Improving the Dynamic Design Philosophy of High-Speed Railway Bridges Using Reliability-Based Methods

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Abstract

Modern railway infrastructures, especially bridges, are exposed to significant vibrations with potential safety implications. In this context, previous studies have shown the inconsistency and inadequacy of some conventional design methods necessitating them to be improved. The assessment of safety inherently deals with uncertainties. Therefore, the current study is dedicated to this objective using reliability-based methods. Of the various possible failure modes, the investigations presented here are limited to running safety and passenger comfort. The investigation of these limit-states requires constructing complex computational models with train-track-bridge interaction capabilities. However, the application of these computationally intensive models in the context of structural reliability does not appear to be feasible. Simplifying the system, the vertical acceleration and the deflection of the bridge serve as implicit limit-state measures. Initially, using First Order Reliability Method (FORM) revealed limitations in the application of the current safety factor, resulting in inconsistent reliability indices. Therefore, probabilistic design curves are proposed, defining minimum required bridge mass and stiffness based on cross-section types, span configurations and train speeds. These results are obtained by formulating a FORM-based optimization. Subsequently, the results are used to investigate the sensitivity of the estimated failure probabilities with respect to the contributing basic random variables. Acknowledging the limitations of FORM, surrogate-assisted simulation-based reliability assessments were used for further investigations. A comparison of the performance of widely used regression-based surrogate models under an identical active learning scheme showed the superior performance of the Kriging method over the others. Within a reliability-based design optimization framework, this Kriging model facilitates the generation of new probabilistic design curves. This is achieved by reformulating the conventional method to account for the dependency between design variables using the copula concept. In addition, the surrogate model aided in calibrating the safety factor associated with the vertical acceleration threshold, leading to a proposal of 1.38 as a new safety factor. Subsequently, the influence of soil-structure interaction on the estimated reliability indices is evaluated using an ensemble of classification-based surrogate models. Results highlighted its beneficial contribution in terms of increased damping for shorter spans, countered by adverse effects due to frequency shortening in longer bridges. Finally, the epistemic uncertainties arising from the limited knowledge of the vertical acceleration threshold are investigated. It is found that neglecting these uncertainties can lead to an overestimation of allowable train speeds by about 13%.

Keywords: High-speed railway bridges; Bridge dynamics; Running safety; Passenger comfort; Structural reliability; Surrogate models; Active learning; Reliability-based design optimization; Partial safety calibration; Epistemic uncertainties.
Sammanfattning


Nyckelord: Höghastighetsjärnvägsbroar; Brodynamik; Trafiksäkerhet; Passagerarkomfort; Konstruktioners tillförlitlighet; Surrogatmodeller; Aktivt lärande; Tillförlitlighetsbaserad optimering; Kalibrering av partialkoefficienter; Epistemiska osäkerheter.
Preface

In the early days of the journey that its end is almost here where I am writing this preface, I had a naive but honest ambition to have a meaningful impact on my field of study. At the moment, that looks more like a fairy tale to me especially when I look back at the point wise data of my mind states, which is best described by an exponentially decaying function. A function which its such a good fit sometimes frightens me of that being over-fitted. This is mostly because of the all moments that I was reading interesting works from great researchers subjecting me to mixed feelings, a joy from a good reading and a damped ambition appreciating all those intelligent minds, again through a good read. This state of mind after all the ups and downs and after all realizations bouncing between safe and failure domains of my research studies over the last five years was personally joyful and probably worth a try. Therefore, I deeply hope that this work falls somewhere in the safe domain (you read it: successful research) or at least becomes a meaningful example to prevent someone somewhere from failure. If it had solely relied on my own capabilities, most likely it would have ended up somewhere in the upper tails of failure, but I was lucky to be around amazing people supporting me in every way. There is a very long list of these people. I am and will be very grateful to them, even if their name is not here in this text. I will always remember them in my heart and in my mind for as long as I can remember anything.

The research presented in this thesis was conducted at the Division of Structural Engineering and Bridges, Department of Civil and Architectural Engineering, KTH Royal Institute of Technology, Stockholm. During this time, I was gratefully supported by the In2Track2 and In2Track3 projects as part of the Shift2Rail Joint Undertaking. They have received funding from the research and innovation program of European Union’s Horizon 2020 with grant agreement Numbers of 826255 and 101101966. Without their support it would not have been possible to focus on research and I sincerely thank them for giving me this opportunity.

Many of the ideas during the last years, including those presented here, were initially like a shot in the dark. It was definitely impossible for me to pursue those without having the full support of my supervisors, Professor Raid Karoumi and Dr. Andreas Andersson. I cannot express enough how much I appreciate their trust on me and for all kind guidance I have received from them over all years. Working with them was an invaluable experience for me, which was not only limited to the scientific aspects, but also included learning the precious personality characteristics of being a good researcher. Qualities that you can hardly find in scientific journals if you are not among humble researchers like them. I am truly grateful for all those experiences.
CHAPTER PREFACE

I hereby express my profound gratitude to Professor John Leander for taking the time to review this work and provide valuable comments. He also kindly answered some of my questions while I was revising my articles, which I truly appreciate them. Special thanks go to Professor Christoph Adam for the time and effort he invested to read the work and leading the discussions during the seminar. I would also like to thank Professor Daniel Straub for his valuable comments during my licentiate seminar, from which I benefited greatly in continuation of my work. In addition, he kindly hosted me in their group at the Technical University of Munich, for which I am very grateful. I also deeply appreciate Professor Pedro Museros for the valuable technical discussions we had on the dynamics of railway bridges and also for his kindness to host me in their group at the Valencia Polytechnic University.

I would also like to thank all my friends and colleagues in the Division of Structural Engineering and Bridges and those from other divisions for all the moments we have spent together. It has been a great pleasure to be with such great people and I thank them all for their kindness. I express my heartfelt thanks to my Iranian friends who have made KTH like a home for me. I will always remember you all, not only for the many moments we laughed together, but also for the sad moments that I could not possibly bear without you. For all the Fikas we had with scientific and non-scientific discussions, for presenting a graph for everything, for the honest advice and for being like family. I have learned a lot from all of you and thanks for all of them.

Last but not least, my soul whispers thank you to my family. I do not even know how to thank my wife Negar, who is also my best friend. I can barely remember a time that I have not known her, probably because the time for me starts after her. Moving to another country is a difficult and uncertain journey, like searching for a land you are not sure even exists. I was very lucky to take the boat with Negar and I will always be grateful to her for all the sacrifices she made, for the moments we spent together and also for the times I could not spend with her because of work. Moreover, words cannot express my gratitude to my parents Parvin and Moharram. Their true love keeps my heart warm and makes the impossible possible. No matter if I fail or succeed, they have always supported me even when I could not be there for myself. There are moments that words cannot transfer a meaning and this seems to be one of them. I wish I could paint my eyes here, maybe it could tell them as it really worth to tell, how much I love them. Therefore, I dedicate this work to my parents and Negar. I should also thank my siblings Ali and Maryam for being the best siblings ever. Finally, I should mention to one of my regrets during PhD studies which is not being always around my nieces Bahar and Baran.

Stockholm, February 2024
Reza Allahvirdizadeh
List of Publications

The current thesis is based on the outcomes presented in the following publications, labelled as Paper I–VII.

**Paper I**  

**Paper II**  

**Paper III**  

**Paper IV**  

**Paper V**  

**Paper VI**  

**Paper VII**  
CHAPTER LIST OF PUBLICATIONS

All papers were planned, implemented and written by Reza Allahvirdizadeh. The co-authors contributed to the papers with comments and revisions.

Other relevant publications:

In addition to the aforementioned papers, the author has contributed to the following related publications which are not included in this thesis.


# List of abbreviations

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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>ADM</td>
<td>Additional Damping Method</td>
</tr>
<tr>
<td>AIC</td>
<td>Akaike Information Criterion</td>
</tr>
<tr>
<td>ANN</td>
<td>Artificial Neural Network</td>
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<tr>
<td>BIC</td>
<td>Bayesian Information Criterion</td>
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<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
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<tr>
<td>CV</td>
<td>Cross-Validation</td>
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<tr>
<td>CVTS</td>
<td>Centroidal Voronoi Tessellations Sampling</td>
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<tr>
<td>DoE</td>
<td>Design of Experiment</td>
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<tr>
<td>EADA</td>
<td>Equivalent Additional Damping Approach</td>
</tr>
<tr>
<td>EFF</td>
<td>Expected Feasibility Function</td>
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<tr>
<td>EI</td>
<td>Expected Improvement</td>
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<td>EIER</td>
<td>Expected Integrated Error Reduction</td>
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<td>FORM</td>
<td>First Order Reliability Method</td>
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<tr>
<td>GPR</td>
<td>Gaussian Process Regression</td>
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<td>KCUQ</td>
<td>Kriging Classification Uncertainty Quantification</td>
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<tr>
<td>kNN</td>
<td>K-Nearest Neighbours</td>
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<tr>
<td>LHS</td>
<td>Latin Hypercube Sampling</td>
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<td>LOO</td>
<td>Leave-One-Out</td>
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<tr>
<td>MCMC</td>
<td>Markov Chain Monte-Carlo</td>
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<td>MCS</td>
<td>Monte-Carlo Simulations</td>
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<tr>
<td>MLP</td>
<td>Multi-Layer Perceptron</td>
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<td>MPP</td>
<td>Most Probable Point</td>
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<td>MSE</td>
<td>Mean Square Error</td>
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<tr>
<td>PCE</td>
<td>Polynomial Chaos Expansion</td>
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<td>PDEM</td>
<td>Probability Density Evolution Method</td>
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<td>PDF</td>
<td>Probability Density Function</td>
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<td>PMA</td>
<td>Performance Measure Approach</td>
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<td>PRF</td>
<td>Potential Risk Function</td>
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CHAPTER LIST OF ABBREVIATIONS

PSD Power Spectral Density
RBDO Reliability-Based Design Optimization
ReLU Rectified Linear Unit
RIA Reliability Index Approach
RMSE Root Mean Square Error
RS Response Surface
SAP Sequential Approximate Programming
SE Squared Exponential
SE-ARD Square Exponential Automatic Relevance Determination
SORA Sequential Optimization and Reliability Analysis
SORM Second Order Reliability Method
SSI Soil-Structure Interaction
SVM Support Vector Machine
TTBI Train-Track-Bridge Interaction
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Chapter 1

Introduction

Sustainability concerns demand that short to medium distance journeys be shifted from other modes of transport to rail in the near future. There is therefore a constant intention to operate trains on modern networks as fast and reliable as possible. As a consequence, modern infrastructure, and in particular the bridges that are the subject of this study, are exposed to increased vibrations. These vibrations can be excessive due to the occurrence of resonance. The latter is the case when the excitation (loading) frequency coincides with the fundamental frequency of the bridge. In this context, the critical (resonance) speed at which resonance occurs reads as Eq.(1.1) (Frýba, 2001; Xia et al., 2006).

\[ v_{cr} = \frac{f_i d}{j}, \quad i = 1, 2, 3, \ldots \quad j = 1, 2, 3, \ldots \]  

(1.1)

where \( f_i \) is the \( i \)th natural frequency of the bridge, \( d \) is the characteristic axle distance and \( j \) is an integer that defines harmonic/subharmonic or primary/secondary resonant speeds.

