Degree Project in Financial Mathematics
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Analysing the Optimal Fund Selection and Allocation Structure of a Fund of Funds

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Abstract

This thesis aims to investigate different types of optimization methods that can be used when optimizing fund of fund portfolios. Moreover, the thesis investigates which funds that should be included and what their respective portfolio weights should be, in order to outperform the Swedish SIX Portfolio Return Index. The funds considered for the particular fund of funds in this thesis are all managed by a particular company. The optimization frameworks applied include traditional mean variance optimization, min conditional value at risk optimization, as well as optimization methods studying alpha in combination with the risk measures tracking error and maximum drawdown, respectively. All four optimization methods were applied on a ten years data period as well as on a five years data period. It was found that while the funds have different strengths and weaknesses, four of the funds were considered most appropriate for the fund of funds. Geography and sector constraints were also taken into account and it was found that, in this particular case, the healthcare sector constraint affected the allocated portfolio weights the most.

Keywords

Master Thesis, Financial Mathematics, Fund of Funds, Portfolio Optimization, Mean Variance Optimization
Analys av optimala fondval och allokeringsstrukturer för en fond i fond

Sammanfattning

Syftet med detta masterexamensarbete är att undersöka olika typer av optimeringsmetoder som kan användas vid optimering av en fond i fond. Vidare är syftet med optimeringen att utvärdera vilka fonder som bör inkluderas och vilka deras respektive portföljvikt bör vara för att prestera bättre än det svenska SIX Portfolio Return Indexet. Optimeringsmetoderna inkluderar traditionell modern portföljteori, minimering av conditional Value at Risk och optimeringsmetoder som studerar alpha i kombination med riskmåttet tracking error respektive maximum drawdown. Alla fyra optimeringsmetoder applicerades på en tio år lång respektive fem år lång dataperiod. Det visade sig att även om fonderna har olika styrkor och svagheter kunde fyra av fonderna anses vara mest lämpliga att inkluderas i fond i fonden. Geografiska och sektoriella begränsningar beaktades och det konstaterades att sektorbegränsningen för hälsovårdssektorn hade störst påverkan på resultatet.

Nyckelord

Masterexamensarbete, finansiell matematik, fond i fond, portföljoptimering, modern portföljteori
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Nomenclature

**CAPM**  Capital Asset Pricing Model. 15

**cVaR**  Conditional Value at Risk. 2, 11, 28, 41, 51, 53

**ES**  Expected Shortfall. 11

**FoF**  Fund of Funds. 1–4, 22–24, 26–28, 30, 39, 41, 42, 44, 45, 51–54

**GICS**  Global Industry Classification Standard. 23, 36

**K-S test**  Kolmogorov-Smirnov test. 17, 34

**MPT**  Modern Portfolio Theory. 11, 14, 15, 20, 21

**NAV**  Net Asset Value. 22, 24–27, 30, 32, 52

**Q-Q plot**  Quantile-Quantile plot. 15, 34, 52

**S-W test**  Shapiro-Wilk test. 17, 34

**SIXPRX**  SIX Portfolio Return Index. 24, 26, 28, 30, 32, 39, 41, 52

**UCITS**  Undertaking for Collective Investments In Transferable Securities. 27

**VaR**  Value at Risk. 10, 11
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Chapter 1

Introduction

1.1 Background

Investors have always desired to hold efficient investment portfolios that are robust under complex environments and generate stable returns. How to compute this portfolio has thus been a question of interest for decades. Investing in different types of assets, including stocks, creates a portfolio for the investor. An investment fund means that the investor invests in a fund (managed by a fund manager) consisting of assets rather than investing in the assets directly. Two types of investment funds are passively managed index funds which aim to track a benchmark representing the general performance of the market, and actively managed mutual funds with the goal to beat the benchmark and outperform the market. This thesis focuses exclusively on actively managed mutual funds. For a fund to outperform the market it needs to generate a higher return than an average benchmark. An investment fund consisting of only stocks, and no other asset types, is often referred to as a stock-based fund.

A Fund of Funds (FoF) is an investment fund consisting of several individual funds containing different assets. Thus, a FoF could be seen as a "pre-made package" of funds to invest in, balancing a portfolio of multiple underlying funds in order to provide diversification benefits. Thereby, the exposure to any single market segment is reduced and the overall stability increased. Consequently, one of the main objectives of a FoF is security, instead of having to trade all funds individually, the investor invests in the FoF and leaves the managing of the individual funds to the professionals. Naturally, investing in a FoF rather than a single fund results in larger diversification of the
investment portfolio. A FoF consisting of only stock-based funds is often referred to as a stock-based FoF.

The investment company Atle is at the beginning of investigating the feasibility of starting a stock-based FoF, consisting of funds from four of the fund companies that the company manages. Consequently, they stand in front of handling the two main steps of portfolio optimization: what funds to include in the FoF and determining the optimal holdings in each fund \[2\]. The company focuses on active investment management concentrating on small niched managers in the Nordic region.

When it comes to which funds to include in order for a FoF to perform as well as possible, there are many factors to consider and optimization methods that can be used. Mean variance optimization is one of the most well known and well used optimization models within portfolio management \[6\]. Meanwhile, the Conditional Value at Risk (cVaR) factor makes the optimization focus mainly on the downside risk \[15\]. These optimization models along with other models taking alpha in combination with the risk measures tracking error as well as maximum drawdown into account will be investigated in this thesis. Furthermore, several allocation constraints will have to be managed in order to evaluate which funds to include in the FoF at Atle.

1.2 Problem Formulation

The many portfolio optimization methods available as well as the complexity of questions such as how to best estimate risk, how to deal with factor exposures and how rebalancing should be handled makes the process of setting up a FoF more complicated than it might seem at first glance. In addition, which optimization methods that are optimal may vary depending on the preferences of the managing company as well as the investors themselves. In addition to the more standard optimization methods, bespoke objective functions might therefore preferably be considered. In other words, each case of setting up a FoF is met with several mathematical, economical, factorial and preference-related decisions that need to be considered and analyzed in detail in order to obtain the most suitable result possible for the individual situation.
1.3 Purpose and Research Questions

The purpose of this thesis is to investigate different types of optimization methods used for portfolio optimization as well as analyzing their respective advantages and disadvantages. Based on the result of using different optimization methods on the potential funds for the FoF at Atle, the purpose is to evaluate which funds that should be included and what their respective portfolio weights should be. This results in the following research questions for the thesis:

- Which portfolio optimization methods are most suitable for determining the optimal FoF composed of mostly medium and small size funds focusing on the Nordic market?
- Which funds should the optimal FoF at the company contain and what will the allocation structure of individual weights be?

1.4 Objectives and Expected Outcomes

The aim of this thesis project is that the outcomes will facilitate and provide a framework for the process of launching a FoF at Atle. By analyzing which of the funds available that ideally would be included in the FoF, a clearer picture of the feasibility of starting a FoF will be obtained. Furthermore, an analysis based on different optimization frameworks and how risk and expected return best are estimated will add valuable insights and perspectives into the work with setting up the FoF.

1.5 Methodology

A literature review about one of the most well known portfolio optimization methods was conducted in order to investigate and illustrate the advantages and disadvantages of using it as well as to gain a better understanding of how to use it in our particular case for optimizing which funds to include in the FoF. How the different optimization methods are used in this thesis are partly based on the results of the literature review. In addition to the standard objective functions used in many optimization methods, bespoke objective functions and constraints according to specific desires and special interest of the company were also obtained.


1.6 Delimitations and Limitations

This thesis focuses on stock based funds and stock based FoF only. Hence, diversification across different asset classes are not applicable. Furthermore, there is a main geographical focus on Sweden and the Nordic countries as well as the rest of Europe and the US. This is a direct consequent of the geographic distributions of the funds managed by Atle as well as the fact that the FoF is planned to be directed towards Swedish investors. Moreover, since the FoF at Atle only will be composed of funds that the company already manages it is assumed that there will be no additional costs of the FoF compared to investing in the funds directly. Moreover, the optimization methods and objective functions used for portfolio allocation in this thesis do not take transaction costs into account.

1.7 Disposition

Chapter 2 presents an in depth theoretical background about portfolio mathematics, risk measures, portfolio optimization methods, distribution tests and performance evaluation methods relevant for the thesis. Chapter 3 includes a literature review of the most common portfolio optimization method, mean variance optimization. Chapter 4 gives an in depth description about the methodology as well as background information about the funds used in the project and the processing of data. Chapter 5 presents findings from the empirical analysis of the data of the individual funds. Chapter 6 presents the results obtained from applying various optimization frameworks to the data set as well as the performance evaluation of the different optimization methods. Chapter 7 deals with the analysis and conclusions of the results along with discussions about future work that could be done in the area.
Chapter 2

Theoretical Background

2.1 Portfolio Mathematics

2.1.1 Linear portfolios

A linear portfolio is a portfolio that can be described as a linear function of its underlying assets. This mannerism entails a number of preferable mathematical relationships. One of the most fundamental and well used mathematical relationships in portfolio theory regarding linear portfolios is that, assuming constant weights of the portfolio assets, the return of a (long positions only) linear portfolio can be represented as the weighted sum of the individual asset returns. Mathematically, this can be expressed as follows,

\[ R = \sum_{i=1}^{k} w_i R_i \]  

(2.1)

where \( R \) is the return of the portfolio, \( w_i \) is the weight of the \( i \):th asset in the portfolio, \( R_i \) is the return of the \( i \):th asset in the portfolio and \( k \) is the number of assets in the portfolio [1].

