *quantitatively* about teaching; the students should complete a number of tasks, going down the list [versus when] the most important is for students to develop the *quality* of their thinking" (p.1, my translation[xxii]). The focus of the traditional approach ends up on calculation, instead of on abstraction (Mann, 2006).

In all, a teacher used to working according to the traditional approach may find the transition to a problem- solving approach daunting, to say the least. To be required to undertake such a transformation on individual level, in an intensely time-pressured operational situation and without the correct project support may be questioned, according to my personal experience from many transformation projects around the world.

A start-up project is much less stressful than a transformation project, since the latter requires implementing change *while* daily operations must be ensured in parallel. As in any transformation project in an operationally intense environment, change resistance behaviours due to a lack of time and resources may be expected – it is most often not a question of "bad" attitude, but of feasibility. Given the seemingly firm roots of today's mathematics teaching, any transformation to e.g. a problem-solving approach could be considered a paradigm shift, since it constitutes such a substantial change of the operational context. In the industrial sector, there is a wide and well-established awareness that such larger changes may require a considerable number of extra project resources, a good, proven knowledge bank (models, time plans etc) and hands-on leadership with experience from transformation projects – it is *not* something that can be done by the operational staff themselves as part of (in addition to) daily operations.

Perhaps, the explanation for the lack of change which Skolverket/Skolinspektionen describes (see section 2.2) is, simply put, lack of *practical feasibility* for many teachers. Change resistance is a human condition and we are all human. Such resistance – sometimes seen as attitude problems that need to be addressed (e.g. Gregoire, 2003) - may of course exist, but according to my experience such problems are primarily inversely proportional to the level of feasibility (or even *ease* of implementation). Resistance to change can thus be heavily reduced if ease of implementation is ensured.

From a leadership perspective, one could question the requirement in the educational world for such a change to take place on an *individual (teacher) level*, or even on school-level, without providing the necessary project support centrally. The target context is not only complex and event-driven, but also perceived by today's teachers as very time-pressured. After many years of change management, my firm view is that if change is to take place, it must not only be *feasible*, but as *easy* to implement as possible for the target population, else change will not occur.

The question surges whether it is possible to find a transitional approach *within* the current paradigm. The overall objective – the paradigm shift – remains, but a step-by-step transition (evolution) could perhaps be more feasible than a revolution. Evolution means gradually including more opportunities for exposing and eliciting quality thinking and – as a by-product – the time necessary to *get used* to a new approach for all parties involved; teachers as well as students. Cultural change takes time, according to my experience from the industry's many transformation projects.

3 Research Questions and Purpose

3.1 Research questions

As mentioned in the Introduction, the primary objective of this study is to examine the *occurrences of activated, exposed and elicited student thinking*, in order to identify potential alternative ways forward; ways that already have been proven in practice. This translates into the following research questions:

1. How is student thinking made *visible*[10]; i.e. to what degree are there observable occurrences of *exposed* student thinking, outside (independently of) paradigm shifts such as problem solving, and how are such occurrences created within the classroom context?
2. How is student thinking *stimulated*; i.e. to what degree are there observable occurrences of *elicited* student thinking, outside (independently of) paradigm shifts such as problem solving, and how are such occurrences created within the classroom context?

Furthermore, in order to substantiate the claim made in this study that a paradigm shift is needed at all, the current status must also be investigated; which is guided by the research question;

3. To what extent are the observed mathematics lessons adhering to the description of Skolverket/ Skolinspektionen in terms of textbook driven and traditional education?

[10] As previously mentioned in section 2.1, student thinking could in theory be elicited/stimulated without making such thinking visible. However, if the teacher during the development is to be able to continuously adapt to a student's current level (cf zone of proximal development, Vygotskij 1978), it is here argued that making student thinking visible, to *expose* the thinking, is an absolute prerequisite for elicitation.

3.2 Purpose of the study

The purpose of this study is therefore to:

- inventory, identify and describe the lesson components (LCs) used during the observed lessons;
- investigate the number and size of the LCs (and compare to the portrayals of the current situation as described by Skolverket/Skolinspektionen);
- analyse the degree of *exposed* and *elicited* student thinking
- describe the LCs – i.e. the teacher actions – which generated a high degree of *exposed* and *elicited* student thinking

3.3 Delimitations

This study does not aim to provide a full statistically representative data set, since it only involves 18 teachers at one (albeit large) school.

However by doing the above, this study can hopefully identify thought-exposing and thought-eliciting teacher actions that are less strenuous to implement than a paradigm shift and hence to contribute a higher transition level as to leaving the old paradigm of traditional education behind.

4 Previous Research

In the literature there is a large number of different concepts, models and tools related to improving student quality thinking. While attempting to inventory these researches, three main groups were identified:

1. Firstly there is a vast volume of studies on problem solving and related concepts (e.g. on problem-posing, problem-based learning, etc), which is described in further detail below in section 4.1.
2. A second group of literature involves a number of concepts outside the first group, but often of similar complexity as to transitional difficulties (e.g. active learning, interactive learning, adventure-based learning, game-based learning, collaborative learning, etc) and is developed in section 4.2.
3. A third group of research, which focusses on student quality thinking without necessarily forming part of an overarching concept is expanded upon in section 4.3.

In all, most of the concepts in the first two groups imply a paradigm shift compared to the prevailing situation (as described in section 2.2). This study claims that such paradigm shift (revolution) requires true transformation project support, for it to be successful (see section 1). Such support is rarely given and it is necessary to instead examine the issue from an evolutionary perspective, allowing a gradual transition. Research from this *evolutionary* perspective has been found to be very limited; also within the third group.

One could argue that some of the literature in all three groups describe (at least examples of) teacher actions which *could* be used also in an evolutionary approach. Sometimes seemingly different concepts even include very similar interventions. It is suspected that several of these *do* have high potential to be used in an evolutionary approach, but it goes beyond this study to do a comprehensive analysis of the literature in this regard; tentatively identifying such – at least today *seemingly disparate* – actions and their conjunctions/ disjunctions[11].

[11] The complexity of the task is furthermore increased by differences between the studies as to theoretical framework, empirical data / research objects, methods etc.

4.1 Literature on problems and problem posing, solving etc in general

The first group of identified research are those using the term *problem-solving* in the article's title or abstract, why this group here is defined as *the explicit group* (e.g. Polya 1945; Schoenfeld 1985, 2013; Blum & Niss 1991; Lester & Cai, 2016; Mason, 2016; Olivares, Lupianez & Segovia, 2021). Some of this literature focus on problem solving as an important *student skill* (be it the skill of problem solving as such or problem-solving as a means to learn something else), while some see problem solving more as a *teaching approach*. Some literature link the two, seeing the teaching approach as a prerequisite for development of the corresponding student skill. In all, there is a vast volume of literature on problem solving and, paradoxically, the more studied a concept is in the literature, the more complex the concept may seem to a teacher (ranging from the theoretical frameworks to the required teacher interventions or prerequisites for the concept to be beneficial).

As described in section 2.2, the transformation from a more traditional teaching approach to a problem-solving approach implies a substantial learning burden (or even a paradigm shift) for the teacher, which in turn constitutes an important change implementation threshold. This group of literature is therefore of limited use to this study.

Other terms are closely related to *problem solving* as described in this study, since they imply a similar paradigm shift (sometimes even also include the same type of teacher interventions) and therefore face the same implementation thresholds. They often also assert the same important effects on thinking skills and/or on empowerment (e.g. McDonald & Smith, 2020). Such terms were therefore also included in this group; for example:

- Problem-posing (e.g. Ellerton, 1986, 2013; McDonald & Smith, 2020)
- Mathematical modelling (e.g. Abassian, Safi, Bush & Bostic, 2020).)
- Model-eliciting activities (e.g. Chamberlin & Moon, 2005).
- Problem-based learning - PBL (e.g. Dochy, Segers, van den Bossche & Gijbels, 2003).
- Inquiry-based learning - IBL (e.g. Pedaste, Maeeots, Siiman, de Jong, van Riesen, Kamp, Manoli & Tsourlidaki, 2015).

4.2 Literature on other tools, methods and concepts

A second group of identified research on students' thinking/thinking skills – *outside* the aforementioned *explicit group* – was observed to relate to studying the use of specific tools, methods and/or concepts. This group comprises literature on concepts such as *collaborative* learning (e.g. Swan, 2005, 2006), *active* learning (e.g. Kerrigan, 2018; Been, 2022), *interactive* learning, *game-based* learning, *adventure-based* learning (e.g. Jabbar & Felicia, 2015), *Realistic Mathematics Education* – RME (e.g. Freudenthal, 1991; Bray & Tangney, 2016), *STM* education, *STEM* education, the *STEAM* concept (e.g. Margot & Kettler, 2019; Kong & Matore, 2022)[12].

The concepts may come either from within the area of mathematics or from other areas; e.g. using computational thinking (Papert, 1980) to improve mathematical thinking. The literature belonging to this second group sometimes *implicitly* relate also to problem solving, but more importantly, the concepts all imply a substantial learning burden for the teacher. They hence share many characteristics with problem solving in terms of transitional difficulties (e.g. Swan, 2007), why also this group was of limited use for this study.

[12] STM is most often defined as Science, Technology and Mathematics; STEM adding Engineering and STEAM also including Arts.

4.3 Other literature on students' thinking

A third group of research focusses on student quality thinking without necessarily forming part of an overarching concept (as the two groups described above). Earlier studies often used the term "mathematical thinking" (e.g. Mason, Burton & Stacey, 1985; Schoenfeld, 1992), while the more recent literature in this group often uses the term "higher-order thinking skills" (HOTS or HOT skills). A few examples of used terminology, in both earlier and more recent studies, are: *(thinking) fluency*, *flexibility*, *originality*, *novelty*, etc and skills of *abstraction* (e.g. Ezeamuzie, Leung & Ting, 2021) *metacognition*, *autonomy*, *self-efficacy*, *agency*, (e.g. Bandura 1977a,b, 1997 onwards; Bjork, Dunlosky & Kornell, 2013), *reasoning*, *proofing*, (e.g. Jeannotte & Kieran, 2017; Dawkins & Weber, 2016) and last but not least, *creativity* (e.g. Balka, 1974). In this group, a difference between on the one hand *understanding* students' thinking and on the other hand *stimulating/promoting* students' thinking was found.

Some of the research on *understanding thinking* may come across as quite abstract, since it often focusses on the definition and/or conceptualization of what these HOTS are (and their delimitations); i.e. which are the "true" HOT skills and which are the "external", but related, skills; which are their respective properties/attributes etc. The similarity to the description of the construct *attitude* by Van Aalderen-Smeets, Walma Van Der Molen and Asma (2012) is striking; as it

> "is not a single unitary concept; it is a construct consisting of multiple dimensions and subcomponents… measuring [it] should consist of measuring the various dimensions and subcomponents ... To date, however, there is no consensus on the number or identity of the dimensions and subcomponents that constitute the construct..." (Van Aalderen-Smeets et al, 2012, p.161).

The complexity of the construct in this study ("thinking" is a higher-order construct and "higher-order thinking" is even higher) similarly implies a vast number of different - and still, within the field of research, mostly unrelated - interpretations. From a usability perspective, a teacher may find it very time-consuming to penetrate. For this reason and due to its rather theoretical character, the literature in this sub-group was found of limited use to this study.

On the other hand, (at least part of) the literature on *stimulating/promoting thinking* could very probably be used in an evolutionary approach. However, firstly, due to the above-mentioned great variety in underlying interpretations, a separate (meta) study would be required in order to do a comprehensive analysis of the literature in this regard; a task that unfortunately cannot be included in this study due to time constraints. Secondly and more importantly, these studies place much emphasis on the higher-level construct as such; often measuring an improvement of learning outcomes, correlating it to the higher-level construct (or one of its perceived subcomponents a/o dimensions) on *"end-level"* (or higher level only). The concept of progression and the use of an evolutionary approach seems very limited.

5 Theoretical Framework

As mentioned previously, the purpose of this study is to explore and identify *yet unknown* actions which expose and elicit student thinking (see section 3). Investigating this issue, the choice of theoretical framework becomes key. In other graduate studies within the mathematics education field, in relation to mathematical thinking, various frameworks are used of which a common one is for example dialogical theory (cf. e.g. Socratic dialogues; Bahktin, 1981; Alexander, 2017). It's a well-integrated theory in the sense that its framework also can be used as analytical method, studying a *predefined* set of parameters/ indicators with a strong focus on language. However, since in this study the objective is to find *any* teacher actions that stimulate students' own thinking in a wider sense – not predefining such parameters – dialogic theory may be considered too narrow. Against the background of Skolverket/ Skolinspektionen's view of quality thinking within mathematics (cf section 2.1), the empowerment theoretical framework fits well for this study: “The process is empowering if it helps people develop skills so they can become independent problem-solvers and decision-makers” (Zimmerman, 2000, p.46).

In the two main research questions, the exposed and elicited student thinking are seen as components of *empowerment as outcome*, while the teacher actions are considered components of *empowerment as process; concepts which are further described below*.

5.1 Empowerment Theory

The term *empowerment* is a western-world construct which came to widespread use during the 80ies and 90ies in many different contexts of society – and not seldom in a loose or casual and uncritical way (e.g. LeCompte & Bennet, 1988/1992; Perkins & Zimmerman, 1995). In my own experience within the business segment, it became a so-called ‘buzz-word’, where empowerment of co-workers was claimed to generate *simultaneously increasing* levels of productivity and quality and it was hence strongly focused – various methods were tried out with varying success. The term has also been used in many different fields of research, e.g. organizational theory, political science, psychology and education, on many different societal levels; nations, communities, groups, and individuals. “In educational research (ERIC), the number of articles on the topic rose from 66 between 1966 and the end of 1981 to an astounding 2,261 from 1982 through March 1994” (Perkins & Zimmerman, 1995, p.571).

In the field of *community psychology* an early definition, seeing empowerment as a *process* and which highlights that empowerment may take place at various levels, is: “Empowerment is viewed as a process: the mechanism by which people, organizations, and communities gain mastery over their lives” (Rappaport, 1984, p.3). That the actions or activities of the process also generates empowerment as *outcome* was included in the concept a few years later (Swift & Levine, 1987).

Research within community psychology furthermore often relate to inequality of power in social relationships – for ethnic, gender, religious, class or other reasons – which is in line with another widespread definition: “Empowerment is an intentional, ongoing process centred in the local community, involving mutual respect, critical reflection, caring, and group participation, through which people lacking an equal share of valued resources gain greater access to and control over those resources” (Cornell Empowerment Group 1989; cited in Zimmerman, 2000, p.43).

In this study, the focus will be on **individual level**, emphasizing individual agency and disregarding social mobilization, that is, the more conservative approach’s primary objective according to LeCompte and deMarrais (1988/1992): “A more conservative, reformist approach focuses on micro-level educational processes and participants within formal, usually public, school settings, where education or pedagogy is the primary objective, and social mobilization is a secondary objective, if present at all” (p.18).

Therefore, the aforementioned general definition of Cornell Empowerment Group is in this study reduced to: "Empowerment is an intentional, ongoing process centred in the local community, involving mutual respect, critical reflection, caring, and group participation” - and in this text with particular focus on “**critical reflection**”. Zimmerman explains the term *individual empowerment* “as a construct that includes only what goes on in the mind." (Zimmerman, 1995, p.582).

All individuals have different characteristics and capabilities and the aim is that any empowerment process must start at the individual’s own level and that the empowerment as outcome will be different for different individuals. Hence “[t]he term *psychological empowerment* takes on different forms in different contexts, populations, and developmental stages and so cannot be adequately captured by a single operationalization, divorced from other situational conditions" (Zimmerman 1995, p.582). Any model of an empowerment process must therefore allow for various *different* entry points.

The objective of empowerment can then be seen as to “manipulate the process of identity formation in order to create "positive," or "empowered," identities” (LeCompte & deMarrais 1988/1992, p.18) as to students’ thinking – which teachers may do in a multitude of different ways, depending on the teachers’ individual characteristics and on the entire learning context.

5.2 To Activate, Expose and Elicit thinking – a three level journey

The above empowerment research is in this study combined with the previously mentioned experience from the attempts in the industry to empower co-workers (see section 1). The often strong financial focus of the industry frequently leads to extensive quality/ continuous improvement systems, with a high number of Key Performance Indicators (KPIs) – financial as well as operational - with predefined target levels, arranged in structural hierarchies. Measurements / follow up is undertaken in a very systematic way. Experience from such activities can hence be considered *proven experience*, in the sense communicated by Skolverket and Skolinspektionen: ”[A]ll experience is not proven experience. Proven experience is systematically tested, documented and generated by many, during longer time periods” (Minten & Kornhall, 2013, p11, my translation)[xxiii].

My personal such experience within several large corporations, rendered a so called “empowerment staircase”, meaning a structured and documented empowerment process which was gradually refined over time by several managers and their co-workers. Empowerment as process *and* outcome was evaluated according to the industrial principles described above; from the co-workers’ perspective as well as from their managers’ perspective, and implications on production results carefully analyzed. The model/method was deemed very successful, so much so that it was considered a competitive tool.

