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Investigation of the Applicability of Fracture Mechanics for Tissue Paper

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Abstract

Tissue paper is a common type of paper material and is used in a variety of products. For tissue paper, several properties are of interest, such as absorbency, softness, bulk and mechanical properties. Embossing is an operation used to apply a pattern on tissue paper. It is used to improve several properties, but is known to reduce mechanical properties. Currently, no models can predict the loss of strength due to embossing. In this report base tissue paper is embossed with two different embossing patterns and tensile tests are conducted with and without edge notches. The edge notch length was varied between 0 mm to 12 mm. From the experiment, a modified Linear Elastic Fracture Mechanics model was applied on both base tissue paper and embossed tissue paper tensile test results. The experimental procedure is described. In total, four different paper qualities were tested. Two that are designed for toilet paper and two that are designed for kitchen paper. The tissue sheets were embossed using 3D-printed plates and conducted in a laboratory environment. Tensile tests with edge-notch specimens were performed. The notch lengths tested were between 0 mm and 12 mm long. It was investigated if any trends of the parameters in the model could be noticed due to embossing. The model worked well for all base tissue qualities. The embossing reduces the material’s tensile strength compared to the base material. With increasing embossing load, longer notches are needed to drop the tensile strength of the specimen. Some general trends were noted. However, the impact of the embossing was different for different paper qualities and the embossing pattern used. The most significant difference between plates was noted in specimens with high embossing load. With increasing embossing load, the edge-notch must also be longer to reduce tensile strength. The model parameters changed more for machine direction (MD) specimens than crossmachine direction (CD) specimens.

Keywords

Tissue Paper, Fracture Mechanics, Embossing
Sammanfattning


Nyckelord

Mjukpapper, Brottmechanik, Prägling
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Stockholm, Sweden, September 2023
Albin Boestad
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<th>Abbreviation</th>
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<td>Crossmachine Direction</td>
</tr>
<tr>
<td>CNT</td>
<td>Central Notch Test</td>
</tr>
<tr>
<td>DCT</td>
<td>Dry Crepe Tissue</td>
</tr>
<tr>
<td>DENT</td>
<td>Double Edge Notch Test</td>
</tr>
<tr>
<td>FPZ</td>
<td>Fracture Process Zone</td>
</tr>
<tr>
<td>LDP</td>
<td>Low Density Paper</td>
</tr>
<tr>
<td>LEFM</td>
<td>Linear Elastic Fracture Mechanics</td>
</tr>
<tr>
<td>MD</td>
<td>Machine Direction</td>
</tr>
<tr>
<td>SENT</td>
<td>Single Edge Notch Test</td>
</tr>
<tr>
<td>TAD</td>
<td>Through Air Dried</td>
</tr>
<tr>
<td>UTM</td>
<td>Universal Testing Machine</td>
</tr>
</tbody>
</table>
List of acronyms and abbreviations
Chapter 1
Introduction

Paper materials are useful and versatile materials whose use is constantly explored and improved. A common daily paper product is tissue paper. Tissue paper generally describes several products, such as toilet paper and kitchen towels. For a tissue paper, there are several properties that are important for the producer as well as the consumer. Such properties are the material’s absorbency capabilities, softness, and mechanical properties. The type of fibers and the operations used in the tissue manufacture depends on the end product. For instance, in a kitchen towel, the wet strength of the tissue is a key property of a functional kitchen towel. In contrast, for toilet paper, the softness of the tissue is important for the consumer. For tissue paper production, a base paper is first produced. Depending on the end product, the base paper undergoes converting operations to obtain the final tissue product. An important converting operation for tissue paper is called embossing. Embossing is the operation of producing a pattern on the tissue sheet. The embossing operation can improve desired properties such as the tissue paper’s absorbency, bulk, softness and porosity. However, mechanical properties such as tensile strength diminish with embossing. Several studies regarding properties to improve tissue paper absorbency or softness exist [1] and [2]. For tissue paper, just a few attempts have been made to apply fracture mechanics theory to paper to capture the material’s strength properties. No attempts have been made to see if fracture mechanics could capture the loss in mechanical properties due to embossing. Applying a linear elastic fracture mechanic (LEFM) theory to paper products is ill-advised since paper as a material does not meet the criteria to act as a linear elastic material, but LEFM is the simplest fracture theory to apply and should be explored. Few current studies focus on how much the tensile strength diminishes with embossing.
1.1 Aim of the thesis

The objectives of this report were to answer the following questions:

• Does a modified LEFM model work on different base tissue papers?
• If it works on base paper does it work on embossed tissue paper?
• Does the parameters in the modified model develop systematically with increased damage introduced by embossing?
• If they do, is it a global or local behavior?
Chapter 2

Background and previous work

In this chapter, some background of tissue paper and previous work is presented.

2.1 Introduction to tissue paper material

Tissue paper is a name for low-density paper (LDP). It is commonly used as toilet paper or household paper. Some key properties of tissue paper are absorbency, softness, bulk and flexibility. There are two standard procedures for creating the base paper for tissue paper. The first method is dry crepe tissue (DCT). For DCT materials, the paper sheet is dewatered by pressing it between rolls and then dried thermally with a Yankee cylinder. The Yankee cylinder is a rotating steam-heated large cylinder. On the Yankee cylinder’s surface, adhesives let the paper web attach to the cylinder. The tissue will dry during the rotation of the cylinder and is then debonded from the cylinder with a blade \[3\]. This final step of the process is called creping.

The other method is through air drying (TAD) tissue. Dewatering and drying are done without compression for the TAD materials. In the forming section of the paper machine, vacuum boxes suck water away from the sheet. After forming, the material is transferred to a fabric through which heated air is blasted. The material is then transferred to the Yankee cylinder to be creped similar to the DCT procedure \[4\].

In the process of making a tissue paper product, the final process before having a final tissue product is called converting. The converting process consists of a series of operations: unwinding, embossing, printing, perforating, rewinding, cutting and packaging. The operations used in the converting process are dependent on the final product. The embossing of the material is an operation
where a pattern is added to the tissue paper. It is generally done by pressing two engraved rolls against each other or by an engraved roll against an elastic surface through a nip, see Figure 2.1.

![Figure 2.1: Conventional embossing setup. Figure taken from [2].](image)

The embossing process is used to improve several properties of the tissue paper such as porosity, thickness and absorbency [5]. Embossing the top layer of a tissue paper is called decorative embossing or deco-embossing. Embossing on the bottom layer is called micro-embossing [6]. Deco-embossing lowers the absorbancy of the tissue paper, while micro-embossing increases absorbancy [1]. The embossing pressure has a significant influence on the embossing result. Too high of a pressure will destroy all the strength properties of the paper. Micro-embossing has a higher impact on thickness, absorbency and mechanical properties compared to deco-embossing [5].