2.1.2 Concavity and Convexity

Concavity and convexity are measures of the curvature of a function’s graph and are thus related to the second derivative \( f''(x) \) of a function \( f(x) \). The relationship is described as follows: \( f(x) \) is said to be strictly concave if \( f''(x) < 0 \) and strictly convex if \( f''(x) > 0 \). Furthermore, this implies that, for a strictly concave function, the value of
the function at two arbitrary points $x_1$ and $x_2$ is larger than the weighted average of the respective function’s values. This results in the fact that a strictly concave function’s graph (in the range of $x_1$ and $x_2$) will always lie above the chord joining the points $f(x_1)$ and $f(x_2)$ together. Hence, the graph gets its concave shape. Mathematically, this can be described as follows,

$$f(px_1 + (1-p)x_2) < pf(x_1) + (1-p)f(x_2) \text{ for every } p \in [0, 1]$$

(2.2)

The same relationship holds true for strictly convex functions, with the exception that the equation contains $>$ (rather than $<$) since the graph of a strictly convex function always lies below the chord joining the relevant points together. Consequently, the graph gets its convex shape. In addition, in some circumstances the strictly requirement is dropped, allowing the second derivative curvature function to take the value zero, resulting in a $\leq$ relationship in equation 2.2 [1].

### 2.1.3 Alpha and Beta

In the world of investing, risk is sometimes divided into two types of risks: diversifiable (also called unsystematic, specific, idiosyncratic or residual) risk and undiversifiable (also called systematic) risk. The terms diversifiable and undiversifiable come from the fact that the former can be reduced by holding a large and diversified portfolio, whereas the latter cannot [2].

In addition, alpha ($\alpha$) and beta ($\beta$) are two frequently used measures in the investment world. The $\beta$ measures a fund’s level of undiversifiable risk against the overall market (and hence the positive or negative returns the fund generates through its exposure to the market as a whole) [11]. Mathematically, $\beta$ can be expressed in the following way,

$$\beta = \frac{\text{Cov}(R, R_m)}{\text{Var}(R_m)}$$

(2.3)

where $R$ is the return of the individual asset and $R_m$ is the return of the market [22].

Moreover, $\alpha$ is a measure of returns that are uncorrelated to the market [4]. Thus, $\alpha$ measures the fund’s performance after allowing for $\beta$ and represents the part of a fund’s return that cannot be explained by responsiveness to the fluctuations of the general market. In other words, $\alpha$ is a measure of excess return (often relative to
a benchmark) after adjusting for systematic risk. Consequently, \( \alpha \) can be seen as a measure of diversifiable risk. Mathematically, there are various ways to compute \( \alpha \) \[11\]. One commonly used regression model for portfolio allocation results in \( \alpha \) being computed as follows,

\[
\alpha = R_t - \beta_1 X_{1t} + \ldots + \beta_k X_{kt} + \varepsilon_t \tag{2.4}
\]

where \( R \) is the return of the asset and \( X_k \) are the returns of the factors.

On a portfolio level, \( \alpha \) and \( \beta \) can be calculated as the weighted sum of each asset’s \( \alpha \) and \( \beta \), respectively. In consequence, choosing portfolio weights such that the portfolio \( \alpha \) and \( \beta \) fulfills the investor’s requirement for return and risk appetite is one of the most central parts of portfolio optimization \[1\].

### 2.1.4 Covariance and Correlation

Covariance and correlation are two measures of dependency between random variables. In the case of portfolio optimization, it is often the dependency between the returns of different portfolio assets that are considered. The covariance measure is determined by both the dependency between the random variables and the size of them. To give an example, monthly returns generally have greater covariance than daily returns of the same asset. In order to disregard the size and only focus on the dependency, correlation can be used instead. Covariance and correlation are closely related. In fact, an \( n \times n \) covariance matrix (measuring the covariance between \( n \) assets of a portfolio) contains the covariances between all asset returns, with the variances on the diagonal. Denoting this covariance matrix \( V \), it holds true that \( V = DCD \), where \( D \) is a diagonal matrix of of the standard deviations of assets returns and \( C \) is the correlation matrix \[1\].

Furthermore, the correlation coefficient takes values in the range of -1 to 1. A correlation coefficient of 1 indicates that the two funds have a perfect positive correlation, meaning that their returns move in perfect synchronization. Contrarily, a correlation coefficient of -1 means that the two funds have a perfect negative correlation. A portfolio with highly correlated assets is considered less diversified than a portfolio with uncorrelated assets.

If a portfolio is comprised of uncorrelated securities, with a correlation denoted by \( \sigma_{ij} = 0 \) for any combination of securities \( i \) and \( j \), the portfolio risk containing \( N \)
securities can be expressed as follows,

\[
\sigma_P^2 = \sum_{i=1}^{N} (w_i \sigma_i^2)
\]  

(2.5)

Assuming the portfolio weights are equally distributed among the \( N \) securities, \((w_1 = w_2 = \ldots = w_N = 1/N)\), the following expression shows that the portfolio risk is inversely proportional to the amounts of securities:

\[
\sigma_P^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{1}{N} \right)^2 \sigma_i^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\sigma_i^2}{N} \right) = \frac{1}{N} \bar{\sigma}^2
\]

(2.6)

On the other hand, assuming the capital available for investment can be distributed equally among the securities, the portfolio risk of a portfolio consisting of correlated securities with correlation \( \sigma_{ij} \neq 0 \) can be calculated as follows,

\[
\begin{align*}
\sigma_P^2 &= \sum_{i=1}^{N} \left( \frac{1}{N} \right)^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \left( \frac{1}{N} \right)^2 \sigma_{ij} \\
&= \frac{1}{N} \left( \sum_{i=1}^{N} \sigma_i^2 \right) + \frac{N-1}{N} \left( \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \frac{\sigma_{ij}}{N(N-1)} \right) \\
&= \frac{1}{N} \bar{\sigma}^2 + \frac{N-1}{N} \bar{\sigma}_{ij} \\
&= \frac{1}{N} \left( \bar{\sigma}_i^2 - \bar{\sigma}_{ij} \right) + \bar{\sigma}_{ij}
\end{align*}
\]

(2.7)

where \( \bar{\sigma}_{ij} \) represents the mean covariance of the portfolio’s securities returns. In result, when the number of securities in a portfolio becomes sufficiently large, the risk associated with each individual security can be effectively eliminated. Consequently, the variance of the portfolio is minimized, reaching a level that is equivalent to the mean variance of the returns of the portfolio’s securities [9].

### 2.1.5 Efficient Portfolios and Efficient Frontier

A portfolio is called efficient if it has a maximal expected return for a given upper bound of risk. Equivalently, a portfolio is efficient if it has a minimal level of risk for a given expected return. The efficient frontier is the part of the optimization curve (that portfolios trace) where the efficient portfolios are located [29].

The relationships described above between risk, return, efficient portfolios, efficient
frontier and global minimum portfolio is illustrated in 2.1.1 obtained from [1]

Moreover, the curvature of the efficient frontier is determined by the correlation of the efficient portfolios’ returns. Perfectly correlated returns result in a straight line between the two efficient portfolios. The convexity of the efficient frontier is then inversely proportional to the correlation between the efficient portfolios. Furthermore, every portfolio on the efficient frontier can be described as a linear combination of any other two efficient portfolios. Lastly, but perhaps most importantly, which of the efficient portfolios (amongst all the portfolios on the efficient frontier) that is chosen depends on the investors’ preferences, such as the risk appetite [1].

2.1.6 Single Objective Optimization

A single objective optimization problem is, as it sounds, an optimization problem with one objective function to minimize or maximize often subject to a number of constraints that has to be fulfilled. In order for the optimization problem to be a convex constrained optimization problem the objective function has to be convex (as described in section 2.1.2) and the constraints need to be either concave (i.e inequalities) or both concave and convex (i.e equalities). Mathematically, an arbitrary single objective optimization problem can be expressed in the following way,

\[
\begin{array}{l}
\text{minimize} f(x) \\
\text{subject to : } g(x) \geq 0 \\
h(x) = 0 \\
x \in X \subset \mathbb{R}^n
\end{array}
\]  

(2.8)
where $f(x)$ is the objective function, $g(x)$ are the inequality constraints and $h(x)$ are the equality constraints [18].