The principles of the industrial model are described in section 5.2.1 below, followed by a critical analysis of how the model is applied and concretized within the educational context, focusing particularly on students thinking.

5.2.1 The underlying model from the industry – its basic principles

A basic principle in the industry, most probably due to its focus on profitability, is to calculate *return on investment* (ROI) on any actions taken. Such calculations lead to the inevitable conclusion, that it is no point in spending time and effort on filling up a bucket with holes – the holes must be fixed first. Spending money on a large marketing campaign to acquire new customers, would be a waste of money, unless first implementing a project which addresses that existing customers are leaving. Else the newly acquired customers most probably would also leave, just as the existing ones. The time and effort to acquire them would be wasted.

Figure 2: Fixing before filling a bucket

The concept of ROI is however not only applicable in finance and for profitability reasons. Also, energy and time can be invested and the term ‘emotional investment’ is well-established within psychology and social sciences. The same ROI principle – and hence “the bucket principle” - can be extended to encompass almost anything which involves investing time and energy to make an improvement happen. The “return” does *not*, furthermore, have to go back to the investor in the form of “profit”: the time and energy saved, can be used to further improve outcome.

Using a step-by-step approach (evolution instead of revolution) means applying the bucket principle on every step of “a staircase”, while avoiding the common error the bucket principle tries to illustrate. Instead of immediately initiating activities to raise the level (spending time and energy), by performing some sort of ‘positive’ actions, we will halt - to first ensure we do not waste the time and energy we are going to spend. On each step we will, in fact, apply the bucket principle *twice*:

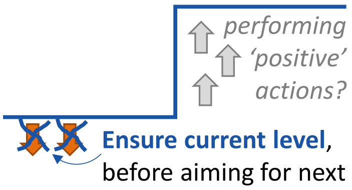

*Figure 3: Decrease negatives **to current level**, before increasing positives*

Illustrating the water levels (existing and desired) in the bucket by a staircase, the first action is identical to the bucket picture above: From the above mentioned ROI-perspective, **ensure the current level**, *before* aiming for next; ensure that the current level isn't sinking before trying to raise it. In short: Fix any "holes" before trying to raise the level.

Continuing the example with the bucket, we have made sure there are no holes, *lowering the level*, but there may be other obstacles to *raising the level* that we haven't noticed.

For example, before pouring the water, make sure there is no (currently invisible) lid on the bucket. Therefore, again before taking positive action, investigate and remove (at least reduce) any obstacles for the change to take place, before trying to raise the level.

*Figure 4: Decrease negatives **to change**, before increasing positives*

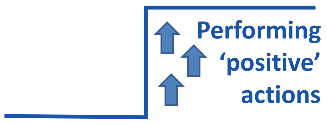

Figure 5: Finally, take positive action

Finally, positive action can be undertaken.

It should be underlined once again that the entire ROI-approach described in this section is for the purpose of the current study *not* used for efficiency in terms of financial gain, but to save the teachers time and energy in a very time-pressured situation.

Simply put: A structured step-by step approach, decreasing negatives before increasing positives, may increase the level of outcome; ensuring spent effort generates effect. In a situation where time and effort are scarce resources, the importance of such an approach increases further.

5.2.2 Applying the industrial principles on student thinking

As discussed above, the aforementioned principles have been used in a number of development situations in the industry, creating step-by-step 'staircases' that have been used – and hence improved over time – in many different cultural contexts. One of these models is a co-worker development staircase:

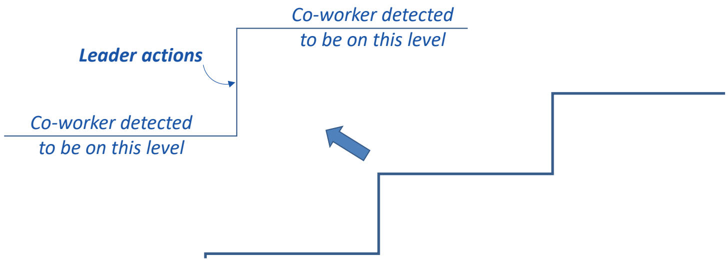

Figure 6: Co-worker development (e.g. empowerment) staircase; repetitive steps.

The basic principle is, as described in the previous section, to detect and ensure the previous level and only then as a leader take action towards the next (decreasing the negative before increasing the positive)[13]. The key is to stay in this stage (the leader repeating a/o intensifying her/his positive actions and from time to time checking on negatives) until the co-worker *is detected* to have reached the next level. Only then is the next step in the staircase addressed. The exact leader

[13] This approach of detection is by no means unique to the industry – cf. so called *hinge questions* as a way to determine what the next step is and when to take it (Wiliam, 2011).

actions and detection criteria will depend on *which* characteristics that are to be strengthened (e.g. area knowledge, customer orientation etc – or, empowerment).

Concretizing the above to the educational context, focusing on thinking, would render the following staircase (the numbers in the picture are further developed below):

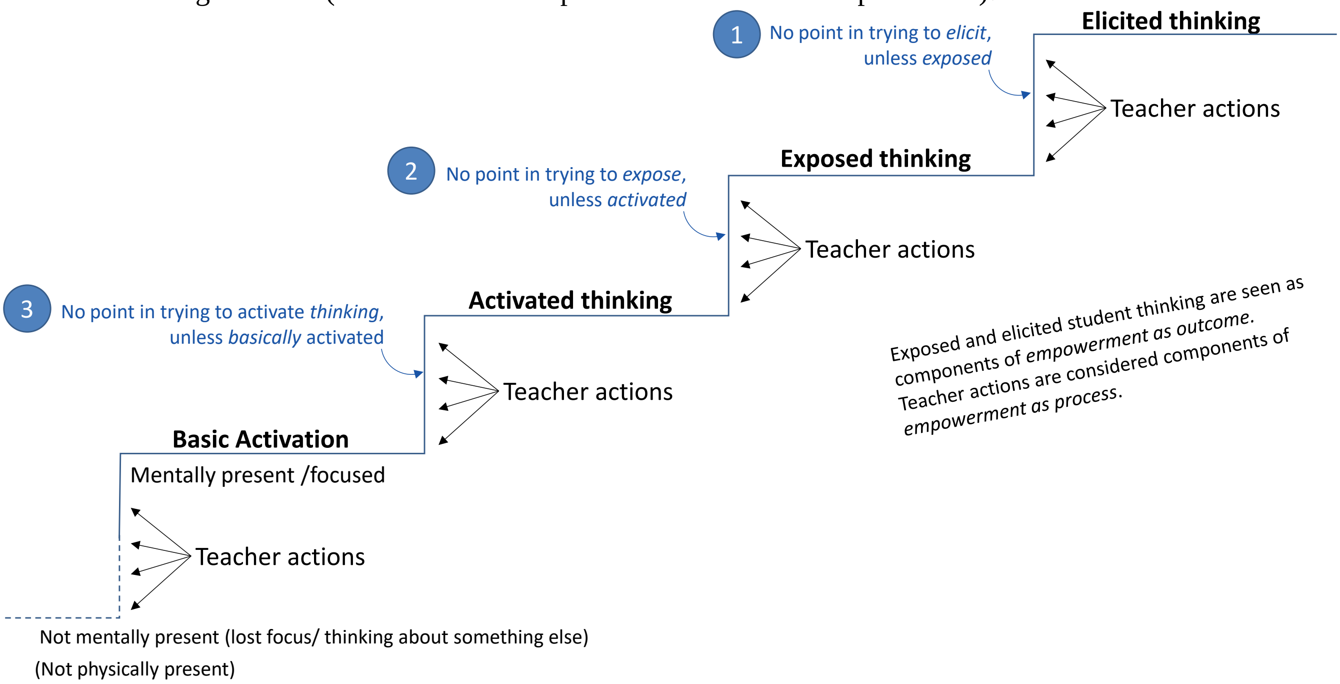

Figure 7: Activating, exposing and eliciting thinking (staircase)

1. As previously mentioned in section 2.1 and 3.1, student thinking could, in theory, be elicited/developed without making such thinking *visible*. However, if the teacher during the development is to be able to continuously adapt to a student's current level (cf. *zone of proximal development*, Vygotskij 1978; and for *formative assessment*, Black and Wiliam, 2009), it is here argued that making student thinking visible, to *expose* the thinking, is an absolute prerequisite for elicitation. This can now also be tied to the industrial model above, as *ensuring the previous step*.

2. In the same way, to try to expose thinking that has not been activated would firstly be inefficient according to the ROI principle (there is not necessarily anything to expose) and secondly rather unfair, since it would expose not only the thinking, but the student on a personal level: without actually thinking (not being activated), the student is asked to expose her thoughts.

3. Again, as a teacher to try and activate a student's *thinking* without her/him being basically activated (mentally present - focussed), would require *first* calling that student back to the "here and now", potentially also calming down (ensuring the previous level)[14].

The objective of choosing the expression "no point" in the picture above, is to raise the alarm so that we don't start initiating actions that has a low probability of positive effect: In the industry it is intended as a help to prioritize the time and effort available to a leader in order to raise awareness and/or performance of co-workers, as they often constitute a quite heterogeneous group. To achieve an increase in outcomes for only a minority of such a group, is then *not* considered an improvement. The change must occur on a general level.

[14] Again, seeing such mental presence (readiness) as being the basic level is not unique to the industry. Already in 1964, Bloom and Krathwohl described such readiness as the first level in the *second* volume of the wide-spread taxonomy (The Affective Domain).

Finally, the teacher actions in the picture above are, as mentioned in section 5.1, seen as *empowerment as process*, while the occurrences of activated, exposed and elicited student thinking are seen as *empowerment as outcome*. Empowerment as outcome *and* as process are the principal elements in the main two research questions presented in section 3.1.

6 Method

6.1 Overview of research design and selection

In order to answer the questions proposed by this study, quantitative as well as qualitative methods have been used. Semi-structured classroom observations constitute the main element, complemented by questionnaires and semi-structured interviews. Based on the purpose of the current research and the research questions, a triangulation mixed-method design was adopted where the quantitative and qualitative data sets complement each other in order to have a holistic picture of the situation under consideration, as proposed by Creswell and Plano Clark (2007).

Classroom observations have been made via visits to all the 18 mathematics teachers at a large upper secondary (age span 16-19) Swedish school in the Stockholm region of Sweden. The number of students at the school is close to 1400 and the number of teachers close to 120 (not counting special pedagogues, maternity tongue teachers, nor teacher assistants/student assistants etc). Even if the study cannot be said to be statistically representative, it is argued that it is analytically representative as to parameters such as Swedish teachers' gender distribution, age and years of experience (Lärarnas riksförbund, 2018; Skolverket, 2020; Sveriges Kommuner och Regioner, 2021). This case is considered to be an analytical typical case (Hansson 2012, 2017).

6.2 Data collection methods

A semi-structured **Classroom Observation Protocol** (COP) was purposely developed for this study. The quantitative observation parameters were mainly of two types: *presence* of …. (e.g. whether or not something occurred) and *amount* of … (e.g. number of … , time spent on …[15]). In an attempt to attend to levels of quality and implementation, sets of *indicators capturing multiple aspects of teaching* were also included according to the categorization of research on classroom instruction by Bostic, Lesseig, Sherman and Boston (2021). Finally, additional qualitative data was captured as free text notes during observations and during the interviews.

The study was done using a three-step approach (see figure below):

[15] Two stop-watches were used alternately to measure time intervals for such parameters.

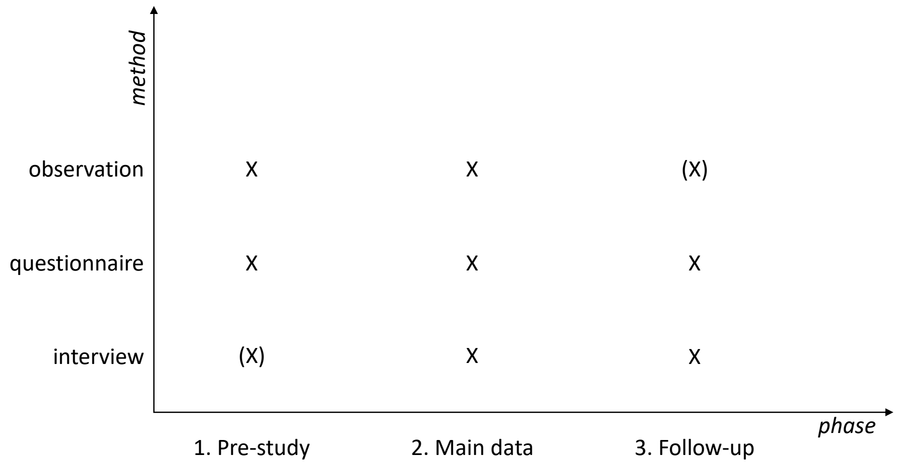

Figure 8: The three-step approach of the study

1. A small pre-study was done, in order to define the observation parameters to be included in the **classroom observation protocol** (COP). Using the theoretical framework, a limited inventory of existing COPs was made and a first COP draft was defined for this study, including quantitative as well as qualitative parameters. The COP was then tested iteratively in class room situations (three teachers selected for differences in teaching style, as communicated by the math teacher community at the school). A few short questions were asked to the teacher after the lesson, for clarifications. Identified shortages were identified and a second COP was developed.
2. The second COP was used at one lesson for *all 18 teachers*. To ensure observation of (according to the teacher's own assessment) a "normal teaching situation", the observation was postponed (also at the very last minute) if the customary teacher was ill or if the lesson due to e.g. re-planning was to be a test, test preparation or any similar activity.

   A set of very short follow-up questions (a **questionnaire/ structured interview**) was answered by *all teachers* directly after the observed lesson.

3. As follow-up, **semi-structured interviews** were conducted with *a subset of the teachers*. The selection criteria for this subset was that the filled-out COP indicated (relatively speaking; in comparison to the overall data) deviating outcome on certain observational parameters. The main objective of the interview was for the teachers to comment on the how and why (underlying reasoning) of their actions in the classroom. In some cases, where the relative difference in the filled-out COP was substantial, revisits were also arranged.

   The majority of these interviews were individual, while four were held in the form of joint discussions or even workshops, where more than one teacher was present at the same time. The workshops were documented jointly on the whiteboard, which was photographed at the end by all participants. An example is found below:

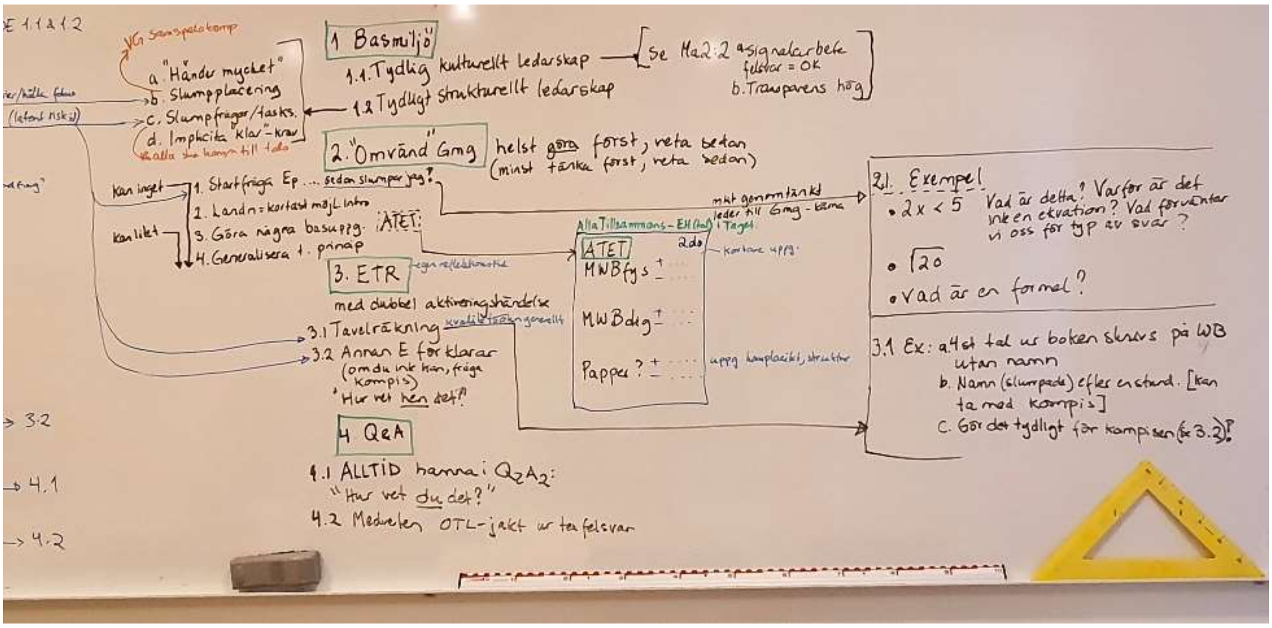

Figure 9: Workshop documentation

The study generated three main **data-sets** (DS); two quantitative (descriptive statistics) and one qualitative, which require different levels of data processing of raw data and different kinds of analysis:

DS1. Contextual *presence of*-parameters (outcome modelled as 0 or 1) were automatically consolidated into frequency tables (absolute numbers and percentages of occurrence, e.g. how many of the teachers did "x") using MS Excel standard functions.

