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Dynamic Modeling of Heat Power System

Modeling of a Heat Power System Using Physical and Data-driven Methods and Investigation of a Moving Boundary Method

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Abstract

Our society is becoming more and more electrified every day. However, a significant portion of the world’s electricity generation relies on the combustion of fossil fuels to produce heat, which is subsequently harnessed to generate electricity. One way of generating electricity from heat is by utilizing a Rankine cycle. The basis of a Rankine cycle is to heat a liquid to its boiling point, which causes an increase in pressure that is used to spin a turbine and a generator. Many industries, such as transportation and manufacturing, produce large amounts of waste heat that needs to be removed from the main process. A Rankine cycle variant called an organic Rankine cycle can be used in a heat power system to generate electricity from lower-temperature waste heat, which increases efficiency since less heat is wasted.

This thesis focuses on constructing a dynamic model of Climeon’s heat power system called HP300. The HP300 utilizes an organic Rankine cycle to generate electricity. Dynamic modeling is valuable because it provides a deeper understanding of the system, which is beneficial for its development and improvement. Moreover, a system model has the potential to enhance the system’s performance by using advanced control methods. The HP300 consists of four main components: a pump, a turbine, an evaporator, and a condenser. Each component will be modeled individually, and the complete model will be constructed by combining the component models. Additionally, an in-depth investigation of an advanced modeling method for heat exchangers is to be conducted.

The constructed model in this thesis has an average error of 4%. The pump and turbine were modeled as steady-state models, and the evaporator and condenser were modeled with data-driven state-space models. The most important output of the model is the power generated by the turbine. The power was modeled with an average error of 6%. The turbine model performs best for pressure ratios of 1.75 and above. The model for the condenser had larger errors than the evaporator since it had fewer input variables. Improving the model of the condenser would decrease the overall errors of the model.

Keywords

System identification, Dynamic modeling, Organic Rankine cycle, Moving boundary, Thermodynamics.
Abstract
Sammanfattning


Nyckelord

System identifikation, Dynamisk modellering, Organisk Rankinecykel, Rörliga randvillkor, Termodynamik.
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Stockholm, October 2023  
Albin Gustafsson
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List of acronyms and abbreviations

**FV**  Finite volume

**HP300**  HeatPower300

**MB**  Moving boundary

**MPC**  Model predictive control

**ORC**  Organic Rankine Cycle

**PHE**  Plate Heat Exchanger

**PNLSS**  Polynomial Nonlinear State-space
Chapter 1

Introduction

The process of electrifying our society is currently underway, with wind and solar power gaining widespread adoption around the world. However, fossil fuels remain the predominant energy source, constituting over 75% of global energy production [1]. The majority of fossil fuel power plants generate electricity by burning fossil fuels which produces heat. This heat is used to vaporize water, leading to a rise in pressure that can be used to turn a turbine and generate electricity. This concept can also be applied in sectors other than power plants; for example, in industrial manufacturing and large engines that generate substantial amounts of heat as a byproduct. This waste heat can be utilized for other purposes, such as indoor heating in factories and heating pools on large cruise ships. Climeon is a company that manufactures heat power systems designed to recover waste heat and use it to generate electricity. Their system can enhance the overall efficiency of a factory or reduce fuel consumption, for instance, on ships, by harnessing energy from waste heat that would otherwise go to waste. This thesis aims to develop a dynamic model of Climeon’s heat power system.

1.1 Background

Climeon’s latest heat power system is called HeatPower300 (HP300). It uses a thermodynamic cycle called a Rankine cycle, which was first developed by William J.M. Rankine in 1859 [2]. At its core, it involves heating a fluid until it vaporizes in a heat exchanger, commonly called an evaporator, and utilizing the pressure increase to turn a turbine which is connected to a generator that produces electricity. The gas is then channeled into another heat exchanger called the condenser which condenses the gas back into a liquid. A pump
then pumps the liquid into the evaporator again to complete the cycle. The fluid that is pumped through the different components is called the media. The evaporator uses hot water to vaporize the media while the condenser uses cold water to condense the media. The hot and cold water are referred to as secondary fluids.

There are different kinds of Rankine cycles, super-critical Rankine cycles use super-heated water well over $100^\circ C$ at high pressure. The super-critical Rankine cycle can be used when the heat source has a very high temperature $[3]$. When the heat source is at or below $100^\circ C$ it is no longer feasible to use water since the Rankine cycle depends on the pressure increase that comes from vaporizing the fluid. In these cases, alternative fluids with lower boiling temperatures, such as toluene, isobutene, or isopentane, must be used. These organic fluids give rise to a type of Rankine cycle known as an Organic Rankine Cycle (ORC)$[4]$ that was first developed in the 1950s by Lucien B. and Harry Zvi T. $[5]$. Climeon’s HP300 is a Organic Rankine Cycle (ORC) system and uses an organic fluid similar to isobutene as its media.

The interest and development in waste heat systems such as ORCs has increased significantly during the 21th century. This can be correlated with the rising demand for efficient and climate-friendly methods for generating energy $[6]$. With the increasing interest in ORCs, the need for dynamic models is also rising. Dynamic models are useful for analyzing the system during its development. With a known model of a system, it is also possible to construct more advanced controllers that utilize the model in the calculations of the control signal for the system.

## 1.2 Purpose

This thesis aims to develop a dynamic model of Climeon’s latest heat power system, the HP300. The model should be capable of simulating dynamic behaviors resulting from changes in system inputs, such as adjustments to pump speed or variations in the temperature of the secondary fluids. The primary objective is to partially replace physical tests during the software development phase of the HP300, thereby saving both time and resources. However, the viability of the model as a substitute for physical testing hinges on its accuracy.

Additionally, the system identification and modeling of an ORC system, like
the HP300, hold significant interest as they open up possibilities for further system developments. Once the system is identified, it is possible to implement enhanced controllers using advanced control theories such as Model predictive control (MPC) that could improve system performance [7, 8].

### 1.3 Goals

The long-term goal for Climeon is to be able to use a dynamic model of their HP300 system to simulate its behavior without needing to conduct as many physical tests. This will enable their software team to spend less time testing new software features and more time developing the system. However, this goal is too broad for this master’s thesis. Therefore, the goals for this thesis are more specific and concrete, demonstrating to Climeon the feasibility of creating a model capable of simulating dynamic behaviors. The desired outcome for Climeon from this thesis is to achieve a proof of concept. If successful, Climeon will likely continue the model’s development towards a functional version that can be integrated into the software development process for their product. The concrete goals for this thesis can be summarized as follows:

**Goal 1.** Develop working models of the four main components of the HP300: Evaporator, condenser, pump, and turbine.

**Goal 2.** Validate the individual components by comparing the model outputs with existing data.
**Goal 3.** Integrate the different components to create a comprehensive model of the entire system.

**Goal 4.** Validate the complete model by comparing its output with existing data.

The thesis work will conclude with the submission of a final written report, an oral presentation, and the delivery of the dynamic model to Climeon.

### 1.4 Research Methodology

The research phase of this thesis involves reviewing prior work related to the modeling of various ORC systems and studying the different methods that have been used. While no new methods will be developed, some existing ones may be modified to ensure their integration with each other. Emphasis has been placed on researching methods that perform well under dynamic conditions, as the model will be used in those scenarios. For each component, an analysis of the identified methods will be conducted, followed by the selection of a method based on its general performance in dynamic conditions.

### 1.5 Delimitations

Climeon’s HP300 is equipped with various sensors that measure parameters such as pressure, temperature, and mass flow. To maintain the simplicity of the chosen modeling approach, some sensors will be omitted from the model. For the purpose of this study, the system will be considered to have seven control inputs: pump speed, and the temperature, mass flow, and pressure of both the hot and cold water entering the evaporator and condenser. The control system dynamics behind these inputs will not be incorporated into the model; rather, they will be assumed as known inputs to the model when simulated.

The different components of HP300 have different dynamics and some are faster than others. The pump and turbine respond much faster than the heat exchangers to changes in input. As the heat exchangers are the slowest parts of the system, there is no need to dynamically model the pump or the turbines [9, 10]. Thus, steady-state models will be created for both the pump and turbine.

The interface between the dynamic model and Climeon’s framework will not be the focus of this thesis. The thesis will only aim to develop the model
and will not involve the integration of the model into Climeon’s framework. This is to ensure that the thesis focuses on applying relevant theory to solve the problem rather than on implementing a finished model into Climeon’s framework.

1.6 Structure of the thesis

Chapter 2 presents relevant background information about the modeling of heat exchangers, pumps, and turbines. Chapter 3 includes an investigation of a physical modeling method for the heat exchangers. Chapter 4 presents the methods used to model the components of the system. Chapter 5 presents the results and analysis of the modeled system. Chapter 6 discusses the result as well as the methods used. Chapter 7 concludes the report and mentions some possible future work. References and an appendix are listed at the end of the report.
6 | Introduction
Chapter 2

Background

This chapter aims to provide the necessary knowledge for understanding the theories and decisions presented in the subsequent chapters of this thesis. First, different types of ORCs will be introduced, followed by an in-depth explanation of HP300, the system to be modeled. Additionally, relevant thermodynamic theory is also introduced. Various methods used in prior work are also briefly described. The chapter ends with a brief overview of related work.

2.1 Description of the system to be modeled

2.1.1 Pump

The pumps used in ORCs are generally either displacement pumps or centrifugal pumps [9]. A displacement pump works by enclosing a fixed volume of liquid at the input of the pump and transporting it to the output side. This can be done using gears, screws, or pistons to name a few [11]. In a displacement pump, the volumetric flow is proportional to the speed of the pump, this is not true for a centrifugal pump. A centrifugal pump works by rotating the liquid around an axis to create a centrifugal force that presses the liquid toward the outer walls of the pump and then to the outlet of the pump. In a centrifugal pump, the volumetric flow is not proportional to the speed of the pump since the pressure at the output works against the centrifugal force. In the system modeled in this thesis, a centrifugal pump is used.
2.1.2 Heat exchangers

The function of a heat exchanger in an ORC is to transfer heat between the media and the secondary fluid. The evaporator transfers heat from the hot water to the media and the condenser transfers heat from the media to the cold water. There are several different types of heat exchangers generally used in ORCs. The most common types are pipe-in-pipe, shell-and-tube, and plate heat exchangers [9]. The system studied uses Plate Heat Exchangers (PHEs), where the media and hot/cold water flow in thin plates stacked together in an alternating order to get a maximum heat transfer. An illustration of a PHE can be seen in Fig. 2.1, where the hot and cold flows are flowing in opposite directions. This is referred to as counter-flow and is generally more efficient since the temperature difference between the two fluids is more uniform throughout the entire heat exchanger [12]. PHEs have a large surface area compared with other heat exchangers, meaning they are capable of fast temperature change, which is highly valued in ORCs [9]. The specifics of the PHEs will be described in Section 2.4.1.