## 2.2 Risk Measures

The uncertainty in the future value of a portfolio is something that investors and risk managers always are cautious about. In order to make this uncertainty problem a bit more "tangible", several risk measures have evolved over the years, with the objective to quantify and forecast this risk in by summarizing and calculating the likelihood of deviations from a specific target or expected future value [2]. Volatility, standard deviation, covariance, correlation, $\alpha$ and $\beta$ are some of the most simple risk measures, but there are many more risk measures available. A number of further relevant and useful risk measures for investing and portfolio management are introduced in this section.

### 2.2.1 Value at Risk

Value at Risk (VaR) is a quantile risk metric measuring the probability of extreme negative losses. This is achieved by forecasting the lower tail (i.e the lower quantile) of the distribution function of the portfolio returns. In this case, a loss is the same as a negative excess return. The distribution can be both empirical or based on historical data, resulting in empirical VaR and historical VaR, respectively. Mathematically, the VaR can be calculated in the following way,

$$\text{VaR}_p(X) = F_{\mathcal{L}}^{-1}(1 - p)$$

(2.9)

where the loss $L$ is calculated as $L = -X/R_0$ with $R_0$ being the risk free rate, $F_{\mathcal{L}}^{-1}$ is the quantile function of $L$ and $p$ is the determined probability level for extreme losses.

Alternatively, in the empirical case, the VaR can be expressed as follows,

$$\hat{\text{Var}}(X) = L_{[np]+1,n}$$

(2.10)

where the loss $L$ is calculated as described above, $p$ is the determined probability level for extreme losses and $n$ is the number of data points of past losses [15].

When calculating VaR it is assumed that the current portfolio weights will remain
constant over the risk horizon, that is the time period the risk measure refers to, normally the time the investor is expected to hold the portfolio. In other words, VaR provides a size of the loss that, with high probability, will not be exceeded given that the portfolio is held static. It is worth noting that portfolio returns are random variable and thus a precise value for the loss cannot be computed, but a confidence level can be associated with every loss. Unless the confidence level is set by an external body, it depends on the risk appetite of the investor [3].

2.2.2 Conditional Value at Risk

One of the main criticisms of the VaR measure is that it is not a subadditive risk measure and the fact that it has the undesirable ability of hiding risk in the left tail at levels beyond \( p \). Consequently, it can be seen as a risk measure that does not favour diversification, one of the main principles of Modern Portfolio Theory (MPT). In result, there is another risk metric very similar to VaR but with subadditive properties called cVaR or Expected Shortfall (ES). In fact, cVaR is a coherent risk measure. Furthermore, cVaR is convex, meaning that all optimization problems with cVaR constraint are convex optimization problems. Mathematically, the cVaR can be calculated as follows,

\[
cVaR_p(X) = \frac{1}{p} \int_0^p \text{VaR}_u(X) \, du
\]

(2.11)

where \( p \) is the probability level for extreme losses that is set beforehand. Similarly to VaR, both historical and empirical cVaR can be computed [15].

2.2.3 Maximum Drawdown

An additional risk measure that is frequently used is maximum drawdown, which measures the largest cumulative loss within a given time period. In other words, it is the largest decline from peak to trough of the time series of a fund’s returns over a given period of time. This time period is sometimes denoted the drawdown period. More intuitively, the maximum drawdown can be seen as a worst case scenario for people who start their investment in the fund within the time period that the maximum drawdown calculation covers. Hence, the maximum drawdown risk measure focuses only on downside risk, that is the returns that perform worse than a target level [16].
2.2.4 Active Risk and Return

When evaluating the performance of a fund it is common to compare the fund’s return (the change in value over a specific time period expressed as a percentage of the present value) or risk with that of a benchmark. Hence, the fund’s risk or return relative the benchmark’s risk or return is more interesting to investigate than the fund’s risk or return in its own. This is called the active or relative risk (return) of the fund and is calculated as the difference between the portfolios risk (or return) and the benchmark’s risk (or return). Furthermore, active asset weights are the difference between that assets weight in the portfolio and its weight the benchmark. In a similar manner, relative alphas and betas can also be calculated [2].

2.2.5 Tracking Error

Tracking error can be seen as a risk measure of active returns since it is the standard deviation of active returns. Tracking errors that are calculated using time series of historical data (rather than a forecast) are called ex post tracking errors. Mathematically, the ex post tracking error can be expressed as follows,

\[ TE = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} (R_t - \bar{R})^2} \]  

where \( T \) is the number of active returns in the time series, \( R_t \) is the active return of the fund at time \( t \) and \( \bar{R} \) is the average active return of the sample. In result, it can be seen that the mean deviations of the active returns, but not the level of the active return itself, affects the tracking error. Consequently, the tracking error of an under- or overperforming fund is small when the variability of the fund’s returns is close to the variability of the benchmark’s return, even though the level of the fund’s returns is much lower (or higher) than the level of the benchmark. In other words, tracking error measures the risk of the relative returns. It does not measure the risk of the fund relative to the benchmark. It is important to take this into account and use the risk measure ex post tracking error carefully for actively managed funds where the goal is to outperform the benchmark [2].
2.2.6 Sharpe Ratio

The Sharpe ratio measures the return of an investment compared with its risk. It is based on the idea that assets without systematic risk must earn the risk free rate or benchmark rate. Likewise, excess returns over the risk free rate is proportional to the systematic risk \[ \frac{1}{\sqrt{1}}. \] Mathematically, the Sharpe ratio can be expressed as follows,

\[
S = \frac{\bar{d}}{\sigma_d}
\]  

(2.13)

where \( S \) is the Sharpe ratio, \( \bar{d} \) is the average of the differential return and \( \sigma_d \) is the standard deviation of the differential return. Moreover, the differential return is calculated as \( \tilde{d} = \tilde{r}_p - \tilde{r}_B \), where \( \tilde{r}_p \) is the random variable return of the portfolio and \( \tilde{r}_B \) the random variable return of the benchmark. One of the most commonly used benchmarks for this is the risk free rate \[ 29. \]

2.2.7 Sortino Ratio

Another ratio that is commonly used to measure risk is the Sortino ratio. It is similar to the Sharpe ratio, with the exception that the excess return of the portfolio over the minimum acceptable return, rather than over the risk free rate is used. Furthermore, the Sortino ratio considers the standard deviation of the downside risk of a portfolio, that is risks lower than a specific benchmark, rather than standard deviation as a risk measure. Consequently, the Sortino ratio is calculated accordingly,

\[
\text{SoR} = \frac{R_P - \text{MAR}}{DD_{\text{MAR}}}
\]  

(2.14)

where \( \text{SoR} \) is the Sortino ratio, \( R_P \) is the return of the portfolio, MAR the minimum acceptable return and \( DD_{\text{MAR}} \) the downside deviation of portfolio returns with respect to MAR \[ 21. \]
2.3 Portfolio Optimization Methods

2.3.1 Modern Portfolio Theory and Mean Variance Optimization

Modern Portfolio Theory (MPT) is a framework introduced by Harry Markowitz in 1952 when his article "Portfolio Selection" was published in *The Journal of Finance* [17]. The two main principles of MPT are as follows: an investor’s goal is to maximize expected return for any level of risk and risk can be reduced by diversification across different assets. Furthermore, it is assumed that investors are risk averse and thus prefers a portfolio with less risk for a given level of return [14].

Mean variance optimization is a mathematical optimization theory that balances risk, in the form of variance, and return, in the form portfolio expected return. In other words, MPT is based on the foundations laid out by mean variance optimization. When variance is seen as measure of risk of a portfolio, quadratic optimization models can be formed to construct efficient portfolios [29].

The maximization of expected return of a portfolio (without a risk free asset) whilst minimizing the variance can mathematically be expressed in the following way,

\[
\max_w \ w^T \mu - \frac{c}{2V_0} w^T \Sigma w \\
\text{s.t. } w^T \mathbf{1} \leq V_0
\]  

(2.15)

where \( w \) is the vector of the weights allocated to each asset in the portfolio, \( \mu \) is the vector of expected returns of the assets om the portfolio, \( \Sigma \) is the covariance matrix of asset returns and \( V_0 \) is the total amount that the investor has to spend [15].

The above is a very basic formulation of the problem with only one constraint that the investor cannot spend more in total on all assets than the amount he has to spend. Another common way of mathematically expressing mean variance optimization is the following,

\[
\min_w \frac{1}{2} w^T \Sigma w \\
\text{subject to } w^T \mu = p \\
\text{and } w^T \mathbf{1} = 1
\]  

(2.16)

where \( w \) is the vector of the weights allocated to each asset in the portfolio, \( \Sigma \) is the covariance matrix of asset returns, \( \mu \) is the vector of expected returns of the
assets on the portfolio and \( p \) is a determined level of portfolio return depending on the investor’s preferences and risk aversion. However, it is possible to add as many relevant constraints to this optimization problem as is suitable for the problem that is to be solved [22].

### 2.3.2 Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM) can be seen as a simplification of MPT that serves the following two purposes: measuring risk with regard to a single market index (not individual assets against each other as in MPT) and providing a handy formula for determining expected return for any type of security [6].