DS2. As for *amount of*-parameters (e.g. number of … , time spent on …), the complexity of the data generated a need for more developed data processing/ consolidation of raw data (also in MS Excel), in order to present, analyse and discuss the results.

DS3. Qualitative data (mainly free text notes from COPs and data from interviews/workshops) was manually consolidated as part of the qualitative analysis.

6.3 Analysis method

For the quantitative data (DS1, DS2), inferential statistical analysis was deemed irrelevant, due to the low number of lessons. Instead descriptive analysis was performed and focus was put on how to organize and describe the collected data.

For the qualitative data (DS3), thematic analysis (Braun & Clarke, 2006) was chosen for this study for several reasons, of which the main ones are listed below:

- Thematic analysis (TA) is not "tied to, or stemming from, a particular theoretical or epistemological position" (Braun & Clarke 2006, p.78) and this study aims to identify yet unknown entities, i.e. the methods used must be relatively *open and flexible* (in opposition to having a high degree of predefined parameters and/or low degree of variability in method application);
- The focus of this study is not on the individual respondents, but on *tendencies* or on *positive deviations* compared to the overall data, across an (entire) data set and TA "involves the searching across a data set - be that a number of interviews or focus groups, or a range of texts - to find repeated patterns of meaning" (Braun & Clarke 2006, p.86).
- This study seeks to generate meaning from what teachers *do* in a broad sense; not only in the terms of what they *said*, nor *how* they said it, it does not matter that TA "does not allow the researcher to make claims about language use, or the fine-grained functionality of talk" (Braun & Clarke 2006, p.97);

- TA can be used as a participatory research approach in the above-mentioned workshops, “with participants as collaborators” (Braun & Clarke 2006, p.97).
- TA is – relative to other analysis methods – “relatively easy and quick… to learn and do” (Braun & Clarke 2006, p.97), since this study includes a fairly large data corpus, both quantitative and qualitative;

TA was firstly used during the development of the Classroom Observation Protocol, in order to identify important themes and thus model (define and structure) COP-parameters perceived as relevant based on the identified themes. The initial themes from the pre-study evolved and changed to a certain extent in the TA of the main data, even if several (sub)themes also remained fairly stable. The analysis was undertaken in accordance to the below figure, although the process was not serial (for example when an important factor was identified, all phases had to be addressed in parallel, in order to assess and ensure consistency):

Table 1 Phases of thematic analysis

Phase	Description of the process
1. Familiarizing yourself with your data:	Transcribing data (if necessary), reading and re-reading the data, noting down initial ideas.
2. Generating initial codes:	Coding interesting features of the data in a systematic fashion across the entire data set, collating data relevant to each code.
3. Searching for themes:	Collating codes into potential themes, gathering all data relevant to each potential theme.
4. Reviewing themes:	Checking if the themes work in relation to the coded extracts (Level 1) and the entire data set (Level 2), generating a thematic ‘map’ of the analysis.
5. Defining and naming themes:	Ongoing analysis to refine the specifics of each theme, and the overall story the analysis tells, generating clear definitions and names for each theme.
6. Producing the report:	The final opportunity for analysis. Selection of vivid, compelling extract examples, final analysis of selected extracts, relating back of the analysis to the research question and literature, producing a scholarly report of the analysis.

Figure 10: Phases of thematic analysis (Braun & Clarke 2006, p.87)

1. Quite some time was spent on “**immersion**”; reading and re-reading the free text notes from the COPs as well as from the interviews; across the entire qualitative data corpse and initial notes were made as to selected patterns and particularly interesting features.

2. Initial **coding** was then made systematically across all instances using colours (a few tentative themes were also sketched with ‘?’). All data with the same code was then grouped (meaning multiple relations; same data could belong to several piles n<->n).

3. Different code-piles were combined into **themes** which were grouped into overarching themes and a picture drawn in parallel on a whiteboard, to maintain the overview.

4. **The tree:** Pile-sorting/moving/grouping (and back-checking), updating the whiteboard, until data sustained the identified relationships themes-codes (1<->n) and codes-data items (n<->n) and patterns were coherent (a total “tree”, without too much overlap).

5. **Tree description:** Naming and describing the scope and specific characteristics of each (sub)theme (a few *key* sentences – no paraphrasing); back-checking relations to the research questions, the theoretical framework etc.

In all, the thematic analysis is underpinned by a *latent theoretical approach* from an *epistemologically constructionist* perspective as described by Braun and Clarke (2006, pp.83-85).

6.4 Method discussion

As for the choice of thematic analysis versus other analytical methods, please see the previous section. A few additional aspects of the methods used are described below:

1. **Validity & reliability of quantitative COP and Questionnaire data:** The main reason to use a COP was to maximize the *consistency* across observations and *replicability* in other contexts (Bryman, 2011). The iterative exercise of defining the COP (during the pre-study) was aimed at increasing its *validity*. The first input consisted in an inductive observational approach when doing the initial observations in order to minimize preconceptions. Secondly, the largest common denominator from a small inventory of existing COPs was identified and compared to the observational data. Thirdly, observational protocols from Skolinspektionen were also used as input to the design of the COP. Various general methodological aspects were finally included - as defined in the Swedish educational context (e.g. Agnafors & Levinsson, 2019; Blomström & Wennerberg, 2018; Skolforskningsinstitutet, 2020; Skolverket, 2020; Persson, 2016; and Vetenskapsrådet, 2017).
2. **Trustworthiness of qualitative data:** A strong focus was put on reducing *observer influence* as well as *bias* as much as possible in the documentation, carefully separating descriptions from any pre-analytical (or even valuing) free text (Bjørndal, 2018). As for *representativity,* it could be claimed that several visits should have been performed per teacher to ensure that each lesson was truly representative. A choice was however made to fully trust each teacher's own assessment of whether the lesson was representative[16]. The question was included in the questionnaire after the lesson, as well as communicated beforehand, when initially talking to all participants. This choice was made for two reasons; firstly and most importantly, as a way to truly trust the professional expertise of the participating teachers. Secondly, although the initial decision was to visit all teachers, it could be noted that theoretical saturation (Eisenhart 1989, p545) occurred before the study was completed (the final cases implied minimal incremental learning).
3. **Validity of the COP itself:** One could argue that using a COP, purposely developed for this study, instead of an established COP, decreases the validity of collected data. Bostic et al (2021) however report in their recent meta-study on peer-reviewed classroom observations that "…61% did not use a formalized classroom observation protocol (COP), 18% developed their own COP, and 20% used a previously developed COP" (Bostic et al 2021, p5), why the approach used in this study is argued to be acceptable.

6.5 Research ethics

The main ethical issues that were addressed in this study include *anonymity*, *voluntary participation*, *informed consent* and *data protection* (Vetenskapsrådet, 2017). No audio or video recordings have been made of ***students***, neither has observational data been gathered/registered on individual student level - not even anonymized individual data. Neither has registry been made on class level (e.g. in which class a lesson was observed), only on course level, in order to maintain anonymity of students. Only behaviour on group level has been registered (e.g. as to increased or decreased level according to the 'staircase' steps, described in section 0), when such a change has occurred *simultaneously for several (unidentified) students* – most often as consequence of teacher action(s).

Classroom observation data related to the 18 observed ***teachers*** has been registered on an individual level, but anonymized. All mathematics teachers were asked individually beforehand and all agreed to an *unannounced* visit, given that changes in lesson planning was occurring more

[16] See section 7.4.1 for a description of how this assessment was made.

frequently due to the Covid-situation. Restrictions implied that absence was required even for simple cold symptoms (both for teachers and students). Hence any pre-planned visit could change so that the lesson no longer consisted in regular classroom teaching by the customary teacher. Typical such situations consisted in a lesson being given by an external replacement teacher or by an already visited colleague, or that the number of absent students required plan changes, so that even if the regular teacher was present, the lesson was dedicated to e.g. a self-test or to test preparation. In all such cases, another *regular* lesson (by another teacher) was visited instead. Without such kind acceptance by the teachers, the slide puzzle of observation planning would have become impossible. The Covid-situation however extended the originally planned observation period and approaching end of term, more and more classes were focusing on preparing for national tests, why regular classroom teaching gradually became scarcer. Observations were therefore continued the following term. When restrictions were reduced, some lessons to be visited could also be planned beforehand.

As mentioned above, a set of very short follow-up questions (a *questionnaire*) was given to *all* teachers directly after the observed lesson. In some cases, the questions were read out loud and the questionnaire was filled out as a service to the teacher (cf. structured interview), but the teacher could always proof read the entries on the questionnaire. Also the questionnaire data was anonymized.

Deeper follow-up *interviews* were, as also mentioned previously, performed with a subset of the teachers, sometimes performed as workshops with more than one teacher participating. In all cases, each teacher was individually and respectfully asked beforehand both as to the extent of the continued participation and as to willingness to share thoughts with others. Interview data was anonymized.

7 Results

Since the relevance of this study to a certain degree depends on whether an evolutionary approach *really* is needed (given the current situation), results will be presented in reverse order, starting with research question 3 in section 7.1 below, with the purpose of setting a background for found results from questions 1-2 (the main research questions).

The results from research questions 1 and 2 (occurrences of activated, exposed and elicited student thinking and their corresponding teacher actions) will be presented according to the "staircase" (presented in section 5.2.2), starting with Basic Activation in section 7.2 and higher steps in section 7.3:

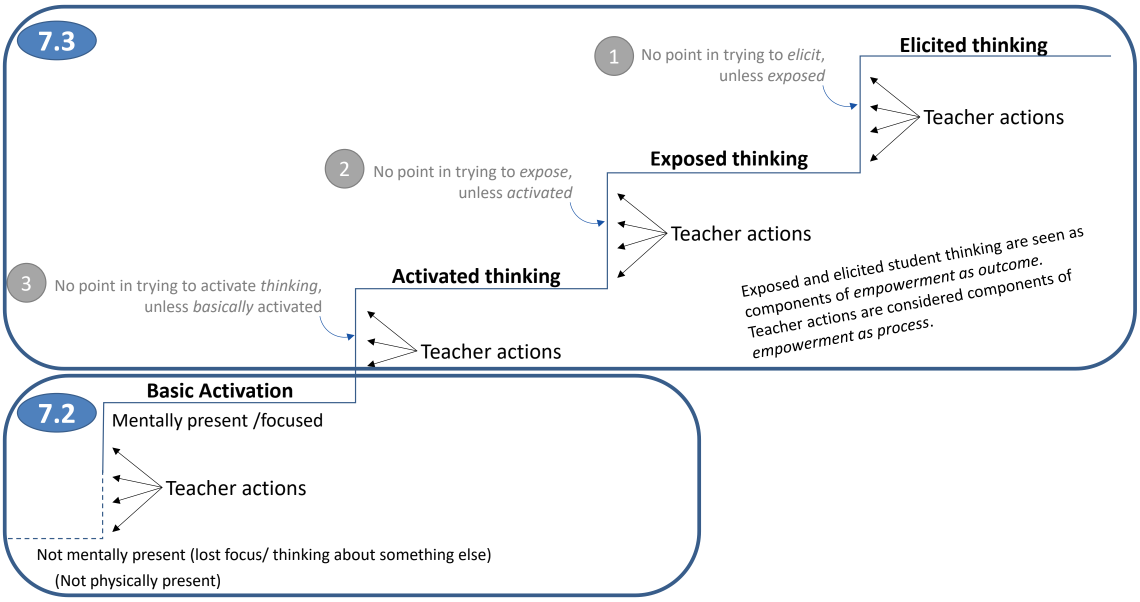

Figure 11: Results presented in sections 7.2 and 7.3

7.1 Analysis of Current situation: Observed maths lessons

Research question 3 was divided into two parts a) and b) in order to include relevant parameters the Classroom Observation Protocol (COP):

RQ3: To what extent are the observed mathematics lessons adhering to the description of Skolverket/ Skolinspektionen in terms of
a) textbook driven and
b) traditional education?

7.1.1 Textbook-driven education

The COP included three parameters related to education being text book-driven or not (outcome was modeled as 0 or 1, where 1 corresponds to "the textbook"):

Content	Whether the knowledge content and tasks given by the teacher come from the text book or from any other source (e.g. invented by the teacher).
Media	Whether, during the lessons, the students read the tasks straight from the book (themselves) a/o from a photocopy / digital website of the same book or via any other method (where it is not distinguishable that the content source is the book).
Order	Whether the lesson followed the order of the book or was planned according to any other principle (e.g. invented by the teacher).

The outcome was (for 18 teachers):

Table 1: Outcome: Text book-driven education as to content, media and order

	Content	Media	Order
Number of	17	17	17
Percentage	94%	94%	94%

(one single teacher created own content, media and order)

The same three parameters were also included as questions to the teachers in the Questionnaire, for that specific lesson as well as whether the same method was used when planning e.g. a longer lesson sequence or even a semester. The outcome was (for all 18 teachers) consistent between the COP-data and the Questionnaire answers.

After thinking a little, 4 teachers added that they, in addition to mainly using content from the book, "sometimes" also used content (and hence also media/order) from *Kunskapsmatrisen*[17]. When asked to estimate how often, all reached the conclusion that it was "much less" (one used the expression "infinitesimal") compared to the number of times using the book.

7.1.2 Traditional education

According to the description by Skolverket/ Skolinspektionen (see chapter 2.2), 'traditional' is here defined as a lesson mainly consisting in what in Sweden normally is referred to as a **'go-through' (GT)** followed by **individual silent task solution (IST).** The COP therefore firstly included three *activity types*:

GT ('go-through')	The teacher talks; explains a "what" / "how to", with or without drawings or other representations, and with or without applications (solving example tasks).
IST (individual silent task-solution)	Individually[18] doing a specified set of tasks from textbook (or similar) at individual pace. The student will, at her/his own discretion, ask for help from the teacher by e.g. raising a hand (or other visual/audial signal).
Other	Any other teaching-learning activity than GT and IST as defined above, for example Think-Pair-Share-activities (TPS).

In order to study time spent on the three activity types above, we also need to define:

Nominal lesson time	The time specified in the schedule (start time and end time), used as base-line during all observations of time
Break	Note: At this school, it is the teacher's *choice* whether breaks are given.
Unproductive time	The time during nominal lesson time when students are not ready to start work; finding their places, getting in order etc, or when a teacher decides to stop the lesson before the nominal end time[19].
Productive time	The nominal lesson time minus Break(s) and Unproductive time(s)

Percentages are used since nominal lesson time could vary: for 16 of the 18 observed lessons, time varied fairly evenly between 60 and 90 minutes (for two teachers, the time was 55 and 105 minutes respectively) with an overall average duration of 73 minutes.

[17] *Kunskapsmatrisen* is a Swedish privately owned web-based tool, initially containing tasks intended for test creation by teachers and later developed into full web-based test-management and additional practice tasks, where students can log in and do/solve the tasks. The content is created and uploaded by the teachers themselves and no central quality control exists.

[18] Working together with neighbour is generally allowed.

[19] This Unproductive time most often increases when a break is given, since the students will have to enter and settle also after the break.

Consolidating time spent per activity type, the outcome was (vertical axis shows % of time, while all observed teachers are listed on the horizontal axis):

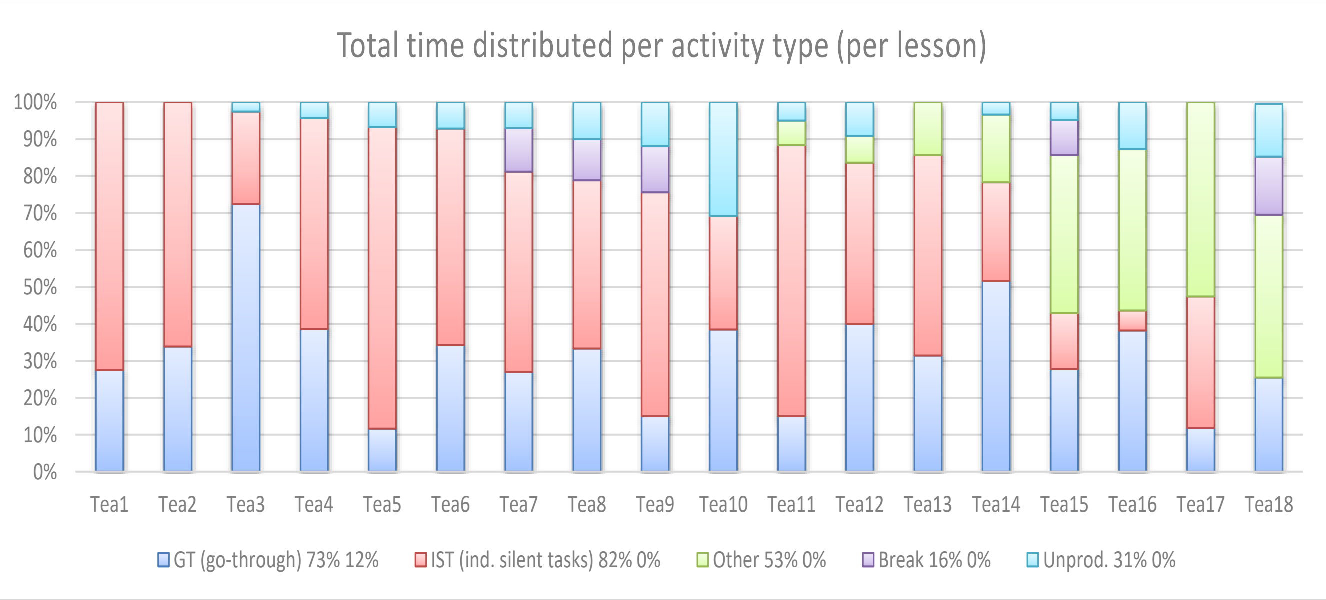

Figure 12: Outcome: Total time spent per activity type for 18 lessons

Maximum- and minimum-values for each activity type, across the whole data set, is indicated in the legend above. It can be noted that:
- all lessons included (at least one) GT segment;
- all lessons *but two* included (at least one) IST segment;
- 5 of 18 teachers gave (at least one) break.