### 2.2 Fracture mechanics

Fracture mechanics is a field of structural mechanics focused on characterizing fracture behavior in materials containing crack-like defects. Fracture mechanics is broadly divided into two theories: Linear and nonlinear fracture mechanics [7]. Linear fracture mechanics theory (LEFM) is the most commonly and simple in-plane fracture mechanics model. LEFM assumes a
linear elastic behavior of the material. LEFM can generally not be applied to paper material unless the nonlinear behavior, such as damage and plasticity, is confined to a small region ahead of the crack tip. This region is usually called a failure process zone (FPZ). If the FPZ is small compared to the characteristic in-plane dimensions of the specimen, Linear Elastic Fracture Mechanics (LEFM) is applicable. With LEFM, the stress and strain field close to the crack tip is characterized by one parameter, the stress intensity factor $K$. For the fracture mechanics of paper, much effort has been spent to test different fracture mechanics concepts applied to paper. These concepts include linear fracture mechanics, J-integral of nonlinear fracture mechanics, crack tip opening displacement and cohesive zone models. For a thorough review of the current state of fracture mechanics of paper the reader is referred to [8].

### 2.3 Fracture behavior determined by anti-floc regions

Previous work by Krasnoshlyk et al. [9] suggested that anti-flocs, regions within a tissue material where the local grammage is lower than the sample’s average grammage, see Figure 2.2, govern the material’s fracture behavior. Krasnoshlyk et al. saw that for a LDP (Low Density Paper), the global fracture force remains the same for small notches. They tested using a TAD tissue paper and suggested a fracture relation between a subscale internal length scale $c$ and the mesoscale fracture behavior. Krasnoshlyk et al. suggested from her experiments that this subscale length $c$ was of the same size as the dominant anti-floc dimensions of the material for low-density paper [9]. She also suggested a relation between the critical stress intensity factor and the internal length $c$,

$$K_{IC} = \frac{\sigma_x \sqrt{c}}{0.3},$$

(2.1)

where $K_{IC}$ is the critical stress intensity factor, $\sigma_x$ is the stress acting perpendicular to the notch. The classical macroscopic expression for fracture toughness for an edge-notched specimen is given by

$$K_{IC} = 1.12\sigma_x \sqrt{\pi a_c}.$$  

(2.2)

In this Equation, $a_c$ is the critical crack length of the edge-notch. Using the
Figure 2.2: Visualisation of the antifloc regions in a tissue paper, the yellow regions are antifloc regions. Images taken from [9].

Equation suggested by Krasnoshlyk et al., the relation

\[ c = 0.113a_c \]  \hspace{1cm} (2.3)

is obtained between \( c \) and \( a_c \).
2.4 Reproduction of Krasnoshlyk et al. work on base tissue paper

In an ongoing project at RISE (yet to be published), attempts have been made to use the method in the work of Krashnolyk et al. [9] for other tissue paper qualities. The attempts were made on base tissue papers of six types. A DCT-Toilet, DCT-Kitchen, TAD-Toilet, TAD-Kitchen, Hybrid-Toilet and Hybrid-Kitchen. The two hybrid materials are made with neither DCT or TAD procedure but instead, a combination. In this production method, the sheet is dewatered through compression and then transferred to a structured fabric which builds up the structure of the sheet [10]. They could not reproduce the result by following the methodology presented by Krashnolyk et al. [9]. In the investigation of the six different tissue papers, none of the calculated subscale lengths corresponded to the anti-floc lengths.

Table 2.1: Results from attempting to reproduce data from Krashnolyk et al. [9] for different base tissue papers.

<table>
<thead>
<tr>
<th>Material</th>
<th>$a_{MD}$</th>
<th>$a_{CD}$</th>
<th>$c_{MD}$</th>
<th>$c_{CD}$</th>
<th>$c_{image}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCT-T</td>
<td>2</td>
<td>5</td>
<td>0.7</td>
<td>1.8</td>
<td>2.3</td>
</tr>
<tr>
<td>DCT-K</td>
<td>3</td>
<td>7</td>
<td>1.1</td>
<td>2.5</td>
<td>2.7</td>
</tr>
<tr>
<td>TAD-T</td>
<td>2</td>
<td>8</td>
<td>0.7</td>
<td>2.8</td>
<td>2.3</td>
</tr>
<tr>
<td>TAD-K</td>
<td>2</td>
<td>6</td>
<td>9.7</td>
<td>2.1</td>
<td>2.4</td>
</tr>
<tr>
<td>Hybrid-T</td>
<td>4</td>
<td>5</td>
<td>1.4</td>
<td>1.8</td>
<td>2.8</td>
</tr>
<tr>
<td>Hybrid-K</td>
<td>4</td>
<td>4</td>
<td>1.4</td>
<td>1.4</td>
<td>3.0</td>
</tr>
</tbody>
</table>

2.5 Modified Linear Elastic Fracture Mechanics for paper material

Coffin et al. [11] suggests a simpler approach to capture the fracture behavior of paper materials. They suggest that the modification is sufficient to model the fracture resistance for different papers. In the model suggested by Coffin et al., two parameters $d$ and $d_s$ are fitted for a material. Here, $d$ is the crack length an isotropic homogeneous material must have to correspond to an unnotched heterogeneous paper sample. The other parameter $d_s$ is a threshold parameter for which a crack in the material starts to affect the strength. They successfully applied the model to many different paper materials, including tissue paper.
However, they did not compare base tissue paper with embossed tissue paper. They also successfully applied the model for different sizes of specimens than the standard specimens for tensile tests [11]. The model was successful for larger specimens as well as smaller specimens. However, smaller specimens were only applicable if the $d$ is also small.
Chapter 3

Theory

The following chapter is dedicated to introducing the fundamental Linear Fracture Mechanic theory and presenting the modification Coffin suggested for paper materials.

3.1 Linear-Fracture Mechanics

For the application of tissue paper, in-plane fracture tests were conducted. In-plane fracture tests on paper materials are typically conducted in one of three specimen geometries: center-notched test (CNT), double edge-notched test (DENT) or single edge-notched test (SENT). The different geometries are shown in Figure 3.1.

From LEFM, the stress intensity factor, $K_I$, is defined as

$$K_I = \sigma_0 \sqrt{\pi a f \left( \frac{a}{W} \right)},$$

(3.1)

where $\sigma$ is the far field stress, $a$ is the length of the notch, $W$ is the width of the specimen and $f \left( \frac{a}{W} \right)$ is a geometry dependent correction factor.
Figure 3.1: Common geometries for in-plane fracture tests of paper materials. Here, \( a \) is the length of the crack, \( W \) and \( 2W \) the widths of the samples and \( 2h \) is the length.