Mathematically, CAPM can be expressed as,

\[
E[R] = r_f + \beta (E[R_m] - r_f)
\]

where \( E[R] \) is the expected return of the security (i.e the portfolio), \( r_f \) is the risk free rate, \( E[R_m] \) is the expected return of the market and \( \beta \) is a measure of systematic risk of the assets computed as described in (2.3) [22].

### 2.4 Distribution Tests

#### 2.4.1 Quantile-Quantile Plots

A Quantile-Quantile plot (Q-Q plot) is a plot of the quantile function of a reference distribution against the quantile function of a data set. Consequently, it is an extremely useful tool for studying and determining the distributional properties of a data set. This method places extra focus on the tails of the distribution, that is the extreme values. By comparing whether or not the tails in the Q-Q plot seem to follow the straight line of the reference distribution it can be analyzed if the data set seems to follow the reference distribution or if the tails are heavier or lighter. Eventual skewness or outliers are also exposed. In other words, a Q-Q plot can be seen as a type of goodness of fit test. The data set can be compared to any type of reference distribution, such as normal distributions, log normal distributions, Students t-distributions or exponential distributions [15]. An illustration of the Q-Q plots characteristics can be found in 2.4.1 obtained from [12].
Figure 2.4.1: QQ-Plot types
2.4.2 Shapiro-Wilk Test

The Shapiro-Wilk test (S-W test) is a hypothesis test used to explore if the distribution of a continuous variable containing the sample data follows a normal distribution. Consequently, the null hypothesis used in the S-W test is that the sample comes from a normal distribution. To test if the null hypothesis can be rejected or not, the S-W test statistic is used. The S-W test statistic is expressed mathematically as follows,

\[
W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2}
\] (2.18)

where \(x_{(i)}\) are the sample values ordered by size, and \(a_i\) are constants calculated by

\[
(a_1, a_2, \ldots, a_n) = \frac{m^T V^{-1}}{(m^T V^{-1} m)^{1/2}}
\] (2.19)

where \(m = (m_1, m_2, \ldots, m_n)^T\) are the expected values of the independent and identically distributed order statistics following a standard normal distribution, and \(V\) is the covariance matrix of the order statistics [25]. Thereafter, in the same way as for any hypothesis test, the p value is calculated and the null hypothesis rejected if the p value is smaller than a given threshold number denoted \(\alpha\) [19].

2.4.3 Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov test (K-S test) is a statistical test used to decide if a sample comes from a population with a specific distribution, which is useful when determining if a sample is normally distributed or not [5]. One of the simplest formulas for this is,

\[
K - S \text{ test statistic} = \sup_x |F_a(x) - F_b(x)|
\] (2.20)

The test is based on the maximum distance between the cumulative distribution function \(F_a(x)\) and \(F_b(x)\), which is the empirical cumulative distribution function of a specific distribution. Figure 2.4.2, obtained from [23], plots this for 100 normal random numbers.
2.5 Performance Evaluation

Beyond its theoretical framework, it is imperative to subject an econometric model to real world evaluation. If an optimization model is used to derive optimal portfolio allocations the model has to be tested by forming portfolios according to its recommendations and by assessing the performance of these portfolios. Backtesting can be characterized as a practical simulation of a trading strategy, gauging its performance through historical data. During this assessment, a hypothetical scenario is assumed, where a point in the past is chosen and the model is estimated accordingly, taking subsequent historical data into account. The utmost caution must be exercised to prevent 'data snooping' or 'look-ahead bias', i.e. using data that is not supposed to be known at the time when the model was estimated [20].

The advantage of operational evaluation is that the model is being assessed in the actual context in which it will be used. The disadvantage is that the best model is likely to be specific to the application. By implementing the portfolio model across shorter time spans, one derives portfolio returns and associated weights for the designated segments. In academic, the testing periods and intervals used in the backtesting are commonly referred to as 'rolling windows'. The resultant portfolio’s return, which mirrors the intervals of backtesting, is subsequently computed by multiplying asset weights with the individual asset returns. Cumulative returns are then determined for both the overall portfolio and each specific strategy, facilitating a comparative analysis against outcomes obtained from the portfolio model [27].
2.6 Rebalancing

Deciding the optimal method for portfolio rebalancing is a frequently encountered and complex problem that holds significance due to its influence on returns and risk. Typically, managers resort to ad hoc rules of thumb, often centred around calendar events (like monthly or quarterly rebalancing) or driven by market volatility (triggered when asset ratios deviate by over 5% from target ratios) [8]. Additional approaches cover notional band rebalancing, range rebalancing, threshold based rebalancing, and tracking error band rebalancing. However, the specific choice of a rebalancing strategy bears relatively limited economic significance since it has been shown that all rebalancing strategies surpass a buy-and-hold approach in terms of key metrics such as Sharpe ratios and Sortino ratios. Historical simulation results bolster this observation, demonstrating the consistent superiority of rebalancing strategies across various trading intervals (yearly, quarterly, and monthly) and investment horizons (5 and 10 years) in comparison to a buy-and-hold approach [7]. Calendar based rebalancing may add value and minimize risk at the time rebalancing occurs, but due to its rule based nature, this approach might not consistently optimally manage risk and create value over an extended period [24].
Chapter 3

Literature Review

3.1 Mean Variance Optimization

The fact that MPT has revolutionized and dominated the financial world for decades means that it is often seen as the optimization tool most widely used in practice. As described in the book *Practical financial optimization: decision making for financial engineers* by Stavros, mean variance analysis could be seen as the starting point that sparked the invention for all optimization models for financial planning during the last 50 years [29]. In addition, Peris describes mean variance optimization as the reigning standards of the investment industry and claims in his book *Getting back to business: why modern portfolio theory fails investors and how you can bring common sense to your portfolio* that almost everyone in the investment management industry uses MPT in one way or another, although they might not always admit it [6].

Moreover, Stavros raises the point that while mean variance optimization might not be considered the most the most versatile tool to support enterprise-wide risk management, it is definitely one of the most used in practice and one that is often used as a starting point. However, the straightforward and simpleness of mean variance is not always seen as a negative aspect as Stavros points out that it is not necessarily true that the most complex model always generates the best results even though the real problem always is more complex than any model built [29].

One of the reasons to why mean variance optimization is an easy tool to apply is because it is essentially a classic quadratic programming model to which you can add any constraints that you need, such as budget and allocation restrictions [29].
A commonly mentioned drawback with mean variance analysis is the fact that it only studies a single period context \[29, 6, 15\]. Another drawback is the fact that most underlying models assumes that data sets are normally distributed. That is, it is assumed that the market is conventional with small and independent fluctuations in price that varies smoothly from one moment to the next. In other words, it is assumed that extreme events are rare, as can be seen in the classic bell shaped curve for normal distribution \[6\]. However, there are many portfolios with future values that are not well summarized by means and variances, making mean variance analysis an inappropriate optimization tool \[15\]. This is an important factor to consider, since if the assumption of normal distribution is wrongly enforced, the optimization model falls apart and might generate a portfolio allocation that is the opposite of optimal, in fact even misleading in the real world situation \[6\].

It is impossible to discuss MPT and mean variance analysis without mentioning the fact that it can be seen as outdated. As Peris puts it in his book, 'All things that rise eventually fall, and history books are filled with examples of high-belief systems and more mundane operational frameworks that came and went'. Consequently, it is worth considering that MPT and mean variance analysis might have had its peak period and is no longer as relevant as it used to be. There is no denying that the investment landscape in the 1950s, when Markowitz MPT was introduced, is completely different to the investment landscape that we have today. Back then, the thought of a portfolio, and not individual stocks, was completely new and diversification almost completely absent. However, this is not the case today and consequently, MPT sometimes leads to absurd levels of diversification at the expense of investors thinking about what they own in an absolute way and why it would be relevant in the real business, not just compared to some relative benchmark. Unfortunately, relative benchmark performance exercised through vast holdings in numerous style boxes, leading to investors holding a very large number of tiny stakes in businesses they might not even want to own, all for the sake of risk-reducing diversification at a theoretical level is not an uncommon thing \[6\].
Chapter 4

Methodology

4.1 Data

4.1.1 Fund Data Sets

In total, 14 stock based funds, named Fund A-N, managed by the company are considered for the FoF. The funds belong to one of the four fund companies Fund company A, Fund company B, Fund company C or Fund company D. More precisely, Fund A to Fund H belong to Fund company A, Fund I & Fund J belong to Fund company B, Fund K & Fund L belong to Fund company C and Fund M & Fund N belong to Fund company D. The Fund company A funds are traded in EUR. All Fund company C, Fund company B and Fund company D funds are traded in SEK. Most of the funds invest in small to medium sized companies.