The outcome was:
- More than three quarters of teachers (14 out of 18) spent more than 80% of *productive time* on **GT+IST**.
- 4 out of 18 spent 50% or more of *productive time* on **Other**.

Teacher characteristics in the respective group furthermore show that the former group was very heterogeneous as to the teacher's age, gender etc. In the latter group, there were two women and two men, all of whom had had many years of teaching experience.

7.2 (basic) Activation

The triangulation mixed method implies that all themes in this study firstly contains the *qualitative* data items from interviews, questionnaires and free text notes from observations which have been thematically analysed (Braun & Clarke, 2006). Examples of main codes are given under each theme description, along with explanations /underlying reasoning if relevant.

Secondly, also *quantitative* data; contextual *presence of*-parameters from the Classroom Observation Protocol (COP; outcome modelled as 0 or 1), as well as some *amount of*-parameters, were added to qualitative data to further underpin the themes. In this latter case, when a corresponding COP-parameter relevant to a theme exists, quantitative results are reported as well, within the theme.

7.2.1 Overview of (basic) Activation and its subthemes

The first (overarching) theme (basic) **Activation** is defined as the level to which a student is active (activated) and expressions such as 'mentally present', 'attentive', 'being here and now' and 'focussed' are considered synonymous. It does not, however, involve any requirements on *thinking* as such – hence the attribute *basic* Activation.

The theme **Activation** consists of several (sub)themes and a slightly different approach to structuring them has been used. In studies using thematic analysis, theme-trees are often created where positive and negative theme-trees are defined and then textually presented *separately*, one after the other (e.g. "Figure 4 Final thematic map, showing final two main themes", Braun & Clarke, 2006, p.91). In this study, a parallel approach was chosen, since many data items are very similar as to content, albeit that some are direct negations of others. This means that the understanding of a theme can be promoted also by its negation / counteracting parameters, seeing them in relation to each other. The approach should not be interpreted as binary, i.e. that data items were "forced" into either side, but that data items could convey *a degree* of the (sub-) theme from "low" on the left to "high" on the right.

The theme **Activation** is therefore modelled as in the figure below. The types of data included in each theme are shown in the structure and further described below the picture.

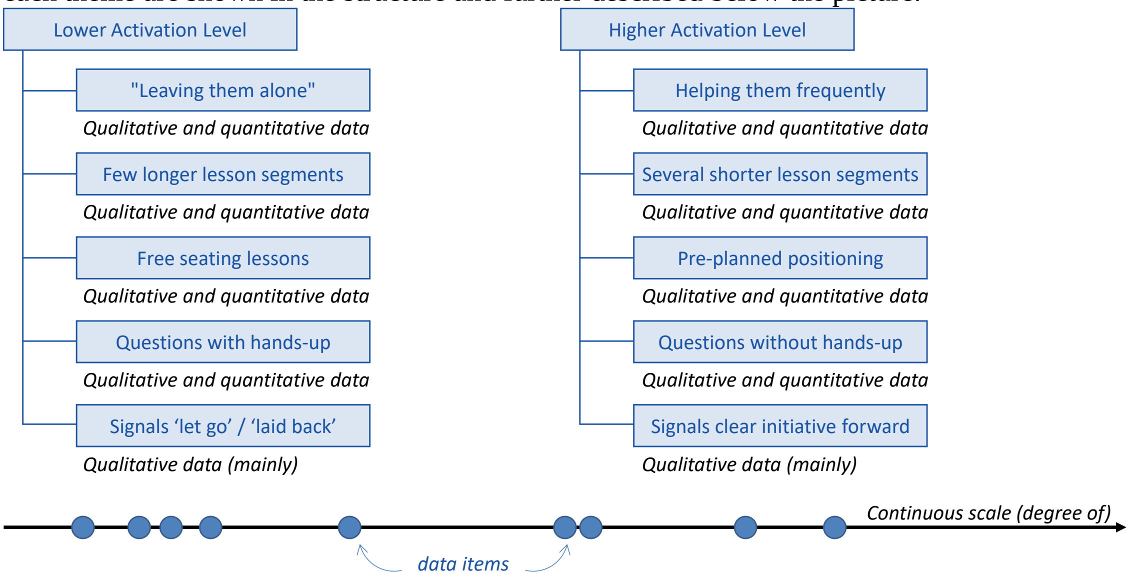

Figure 13: The theme Activation and its subthemes.

The themes in the picture above are developed in the subsections below, grouped so that the two opposites (horizontal items in the picture) are described together. Each theme-pair is considered as a continuous scale, and its data items can be ordered along this scale.

One of the subthemes, **"Leaving them alone"**, was however chosen, due to its central nature, for illustrative purposes of the triangulation mixed method. It is therefore firstly presented separately (without its opposite) and in more depth in the next section.

7.2.2 Triangulation thematic analysis – a central theme as example: "Leaving them alone"

To clarify its name, this theme is not about loneliness, but about students being left "in peace":

"Leaving them alone"	The students are, after explicit or implicit instructions, "left alone", to their own devices, not necessarily intentionally, but the students - in practice - have the opportunity to do (or not do) whatever they like and/or is able to. The student has become the *owner of initiative* (to work during IST, to ask for help from the teacher, to take notes during GT etc.).

"Leaving them alone" can firstly be illustrated[20] by a few answers in relation to a follow-up question during **interviews**: "What do you think is important – or works well and not - during IST?". The selection of the below quotes was made for their differences in underlying causes – but where the result is the same: many students in the classroom were **"left alone"**, which also was observed during the lesson's IST:

- "…leaving students alone is a matter of respect … they are almost adults and should be able to handle this…"
- "…it's important that students with difficulties are allowed to set their own pace so I usually leave them alone until they want help… I don't want to stress them…"
- "…I wish I had more time to go around and check on how they are doing… but… I simply don't have time since there are so many hands in the air…"

As for data from **observations**, **Leaving them alone** was firstly detected by observing *teachers*, who during IST e.g. sat down at the desk or stood in a classroom corner while students did their IST, waiting for students to ask for help (outcome of COP-data is presented in the next section 7.2.3 below, where its opposite is added for comparison).

Secondly, it was identified from free text notes regarding *students* who during the lesson started doing something other than what the teacher intended. Two type cases that were identified during IST as well as during the GT were:

- One or more student pairs/groups talk to each other for more than a few seconds. During GT, it is mostly (at least initially) in a whispering tone, but during IST, it can be at full volume, depending on the didactic contract. The conversation topic was not related to the lesson.
- More than a few (most often three) individual students, in different locations in the classroom, *observably* stop paying attention to the teacher/ the task at hand for more than a few seconds. Cases observed were when the student sat looking out the window, or at his/her mobile (sometimes openly, sometimes under the table) – or simply left the room with a low but audible comment to the neighbour that the student was going to the kiosk to buy a drink or similar (sometimes followed by a louder comment to the teacher that a bathroom visit was necessary).

Generally when this occurred, the number of students showing this behaviour increased with time, unless the teacher reacted to (and acted upon) the first few cases. **Leaving them alone** was therefore considered to occur if the students showed such behaviour and the teacher did ***not*** act upon it.

7.2.3 Leaving them alone vs. Helping them frequently/ immediately/ permanently

In order to see the pair together, the description of **Leaving them alone** is repeated here:

"Leaving them alone"	The students are, after explicit or implicit instructions, "left alone", to their own devices, not necessarily intentionally, but the students - in practice - have the opportunity to do (or not do) whatever they like and/or is able to. The student has become the *owner of initiative* (to work during IST, to ask for help from the teacher, to take notes during GT etc.).

Its key constituents (main codes) are for example:

1. During IST: The teacher does not approach students unless asked for help (can be intentional or not)
2. The teacher hesitates to act (or does not act at all) on counterproductive behaviour (loss of focus)

[20] Note that although often quite vivid or contrasting or examples are *presented for illustrative purposes*, this does not imply that the underlying theme definition has been made using an "anecdotal approach" (Braun & Clarke, 2006, p.96).

Its opposite is a situation where:

Helping them *immediately / frequently / permanently*	The teacher "reminds"(or more generally; helps) the student to "stay" in the classroom, in terms of being *mentally* present (focussed). This can firstly occur reactively, if focus is lost, and the teacher then acts **immediately**. Secondly, action can be undertaken proactively; the teacher *foresees* that focus (likely) will (soon) be lost, due to e.g. time passing, or due to the nature of the situation, and is then done **frequently**. It can also be done **permanently**, if a single initial action affects the entire lesson.

Its key components (main codes) are for example that the teacher:

1. Frequently acts during any extended lesson segment; for example:
   During IST: The teacher systematically walks around the entire IST-time, showing clearly that she/he looks at students (and their work)
2. Immediately acts on first instance of counterproductive behaviour (loss of focus)
3. Permanently; i.e. takes one proactive action *at the start* of the lesson, which will reduce (or remove) the need to act either frequently or immediately *during* the lesson

From COP-data, 14 of 18 teachers **left them alone** during IST, in contrast to 4 teachers who **helped them *frequently;*** constantly and systematically during the *entire* IST, was walking between the tables, actively (and obviously) looking at the students work and frequently asking students how they were doing.

Two contrasting examples of qualitative data from observations and follow-up interviews were:

- In a situation where a student went up to the board to show her/his solution, the sound level instantly rose to a fairly high level (students started talking to each other about other things). The teacher then **helped them *immediately*** …by directly saying "Hey, we have a comrade offering to share the solution with us – we then focus out of respect"

- In another lesson, the sound levels during IST rose to very high levels when a group of five male students (dominant from a group-psychological/dynamical perspective) for several minutes were laughing and joking. Other students were clearly disturbed, looking intermittently at the group and at the teacher, who continued helping another student. Later, during the interview, the teacher commented on the IST saying "…that bunch is a bit …energetic…<smiles>… and I don't want to be too old-fashioned… too harsh. Normally they kind of calm down by themselves after a little while". According to the theme definition, not only the rest of the class were **left alone,** but also the group who forgot to keep focus.

An example of a proactive measure where the teacher **helps them *permanently*** (during the lesson; without additional teacher actions), is when the teacher gathers all the students' mobile phones at the beginning of class (one single initial action affects the entire lesson). COP-data showed 4 of 18 teachers using this approach (it was observed in four of the first year classes). Several of the teachers who did not collect the mobiles, said that it was not needed for older students and/or that they preferred to manage it if/when the situation arose, i.e. **helping them *immediately***. One of the teachers who *did* gather the phones commented that it was absolutely necessary, else all the lesson time would be needed to handle each incident, but that it "very much depends on class characteristics" whether the measure is needed on such high level of proactivity.

7.2.4 Number and duration of lesson segments

The number (and therefore the duration) of lesson segments is another theme which was identified as relevant to *Activation*. A lesson segment is here defined as an activity of a certain type (such as a GT or an IST, or any other activity) which lasts until change is made into *another* activity type.

Few longer lesson segments	The lesson is designed with very few lesson segments. If using only the two types GT and IST, the smallest possible number of segments during a lesson is two; one GT-segment followed by one IST-segment. A longer segment will increase the need for some sort of "**helping them *frequently*"** if students are to stay focussed (cf. the example used in the previous section of a teacher walking constantly during IST).

Its key components (main codes) are for example that the lesson has:

1. At least one *long* IST segment (20 min or longer)[21] *without* the teacher **helping them *frequently*** to maintain focus (e.g. walking or similar);
2. *Only one or two* GT segments, since e.g. a lesson conclusion after IST can be considered a second GT segment[22].

Its opposite is:

Several shorter lesson segments	The lesson is designed with several shorter lesson segments. If using only the two types GT and IST, an interwoven GT-IST-GT-IST-GT-IST-… chain could be created. Each *change* in such a sequence here represents a new segment and such a change was *in itself* in this study observed to help to maintain focus, why the requirements on the teacher to implement **frequent help** would decrease.

By contrast, its main codes are for example that the lesson has:

1. IST segments which are shorter than 20 minutes and/or the teacher is **helping them *frequently*** during the entire IST segment to maintain focus (e.g. by walking or similar);
2. Three or more GT segments, since this is interpreted as a sign of consciously splitting the content into segments (up to and including 2 GT segments *could* imply one "true" GT segment and a lesson conclusion last).

An example from observational notes, where the teacher suddenly changed into a new segment:

- After doing two application examples (task solutions) on the whiteboard during a GT, a teacher suddenly stopped, looked at the class and said: "I get tired of my own voice. Has someone done task 1234 and can do it on the board please"? A student volunteered saying "I have done it but have no idea if it's correct" and went up to the board.

From the interviews ("What do you think is important – or works well and not - during the GT?"), two contrasting examples are (where observational notes were observed to coincide with the principle expressed verbally):

- **Fewer longer lesson segments:** "I seriously try to give a really good and full explanation before they start with the tasks… so they can see how different things interrelate…. I try to use many different representations… so it can take some time".
- **Several shorter lesson segments:** "I try to talk as little as possible… just giving them what they absolutely need to get started – I mean they can't keep focus for long anyway during a GT. It's better that they get going *doing* something instead of listening - as soon as possible"

All COP data related to this theme could be distributed into the two opposites above by using their coding for the theme pair above (see table below):

- **Fewer longer lesson segments:** 11 of 18 teachers (Tea1-Tea11) have only 1-2 GT segments as well as longer IST segments (20 minutes or longer; the longest in the whole data set being 51 min) *without* **helping them *frequently*** (walking).

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[21] The time limit 20 minutes was chosen due to that two experienced teachers in the study independently mentioned it as an absolute maximum for keeping focus (without help) for most students.

[22] The observant reader may ask why there is no "activation activity" specified for a GT, similar to the IST walking. It is in this study argued that a GT segment always – due to its nature of being a GT - is a flow of knowledge transmission, where at some point the cognitive reception capacity of any human being will be saturated, regardless of activation. Hence the higher importance of its length.

- **Several shorter lesson segments:** 7 of 18 teachers (Tea12-Tea18) have 4 or more GT segments and IST segments shorter than 20 minutes and/or **helping them *frequently*** (by walking around). The extreme was 19 GT segments where the longest lasted 30 seconds (Tea18).

The below table describes some more details for the activity types GT (go-through) and IST (individual silent task solution). The number of segments, the longest segment (minutes) and the accumulated total (minutes) per activity type is listed, along with whether the teacher helps them by continuously walking around during IST. The sorting order below is the same as in *Figure 12* above.

Table 2: Number of and max. duration of GT and IST segments for 18 teachers

	More on GT			***More on IST***			
	G-#segm	G-max(m	G-tot (m)	I-#segm	I-max (m)	I-tot (m)	*I-walks*
Tea1	2	11,0	22,0	2,0	51,0	58,0	
Tea2	2	15,0	22,0	2,0	38,0	43,0	
Tea3	2	34,0	58,0	1,0	20,0	20,0	
Tea4	1	27,0	27,0	1,0	40,0	40,0	
Tea5	1	7,0	7,0	1,0	49,0	49,0	
Tea6	1	26,0	26,0	1,0	41,0	41,0	
Tea7	2	15,0	23,0	2,0	33,0	46,0	
Tea8	2	25,0	30,0	2,0	29,0	41,0	
Tea9	2	6,0	12,0	3,0	25,0	48,5	
Tea10	1	25,0	25,0	1,0	20,0	20,0	
Tea11	2	6,0	9,0	1,0	44,0	44,0	
Tea12	4	6,0	22,0	1,0	24,0	24,0	Yes
Tea13	6	9,3	20,8	3,0	33,0	44,0	Yes
Tea14	5	17,0	31,0	1,0	16,0	16,0	
Tea15	12	4,0	29,1	1,0	0,0	16,0	Yes
Tea16	5	13,0	21,0	1,0	3,0	3,0	
Tea17	6	4,0	10,7	2,0	18,0	32,0	Yes
Tea18	19	0,5	3,8	0,0	0,0	0,0	

Legend:

- G-#segm: Number of GT segments
- G-max (m): Duration of longest GT segment in minutes
- G-tot (m): Total (accumulated) time spent on activity type GT in minutes
- I-#segm: Number of IST segments
- I-max (m): Duration of longest IST segment in minutes
- I-tot (m): Total (accumulated) time spent on activity type IST in minutes
- I-walks: The teacher walks among the students during the entire IST, also when no one is asking a question.