Figure 2.1: An overview of a plate heat exchanger, with counter-flowing fluids, showing the hot and cold fluids and the plates between in which the heat is transferred.
2.1.3 Turbine

The turbine is commonly referred to as the expander because its primary function is to expand the gas, reducing its pressure, and consequently, the vaporization point. This allows the gas to decrease in temperature without transitioning into a liquid phase, enabling the extraction of thermal energy from the gas. In ORC systems, the most prevalent types of expanders are volumetric and turbo-expanders [13], with HP300 utilizing a turbo-expander.

2.1.4 System assumptions

HP300 also includes several valves that are controlled during system start-up and shut-down. In this thesis, the valves will be assumed to be fully open to reduce model complexity. Therefore, the model developed in this thesis is designed for use when the system is already in operation. Moreover, certain components are cooled using the media, which causes a slight increase in the media’s temperature. However, it is assumed that the added heat is negligible.

2.2 Thermodynamics and enthalpy

This section delves into specific thermodynamic principles that will be necessary when constructing models for the various components of the system. When modeling an ORC system from physical equations it is necessary to understand the thermodynamics that describe the different components. First, it is important to have an understanding of the concept of enthalpy.

**Definition 1 (Enthalpy)**  Enthalpy is a measure of the energy of a system and is thus measured in Joules [J]. The equation for enthalpy is commonly written as

\[
H = Q + pV, \tag{2.1}
\]

where \(Q, p, V\) are the internal energy, pressure, and volume of a system respectively.

In the context of ORCs, the internal energy \(Q\) can be thought of as thermal energy and can be further broken down to \(Q = mc_p T\), where \(m\) is the mass, \(T\) is the temperature and \(c_p\) is the specific thermal capacity quantifies the amount of energy (heat) required to change the temperature of a material per kg and is typically measured in \([J/(kg\,K)]\). The models in this thesis all have specific enthalpy as inputs and outputs. The specific enthalpy of a system is its enthalpy
divided by its mass, \( \frac{J}{kg} \),

\[ h = c_p T + \frac{P}{\rho}, \tag{2.2} \]

where \( \rho \) is density. The models discussed in this thesis all use specific enthalpy as inputs and outputs, so in the remaining part of this thesis, specific enthalpy will be referred to as enthalpy.

### 2.2.1 Pressure-enthalpy (p-h) diagram

While (2.1) is useful for gaining an understanding of the concept of enthalpy, throughout this work, we will mostly calculate enthalpy using a pressure-enthalpy (p-h) diagram instead. A p-h diagram illustrates how thermodynamic properties change for a specific fluid under varying pressures and enthalpies. It features enthalpy on the x-axis and pressure on the y-axis, and also includes lines representing density, temperature, and thermal capacity. Given a particular pressure and temperature, it is straightforward to determine enthalpy, density, and thermal capacity from a fluid’s p-h diagram. The p-h diagram for the water can be seen in Figure 2.2 [14]. A similar p-h diagram was also used for the media in the HP300 system but as the specific media is a company secret it is shown here.

The p-h diagram can be divided into three sections: subcooled liquid, saturated liquid/gas, and superheated gas. When the liquid is subcooled, its temperature is below its boiling point. When it reaches its boiling point for a given pressure, it becomes a saturated liquid, meaning that any increase in energy will not increase the temperature but instead vaporize the liquid [15]. It is only when the entire liquid is vaporized that the gas becomes superheated since it has a temperature above its boiling point. Between the subcooled phase and the saturated liquid phase, a saturated liquid line can be drawn, visualized as the blue line in Figures 2.2 and 2.3. On this line, the liquid starts to boil. A saturated gas line can be drawn at the boundary between the saturated liquid/gas phase and the superheated gas phase; here, all the liquid has vaporized, visualized as the red line in Figures 2.2 and 2.3.

In Figure 2.2, it is apparent that for pressures exceeding 3000 kPa, there is no distinct separation between the subcooled liquid region and the superheated gas region. This phenomenon is known as the critical point, and under such extreme conditions, it becomes impossible to differentiate between the liquid and gas phases [16]. The critical point can be observed in Figure 2.3.
Figure 2.2: The p-h diagram for water, with the saturated liquid line in blue and the saturated gas line in red. The media used in HP300 has a similar p-h diagram.

### 2.3 Static modeling approaches for turbine and pump

Both the pump and turbine will be modeled using static modeling methods. This is in accordance with the literature since the two PHEs in the system have far slower dynamics than the pump and turbine [9]. This implies that the dynamic behaviors of the pump and turbine are negligible, allowing them to be modeled with static methods. During the pre-study phase, three different methods for modeling static components in the ORCs system were investigated and will be briefly explained in this section.

#### 2.3.1 Constant-efficiency method

The constant-efficiency method assumes a constant efficiency of the component at all operating points, making it a simple and computationally inexpensive method. However, this simplification comes with its consequences. This method is useful when the system operates close to its nominal operating point.
The pump enthalpy output could then be modeled as (2.3), assuming a constant efficiency $\eta$ [17],

$$h_{out} = h_{in} + \frac{c_p(P_{in} - P_{out})}{\eta},$$

(2.3)

where the $c_p$ is the thermal capacity of the fluid in the pump. However, for models that are expected to maintain consistent accuracy across a wider range of operating points, other methods are recommended [9].

### 2.3.2 Polynomial-regression method

The polynomial-regression method involves creating polynomial regression models of the efficiency with multiple variables. R. Dickes et al. [18] used quadratic functions to minimize overfitting while also ensuring that non-linear behavior was captured. The regression can be expressed as,

$$\epsilon = \sum_{i=0}^{n_x} \sum_{j=0}^{n_y} \sum_{k=0}^{n_z} a_{ijk}X^iY^jZ^k$$

(2.4)

where X, Y, and Z are the variables with the most impact on the efficiency. However, [18] found that the regression method showed limited accuracy when multiple components were implemented into the entire system.
2.3.3 Semi-empirical method

The third method called the semi-empirical method is the most widely used. The premise is to use simplified physical equations and then use reference data of the component to fit the parameters in the physical equations. A pump was modeled using a semi-empirical method in [7] and [10]. For pumps and turbines, it is common to use mass and energy balance equations. In the study conducted by R. Dickes et al. [18] the semi-empirical method showed the best performance when coupling all components of the ORC together.

2.4 Modeling of plate heat exchanger

2.4.1 Physical modeling of plate heat exchangers

As introduced in Section 2.1.2, in a PHE, heat is transferred between the media and hot/cold water through a metal plate, commonly referred to as the wall. When modeling PHEs it is useful to use two types of balance equations. A mass balance equation is used to describe the mass flow of the two liquids. The second type of equation is energy balance equations that describe both the heat transfer between the media and plate wall, and the wall and the secondary fluid. Two different methods for modeling PHEs will be described in this section, the Finite volume (FV) method and the Moving boundary (MB) method. Both methods rely on the physical equations that describe the mass and energy balance in the PHE. Satyam Bendapudi et al. [19] found that the MB was up to three times faster than FV while having similar accuracy. However, the MB method was less robust during load change transients of the studied system. In their simplest form, the mass and energy balance equations can be articulated as follows:

Mass Balance Equation:

$$\frac{dm}{dt} = ˙m_{in} - ˙m_{out}. \quad (2.5)$$

Energy Balance Equation:

$$\frac{dH}{dt} = \frac{dQ}{dt} + \frac{d(pV)}{dt}. \quad (2.6)$$

Here, the mass balance equation (2.5) represents the mass flow with respect to time, where \(\dot{m}_{in}\) is the incoming mass flow rate and \(\dot{m}_{out}\) is the outgoing mass flow rate. The energy balance equation (2.6) is the time derivative of
the enthalpy (2.1), which portrays the rate of change of enthalpy with respect to time. $\frac{d\varepsilon}{dt}$ is the time derivative of internal energy called heat flux that can be thought of as heat flow, and $p\dot{V}$ is the change in energy that comes from changes in pressure and volume.

### 2.4.1.1 Finite-volume

The FV method divides the PHE into $N$ number of cells and sets up the mass and energy balance equations for each cell. Here the output from cell $n$ is used as input in cell $n + 1$. This allows for high-resolution modeling of the mass flow inside the PHE, which could be useful for tracking the distribution of the media in the system [20].

### 2.4.1.2 Moving boundary

The MB method divides the PHE into at most three cells corresponding to the three zones that the media can be in. The three zones are the liquid-, two-phase, and gas zones. This can be done with limited impact in accuracy because the media behaves similarly within each phase. [21]

### 2.4.2 Data-driven modeling of plate heat exchangers

In [22] P. Csurcsia created a linear state-space model using a frequency response function. That linear state-space model was then used to initialize a Polynomial Nonlinear State-space (PNLSS) model. A PNLSS is a modified linear state-space model with added nonlinear terms of the inputs and states. In [22], a physical model, a linear state-space model, and a PNLSS model were compared, and found that the PNLSS model performed the best and the linear state-space model performed the worst. Additionally, it took more than 5 times longer to create the PNLSS model than the linear state-space model. Both the linear state-space model and PNLSS model had vastly shorter simulation times than the physical model since no advanced calculations are done in those models.

#### 2.4.2.1 State-space system identification

System identification on linear state-space form results in a simpler model. It is also significantly less computationally expensive to create the models. When identifying a system on state-space form, a common system identification method is to use the $n4sid$ algorithm. The $n4sid$ algorithm is an advanced
Background

subspace system identification algorithm created by P. Overschee and B. Moor in 1993 [23]. A subspace system identification algorithm creates a state-space model from a set of input and output data [24]. Generally, these algorithms use QR-decomposition and singular-value-decomposition of a weighted matrix containing the data to identify the system [25].

2.5 Related work

This section provides an overview of other research where ORCs were modeled using various methods. Additionally, some research involving the development of controllers for ORCs will be discussed.

2.5.1 Modeling of components in ORCs

R. Dickes et al. [18] studied and compared the three static modeling methods; constant-efficiency (CE), polynomial-regression (PR), and semi-empirical (SE). They implemented the three methods on the heat exchangers, pump, and turbine. When analyzing on a component level they found acceptable levels of accuracy for all methods when tested inside their nominal operational point. When testing the methods at component-level, but outside their nominal operational point, the CE method showed large errors. The RP method gave larger errors when coupling the components together and analyzing errors at a system level. The SE method had the best overall performance and acceptable accuracy on a system level and outside nominal operation. Salam K et al. [26] created a dynamic model of a PHE using a linearized energy balance equation that was then transformed into the Laplace domain. The model has been described by a first-order lead and second-order lag transfer function. They found that the model fits well enough with experimental data for the model to be validated. Two controllers were also created, a classic PID and a fuzzy controller. The conclusion was that the fuzzy controller had better performance.