The data set obtained contained Net Asset Value (NAV) for the different funds over various time periods depending on the age of the fund and/or how long they have been managed by the company. Daily NAV data was obtained for all the funds except for the Fund I that was only traded monthly until 2017-11-01. In the table below, the 14 stock based funds as well as the currency they are traded in and the starting point of the NAV data set that was obtained is presented.
4.1.1 Fund Raw Data Sets Information

<table>
<thead>
<tr>
<th>Fund Name</th>
<th>Currency</th>
<th>Start Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fund A</td>
<td>EUR</td>
<td>1997-09-10</td>
</tr>
<tr>
<td>Fund B</td>
<td>EUR</td>
<td>1998-09-14</td>
</tr>
<tr>
<td>Fund C</td>
<td>EUR</td>
<td>1997-04-07</td>
</tr>
<tr>
<td>Fund D</td>
<td>EUR</td>
<td>2006-08-29</td>
</tr>
<tr>
<td>Fund E</td>
<td>EUR</td>
<td>2011-05-19</td>
</tr>
<tr>
<td>Fund F</td>
<td>EUR</td>
<td>2014-04-01</td>
</tr>
<tr>
<td>Fund G</td>
<td>EUR</td>
<td>2017-03-07</td>
</tr>
<tr>
<td>Fund H</td>
<td>EUR</td>
<td>2018-06-14</td>
</tr>
<tr>
<td>Fund I</td>
<td>SEK</td>
<td>2013-01-31</td>
</tr>
<tr>
<td>Fund J</td>
<td>SEK</td>
<td>2022-12-29</td>
</tr>
<tr>
<td>Fund K</td>
<td>SEK</td>
<td>2009-11-30</td>
</tr>
<tr>
<td>Fund L</td>
<td>SEK</td>
<td>2014-04-01</td>
</tr>
<tr>
<td>Fund M</td>
<td>SEK</td>
<td>2019-02-04</td>
</tr>
<tr>
<td>Fund N</td>
<td>SEK</td>
<td>2020-06-12</td>
</tr>
</tbody>
</table>

4.1.2 Geography, Sector and Company Holdings Data for the Funds

In order to be able to measure the diversification and obtain relevant information about the FoF as well as fulfilling model requirements, information about the individual fund’s company holdings, geography and sectors needed to be obtained. The information about each fund's company holdings and the sizes in percent of each holding was obtained from the 2022 annual report for each fund. The number of companies a fund had holdings in varied between 24 and 61 for the different funds. Moreover, the industry sector and country that each company belonged to was retrieved. We analyzed eleven industry sectors as defined by the Global Industry Classification Standard (GICS): information technology, healthcare, industrials, materials, utilities, consumer discretionary, consumer staples, energy, real estate, communication services, and financials [13]. In total, eleven sectors and 24 countries are represented among the funds. When the country and sector for each company had been retrieved they were multiplied by the company's percentage of
the fund’s total holdings and summed in order to be able to calculate accumulated geography and sector holdings for each fund.

Lastly, the market cap in billion SEK was retrieved for each of the companies, so that the size of the fund could be determined in terms of micro cap, small cap, or large cap. Based on the market value of the Stockholm Stock Exchange being 10 000 billion SEK, small cap represents 1% of that value, and micro cap represents 0.1% of that value.

### 4.1.3 Benchmark Data Set

The SIX Portfolio Return Index (SIXPRX) is an index of the average return for stock based funds on the Stockholm Stock Exchange [26]. SIXPRX has been chosen by the company as the benchmark that the FoF should aim to outperform. It is a suitable index since the FoF that the company plans to launch will be a stock based FoF listed on the Stockholm Stock Exchange and primarily aimed at Swedish investors. Daily NAV data was obtained for SIXPRX from 1999-01-09.

### 4.2 Preprocessing Data

#### 4.2.1 Time period

The various time periods of the NAV data obtained, especially the very short ones, required some attention. The data set for Fund M only contained data from 2019-02-04. However, another similar 'outside' fund was managed by the same managers at the same company as Fund M during the time period 2014-05-02 to 2019-02-04. Assuming that the fund managers were managing these similar funds in the same way, and thus that the performance and index values of Fund M would have followed a similar pattern to that of the similar 'outside' fund during this period, data for this time period could be obtained in a pro forma manner for Fund M. Consequently, interpolation of NAV time series for the similar 'outside' fund (time period 2014-05-02 to 2019-02-04) and Fund M (time period 2019-02-04 onwards) was performed. The process is illustrated in figure 4.2.1.

Fund J fund was treated in a similar pro forma way to Fund M. This fund is very new, however it is managed by the same manager as a second 'outside' healthcare fund was from 2007 until recently. Consequently, daily NAV data for Fund J from 2007-11-30...
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(a) The similar ‘outside’ fund data

(b) Fund M data before interpolation

(c) Fund M data after interpolation

Figure 4.2.1: Comparison of Fund M data before and after interpolation

was obtained. On the contrary, the whole time series for Fund L was not relevant since only data from 2022-02-01 was representable for the fund today. These interpolations and cut in combination with the fact that Fund I only has been traded daily since 2017-11-01 resulted in start dates for daily NAV that could be used for optimization. These start dates are presented in table 4.2.1. The end date for the time series used was set to 2022-12-30 since this is the most recent ‘full calendar year’.

Lastly, all week end dates and Swedish holiday dates on week days when the Stockholm Stock Exchange was closed was omitted from the time series. In result, only relevant traded dates were included in the NAV time series and all missing data points were omitted since the only missing data points included in the data sets were on dates when the stock exchange had been closed.

Based on the preprocessed daily NAV time frames as well as how far back past performance can be considered relevant for indicating future performance, the decision was made to apply optimization methods based on historical data for the past ‘almost ten years’ (2014-04-01 to 2022-12-30) as well as the past ‘a little over five years’ (2017-11-01 to 2022-12-30). Consequently, nine and eleven funds, respectively,
Table 4.2.1: Individual start dates of the funds after preprocessing the data

<table>
<thead>
<tr>
<th>Fund Name</th>
<th>Start Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fund A</td>
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<tr>
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<td>2022-02-01</td>
</tr>
<tr>
<td>Fund M</td>
<td>2014-05-02</td>
</tr>
<tr>
<td>Fund N</td>
<td>2020-06-12</td>
</tr>
</tbody>
</table>

were included in the optimization model computations. Although the three funds with the shortest data were not included in the optimization framework they were still compared individually to SIXPRX in the same way as the rest of the funds in order to provide relevant information for when they might be considered for the FoF in the future when they are a bit older.

### 4.2.2 Currency Conversion

Since there was a mixture of funds traded in EUR and funds traded in SEK, a currency conversion was needed in order to obtain comparability among all the funds. Since the FoF will be traded in SEK, the NAV of the Fund company A funds (that are traded in EUR) were converted to SEK. The daily exchange rate for EUR to SEK was obtained from the European Central Bank for the time period 1999-01-04 and forward.
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4.2.3 Indexing

All NAV data of the funds were indexed in order to facilitate the benchmarking of fund performance. The index level was initialized to a base value of 100 at the starting point of the measurement period. Moreover, the index calculations incorporate an implicit assumption of reinvestment, where gains from one period are reinvested into the index in the subsequent period, ensuring the inclusion of any accumulated returns in the index computation. Time series of the index levels serve as a reliable tool for evaluating market performance during a selected time frame [10].

The cumulative index return over a specific time period expressed as a percentage was calculated using the following formula,

\[
\text{Cumulative index return} = \left( \frac{\text{Index level end of period}}{\text{Index level begin of period}} - 1 \right) \times 100
\]  
(4.1)

The annualized index return, taking into account the number of trading days in the period, was computed by the following formula,

\[
\text{Annualized index return} = \left( 1 + \text{Cumulative index return} \right)^{\frac{1}{\text{num trading days}}} - 1 \times 100
\]  
(4.2)

4.3 Portfolio Constraints

There were a number of portfolio constraints obtained by the company that needed to be applied to the FoF in order for it to be relevant for its purpose as well as to fulfill all laws and requirements. The FoF that the company plans to launch will be an Undertaking for Collective Investments In Transferable Securities (UCITS) FoF, meaning that it will be obliged to follow UCITS directives. A UCITS FoF cannot have a single fund holding larger than 20%. Consequently, this is a constraint of the optimization. Furthermore, since the FoF launched by the company will be aimed at Swedish investors and will strive to be well diversified across sectors there are constraints regarding this. These geography and sector constraints (together with the UCITS constraint) are as follows:

- maximum 20% in any fund
- maximum 70% in Sweden
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- maximum 40% in any other country
- minimum 40% in the Nordic countries
- maximum 30% in any sector

The Nordic countries are Sweden, Finland, Norway, Denmark and Iceland. Naturally, these constraints needed to be applied regardless of optimization method.