Such excessive vibrations can cause structural and non-structural damage, such as the crack initiation and propagation, fatigue, loss of contact between wheels and rails, track misalignment or disturbing comfort of passengers. It should be emphasized here that the present study aims to improve the design philosophy of dynamic systems with respect to the above-mentioned possible failure modes; however, it could not afford to focus on all possible scenarios. The discussions are therefore limited to some of these limit-states, although the methods presented seem to be applicable to others as well. Among the failure modes mentioned, loss of contact can jeopardize the safe passage of trains over bridges and increase the possibility of derailments and is often referred to as running safety. Various approaches can be used to assess the running safety, such as the evaluation of the derailment coefficient (evaluation of the ratio of lateral to vertical load at the contact point) and the unloading ratio (evaluation of the reduction of the dynamic vertical load to the static load) (EN 14067-6, 2016; Nadal, 1908). The detailed investigation of such approaches is beyond the scope of the current study; however, the interested reader can find a detailed overview in Montenegro et al. (2021).

Discussing about failure often resembles an ultimate situation in which a system can no longer withstand any further load, which is synonymous with a breaking point. However, in the case of disturbing passenger comfort, for example, the fail-
ure is more related to the proper operating conditions (serviceability) of the bridge. Passengers would probably feel uncomfortable if they experience excessive acceleration. Therefore, similar to running safety, the study of passenger comfort requires the construction of complex models capable of capturing seat-level accelerations.

The dynamic system involved here is a combination of various subdivisions and their interactions, including the bridge structure, the track, the passing train and the boundary conditions. This system is shown schematically in Figure 1.1; which the individual subsections are briefly explained below.

Figure 1.1: Schematic representation of the considered physical model and its different subsystems.

Considering these subsystems, the computational models associated with the running safety and passenger comfort should at least be able to account for Train-Track-Bridge-Interaction (TTBI). These models are able to evaluate the response of the train (coach) and track-bridge subsystems simultaneously. They therefore offer detailed investigations which, however, require considerable computational
effort. The governing equations of motion for such models read as Eq.(1.2).

\[
\begin{align*}
\mathbf{M}_{\text{TB}} \ddot{\mathbf{u}}_{\text{TB}} + \mathbf{C}_{\text{TB}} \dot{\mathbf{u}}_{\text{TB}} + \mathbf{K}_{\text{TB}} \mathbf{u}_{\text{TB}} &= \mathbf{p}_{\text{TB}} \\
\mathbf{M}_{\text{V}} \ddot{\mathbf{u}}_{\text{V}} + \mathbf{C}_{\text{V}} \dot{\mathbf{u}}_{\text{V}} + \mathbf{K}_{\text{V}} \mathbf{u}_{\text{V}} &= \mathbf{p}_{\text{V}}
\end{align*}
\] (1.2)

where \( \mathbf{M}, \mathbf{C} \) and \( \mathbf{K} \) are mass, damping and stiffness matrices respectively. In addition, \( \mathbf{u} \) represents the displacement vector and \( \mathbf{p} \) denotes the force vector. Moreover, the indices of \( \text{TB} \) and \( \text{V} \) correspond to the track-bridge and the vehicle (train) respectively. It should be noted that the above equations of motion are coupled by the contact force between two subsystems, which should be solved by an iterative algorithm or by merging the matrices of the subsystems into a larger matrix to avoid the iterative approach (Arvidsson, 2018; Wu, 2000; Yang and Yau, 1998; Zhai et al., 2013). The former involves assuming a displacement at the contact point, solving the equation of motion of the vehicle to calculate the contact force, substituting that into the equation of motion of the track-bridge, and then updating the displacement at the contact point (Liu et al., 2014; Yang and Fonder, 1996; Yang et al., 2004).

As mentioned earlier, performing such analyses can provide a detailed overview of the system’s response, including the passing train, the track, the bridge and their interactions. However, as will be discussed in the next chapters, the use of such computational models in the context of structural reliability (probabilistic assessments) would be computationally intractable. This is because reliability analyses often require a significant number of calls to the computational model. The order of these calls in one of the most widely used reliability assessment methods, namely crude Monte-Carlo simulations (discussed in the next chapter), and for a problem with failure probability of \( 10^{-q} \) would be in order of \( 10^{q+2} \) (Sudret, 2012). The failure probability of a railway bridge depends on many aspects, in particular the operating speed of the trains. However, assuming a typical safety level in the range of \( 10^{-4} \) for such systems, it becomes clear how demanding it would be to perform reliability analyses using models that consider TTBI. This encourages the use of simplified, less computationally expensive computational models for the objectives of the current study.

In the earlier simplified modelling strategies, the passing trains were considered as massless moving loads or as a lumped mass on a simply-supported beam. The interested reader can find detailed information on such studies for example in Akin and Mofid (1989); Gbadeyan and Oni (1995); Stanišić (1985); Stanišić and Hardin (1969); Ting et al. (1974); Wang (1997). In this context, it has been shown that the moving forces approach can acceptably predict the dynamic response of bridges when the mass of the train is negligible compared to that of the bridge (Yang and Yau, 1998). Therefore, they are often used when only the assessment of the dynamic response of the bridge is of interest. Under these circumstances, the passing train
can be modelled as a series of moving forces, neglecting the inertial effects of the coaches. Thus, the equation of motion of the system reduces to Eq.(1.3).

\[
    m_B \ddot{u}_B(x,t) + c_B \dot{u}_B(x,t) = E_B I_B u_B(x,t) = \sum_{j=1}^{N_{\text{coach}}} p \left[ \mathcal{P}_j(t, v, L) + \mathcal{P}_j(t - d_c/v, v, L) \right]
\]

where \( m_B \) represents the linear mass of the bridge, \( c_B \) is the viscous damping of the bridge, \( E_B I_B \) denotes the flexural rigidity of the bridge, \( N_{\text{coach}} \) is the number of train coaches, \( p \) is the axle load of the train, \( v \) is the train speed and \( d_c \) is the distance between the front and rear wheels of coaches. Furthermore, \( \mathcal{P}_j(t, v, L) = \delta \left[ x - v(t - t_j) \right] h(t - t_j) - h(t - t_j - L/v) \) moves the axle load along the bridge, activating it when the load reaches the bridge and deactivating it when it leaves the bridges. Therefore, \( \delta(\cdot) \) is the Dirac delta function, \( h(\cdot) \) is the Heaviside function and \( t_j = (j-1)D/v \), where \( D \) is the coach length. It is worth noting that considering this approach and modeling the bridge as a simply-supported Euler-Bernoulli beam led to solving this equation analytically by superposition of the previously developed solutions for the single moving force problem in Frýba (2001); Yang et al. (2004, 1997).

Such solutions would be computationally advantageous to the reliability assessment objectives, although they do not provide direct estimates of the responses of interest. This concern can be addressed by implicitly modelling the failure modes using the results of the simplified modelling strategy (e.g., deflections, rotations, or accelerations). In this context, the design regulations control passenger comfort by limiting the deflections of the bridge (EN 1990, 2002). In addition, it is assumed that the sleepers will move laterally if the ballast particles become unstable due to the vibrations experienced, which can disrupt the load path from the rail level to the bridge deck and cause the passing trains to derail under severe circumstances. Theoretically, ballast particles become unstable when the induced inertial forces overcome the resistance forces (e.g. friction or interlocking between the particles). Therefore, it seems possible for the running safety limit-state to be indirectly formulated based on the vertical acceleration levels of the bridge deck. This assumption is consistent with the observations made on the earlier high-speed lines, in particular on the line between Paris and Lyon in France, where it was found that ballast instability occurs at vertical accelerations in the range of 0.7-0.8 g (ERRI D 214/RP 9, 1999). These observations were later confirmed by conducting a series of shaking table tests, where significant lateral movements of the sleepers were considered an indicator of ballast instability (ERRI D 214/RP 8, 1999; Zacher and Baeßler, 2009). The observed responses then led to the suggestion of 7.0 m/s\(^2\) as the threshold which ballast instability occurs at accelerations above this value. It should be emphasized here that similar conclusions were also drawn in (Kumakura et al., 2010; Nakamura et al., 2011).
It should be mentioned here that a train derailment due to a local (single point) ballast instability seems very unlikely. In this context, EN 1990 (2023) allows the design limit for vertical acceleration to be increased by about 40% (i.e. from 3.5 m/s² to 5.0 m/s²) if the calculated vertical accelerations exceed the threshold value at a distance along the track that is less than the spacing between two consecutive sleepers. Disregarding this aspect may result in the formulation of the running safety limit-state based on the vertical acceleration of the track being conservative. This statement is emphasized in Rocha (2015), where it was found that the instability of the ballast almost always governs the wheel-rail contact loss criterion under current normative approaches. Nevertheless, the results presented in this study neglect this aspect for reasons of conservatism.

The adopted modelling strategy would then be suitable for carrying out the intended reliability analyses. However, the influence of its simplifications should be evaluated and, if possible, compensated for before proceeding with such investigations. As mentioned above, the employed approach models the bridge as an Euler-Bernoulli beam and neglects the contribution of ballast and rails to stiffness. This could lead to negligible errors in the predictions as they either have a very low modulus of elasticity or small dimensions. However, the mass of the tracks cannot be neglected. Therefore, the linear mass of the bridge can be modified to account for the contribution of the track. In addition, the structure of the track distributes the concentrated loads acting at the rail level along the bridge deck. Previous studies have shown that such a distribution can significantly reduce the experienced vibrations by the bridge, especially for shorter span bridges (Axelsson et al., 2014; Jin et al., 2018; Museros et al., 2002). In this context, a variety of approaches have been proposed to implement such reductions in the moving load modelling strategy. These include the distribution of the axle load over three successive thresholds with 25%-50%-25% portions, the use of a triangular distribution with a slope of 4:1 and the application of a graphical reduction factor as a function of the wavelength (EN1991-1, 2003; ERRI D 214/RP 9, 1999). The latter is used in this study and can be formulated as a polynomial function, which is shown in Eq.(1.4).

\[
R(\lambda) = -0.00011\lambda^4 + 0.0553\lambda^3 - 0.083\lambda^2 + 0.57\lambda - 0.54, \quad 0.14 \leq R(\lambda) \leq 1.0 \quad (1.4)
\]

where, \(\lambda = v/f_1\) is the wavelength and \(f_1\) represents the fundamental frequency of the bridge.