2.5.2 Dynamic modeling of ORC

M. Yousefzadeh et al. [10] modeled a ORC system with a liquid tank that collects the liquid media between the condenser and the pump. The purpose of this tank is the ensure that the pump has a steady flow of media to its inlet. The semi-empirical method was used for the pump and turbine while the FV method was used to model the PHEs. In the PHE model, the density of the
media was modeled as a function of the pressure and enthalpy. Since the FV method was used, the mass flow in each defined cell in the PHE was modeled. It was thus possible to see that the mass contents of the evaporator and liquid tank were coupled as long as the liquid tank was not full. When the liquid tank was full it was observed that the mass contents of the condenser increased.

2.5.3 Dynamic modeling and controlling of ORC systems

Zhang et al. [8] modeled a ORC system and constructed a multivariable predictive control for it. They used the moving boundary method (MB) and made the assumption that there was no pressure drop in the media in the PHEs, which simplifies the calculations when creating the model. When modeling the turbine and pump a constant-efficiency method was used. The created constrained MPC was able to keep the relevant outputs within safe operational limits. M. Imran et al. [13] also developed a dynamic model of a ORC system using different methods and discussed their advantages and drawbacks. They stated that the heat exchanger is responsible for a majority of the lag of the system. They also conducted a comparison of different control methods for the ORC system, where they stated that the complexity of the controllers was dependent on whether the system was connected to the grid or not. Advanced controllers such as nonlinear MPC perform well because of the nonlinear nature of the system. The use of multiple controllers was discussed to decrease the computational time of the nonlinear MPC. The report concluded that the dynamic modeling of ORCs is still a relatively new field and no standard design rules are established yet.
Chapter 3

An investigation of modeling PHEs using the MB method

The MB method was chosen for further investigation as it was the simplest method among those mentioned in Section 2.4.1, with only a slight detriment in accuracy [19]. In this chapter, the theory behind the MB method will be described. First, certain necessary assumptions and simplifications are introduced. Subsequently, the fundamental balance equations are reformulated generically and then in detail for the single-phase regions. Finally, added complexities regarding the two-phase region is detailed.

3.1 In-depth description of the MB method

The series of plates that the media and secondary fluid flow through in the PHE would be very complex to model without any simplifications. The simplifications that are included in the MB method are:

1. The fluid flow through the PHE is one-dimensional.

2. There is no pressure drop in either fluid.

3. There are no external heat losses in the metal wall or either fluid.

With these assumptions, it is possible to model the PHEs as follows: an inner cylinder filled with the media is encased by a hollow, thin-walled metal cylinder, which is further surrounded by an outer cylinder containing the secondary fluid, see Figures 3.1 and 3.2. The cylinder is created so that the contact area between the media and metal wall is consistent with the actual PHE, the same goes for the secondary fluid. The volumes of the three cylinders
are also consistent with the actual volumes in the PHE. The assumption that the pressure is constant is not technically true. There is a pressure offset between input and output pressure. This pressure could be modeled by calculating the liquid column in the PHE. This cylinder is then divided lengthwise into three regions corresponding to each phase the media can be in: the sub-cooled liquid region, the saturated liquid/gas region, and the super-heated gas region. The boundary between the sub-cooled region and the saturated liquid/gas region is where the media starts to vaporize, which can be visualized in a p-h diagram such as in Figure 2.3 as the media being on the saturated liquid line. The boundary between the saturated liquid/gas region and the super-heated gas region is then on the saturated gas line in the p-h diagram. For a given pressure, the enthalpy at the boundary of each region is thus known by inspecting the p-h diagram. It is instead the lengths of the regions that change with time, hence the name "moving boundary". For each region, a total of five equations will be set up: A mass and energy equation for the media, an energy equation for the metal wall since there is no mass flow in the wall, and a mass and energy equation for the secondary fluid. The same mass and energy equation will be used for all three parts, but they will be integrated over their respective lengths resulting in different equations.
### 3.2 Inputs and outputs of the model

The variables that are the inputs and outputs in the PHEs depend on the models for the pump and turbine. In the literature, it is common to use the pump’s output enthalpy and mass flow and the turbine’s mass flow. This results in the PHE models outputting the pressure across the PHEs and the output enthalpy [21]. The pump model developed in this thesis outputs the enthalpy, mass flow, and pressure while the turbine model does not output a mass flow. So if a MB model were to be developed it would be more suitable to design it such that it takes the pump pressure as input and outputs the mass flow over the turbine instead. The pump and turbine models will be described in detail in Section 4.4. The inputs and outputs to the evaporator, using the MB method, can be seen in Figure 3.2. The inputs are the variables in green and the outputs in red. The 16 variables present in Figure 3.2 can be computed as a result of the 16 equations presented ahead in Section 3.6.

![Figure 3.2: A lengthwise cross-section of the cylinder model of the evaporator. The media can be seen in the inner cylinder in green, surrounded by the metal wall in gray, and encased by the hot water outer cylinder in red. Green variables are known, and red variables are unknowns that need to be calculated.](image)

### 3.3 Differential mass balance equation

A mass balance equation is needed for the mass flow in both the media and secondary fluid. However, the mass balance needs to be rewritten as a differential mass balance equation since it is used to integrate over each region
in the PHE. Starting from the standard mass balance equation, the derivation of the differential mass balance equation is as follows:

\[
\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}. \tag{3.1}
\]

The mass \( m \) can then be expressed as the product of the density \( \rho \) and volume \( V \) as \( m = \rho V \), where the volume \( V \) is the volume of either the inner cylinder with media or the outer cylinder with secondary fluid. The volume can also be expressed as the cross-sectional area of the inner or outer cylinder times the length of the cylinder, \( V = A_c z \), which gives,

\[
z A_c \frac{\partial \rho}{\partial t} = \dot{m}_{in} - \dot{m}_{out}. \tag{3.2}
\]

The partial derivative of the density \( \frac{\partial \rho}{\partial t} \) can, using the chain rule, be expressed as,

\[
\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial p} \frac{dp}{dt} + \frac{\partial \rho}{\partial h} \frac{dh}{dt}, \tag{3.3}
\]

where the enthalpy \( h \) is the mean enthalpy in the region. Since the pressure \( p \) is assumed to be constant through the entire PHE and the mean enthalpy is per definition constant in a region, \( \frac{\partial \rho}{\partial t} \) is not dependent on the length \( z \). Before integrating over each region, the derivative of (3.2) is taken with respect to the length of the corresponding region \( z \). In accordance with [8, 21, 27, 28], the differential mass balance equation is then,

\[
A_c \frac{\partial \rho}{\partial t} + \frac{\partial \dot{m}_{in}}{\partial z} = 0. \tag{3.4}
\]

### 3.4 Differential energy balance equation

The differential energy balance equation is derived similarly to the differential mass balance equation. Starting with the standard energy balance equation (2.6), the differential energy balance equation can be derived as,

\[
\frac{dH}{dt} = \frac{dQ}{dt} + \frac{d(pV)}{dt}. \tag{3.5a}
\]

Rewriting the enthalpy using the mass and specific enthalpy in addition to the volume \( V = \rho A_c z \), the equation can be stated as,

\[
\frac{d(mh)}{dt} = \dot{Q} + \frac{d(ppA_c z)}{dt}. \tag{3.5b}
\]
By using the product rule on both sides, the following is achieved:

\[ m \frac{dh}{dt} + \frac{dm}{dt} h = \dot{Q} + \frac{dp}{dt} \rho A_c z + p \frac{dz}{dt} \rho A_c, \quad (3.5c) \]

with the mass \( m = zA_c \rho \) and the \( \frac{dp}{dt} \) term moved to the left-hand side, we get

\[ zA_c \left( \rho \frac{dh}{dt} - \frac{dp}{dt} \right) + \dot{m} h = \dot{Q} + p \frac{dz}{dt} \rho A_c, \quad (3.5d) \]

with \( \dot{Q} = \pi D \alpha (T_w - T_m) \), where \( \alpha \) is the heat transfer coefficient which measures how much heat can be transferred per area and temperature difference, whose unit is given in \([\frac{W}{m^2 K}]\) and \( D \) is the diameter of the cylinder model of the PHE. Finally, differentiating (3.5d) with respect to \( z \), results in the differential energy balance equation, in accordance with [8, 21, 27, 28],

\[ A_c \frac{\partial (\rho h - p)}{\partial t} + \frac{\partial \dot{m} h}{\partial z} = \pi D \alpha (T_w - T_r). \quad (3.6) \]

### 3.5 Heat transfer coefficients

The heat transfer coefficient depends on which materials the heat is transferring between, the geometry of the heat exchanger, and the phases of the materials [29]. For each PHE in HP300 a total of four heat transfer coefficients need to be identified: one for the sub-cooled liquid region on the media side, one for the saturated liquid/vapor region, one for the super-heated gas region, and one for the secondary fluid side. This work found three methods of determining the heat transfer coefficients: the first option is to contact the heat exchanger manufacturers to see if they can supply the heat transfer coefficients. The second option involves setting the time derivative terms in all the differential balance equations to zero to simulate a steady-state for a given set of constant inputs and solving for the different \( \alpha \)s. The third option is to use heat transfer coefficient correlations that are developed and validated against testing data [29]. These correlations can vary greatly in both accuracy and complexity, generally, the more complex correlations are for two-phase flows such as in the saturated liquid/vapor region. The correlations often use dimensionless numbers such as the Reynolds number and the Prandtl number. They are numbers that describe the behavior of a fluid, which are dependent on parameters such as density, velocity, viscosity, and thermal conductivity [30, 31]. These correlations are then compared empirically with each other to find the one with the best accuracy.
3.6 Integration of balance equations

To get the final equations that describe the dynamics of the PHEs, the differential balance equations are integrated over the three regions separately. In each region, there is a mass and balance equation for the media, an energy equation for the wall, and a mass and energy equation for the secondary fluid. There is no need for a mass equation in the wall since there is no mass flow in the metal. Totaling five equations per region for a total of 15 equations for the entire PHE. A 16th equation ensures that the lengths of the regions add up to the total length of the PHE, \( L_1 + L_2 + L_3 = L_{\text{total}} \). These 16 equations are interconnected and are solved as a system of equations. From these equations, 16 states are calculated: The six temperatures of the wall and secondary fluid in each region. As well as the four mass flows between each region on both the media and the secondary fluid and the two output mass flows. Finally, the lengths of the three regions are also calculated. The calculation is similar for each region with the second region with saturated liquid/vapor being more complex due to the fact that a phase change is undergoing in that region. This will be described in more detail in Section 3.6.3.