4.4 Optimization Framework

The first optimization framework used was mean variance optimization, expressed as maximum return for a given level of risk. The target risk level was set to the average annual risk of the benchmark SIXPRX for the studied time period, since the goal is to outperform this index. The second optimization framework used was min cVaR optimization, since finding the portfolio with minimal cVaR controls the size of the worst case scenarios for the FoF. Lastly, maximum \( \alpha \) was used for optimization in two different ways. Both in combination with tracking error as a risk measure and in combination with maximum drawdown as a risk measure. The maximization of \( \alpha \) was used for optimization since, as described in 2.1.3, \( \alpha \) is often seen as a measure of performance after allowing for systematic risk. The objective functions for these optimization models were \( f(x) = \lambda_{11}\alpha - \lambda_{21}\text{tracking error} \) and \( f(x) = \lambda_{12}\alpha - \lambda_{22}\text{maximum drawdown} \), respectively. The size of the coefficients \( \lambda_{11} \), \( \lambda_{21} \), \( \lambda_{12} \) and \( \lambda_{22} \) were determined taking the specific preferences for this particular FoF into consideration. Naturally, the \( \alpha \) coefficients were set to \(-1\) since the optimization was formulated as a minimization problem and the goal is to maximize alpha. Thereafter, \( \lambda_{21} \) and \( \lambda_{22} \) were set to 0.2 in order to gain a suitable trade off between the maximization of \( \alpha \) and the minimization of the risk measure. The aim was to give maximizing \( \alpha \) a stronger impact than minimizing tracking error or maximum drawdown, since this was more valuable for the company. The tracking error was calculated as described in equation 2.12 and the maximum drawdown according to the process described in subsection 2.2.3. All optimization frameworks used were applied both on the ten years data period (2014-04-01 to 2022-12-30) and on the five years data period (2017-11-01 to 2022-12-30).
4.5 Performance Evaluation

A backtesting method using rolling windows was implemented to the four optimization strategies. The rolling windows have a length of 252 days, equivalent to average number of trading days in a year. Consequently, rather than computing over the entire period from 2014-05-02 to 2022-12-30 once, the process is iterated with a total of 1920 rolling windows. This approach enables an assessment of how the strategy would have performed over different periods. Furthermore, it accounts for changes in market conditions by recalculating the optimal asset weights at each rolling window. The plotted cumulative returns provide insight into the historical performance of the portfolio strategy.

4.6 Calendar Based Rebalancing

A calendar based rebalancing approach was used, meaning periodically adjustments of the portfolio’s asset allocations taking all portfolio constraints into account. More specifically, quarterly rebalancing, i.e recalculating the optimal portfolio at the beginning of each quarter by adjusting the asset weights based on the current market conditions, was performed. Each iteration added a new quarter’s worth of data to the previous data used for optimization. This way, the models were updated and re-optimized using an expanding data window. The portfolio remains unchanged throughout each quarter. The intention, when implemented in a real world setting in the future when even more historical data is available, is not to gather data over an indefinite time period. As the data set expands, considerations regarding computational efficiency, data relevance, and changing market dynamics come into play. In other words, the largest data window used in the ten years scenario should stay at ten years and the largest data window used in the five years scenario should remain five years.
Chapter 5

Empirical Data

5.1 Overview of Historical Performance

The past performance of all funds, in terms of NAV returns, can be seen in figures 5.1.1 and 5.1.2. The former takes historical NAV values from 2014-05-02 until 2022-12-30 into account, henceforth referred to as the ten years scenario, whereas the latter takes historical values from 2017-11-01 until 2022-12-30 into account, hereafter referred to as the five years scenario. The NAV shown in 5.1.2 and 5.1.1 were all indexed as described in 4.1. The historical performance of SIXPRX is also included in both graphs, printed in red, for easy comparison of the funds historical performance relative to the benchmark that the FoF will strive to outperform. Evidently, there is a presence of both bullish phases (characterized by upward trends) and bearish phases in the market. Each fund’s past performance was also plotted individually against the SIXPRX benchmark from each fond’s individual start date. These plots can be found in appendix 7.6. Moreover, the annualized mean expected return of the historical NAV returns for both the five years period and ten years period are presented in table 5.1.1.

5.2 Correlation and Covariance

A correlation matrix of five days rolling daily NAV returns for all funds included in the five years version (and hence also in the ten years version) as well as SIXPRX is displayed in figure 5.2.1. It can be seen that the Fund company B funds have the
CHAPTER 5. EMPIRICAL DATA

Figure 5.1.1: Plot of 9 funds and SIXPRX benchmark indexed to 2014-05-02

Figure 5.1.2: Plot of 11 funds and SIXPRX benchmark indexed to 2017-11-01
Table 5.1.1: Annualized Mean Expected Return (AMER) for the five and ten years scenario

<table>
<thead>
<tr>
<th>Fund Name</th>
<th>AMER 5 years</th>
<th>AMER 10 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fund A</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>Fund B</td>
<td>0.05</td>
<td>0.10</td>
</tr>
<tr>
<td>Fund C</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>Fund D</td>
<td>0.05</td>
<td>0.09</td>
</tr>
<tr>
<td>Fund E</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>Fund F</td>
<td>0.02</td>
<td>0.07</td>
</tr>
<tr>
<td>Fund G</td>
<td>0.04</td>
<td>-</td>
</tr>
<tr>
<td>Fund I</td>
<td>0.08</td>
<td>-</td>
</tr>
<tr>
<td>Fund J</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>Fund K</td>
<td>0.15</td>
<td>0.16</td>
</tr>
<tr>
<td>Fund M</td>
<td>0.09</td>
<td>0.19</td>
</tr>
<tr>
<td>SIXPRX</td>
<td>0.09</td>
<td>0.10</td>
</tr>
</tbody>
</table>

lowest correlation both compared to the SIXPRX benchmark as well as to other funds. In contrast, all the Fund company A funds have high correlations toward each other and Fund K is the fund with highest correlation to the benchmark.

Furthermore, the covariance matrix of daily NAV returns of the funds contains only positive eigenvalues, in particular the eigenvalues presented in 5.1. Consequently, the covariance matrix is positive definite and thus invertible, which is crucial for the mean variance framework.

\[
\lambda = \begin{bmatrix}
0.208 \\
0.020 \\
0.016 \\
0.012 \\
0.007 \\
0.005 \\
0.003 \\
0.003 \\
0.003
\end{bmatrix}
\] (5.1)
Figure 5.2.1: Rolling correlation of daily NAV returns on a 5 years period
CHAPTER 5. EMPIRICAL DATA

5.3 Normal Distribution Tests

5.3.1 Shapiro-Wilk and Kolmogorov-Smirnov

The results from the statistical S-W test and K-S test are shown in table 5.3.1. The results show that the p-value is less than a significance level of 0.001, which gives substantial evidence to reject the null hypothesis, indicating that the data does not follow a normal distribution. While the S-W test statistics, which range between about 0.89 and 0.96 for most funds, indicate that the data is near a normal distribution although smaller deviations from a normal distribution (indicated by the value of 1) occurs, though perhaps quantitatively minor. The range of the K-S test statistics 0.48 to 0.51 indicates a moderate departure from the normal distribution.

<table>
<thead>
<tr>
<th>Fund Name</th>
<th>S-W Statistic</th>
<th>p</th>
<th>K-S Statistic</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fund A</td>
<td>.91</td>
<td>&lt; .001</td>
<td>.49</td>
<td>&lt; .001</td>
</tr>
<tr>
<td>Fund B</td>
<td>.91</td>
<td>&lt; .001</td>
<td>.48</td>
<td>&lt; .001</td>
</tr>
<tr>
<td>Fund C</td>
<td>.93</td>
<td>&lt; .001</td>
<td>.51</td>
<td>&lt; .001</td>
</tr>
<tr>
<td>Fund D</td>
<td>.89</td>
<td>&lt; .001</td>
<td>.48</td>
<td>&lt; .001</td>
</tr>
<tr>
<td>Fund E</td>
<td>.93</td>
<td>&lt; .001</td>
<td>.49</td>
<td>&lt; .001</td>
</tr>
<tr>
<td>Fund F</td>
<td>.94</td>
<td>&lt; .001</td>
<td>.49</td>
<td>&lt; .001</td>
</tr>
<tr>
<td>Fund J</td>
<td>.96</td>
<td>&lt; .001</td>
<td>.49</td>
<td>&lt; .001</td>
</tr>
<tr>
<td>Fund K</td>
<td>.91</td>
<td>&lt; .001</td>
<td>.49</td>
<td>&lt; .001</td>
</tr>
<tr>
<td>Fund M</td>
<td>.95</td>
<td>&lt; .001</td>
<td>.50</td>
<td>&lt; .001</td>
</tr>
<tr>
<td>SIXPRX Index</td>
<td>.94</td>
<td>&lt; .001</td>
<td>.49</td>
<td>&lt; .001</td>
</tr>
</tbody>
</table>

S-W: Shapiro-Wilk, K-S: Kolmogorov-Smirnov

5.3.2 Quantile-Quantile Plots

The Q-Q plots were constructed to compare the quantiles of daily (5.3.1) and monthly (5.3.2) logarithmized returns against the quantiles of a standard normal distribution. The plots show that the monthly log-transformed data aligns more closely with the normal distribution compared to the daily log-transformed data. These plots suggest that the monthly fund data exhibits a distribution that is more normally distributed than its daily counterpart since they are not as heavy tailed.
Figure 5.3.1: QQ-Plots of fund’s daily logarithmized returns against the theoretical normal quantile

Figure 5.3.2: QQ-Plots of fund’s logarithmized monthly returns against the theoretical normal quantile
CHAPTER 5. EMPIRICAL DATA

5.4 Geography and Sector Allocations

The country and GICS sectors distributions of the funds are presented in figures 5.4.1 and 5.4.2, respectively. The country codes used for geographic locations can be found in appendix table 7.4.1 if needed. The sum of a funds company holdings stated in the annual report do not always sum up to 1 (presumably since a small percentage is held in cash). Consequently, a small percentage of unspecified countries have been added to the graphs in these cases in order to reach a total of 100% making the pie charts appropriate to read. A colour code for the countries and sectors, respectively, are presented at the top of each figure.
CHAPTER 5. EMPIRICAL DATA

Figure 5.4.1: Geographical allocation of the funds
Figure 5.4.2: Sector allocation of the funds
Chapter 6

Results

6.1 Portfolio Allocations

This results section presents the results of the portfolio allocation weights, i.e the weights assigned to each fund in the FoF for the four different optimization frame works applied. Portfolio weights are presented for each optimization framework for both the ten years scenario and the five years scenario. It is worth remembering that the latter scenario has two more funds to chose from when allocating weights since these two funds have a track record five years back in time but not ten years.