As already mentioned, the rails are not explicitly modeled in the discussed approach, basically assuming them to be smooth, i.e. the geometry of the rails along the track and at all points is assumed to be ideal geometry. It is obvious that this is not always the case. Depending on the quality of the track, the real geometry can deviate from the ideal situation, which is also known as rail irregularity. In such
cases, the vertical displacement of the wheel at the contact point would be equal to the sum of the track-bridge deflection at that point and the rail irregularity (Zhang et al., 2010). Rail irregularities are often implemented in models with TTBI as realizations of a stochastic Gaussian process (Fryba, 1996). The latter is defined based on its amplitude, which is described by the Power Spectral Density (PSD) fitted to the in-situ measurements and a random phase (Claus and Schiehlen, 1998). It has been shown that the presence of rail irregularities can significantly amplify the experienced vibrations (especially accelerations) compared to smooth rails (Cantero et al., 2016; Salcher et al., 2019a, 2016). Therefore, an amplification factor is introduced in the design regulations as Eq.(1.5) to take such effects into account in the moving loads modelling strategy (EN1991-1, 2003).

\[
\chi_{RI} = 1 + \varphi'' = 1 + \frac{\alpha}{100} \left[ 56 \exp \left( -\frac{L}{10} \right)^2 + 50 \left( \frac{L_f}{80} - 1 \right) \exp \left( -\frac{L}{20} \right)^2 \right] \quad (1.5)
\]

where \(\varphi''\) is the amplification coefficient and \(\alpha = \min \{v/22, 1\}\). It should be noted that \(\varphi''\) can be reduced by 50% for carefully maintained tracks. The applicability of this relationship is examined in Salcher et al. (2019a) following a probabilistic framework, which revealed its adequacy for displacements while showing the possibility of underestimation for accelerations. This inadequacy led to propose probabilistic amplification factors modelled by a Gaussian distribution for deflections and a log-normal distribution for acceleration in Salcher et al. (2022).

In addition to the above, consideration of TTBI has been shown to reduce the vibration experienced. Such reductions appear to depend on the ratio of bogie to bridge frequency, the ratio of bogie to bridge mass and the ratio of coach to bridge length being increased by the former and reduced by the latter (Arvidsson and Karoumi, 2014; Doménech et al., 2014; Liu et al., 2009; Museros et al., 2002). Therefore, an Additional Damping Method (ADM) was introduced in the design codes to implicitly account for such beneficial contributions (EN1991-1, 2003), although later studies indicated the possibility that this method overestimates the system damping (Arvidsson et al., 2014; Calgaro et al., 2010). This concern is addressed in Yau et al. (2019a) by modifying ADM as the Equivalent Additional Damping Approach (EADA), which is formulated as Eq.(1.6).

\[
\Delta \xi = \mu_1 r_1 \left| \frac{r_1 + 2\xi_{v_1} i}{(1 - r_1)^2 - 2\xi_{v_1} r_1} \right| \approx \mu_1 r_1 \sqrt{r_1^2 + (2\xi_{v_1})^2} \quad (1.6)
\]

where \(\mu_1\) is the modal mass ratio between coach and bridge, \(r_1\) is the fundamental frequency ratio of coach and bridge and \(\xi_{v_1}\) is the effective damping ratio of the suspension.

Furthermore, the previous studies mostly disregarded the contribution of Soil Structure Interaction (SSI) effects. It is assumed that the consideration of SSI increases the damping of the system; therefore, neglecting these effects would be a conservative simplification. However, this assumption neglects the detrimental effects of
frequency shortening due to the increasing flexibility of the system. Considering these two opposing effects, bridges with shorter spans may benefit from the additional damping provided by SSI; however, bridges with longer spans are likely to suffer from the increase in their flexibility (Lind Östlund et al., 2020). Therefore, accurate prediction of the dynamic response of high-speed railway bridges may require consideration of SSI effects (Takemiya and Bian, 2007; Ülker-Kaustell et al., 2010). In order to include SSI effects in the assessment of railway bridges, different modelling strategies are compared in Zangeneh Kamali (2021). It was shown that simplified approaches using frequency-dependent lumped springs and dashpots can acceptably reproduce the experimental observations.

In this context, the relative frequency parameter (Eq.(1.7)) is introduced as the ratio of the natural frequency of the simply-supported beam to the stratum.

$$\phi = \frac{f_{1,ss}}{f_{so}} = \frac{4Hf_{1,ss}}{v_{L_a}} = \frac{4H\pi(1 - \nu_s)f_{1,ss}}{3.4c_s}$$  \hspace{1cm} (1.7)

where $f_{1,ss}$ is the fundamental frequency of the simply-supported bridge, $f_{so}$ represents the frequency of the deposit, $H$ is the depth of the stratum, $v_{L_a}$ is the Lysmer’s analog wave velocity (Dobry and Gazetas, 1986), $\nu_s$ is the Poisson’s ratio of the soil, $c_s$ is the shear wave velocity of the soil. Based on the relative frequency parameter, the stiffness and damping of the associated springs and dashpots can be categorized into different ranges. It has been found that the additional damping due to SSI effects decreases for small values of the relative frequency, which means that the SSI effects can be neglected at $\phi \leq 0.1$. For larger values of the relative frequency (i.e. $\phi \geq 1.5$), the behaviour of the system resembles that of a beam resting on a half-space medium, resulting in the equivalent frequency-independent springs and dashpots being estimated as Eq.(1.8) and Eq.(1.9), respectively.

$$k_{hs} = \frac{\rho_s c_s^2 A_f}{2B_f (1 - \nu_s)} \left( 0.73 + 1.54 \frac{B_f}{L_f} \right)^{3/4}$$ \hspace{1cm} (1.8)

$$c_{hs} = \frac{3.4\rho_s c_s A_f}{\pi (1 - \nu_s)} + \frac{\xi_s k_{hs}}{\pi f_{1,ss}}$$ \hspace{1cm} (1.9)

where $\rho_s$ is the mass density of the soil, $A_f$ is the area of the foundation, $B_f$ is the semi-width of the foundation, $L_f$ is the semi-length of the foundation and $\xi_s$ is the material damping ratio of the soil. In addition, in cases with $0.1 \leq \phi \leq 0.5$, the system behaves similarly to a surface foundation on a stratum, which means that the frequency-independent springs and dashpots can be approximated using Eq.(1.10) and Eq.(1.11) respectively.

$$k_{st} = k_{hs} \left( 1 + \frac{B_f/H}{0.5 + B_f/L_f} \right)$$ \hspace{1cm} (1.10)

$$c_{st} = \frac{\xi_s k_{st}}{\pi f_{0,ss}}$$ \hspace{1cm} (1.11)
Moreover, damping and frequency shift seem to be maximized around the resonance frequency of the deposit (i.e. $1.0 \leq \phi \leq 1.5$). Therefore, the proper approximation of springs and dashpots requires the construction of more detailed finite element models (Zangeneh Kamali, 2018; Zangeneh Kamali et al., 2019).

As can be seen from the above, the natural frequency of the bridges plays a decisive role in their dynamic behavior. Therefore, it is important to investigate the effects of factors that influence the natural frequency. One of these factors is environmental impacts and in particular temperature variations. Previous experimental observations have shown that temperature fluctuations can significantly change the natural frequency of railway bridges, especially those with ballasted tracks. In this context, the natural frequency of a case study bridge was compared between winter and summer days in Gonzales et al. (2013), where a frequency variation of up to 35% was observed. Such temperature variations are stochastic in nature; therefore, a probabilistic investigation is performed in Salcher et al. (2016) to assess their impact on the failure probability of railway bridges corresponding to the running safety limit-state. Despite this importance, the current study neglects such effects in the investigations carried out.

### 1.1 State of the art

Most of the studies mentioned above have investigated various aspects of the dynamic response of railway bridges following deterministic approaches. However, the evaluation of the safety of a system is inevitably related to its probability of failure. Therefore, to improve the design philosophy of a system, it seems essential to rely on the results of a probabilistic investigation. With this in mind, this section provides a brief overview of previous probabilistic studies that have been carried out in the context of railway bridge dynamics. Most of these studies, such as Adasooriya (2016); Imam et al. (2008); Leander and Al-Emrani (2016); Leander et al. (2015); Li et al. (2016) have been dedicated to the fatigue problem. This aspect does not fall within the scope of the current study, so the details are not discussed further here.

The estimated failure probability of a railway bridge obviously depends on the operating speed of the trains passing over it. In this context, several reliability measures for running safety limit-state, namely the probability of failure for a given train speed as Eq.(1.12), the maximum envelope acceleration as Eq.(1.13), envelope of probabilities as Eq.(1.14) and weighted probability as Eq.(1.15) were introduced in Hirzinger et al. (2020). Among these, the upper bound of the failure probability as the most conservative measure seems to be a better measure to improve the design philosophy of such systems.

\[
p_{f_i} = \mathbb{P} \left[ a_{\text{max}}(v_i) \geq a_{\text{lim}} \right] \tag{1.12}
\]
1.1. STATE OF THE ART

\[
p_{f_{\text{max}}} = \mathbb{P} \left( \max_{v_l \leq v \leq v_u} a_{\text{max}}(v) \geq a_{\text{lim}} \right) \tag{1.13}
\]

\[
p_{f_{\text{env}}} = \max_{v_l \leq v \leq v_u} \left[ \mathbb{P} \left( a_{\text{max}}(v) \geq a_{\text{lim}} \right) \right] \tag{1.14}
\]

\[
p_{f_{w}} = \int_{v_l}^{v_u} \mathbb{P} \left( a_{\text{max}}(v) \geq a_{\text{lim}} \right) f_V(v) dv \tag{1.15}
\]

where \( a_{\text{max}}(\star) \) is the maximum vertical deck acceleration resulting from the computational model, \( v_l \) is the lower bound of the operating speed, \( v_u \) is the upper bound of the operating train speeds and \( f_V(v) \) denotes the probability density function of the operating train speed.

Cho et al. (2010) applied the First-Order Reliability Method (FORM) with limit-state functions approximated by polynomial response surfaces to evaluate the safety of a two-span continuous box-girder bridge with respect to running safety and passenger comfort limit states. The bridge under consideration was subjected to vibrations induced by a train passing at 300 km/h speed and it does not appear to be susceptible to vibrations. As a result, the calculated failure probabilities are very low. However, it was found that under the limit-states considered, running safety appears to be decisive.