3.6.1 Leibniz’s integral rule

The Leibniz’s integral rule is commonly used in the relevant literature when calculating the mass and energy balance equations of the PHE [21, 27, 32]. The rule will be used when integrating over the differential balance equations (3.4) and (3.6).

\[
\int_{\alpha}^{\beta} \frac{\partial f(x)}{\partial t} dz = \frac{d}{dt} \int_{\alpha}^{\beta} f(x)dz - f(\beta) \frac{d\beta}{dt} + f(\alpha) \frac{d\alpha}{dt}. \tag{3.7}
\]

3.6.2 Single phase region calculations

In this section, the mass and energy equations are derived for the sub-cooled region on the media side, as well as the energy equation for the wall. We start by taking the integral over the length of the sub-cooled region for each equation.
3.6.2.1 Media mass equation: sub-cooled region

We take the length integral of (3.4) with bounds 0 to $L_1$ since we are integrating over the first region,

$$
\int_0^{L_1} \left( A_c \frac{\partial \rho}{\partial t} + \frac{\partial \dot{m}}{\partial z} \right) dz = 0.
$$

(3.8a)

The mass flow term is easy to compute, and Leibniz’s rule is applied to the $\rho$ term to get,

$$
A_c \left\{ \frac{d}{dt} \int_0^{L_1} \rho dz - \rho_{12} \frac{dL_1}{dt} \right\} = \dot{m}_{in} - \dot{m}_{12},
$$

(3.8b)

where $\rho_1$ and $h_1$ are the mean density and enthalpy in region 1. The subscript of $\dot{m}_{12}$ symbolizes the mass flow at the boundary between the first and second regions. Solving the integral, derivative, and moving terms results in

$$
A_c \left\{ L_1 \frac{d\rho_1}{dt} + (\rho_1 - \rho_{12}) \frac{dL_1}{dt} \right\} = \dot{m}_{in} - \dot{m}_{12}.
$$

(3.8c)

$d\rho_1/dt$ can be expanded using (3.3). In the media sub-cooled region and all the secondary fluid regions, the density is independent of the pressure, see the $p$-$h$ diagram in Figure 2.2. This gives us

$$
A_c \left\{ L_1 (\frac{d\rho_1}{dh_1} \frac{dh_1}{dt}) + (\rho_1 - \rho_{12}) \frac{dL_1}{dt} \right\} = \dot{m}_{in} - \dot{m}_{12},
$$

(3.8d)

here $h_1$ is the mean enthalpy in the first region, which can be written as $\frac{h_{in} + h_{12}}{2}$. The saturation enthalpy $h_{12}$ is strictly dependent on the pressure in the PHE. Inserting these facts in the previous equation gives

$$
A_c \left\{ (\rho_1 - \rho_{12}) \frac{dL_1}{dt} + \frac{1}{2} L_1 \frac{d\rho_1}{dh_{12}} \frac{dh_{12}}{dp} \frac{dp}{dt} + \frac{1}{2} L_1 \frac{d\rho_1}{dh_{in}} \frac{dh_{in}}{dt} \right\} = \dot{m}_{in} - \dot{m}_{12}.
$$

(3.9)

This is the final mass balance equation for the media in the sub-cooled region. The calculations are similar to the mass balance equations for the secondary fluid. The main difference is that there is no saturation enthalpy in the secondary fluid since it never undergoes a phase change. So the output enthalpy of each region needs to be calculated for the secondary fluid.
3.6.2.2 Media energy equation: sub-cooled region

The energy equation is derived similarly to the mass equation (3.9). By integrating the differential energy balance equation (3.6) from 0 to \( L_1 \) for the first region as

\[
\int_0^{L_1} \left( A_c \frac{\partial (\rho h - p)}{\partial t} + \frac{\partial \dot{m} h}{\partial z} \right) \, dz = \int_0^{L_1} \left( \pi D_i \alpha (T_{w1} - T_{m1}) \right) \, dz. \tag{3.10a}
\]

By applying Leibitz’s rule on the left-hand side’s first term and simplifying the remaining terms we get

\[
A_c \left\{ \frac{d}{dt} \int_0^{L_1} (\rho h) \, dz - \rho_{12} h_{12} \frac{dL_1}{dt} + \frac{d}{dt} \int_0^{L_1} p \, dz - p \frac{dL_1}{dt} \right\} \\
+ \dot{m}_{12} h_{12} - \dot{m}_{in} h_{in} = L_1 \pi D_i \alpha_1 (T_{w1} - T_{m1}). \tag{3.10b}
\]

Solving the integrals and derivatives and then gathering terms gives us

\[
A_c \left\{ \left( \rho_1 h_1 - \rho_{12} h_{12} \right) \frac{dL_1}{dt} + \left( L_1 h_1 \frac{dp_1}{dh_1} + L_1 \rho_1 \frac{dh_1}{dt} + L_1 \frac{dp}{dt} \right) \right\} \\
+ \dot{m}_{12} h_{12} - \dot{m}_{in} h_{in} = L_1 \pi D_i \alpha_1 (T_{w1} - T_{m1}). \tag{3.10c}
\]

Substituting the mean enthalpy \( h_1 \) for \( \frac{h_{in} + h_{12}}{2} \) and sorting terms gives

\[
\frac{1}{2} A_c \left\{ \left( \rho_1 (h_{in} + h_{12}) - 2 \rho_{12} h_{12} \right) \frac{dL_1}{dt} + \left( \rho_1 L_1 + \frac{1}{2} L_1 (h_{in} + h_{12}) \frac{dp_1}{dh_1} \right) \frac{dh_{in}}{dt} \right. \\
+ L_1 \left( \rho_1 \frac{dh_{12}}{dp} + (h_{in} + h_{12}) \left( \frac{1}{2} \frac{dp_1}{dh_1} \frac{dh_{12}}{dp} - 2 \right) \frac{dp}{dt} \right) \right\} = \\
\dot{m}_{in} h_{in} - \dot{m}_{12} h_{12} + \pi D_i L_1 \alpha_1 (T_{w1} - T_{m1}), \tag{3.11}
\]

which is the final expression. Here \( D_i \) is the inner diameter of the cylinder and \( T_{w1} \) and \( T_{m1} \) are the mean temperatures of the wall and media in the first region respectively.

3.6.2.3 Wall energy equation: sub-cooled region

The differential energy balance equation (3.6) can be simplified for the wall since there is no mass flow [21, 27]. The simplified equation is written as

\[
c_w \rho_w A_w \frac{\partial T_w}{\partial t} = \alpha_{sf} \pi D_o (T_{sf} - T_w) + \alpha_1 \pi D_i (T_w - T_m), \tag{3.12a}
\]

\[
\frac{d}{dt} \int_0^{L_1} (\rho h) \, dz - \rho_{12} h_{12} \frac{dL_1}{dt} + \frac{d}{dt} \int_0^{L_1} p \, dz - p \frac{dL_1}{dt} \\
+ \dot{m}_{12} h_{12} - \dot{m}_{in} h_{in} = L_1 \pi D_i \alpha_1 (T_{w1} - T_{m1}). \tag{3.10b}
\]

\[
A_c \left\{ \left( \rho_1 h_1 - \rho_{12} h_{12} \right) \frac{dL_1}{dt} + \left( L_1 h_1 \frac{dp_1}{dh_1} + L_1 \rho_1 \frac{dh_1}{dt} + L_1 \frac{dp}{dt} \right) \right\} \\
+ \dot{m}_{12} h_{12} - \dot{m}_{in} h_{in} = L_1 \pi D_i \alpha_1 (T_{w1} - T_{m1}). \tag{3.10c}
\]

\[
\frac{1}{2} A_c \left\{ \left( \rho_1 (h_{in} + h_{12}) - 2 \rho_{12} h_{12} \right) \frac{dL_1}{dt} + \left( \rho_1 L_1 + \frac{1}{2} L_1 (h_{in} + h_{12}) \frac{dp_1}{dh_1} \right) \frac{dh_{in}}{dt} \right. \\
+ L_1 \left( \rho_1 \frac{dh_{12}}{dp} + (h_{in} + h_{12}) \left( \frac{1}{2} \frac{dp_1}{dh_1} \frac{dh_{12}}{dp} - 2 \right) \frac{dp}{dt} \right) \right\} = \\
\dot{m}_{in} h_{in} - \dot{m}_{12} h_{12} + \pi D_i L_1 \alpha_1 (T_{w1} - T_{m1}), \tag{3.11}
\]

which is the final expression. Here \( D_i \) is the inner diameter of the cylinder and \( T_{w1} \) and \( T_{m1} \) are the mean temperatures of the wall and media in the first region respectively.
where $c_w$ is the heat capacity of the wall, and $A_w$ is the cross-sectional area of the wall cylinder. $D_i$ and $D_o$ are the inner and outer diameters of the wall cylinder respectively. $\alpha_{sf}$ is the heat transfer coefficient between the secondary fluid and the wall. $T_{sf}$ and $T_m$ are the secondary fluid and media temperatures respectively. Continuing by integrating over the region length gives

$$
\int_0^{L_1} \left( c_w \rho_w A_w \frac{\partial T_w}{\partial t} \right) = L_1 \alpha_{sf} \pi D_o (T_{sf} - T_w) + L_1 \alpha_1 \pi D_i (T_w - T_m), \quad (3.12b)
$$

and using Leibnitz’s rule (3.7), the equation can be written as

$$
c_w \rho_w A_w \left\{ L_1 \frac{dT_{w1}}{dt} + (T_{w1} - T_w(L_1)) \frac{dL_1}{dt} \right\} = \alpha_1 \pi D_i L_1 (T_{r1} - T_{w1}) + \alpha_{sf} \pi D_o L_1 (T_{sf} - T_{w1}). \quad (3.13)
$$

### 3.6.3 Void coefficient in two-phase region

The media and wall equations for the saturated liquid/vapor region can be found in Appendix A. In the saturated liquid/vapor region the equation becomes more complicated since the vaporization causes the assumption that the enthalpy in a given region is linearly distributed to break down. This means that the mean enthalpy $h_2 \neq \frac{h_{12} + h_{23}}{2}$. Instead, the mean enthalpy is calculated using a so-called void-coefficient $\gamma$, where $h_2 = h_{12}(1 - \gamma) + h_{23}\gamma$. The mean density is also calculated using the same coefficient, as $\rho_2 = \rho_{12}(1 - \gamma) + \rho_{23}\gamma$. The void-coefficient $\gamma$ describes the gas-to-liquid ratio in the saturated liquid/vapor region. $\gamma = 0$ means that the entire region is liquid while $\gamma = 1$ means that the region is filled with gas. Jensen et al. calculated $\gamma$ with the use of some additional assumptions and a steady-state model of the PHE. They also used a slip velocity ratio that describes the ratio of liquid and gas velocities to model the void coefficient [21]. Cecchinato et al. discussed correlations to approximate the void coefficient [32]. Prasad Datta et al. presented a method for calculating the void coefficient by using the densities and velocities of both the liquid and gas to calculate the evaporation rate in the two-phase region [33].