7.2.5 Positioning of students

The positioning of students (where they physically seated in the classroom or where positioned in work groups; sitting or standing) has been identified as a relevant factor to activation:

Free seating	Free seating (or positioning) consists in students entering the classroom and choosing where to sit themselves. Most often friends then sit together / close to each other and subgroups can often be observed in the classroom.
Pre-planned positioning	Planned positioning consists in the teacher deciding where the individual students are to sit / work, either by physically putting simple place cards (printed on regular paper) on the tables before students enter the classroom, or by digitally presenting a seating map on the projector when the door is opened. In both cases the teacher can plan the positioning manually or use a randomizing generator.

Two contrasting data items from interviews were:

- **Free positioning**: "I don't want to use pre-planned positioning since I have two students with diagnoses who need to sit next to each other".
- **Pre-planned positioning**: "I always use randomized positioning in all classes - when a student has special needs, I fix that manually in the seating chart before the lesson".

From COP-data, half of the teachers (9 of 18) used **free positioning** and the remaining 9 teachers used **pre-planned positioning**, where 6 used physical place cards and 3 used a digital seating map on the projector, showed initially. Several of the teachers using pre-planned positioning commented that the main reason for using it was to break up talkative groups.

Observation notes were consistent with those comments and showed that sound levels were notably higher in **free positioning** classrooms (all lessons with free positioning had the indicator "high general sound level" crossed at least once during the lesson, several had the indicator crossed multiple times), while most of the **pre-planned positioning** lessons did not.

One teacher furthermore observed that there are multiple benefits with pre-planned positioning, besides breaking up talkative groups: it is a means to create ***better class atmosphere and work with values*** (since all students were forced to sit next to *anyone* - and therefore got used to working with *anyone*) and also train ***dialogical skills***, since "communication is much easier if you only need to communicate with close friends".

7.2.6 Hands-up or not

A common element in any classroom is when the teacher asks the class a question. The appointment of who is to answer can be done in a variety of ways, for example using hands-up or not. In this section, the subject of "questions" is delimited to this *appointment*, since it can be observed to affect the *degree of activation* (more on how questions were used in order to expose a/o elicit thinking, is found in section 7.3).

Hands-up	When asking a question (audible/visible to the entire class), the teacher waits for (some of) the students to raise their hand, as a sign that they are willing to answer.
No hands-up	When asking a question (audible/visible to the entire class), the teacher can manually appoint a student to answer or use some kind of randomizing system (physical or digital).

From COP-data, 16 of 18 teachers did ask the class at least one question during class, where such *appointment* of who was to answer occurred. Of these 16 teachers, 14 used hands-up. A comment made by a teacher in this group (in relation to the questionnaire) was:

- ”Yeah I always use hands-up…. but of course, it's always the same few students who answer… I actually tried out the ice cream sticks a few times but it was too messy”[23].

Observation notes indicated that it was indeed the same few students who answered in *all* classes using hands-up. The rest of the class (who did not raise their hand) varied very much in attention level – a large number was inattentive.

The two teachers who did not use hands-up, used different methods:

[23] The "ice-cream sticks" mentioned refer to a method implemented by Dylan Wiliam, who during a classroom experiment took over a class for a term. The appointment of who was to answer, was then made using a mug of wooden ice-cream sticks, marked with the students' names. BBC has produced two films on the experiment (Wiliam, 2012a,b) and in the first film Wiliam states that "hands-up is one of the most damaging things that happen in a classroom" (Wiliam, 2012a, 05:55).

- ”I don't use a random generator – but I always decide on which student who should answer… by saying a name or pointing …. <thinking pause>… and I really try to make it fairly and evenly distributed…” (comment made in relation to the questionnaire).
- “I always use a random generator on my mobile - it keeps everyone attentive and they know it’s fair… equal. I most often spin the wheel myself - which gives them a little extra time to think, which I think is very important... and me the time to respin if a too hard question lands on a weak student <said with a smile> ... and sometimes I ask a student to spin the wheel and read out the name... but I only do that for the easiest questions, since it can land anywhere” (comment made during interview).

7.2.7 Leadership signalling

A teacher can see leadership styles as more or less important, be more or less aware of *the subject matter leadership* itself and also, more or less consistent in *implementing* her/his own behaviour in this regard. What a leader says and does may furthermore signal messages far beyond the direct consequences of such actions. For these reasons, codes were defined and grouped into themes as described below. A recommendation to the reader is to read the corresponding numbered items together (seeing examples of the opposite themes together).

Signals ‘let go’ / ‘laid back’	The teacher may during class let go of initiative ownership, or even not clearly signal such ownership from the start. There may exist inconsistent signals as to whether the teacher assumes the leader role, whether the lesson (content) is important and/or whether communicated classroom rules/values are important.

Its components and a few, very common examples (main codes with illustrative data items) are that the teacher:

1. Opens door *no earlier than* nominal lesson start-time and does not rush the students, but patiently waits until they are settled.
    - 14 of 18 lessons had unproductive time, as described in section 7.1.2 (see *Figure 12*).
    - 10 of 18 lessons started *without anything* on the whiteboard, on the projector, or on desks, when the students were let in.
2. Does not seem to focus on time management and does not clearly *verbalize* time expectations (regardless of whether a e.g. a task sequence is written on the board)[24] nor supports students’ time management in other ways.
3. Hesitates to act (or does not act at all) on counterproductive behaviour (loss of focus)[25]
    - *See section 7.2.3 for examples.*
4. Seems to focus *less* on making the students talk/act than talking/acting her/himself.
    - Comments: while talking, the teacher mainly touch on *own* actions and does not mention how to activate *students*, e.g. “I think a lot about how I should do the GT… which examples I should present…”
    - Observational notes: Some teachers seemed to enjoy being the centre of attention, creating focus upon her/himself, rather than on the students and their work.
5. Behaves inconsistently compared to the values/rules verbally communicated to students

[24] Such task sequences were sometimes written by teachers on the board, but unless the teacher clearly stated time expectations on their fulfilment, they were observed to be interpreted by *both students and teachers* as “everyone works as far as they are able to during this lesson”.

[25] These items were also coded in section 7.2.3, but there ‘leaving them alone’ is a direct consequence of the teacher’s action, while they are here coded for their contribution to conveying a *lack of importance* on general level.

- One teacher, doing application examples in front of the class, wrote so shortly/ schematically in all these examples that it was almost incomprehensible - to be compared to previously during the lesson, having reminded a few students to “write clearly on tests, else the I cannot follow the solution” (which was why they had obtained 0 points).
- During IST, one teacher was observed to stop to (smilingly) talk to, and hence *become part of*, a group who during IST were laughing and joking, obviously disturbing other students.

6. Unclear signals as to whether the teacher adopts a personal position as *friend* – or as leader.
    - The last example in the previous code also serves as example for such signalling.

Signals clear initiative	Initiative ownership is clearly held (and upheld over time) by the teacher, during the entire lesson. The teacher assumes the leader role, signals that the lesson and its content are important and also leads by example; behaving consistently with communicated classroom rules/values.

By contrast, its key components and a few examples (main codes with illustrative data items) are for example that the teacher:

1. Opens door a few minutes *before* nominal lesson start-time (in order to settle the students before nominal lesson start time), with an air of being prepared and ready.
    - 4 of 18 lessons had no unproductive time, as described in section 7.1.2 (see *Figure 12*).
    - 8 of 18 teachers had prepared *something*[26] on the whiteboard and/or showed something on the projector and/or had materials distributed on desks already when the students were let in.
    - One teacher very carefully prepared the board in advance (had a clear plan for board usage), consistently using different colours for facts/key words; calculations etc. A comment made in the questionnaire was that “I almost always arrive 15 minutes before to prepare everything”.
2. Expresses tasks using clear time directives (regardless of whether a task sequence is written on the board)[27] or supports students’ time management in other ways:
    - "You have one minute to..."
    - “By the end of today, *everyone* should be ready with chapter/tasks xyz”
    - When a student said “I can’t take to do this any longer – it’s sooo boring- can’t I take a break now?” The teacher said: “OK, but to *train* your ability to keep your focus – you need that for tests! - , do at least two more tasks before taking that break.”
    - One teacher *governed time* for the students during the *entire* lesson, by using physical mini-whiteboards (MWBs) for all students. All tasks were done at a common tempo, together as to time, but individually as to performance (no IST using the book was done at all during this lesson).

[26] The degree of such preparation could vary; from only lesson start time and end time with a word or two to describe lesson content, to a carefully planned whiteboard with content written, using a variation of colours etc.

[27] Such task sequences were often written by teachers on the board, but unless the teacher clearly stated time expectations, they were observed to be interpreted by *both students and teachers* as “everyone works as far as they can during this lesson”.

3. Immediately acts on first instance of counterproductive behaviour (loss of focus)[28].
   - *See section 7.2.3 for examples.*
4. Seems to focus more on making the students talk/act than talking/acting her/himself (mentions thinking about "how to activate them" or similar expressions in comments and/or showed such behaviour during observations);
   - "I think a lot about how to get them going – and how to *keep* them focussed"
   - "I spend quite some energy on finding out where they are, so I adapt content to their starting point"
   - During class, 4 of 18 teachers clearly assumed a more withdrawn and tacit position, focussing on placing focus on the students and on their work, rather than on her/himself and also commented upon it in the questionnaire.
5. The teacher's own behaviour is consistent with the values/rules communicated to students; leading by example.
   - One teacher constantly whispered when helping students during IST; behaving consistently with the comment made to students at start of IST: "Let's keep our voices down to help each other to stay focussed" (leading by example).
6. Signals to adopt a professional position as *friendly leader* - rather than a personal position as *friend*.
   - The teacher was observed to irradiate qualities such as friendliness, empathy and helpfulness, without seeming to personally need/ strive for 'being liked' or 'becoming friends'.

In all, more than 50% of lessons (10 of 18) were clearly characterized by **signalling 'let go' / 'laid back'**, since they fulfilled almost all of the characteristics (codes) belonging to this theme (many in this group fulfilled all), while 4 of 18 displayed a majority of the above and were considered to **signal clear initiative.** The remaining teachers formed a mixed group, not clearly belonging to either opposite.

7.3 Activation, exposure and elicitation of thinking

7.3.1 Overview and subthemes

The second (overarching) theme, **Activate, expose & elicit thinking** is presented in this section. With reference to the theoretical framework in section 0, student thinking is in this study considered to be **activated** (prompted) by the questions asked (a/o tasks given – or any other action performed) by the teacher[29] and then **exposed** (using different forms of expression; verbal a/o written). Different types of activation (i.e. different types of questions/tasks/actions[30]), as well as their "size", has been identified to activate different types of thinking. They are all described in the subsections below.

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[28] The same items are also coded in section 7.2.3, but there the interest is the direct consequence of the teacher's action, while these items here are coded for their contribution to conveying a signal of *importance* on general level.

[29] Student thinking prompted by the textbook is not considered in this study.

[30] In the continuation, the words (teacher's) *question, task* and *action* are used synonymously in the theme descriptions and their explanations. In exemplifying data items, the word relevant to the specific observation/comment is used.

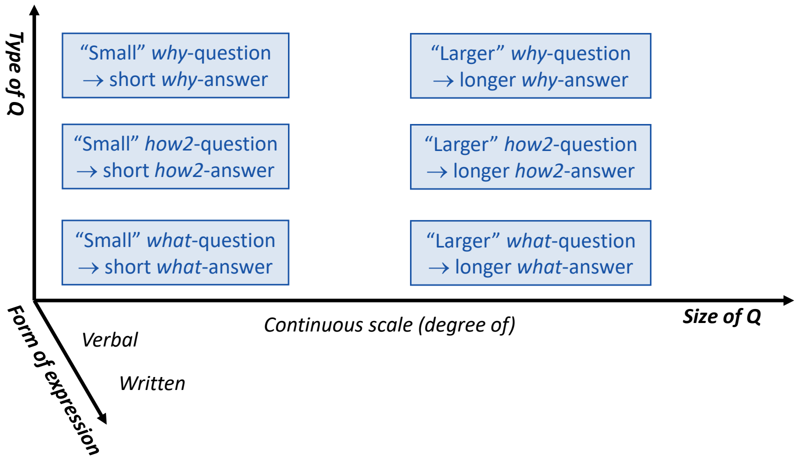

Figure 14: The theme 'Activate, expose & elicit thinking' and its subthemes (prompting types).

7.3.2 Forms of expression and Size of questions/tasks

Two forms of expression (how exposure takes place) have been identified in this study:

Verbal expression	'Verbal' is here defined so that the student will expose her/his thinking (e.g. give an answer to a question or a solution to a task) in verbal form, *regardless* of whether the prompting (the question or task given) by the teacher was given in verbal, written, or any other form.

Written expression	'Written' is here defined so that the student will expose her/his thinking (e.g. give an answer to a question or a solution to a task) in written form, *regardless* of whether the prompting (the question or task given) by the teacher was given in verbal, written, or any other form.

Observed examples of different types of expression forms were (quantitative outcomes from COP data presented per illustrative example):

- A typical, frequently observed, example is when a teacher asks a **verbal question** during a GT, and the students are expected to answer verbally. As already mentioned in section 5.2.6, COP data showed 16 of 18 teachers using verbal exposure at least once during the lesson[31].
- Another example is when students during e.g. a pair-exercise **verbally discuss** the solution.
- Also, as mentioned previously, some (4 of 18) teachers were walking around constantly during IST, looking at the **students' own notes**. In addition to the *basic activation* effect (see section 7.2), two of these teachers during interviews also commented that an additional purpose was to *actively* gather information on each student's progress and level of understanding. One of these teachers was also observed to clearly monitor *the way* the students were writing and give feedback (development of written expression skills; as to creating a clear solution).
- Students doing written solutions **on the board** were observed quite rarely (occurred in 2 of 18 lessons).
- The use of **mini-whiteboards** (MWBs; either physical or digital), when students wrote the answer to questions (and/or the solution to tasks) on the MWB and then showing the teacher, was equally rare (occurred in 2 of 18 lessons). One of these two teachers *solely* used such written exposure (no verbal exposure).

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[31] What *type* of thinking it exposes, will be developed in the next section.

- Finally, in one case (1 lesson of 18), students exposed in written form using a written exit ticket (digital, in the *Classroom*-digital platform).

In order to analyse the types of thinking, the **size** of the prompting (tasks / questions given by the teacher) was identified as an important factor:

***Size* of questions/ tasks: "small" or "large"**	All questions/tasks in this section, are considered to have "size", that is, a "small" question/task is expected to generate a short answer/solution, while a "larger" one should (is expected to) generate a longer answer/solution. The teacher's expectations in this regard was clearly shown by her/his confirmation of the student's answer – or in some cases, by a follow-up question "anything more?" if such "size" expectations were not fulfilled.

In theory, depending on the expected length of the student's answer (the "**size**" of the question), a varying degree of:

- *expression skills*[32] could be considered to be practiced, where a larger question is expected to provide more such practice, while a very small question provides limited practice.

Besides the size of questions, a number of different activation (question/task) **types** were identified in the data. The types are further described below, with differentiation of examples as to size and form of expression.

7.3.3 *'What'*-questions activate, expose and elicit *'What'*-thinking'

The first type of activation is achieved by *what*-questions:

***What*-thinking** (prompted by *what*-questions)	A student is prompted to think about the *what*; most often by a verbal *what*-question from the teacher. It consists in ***remembering*** something the teacher has previously taught and ***repeat it back***, e.g. to remember *what something is* (a mathematical term, a definition or similar; be it in colloquial or mathematical language) or giving (only) the *answer* to a standard task (remembering the standard procedure and giving the answer). Generally, there is one correct answer to *what*-questions (they are therefore considered 'closed' questions).

In theory, depending on the **size** of a *what*-question, a varying degree of:

- *memory (recall) skills* could be considered to be practiced, where a larger question is expected to provide more such practice, while a very small question provides limited practice.

Only one teacher was observed to work with *memory (recall) skills*:

- Directly at the beginning of the class, a few minutes were spent on students doing a very short memory test, including questions such as "What are the names of these: a) 2x+3 =5" b) y = 2x+3; c) 2x+3 >5" (answer "Equation", "Function", "Inequality") as well as three graphical representations in a coordinate system of the same three expressions, identifying which was which.