6.1.1 Mean Variance Optimization

This subsection presents the results from the mean variance optimization framework, formulated as maximum return for a given level of volatility. In the five years scenario the volatility level was set to 19.8% since this was the average annual volatility of the benchmark SIXPRX for this time period. In the ten years scenario the volatility was set to 18.3% for the same reason. Figure 6.1.1 shows the fund weights for the mean variance optimization framework applied to the five years scenario. It can be seen that Fund C, Fund E, Fund M and Fund K all hit the maximum weight possible, which is 20%. On the contrary, Fund A, Fund B, Fund F, Fund G and Fund I were not included in the FoF at all.

The portfolio weights result of the mean variance optimization on the ten years old time series data set are presented in figure 6.1.2. Interestingly, the FoF for the ten years scenario obtains the same allocation weights as for the five years scenario.
CHAPTER 6. RESULTS

Figure 6.1.1: FoF weights Mean Variance (5 years, daily data)

Figure 6.1.2: FoF weights Mean Variance (10 years, daily data)
CHAPTER 6. RESULTS

For the mean variance model, the portfolio optimization was also performed on monthly data. The results were identical to their respective daily data results presented in 6.1.1 and 6.1.2.

6.1.2 Min Conditional Value at Risk Optimization

This subsection presents the results from the min cVaR optimization framework. In both the five years scenario and the ten years scenario a 95% cVaR level was considered. Figure 6.1.3 shows the allocated portfolio weights for the five years scenario. The maximum possible fund holding of 20% was assigned to Fund D, Fund C, Fund G and Fund E. The min cVaR value obtained for the FoF was 2.45%.

![Figure 6.1.3: FoF weights Min cVaR (5 years, daily data)](image)

The results for the ten years scenario are shown in figure 6.1.4. Notable differences from the five years scenario are that the weight of Fund J increases up to the maximum possible weight and that Fund B as well as Fund K now have substantial holdings in the FoF. The min cVaR value obtained for the FoF was 2.29%.

6.1.3 Alpha Tracking Error Optimization

This subsection presents the results from the model maximizing alpha while also taking tracking error into account. The fund weights for the five years scenario is presented in figure 6.1.5. Interestingly, this optimization framework allocated weights similar to those of the maximal return given SIXPRX level of volatility optimization. The
annual tracking error for the FoF was 15.3%. The results for the ten years scenario are presented in figure 6.1.6, notable is that the weights for the ten years scenario are identical to those of the five years scenario. The annual tracking error for the FoF was 14.2%.

6.1.4 Alpha Maximum Drawdown Optimization

This subsection presents the results optimization framework maximizing alpha while also taking maximum drawdown into account. The fund weights for the scenario
based on data from five years back was used is presented in figure 6.1.5 and results for ten years back are presented in 6.1.8. Again, the allocation weights turned out to be similar for both time scenarios as well as to some of the results obtained by the other optimization models.
**6.2 Overview**

An overview of the portfolio weights allocation results when applying the four different optimization frameworks are displayed in table 6.2.1 for the five years scenario and 6.2.2 for the ten years scenario. The right most column (num) in these tables indicates in how many of the four optimizations each fund was included in the FoF (i.e was assigned a weight larger than zero).

<table>
<thead>
<tr>
<th>Five years</th>
<th>MeanVar</th>
<th>MinVaR</th>
<th>Alpha TE</th>
<th>Alpha MDD</th>
<th>num</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fund A</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>Fund B</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>Fund C</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>4</td>
</tr>
<tr>
<td>Fund D</td>
<td>0.02</td>
<td>0.20</td>
<td>0.02</td>
<td>0.02</td>
<td>4</td>
</tr>
<tr>
<td>Fund E</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>4</td>
</tr>
<tr>
<td>Fund F</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>Fund G</td>
<td>0.00</td>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
<td>1</td>
</tr>
<tr>
<td>Fund I</td>
<td>0.00</td>
<td>0.06</td>
<td>0.00</td>
<td>0.00</td>
<td>1</td>
</tr>
<tr>
<td>Fund J</td>
<td>0.18</td>
<td>0.13</td>
<td>0.18</td>
<td>0.18</td>
<td>4</td>
</tr>
<tr>
<td>Fund K</td>
<td>0.20</td>
<td>0.00</td>
<td>0.20</td>
<td>0.20</td>
<td>3</td>
</tr>
<tr>
<td>Fund M</td>
<td>0.20</td>
<td>0.00</td>
<td>0.20</td>
<td>0.20</td>
<td>3</td>
</tr>
</tbody>
</table>
Moreover, a presentation of expected annual return, average volatility as well as average Sharpe ratio calculated for the FoF optimizations are shown in table 6.2.3.

Table 6.2.3: Performance Metrics for Different Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Expected Annual Return</th>
<th>Average Volatility</th>
<th>Average Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Variance</td>
<td>0.14</td>
<td>0.15</td>
<td>1.43</td>
</tr>
<tr>
<td>Min cVaR</td>
<td>0.09</td>
<td>0.15</td>
<td>0.90</td>
</tr>
<tr>
<td>Alpha Tracking Error</td>
<td>0.12</td>
<td>0.14</td>
<td>1.28</td>
</tr>
<tr>
<td>Alpha Maximum Drawdown</td>
<td>0.12</td>
<td>0.14</td>
<td>1.28</td>
</tr>
<tr>
<td>SIXPRX index</td>
<td>0.11</td>
<td>0.18</td>
<td>0.62</td>
</tr>
</tbody>
</table>

### 6.3 Geography, Sectors and Company Holdings

The geography as well as sector distributions for the FoF were calculated for all optimization frameworks applied and the results are presented in figures 6.3.1 and 6.3.2 respectively. The two digits country codes used are explained in appendix 7.4.1. A colour code for the countries and sectors, respectively, are presented at the top of each figure. Since no substantial differences in distributions were observed between the ten years and five years scenario, pie charts are only shown for the five years scenario in order to avoid repetitions. The number of companies the FoF had holdings in varied between 222 and 299 companies.
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Figure 6.3.1: FoF Geography 5 years
CHAPTER 6. RESULTS

(a) Mean Variance

(b) Min cVaR

(c) Alpha Tracking Error

(d) Alpha Maximum Drawdown

Figure 6.3.2: FoF Sectors 5 years
6.4 Backtesting

The plotted results of the backtesting of the models over the same rolling windows are shown in figure 6.4.1. Furthermore, the results of calculations of expected annual return for each model in the backtesting were the following:

- Model 1 (Mean Variance Optimization) had an expected annual return of 13.57%.
- Model 2 (Min cVaR) had an expected annual return of 8.56%.
- Model 3 (Alpha Tracking Error) had an expected annual return of 12.49%.
- Model 4 (Alpha Maximum Drawdown) had an expected annual return of 12.49%.
- The benchmark SIXPRX had an expected annual return of 11.31%.

![Portfolio Cumulative Returns over Time](image)

Figure 6.4.1: Backtesting of four models, plot with SIXPRX index

6.5 Efficient Frontier

The efficient frontiers for the mean variance optimization are presented in figure 6.5.2 for the five years scenario and figure 6.5.1 for the ten years scenario. Looking at the start of the efficient frontier on the left in 6.5.2 and 6.5.1 respectively, it is evident that the five years scenario exhibits a higher risk and yields a lesser return per unit of risk taken. The point of the portfolio with the maximum Sharpe ratio is also
CHAPTER 6. RESULTS

differently positioned in the two plots, situated at a higher volatility for the five years scenario.