### 3.7 MB method summary

The MB method models a PHE as a cylinder, that can be divided into three regions: the sub-cooled region, the saturated liquid/vapor region, and
the super-heated region. Each region symbolizes a part of the evaporating alternatively condensing process of the media in the heat exchanger. A total of 16 equations are solved to model the PHE, which gives an insight into the internal state of the PHE since the internal lengths of each region are calculated. The MB method was not used to model the PHEs in the final model of the system, the reasons for this are explained in Section 6. The modeling methods that were used to model the PHEs are described in Section 4.5.
Chapter 4

Modeling components in HP300

This chapter describes the different methods used to model each component in the HP300 system. The chapter starts by explaining how the components are connected and how the different component’s inputs and outputs are coupled. Then the data collection process is described followed by an explanation of how various functions were created by using fitting functions from data collected from the p-h diagram. Subsequently, the pump and turbine modeling methods are described including a description of how the efficiency curves are created for both the pump and turbine models. Finally, the modeling method for the two PHEs is explained.

4.1 Inputs and output to the model

As mentioned in Section 1.1, the system can be divided into 4 components: the pump, evaporator, turbine, and condenser. These components are all modeled separately and then coupled through their inputs and outputs. In Figure 4.1, the flow of both the media and the two secondary fluids is shown, as well as, each component’s inputs and outputs. The inputs and outputs of each component are further clarified in a set of lists in appendix C.

4.2 Data Collection

The relevant data was gathered from Climeon’s Cloud storage where they log all data from their systems. Since this work only aims to model the system in its operational state it was necessary to make sure that all valves were completely open during data gathering. The data sets range in length from 30 minutes to 3 hours and with a sample rate of 1 Hz. A total of 17 data
sets were gathered from Climeon’s Cloud storage. Out of these 17, three were excluded since the sensor measuring the input pressure of the condenser, \( P_4 \), was malfunctioning at the time of data collection. A potential solution that circumvents the malfunctioning sensor problem is discussed in Section 6. Most of the remaining 14 data sets had high-frequency noise, so a lowpass filter with normalized frequency \( \nu = 0.01 \) was applied to all relevant data signals to get a smoother signal. The filtering of the data can be seen in Figure B.2. In total, there are 14 data sets from the same HP300 machine collected over a period of four months.

### 4.3 Function fitting from graphs

Many functions had to be fitted during this thesis. Most notably the system curve for the pump, but also the p-h diagram. See Section 4.4.2 for a description of the system curve. Using the p-h diagram 2.2, it was possible to create functions that output density, enthalpy, and heat capacity depending on the pressure and temperature. This was heavily used in all parts of the modeling of the components since all models depend on those parameters. To create the functions from the p-h diagram a tool called WebPlotDigitizer was used [34]. With the tool, it was possible to export any lines from the p-h diagram as a data list with x and y coordinates. These coordinates were used to fit polynomial functions that then were used in the various models in the system. The polynomial functions were fitted using the Matlab function *fit,*
see Figures 4.2 and 4.3 that shows the functions for enthalpy and temperature respectively, in the super-heated region. The red lines in Figures 4.2 and 4.3 are similar to the temperature lines that can be seen in the p-h diagram in Figure 2.2.

![Super-heated enthalpy function](image)

Figure 4.2: This function takes the pressure and temperature of the super-heated media and returns the enthalpy according to the p-h diagram.

### 4.4 Pump and turbine models

As mentioned in Section 1.5, the dynamics of the heat exchangers are slower than that of the pump and turbine. This means that the pump and turbine can be modeled with static models i.e. assuming that the components are in steady-state [9, 10]. This simplifies the modeling significantly since there are now only a couple of simple equations that govern the behavior of the static components. Both the pump and turbine models will take the enthalpy and pressure at the inlet as inputs. The inputs are $h_1$, $P_1$ and $h_3$, $P_3$ respectively. The turbine will also have the outlet pressure $P_4$ as input from the condenser since the turbine model does not include the output pressure. The pump model also includes the speed of the pump $rpm_{pump}$ as an input. The enthalpies and pressures are outputs from the two PHEs models. The pump speed is used to control the operating point of the entire HP300

The pump and turbine models were created using different variations of the three static modeling methods mentioned in Section 2.3. The pump mass flow and output pressure were modeled using a pump performance curve, which will be explained in Section 4.4.2. The accuracy of the modeled outputs was
further improved using polynomial regressions to model the efficiency of the pump. The temperature increase caused by the pump is small and could thus be modeled with a constant efficiency for the sake of simplicity as in (2.3). The turbine model was modeled using semi-empirical and physical equations. The efficiencies of the outputs from the turbine model were also modeled using polynomial and nonlinear regressions, see Section 4.4.1.

### 4.4.1 Efficiency curves

The purpose of the efficiency curves is to increase the accuracy of the models. The efficiency curves are dependent on the pressure ratio over either the pump or turbine and are coefficients applied to the steady-state models, to increase accuracy. Each modeled output has a separate efficiency curve apart from the enthalpy output from the pump that has a constant efficiency, see Section 4.4.2 for the motivation behind that decision. Some outputs only achieve a slightly increased accuracy, while for other outputs the efficiency curves were absolutely necessary to achieve acceptable accuracy.

An efficiency curve is created using a model’s output from a set of data and the data itself. The efficiency curves are functions that are fitted over what is referred to as the ‘true’ efficiency of a model. The ‘true’ efficiency is calculated by dividing the modeled output by the actual output of a data set. In Figure 4.4 the ‘true’ efficiency of the pump mass flow was plotted against the pressure

Figure 4.3: This function takes the pressure and enthalpy of the super-heated media and returns the temperature according to the p-h diagram.
ratio for data set 4. Although the 'true' efficiency data is very spread out, several clusters can be distinguished from the data. The spread in the data is a result of step changes in the pump speed. The *kmeans* function in *Matlab* was used to identify the clusters, see red crosses in Figure 4.4. The number of clusters was manually selected based on each data set. When this is done for

![Figure 4.4: This Figure shows the 'true' efficiency in blue and the clusters created from the *kmeans* function in red.](image)

all 14 data sets, see Figure 4.5 a clear trend is observed that makes it possible to choose what type of function should be used to fit a function from the clusters. In this case, a first-order polynomial is the obvious choice. It should be noted that the pressure ratio over the pump, i.e. \( \frac{P_{out}}{P_{in}} \) is not an input to the pump model since the pump models the output pressure. So when calculating the pressure ratio for the pressure efficiency curve for the pump, the unaltered (no efficiency curve) output pressure is used.

### 4.4.2 Modeling of the pump

The pump model makes use of the pump performance curve which was supplied by the manufacturer. The pressure and mass flow are modeled using a performance curve. The performance curve, in Figure 4.6, has the volumetric flow \( \left[ \frac{m^3}{h} \right] \) on the x-axis and the head pressure generated by the pump on the y-axis [m]. Volumetric flow can be converted to mass flow by using (4.1). Head pressure is measured in meters and can be converted to pascal \( (Pa) \) with (4.2). With the head pressure \( H \), gravity \( g \), density \( \rho \), and the units displayed in the square brackets.
Figure 4.5: The fitted function for the mass flow efficiency curve for the pump model. Here all 14 data sets are used to create the fitted function.

\[
\dot{m} \text{[kg]} = \rho \cdot \dot{v} \left[ \frac{kg \cdot m^3}{60 \cdot 60 \cdot m^3 \cdot h \cdot 60 \cdot 60} \right] \tag{4.1}
\]

\[
P \text{[Pa]} = \left[ \frac{kg}{m \cdot s^2} \right] = H \cdot g \cdot \rho \left[ \frac{m \cdot m}{s^2} \cdot \frac{kg}{m^3} \right] \tag{4.2}
\]

The performance curve in Figure 4.6 contains a system curve that describes how the pump performs in that specific system, in this case, HP300. The system curve is created by running the pump in the desired system and recording the achieved pressure and mass flow for a set of pump speeds, both take only the pump speed as input. The black crossed points in the performance curve are the system curve. From these points, two fitted curves were created, one for the mass flow and one for the pressure difference. The fitted curves were created using Matlab’s fit function and the WebPlotDigitizer tool [34], see Figure 4.7. The pump mass flow and pressure output models were improved upon by using efficiency curves. The efficiency curves for the pump mass flow and the pressure output are found in Figures 4.5 and 4.8 respectively. The remaining output of the pump is the enthalpy. The enthalpy was not modeled using the performance curve and instead used some simple thermodynamic equations based on the fact that a 100% efficient pump (isotropic) would do the work,

\[
W_i = \frac{\dot{m}(P_{out} - P_{in})}{\rho}, \tag{4.3}
\]
with no work going toward increasing the temperature of the media. The non-isotropic work done by the pump can be described by,

\[ W_a = \dot{m}(h_{out} - h_{in}). \]  \hspace{1cm} (4.4)

Here the work goes towards increasing the enthalpy indicating that both the pressure and temperature are increasing. Since no pump is 100% efficient, the enthalpy efficiency of the pump can be written as the isotropic work divided by the actual work to get the enthalpy efficiency of the pump \( \epsilon_p \),

\[ \epsilon_{pump,h} = \frac{W_i}{W_a} = \frac{(P_{out} - P_{in})}{\rho(h_{out} - h_{in})}. \]  \hspace{1cm} (4.5)

(4.5) can be rewritten with the output enthalpy on the left-hand side, similarly to in [28], as,

\[ h_{out} = h_{in} + \frac{P_{out} - P_{in}}{\rho \epsilon_{pump,h}}. \]  \hspace{1cm} (4.6)

The enthalpy efficiency used was not dependent on the pressure ratio and was instead set to a constant of 0.96. This was done since the enthalpy difference over the pump is very small and sufficient accuracy was achieved with a constant efficiency.
4.4.3 Modeling of the turbine