Later, Rocha et al. (2012) investigated an existing filler-beam short span bridge using simulation-based reliability assessment methods assisted by extreme value theory (tail fitting). It is worth noting that the applicability of this approach was investigated in a previous study by Rocha et al. (2014). The computational models used in these studies were able to account for TTBI effects and were subjected to a wide range of train speeds. The results showed that the application of probabilistic methods for the considered case study bridge can lead to an increase in the operating speed of trains by 40 km/h compared to the conventional normative assessment methods. In addition, the wheel-rail contact loss limit-state is compared with the ballast instability, which shows that the latter dominates the design (Rocha et al., 2015, 2016).

Subsequently, a filler-beam single-span bridge in Ferreira et al. (2022) is considered in order to compare the normative partial safety factor associated with the running safety limit-state with the one obtained from probabilistic investigations. The latter is calculated as the ratio between the maximum acceleration corresponding to the minimum train speed at which the probability of running safety violation exceeds the target value of 10\(^{-4}\) and the deterministic resistance value (i.e. 7.0 m/s\(^2\)). This approach led to the conclusion that the current partial safety factor of 2.0 can be reduced to a factor of about 1.5. In addition, similar results are presented in Ferreira et al. (2021).
The reliability analyses described above are computationally intensive. Therefore, a comparison between advanced simulation-based techniques, namely line sampling, subset simulation and asymptotic sampling was performed in Hirzinger et al. (2019) with respect to the conventional simulation-based methods. In this study, two different bridges were investigated where resonance and track irregularities control the behaviour. It was found that the considered methods can significantly reduce computational costs without sacrificing accuracy when the track irregularities are neglected. However, their efficiency decreases when the dimensionality of the problem in the second model increases. This concern is addressed later in Li et al. (2023) by proposing an improved response surface method. Furthermore, an equivalent linearization technique is implemented in Jin et al. (2015) to increase the computational efficiency of the model with vehicle-bridge interactions. This method is then applied using crude Monte-Carlo simulations to assess the running safety over a simply-supported bridge. Although it is computationally efficient compared to conventional iterative methods for solving models with TTBI, it would still require a significant number of calls to the computational model. Furthermore, the Probability Density Evolution Method (PDEM) is used in Mao et al. (2019) to estimate the time-varying probability of the model with TTBI and rail irregularities under excitation by random passing train loads. Using the same method, dimensionality reduction methods are adopted in Wang et al. (2023) to reduce the contributing basic random variables of the same problem while lateral wind loads coexist with the rail irregularities.

A detailed investigation of the influences of random rail irregularities was later carried out in Salcher and Adam (2020); Salcher et al. (2019b). It was found that the dynamic amplification factor of deflections related to rail irregularities can be described by a Gaussian distribution, while extreme value or log-normal distributions better model those related to accelerations. Thus, stochastic amplification factors were calculated for both deflections and accelerations, illustrating the inadequacy of normative amplification factors for accelerations.

1.2 Aims and scope

This study is dedicated to the task of evaluating and possibly improving the conventional dynamic design methods of high-speed railway bridges from a probabilistic point of view. It includes understanding the influence of various aspects of the problem on system safety, the computational issues involved in performing the associated reliability analyses, and also the presentation of easily applicable, safety-compliant design methods.

Conventional design methods seem to rely on a limited amount of information, mainly due to the restricting computational resources at the time of development. In this context, previous studies have highlighted some of their inconsistencies and
shortcomings. This study also addresses some of these concerns by using reliability-based approaches. Therefore, the specific aims of the study can be formulated as follows:

- Realize the uncertainties involved by collecting representative information on the contributing variables and using statistical approaches to assign the appropriate theoretical probability distribution functions.

- Evaluate the adequacy of current dynamic design methods for high-speed railway bridges using simplified probabilistic methods to identify possible aspects for further improvement.

- Investigate the computational aspects of the intended reliability analyses to identify the most efficient methods for further analysis.

- Evaluate the influence of different aspects such as boundary conditions and available data on the estimated safety levels.

- Suggest straightforward, improved design methods to overcome the safety concerns of current approaches.

Moreover, this study is subjected to the following limitations.

- The results presented are limited to reinforced concrete bridges.

- The information collected on the geometric properties of the bridges does not necessarily refer to bridges located on modern high-speed lines.

- There was no reliable information available for the author to calculate the corresponding target reliability levels, so these were selected based on recommendations of current guidelines and the author’s judgment.

- Preliminary studies, used simplified approximate reliability assessment methods. Such methods may provide less accurate results under certain conditions. However, these possible shortcomings are disregarded here in order to make the intended reliability analyses computationally feasible.

- The computational models used were kept as simple as possible by constructing 2D models, modelling bridges as Euler-Bernoulli beams, neglecting TTBI and indirectly implementing rail irregularities, load distribution in the track and boundary conditions. The predictions of such models may deviate from reality, which is attempted to be compensated for by introducing a model uncertainty variable based on judgments of the author. With such simplifications, the employed modelling strategy significantly reduces the computational costs of the reliability analyses and allows the conclusions presented to be derived.
CHAPTER 1. INTRODUCTION

- The distributions assigned to the resistance terms of the considered failure modes are obtained on the basis of limited information. This increases the epistemic uncertainties associated with these terms. This aspect is investigated in one of the studies; however, its impact on other studies, and in particular those aimed at improving current design methods, is neglected.

- The chosen modelling strategy resulted in the intended limit-state functions being formulated based on indirect measures. As a result, the considered limit-states do not directly describe the desired failure modes. This obviously leads to a certain degree of error in the estimated safety levels; however, the author believes that this is the best of practice considering the computational costs involved.

- The probability distributions associated with the modelling parameters of the passing trains are based on the data available in the literature and some measurements. The former do not necessarily refer to the real trains, as access to such information is often not possible.

- The effects of soil-structure interaction on system safety are assessed using approximate modelling strategies. The chosen modelling strategy is subjected to a considerable degree of uncertainty; however, it appears to be suitable for deriving general qualitative conclusions, as was intended here. In addition, the soil parameters present a considerable degree of uncertainty. It was therefore decided to isolate their effects from the other random variables by assigning them a set of deterministic values.

1.3 Scientific contribution

The research presented in this dissertation, which is accompanied by the appended papers, has led to the following scientific contributions:

- Assessing the adequacy and consistency of conventional design methods from a safety perspective and evaluating the sensitivity of the failure probability of railway bridges with respect to the contributing variables using approximate probabilistic methods (Paper I).

- Proposing probabilistic design curves including the minimum required linear mass and stiffness as a function of bridge cross-section type, span length, number of spans and maximum permissible operating speed of trains to meet the target safety level of running safety and passenger comfort design limit-states (Paper II).

- Comparing the performance of different regression-based surrogate models for reliability assessment of dynamic systems under identical active learning
strategies and proposing the best model to be used in further studies (Paper III).

- Assigning appropriate probability distribution functions for basic random variables contributing to the dynamic behaviour of high-speed railway bridges (Paper IV).

- Using surrogate-assisted reliability-based design optimization to propose minimum design requirements for ballasted railway bridges considering the dependency between the design parameters (Paper IV).

- Using surrogate-assisted reliability assessment methods to calibrate the associated safety factor of the running safety design limit-state as a function of the maximum allowable operating train speed (Paper V).

- Employing an ensemble of classification-based surrogate models to investigate the influence of considering or neglecting soil-structure interaction effects on the safety of high-speed railway bridges (Paper VI).

- Using surrogate models to evaluate the influence of lack of knowledge (epistemic uncertainties) associated with the resistance (threshold) terms of the limit-state function and model uncertainty on the estimated failure probability of high-speed railway bridges (Paper VII).

1.4 Outline of the thesis

This thesis is based on seven appended papers. It therefore contains a detailed summary of the concepts discussed in these papers. Chapter 1 gives an overview of the fundamentals of dynamic analysis of railway bridges and previous probabilistic studies on this topic. Chapter 2 gives an overview of the adopted reliability assessment methods and Chapter 3 summarizes the main contents of the appended papers. The derived conclusions and recommendations for further studies are then presented in Chapter 4.

Paper I employed FORM to perform a computationally inexpensive and preliminary reliability assessment of the running safety of high-speed railway bridges. The main objective at this stage was to investigate the consistency of current design methods over a wide range of operating train speeds and bridge spans, rather than to calculate the absolute values of the safety indices. Therefore, FORM was selected despite its inherent limitations. This study compares the safety resulting from the current design methods with the desired level of safety to highlight the possible inconsistencies. In addition, the directional cosine values resulting from the FORM were provided as a sensitivity measure for the assessed probability of failure with respect to the contributing random variables.
CHAPTER 1. INTRODUCTION

Paper II formulated a simple line search optimization algorithm based on the FORM results to propose minimum requirements for the design of railway bridges. The obtained design curves are classified as a function of bridge cross-section type, maximum permissible operating train speed, number of spans and bridge span length, and have minimum linear mass and stiffness. The fulfillment of the presented values in these probabilistic design curves leads to design bridges that meet the desired target reliability levels with respect to running safety and passenger comfort design limit-states. Furthermore, the sensitivity of the estimated failure probabilities with respect to the contributing basic random variables is investigated.

Paper III compares the performance of widely used regression-based surrogate models in the context of structural reliability assessment of dynamic systems. The comparison is performed from the perspective of accuracy and under identical training situations. Therefore, a universal active learning strategy with a stopping criterion based on the size of the training data set is used. The best performing surrogate model is then recommended for further investigations.

Paper IV proposes minimum design requirements, namely mass per length and stiffness for single-span, simply-supported high-speed railway bridges using a surrogate-assisted reliability-based design optimization method. To achieve this goal, in the first part of the study, information on different variables was collected and corresponding probability distribution functions were assigned. Then, the conventional reliability-based design optimization method is reformulated to automatically consider the dependency between the design variables, i.e. disregarding the practically infeasible solutions. This dependency is implemented in this study using the Copula concept.

Paper V applies a surrogate-assisted method to calibrate the safety factor associated with the running safety limit-state. In this context, a series of design scenarios are considered, which include different cross-section types, number of spans, wide range of bridge span lengths and three different permissible operating speeds of trains. An active learning strategy is then followed to train the corresponding surrogate models. The reformulation of the limit-state function allows the estimation of the safety index corresponding to each considered safety factor. Thus, the optimal safety factor is determined by minimizing the variation between the safety indices of the considered design scenarios and the target reliability level.