The geometric correction factors for the in-plane fracture test types are

\[
\text{CNT} \quad \frac{f(a)}{W} = (1 - 0.025\left(\frac{a}{W}\right)^2 + 0.06\left(\frac{a}{W}\right)^4)\sqrt{\text{sec}\left(\frac{\pi a}{2W}\right)}, \tag{3.2}
\]

\[
\text{SENT} \quad \frac{f(a)}{W} = \left(0.752 + 2.02\frac{a}{W} + 0.37(1 - \sin\left(\frac{\pi a}{2W}\right))^3\right)\frac{2W\tan\left(\frac{\pi a}{2W}\right)}{\pi a}, \tag{3.3}
\]

\[
\text{DENT} \quad \frac{f(a)}{W} = \sum_{i=0}^{6} \frac{\alpha_i\left(\frac{a}{W}\right)^i}{\sqrt{1 - \frac{a}{W}}}, \tag{3.4}
\]

with \( \alpha_i = [1.1215, -0.5699, -0.7056, 2.4748, -3.1194, 1.8945, -0.4594][7]. \)

### 3.2 Coffin compensation

To predict the strength of paper materials, Coffin et al. suggest the following modification of LEFM. They assume that failure occurs when the stress at the crack tip reaches a critical level. That would imply that the stress intensity factor is the same regardless of the notch length. Paper as a material is not linear nor isotropic. To compensate for the anisotropy and heterogeneity
of paper materials, it is assumed that a paper material has an inherent characteristic fracture process zone (FPZ) with a characteristic dimension \( d \). Equation (3.1), modified to solve for the applied far-field force \( F_0 \) for an unnotched specimen is then obtained as

\[
F_0 = \frac{K_I W t}{f\left(\frac{d}{W}\right)\sqrt{\pi d}}.
\]  

(3.5)

Here, \( t \) is the specimen thickness. For any notch larger than \( d \) the predicted max force can be expressed as

\[
F = \frac{K_I W t}{f\left(\frac{d+a}{W}\right)\sqrt{\pi (d+a)}}.
\]  

(3.6)

Since the stress intensity factor \( K_I \), the thickness \( t \) and the width are the same, a load ratio expression between unnotched and notched specimens can be obtained and described as

\[
\frac{F}{F_0} = \frac{f\left(\frac{d}{W}\right)}{f\left(\frac{d+a}{W}\right)} \sqrt{\frac{d}{d+a}} \forall a < W - d.
\]  

(3.7)

The inherent fracture process dimension \( d \) is assumed to be composed of a structural component, \( d_s \), and a material component, \( d_m \). Thus, \( d \) can be written as \( d = d_m + d_s \). For notch length less than \( d_s \), the load ratio will be unity with small variations. The load ratio is significantly reduced only when the notch length is larger than the structural component \( d_s \). The final expression is

\[
\frac{F}{F_0} = \begin{cases} 
1 & \forall a \leq d_s \\
\frac{f\left(\frac{d}{W}\right)}{f\left(\frac{d+a-d_s}{W}\right)} \sqrt{\frac{d}{d+a-d_s}} & \forall d_s < a < W - d.
\end{cases}
\]  

(3.8)

In this work, we will redefine the expression of \( d, d_s \) and \( d_m \). The parameter \( d \) is seen as an effective crack length representing the flaw of the material, \( d_s \) is similar to Coffin et al. [11] described as a structural flaw of the network and \( d_m \) is the size of the fracture process zone.
Chapter 4

Method

Tests were performed to investigate the viability of a modified linear fracture theory on tissue paper. Four base tissue papers were embossed using a universal testing machine. The embossing pattern plates were square 15×15 cm plates that were manufactured by 3D printing. These plates are the same as used in RISE Bioeconomy Report No:60 [12]. The plate embossing was done using an Universal Testing Machine (UTM). In total, three plates were used at three embossing loads. Next, a punch press was used to cut the samples into the correct size. A scalpel and a template were used to create notches of different lengths. A tensile testing machine was used to perform the tensile test to obtain the fracture load at different notch lengths.

4.1 Material

Four different base tissue papers were tested. Two DCT tissue paper and two TAD tissue paper. The papers were leftover papers from a previous project at RISE. Table 4.1 presents the base paper material’s thickness, grammage, stiffness and strength anisotropy.

Table 4.1: Tissue paper used in the investigation.

<table>
<thead>
<tr>
<th>Qualities</th>
<th>Thickness [µm]</th>
<th>Grammage [g/m²]</th>
<th>Stiffness anistropy</th>
<th>Strength anistropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCT-T</td>
<td>118</td>
<td>19.4</td>
<td>0.34</td>
<td>1.79</td>
</tr>
<tr>
<td>DCT-K</td>
<td>87.0</td>
<td>18.4</td>
<td>0.38</td>
<td>2.10</td>
</tr>
<tr>
<td>TAD-T</td>
<td>228</td>
<td>19.8</td>
<td>0.27</td>
<td>1.03</td>
</tr>
<tr>
<td>TAD-K</td>
<td>412</td>
<td>21.9</td>
<td>0.79</td>
<td>1.26</td>
</tr>
</tbody>
</table>
4.2 Embossing

3D-printed embossing plates were used for embossing the tissue papers. The plates were provided from a previous project at RISE [12]. Three different plates were used and the properties are given in Table 4.2. Embossing area

Table 4.2: Embossing plates used in the report. Here, $n$ is the dot density, $d$ is the diameter of the dot and $z$ is the depth of the dots.

<table>
<thead>
<tr>
<th>Description</th>
<th>$d$ (µm)</th>
<th>$n$ (dots/cm$^2$)</th>
<th>$z$ (mm)</th>
<th>Embossing area</th>
<th>Edge length (mm/cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>1000</td>
<td>15</td>
<td>1.4</td>
<td>12%</td>
<td>47</td>
</tr>
<tr>
<td>EL+</td>
<td>750</td>
<td>26.7</td>
<td>1.4</td>
<td>12%</td>
<td>63</td>
</tr>
<tr>
<td>Z+</td>
<td>1000</td>
<td>15</td>
<td>2.4</td>
<td>12%</td>
<td>47</td>
</tr>
</tbody>
</table>

(EA) and edge length were calculated from

$$EA = n_{tot} \frac{\pi d^2}{4A} \quad (4.1)$$

and

$$EL = n_{tot} \frac{\pi d}{A}. \quad (4.2)$$

Here $n_{tot}$ is the total number of dots on the plate, $A$ is the total area of the surface with dots. For plate-based embossing, an MTS universal testing machine. The rubber plate was the same for all samples.

4.3 Tensile tests

From the embossed paper sheet, 50×100 mm specimens were punched out using a punch form and a punch press to avoid bending the sides. The tensile tests were done using a Lorentzen and Wettre tensile tester, see Figure 4.1. All the settings followed SS-EN ISO 12625-4:2017 [13]. The tensile test used crack lengths of 0, 1, 2, 3, 4, 6, 8, 10 and 12 mm, respectively.
4.4 Matlab analysis

Based on the tensile test result, a least square curve fit was used to fit the data to Equation (3.8). For the curve-fitting, a lower-bound constraint was set so that $d$ and $d_s$ are always positive values since it would not be reasonable to have a negative internal characteristic dimension for the fracture process zone.