No performance curve was used for the turbine so each output needs to be computed by physical or semi-empirical equations. In the turbine model the enthalpy output, mass flow, and power output are calculated. Although there is no mass flow sensor in the turbine to validate the mass flow model, it is still necessary to model it since the power output of the turbine is dependent on the mass flow through it. The enthalpy output was modeled with the semi-empirical equation (4.7) from [35, 36]. Here the input media is assumed as a super-heated gas and thus regarded as an ideal gas,

\[
h_{\text{out}} = h_{\text{in}} - \epsilon_{h,\text{turbine}} \left( \frac{P_{\text{in}}}{P_{\text{out}}} \right) C_p T_{\text{in}} \left[ 1 - \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right)^{\frac{1 - \gamma}{\gamma}} \right], \tag{4.7}
\]

where \( \epsilon_{h,\text{turbine}} \) is the turbine enthalpy efficiency and \( \gamma = \frac{c_p}{c_v} \), with \( c_p \) as the specific heat capacity for constant pressure and \( c_v \) as the specific heat capacity for constant volume [37]. In spite of this, \( \gamma \) was hand-tuned for better performance. The enthalpy efficiency curve for the turbine, seen in Figure 4.9, was fitted as a logistic curve,

\[
\epsilon_{h,\text{turbine}}(PR) = \frac{a}{1 + be^{c PR}}, \tag{4.8}
\]

where \( PR \) is the pressure ratio over the turbine and \( a, b, c \) are tuned to fit the clusters. A logistic function is plotted in Figure D.1.
Figure 4.8: The pump pressure efficiency curve was fitted as a second-order polynomial with sufficient accuracy.

Figure 4.9: The turbine enthalpy efficiency curve. Although the fit does not look too good the performance of the turbine enthalpy output is very accurate. Thus no effort was put into improving the fit.

The turbine mass flow was modeled using the semi-empirical Stodola equation used in [36, 38],

\[
\dot{m}_t = K_s \left( \frac{P_{in}}{P_{out}} \right) \sqrt{\frac{P_{in}}{\rho_{in}} P_{in} \left[ 1 - \frac{P_{out}}{P_{in}} \right]^2},
\]

(4.9)

where \( K_s \) is the mass flow efficiency that is dependent on the pressure ratio. \( K_s \) is the product of the cross-sectional area at the turbine inlet and what is called in [36] as the coefficient of discharge.
Figure 4.10: The turbine mass flow efficiency curve. Apart from three outlier clusters, the curve has a very good fit.

The power equation is just the difference in specific enthalpy times the mass flow to get the difference in total enthalpy/energy flow over the turbine. Energy flow, i.e. energy over time is the power extracted from the turbine. Multiplying with the power efficiency $\epsilon_W$ gives the power $W$ generated by the generator [36],

$$W = \epsilon_W \left( \frac{P_{in}}{P_{out}} \right) \dot{m}(h_{in} - h_{out}). \quad (4.10)$$

The turbine output power efficiency curve seen in Figure 4.11 is fitted using a logistic function as seen in (4.8).

Figure 4.11: The turbine power efficiency curve. Although the fit seems to be a good fit, the steep slope starting at a pressure ratio of $\approx 1.75$ is one of the main drawbacks in the system model.
4.5 PHE Modeling

In this thesis, the MB method was not used to model for the two PHEs. The reason for this is discussed in Chapter 6. The models were instead created using data from the 14 data sets gathered from Climeon’s Cloud storage. A linear state-space model was created using the \textit{n4sid} algorithm introduced in Section 2.4.2.1. The order of the models was decided by analyzing the Hankel singular values. Using additional states with low singular values does not increase the accuracy of the model [23]. A model order of 6 was chosen. The inputs to the evaporator are the pump output temperature, pressure, and mass flow as well as the hot water input temperature, pressure, and mass flow. Similarly, the condenser inputs are the turbine output temperature, the pump mass flow, and the cold water temperature, pressure, and mass flow. The models for the PHEs are discrete state-space models on the form,

\begin{align}
    x_{k+1} &= Ax_k + Bu_k, \quad (4.11) \\
    y_{k+1} &= Cx_k + Du_k. \quad (4.12)
\end{align}

This means that the two PHE models calculate their respective outputs for the next timestep. This is not true for the pump and turbine models, which both calculate their outputs for the current timestep.

4.6 Coupling components

The components were connected together with their inputs and outputs as described in Section 4.1 and Appendix C. Since the model for the evaporator and condenser were modeled as discrete state-space models, their outputs were used in the pump and turbine inputs in the next timestep. The outputs from the models for the pump and turbine were used as the inputs for the evaporator and condenser at the same timestep.
Chapter 5

Results and Analysis

This chapter presents the results of simulations of the modeled system. The chapter starts with a description of how the models are simulated and how the model errors are calculated. The chapter then continues to present simulations of each component separately as well as the entire system in order to see how each component is affected by errors in the other components. The system results are also presented for data sets with a high turbine pressure ratio. The chapter concludes with an analysis of a simulation of a specific data set.

5.1 Simulation set-up

5.1.1 Cross-validation

Since the PHE models are data-driven models and the models for the pump and turbine are partially data-driven, it was important to make sure that the models use different training data than the data used to test and validate the models. In the simulation environment for the system, the 14 data sets could be separated into a training data set and a validation data set. This separation was done randomly and many times to show how the system behaved depending on what data was used to train and test it, which is a method called cross-validation. For most simulations, the total data was divided into 10 training data sets and 4 validation data sets. This means that 71% of the data was used for training and the remaining 29% was used for validation. This is done to show the model’s ability to predict the outputs from new data.
5.1.2 Error measurements

When simulating the modeled system the mean error was measured in all 12 variables. The error of variable $X$ in a simulation was measured as $\frac{|X_{\text{data}} - X_{\text{model}}|}{X_{\text{data}}}$.

5.2 Individual component results

In this section all components are modeled separately, meaning that they all take their respective inputs from the actual data instead of each other. Here the 100 simulation average performance of the four modeled components of the HP300 system is presented. In the following figures, the results of each data set are displayed as a colored set of dots to ease the analysis of the results. Each dot represents the mean error of a simulation in the given variable for a certain cross-validation setup.

5.2.1 Pump results

![Pump Errors](image)

Figure 5.1: Average errors when simulating the pump model 100 times. $P2$ and $T2$ refer to the pump output pressure and temperature respectively.

The average pump performance is shown in Figure 5.1. It can be seen that the pump model performs very well with no error over 5%. A very small
temperature $T_2$ error is observed since the difference in temperature over the pump is very small to begin with. The mass flow $m_{\text{pump}}$ and pressure output $P_2$ are both derived from the pump performance curve which results in a good correspondence to the data.

5.2.2 Evaporator results

![Evaporator Errors](image)

Figure 5.2: Average errors when simulating the evaporator model 100 times. $P_3$ and $T_3$ refer to the evaporator output pressure and media temperature respectively. $T_{h,\text{out}}$ is the hot water output temperature.

When modeling the evaporator, errors similar to that of the pump are observed for the pressure $P_3$ and temperature $T_3$, with a large spread of the error in hot water temperature $T_{h,\text{out}}$ as shown in Figure 5.2. This means that some combinations of training and validation data cause the model to perform poorly. This indicates that the hot water output temperature dynamics are more complicated than a linear state-space can capture and that it needs to be modeled with a more complex modeling method. Since the hot water output is not an input to any other component this error will not propagate in the system. The data set in light blue has a significantly higher error than the rest. This is data set 5 and it has the lowest pressure ratio over the turbine out of all data sets.
5.2.3 Turbine results

The output temperature $T_4$ error of the turbine is small but the power output $W_{\text{turbine}}$ error can get very large, see Figure 5.3. The power efficiency curve has a very steep slope for pressure ratios under $\approx 1.75$, see Figure 5.9. This gives poor accuracy for low pressure ratios since the fit for the power efficiency is not perfect and a small error in the fit is amplified due to the slope. All data sets with power errors above 20% have a pressure ratio under 1.75. This error is magnified when the inlet and outlet pressures of the turbine are also modeled by the two PHE models in Section 5.4.3.

5.2.4 Condenser results

In Figure 5.4, the errors in the condenser are generally larger than in the other components, which is mainly due to the condenser model not having the inlet pressure $P_4$ as an input to the model as the evaporator has. Instead, the condenser models both the inlet and outlet pressures, $P_4$ & $P_1$. This is because the turbine model depends on the pressure ratio over the turbine, and the outlet pressure of the turbine is thus not included in the turbine model. The condenser models the pressures better than the media and cold water output temperature.
5.3 Component’s results summarized

In Table 5.1 the component results are presented with the minimum, average, and maximum errors for each of the 12 variables.

Figure 5.4: Average errors when simulating the condenser model 100 times. $P_1$ and $P_4$ are the inlet and outlet pressure respectively. $T_1$ is the outlet temperature of the media and $T_{c,out}$ is the cold water outlet temperature.

which is good since the turbine is heavily dependent on the accuracy of the pressures of the system.
<table>
<thead>
<tr>
<th>Variables</th>
<th>Min %</th>
<th>Median %</th>
<th>Mean %</th>
<th>Max %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>0.7</td>
<td>2.9</td>
<td>3.6</td>
<td>11.3</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>2.1</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.3</td>
<td>0.7</td>
<td>0.9</td>
<td>5.5</td>
</tr>
<tr>
<td>$P_4$</td>
<td>0.6</td>
<td>3.1</td>
<td>3.9</td>
<td>13.9</td>
</tr>
<tr>
<td>$T_1$</td>
<td>0.9</td>
<td>5.2</td>
<td>7.8</td>
<td>31.4</td>
</tr>
<tr>
<td>$T_2$</td>
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<td>0.3</td>
<td>0.3</td>
<td>0.7</td>
</tr>
<tr>
<td>$T_3$</td>
<td>0.5</td>
<td>1.1</td>
<td>1.2</td>
<td>2.0</td>
</tr>
<tr>
<td>$T_4$</td>
<td>0.1</td>
<td>0.7</td>
<td>0.8</td>
<td>2.0</td>
</tr>
<tr>
<td>$m_{pump}$</td>
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<td>2.0</td>
<td>2.0</td>
<td>4.2</td>
</tr>
<tr>
<td>$W$</td>
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<td>2.3</td>
<td>5.0</td>
<td>74.3</td>
</tr>
<tr>
<td>$T_{h,\text{out}}$</td>
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<td>3.4</td>
<td>4.8</td>
<td>23.1</td>
</tr>
<tr>
<td>$T_{c,\text{out}}$</td>
<td>1.1</td>
<td>4.1</td>
<td>5.1</td>
<td>17.9</td>
</tr>
</tbody>
</table>

Table 5.1: Minimum, median, mean, and maximum errors for simulation of the individual components separately for all 12 variables. Average error: 3.0%
5.4 System results

Here the simulation results of the entire system are presented. In general, the errors for all components are slightly larger with the turbine and condenser being affected the most since they were the most sensitive to errors in other components.