Paper VI uses the concept of stack generalization to train an ensemble of classification-based surrogate models. For this purpose, support vector machines, $k$-nearest neighbours and decision trees are used. The main objective of the study was to investigate the influence of soil-structure interaction on the estimated reliability of high-speed railway bridges with respect to the running safety limit-state. In this context, a simplified modelling technique based on lumped springs and dashpots is used to implement the effects of soil-structure interaction on the dynamic response.
1.4. OUTLINE OF THE THESIS

of railway bridges. The large uncertainties associated with soil properties led to their isolation from other basic random variables by assigning a deterministic feasible range to the parameters of the stratum. Results are then presented for a wide range of operating train speeds normalized to the corresponding simply-supported cases.

Paper VII Investigates the influence of the associated epistemic uncertainties on the vertical acceleration threshold in running safety limit-state and the model uncertainty on the estimated safety of railway bridges. The epistemic uncertainties are modelled using parametric probability-boxes whose parameters are determined based on bootstrap sampling from the available data. The computational costs of performing such analyses is significantly reduced by substituting the computational model with trained surrogate models. The associated computational model in this study does not depend on variables with epistemic uncertainties, so that the above approach is possible. Subsequently, the influence of epistemic uncertainties or lack of knowledge on the estimated allowable operating speed of trains and its impact on the safety factor of the system is investigated.
Chapter 2

Reliability-Based Methods for Design Improvement

Structural reliability assessment methods are generally referred to as methods that estimate the safety of a system/component with respect to a particular or a set of failure modes. A safe system can be considered to be that for which the probability of its induced actions being exceeded by its corresponding resistance is below an acceptable threshold. This probability is often referred to as the probability of failure, which should be determined by solving a multidimensional integral such as Eq.(2.1). All further decisions based on reliability-based methods appear to rely (directly or indirectly) on the estimated probability of failure corresponding to each scenario, which is mathematically described by the limit-state function. The latter basically defines the boundary between failure and safe regions in the problem domain.

\[
 p_f = \mathbb{P}[g(x) \leq 0] = \int_{D_f} f_X(x) \, dx \quad (2.1)
\]

where \( g(\cdot) \) is the limit-state function, often represented as the subtraction of the action from the resistance, \( D_f \) is the failure domain and \( f_X(x) \) is the joint probability distribution of the contributing random variables. It is important to note that the safety index is often given instead of the estimated probability of failure. This reliability measure reads as \( \beta \approx -\Phi^{-1}(p_f) \), where \( \Phi(\cdot) \) is the Cumulative Distribution Function (CDF) of the standard Normal distribution.

In light of the above discussion, structural reliability assessment methods deal with uncertainties, which can generally be performed at the following four levels (Madsen et al., 2006):

- **Level I** models the variables with their characteristic values by including partial safety factors. It is therefore a semi-probabilistic approach that is used in design guidelines.

- **Level II** models the random variables with their first and second moments and also takes into account the correlation coefficients between them.

- **Level III** calculates the probability of failure as a reliability measure using the joint probability distribution functions between the variables.
CHAPTER 2. RELIABILITY-BASED METHODS FOR DESIGN IMPROVEMENT

- **Level IV**, also known as the risk-based method, takes into account the consequences of the occurrence of a failure, such as financial losses or fatalities.

In this study, Level II and Level III methods are used to evaluate the reliability of current design methods for railway bridges and to propose improved design philosophies. In addition, the Level I method is also considered to modify the conventional normative design methods. It should be noted that no risk-based approaches are taken here; therefore, only a brief overview of Level I-III methods is given below.

2.1 Partial safety factors: implementation and calibration

Often the nominal (characteristic) values of the design parameters are solely known during the design phase of each system. This has led the design guidelines to follow a semi-probabilistic approach through partial safety factors, also known as the limit-state format. In this context, the design equation is formulated as Eq.(2.2).

\[
\frac{R_k}{\gamma_R} \geq \gamma_S S_k \tag{2.2}
\]

where \(R_k\) and \(S_k\) represent the characteristic values of capacity and demand respectively in relation to a design scenario. In addition, \(\gamma_R\) and \(\gamma_S\) are the corresponding partial safety factors. It should be noted that the characteristic values are represented as quantiles of the associated probability distribution functions. In this context, the lower quantiles (often 1.0 – 5.0%) for the capacity-related variables are considered for reasons of conservatism. Similarly, the upper percentiles (often 95.0 – 98.0%) for the induced actions are adopted (Sørensen, 2002).

A system designed according to this approach should fulfill a desired safety level, also referred to as the target reliability level (hereafter represented as \(p_t^f\) or \(\beta_t\)). To take this into account, each partial safety factor used should be calibrated by a higher-level reliability analysis. In addition, a suitable target reliability should be selected as a reference for such calibrations. A variety of methods have been proposed for the selection of target reliability levels, including those based on previous failure experiences and those based on cost optimization (Ghasemi and Nowak, 2017). In the latter, the target reliability level is calculated as the optimal point that maximizes the utility of the structure (see Eq.(2.3)) or minimizes its expected total cost (see Eq.(2.4)).

\[
\beta_t^* = \arg \max_{\beta_t} \quad B - \left( C_I(\beta_t) + C_R(\beta_t)p_d + C_F(\beta_t)p_f \right) \tag{2.3}
\]

\[
\beta_t^* = \arg \min_{\beta_t} \quad C_I(\beta_t) + C_R(\beta_t)p_d + C_F(\beta_t)p_f \tag{2.4}
\]

where \(B\) denotes the total benefit of the system during its service life. Moreover, \(C_I\), \(C_R\) and \(C_F\) stand for the initial construction costs, possible repair costs and
2.1. PARTIAL SAFETY FACTORS: IMPLEMENTATION AND CALIBRATION

the expected failure costs. In addition, $p_d$ is the probability of structural deterioration and $p_f$ is the probability of failure with respect to the limit-state of interest (Rackwitz, 2000; Sørensen et al., 1994).

In the above approaches, the target reliability level is estimated using monetized formulations. This raises ethical concerns as human life is assigned a value (Liu et al., 2021). Furthermore, it would be very difficult to collect reliable information about the costs in real-world problems, which limits the application of the discussed approaches.

Taking these aspects into account, relying on previous experiences (safety of existing structures) appears to be widespread when selecting the target reliability level. This approach assumes that construction technology is constantly evolving by learning from its mistakes. Therefore, the majority of existing structures are satisfactorily safe from a societal perspective. This leads to the target reliability level being chosen as the mean, median or minimum of the safety indices resulting from a representative set of cases designed on the basis of current practice (Melchers and Beck, 2018). In this context, the recommendations of design guidelines such as JCSS (2001), EN 1990 (2002), ASCE (2010), ISO 2394 (2015) and TSFS 2018:57 (2018) can also be used in cases where detailed information was not available. In general, these guidelines specify the target reliability levels depending on the expected consequences of a failure.

After determining the target reliability level, the associated partial safety factors can be calibrated as the optimal values that minimize the difference (or standard of deviation) between the system safety and the target reliability level. For this purpose, a variety of cost functions have been developed, some of which are as presented in Eqs.(2.5)-(2.7) (Gayton et al., 2004; Melchers and Beck, 2018; Sørensen, 2011).

$$\gamma^* = \arg\min_{\gamma} \sum_{i=1}^{N'} w_i \left( \beta_i(\gamma_i) - \beta_t \right)^2$$ \hspace{1cm} (2.5)

$$\gamma^* = \arg\min_{\gamma} \sum_{i=1}^{N'} \left( r \left[ \beta_i(\gamma_i) - \beta_t \right] + \exp \left[ -r(\beta_i(\gamma_i) - \beta_t) - 1 \right] \right)$$ \hspace{1cm} (2.6)

$$\gamma^* = \arg\min_{\gamma} \sum_{i=1}^{N'} w_i \left[ \beta_i(\gamma_i) - \beta_t \right]^2 + r' \sum_{j=1}^{N''} \left[ \gamma_j - \gamma_{ij} \right]^2 \hspace{1cm} (2.7)$$

s.t. \quad \left\{ \begin{array}{l} \beta_i(\gamma) \geq \beta_t^\text{min} \\
\gamma^\text{l} \leq \gamma_j \leq \gamma^\text{u} \\
i = 1, ..., N' \\
j = 1, ..., N'' \end{array} \right.$$

where $N'$ denotes the number of design scenarios considered, $\gamma$ is the partial safety factor applied, $\beta(\gamma)$ represents the resulting safety index of the design scenario.
CHAPTER 2. RELIABILITY-BASED METHODS FOR DESIGN IMPROVEMENT

obtained from the application of the partial safety factor of $\gamma$ and $w$ is the relative frequency of each design scenario. It should be noted that Eq.(2.5) does not penalize the situations with safety indices smaller than the target reliability value. Therefore, Eq.(2.6) is developed with $r > 0$ as the curvature parameter. Similarly, a lower bound ($\beta_t^{\min}$) is imposed in Eq.(2.7). This function is able to calibrate partial safety factors that refer to multiple design limit-states (here referred to as $N''$). This is achieved by introducing a relative importance parameter of $r'$ and $\hat{\gamma}_{ij}$ as a partial safety factor for the $i$th design situation by considering the combination $j$ in isolation. Furthermore, an additional constraint is imposed on the problem by defining lower ($\gamma_l$) and upper ($\gamma_u$) bounds on the partial safety factors to force the optimal solutions towards the definition of the partial safety factors.

The above approaches can be used to calibrate the partial safety factors of complex problems. However, in Melchers and Beck (2018), a simplified approach to approximate the associated safety factors of a limit-state function is also proposed, which is formulated as $g(x) = R - S$. This is achieved by introducing a separation factor (here denoted as $\tau$) to linearize the standard deviation of the limit-state function, i.e. $\sigma_g = (\sigma_R^2 + \sigma_S^2)^{1/2} \approx \tau(\sigma_R + \sigma_S)$. It should be noted that the consideration of $\tau = 0.75 \pm 0.06$ leads to errors of less than 10% for the cases in which $1/3 \leq \sigma_R/\sigma_S \leq 3.0$. The characteristic safety factor then reads as Eq.(2.8).

\[
\frac{R_k}{S_k} \geq \gamma_R \gamma_S = \left( \frac{1 - \kappa_R V_R}{1 - \tau \kappa_R V_R} \right) \left( \frac{1 + \tau \beta V_S}{1 + \kappa_S V_S} \right) \tag{2.8}
\]

where $V_R$ and $V_S$ are the coefficients of variation of the resistance and demand terms respectively. In addition, $\beta$ is the safety index, which can be regarded as the target reliability level, and $\kappa_R$ and $\kappa_S$ are standardized characteristic values corresponding to the lower and upper quantiles of the resistance and demand, respectively.