4.5 Testing matrix

The embossing and testing of the material was done in batches. First, a DCT-K material was embossed and tested in MD and CD for the reference plate. The material was embossed at three embossing load levels. Next, the same material was embossed with an EL+ plate, which unfortunately cracked during the embossing procedure. It resulted in only MD samples being tested. Next, a DCT-T sample was embossed with the reference plate and the Z+ plate. Only MD samples were used to save time and for a first comparison. MD
samples of DCT-K quality were obtained in the same embossing batch. The tensile tests for toilet materials, DCT-T and TAD-t, samples made at a 25 kN embossing pulse failed due to samples being too weak for the tensile test machine to measure the applied load. The testing matrix can be seen in Table 4.3.

Table 4.3: Testing matrix for the embossing. The symbol * denotes that only MD samples were used.

<table>
<thead>
<tr>
<th>Material</th>
<th>Reference plate</th>
<th>EL+</th>
<th>Z+</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.5 kN</td>
<td>15 kN</td>
<td>25 kN</td>
</tr>
<tr>
<td>DCT-T</td>
<td>x*</td>
<td>x*</td>
<td>-</td>
</tr>
<tr>
<td>DCT-K</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>TAD-T</td>
<td>x</td>
<td>x</td>
<td>-</td>
</tr>
<tr>
<td>TAD-K</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>
Chapter 5

Experimental Procedure

The following chapter describes the laboratory setup and procedure for embossing the material, obtaining the specimen dimensions and the tensile test of the materials. Table 5.1 presents a summary of material and equipment.

Table 5.1: Materials and equipment used in the experimental procedure.

<table>
<thead>
<tr>
<th>Material</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCT-T</td>
<td>Testing material</td>
</tr>
<tr>
<td>DCT-K</td>
<td>Testing material</td>
</tr>
<tr>
<td>TAD-T</td>
<td>Testing material</td>
</tr>
<tr>
<td>TAD-K</td>
<td>Testing material</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotatrim</td>
<td>A rotary cutting machine</td>
</tr>
<tr>
<td>Embossing plates</td>
<td>3D printed embossing plates</td>
</tr>
<tr>
<td>UTS</td>
<td>Universal Testing Machine for embossing</td>
</tr>
<tr>
<td>Punching form</td>
<td>A punching form that punches samples of 50 by 100 mm</td>
</tr>
<tr>
<td>Punch Press SP2</td>
<td>Punching press from TJT-teknik AB</td>
</tr>
<tr>
<td>Scalpel</td>
<td>Scalpel for creating notches</td>
</tr>
<tr>
<td>Crack template</td>
<td>A template to create notches of different sizes</td>
</tr>
<tr>
<td>Tensile tester</td>
<td>Tensile testing machine from ABB Lorentzen &amp; Wettre</td>
</tr>
</tbody>
</table>
5.1 Embossing setup

Before embossing, specimens need to be cut from the material rolls. Test specimens of size $21 \times 21$ cm were cut out using a rotary cutting machine from Rotatrim Ltd. For each specimen sheet, the machine direction is indicated by an arrow. For the embossing 3D-printed embossing pattern plates of size $15 \times 15$ cm were used. The properties of the plates were presented in Table 4.2.

A universal testing machine (UTM) from MTS Systems Corporation was used to emboss the samples. The embossing plate was attached to an aluminum plate with tape and then attached to the actuator arm of the UTM. A similar aluminum plate was attached to the load cell. On top of the lower aluminum plate, the rubber material was placed, see Figure 5.1. The rubber material has a Shore-A value of 52 A. The embossing process followed a procedure where first, the embossing plate is lowered until the load cell detects a force of 10 N. Next, the machine applies a 0.25 second sinus-shaped pulse load. The embossing loads tested were 7.5 kN, 15 kN and 25 kN. Afterward, the plate was raised, the sample was removed, and replaced with a new sample.

![Figure 5.1: Plate-based embossing setup. Pictures taken from RISE Bioeconomy Report No:60 [12].](image-url)
5.2 Preparation before tensile and fracture tests

The tensile and fracture tests were done in a separate laboratory. Before conducting the tensile and fracture test, the specimen needs to be prepared by conditioning in the laboratory environment for at least two days before the experiments are conducted.

5.2.1 Punching of the samples

Following ISO 12624-4, sample dimensions for tensile and fracture tests for tissue paper are $50 \times 100$ mm [13]. A punch form was bought and a punch press machine model SP2 from TJT-Teknik AB was used to get samples in the desired dimensions. The machine is seen in Figure 5.2 (a). When using a plate-based embossing setup, the center of the specimen will become more embossed than the edges of the embossed area. This is due to the center of the specimens being aligned with the actuator arm of the testing machine. Therefore, a central strip is punched out from each embossed sample. The embossed sheet was placed on the plastic plate so that the edge of the sheet aligns with the line on the plastic plate, see Figure 5.2 (b). The punch plate was aligned in the center of the specimen. After the specimen was punched out, an arrow indicates the machine direction on the specimen.
5.2.2 Creating cracks for fracture test

To conduct the fracture test, an edge-notch needs to be introduced to the samples. Notches of length from 1 mm to 12 mm were created using a scalpel and a metal template that has cracks up to 25 mm, see Figure 5.3. The center of the specimen is marked with a pen. The intended crack-length slide is aligned with the central mark. The scalpel is then used to create the crack.

5.3 Tensile test and fracture test

The testing machine used was an ABB Lorentzen & Wettre Tensile tester. The machine setup follows ISO 12624-4 [13]. A specimen is placed in between the clamps. The center of the specimen is aligned with the center of the span between the clamps. The clamping pressure is set to 2.0 bar. The force and displacement are saved in .txt files and then exported to a computer for post-processing.
Figure 5.3: (a) Crack template used for creating cracks for the fracture tests. (b) Scalpel used to create the crack
Chapter 6

Results

The following chapter presents the results of applying Equation (3.8) to the result of the tensile tests at different notch lengths. Equation (3.8) will be referred to as the Coffin model. A nonlinear curve-fit was used to obtain the optimal $d$ and $d_s$ of the Coffin model, Equation (3.8). Also, in this chapter, "0 kN" means base material. MD and CD, respectively, denotes the material direction in which the tensile test machine applied the load. For MD samples, the notch will be perpendicular to MD, i.e., oriented in the cross direction (CD). The notations 7.5 kN, 15 kN and 25 kN refer to the embossing load.

6.1 Tensile test base material

First, the model was tested for the base paper. All the tensile tests for base papers had already been done in a previous project at RISE (yet to be published). Ten specimens were tested for every notch length and the average tensile load at failure and the standard deviation were calculated.
Figure 6.1: Fitted Coffin model fitted to the experimental results for base DCT-T material. In the graphs, the circles are the mean value, and the vertical bar at each circle is the standard deviation.

Figure 6.2: Coffin model fitted to experimental results for base DCT-K material. In the graphs, the circles are the mean value, and the vertical bar at each circle is the standard deviation.