5.4.1 Pump results

The average system performance of the pump is similar to its performance when simulating the pump separately. The main difference is the large spread in error for the output temperature ($T_2$). This is entirely caused by the condenser output temperature’s ($T_1$) error in Figure 5.8. Since the pump model models the difference in enthalpy over the pump, $\Delta h$, the error in the condenser is passed on to the pump because the pump model simply calculates the output enthalpy as, $h_{out} = h_{in} + \Delta h$. In the sub-cooled region that the pump operates in, the enthalpy and temperature are directly related. Notice the higher maximum error in data set 5 (light blue, fifth from the left): the error from the turbine for low pressure ratios propagates to the pump, see Section 5.2.3.

![Figure 5.5: System simulation, pump errors](image)
5.4.2 Evaporator results

The evaporator also performs similarly in the system simulations as it does when it is simulated separately, apart from data sets with lower pressure ratio over the turbine. Additionally, although the condenser temperature error is passed to the pump output temperature, it seems like the pump output temperature is not a significant input to the evaporator output temperature as can be seen in Figure 5.6, since the evaporator has a small error in output temperature. Instead, the hot water output temperature has a large error. This is likely due to the fact that the media output temperature reaches the same temperature as the hot water input temperature so it does not matter what input temperature the media has. The error is thus passed to the hot water output temperature. This can be seen as a good thing for the overall accuracy of the model since the condenser media output error does not propagate through the system and feedback to itself.

5.4.3 Turbine results

The output temperature of the turbine model has a slightly larger mean error when the complete system is modeled. The power output also has a larger mean error which is expected but interestingly a slightly smaller maximum
Figure 5.7: System simulation, turbine model errors

error, see Figure 5.7. This could be due to the randomness in the cross-validation or it could be the case that the inevitable errors in the PHE models cause the modeled pressure ratio to differ from the real value resulting in a cancellation of errors. The pressure ratio error could compensate for the error in fitting of the power efficiency curve.

5.4.4 Condenser results

The errors of the condenser model when simulating the complete system are also increased when compared with simulating it alone, see Figure 5.8. The overall poor performance of the condenser model is due to the fact that the condenser model does not have a pressure input, meaning that it has less information to predict the outputs than the evaporator has. It should be added that the average performance of the condenser is much lower than the maximum, indicating that some combinations of training and validation data have poor performance while other combinations perform better. This is discussed further in Section 7.2 about future work.
5.5 System results summarized

Here a summary of the results presented above can be found, see table 5.2. Most variables have an unfortunately large maximum error but a mean error that is much lower. This means that while some simulations performed poorly, the majority performed closer to the mean. This can be further visualized by observing that the median is below the mean in all result figures meaning that a majority of the simulations have an error below the mean. The overall performance of the system model can be summarized with the mean error in all 12 variables, the total mean error is then 4.0%.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Min %</th>
<th>Median %</th>
<th>Mean %</th>
<th>Max %</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
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<td>3.7</td>
<td>14.8</td>
</tr>
<tr>
<td>P2</td>
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<td>1.5</td>
<td>1.8</td>
<td>5.7</td>
</tr>
<tr>
<td>P3</td>
<td>0.4</td>
<td>1.8</td>
<td>2.0</td>
<td>9.6</td>
</tr>
<tr>
<td>P4</td>
<td>0.5</td>
<td>3.2</td>
<td>4.1</td>
<td>16.6</td>
</tr>
<tr>
<td>T1</td>
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<td>5.2</td>
<td>7.1</td>
<td>26.0</td>
</tr>
<tr>
<td>T2</td>
<td>1.0</td>
<td>5.4</td>
<td>7.2</td>
<td>25.7</td>
</tr>
<tr>
<td>T3</td>
<td>0.5</td>
<td>1.1</td>
<td>1.1</td>
<td>2.0</td>
</tr>
<tr>
<td>T4</td>
<td>0.3</td>
<td>1.7</td>
<td>1.7</td>
<td>4.7</td>
</tr>
<tr>
<td>mpump</td>
<td>0.3</td>
<td>2.3</td>
<td>2.4</td>
<td>5.0</td>
</tr>
<tr>
<td>Wturbine</td>
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<td>5.6</td>
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<td>Tc,out</td>
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<td>5.5</td>
<td>23.8</td>
</tr>
</tbody>
</table>

Table 5.2: Minimum, median, mean, and maximum errors for simulation of the complete system model for all 12 variables. Average error: 4%

5.6 High pressure ratio simulation results

The turbine model has very large errors for data sets with a low pressure ratio (< 1.75). When the system model is simulated only using data with a high turbine pressure ratio (> 1.75), better accuracy is observed. In Figure 5.9 the data sets were sorted into high or low pressure ratios. The overlap in red and green pressure ratio clusters is due to the fact that some data sets have multiple clusters, some above and some below a pressure ratio of 1.75. Data sets with pressure ratio clusters both below and above 1.75 are classified as low pressure ratios. The high pressure ratio data sets are in green while the low ratios are in red. The simulation results when the validation data is exclusively the high pressure ratio data are represented in the following figures.
Figure 5.9: The power efficiency of the turbine, showing the distinction between high (green) and low (red) pressure ratio clusters.

5.6.1 Pump results: high pressure ratio

Figure 5.10: High pressure ratio system results for the pump model
The pump results with only high pressure ratio data are very similar to the results with all data. This is because the pump model has very good accuracy relative to the other models. In Figure 5.10 the pump results are shown and it can be seen that the remaining data sets show smaller mean and maximum errors, meaning that the errors have a smaller spread. The large error in output temperature $T_2$ is passed on from the condenser output temperature.

### 5.6.2 Evaporator results: high pressure ratio

![Evaporator Errors](image)

Figure 5.11: High pressure ratio system results for the evaporator model

The difference in evaporator results when removing the high pressure ratio data set is similar to that of the pump results; smaller errors with a tighter spread but to a greater extent since the errors in the evaporator model were larger, to begin with.
5.6.3 Turbine results: high pressure ratio

The turbine model performs significantly better for data with high pressure ratios due to the power efficiency curve being less sensitive to pressure ratio modeling errors. The mean error of the power was decreased from 7.2% to 6.2% when excluding low pressure ratio data sets from the validation data. The maximum error decreased from 70.4% to 16.6%. In Figure 5.9 it can be seen that for high pressure ratios the power efficiency curve is flat, so power modeling errors $W_{turbine}$ are caused by inaccurate curve fitting of the power efficiency.

Figure 5.12: High pressure ratio system results for the turbine model
5.6.4 Condenser results: high pressure ratio

The most interesting result from the high pressure ratio simulations is that of data set 1. It regularly has some of the larger errors in the remaining data sets even though it has the highest pressure ratio of all data. This might be exactly the reason for the larger errors. Some dynamics present for larger pressure ratios might not have been captured in the data apart from data set 1, making it difficult to model using the other data sets.

5.7 High pressure ratio simulation results summarized

Simulation of only data sets with a pressure ratio $> 1.75$ generally results in lower maximum and mean errors. This is especially true for the turbine model power output which saw a four-fold decrease in modeling error. In Table 5.3 the minimum, median, mean, and maximum errors of all variables for high pressure ratio simulations. Additionally, data set 1 was observed to give higher errors than the other data sets in the condenser model. Data set 1 has the highest pressure ratio of all data sets. The condenser model has fewer inputs than the evaporator model which could result in the model not being
able to capture the dynamics of the condenser when modeling outlier data, leading to a larger error.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Min %</th>
<th>Median %</th>
<th>Mean %</th>
<th>Max %</th>
</tr>
</thead>
<tbody>
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<td>1.8</td>
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</tr>
<tr>
<td>$P_4$</td>
<td>0.7</td>
<td>3.7</td>
<td>4.8</td>
<td>14.9</td>
</tr>
<tr>
<td>$T_1$</td>
<td>0.5</td>
<td>6.8</td>
<td>8.6</td>
<td>20.9</td>
</tr>
<tr>
<td>$T_2$</td>
<td>0.6</td>
<td>7.2</td>
<td>8.7</td>
<td>20.9</td>
</tr>
<tr>
<td>$T_3$</td>
<td>0.6</td>
<td>1.0</td>
<td>1.1</td>
<td>1.5</td>
</tr>
<tr>
<td>$T_4$</td>
<td>0.5</td>
<td>1.8</td>
<td>2.0</td>
<td>4.7</td>
</tr>
<tr>
<td>$m_{pump}$</td>
<td>0.5</td>
<td>2.5</td>
<td>2.5</td>
<td>4.2</td>
</tr>
<tr>
<td>$W_{turbine}$</td>
<td>1.6</td>
<td>5.3</td>
<td>6.2</td>
<td>16.6</td>
</tr>
<tr>
<td>$T_{h,out}$</td>
<td>0.3</td>
<td>2.8</td>
<td>2.7</td>
<td>6.1</td>
</tr>
<tr>
<td>$T_{c,out}$</td>
<td>1.7</td>
<td>4.4</td>
<td>6.2</td>
<td>19.4</td>
</tr>
</tbody>
</table>

Table 5.3: High pressure ratio system simulation results. Total average error: 4.2%

5.8 System simulation example

In this section, the results for the simulation of data set 3 are presented. The training and validation data sets were data sets 1, 2, 4, 5, 6, 7, 8, 9, 11, 12, and 3, 10, 13, 14 respectively. Data set 3 is considered a high pressure ratio data set. Relevant plots are shown and analyzed. The average error in each variable can be seen in the title of each simulation plot.

5.8.1 Pump simulation

In Figure 5.14 the pump model performs well with the exception of the pump output temperature $T_2$ which is a result of the condenser output temperature $T_1$ error, see Figure 5.16.
5.8.2 Evaporator simulation

The evaporator simulates data set 3 with good accuracy in Figure 5.15. In the evaporator model outputs, a brief transient period is observed at the start of the simulation, which is caused by the fact that the internal states in the evaporator model were not a perfect representation of the real state of the evaporator at the beginning of the simulation. A better initialization of the evaporator model would limit this behavior. Similar behavior can be seen in the condenser model in Figure 5.16.
Figure 5.15: The evaporator model’s outputs are compared to the actual outputs from data set 3.