As it is evident, the calibration process requires the calculation of Eq.(2.1). This can be achieved using a variety of approaches. However, the First Order Reliability Method (FORM) seems to be the most commonly used method for this objective. Therefore, FORM is briefly described in the following sections along with some of the other methods used in this study.

2.2 First order reliability method

The solution of Eq.(2.1) can be computationally intractable in real engineering problems. This is because the failure domain can often not be formulated explicitly. Rather, only pointwise estimates of the problem area are usually available, which result from calling the associated computational models (e.g. finite element models, discrete element models or computational fluid dynamic models). It should be noted that obtaining even a single one of these estimates may require a considerable computational budget. On the other hand, the joint probability distribution
between random variables is often unknown. In view of these concerns, a variety of methods have been proposed to evaluate this integral, which can generally be classified into two categories, those of approximate nature and simulation-based methods.

The first group of the above methods approximates the limit-state function around the point with the largest contribution to the probability of failure or Most Probable Point (MPP). This is often achieved using the first/second order Taylor series expansion around the MPP, resulting in the methods being known as the First Order Reliability Method (FORM) or Second Order Reliability Method (SORM) respectively. Among these methods, the FORM seems to be the most commonly used method in the context of structural reliability. At this point, it should be emphasized that the approximation of the limit-state function around the MPP has clear computational advantages. However, depending on its shape, an unreliable estimate of the failure probability can be obtained for highly concave, convex or nonlinear limit-states.

FORM takes the shortest distance from the point in the problem domain with the largest likelihood to the MPP as a measure of reliability. It has been shown that the use of this definition may result in the estimation of different reliability levels for a given problem, being formulated differently; which contradicts the requirement that a safety measure must be invariant. This concern was addressed by introducing the Hasofer-Lind transformation, which transforms the problem from its original space to the equivalent standard Normal space using Eq.(2.9) (Hasofer and Lind, 1974).

\[
\begin{align*}
    u_i &= \frac{x_i - \mu_i}{\sigma_i} \quad (2.9)
\end{align*}
\]

where \( \mu_i \) and \( \sigma_i \) are the mean and standard deviation of the \( i \)th basic random variable. It is worth mentioning here that FORM only requires the first two moments of each basic random variable and can therefore be categorized as a level II reliability assessment method.

With this transformation, the point with the largest contribution would be transferred to the origin of the new space. In addition, the basic random variables would follow the Gaussian distribution, resulting in the linearized limit-state function also following a Gaussian distribution. Therefore, Eq.(2.1) can be approximated as Eq.(2.10).

\[
\begin{align*}
    p_f &= P[g(X) \leq 0] \approx \int_{-\infty}^{0} f_{g_i}(\mathbf{u})d\mathbf{u} = \Phi \left( \frac{0 - \mu_{g_i}}{\sigma_{g_i}} \right) \\
    &= \Phi \left( \frac{\nabla g(\mathbf{u}^*)^\top (\mathbf{u}^* - \mathbf{u})}{\| \nabla g(\mathbf{u}^*) \|} \right) \\
    &= \Phi(-\alpha^\top \mathbf{u}^*) = \Phi(-\beta) \quad (2.10)
\end{align*}
\]

where \( \mu_{g_i} \) and \( \sigma_{g_i} \) represent the mean and standard deviation of the linearized limit-state function around the MPP reading as \( g_i(\mathbf{u}) = g(\mathbf{u}^*) + \nabla g(\mathbf{u}^*)^\top (\mathbf{u} - \mathbf{u}^*) \).
Furthermore, \( u^* \) is the MPP, also known as the design (checking) point. Moreover, \( \alpha \) is the direction of the vector from the origin of the standard Normal space to the MPP, which consequently reads as \( \alpha = -\nabla_g(u^*)/\|\nabla g(u^*)\| \).

Taking into account the relationship between the safety index and the directional cosines, the latter can provide a measure of the importance (sensitivity) of each basic random variable with respect to system safety without incurring further computational costs, i.e. \( \partial \beta / \partial u_i = \alpha_i \). It has been shown that considering the random variables with \( |\alpha_i| < 0.14 \) as deterministic leads to an error of less than 1% in the estimated safety index (Sørensen, 2011). Furthermore, using the obtained directional cosines, the omission sensitivity factor is introduced by Madsen (1988) as the ratio between the safety index when a basic random variable is considered deterministic (denoted here as \( \beta'_i \)) and the safety index when all variables are stochastic. This sensitivity measure reads as Eq.(2.11). Similar to the importance measure, a random variable with \( \zeta_i < 1.01 \) can be regarded as a deterministic variable (Sørensen, 2011).

\[
\zeta_i = \frac{\beta'_i}{\beta} = \frac{1}{\sqrt{1 - \alpha_i^2}}
\]  

(2.11)

Considering the above discussions, FORM can be formulated as an optimization problem, which is shown in Eq.(2.12).

\[
\beta = \arg \min_{\mathbf{u}} \left( \mathbf{u}^\top \mathbf{u} \right)^{1/2} \quad \text{subject to} \quad g_l(\mathbf{u}) = 0
\]  

(2.12)

In this context, the Hasofer-Lind-Rackwitz-Fiessler (HLRF) method is proposed as a sequential optimization method to solve it iteratively (Rackwitz and Flessler, 1978). This approach estimates the next design point based on the safety index, the directional cosines and the derivatives of the limit-state at the previous iteration, which reads as Eq.(2.13).

\[
\mathbf{u}^*_j(\mathbf{j}+1) = \left[ \alpha(j) \mathbf{u}^*_j(j) + \frac{g(\mathbf{u}^*_j(j))}{\|\nabla g(\mathbf{u}^*_j(j))\|} \right] \alpha(j)
\]  

(2.13)

This procedure terminates when the estimated safety index and the design point stabilize, i.e. \( |\beta_{(j+1)} - \beta_{(j)}| \leq \epsilon_\beta \) and \( \|\mathbf{u}^*_{(j+1)} - \mathbf{u}^*_{(j)}\| \leq \epsilon_u \); where the stopping criteria are often assumed to be \( 10^{-3} \).

Considering the presented formulation, FORM is a computationally moderate reliability analysis that not only provides an estimate of the reliability of physical system but also gives a measure of the importance of the contributing random variables. Considering these aspects, it can be used in early stages of the studies to investigate the problem of interest in general.
2.3 Simulation-based methods

From another perspective, numerical integration methods can be used to solve the reliability integral of Eq. (2.1). These methods use the pointwise estimates of the limit-state function through the realization of the joint probability distribution function to approximate the failure domain, which is why they are also known as simulation-based methods. In their most basic approach, an indicator function is used to translate the problem to the calculation of an expected value. This approach is known as crude Monte-Carlo Simulations (MCS), which reads as Eq. (2.14).

\[
p_f = \int_{-\infty}^{+\infty} \mathbb{I}[g(x) \leq 0] f_X(x) dx = \mathbb{E}\left[\mathbb{I}[g(x) \leq 0]\right] \approx \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}[g(x_i) \leq 0] = \hat{p}_f
\]

(2.14)

where \(\mathbb{I}(\cdot)\) denotes an indicator function that takes the value 1 if its statement is true and 0 otherwise. In addition, \(N\) is the number of realizations, i.e. the number of evaluations of the limit-state function. Considering the strong law of large numbers, the estimated probability of failure \(\hat{p}_f\) converges to the true probability of failure as \(N \to \infty\) leading MCS to provide an unbiased predictor of the failure probability.

It is important to note that the algorithms used to generate the required sample points follow a deterministic approach to generate a sequence of numbers. In this context, the most basic approaches, called pseudo-random approaches, generate a set of non-negative integers using the congruence relation and considering an arbitrary integer called a seed (Hull and Dobell, 1962). The pseudo-random approach has no control over the discrepancy of the sample points generated. Therefore, there is a high probability that the generated sample points are located in regions that make only a small contribution to the reliability problem, which has a negative effect on the convergence of MCS. This aspect can be evaluated by calculating the coefficient of variation of the estimated failure probability using MCS as an error measure, which results as Eq. (2.15) As can be seen, the accuracy of MCS is inversely proportional to the squared number of realizations, i.e. it has a convergence rate of \(O(N^{-0.5})\).

\[
CoV_{\hat{p}_f} = \sqrt{\frac{1 - \hat{p}_f}{N\hat{p}_f}}
\]

(2.15)

The disadvantage of the low convergence rate resulting from the sampling method has been overcome by quasi-random sampling (also known as low discrepancy sequences). An example of this are space-filling methods, e.g. Latin Hypercube Sampling (LHS) and its improved versions (Beachkofski and Grandhi, 2002), and Centroidal Voronoi Tessellations Sampling (CVTS). The former partitions the space into non-overlapping intervals (or hypercubes) with equal probability and then draws a sample from each of them. In the second method, the space is subdivided using Voronoi diagrams and then a sample is drawn from each of these diagrams.
(Saka et al., 2007). It should be emphasized here that these approaches require the solution of an optimization problem for sample generation, which slows down their application for higher dimensional problems.

Despite the considerable computational advances that have resulted from quasi-random sampling methods, a significant number of calls to the computational model are still required to estimate a sufficiently accurate probability of failure using simulation-based methods. This makes the application of such methods to real-world engineering problems a prohibitively expensive task, as a considerable computational budget often needs to be invested in such problems, even for a single evaluation. This is especially true for rare events, i.e. problems with a very low probability of failure. This problem has been the subject of numerous studies that have proposed a variety of methods. A detailed review of these methods is beyond the scope of this study; therefore, only a brief overview of the methods used is provided in the following sections. It should be mentioned here that an illustrative example is presented in Allahvirdizadeh (2021) to examine the applicability of the discussed methods in the context of this study. Therefore, for the sake of brevity, the same study is not presented here and the interested reader is referred to the aforementioned study. Moreover, in Allahvirdizadeh et al. (2021), a comparison of both accuracy and computational efficiency between the crude MCS, subset simulation and surrogate-assisted MCS for the running safety and passenger comfort criteria of railway bridges was performed, highlighting the acceptable performance of the alternative reliability assessment methods compared to the crude MCS.