In Figure 6.1 and 6.2, the Coffin model has been applied to the result of base DCT materials. For both materials, the structural flaw $d_s$ is smaller in MD than in CD, meaning that an edge-notch affects the strength in MD for shorter notch lengths than in CD. The effective crack length of the material $d$ is larger in CD than MD, indicating the specimen is weaker in CD than in MD.
In Figures 6.3 and 6.4, the Coffin model has been applied and fitted to TAD materials. For TAD-T, the effective crack length $d$ and the structural flaw $d_s$ are similar in MD and CD, while the TAD-K material behaved more similar to the DCT material with a lower $d$ and $d_s$ in MD compared to CD.

In Table 6.1 a summary of the results for all base materials are presented.
Table 6.1: Summary of $d$, $d_s$ and $d_m$ for the base tissue papers tested. The unnotched tensile load at failure is also presented as well as the load anisotropy.

<table>
<thead>
<tr>
<th>Material</th>
<th>$d$ [mm]</th>
<th>$d_s$ [mm]</th>
<th>$d_m$ [mm]</th>
<th>Tensile load [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MD</td>
<td>CD</td>
<td>MD</td>
<td>CD</td>
</tr>
<tr>
<td>DCT-T</td>
<td>4.6</td>
<td>12.7</td>
<td>0.0</td>
<td>5.4</td>
</tr>
<tr>
<td>DCT-K</td>
<td>3.9</td>
<td>13.9</td>
<td>0.4</td>
<td>7.6</td>
</tr>
<tr>
<td>TAD-T</td>
<td>12.4</td>
<td>12.6</td>
<td>3.3</td>
<td>3.9</td>
</tr>
<tr>
<td>TAD-K</td>
<td>6.9</td>
<td>12.3</td>
<td>0.0</td>
<td>5.1</td>
</tr>
</tbody>
</table>

6.2 Tensile test of embossed material

In this section, only the graphs for TAD-K will be presented. The graphs for the other materials can be found in Appendix A. DCT-K was embossed and tested in MD and CD with the reference plate. For the increased edge length plate (EL+) and increased depth plate (Z+) DCT-K testing was made only in MD. DCT-T was tested only with the reference plate and increased depth plate (Z+) in MD. TAD-K and TAD-T material were tested with the reference plate and the increased depth (Z+) plate in both MD and CD. For both toilet materials, DCT-T and TAD-T, embossing up to 15 kN was only tested because at higher embossing load, the material could barely sustain any applied load from the tensile testing machine.

Figure 6.5: Tensile load at failure for increasing crack length $a$. The embossing plate used is the reference plate. In the graphs, the circles are the mean value, and the vertical bar at each circle is the standard deviation.

Figure 6.5 shows the tensile test result for TAD-K material with reference embossing. In both MD and CD specimens, a huge drop in tensile strength
occurs for the 25 kN embossing load.

Figure 6.6: Coffin model applied to TAD-K material for reference plate embossing using a pulse embossing load of 7.5 kN. In the graphs, the circles are the mean value, and the vertical bar at each circle is the standard deviation.

Figure 6.6 shows the applied model for embossed TAD-K material. MD and CD specimens had similar values of the effective crack length, $d$, implying that MD and CD specimens have a similar unnotched tensile strength. MD specimens had a lower structural flaw $d_s$. This would indicate that a smaller edge-notch would affect the tensile load in MD specimens than in CD specimens, which was also seen for the base TAD-K material. Compared to the base TAD-K material, the effective crack length, $d$, increased for both MD and CD specimens. The effective crack length of MD specimens increased by 5.6 mm, while in CD it was increased by only 0.5 mm.

The result of the model fitted to a TAD-K material with a reference embossing at an embossing load of 15 kN can be seen in Figure 6.7. A small decrease in $d$ occurred in both MD and CD specimens and a small increase in $d_s$ for both MD and CD samples.

In Figure 6.8, the fitted model for TAD-K embossed at 25 kN is seen. Here, a significant loss in tensile load at failure first occurs when the notch is 15% of the width of the specimen.

Next, the result of TAD-K embossed with the Z+ plate is presented. The difference between the plates is the depths of the dots on the plate, see Table 4.2.
Results

Figure 6.7: Coffin model applied to TAD-K material with a pulse embossing load of 15 kN. At every notch length, ten samples were tested. The standard deviation is indicated by the error bar at each notch length.

Figure 6.8: Coffin model applied to TAD-K material with a pulse embossing load of 25 kN. At every notch length, ten samples were tested. The standard deviation is indicated by the error bar at each notch length.

As seen in Figure 6.9, a similar behavior as for the reference embossing was obtained from the Z+ embossing. Embossing at 0 to 15 kN follows a similar behavior, while at an embossing load of 25 kN a significant loss in strength occurred in both MD and CD.

When TAD-K was embossed at 7.5 kN with the Z+ plate a similar jump in $d$ occurred in MD and CD specimens as for the reference plate. As seen in Figure 6.10, the internal length was 12.0 mm and 12.5 mm in MD and CD, respectively, and $d_S$ was 3.5 and 3.8 mm in MD and CD, respectively.

For TAD-K material embossed at 15 kN with Z+ plate, an increase in both $d$ and $d_S$ occurred for MD and CD specimens, compared to the result for 7.5
Figure 6.9: The results of tensile load at failure, $F$, against increasing notch length $a$ for TAD-K material embossed with Z+ plate at different embossing loads. At every notch length, ten samples were tested. The standard deviation is indicated by the error bar at each notch length.

Figure 6.10: Coffin model applied to TAD-K material with a pulse embossing load of 7.5 kN using Z+ plate. At every notch length, ten samples were tested. The standard deviation is indicated by the error bar at each notch length.

kN embossing load with the same plate. It is seen in Figure 6.11 that small notches have less of an impact on the tensile force.

Figure 6.12 shows the result of the Coffin model applied to the experimental data for TAD-K embossed at 25 kN with the Z+ embossing plate. The model seems to be able to follow the behavior of MD specimens. In MD, $d_s$ has slightly increased compared to the case with a 15 kN embossing load. In CD, only large notches affected the tensile force compared to an unnotched specimen. When the notch length $a$ is less than 10.2 mm, the tensile load would remain the same as for a specimen with no edge-notch. For CD, a considerable
jump in the effective crack length $d$ occurred, corresponding to an internal length of 27.1 mm, more than half the width of the specimen.

The change in the fracture process zone, $d_m$, for TAD-K quality is presented in Figure 6.13. For both MD and CD specimens, $d_m$ did not change with increasing embossing except for the CD specimen at an embossing load of 25 kN for the Z+ plate.
Figure 6.13: Evolution of fracture process zone length $d_m$, for TAD-K quality with increasing embossing load for both plates.

Figure 6.14: Evolution of structural flaw, $d_s$, for TAD-K quality with increasing embossing load for reference and Z+ plates.