### 5.8.3 Condenser simulation

The condenser model models the pressures $P_1$ and $P_4$ quite well with a slightly larger error in the cold water output temperature $T_{c,\text{out}}$, see Figure 5.16. The media output temperature $T_1$ has a large error which indicates that the dynamics behind the media temperature are not captured in the linear state-space model.
Figure 5.16: The condenser model’s outputs are compared to the actual outputs from data set 3.

5.8.4 Turbine simulation

The turbine model performs better than average with a mean power error of 5.6%. Although the pressure ratio in Figure 5.18 is not modeled perfectly, the turbine power output error does not depend significantly on the pressure ratio since the ratio is above 1.75. The turbine power error can instead be explained by analyzing the turbine power efficiency curve in Figure 5.9. It is for pressure ratios $\approx 1.75$ and lower that have an extremely sensitive power efficiency curve because of the steep slope of the curve.
Figure 5.17: The turbine model’s outputs are compared to the actual outputs from data set 3.

Figure 5.18: The modeled and true pressure ratio over the turbine for simulation of data set 3.
Chapter 6

Discussion

In this chapter, some of the decisions made in this work are discussed, along with changes that could have been made to improve the accuracy of the model and finally analyzing the method used when creating the efficiency curves.

Choice of data-driven modeling method

A linear state-space model was created to model the system’s two PHEs. Although the actual dynamics would be better captured by a nonlinear model, state-space models were chosen for their relatively good accuracy along with the fact that they are computationally inexpensive to create from a set of data. Nonlinear ARX models were briefly compared with state-space models and it was concluded that the nonlinear ARX models had better accuracy. However, the time to create the nonlinear ARX models was magnitudes slower than that of the state-space models. This would make cross-validation and testing infeasible to do with a nonlinear ARX model. The state-space model was thus chosen as the data-driven modeling method.

Multiple linear models

Instead of creating a nonlinear model. It might be possible to divide the data into similar segments with similar dynamics. From these segments, a set of linear models could be created. A method for evaluating which segment a given data set belonged to would then need to be developed, then depending on the segment the appropriate linear model could be used.
**Attempt to improve PHE model performance**

One attempted way of improving the performance of the evaporator and condenser models was to divide each output into separate models and to model one output with the other outputs as inputs. This seemed to improve performance when validating the models by using purely the output data. But when the actual modeled output was used as input for the other outputs, large oscillations occurred that could output negative pressures and in some cases cause the entire model to crash. This implementation was not used in the final models.

**Switching sensor for condenser inputs pressure**

Three data sets were excluded from being used in training the models in this thesis. This was done since the pressure sensor used to measure the inlet pressure of the condenser was not working correctly leading to faulty data. The HP300 system has multiple sensors that measure the pressure along the pipe between the components. A different sensor could have been used to be able to include the previously excluded data. This could have led to better performance and validity, especially during cross-validation. See Figure B.1.

**Mass flow in system**

The turbine mass flow efficiency curve is created with the use of the pump mass flow data since there is no data on the turbine mass flow. Because the mass flow modeled by the pump model is very accurate it might be wise to discard the modeled turbine mass flow and instead use the pump mass flow in the model for the power output of the turbine. This was done for the condenser model. This would require the assumption that the mass flow is uniform in the entire system.

**Efficiency curve clustering**

As can be seen in Figures 6.1 and 6.2, the \texttt{kmeans} function has limitations when it comes to clustering the 'true' efficiencies. This affects the final efficiency curves which could decrease the accuracy of the models. A more advanced method for calculating the efficiency curves could increase accuracy.
Figure 6.1: Pump mass flow efficiency curve alongside the 'true' efficiency from data set 9 and the cluster calculated by the \textit{kmeans} function.
Figure 6.2: Pump pressure efficiency curve alongside the 'true' efficiency from data set 9 and the cluster calculated by the *kmeans* function.
Chapter 7

Conclusions and Future work

In this chapter, the conclusion is presented along with potential future directions to improve or build upon the designed model presented in this thesis.

7.1 Conclusions

The main goal of this thesis was to create a dynamic model of the ORC system, HP300. This was achieved with two steady-state models for the pump and turbine, along with two linear state-space models created from data from the evaporator and condenser. These four models were connected to form the complete system model. The thesis started with a pre-study phase where relevant literature was studied to get a good grasp of how general ORCs systems work, the relevant thermodynamics of ORCs, and the various components in the HP300 system. This was followed by researching relevant work where similar ORCs were modeled.

Modeling of ORC systems is a complex endeavor. In this thesis, an investigation into the MB method was conducted. The MB method is an advanced heat exchanger modeling method based on physical balance equations. A simpler data-driven method was implemented and evaluated. At the start of this thesis, the goal was to implement the MB method. However, halfway through it was concluded that the MB method was too complex and time-consuming for this thesis. The data-driven method was then researched and implemented.

The results of the modeled system have varied accuracy, depending on which data set was simulated. The average error in the model across all outputs over multiple simulations was 4.2%. The maximum error in the final model was in
the condenser cold water output \( T1 \) at 20.9%. The main output is the power produced by the turbine. The final model had an average power error of 6.2% and had a maximum of 16.6%. If this project were to be conducted again, it would be wise to start with the simpler method, and then if time allows a more advanced method could be tried. Perhaps the most important output of the model is the power produced by the turbine, which has a low accuracy for low pressure ratios. The designed model can be used in software development by the Climeon software team, but for other applications, the accuracy might not be sufficient and can instead be seen as a proof of concept to be improved upon.

### 7.2 Future work

Future work can be divided into two approaches. Further investigation and eventually implementation of a MB method for the evaporator and condenser. Or to improve the performance of the data-driven models developed in this thesis. The available data could be analyzed and divided into sections and linear models could be created for each section, or a nonlinear model could be created for the entire data set. Work could also go towards improving upon the turbine model to reduce the error by using a different fitting function for the power efficiency curve or choosing a different method entirely.
References


[27] X. Ma, P. Jiang, and Y. Zhu, “Dynamic simulation model with virtual interfaces of supercritical working fluid heat exchanger


[33] S. Datta, R. Chandra, and P. Das, “An appraisal of system mean void fraction and its application for the moving boundary simulation of phase-


Appendix A

Saturated liquid/vapor equations

Media: mass equation

\[ A_{eq}\left\{ (\rho_1 - \rho_{12})\frac{dL_1}{dt} + (1 - \gamma)(\rho_{12} - \rho_{23})\frac{dL_2}{dt} \\
+ (\gamma \frac{d\rho_{23}}{dp} + (1 - \gamma)\frac{d\rho_{12}}{dp})\frac{dp}{dt} \right\} = \dot{m}_{12} - \dot{m}_{23} \quad (A.1) \]

Media: energy equation

\[ A_{eq}\left\{ L_2\left( \gamma \frac{d\rho_{23}h_{23}}{dp} + (1 - \gamma)\frac{d\rho_{12}h_{12}}{dp} - 1 \right)\frac{dp}{dt} \\
+ \left( \gamma \rho_{23}h_{23} + (1 - \gamma)\rho_{12}h_{12} \right)\frac{dL_1}{dt} \\
+ \left( (1 - \gamma)(\rho_{23}h_{23} + \rho_{12}h_{12}) \right)\frac{dL_2}{dt} = \right\} = \dot{m}_{12}h_{12} - \dot{m}_{23}h_{23} + \pi D_{eq}L_2\alpha_2(T_{w2} - T_r2) \quad (A.2) \]

Wall: energy equation

\[ C_w\rho_wA_w\left\{ L_2\frac{dT_w^2}{dt} + (T_w(L_1) - T_{w2})\frac{dL_1}{dt} + (T_{w2} - T_w(L_1 + L_2))\frac{dL_1}{dt} \right\} \]
\[ = \alpha_3\pi D_{eq}L_2(T_{r2} - T_{w2}) + \alpha_w\pi D_wL_2(T_{h2} - T_{w2}) \quad (A.3) \]
Figure B.1: Malfunctioning input pressure sensor to the condenser (blue) and the other functioning sensor (orange). Both sensors measure the pressure $P_4$ but the sensor functioning has slightly different placement in the machine, resulting in the offset in pressure.
Figure B.2: The lowpass filter removes unwanted noise from the signal. The unfiltered signal is in blue and the filtered signal is in red.
Appendix C

System model input & output lists

C.1 Pump inputs and outputs

The inputs to the pump model are:

- Input 1: condenser output pressure ($P_1$)
- Input 2: condenser output enthalpy ($h_1$)
- Input 3: pump speed ($\text{rpm}_{\text{pump}}$)

The outputs from the pump model are:

- Output 1: pump output pressure ($P_2$)
- Output 2: pump output enthalpy ($h_2$)
- Output 3: pump mass flow ($\dot{m}_{\text{pump}}$)

C.2 Evaporator inputs and outputs

The inputs to the evaporator model are:

- Input 1: pump output pressure ($P_2$)
- Input 2: pump output enthalpy ($h_2$)
- Input 3: pump mass flow ($\dot{m}_{\text{pump}}$)
- Input 4: hot water pressure ($P_h$)
• Input 5: hot water input enthalpy ($h_{in}$)
• Input 6: hot water mass flow ($\dot{m}_c$)

The outputs from the evaporator model are:
• Output 1: evaporator media output pressure ($P_3$)
• Output 2: evaporator media output enthalpy ($h_3$)
• Output 3: hot water output enthalpy ($h_{out}$)

C.3 Turbine inputs and outputs

The inputs to the turbine model are:
• Input 1: evaporator output pressure ($P_3$)
• Input 2: evaporator output enthalpy ($h_3$)
• Input 3: condenser input pressure ($P_4$)

The outputs from the turbine model are:
• Output 1: turbine output enthalpy ($h_4$)
• Output 2: power generated by the turbine ($W$)

C.4 Condenser inputs and outputs

The inputs to the condenser model are:
• Input 2: turbine output enthalpy ($h_4$)
• Input 3: pump mass flow ($\dot{m}_{pump}$)
• Input 3: cold water pressure ($P_c$)
• Input 4: cold water enthalpy ($h_{cin}$)
• Input 5: cold water mass flow ($\dot{m}_c$)

The outputs from the condenser model are:
• Output 1: condenser media inlet pressure ($P_4$)
• Output 2: condenser media output pressure \( P_1 \)
• Output 3: condenser media output enthalpy \( h_1 \)
• Output 4: cold water output enthalpy \( h_{cout} \)
Appendix D

Efficiency related figures

D.1 Logistic function

Figure D.1: A basic logistic function. This function type was used to fit some efficiency curves in the turbine model.