Figure 6.5.1: Efficient frontier of the mean variance optimization (10 years, daily data)

Figure 6.5.2: Efficient frontier of the mean variance optimization (5 years, daily data)

6.6 Portfolio Risk Contribution

The risk contribution of the different funds were analyzed for the four different optimization models, generating the risk contribution per asset stated in terms of
shares. The results of this analysis for all optimization models in both time scenarios can be found in appendix 7.5.
Chapter 7

Discussion

7.1 Analysis of Results

7.1.1 The Number of Times a Fund is Included in the Fund of Funds

Studying the overview tables 6.2.1 and 6.2.2 it can be seen that Fund C, Fund D and Fund J are the three funds that are included in all FoF regardless of optimization framework applied or historical time period. On the contrary, Fund A is the only fund that is never included in the FoF regardless of optimization framework and time period. However, Fund F and Fund I are only included once each with the very small weights of 0.01 and 0.06, respectively. Another vital observation is that, while not included in all FoF possible, Fund M, Fund K and Fund E only miss out on one possible FoF. In addition, taking the results of the backtesting presented in 6.4.1 into account, the min cvaR optimization model seems to be the one that performs the least well and this is the only model (and only in the five years scenario) that does not include Fund M and Fund K in the FoF.

Furthermore, it is worth taking into consideration that the five years scenario includes two more funds than the ten years scenario. This means that differences in portfolio allocation compared to the equivalent model for the ten years period might not only be because a fund has performed better or worse over the past five years compared to the five years before that, the reason for the differences can also be simply because one of the two ‘new choices’ in the five years scenario were better. It is also worth
noting that in many cases there are several similarities between the five years scenario and the ten years scenario. Like with everything when it comes to investing, whether a ten years scenario, five years scenario or a combination of both as in this project is deemed suitable for historical data to still be relevant should be adapted to the specific situation.

### 7.1.2 Daily and Monthly NAV Returns

In general, daily NAV returns were used throughout this project in order to be able to obtain a more detailed picture when appropriate and calculating annualized values from the daily data when this was more suitable. However, one of the cases where there might have been better to use monthly data than daily data were for the mean variance optimization framework. Studying the daily Q-Q plots in 5.3.1 and monthly Q-Q plots in 5.3.2 it can be seen that the monthly returns are more closely following a normal distribution than the daily returns. While the daily returns are not very far from it with only some heavy tales, the monthly returns are unquestionably closer. The mean variance optimization model is the only optimization model out of the four used in this study that assumes that data sets are normally distributed. Consequently, after this framework had been applied to daily data a version with monthly data were performed. Since the two versions obtained the same results, only daily data was used for the other models since their results would not have to be compared to results obtained from monthly data in the mean variance model.

### 7.1.3 Geography and Sector Constraints

Looking at figure 6.3.2, it is evident that the healthcare sector hits the maximum possible sector allocation of 30% for all FoF. Consequently, if the sector constraint was released the portfolio allocation could have looked very different. One interesting example of this can be seen when studying the individual funds’ performance against the SIXPRX benchmark included in appendix 7.6. In particular, Fund B outperforms the benchmark more than Fund D. However, Fund D is included more in the FoF than Fund B. Looking at figure 5.4.1 one can observed that Fund B have a significantly larger part of its holdings within the healthcare sector than Fund D. Thus, taking other things into consideration than outperformance, such as portfolio constraints, a fund with slightly lower performance was deemed more suitable for the FoF in this case.
Another clear example of when the sector constraint for healthcare affects the result is the fact that Fund I is not included often even though it performs well compared to benchmark. The presence of another 100% healthcare fund turns this part of the weight allocation more into a question of Fund I or Fund J.

Furthermore, in figure 6.3.1, the geographical allocation is not as clearly dominated by a single country as it is in the sector case with healthcare, since no country reaches its maximum allocation. However, Finland is fairly heavily represented taking into consideration that this FoF is mainly aimed for Swedish investors. This is probably due to the frequent inclusion of Fund C in the FoF. In addition, there is indeed a constraint of minimum 40% in the Nordic countries in order to keep a Nordic profile.

7.1.4 Risk Profile

When examining the risk profiles, an interesting observation emerges. By allowing the mean variance optimization to reach the volatility of SIXPRX (18.3%), the optimization process still gravitates towards a lower volatility for the FoF. This suggests that there may not be significant advantages in embracing higher risk levels. The efficient frontier in figure 6.5.1 further corroborates this finding: the portfolio with the maximum Sharpe ratio is not situated above an 18.3% volatility but hovers around 15.4%. For a comprehensive perspective, the index SIXPRX would have yielded an expected annual return of 12% for a volatility of 18.3%. In contrast, the mean variance optimization anticipates an annual return of 13.5% but at a notably reduced volatility of 15.4%.

7.1.5 Fund of Funds or Individual Funds

It is worth analyzing what the advantages of a FoF in this case are compared to the individual funds. In other words, why invest in the FoF rather than some of the individual funds directly. By observing the annualized mean expected returns of the individual funds in 5.1.1 and for the FoF in 6.2.3 it can be noted that all optimization frameworks (except min cVaR which also performs the worst in the backtesting), while having higher expected return than the benchmark, also has a higher expected return than all funds except three (Fund K, Fund M and Fund J). Hence, in terms of expected return, only three of the individual funds might even be argued to be a better investing choice than the FoF. It is worth noting that these funds have indeed maximum weights in the FoF and there are many other diversification and expertise advantages to the FoF.
that cannot be obtained by the individual funds. One clear measure of diversification, 
apart from the factors already mentioned, is the number of company holdings in the 
FoF compared to the individual funds. The individual funds each have holdings in 
between 24 and 61 companies while the FoF have holdings in between 222 and 299 
companies.

7.2 Conclusions

In conclusion, there are several funds that have performed well and the majority 
of the funds considered can be argued suitable candidates to include in the FoF. 
Taking all factors most important in this particular case in to account, Fund J, Fund 
M, Fund K and Fund E are funds that are all very favourable. On a more general 
scale, looking at the performance of the different optimization frameworks it can be 
seen that traditional mean variance optimization when putting the benchmark risk as 
target volatility can be seen as an effective model. In addition, optimization methods 
considering maximization of alpha in combination with risk measures tracking error 
and maximum drawdown are suitable for this type of FoF allocation mainly focusing 
on smaller companies on the Nordic market.

7.3 Future Work

In our analysis, we have mentioned various rebalancing models as highlighted in the 
thoretical framework. For future research, it would be wise to delve deeper into 
these models and evaluate their efficacy in different market scenarios. Moreover, it is 
suggested to incorporate trigger strategies into the rebalancing process. Such methods 
are attractive because they are outcomes of an optimization problem, meaning that 
they minimize the possibly unnecessary (and redundant) costs of rebalancing on a 
purely calendar basis.

While the optimization frameworks have only been tested on the eleven funds included 
in this project. It would be interesting to further investigate if the results of how 
the optimization frameworks perform would be similar both when testing on funds 
with similar characteristics to those in this project as well as other types of funds, for 
example with a focus in other geographical areas or expanding into other types of asset 
classes than exclusively stock-based funds.
## Appendix

### 7.4 Appendix A: Country Codes

Table 7.4.1: Country Codes

<table>
<thead>
<tr>
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7.5 Appendix B: Risk Contributions

7.5.1 Mean Variance Optimization

Figure 7.5.1: Risk Contribution per Asset of the Mean Variance Optimization Portfolio (5 years data)

Figure 7.5.2: Risk Contribution of the funds in the Mean Variance Optimization Portfolio (10 years data)
7.5.2 Min Conditional Value at Risk Optimization

![Figure 7.5.3: Risk Contribution Per Asset for Min cVaR Optimization (5 years data)](image)

![Figure 7.5.4: Risk Contribution Per Asset for the Min cVaR Optimization (10 years data)](image)
7.5.3 Alpha Tracking Error Optimization

Figure 7.5.5: Risk Contribution per Asset for the Max Alpha - Tracking Error Optimization (5 years data)

Figure 7.5.6: Risk Contribution per Asset for the Max Alpha - Tracking Error Optimization (10 years data)
7.5.4 Alpha Maximum Drawdown Optimization

Figure 7.5.7: Risk Contribution per Asset for the Max Alpha - Maximum Drawdown Optimization (5 years data)

Figure 7.5.8: Risk Contribution per Asset for the Max Alpha - Maximum Drawdown Optimization (10 years data)
7.6 Appendix C: Separate plots

Figure 7.6.1: Separate plot of Fund A and SIXPRX index

Figure 7.6.2: Separate plot of Fund B and SIXPRX index
Figure 7.6.3: Separate plot of Fund C and SIXPRX index

Figure 7.6.4: Separate plot of Fund D and SIXPRX index
Figure 7.6.5: Separate plot of Fund E and SIXPRX index

Figure 7.6.6: Separate plot of Fund F and SIXPRX index
Figure 7.6.7: Separate plot of Fund G and SIXPRX index

Figure 7.6.8: Separate plot of Fund H and SIXPRX index
Figure 7.6.9: Separate plot of Fund I (monthly NAV data) and SIXPRX index

Figure 7.6.10: Separate plot of Fund J and SIXPRX index
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Figure 7.6.11: Separate plot of Fund K and SIXPRX index

Figure 7.6.12: Separate plot of Fund L and SIXPRX index
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Figure 7.6.13: Separate plot of Fund M and SIXPRX index

Figure 7.6.14: Separate plot of Fund N and SIXPRX index
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