The change in structural flaw length, $d_s$, from embossing is presented in 6.14 for TAD-K quality. MD specimens show increased structural flaws for both plates with the increasing embossing load. The structural flaws of CD specimens was unaffected with the reference plate, while the Z+ plate affected the structural flaw at higher embossing loads.

Graphs of the result for the other materials (DCT-T, DCT-K and TAD-T) can be found in Appendix A.

Below, a summary of the fitted values $d$ and $d_s$ and the unnotched tensile load at failure, $F_0$, is presented in Tables 6.2 to 6.5 for each material and the
embossing plates tested.
Table 6.2: Summary of $d$ and $d_s$ for reference plate and Z+ plate. The tensile load at failure, $F_0$, and the tensile strength for unnotched specimens at different embossing loads are presented.

<table>
<thead>
<tr>
<th>Embossing load kN</th>
<th>d MD mm</th>
<th>d CD mm</th>
<th>ds MD mm</th>
<th>ds CD mm</th>
<th>$d_m$ MD mm</th>
<th>$d_m$ CD mm</th>
<th>$F_0$ N</th>
<th>Tensile strength N/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.6</td>
<td>12.7</td>
<td>0.0</td>
<td>5.4</td>
<td>4.6</td>
<td>7.3</td>
<td>6.1</td>
<td>3.4</td>
</tr>
<tr>
<td>7.5</td>
<td>5.2</td>
<td>-</td>
<td>0.0</td>
<td>-</td>
<td>5.2</td>
<td>-</td>
<td>5.6</td>
<td>-</td>
</tr>
<tr>
<td>15</td>
<td>12.4</td>
<td>-</td>
<td>1.8</td>
<td>-</td>
<td>10.6</td>
<td>-</td>
<td>3.4</td>
<td>-</td>
</tr>
</tbody>
</table>

DCT-T: Z+ Plate

<table>
<thead>
<tr>
<th>Embossing load kN</th>
<th>d MD mm</th>
<th>d CD mm</th>
<th>ds MD mm</th>
<th>ds CD mm</th>
<th>$F_0$ N</th>
<th>Tensile strength N/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.6</td>
<td>12.7</td>
<td>0.0</td>
<td>5.4</td>
<td>4.6</td>
<td>7.3</td>
</tr>
<tr>
<td>7.5</td>
<td>5.6</td>
<td>-</td>
<td>0.7</td>
<td>-</td>
<td>4.9</td>
<td>5.3</td>
</tr>
<tr>
<td>15</td>
<td>12.0</td>
<td>-</td>
<td>3.5</td>
<td>-</td>
<td>8.5</td>
<td>3.4</td>
</tr>
</tbody>
</table>

In Table 6.2, the result for the DCT-T material is presented. The embossing of the material increased the values of $d$ and $d_s$ with increasing embossing load for both the reference and Z+ plates. The unnotched tensile load at failure, $F_0$, decreased with increasing embossing for both plates. The Z+ plate decreased $F_0$ more for 7.5 kN embossing load compared to the reference plate. However, at 15 kN embossing load, the unnotched tensile load was the same for the two plates.

In Table 6.3, the summary of $d$ and $d_s$ for DCT-K material at different embossing plates and embossing loads is presented. For DCT-K with increasing embossing load for the plates the value of $d$ increased gradually for all the plates. At the first embossing load level of 7.5 kN, Z+ and El+ plates decreased $d_s$ compared to the reference plate, meaning that a notch of only 0.5 mm will lower the tensile load at failure for the material. The Z+ plate decreased the unnotched tensile load $F_0$ more than the reference and EL+ plates. The reference plate and Z+ plate had a significant loss in unnotched tensile load when embossed with a load of 25 kN, where the Z+ plate is the worst of the two plates. However, even if there is a lower $F_0$ for the Z+ plate than the reference plate, the values of $d$ and $d_s$ were higher for the Z+ plate than the reference plate. The tensile load was preserved better with the EL+ plate.
Table 6.3: Summary of the resulting fitted values of $d$, $d_s$, $d_m$, and the unnotched tensile load at failure $F_0$ for DCT-K material embossed with the reference plate, EL+ plate and Z+ plate.

<table>
<thead>
<tr>
<th>Embossing load</th>
<th>$d$ mm</th>
<th>$d_s$ mm</th>
<th>$d_m$ mm</th>
<th>$F_0$ N</th>
<th>Tensile Strength N/m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MD</td>
<td>CD</td>
<td>MD</td>
<td>CD</td>
<td>MD</td>
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<tr>
<td>0</td>
<td>3.9</td>
<td>13.9</td>
<td>0.4</td>
<td>7.6</td>
<td>3.5</td>
</tr>
<tr>
<td>7.5</td>
<td>5.6</td>
<td>13.5</td>
<td>0.9</td>
<td>7.3</td>
<td>4.7</td>
</tr>
<tr>
<td>15</td>
<td>11.1</td>
<td>14.2</td>
<td>2.3</td>
<td>7.9</td>
<td>8.8</td>
</tr>
<tr>
<td>25</td>
<td>14.2</td>
<td>12.7</td>
<td>9.2</td>
<td>6.8</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Table 6.4: Summary of the resulting fitted values of $d$, $d_s$, $d_m$, and the unnotched tensile load at failure $F_0$ for TAD-T material embossed with the reference plate and Z+ plate.

<table>
<thead>
<tr>
<th>Embossing load</th>
<th>$d$ mm</th>
<th>$d_s$ mm</th>
<th>$d_m$ mm</th>
<th>$F_0$ N</th>
<th>Tensile Strength N/m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MD</td>
<td>CD</td>
<td>MD</td>
<td>CD</td>
<td>MD</td>
</tr>
<tr>
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<td>12.4</td>
<td>12.6</td>
<td>3.3</td>
<td>3.9</td>
<td>9.1</td>
</tr>
<tr>
<td>7.5</td>
<td>12.3</td>
<td>12.7</td>
<td>3.2</td>
<td>5.6</td>
<td>9.1</td>
</tr>
<tr>
<td>15</td>
<td>12.2</td>
<td>13.5</td>
<td>4.9</td>
<td>5.8</td>
<td>7.3</td>
</tr>
</tbody>
</table>

For the TAD-T material the result was different compared to the DCT.
materials. As seen in Table 6.4 for the reference plate, the value of $d$ barely changed with the embossing for both MD and CD specimens. The parameter $d$ for MD specimens even decreased compared to the base material while $d_s$ increased and $F_0$ decreased.

Table 6.5: Summary of the resulting fitted values of $d$, $d_s$, $d_m$ and the unnotched tensile load at failure $F_0$ for TAD-K material embossed with the reference plate and Z+ plate.