**Subset simulation**

One of the methods developed to improve the performance of simulation-based reliability assessment methods from a computational perspective is Subset Simulation (SS). This method divides the failure domain into frequent intermediate failure events (i.e. \( p_{f_j} < p_f \) and even \( p_{f_j} \ll p_f \)), where each of these events represents a subset of the higher events. Furthermore, the union of all these sets corresponds to the original failure domain. Therefore, the probability of failure can be estimated as the product of the conditional failure probabilities of the intermediate events, which is formulated as Eq. (2.16) (Au and Beck, 2001).

\[
p_f = \mathbb{P}(g_1(x) < 0) \prod_{i=1}^{m-1} \mathbb{P}(g_{i+1}(x) < 0 \mid g_i(x) < 0) = p_{f_0} \prod_{i=1}^{m-1} p_{f_i}, \tag{2.16}
\]

where \( m \) denotes the number of intermediate failure events and \( p_{f_0} \) is the failure probability of the first intermediate event, which is determined using the conventional MCS. In addition, \( p_{f_i}, \quad i = 1, \ldots, m-1 \) represents the failure probability of the other intermediate events, for which it is recommended to assume a predetermined constant value, denoted as \( p_0 \). Considering this recommendation, the boundary of each event can be adaptively determined to be consistent with \( p_0 \). Assuming
that $N$ is the number of simulations for each event, the threshold corresponding to this boundary would be the one in the rank of $N p_0$. Following this approach, the performance of the subset simulation depends on $N$ and $p_0$. In this context, a small value of $p_0$ will result in the intermediate event being rare. Moreover, a large value of $p_0$ makes it necessary to divide the failure domain into many subdivisions.

As already mentioned, the intermediate failure probabilities, and not the first term, are conditional probabilities. Therefore, their estimation requires sampling from the conditional joint probability distribution, which was specified as the defined failure event in the previous step and is denoted here as $f_X^{(i)}(x|F_{i-1})$. The latter is an unknown posterior distribution, which leads to the Markov Chain Monte-Carlo (MCMC) with the modified Metropolis-Hastings algorithm being used for sampling.

MCMC generates a sequence of samples, each of which depends only on the previous sample, whose density of final samples is stationary and converges to the intended posterior distribution with $N \to \infty$. Each new sample in this approach is proposed based on a proposal distribution that represents the probability of generating $x_{j+1}$ from $x_j$. This Markovian property causes the generated samples to be interdependent, which can be solved by a thinning procedure or selecting a sample in each given $t$ step. It should be emphasized here that SS uses a uniform distribution centered around the previous sample as the proposal distribution and propagates in both directions with a length equal to the standard deviation of the random variable (Au and Beck, 2001).

The procedure therefore begins with the generation of a sample from the proposal distribution. The probability of moving (flow) from the previous point to the new point can be expressed by Eq.(2.17).

$$P(x_{j+1}|x_j) = q(x_{j+1}|x_j)T(x_{j+1}|x_j)$$

(2.17)

where $T(x_{j+1}|x_j)$ is the transition probability that determines whether the new sample is accepted or rejected. To fulfill the stationary condition, the probability of moving from the previous point to the new point should be equal to the reverse movement (referred to as detailed balance), which reads as Eq.(2.18).

$$P(x_{j+1}|x_j)f_X(x_j) = P(x_j|x_{j+1})f_X(x_{j+1})$$

(2.18)

Since the considered proposal distribution is symmetric, the transition probability can be expressed as the ratio of the joint probability distribution. Thus, the new sample is accepted if the transition probability is greater than a generated random number with uniform distribution and if it is a member of the failure domain of the previous intermediate failure event. Otherwise, the previous point would be considered as a sample in the next state.
Surrogate-assisted simulation-based methods

As discussed, the conventional (crude) MCS is computationally less favorable, as the associated computational models have to be called many times. This encouraged to substitute the limit-state function with easy-to-evaluate approximate functions called surrogate models (or meta-models, represented here as \( \hat{M}(x) \)). After training surrogate models, the reliability integral can be evaluated using any simulation-based method without imposing further significant computational costs. However, it should be noted that their approximate nature increases the epistemic uncertainty of the evaluated reliability. In other words, surrogate models do not necessarily provide an unbiased estimate of the probability of failure (Dubourg et al., 2013; Sudret, 2012).

Referring to the simulation-based method for evaluating the reliability integral presented in Eq.(2.14), only the sign of the limit-state function is required. In view of this, the surrogate models can be classified into two main groups, namely regression-based and classification-based models. The former predict the real value of the response, i.e. \( \hat{M} : \mathbb{R}^{N_{d}} \rightarrow \mathbb{R} \); while the latter determines whether each sample point is in the safe domain or in the failure domain, i.e. \( \hat{M} : \mathbb{R}^{N_{d}} \rightarrow \{-1, +1\} \). In this context, different models have been proposed within each category, which are briefly presented in the following sections.

It should be noted that these surrogate models were mostly adapted from statistical learning theory. Therefore, their training phase seems to be similar to machine learning models, which leads to the need for a training dataset. To this end, the corresponding responses of a Design of Experiment (DoE) obtained from the true computational model are collected and form a set of discrete estimates of the output as a function of the input vector, i.e. \( D = \{(x^{(i)}, y^{(i)}), \; i = 1, 2, ..., N\} \). It is clear that the size of this training set should be minimal, otherwise this contradicts the main purpose of using surrogate models. This is the main concern of a concept called active learning, which is discussed in more detail in the following sections.

Gaussian process regression (Kriging)

Gaussian Process Regression (GPR) is one of the most commonly used surrogate models in the context of structural reliability, which extends the concept of Normal distribution to functions. It is therefore a collection of random variables, each subset of which also follows a Gaussian distribution. This section gives a brief overview of the main concepts of GPR; therefore, the interested reader is referred to Jones et al. (1998); Rasmussen and Williams (2006) for more detailed information.

The GPR consists of two parts, namely the regression term and the prediction error, which is formulated as Eq.(2.19). The first part describes the general trend of the response of interest. In addition, conventional regression analyses assume
2.3. SIMULATION-BASED METHODS

an independent prediction error between different points; however, GPR assumes a spatial correlation.

\[ \hat{M}(x) = H(x|w) + Z(x|\theta, \sigma^2_z) \]  

(2.19)

where \( H(x|w) = H^\top(x)w \) represents the regression term with \( h(x) \) as the vector of the preselected basis functions and \( w \) as the corresponding regression coefficients. In addition, \( Z(x|\theta, \sigma^2_z) \) is the Gaussian process that defines the prediction error with \( \theta \) as the set of hyperparameters and \( \sigma^2_z \) as the process variance. The trained model should provide an unbiased estimate of the response, which results in the mean of \( Z(x|\theta, \sigma^2_z) \) being assumed to be zero; however, the covariance is modeled using the kernel trick, i.e. \( Z(x|\theta, \sigma^2_z) \sim GP(0, \Sigma(x|x', \theta, \sigma^2_z)) \), where \( \Sigma(x,x'|\theta, \sigma^2_z) = K(x, x'|\theta, \sigma^2_z) \).

The kernel function is a preselected autocorrelation function that models the covariance between two points based on a distance measure. Therefore, the prediction uncertainty of each new point depends on its distance from the training DoE. In this context, a variety of kernel functions have been proposed, all of which construct a positive definite covariance matrix. Among these, the Squared Exponential (SE) kernel function, as expressed in Eq.(2.20), seems to be the most widely adopted kernel function for meta-modelling purposes. The multiplication of SE kernels over different dimensions results in the Square Exponential Automatic Relevance Determination (SE-ARD) kernel as a universal kernel function. This kernel function assigns different hyperparameters in the different dimensions. The discussion of the various properties of kernel functions is beyond the scope of this study; therefore, the interested reader is referred to Duvenaud (2014) for further detailed information in this context.

\[ K(x,x'|\theta, \sigma^2_z) = \sigma^2_z \exp\left(-\frac{\|x - x'\|^2}{2\theta^2}\right) \]  

(2.20)

where \( \theta \) is a hyper-parameter, which in the case of the SE kernel function is referred to as the characteristic scale length.

To train the GPR, the regression coefficients, the process variance and the hyperparameters of the kernel function must then be determined, the first two depending on the latter, which are expressed as Eq.(2.21) and Eq.(2.22).

\[ \hat{w}|\hat{\theta} = (H^\top\Gamma^{-1}H)^{-1}H^\top\Gamma^{-1}y \]  

(2.21)

\[ \hat{\sigma}^2_z|\hat{\theta} = \frac{1}{N}(f - H\hat{w})^\top\Gamma^{-1}(y - H\hat{w}) \]  

(2.22)

where \( H, \Gamma \) and \( y \) are the matrices that collect the value of the basis functions, the correlation and the true responses at training DoE. In addition, \( \hat{\theta} \) are the estimated hyperparameters of the kernel function, which are often determined using the maximum likelihood method.
CHAPTER 2. RELIABILITY-BASED METHODS FOR DESIGN IMPROVEMENT

The above model parameters are obtained from a limited amount of information. They can therefore be considered as random variables. Taking this into account, the predicted response (denoted here as $y^*$) would also be random. The prediction probability of an unseen point of $x^*$ can then be estimated using the Bayesian approach as in Eq.(2.23).

$$P(y^* | x^*, X, y) = \int P(y^* | x^*, \Omega)P(\Omega | X, y)d\Omega$$  \hspace{1cm} (2.23)

where $\Omega$ is the vector of all defining parameters of the model. It is important to note that the integration is computed over all values of the model parameters, so GPR would be a non-parametric model. Despite this fact, the characteristic of GPR allows an explicit estimation of the parameters of the predictive distribution, which would also follow a Gaussian distribution. Consequently, the mean and variance of the predictive distribution would be as Eq.(2.24) and Eq.(2.25), respectively.

$$\hat{\mu}(x^*) = h^T(x^*)\hat{w} + r^T(x^*)\Gamma^{-1}(y - H\hat{w})$$  \hspace{1cm} (2.24)

$$\hat{\sigma}^2(x^*) = \hat{\sigma}^2 \left[1 - r^T(x^*)\Gamma^{-1}r(x^*) + B(x^*)^TQ^{-1}B(x^*)\right]$$  \hspace{1cm} (2.25)

where $r(x^*)$ represents the correlation between the unseen point and the training samples, and $B(x^*) = H^T\Gamma^{-1}r(x^*) - h(x^*)$.