A few typical examples from observations and interviews of ***small*** verbal *what*-questions, with *direct* answers (no explicit allocation of thinking time[33]) are:

[32] Note that *expression skills* are different skills sets in verbal and written form; e.g. verbal explanations vs. clearly written mathematical solutions.

[33] The teacher *immediately* (the moment there was a hand in the air) appointed who was to answer (as a contrast, during other lessons, a teacher could explicitly say e.g. "for one minute, think about…").

- Observation notes: The teacher writes 2x + 1 = 3 on the board and asks “anyone remember what this is?” A student says (after hands-up) “an equation”. The teacher continues: “yes, and what’s the answer?” The student says “1” and the teacher says “correct”. The teacher then asks a rhetorical question: “so how do we do this?” and solves the equation on the board, using the standard procedure and carefully explaining what to *do* in all the steps. After solving a few more examples on the board, frequently including another *what*-question (using hands-up), the teacher writes a task sequence from the text book on the board and says “now it’s time to work”.
- During the interview (“What do you think is important – or works well and not - during the GT?”) the same teacher stated that “It’s important to ask questions during the GT to get them involved so they start thinking”.
- Another teacher was observed to use similar what-questions (with hands-up) during the GT; asking *a series* of small *what*-questions and then every time confirming the student’s short answer with “yes, and then…”; *elaborating* on the answer (i.e. the teacher used the answer as a ‘stepping stone ’to further develop the theme, instead of asking the student to do so).

COP data indicated that **9 teachers used *only* such *short what*-questions** during the lesson (hence only potentially exposing *what*-thinking), all *without* allocation of thinking time and using hands-up, why only a minority of students’ thinking was in reality activated and exposed. No *longer what*-questions were observed. Neither *memory (recall) skills*, nor *expression skills* could therefore be considered to be practiced to any larger extent. Nor did this group of teachers use any other *type* of questions (see below).

7.3.4 *‘How2’*-questions activate, expose and elicit *‘How2’*-thinking’

A second activation type, which in this study is argued to be a slightly higher level of thinking than *what*-thinking (albeit still repeating back the teacher’s transmitted knowledge), is *how2*-questions:

***How2*-thinking** (prompted by *how2*-questions)	A student is prompted to think about the *how-to*; most often by a verbal *how-to*-question from the teacher. It consists in ***remembering*** something the teacher has previously taught and ***repeat it back***, e.g. to remember *how to do something* (most often a standard procedure; be it in colloquial or mathematical language). In theory, there can be more than ‘one correct answer’ since the answer consists in a more personal description. There is however most often one ‘correct procedure’ to describe as answer to a *how2*-question since the teacher has previously shown exactly what it is (they are therefore considered also ‘closed’ questions).

The size of a *how2*-question can range from just ‘the next step’ in the standard procedure to a full description of the entire procedure. In theory, depending on the **size** of a *how2*-question, a varying degree of (at least the first step of):

- *memory (recall) skills* could be considered to be practiced, where a larger question is expected to provide more such practice, while a very small question provides limited practice.
- *meta skills*[34] could be considered to be practiced, where a larger question is expected to provide more such practice, while a very small question hardly gives any such practice at all.

[34] *Meta skills* is outside the primary scope of this study, but is here seen (briefly defined) as a multi-level journey in itself, of which the first step is the ability to “see oneself from above” and *describe one’s own work process*, step by step, knowing at each step what the next step is. At later stages, aspects such as generalization, transferral to other situations, self-monitoring and self-regulation are included among the meta-skills.

One teacher was observed working consciously with meta skills (albeit on basic level), using a digital form that students filled out individually every week or every two weeks. At the end of this lesson, the students were reminded to fill it out:

Table 3: One teacher's example of working with students' meta skills

Week	1st lesson of week: what did we work on??	2nd lesson of week: what did we work on?	3rd lesson of week: what did we work on?	How safely has it landed? red/ yellow/ green
40				
41				

The entire group of teachers who used verbal *how2*-questions (8 of 18 teachers), also used the aforementioned *what*-questions. A few illustrative examples, specific to the *how2*-questions, are:

- Two teachers were (independently) observed using a very similar approach (both independently later commented that the objective was to test that the students had captured enough of a GT to be able to start the IST – one said "else there are too many hands in the air during IST"). **After the GT,** the teacher wrote a typical task on the board (standard procedure explained during the GT) and asked "How do you do this?" Take one minute and think about it alone, before discussing – and solving it - with your neighbour." After checking that students were done, the teacher did a full solution on the board. Only *after* this exercise, the IST started. One of these teachers continuously walked around during the pair-exercise, constantly checking for comprehension.

- Another teacher (continuing from previous lesson) immediately started the lesson, **before the GT**, by writing an equation on the board, turning to the class and asking the same question: "How do you do this?" Take a minute or two and think about it alone, before discussing with your neighbour." After a few minutes, the teacher asked "Are you done?" and when some students said "yes", the teacher did the GT of the *how-to*. It was observed that not all students were in fact ready. The IST followed immediately after the GT.

This approach, to do the GT *after activating* the students' *how2*-thinking and preferably also make them *do* (at least one task) *before listening* is in this study named '**reversed GT**'. This *reversed GT* is a central finding in this study.

- Three other teachers in this group used a very similar approach (reversed GT) with the only difference that instead of asking students whether they were ready, they continuously walked around during the students' work and hence assessed class readiness *without* asking them. These teachers also used the approach *several times* (several shorter reversed GT-segments); the next segment could start once the teacher had confirmed readiness – or a short GT segment was added as explanation of identified common problems during the walk.

- Another example (this time via written exposure) is a teacher who, after initiating a new knowledge area, made all students use digital MWBs for a limited set of initial tasks. All the MWBs were shown in parallel on the projector (anonymized) and the teacher later commented that the objective was two-fold: weaker students could see what the others did and learn from them, while the teacher could assess understanding *for all* at a glance –to ensure that everyone had captured the basics before starting the IST[35]. The teacher later commented that it was extremely important that the weak students should govern the speed during this initial MWB session.

The group of 8 teachers who used *how2*- as well as *what*-questions, thus also potentially exposed *how2*-thinking. Compared to the group of 9 teachers in the 'pure' *what*-group, question size was

[35] This example was also coded for *time governance*, see section 7.2.7.

generally slightly larger and own thinking time was allocated for both types of questions, why *memory (recall) skills* can be argued to have been practiced and, due to think-pair exercises, verbal *expression skills* as well. All such practice applied to a majority of the students. As for exposure *of thinking*, especially in the cases where the teachers constantly walked during the students' work, listening as well as looking, exposure of thinking of most (if not all) students' thinking can be argued.

7.3.5 *'Why'*-questions activate, expose and elicit '*Why'*-thinking'

A third activation type, *why*-thinking, is in this study is argued to be an even higher level of thinking than *what*- and *how2*- thinking.

***Why*-thinking** (prompted by *why*-questions)	A student is prompted to think about the *why*; most often by a verbal *why* -question from the teacher. To answer, it is *not* enough to remember something the teacher has previously taught and repeat it back. There can be several correct answers to a *why*-question (they are most often open questions). Note: *Why*-questions are in this study defined as *semantically* containing the questions such as "why is that true?", "why is this (or isn't)…" or "why do we do it this way?", *regardless* of which interrogative pronouns that are actually used.

The group of teachers who used verbal *why*-questions (4 of 18 teachers), also used the aforementioned *what- and how2*-questions. A few illustrative examples, specific to the *why*-questions, are:

- The most common examples (used by all teachers in this group) was "how does one [do we | do you] know that?" followed by "why does one [do we | do you] do that – why is it important to do it?".
- Two of the teachers in this group seemed to use the *why*-questions in a carefully distributed manner, a few (2-3) times during the lesson, related to a key concept that was explained during GT or during a task that was solved and involved such a key concept.
- The two other teachers seemed to use such questions much more frequently (7-12 times) and, above all, in a more integrated way; it seemed part of their normal "classroom talk" and **all** *what*- and *how2*-questions were *always* followed by (or integrated with) a *why*-question – the two first types of questions did not exist on their own.
- One teacher *combined* several *what- /how2-/ why*-questions as part of the *reversed GT* mentioned in the previous section; always activating thinking *before* starting all GT segments during that lesson:
  After writing 2x > 5 on the board, the teacher said: "Now take 3 minutes to think and discuss with your neighbour: What is this? Why isn't it an equation? What kind of answer could we expect? How could we solve it?". The teacher then walked around listening to the students' discussions. After the thinking time, via a random generator, each appointed student pair contributed to the joint answer. The teacher then gave a very short GT, landing all the contributions, complementing them with more formal knowledge (vocabulary, principles etc) and gave the students (again in pairs) the task to solve the inequality.

A final example is related to why-thinking in *combined* written and verbal form (1 of 18 teachers):

The IST was initiated (as in most cases) by the teacher writing a task sequence on the board. However, three of the task numbers from the sequence were also written alone, on top of other whiteboards in the class room. The students were observed to carefully note which they were. After approximately five minutes (the teacher was walking among the students during the IST), names were added under each separate task on the boards via the use of a randomizing app on the teachers mobile. The teacher said: "OK, names are up now. Bring a friend if you want to". Three individual students went up to the three boards and wrote their solutions to the tasks (without speaking) and then they sat down.

The teacher then wrote three new names on the same boards. After a few minutes more the teacher said, “ok now it’s time for explanations”. Another student (the second name on the board) went up and started looking at the first one’s solution. “OK”, said the teacher, “X, can you figure out what Y has done, can you try and explain how he was thinking?. The second student tried explaining how the first one was thinking and how it could be observed, from time to time prompted by the teacher’s questions “how do you know that?”, “why do you think he did it that way?”, etc.

No student had thus to answer for her/his own solution; another student assumed the task of explaining it, talking to the class. Sometimes misunderstandings occurred – Y correcting X’s explanation, saying “No, I meant that…. At the end the teacher asked, “Ok, how could you Y have helped X to understand what you meant?” “I should have drawn an arrow and… (the first student suggested improvements of his own written expression).

- The above procedure was also commented upon by a student group waiting for another class (again talking as a group): “it would be really scary to go to the board and present your own solution…” “yeah – what if it’s wrong?” “yeah explaining someone else’s is cool… it’s more of a guessing game – trying to figure it out”. Another student finally added “it’s the only time you really pay attention to writing clear solutions, to help the one coming after”.

7.3.6 *‘Non’*-questions do not activate thinking

A final type of questions is *‘non’*-questions which do not activate thinking*:*

***Non*-thinking**	Questions can also be asked *without* the purpose to prompt thinking – for example when the more or less conscious objective of the teacher is *basic activation* (se section 7.2). Rhetorical questions (questions which are immediately answered by the teacher her/himself) and questions asked *without waiting* for an answer could potentially also be considered belonging to this group.

A few examples from observations are:

- During GT, a teacher was solving examples on the board and from time to time (four times) asked “can someone help me on the calculator – please do ….”, upon which a student pushed the buttons on the calculator and gave the numerical answer.
- Rhetorical questions (e.g. the “so how do we do this?” mentioned above) were quite frequent during GTs, along with variants such as “what is the next step then?”, upon which the teacher answered the question her/himself – as a way to continue the GT.
- During GT, several teachers from time to time used expressions as “are you with me?” or “OK?”, but *without* waiting for an answer. The extreme was a teacher who wove “OK?” into the GT 18 times – not waiting for an answer one single time.

7.3.7 A word on Non-*answers*

In the above sections, the focus has been on the different types of questions asked by teachers to students. Finally, a few examples of how teachers handled students’ *non-answers*:

- One teacher asked a *what*-question during the GT. No hands were raised. The teacher – observably flustered – moved on to the next subject of the GT without further comments. No more questions were asked during the lesson.
- Another teacher, asked a *how2*-question but the class was immobile (no hands raised). The question was repeated with the addition “come on!” – with the same result. The teacher then turned to the board and answered the question himself in the form of a GT-explanation.
- A third teacher asked a question and no one seemed to be able to answer. The teacher then smiled and said: “You have two minutes to discuss it with your neighbour! We won’t give

up until we together built up an answer!" The students all started discussing the task in pairs.

7.4 Non result-related parameters

7.4.1 Teachers assessing their lesson representativity

The questionnaire included a set of yes/no questions (yes modelled as 1) as to the lesson's representativity and some free space for additional comments, next to each question. Free text space was also left below the questions with the text "Anything else you would like to add or comment upon?".

The outcome was (for 18 teachers):

Table 4: Teachers assessing lesson representativity

#	Question	Outcome
	Was the lesson **representative** for how you normally do[xxiv];	---
1	• Generally – on an overview level?	18
	• Specifically, representative in terms of:	---
2	• Positioning (randomized, free etc)?	18
3	• Adhering to book order; on lesson as well as for longer sequences (term)?	18
4	• Usage of board vs projector?	18
5	• Usage of digital tools	18
6	• Time spent on IST	18
7	• Time spent on GT	18
8	• Questions to students (hands-up or not)	18
9	• Working with terminology?	17
10	• Working with Think-Pair-Share?	16
11	• Sound level during class?	15

Some teachers chose to add comments to the questions above. In answer to questions 9-10, the teachers saying it was *not* representative, also commented that there is normally *more* work with terminology/vocabulary and in Think-Pair-Share form and/or that it was unusually little in the lesson which was observed. Similarly, the three teachers commenting on the sound level in question 11 were the same ones who stated that it was not representative and that normally, the sound level is *lower*.

8 Discussion

Just as for the Results section above, the discussion will be presented below in reverse order, starting with research question 3 (current status), with the purpose of setting a background for found results from questions 1-2 (the main research questions). Due to the limited amount of previous research found (as described in section 3), there is no consolidated research background which can be used as sounding board to the findings of this study. An attempt has been made to use relevant research where possible.

8.1 Traditional and textbook-driven education is still prevalent

This study advocates that an evolutionary, instead of revolutionary, approach is necessary for teachers to *be able to* – in a time-pressured reality - leave the 'traditional education' behind, and instead, activate, expose and elicit students 'quality thinking'.

The above claim would of course be irrelevant, unless such 'traditional education' truly exists – and needs to be replaced - today and in many of the classrooms. With that in mind, it was first investigated to what extent the observed mathematics lessons were adhering to the description by Skolverket/ Skolinspektionen, that Swedish mathematics education today is traditional, in terms of being textbook driven and spending most of lesson time on 'go-through' (GT) followed by individual silent task solution (IST) - or even 'calculation'.

The claims made by Skolverket/ Skolinspektionen are substantiated by this study:

- All the observed lessons included (at least one) GT segment and all lessons *but two* included (at least one) IST segment. More than 75% of teachers (14 out of 18) spent more than 80% of *productive time* on GT+IST.
- 17 of 18 teachers followed the textbook in all of the three aspects that were studied (whether the learning content came from the book; whether the book was used as transmission media; and whether the order of the book was adhered to). Comments and observations (qualitative data) was fully consistent with the quantitative data from the Classroom Observation Protocol (COP).
- All 18 teachers assessed their lessons to be representative for their way of teaching on overview level and very few exceptions were expressed on detailed level.

The above could however be questioned to a certain extent. Firstly, the way the quantitative results were presented (see 7.1.2), of course influences their interpretation. Since Skolverket/ Skolinspektionen measure *time on accumulated level*, this method was chosen also in this study. However, this study found that there were a few cases of fairly substantial ISTs on accumulated level, which consisted of several shorter segments and /or other thought-activating teacher actions – a fact which may render questionable any unambiguous conclusions based on cumulative times only. In all, the strong focus of Skolverket/ Skolinspektionen on certain measurement parameters (e.g. accumulated time spent, or specific teacher actions such as class introductions and conclusions) as communicated in the reports (see section 2.2), does not coincide with the priorities found in this study; which sees *activation* as the most important factor, on two levels:

- *Basic* activation (rendering a mentally present student);
- Activation of *thinking* (*before* exposure and elicitation)

(The high importance of *basic* activation has not been identified in any found literature. The importance of activation of *thinking* is very much present in research, but defined in many different ways. These two topics are further developed in the sections below.)

Secondly, the strongly (albeit sometimes implicitly) conveyed message by Skolverket/Skolinspektionen that so much time is spent on IST is negative, should perhaps be balanced. Some (especially younger) teachers in the study declared that they tried to "avoid" normal IST and strived towards "Other" activities (as defined in section 7.1.2). Two consequences can be observed:

- Removing (or substantially reducing) IST also removes the students chance to *practice* for individual written tests. Since almost all student assessment in mathematics is made by such written, individual tests, this study claims this is a clear negative – a student must have the chance to practice not only the same content, but also in the same *form*, as the upcoming test; which is consistent to the concept of *constructive alignment* (Biggs, 1996, 2003): A balance must be strived for between practice opportunities and student activation.