<table>
<thead>
<tr>
<th>Embossing Force kN</th>
<th>d MD mm</th>
<th>d MD mm</th>
<th>d MD mm</th>
<th>d MD mm</th>
<th>$F_0$ N</th>
<th>Tensile Strength N/m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6.9</td>
<td>12.3</td>
<td>0.0</td>
<td>5.1</td>
<td>14.6</td>
<td>11.6</td>
</tr>
<tr>
<td>7.5</td>
<td>12.0</td>
<td>12.4</td>
<td>3.4</td>
<td>7.9</td>
<td>9.7</td>
<td>7.7</td>
</tr>
<tr>
<td>15</td>
<td>13.3</td>
<td>12.9</td>
<td>6.7</td>
<td>6.8</td>
<td>6.6</td>
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</tr>
<tr>
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<td>6.9</td>
<td>12.3</td>
<td>0.0</td>
<td>5.1</td>
<td>14.6</td>
<td>11.6</td>
</tr>
<tr>
<td>7.5</td>
<td>12.0</td>
<td>12.5</td>
<td>3.5</td>
<td>3.8</td>
<td>8.5</td>
<td>8.7</td>
</tr>
<tr>
<td>15</td>
<td>12.2</td>
<td>13.0</td>
<td>4.8</td>
<td>5.2</td>
<td>7.4</td>
<td>7.8</td>
</tr>
<tr>
<td>25</td>
<td>12.7</td>
<td>27.1</td>
<td>5.0</td>
<td>10.2</td>
<td>7.7</td>
<td>16.0</td>
</tr>
</tbody>
</table>

For TAD-K quality, the embossing of the material increased $d$ at the first load level of embossing for both plates and MD and CD specimens. However, with increasing load levels, the increase in $d$ is much smaller than the initial change. For the Z+ plate at 25 kN, CD specimens got a significant jump in $d$ to represent an internal length of 27.1 mm.
Chapter 7

Conclusion

The model suggested by Coffin et al. [11] could fit tensile tests of notched specimens for four 1-ply base toilet and kitchen tissue papers. It can be seen in Table 6.1 that for all materials except TAD-T, the values of $d$, $d_s$ and $d_m$ are different in MD and CD specimens. For MD specimens, the internal structural flaw, $d_s$, was close to zero for all four materials, which means that any length of an edge-notch will affect the material’s tensile strength. For CD samples, it is seen that longer edge-notches are needed before a loss in tensile strength occurs due to the edge-notch. For TAD-T, all parameters were similar in MD and CD, which coincide with the strength anisotropy of the TAD-T quality. This implies that if the material behaves similarly in MD and CD, the model will find the same value for the effective crack, $d$, structural flaw, $d_s$ and the material’s fracture process zone length, $d_m$. The model suggested by Coffin et al. [11] followed the behavior of the MD specimen better than the CD specimen for all paper qualities.

When the paper qualities were embossed, some of the parameters in the model changed with increased embossing load. However, between the materials, they behaved differently. The DCT-T quality embossed with both plates, see Table 6.2, showed that the structural component, $d_s$, was unaffected by increasing embossing load for MD specimen while the fracture process zone, $d_m$, instead increased. The effective internal crack of the material, $d$, increased with the embossing load, indicating a more damaged material. Since the structural flaw, $d_s$, remained unchanged, that would suggest that the quality was still sensitive to edge notches and that failure localizes at the edge notch for even small notches. For DCT-K quality, the opposite behavior regarding $d_s$ and $d_m$ occurred. The fracture process zone remained roughly the same with increasing embossing load and the structural component kept increasing.
with the embossing load. This would indicate that the fracture process zone was unaffected while the embossing changed the network structure. The effect is then that for embossed DCT-K material edge notches have less impact on the tensile strength of the specimen, resulting in the edge notch having to be longer to affect the tensile strength of the specimen. The model parameters for CD samples of the DCT-K quality were unaffected. This suggests that the embossing of the material did not affect the strength of the CD samples. Of all the qualities tested, TAD-T behaved the most similarly in MD and CD. Overall, the embossing of the quality did not change the parameters of the model, TAD-T was also the initial weakest of the base materials.

A sizeable effective crack length, \( d \), usually means low tensile strength. However, the value of the effective crack, \( d \), for the same material but with different plates did not result in the same tensile strength. For instance the value of \( d \) for TAD-K embossed with reference plate at 15 kN and Z+ plate at 7.5 kN was 12.0 mm, but the tensile load \( F_0 \) was 10.0 N and 11.6 N, respectively. With highly embossed specimens (15 kN embossing load for toilet qualities and 25 kN embossing load for kitchen qualities), there is a significant drop in the tensile strength for MD and CD specimens. The parameters of the Coffin model do not capture this in a general way for the different types of materials. Overall, the embossing of the material decreased the impact of small edge notches on the tensile strength. For all qualities, embossing with the reference or the Z+ had a similar effect on the material’s tensile strength at low embossing loads. At higher embossing load, the Z+ plate had a more significant effect on the loss in tensile strength.
Chapter 8

Discussion and future work

While the model suggested by Coffin et al. [11] seems to be able to follow the result of the test performed in this report for both base and embossed materials, the values of the effective crack length $d$ were not comparable and particularly, they were not comparable with embossing. The embossing affects the values as expected since embossing affects the material’s internal structure. For embossed materials, the structural length scale, $d_s$, is expected to increase compared to the base paper since embossing changes the structure of the sheet. The value of $d_s$ indicates how long a physical notch needs to be before a decrease in tensile strength occurs. This behavior was seen for all materials except DCT-T, but the amount of change seemed to vary between the materials.

For DCT-K, the plate with increased edge length (EL+) was also tested and showed promising performance regarding the tensile strength. Initially, it was intended that an EL+ plate was to be used for all the material. During the embossing of DCT-K the plate cracked and could not be used further. Nonetheless, the results of the EL+ plate imply that with an increasing edge length of the embossing pattern, more of the tensile strength is preserved at increasing embossing load. A dot pattern with many smaller dots could be beneficial to reduce the loss in tensile strength from embossing.

During the tensile tests, samples did not necessarily break at the center of the span. When the edge notch was long, the crack growth localized at the notch tip end grew into the center of the specimen. The failure generally happened in the center for the long and short notches for the MD samples. Sometimes, the CD samples failed at the clamps. The results of those samples were discarded and not used in this report. Another problem was that the machine sometimes stopped in the middle of a tensile test usually during the
first second of a test, without any apparent reason. For those tests, as long as the notch had not visibly increased in length, the specimen was tested again otherwise the specimen was not used. These problems most often occurred with highly embossed samples. The problem occurred as much for both plates used in the report. Regularly, pressurized air was used to blow away dust around the clamps of the tensile tester and it seemed to reduce this problem, but it was not a guarantee that it would not happen.

With increasing embossing load, longer notches were needed to see a loss in tensile strength. This might seem odd since more damage is expected with embossing. However, this is the reason why the notch has less of an impact. Localization of stress at the notch tip only occurs when the defect gets sufficiently large. Since the model depends on the fact that a notch will affect the material’s load-carrying capacity, the model becomes less applicable with higher embossing. This can be seen more clearly in CD samples and MD samples embossed at 25 kN (see figure A.9 in Appendix A). Where a loss in tensile strength occurs only at the last notch length. Longer notches should then be used to have more data points to obtain a better fit.