- The definition of such "Other" activities seemed unclear on classroom level. The same teachers could describe this "Other" as that students were encouraged to do the IST-tasks in pairs, referring to this way of working as *think-pair-share* (Lyman, 1981). Contrary to the principles of *think-pair-share,* there was however no initial individual thinking, nor final "landing" by the teacher (e.g. the conclusions the students had reached on their own were not exposed, nor complemented/adjusted by the teacher) in these observed cases; the teachers' definition of this concept was very different from the original definition by Lyman (*ibid*). Even though the teachers were *activating students as instructional resources for one another* (one of the five key strategies in *formative assessment*; Black & Wiliam, 2009), it can be argued that the students *as a pair* were equally '**left alone**' (see section 7.2.2) *by the teacher* as the individual would be during traditional IST. Another student cannot replace the teacher according the concept of formative assessment (*ibid*). These time segments were therefore still coded as IST, despite the teachers comments.

Fully acknowledging that the borders between the used GT, IST and Other are difficult to clearly define (when does a GT- or IST-segment become "Other"? When is a claimed "Other" in fact a 'disguised' IST?), this study advocates that the concept of '**leaving them alone**' (see next section) is a powerful tool to help distinguishing these borders.

8.2 An unexpected low level of *basic activation*

This study initially set out to spend most of its focus on occurrences of activated, exposed and elicited thinking. However, the study found an unexpected low level of *basic activation* among the studied 18 lessons (even under the presumption of 'traditional education'); a vast majority of the students were **'left alone'**. Since such basic activation here is claimed to be an *absolute pre-requisite* for activation, exposure and elicitation of *thinking,* a larger effort than initially planned was firstly dedicated to investigating this basic activation (see section 7.2, from which the following figure is repeated)

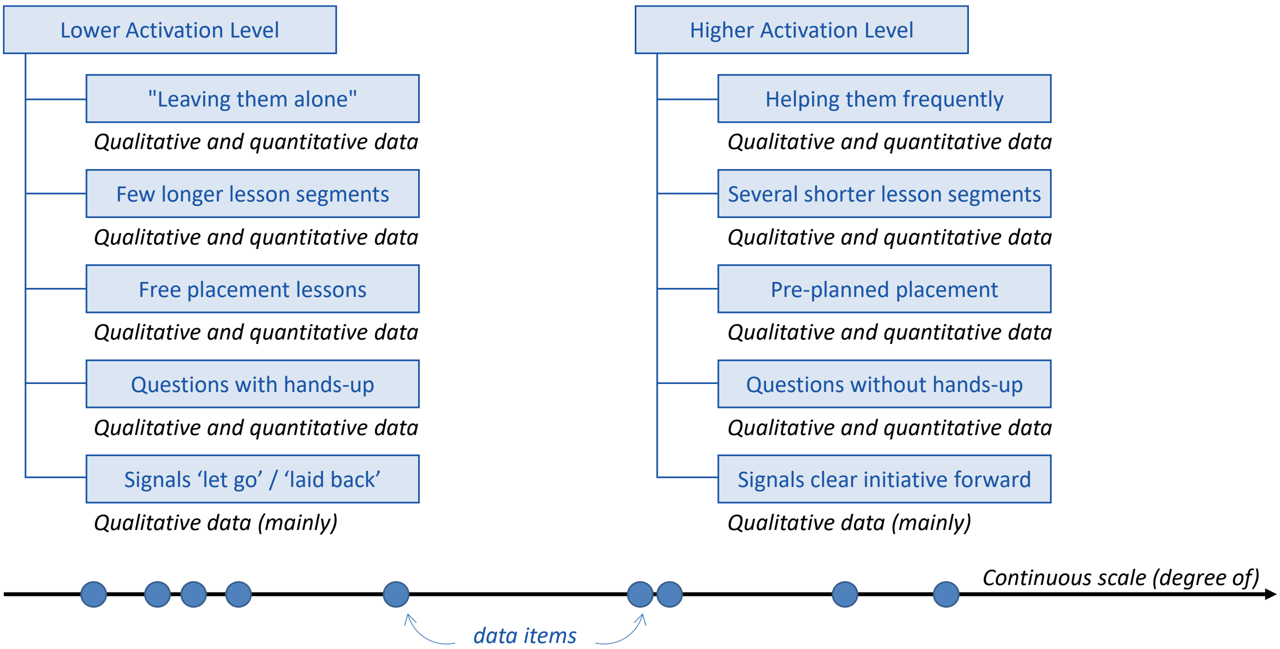

Figure 15: Overview of the factors observed to be contributing to increasing/decreasing basic activation.

This study found that more than 75% of teachers (14 out of 18) **left them alone** during IST, more than 60% used **fewer longer lesson segments**, 50% of the teachers (9 of 18) used **free positioning**; almost 90% of teachers asking questions used **hands-up**; and more than 50% of lessons (10 of 18) were characterized by a **'let go' / 'laid back'** leadership type.

An underlying assumption underpins the entire theme of *basic activation* (and all the identified subthemes stem from this hypothesis): It is assumed that we are all human; i.e. unless very interested in something and/or having a very well-trained capability of keeping focus by pure self-discipline, extensive and/or monotonous activities *will unavoidably* cause us to lose focus[36]. Firstly, this underlying assumption means we need all help we can get to stay mentally present/active.

One teacher commented that "I put a lot of focus on generating interesting lesson initiations" in order to provide such help. It was however observed that students lost focus over time during this lesson, on a similar level compared to other lessons. It is in this study argued that addressing (basic) activation by selecting and/or designing tasks that are fun, interesting etc to the students may be of less importance - or possibly even detrimental. This study adheres to the view on the relation to the brain's reward system: that creating a habit where attention is linked to "fun" or personal interest, may cause students to disengage once the external rewards are no longer provided, which is in line with the view by Barton (2018). The task of the teacher to provide *continuous* "fun" during a lesson is here considered impossible. If our aim is to keep students activated *over time*, it is here claimed that other ways of working may be more effective.

To include opportunities to train the students' abilities to uphold focus/attention is a possible way forward – which only one single teacher seemed to do explicitly (see section 7.2.7). Another way is for the teacher to work in an integrated way, seeing the subthemes in *Figure 15* above as being *enablers* or *disablers* (on a continuous scale) for such continued activation. These themes furthermore tie in to the theoretical framework in the sense that *decreasing the negative* (see section 5.2.2) implies decreasing the identified disablers (the left side in *Figure 15* above) and *increasing the positive* means increasing the identified enablers (the right side).

Secondly, this integrated way of working means that the themes above also can be seen as interrelated as to meaning: a long GT, where the teacher talks without (intentionally or not) activating a *substantial* amount of the students, could also be considered as the teacher **leaving them alone**. Similarly, using hands-up for questions (letting *students* chose whether they want to answer), implies that a majority of the class is **left alone,** since it was observed that in *all* classes, the same few students chose to raise their hand. Another example is that **pre-planned positioning** can be seen as a proactive measure to help them keep focus, i.e. *not* **leaving them alone**. Due to these interrelations, the theme **leaving them alone** is considered very central to this study and **the theme basic activation can be seen as a means to take a first step away from the traditional**, inactivating teacher role as " 'dispenser of knowledge' and arbiter of mathematical 'correctness'" (Stein, Engle, Smith & Hughes, 2008)

Despite the limited found research about the concept of *basic activation*, a critical analysis of the results has identified the following important factors to such *basic activation*:

- *Helping them immediately/frequently/permanently* - instead of *Leaving them alone*
- Several shorter lesson segments - instead of few and longer
- Pre-planned positioning - instead of free seating
- Questions without hands-up - instead of using hands-up
- Signalling clear forward initiative - instead of 'let go' / 'laid back'

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[36] Lost focus is, as described in the theoretical framework (section 5.2.2), considered the lowest possible level of the framework, before the *basic* activation step; the student not being mentally present/ attentive at all.

8.3 Good examples: Activation, exposure and elicitation of *thinking*

Several teachers in this study been observed to, within a sea of traditional education, create islands of thought-developing activities and other contextual factors that can be considered beneficial for activating, exposing and eliciting students thinking. In this section such good examples (the islands) will be discussed against the background of found results on general level (the sea), after a few observations on general level.

As mentioned previously, this study considers that thinking must be activated, before it is exposed, which is in line with that the way the teachers ask questions will affect students' levels of activity (Black & Wiliam, 2009). It is in this study further claimed that different types of activation (i.e. different types of questions/tasks) have been identified to activate *different types of thinking*, which is consistent with that "[s]tudents learn what they have the opportunity to think about; what they have the opportunity to think about is bounded by the *tasks* they are assigned to work on" (Tekkumru-Kisa, Stein & Doyle, 2020).

Also, depending on the expected *length* of the student's answer (the "**size**" of the question /task /other teacher action), the degree of activation could firstly be observed to differ. Secondly, a varying degree of other skills was observed to be practiced and/or identified by the teachers themselves as related to the questions asked, for example (see *Figure 16* below):

- *memory (recall) skills*
- *expression skills*
- *meta skills*

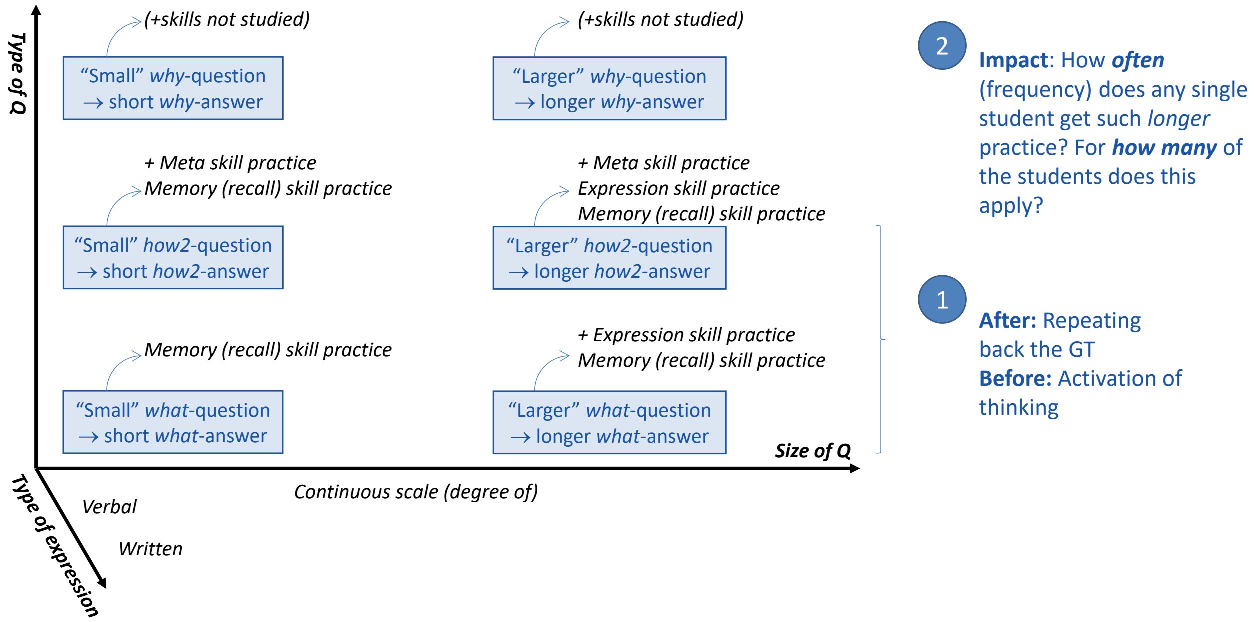

Figure 16: Overview of types of activation of thinking and related practiced skills.

Two observations on overview level are:

1. **Before** or **after**: When asked *after* a GT (perhaps on next GT), *what*- as well as *how2*-questions mainly exposes memory (recall) of what the teacher has said earlier. Using these types of questions *before* entering into a new knowledge area, constitutes activation of thinking to a higher degree and is in this study called a '**reversed GT**': asking students to think and/or do before telling them anything about this topic.
2. In all observed cases, a central factor was **impact** of teacher questions; as to both potential outcomes (activation of thinking and practice of other related skills): How often (frequency) does any single student get to practice? Is it long enough to actually constitute practice? For *how many* of the students does this apply?

The observed lessons have been grouped, using qualitative as well as quantitative data, identifying three groups of lessons/teachers; A, B and C.

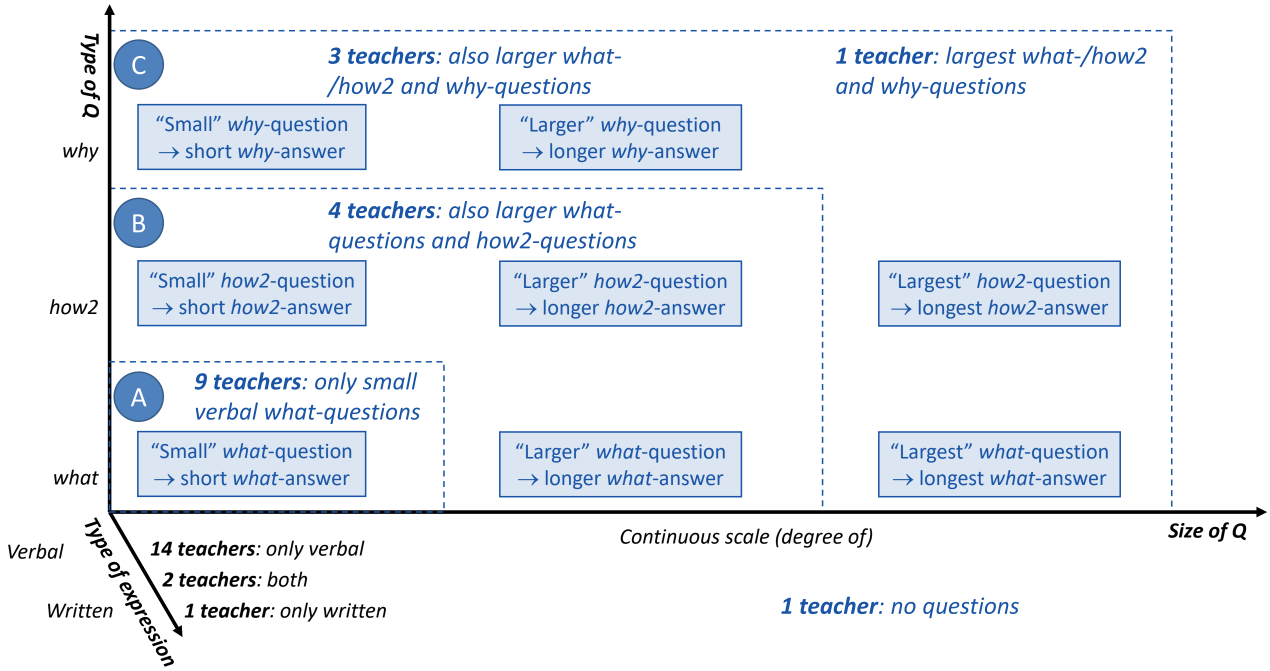

Figure 17: Overview of groups of lessons A, B and C.

From COP-data, 17 of 18 teachers were identified to use questions/tasks (regardless of exposure form) during a lesson, while 1 of 18 teachers did not ask any questions at all during the lesson (neither verbal, nor written).

14 teachers used only verbal exposure (at least once during the lesson), while 1 teacher solely used written exposure (MWB). 2 teachers used *both* written and verbal exposure during the lesson.

8.3.1 Group A; *What*- thinking and memory skills

Short *what*-questions can be seen as a typical examples of mathematics *as product, not as process*: "…the product does not represent the process of coming to know" (Lampert, 1990, p.30).

Of the 16 teachers using verbal exposure, 9 teachers used **only** *short what*-questions during the lesson (hence only potentially exposing *what*-thinking). In all these lessons, this occurred without allocation of thinking time and using hands-up, why only a minority of students' thinking was in reality exposed. Often answers consisted of a single word; no *longer what*-questions were observed at all in this group. Neither *memory (recall) skills*, nor *expression skills* could therefore be considered to be practiced to any larger extent. Nor did this group of teachers use any other *type* of questions. This group is named **Group A** (see *Figure 17*).

In the comments, these teachers often referred to these *what*-questions as a way to make students remember facts, but given the low *impact* (as defined in the previous section), this view could firstly potentially be questioned. One teacher was however observed to work more systematically with *memory (recall) skills* with focus on **vocabulary quizzes as lesson initiation** (see section 7.3.3), in a form that directly follows the principle "Give a low-stakes quiz at the beginning of each class and focus on the most important material. Consider calling it a "review" to make it less intimidating" (Dunlosky, 2013, p.21).

This study secondly argues that having (remembering) at least a basic vocabulary is a prerequisite for the next step in the staircase. Perhaps the use of such short vocabulary quizzes could be used *instead* of using *what*-questions as described above (especially in combination with hands-up), since so few students else will get true opportunities for training. This is fully in line with the claim made by Dunlosky, Rawson, Marsh, Nathan and Willingham (2013): “Note, however, that although gaining factual knowledge is not considered the only or ultimate objective of schooling, we unabashedly consider efforts to improve student retention of knowledge as essential for reaching other instructional objectives; if one does not remember core ideas, facts, or concepts, applying them may prove difficult, if not impossible” (p.7) – which is also in keeping with Craig Barton’s focus on the importance of a solid foundation of such knowledge (Barton, 2018).

Another observed way of working with memory was that two teachers **split the IST in two parts, pushing the second half to act as start of the next lesson**, one commenting that students “remembered stuff better that way”. This view could be seen as so called *distributed practice*, and is in line with that “Students will retain knowledge and skills for a longer period of time when they distribute their practice…[which] typically refers to distributing the practice of the same problem across time” (Dunlosky, 2013, p.15).

8.3.2 Group B; *How2*- thinking and meta-skills

The remaining **8 teachers** sometimes used *how2*- as well as *what*-questions (hence also potentially exposing *how2*-thinking). Generally in this group, question size was slightly larger and own thinking time was more often allocated, why *memory (recall) skills* can be argued to have been practiced – for the students answering. Due to the larger question size (and also to think-pair exercises, when such exercises occurred) verbal *expression skills* were practiced as well. Such practice also applied to a larger share of the students in the classes. As for exposure of *thinking*, the impact was higher than in group A, especially in the cases where the teachers constantly walked during the students’ (verbal or written) work, exposure of thinking of most (if not all) students thinking can be argued. 4 teachers in this group did however not use the next *type* of question (*why*-questions) and are delimited as **Group B** (see *Figure 17*).

Group A and B (and the teacher not asking questions at all) are all (14 of 18 teachers) considered as having a **procedural focus** in this study, as they were observed to not move *beyond* the “how-to” level.

It could tentatively be claimed that two “flavours” existed among the questions/tasks within group B:

- one more “theoretical” (cf. trying to remember the recipe for a cake and thinking about the steps to make it, in order to share the recipe with someone else). Exposure is verbal; describing / explaining a step (or several steps).
- one more “practical”; integrating the knowledge into actually *doing* it (cf. doing the thinking about next step *while making* the cake). Exposure takes place by *doing*, without the demand to verbalize the steps, under the condition that such doing is observable, step by step, by the teacher[37].

One teacher underlined that the practical variant above was absolutely crucial for the more practically oriented classes (called Mathematics A in Sweden) and that it was “necessary to avoid the more theoretical verbalization, else no energy would be left for learning the *mathematics*”. After thinking, the same teacher added that also in the more theoretical programmes, the theoretical verbalization “wasn’t worth anything unless it was *also* shown by doing”. It could be reasoned that this view aligns with a perspective where knowledge never can be passive, but always is about doing: “JATD [the Joint Action Theory in Didactics] considers knowledge as a power of acting, in a given situation, within a situation” (Sensevy, 2012, p.514).

[37] It should be noted that in this study, such doing not include e.g. taking notes during a GT (however intensely and attentively); the doing must involve a *mathematical task*.

Two other teachers used the more 'practical flavour' differently; letting all students do the same tasks on a (physical or digital) mini whiteboard at a common pace. One teacher used it all through the lesson, as a substitute for the IST, so that the GT was divided into extremely short segments, between the tasks. The other used it after introducing a new knowledge area, for a limited set of initial tasks (see section 7.3.4). It may be argued that using *only* such a practical approach – without ever prompting the student to describe it - may deprive the students of any chance to practice meta skills, besides breaking the principle of *constructive alignment* (Biggs 1996, 2003), getting practice for written tests, as mentioned in section 8.1. Whether such meta-skills are required will however depend on the math course taken.

8.3.3 Group C; *Why-* thinking

The importance of *activation of thinking* is very much present in the literature. *Why*-thinking could be seen as what generally is called Higher Order Thinking Skills (HOT skills or HOTS) in more recent research, but as mentioned in section 4.3, such thinking is defined in many different ways, using many different theoretical frameworks, research methods, etc. It has therefore been difficult, to within the time frame of a degree project, identify the *relevant* literature.

Of the previously mentioned 8 teachers, **4 teachers** also used *why*-questions and constitute **Group C** (see *Figure 17*). Since these 4 are indeed a subset of the 8, the same positive factors apply (as described in the previous section). *In addition*, what in this study previously has been called "quality thinking" was activated, exposed and elicited – albeit to a varying degree. Whether any additional skills (in addition to the memory, expression and meta skills mentioned in the previous sections) were developed as by-products, has not been investigated in this study[38].

The most frequent type of why-questions used, "why this fact is true", adheres fully to so-called *elaborative interrogation* (Dunlosky 2013, s18). One teacher in group B commented that such *why*-questions was "above the level of the students in this class", perhaps conveying an implicit message of her/his expectations on the students' abstraction capabilities. However, a teacher in group C put up two examples of fairly basic tasks on the projector (showing them side by side) and asked "why is the second task more difficult than the first?", which was a question perceived as fairly concrete by the students in that class, observing their answers which came fluently, without breaks. The two lessons were for students within the *same* programme and this study argues that why-questions can be used on a general level, for all. Two observed ways of raising the level of prompting may be by gradual transfer in wording on *how2*-level, as preparation for the *why*-level:

- the difference in *pronominal use* between teachers "how *is it* done" (passive form) "how does *one* do this" (active but anonymous) "how do *we* do this" (active but collective) "how do *you* do this" (active and personal);
- the difference in *verbal use* (which verb is important); "how do xxx *do* this" versus "how do xxx *think*?"

The teacher on the highest level (see top right in *Figure 17*), used personal and active pronouns and the verb *think* consistently. The *elaborative interrogation* (Dunlosky 2013, s18), hence became personal, but the teacher continuously underlined that many ways of thinking were necessary to get "the full picture".

The board technique used to let one student (silently) share a written solution on the board and then *another student* verbally explain it in front of the class (see section 7.3.5), was observed to reduce stress of personal student exposure and also created a situation when students paid

[38] As mentioned in section4, an in-depth study of previous research related to student thinking would most probably be needed before undertaking a study of such related skills.

attention to writing clear solutions, to help the student coming after. To explain someone else's solution can be seen as a less stressful variant fully in line with the concept *self-explanation* by Dunlosky et al (2013): "Self-explanation is similar to elaborative interrogation, except that educators ask students to explain their thought processes, rather than just a concept" (Digital Promise, 2015, p.6). Due to the teacher activity during that explanation; the teacher guiding the process via *why*-questions to the student as well as to the class, it also generated "public discussion about the comprehension process" (Sensevy, 2014, p.589).

Two teachers clearly refrained from assuming centre stage; generally restraining own talk-time, making students talk instead. They firstly thus generally kept "a low *topogenetic profile*" (Sensevy, 2014, p.590). Secondly, they generated debate, by for example saying "who thinks this and who thinks that?" (without clarifying which was true), fully in line with that "the teacher does not agree to adopt a high topogenetic position until the debate has been expanded and largely resolved by the students themselves." (Sensevy, 2014, p.593).

As a final note on questions, this study has focussed on *teacher-initiated* questions, tasks and actions; their type, size and effects on activation. It is noteworthy that **not one single student-initiated question** related to learning content was observed during any of the 18 lessons[39]. When informally talking to a group of students[40] outside a classroom, I introduced myself as studying to become a teacher and asked if it was nowadays common that students ask questions in class. They started laughing – said "noooo!" - and asked if I was active on social media. When I looked puzzled, they explained (all at once, finishing each other's sentences) that asking questions on own initiative was stupid unless you wanted "not so positive" comments on social media. When I asked about answering a teacher's question – whether that gave the same result – a student rolled her/his eyes and answered "No, not at all - 'cause that's not one's own fault – what the *teachers* make us do is not the *student's* fault of course!!!"… "unless really, really stupid" another student filled in, giggling. If such symptoms are found also within other contexts, the importance of teachers activating students grows even further. There may exist strong inhibitory factors which makes expecting (and stimulating) them to act *on own initiative* (as mentioned in the reports by Skolverket/Skolinspektionen) unrealistic: There are heavy obstacles to filling the bucket (see section 5.2.1).

8.4 Conclusion

This study has identified qualitative and quantitative data that underpins an evolutionary approach to leaving the old paradigm of 'traditional mathematics education' behind. Today's teachers – to a varying degree adhering to the old paradigm - could use the teacher actions identified in this study as inspiration for evolution towards the new paradigm. By furthermore using a step-by-step approach, based on the industrial principle of return on investment for any time and effort spent, ***basic* activation** is seen as the first step on such a journey. As for **activation, exposure and elicitation of *thinking***, the following has been argued:

- *(basic)* Activation is seen as a first level, i.e. previous to Activation of *thinking.* A student must be mentally present in order for activation to be possible;
- Activation of thinking must take place before it is exposed; true exposure is argued to *not* occur (there is "nothing to expose"), unless thinking has been activated beforehand. Also, true exposure *cannot* occur when the (implicitly or explicitly) communicated thinking time (after activation/prompting) is insufficient;
- Exposure of thinking is seen as a prerequisite for *developing* thinking.

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[39] The only questions asked by students were related to upcoming written tests; e.g. "will this be on the test?", "is the test on Monday or Tuesday?" etc.

[40] The student group did not belong to this study (had class in a classroom next door).

This report furthermore claims that any substantial transformation project (such as a paradigm shift) must have project support on central level for it to be *feasible* at all for the vast majority of teachers in today's time-pressured reality. Curriculums have been changed in most countries to involve quality thinking. There is much less on *how* to go about such change and a greater focus on implementation feasibility is needed. Academic quality intrinsically involves creating multiple perspectives, but such diversification may imply a substantial learning threshold for teachers. Striving to find common grounds and/or approaches could be useful.

"When countries are concerned about education, there are intense and ongoing debates about *what* should go into the curriculum. There is much less discussion as to *how* to get it to happen. … Curriculum changes …involve fairly profound changes in the professional practice of many people across a range of constituencies: textbook writers, test designers, professional development leaders and, particularly, teachers. All these need to be not just *motivated* but *enabled* to meet the new challenges." (Burkhardt, 2014, p.28).

8.5 Suggestions for future research

This study should be regarded as exploratory - above all of whether the combination of the two theoretical frameworks can be regarded as a fruitful approach: the power of a thematic analysis will always be heavily dependent on the theoretical framework - as to its analytical claims (Braun & Clarke, 2006).

Activation of thinking is here regarded as a fundamental prerequisite for learning. How such activated thinking best can be studied should be further developed. A deeper literature review on thinking skills is firstly recommended, as mentioned in section 4.3. Such a study should ideally however include also *progression of thinking* and not solely focus on the very highest orders of thinking, as seems too often be the case today. Secondly, since this study took place at one single school, a natural extension would be to extend the study to other schools. If so, in order to reduce potential observer bias, such a larger study would preferably by carried out by several different observers. Thirdly, if such a larger study is performed, potentially a large enough data set could be obtained where inferential statistical analysis would be relevant, possibly correlating what this study calls *enablers* and *disablers* to outcome. Such proven outcome – if identified - would most probably be a prerequisite for a more nomothetic approach as to the "how to", truly generating benefit from a feasibility (or even ease of implementation) perspective – and thus for the paradigm shift to take place.

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Appendix: Original quotes in Swedish

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[i] ”Förändringar i arbetslivet, digitaliseringen och den tekniska utvecklingen, internationaliseringen samt miljöfrågornas komplexitet ställer nya krav på människors kunskaper och sätt att arbeta. Skolan ska stimulera elevernas kreativitet, nyfikenhet och självförtroende samt vilja att pröva och omsätta idéer i handling och att lösa problem (SKOLFS 2010:144, s4)

[ii] ”Skolan ska ansvara för att varje elev efter genomgången grundskola … kan använda sig av matematiskt tänkande för vidare studier och i vardagslivet” (SKOLFS 2010:144, s7)

[iii] ”Eleverna ska träna sig att tänka kritiskt” (SKOLFS 2010:144, s4)

[iv] ”… skolans ansvar att varje elev … kan använda sina kunskaper som redskap för att
– formulera, analysera och pröva antaganden och lösa problem,
– reflektera över sina erfarenheter och sitt eget sätt att lära,
– kritiskt granska normer, påståenden och förhållanden, och
– lösa praktiska problem och arbetsuppgifter, … (SKOLFS 2010:37, s6)

[v] ”Matematisk verksamhet är till sin art en kreativ, reflekterande och problemlösande aktivitet…” (SKOLFS 2010:37, s29)

[vi] ”…formulera och lösa problem… reflektera över och värdera valda strategier, modeller och resultat… argumentera logiskt och föra matematiska resonemang… kommunicera om matematik.:. se matematikens sammanhang och relevans…” (SKOLFS 2010:37, s30)

[vii] ”Ytterst handlar matematiken om att upptäcka mönster och formulera generella samband” (SKOLFS 2010:261, s108)

[viii] Undervisningen ska
- ”utmana eleverna och ge dem erfarenheter av matematikens logik, generaliserbarhet, kreativa kvaliteter och mångfacetterade karaktär …”
- ”stärka elevernas tilltro till sin förmåga att använda matematik i olika sammanhang och ge utrymme åt problemlösning som både mål och medel” (SKOLFS 2010:261, s108).

[ix] "De matematiska förmågorna handlar till stor del om att kunna se samband och mönster, bedöma och värdera, uppskatta rimlighet och resonera sig fram till en lösning med mera" (Skolinspektionen 2016, s18).

[x] "Den i särklass vanligaste arbetsformen är att eleverna sitter och arbetar var för sig med lärobokens uppgifter…. Det händer sällan att man diskuterar matematik i klassen, vare sig under lärarens ledning eller mellan elever sinsemellan" (Skolverket 2003, s45-46)

[xi] ”…individuella arbetsformer tar allt mer tid i anspråk medan undervisning i helklass får mindre utrymme” (Skolverket 2010, s70)

[xii] ...en övervägande del av undervisningstiden i svenska matematikklassrum ägnas åt så kallad tyst räkning…” (Skolinspektionen 2016, s18)

[xiii] ”En observation är att med ökad ålder på eleverna så minskar förekomsten av matematiska resonemang samtidigt som fokus på att hantera procedurer ökar” (Skolinspektionen 2009, s22)

[xiv] ”Undervisningen behöver ge stöd för att utforska matematiska problem och stimulera abstrakt tänkande” (Skolinspektionen 2016, s6)

[xv] "Undervisningen är starkt styrd av läroboken. Det får konsekvensen att eleverna får små eller inga möjligheter att utveckla sin kompetens i problemlösning, sin förmåga att använda logiska resonemang och sin förmåga att sätta in matematiska problem i sammanhang" (Skolinspektionen 2009, s9)

[xvi] "Det finns ... en stark negativ korrelation mellan [användning av] läroboken och övriga [ej procedurrelaterade] kompetenser" (Skolinspektionen 2009, s16)

[xvii] ”Lärarna tycker visserligen att problemlösning är viktigt att lära sig men uppger att det får stryka på foten eftersom de anser att tiden inte räcker till” (Skolinspektionen 2016, s12).

[xviii] ”det är individuellt vad som är ett problem” (Larsson, 2013, s2)

[xix] 1. ”Problemet ska introducera till viktiga matematiska idéer eller vissa lösnings-strategier.
2. Problemet ska vara lätt att förstå och alla ska ha en möjlighet att arbeta med det.
3. Problemet ska upplevas som en utmaning, kräva ansträngning och tillåtas ta tid.
4. Problemet ska kunna lösas på flera olika sätt, med olika strategier och representationer.
5. Problemet ska kunna initiera en matematisk diskussion utifrån elevernas skilda lösningar, en diskussion som visar på olika strategier, representationer och matematiska idéer.
6. Problemet ska kunna fungera som brobyggare.
7. Problemet ska kunna leda till att elever och lärare formulerar nya intressanta problem.” (Taflin, 2007, s56).

[xx] ”…möjlighet att formulera och pröva hypoteser och antaganden…” (Skott et al, 2010, s17)

[xxi] ”’ta över’ tänkandet och resonemangen och tala om för eleverna hur man löser problemet” (Larsson, 2013, s6).

[xxii] ”man talar och tänker *kvantitativt* om undervisning; eleverna ska räkna igenom, ”beta av” ett visst antal uppgifter [versus när] det centrala är att eleverna utvecklar *kvaliteten* på sitt tänkande.” (Helenius, 2019, s1)

[xxiii] ”[A]ll erfarenhet är inte beprövad erfarenhet. Beprövad erfarenhet är systematiskt prövad, dokumenterad och genererad under en längre tidsperiod och av många” (Minten & Kornhall, 2013, p11).
[xxiv] Var lektionen representativ för hur du brukar göra;
¤ Allmänt sett - generellt?
¤ Specifikt - representativ vad gäller:
- Placering (fri/slumpad etc)
- Bokens ordning (både längre o detta pass)
- Användning av tavlan vs projektor
- Användning av digitala verktyg
- Tiden för egen räkning
- Längden på genomgången
- Arbete EPA
- Arbete med begrepp
- Frågor t. eleverna (hands up eller ej)
- Ljudnivå