When comparing the resulting maximal tensile load $F_0$ against notch length for different embossing plates and at different embossing loads, it can be seen that the embossing loads up until 15 kN seem to follow the same trend, see Figure A.1. For a 25 kN embossed material, a significant change in load-carrying capacity occurred due to the embossing, and the model is no longer applicable. The largest difference between the plates occurred at the highest embossing loads. This would indicate that at a high embossing load, the embossing pattern is more dominant for the tensile strength of the specimens.

For CD samples and highly embossed MD samples, notches of 12 mm seem insufficiently long to capture the loss in tensile strength. Longer notches could be used to get a more accurate description of the loss. A suggestion for further work would be to investigate longer notches for samples embossed at high embossing loads in both MD and CD specimens. A natural progression from his work would be investigating if the result is transferable to larger specimens. In the work by Coffin et al. [11] they successfully used the model for larger specimens than the standard 50 mm × 100 mm specimens. Another area of interest is investigating a more extensive range of embossing patterns. It would be interesting to see if a pattern aligned in the CD direction would follow the same behavior as noted with the dotted plates used here.
Bibliography


Appendix A

Result from tensile tests

Figure A.1: MD tensile result for DCT-T material for different notch lengths and at different embossing loads using the reference plate as the embossing pattern. At every notch length, five samples were tested. The standard deviation is indicated by the error bar at each notch length.
Figure A.2: Coffin model applied to experimental MD data for DCT-T material embossed with the reference plate at embossing loads: a) 7.5 kN and b) 15 kN. At every notch length, five samples were tested. The standard deviation is indicated by the error bar at each notch length.
Z+ DCT-T MD samples

- DCT-T: MD - 0 kN
- DCT-T: MD - 7.5 kN
- DCT-T: MD - 15 kN

Figure A.3: MD tensile result for DCT-T material for different notch lengths and at different embossing loads using the Z+ plate as the embossing pattern. At every notch length, five samples were tested. The standard deviation is indicated by the error bar at each notch length.
Figure A.4: Coffin model applied to experimental MD data for DCT-T material embossed with the Z+ plate at embossing loads: (a) 7.5 kN and (b) 15 kN. At every notch length, five samples were tested. The standard deviation is indicated by the error bar at each notch length.

Figure A.5: Evolution of fracture process zone length $d_{FPZ}$ and structural flaw $d_{S}$, for DCT-T quality with increasing embossing load for both plates.
Figure A.6: Tensile result for DCT-K material for different notch lengths and at different embossing loads using the reference plate as the embossing pattern with a) MD results and b) CD results. At every notch length, ten samples were tested. The standard deviation is indicated by the error bar at each notch length.

Figure A.7: Coffin model applied to experimental data for DCT-K material embossed with the reference plate at the embossing load of 7.5 kN with a) MD results and b) CD results. At every notch length, ten samples were tested. The standard deviation is indicated by the error bar at each notch length.
Figure A.8: Coffin model applied to experimental data for DCT-K material embossed with the reference plate at the embossing load of 15 kN with a) MD results and b) CD results. At every notch length, ten samples were tested. The standard deviation is indicated by the error bar at each notch length.

Figure A.9: Coffin model applied to experimental data for DCT-K material embossed with the reference plate at an embossing load of 25 kN with a) MD results and b) CD results. At every notch length, ten samples were tested. The standard deviation is indicated by the error bar at each notch length.
Figure A.10: MD tensile result for DCT-K material for different notch lengths and at different embossing loads using the Z+ plate as the embossing pattern. At every notch length, five samples were tested. The standard deviation is indicated by the error bar at each notch length. The x-axis has been normalized by the width of the specimen.
Figure A.11: Coffin model applied to MD experimental data for DCT-K material embossed with the Z+ plate at an embossing loads: a) 7.5 kN and b) 15 kN. At every notch length, five samples were tested. The standard deviation is indicated by the error bar at each notch length.

Figure A.12: Coffin model applied to MD experimental data for DCT-K material embossed with the Z+ plate at an embossing load of 25 kN. At every notch length, ten samples were tested. The standard deviation is indicated by the error bar at each notch length.
Figure A.13: MD tensile result for DCT-K material for different notch lengths and at different embossing loads using the EL+ plate as the embossing pattern. At every notch length, five samples were tested. The standard deviation is indicated by the error bar at each notch length. The x-axis has been normalized by the width of the specimen.
Figure A.14: Coffin model applied to MD experimental data for DCT-K material embossed with the EL+ plate at an embossing loads: (a) 7.5 kN and (b) 15 kN. At every notch length, five samples were tested. The standard deviation is indicated by the error bar at each notch length.

Figure A.15: Coffin model applied to MD experimental data for DCT-K material embossed with the EL+ plate at an embossing load of 25 kN. At every notch length, five samples were tested. The standard deviation is indicated by the error bar at each notch length.
Figure A.16: Evolution of fracture process zone length $d_m$, for DCT-K quality with increasing embossing load for all plates.

Figure A.17: Evolution of structural flaw $d_s$, for DCT-K quality with increasing embossing load for all plates.
Figure A.18: Tensile result for TAD-T material for different notch lengths and at different embossing loads using the reference plate as the embossing pattern. with a) MD results and b) CD results. At every notch length, ten samples were tested. The standard deviation is indicated by the error bar at each notch length.

Figure A.19: Coffin model applied to experimental data for TAD-T material embossed with the reference plate at an embossing load of 7.5 kN with a) MD results and b) CD results. At every notch length, ten samples were tested. The standard deviation is indicated by the error bar at each notch length.
Figure A.20: Coffin model applied to experimental data for TAD-T material embossed with the reference plate at an embossing load of 15 kN with a) MD results and b) CD results. At every notch length, ten samples were tested. The standard deviation is indicated by the error bar at each notch length.

Figure A.21: Tensile result for TAD-T material for different notch lengths and at different embossing loads using the Z+ plate as the embossing pattern, with a) MD results and b) CD results. At every notch length, ten samples were tested. The standard deviation is indicated by the error bar at each notch length.
Figure A.22: Coffin model applied to experimental data for TAD-T material embossed with the Z+ plate at an embossing load of 7.5 kN with a) MD results and b) CD results. At every notch length, ten samples were tested. The standard deviation is indicated by the error bar at each notch length.

Figure A.23: Coffin model applied to experimental data for TAD-T material embossed with the Z+ plate at an embossing load of 15 kN with a) MD results and b) CD results. At every notch length, ten samples were tested. The standard deviation is indicated by the error bar at each notch length.
Figure A.24: Evolution of fracture process zone length $d_m$, for TAD-T quality with increasing embossing load for both plates.

Figure A.25: Evolution of structural flaw $d_s$, for TAD-T quality with increasing embossing load for both plates.