It should be emphasized here that the predictive mean minimizes the expected loss, so it can be considered the best prediction for an unseen point. In addition, GPR provides an estimate of the uncertainty of the prediction through the prediction variance. This capability allows the trained model to measure its epistemic uncertainty with respect to the unseen points. This capability is used in active learning to distinguish the regions in the problem domain where the model is less confident about them. This concept is explained in the following sections of this chapter. Moreover, it can be used to adjust the calculated probability of failure by taking into account the epistemic uncertainties introduced by using the GPR as a surrogate model; this can be expressed as Eq.(2.26) (Jiang et al., 2013).

$$p_f = P\left[g(x) \leq 0 \right] \approx \frac{1}{N} \sum_{i=1}^{N} \int_{-\infty}^{0} P\left[g(x_i) \right] dg = \frac{1}{N} \sum_{i=1}^{N} \Phi\left(-\frac{\hat{\mu}(x_i)}{\hat{\sigma}(x_i)}\right)$$  \hspace{1cm} (2.26)

Polynomial chaos expansion (PCE)

The decomposition of a function into a summation of orthogonal functions, such as modal analysis in structural dynamics, is a concept widely used in engineering applications. It is worth noting that two functions in Hilbert space are considered orthogonal if their inner product is equal to a multiplication of the Kronecker delta. In this context, the Polynomial Chaos Expansion (PCE), as a surrogate
model mostly used for uncertainty quantification, adopts the same concept. This section gives a brief overview of PCE, which is largely adapted from Blatman (2009); Blatman and Sudret (2011); Sudret (2008); Xiu and Karniadakis (2002).

PCE uses a set of orthogonal polynomials satisfying a three-term recursion relation, as Eq.(2.27), where the weighting function of their inner product is a certain probability distribution.

\[ A_n \phi_{n+1}(x) = (A_n + B_n - x)\phi_n(x) - B_n \phi_{n-1}(x), \quad n \geq 1 \]  

(2.27)

where \( A_n \) and \( B_n \) are non-zero constants, \( \phi_{-1}(x) = 0 \) and \( \phi_0(x) = 1 \).

They were originally developed for independent random variables with the Gaussian distribution as weighting function and called Wiener polynomial chaos. However, they were extended for other widely used probability distributions and called Wiener-Askey polynomial chaos. Some of the most commonly used are listed in Table 2.1. Nevertheless, random variables with other distributions can also be used in this context by simply transforming them into the known distributions.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Wiener-Askey polynomial chaos</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>Hermite</td>
<td>((-\infty, \infty))</td>
</tr>
<tr>
<td>Uniform</td>
<td>Legendre</td>
<td>([-1, 1])</td>
</tr>
<tr>
<td>Beta</td>
<td>Jacobi</td>
<td>([-1, 1])</td>
</tr>
<tr>
<td>Gamma</td>
<td>Laguerre</td>
<td>([0, \infty))</td>
</tr>
</tbody>
</table>

It has been shown that any function (the limit-state function in the context of structural reliability) in the \( L_2 \) sense (i.e. finite variance) can be expanded as the summation of the tensor product of the corresponding Wiener-Askey polynomial chaos functions. This equality is valid if an infinite degree of polynomials is considered, which is obviously not feasible from a computational point of view. Therefore, an approximation of the true function can be obtained by truncating the degree of the polynomials to a prescribed degree (denoted here as \( q \)), which reads as Eq.(2.28).

\[ g(x) \approx \hat{M}(x) = \sum_{|\alpha| \leq q} c_\alpha \Psi_\alpha(\xi) \]  

(2.28)

where \( c_\alpha \) is the set of coefficients to be determined, \( \Psi_\alpha(\xi) \) is the tensor product of the basis polynomial chaos functions and \( \xi \) is the vector of the reduced form of the basic random variables. This approach leads to the cardinality of the total degree (number of expansion terms) being equal to \( \binom{d+q}{q} \), where \( d \) is the dimensionality of the problem. Therefore, the number of unknowns during the training of PCE
increases exponentially with the dimensionality of the problem, so this surrogate model suffers from the curse of dimensionality. Similarly, the accuracy of the trained PCE can be improved by increasing the degree of truncation, which in turn increases the computational cost of training. To summarize, training PCE is a trade-off between accuracy and computational cost.

Considering this drawback, another truncation scheme is presented by Blatman (2009). The proposed approach defines a hyperbola with the prescribed degree $0 < \gamma \leq 1$, which excludes all terms outside of it and reduces the cardinality of the total degree to $(\sum_{i=1}^{d} \alpha_{i}^{\gamma})^{1/\gamma}$.

Therefore, the coefficients of the polynomials are often determined either by projection or by least square minimization methods. The former uses the property of orthogonality to remove all coefficients except the one corresponding to the desired polynomial, which is formulated as Eq.(2.29). It should be noted that the right-hand side of this equation must be estimated using numerical integration methods, which increases the computational cost of training.

\[
    c_{\alpha} = \mathbb{E} \left[ \sum_{|\alpha| \leq q} c_{\alpha} \Psi_{\beta}(x) \Psi_{\alpha}(x) \right] = \sum_{|\alpha| \leq q} c_{\alpha} \mathbb{E} \left[ \Psi_{\beta}(x) \Psi_{\alpha}(x) \right] \approx \mathbb{E} \left[ \Psi_{\beta}g(x) \right] \quad (2.29)
\]

On the other hand, the least square minimization method determines the coefficients by minimizing the sum of the residual squares; which reads as Eq.(2.30).

\[
    C = (F^{\top}F)^{-1}F^{\top}y \quad (2.30)
\]

where $y$ is the vector collecting the true values and $F$ denotes the corresponding values resulted from the polynomials.

**Artificial neural networks (ANN)**

Artificial Neural Networks (ANN) are inspired by biological neurons, in which a set of interconnected nodes (called neurons) in multiple layers construct a nonlinear function to map the input vector to the desired responses. The intermediate layers are referred to as hidden layers and ANN models with many hidden layers are often called as deep learning. The number of hidden layers and also the number of neurons per layer formulates the structure of the ANN model, which is known as ANN architecture and is often defined by cross-validation techniques. An example of an ANN architecture is shown in Figure 2.1. It should be mentioned here that the overview presented in this section was mostly adapted from James et al. (2013), to which the interested reader is referred for further reading.

The input vector is transformed into the output layer by passing through all neurons. However, it should be emphasized here that this statement is not necessarily valid for all ANN architectures; however, it seems to be true for the Multi-Layer
Perceptron (MLP) architecture, which is the focus of this study (see Figure 2.1). Therefore, the input of each neuron is a transformed weighted combination of the outputs of the neurons in the previous layer; the transformation is achieved by a preselected nonlinear basis function called the activation function. This statement for the $i$th neuron in the $j$th hidden layer can be formulated as Eq.(2.31).

$$h_i^{(j)} = A\left[\sum_{k=1}^{N_{(j-1)}} w_{ki}^{(j)} h_k^{(j-1)} + b\right]$$

(2.31)

where $N_{(j-1)}$ is the number of neurons in the previous layer, $w_{ki}^{(j)}$ is the weight of the $k$th neuron in the previous layer fed to the $i$th neuron in layer $j$, $b$ represents the bias term and $A(\cdot)$ is the activation function.

A variety of activation functions have been proposed, such as the sigmod function, the hyperbolic tangent, the Rectified Linear Unit (ReLU) and the Leaky ReLU, of which the ReLU, expressed in Eq.(2.32), is the most commonly used.

$$A(x) = \begin{cases} 0, & x < 0 \\ x, & \text{otherwise} \end{cases}$$

(2.32)

Training the ANN surrogate model therefore requires tuning the weights and bias terms (i.e. $\theta = [w, b]$). This is achieved by a procedure known as feed-forward-back propagation. This procedure involves feeding the training DoE into the model, estimating the loss function (e.g. Mean Square Error - MSE - as Eq.(2.33)) as a measure of the prediction error, calculating the gradients of the loss function in relation to the model parameters (shown here as $\delta = \nabla_\theta \mathcal{L}(\mathbf{x}|\theta)$) using the chain rule and adjusting the new model parameters using a learning rate parameter ($\alpha_\delta$), i.e. $\theta_{i+1} = \theta_i + \alpha_\delta \delta$. This one training cycle is often referred to as an epoch and
the training of the ANN terminates when a certain prescribed number of epochs is reached or the variation of the loss function becomes smaller than a threshold value.

$$\mathcal{L}_{\text{MSE}}(\mathbf{x}|\theta) = \frac{1}{N} \sum_{i=1}^{N} \left( g(x_i) - \hat{M}(x_i) \right)^2$$ \hspace{1cm} (2.33)

Support vector machines (SVM)

Support Vector Machines (SVM) distinguish the different classes by finding a separating hyperplane. Such a hyperplane is not necessarily unique if it exists. Therefore, additional constraints are imposed on the training phase by maximizing the distance between the classes (margin). Without compromising the generality of the method, it is also assumed that the multiplication of the class label by the distance of the closest points in each class to the hyperplane is equal to 1. These points are called support vectors. Considering these factors, training the SVM involves solving an optimization problem that minimizes the norm of the normal vector of the separating hyperplane. For reasons of brevity, however, the details are not described here. In this context, the interested reader is referred to Cortes and Vapnik (1995); Hurtado (2013).

After training, the prediction is made based on the location of the point with respect to the separating hyperplane. It is worthwhile to mention that the SVM does not directly provide the associated probability of belonging to each class. In this context, the maximum likelihood method can be used to fit a transformation function (e.g. a sigmoid function) that transforms the distance from the separating hyperplane (referred to as the classification score) into a probability space.

The presented algorithm was originally developed for linearly separable problems, which is not often the case. However, the reformulation of the optimization problem in Lagrangian format revealed that it depends only on the Lagrange multipliers and the inner product of the training points. Therefore, the kernel trick is used to arbitrarily transform the problem space into higher dimensions where the probability of a linearly separable problem is higher. A variety of kernel functions have been proposed, of which Gaussian kernel (also known as RBF kernel) and polynomial kernel are the most commonly used. The RBF kernel function is similar to the SE kernel function presented in Eq.(2.20), while the polynomial kernel reads as Eq.(2.34).

$$\mathcal{K}(\mathbf{x}, \mathbf{x'}) = (1 + \mathbf{x}^\top \mathbf{x'})^p$$ \hspace{1cm} (2.34)

where $p$ is the degree of the polynomial and is usually defined using the cross-validation technique.

Therefore, the training of the SVM leads to the solution of the optimization problem
as Eq.(2.35), which is reduced to the determination of the Lagrange multipliers.

\[
\hat{\alpha} = \arg \max_{\alpha} \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} y(x_i) \alpha_i K(x_i, x_j) \alpha_j y(x_j)
\]

s.t. \[
\begin{align*}
\alpha_i &\geq 0, \quad i = 1, \ldots, N \\
\sum_{i=1}^{N} \alpha_i y(x_i) &= 0
\end{align*}
\] (2.35)

where \(y(x) = \text{sign}[g(x)]\) denotes the associated class of each data point, \(\alpha\) is the vector of Lagrange multipliers. They are usually equal to zero, except for support vectors. The trained surrogate models are then as in Eq.(2.36).

\[
\hat{M}(x) = \text{sign} \left[ \sum_{i=1}^{N} \alpha_i y(x_i) K(x, x_i) \right]
\] (2.36)

**K-nearest neighbours (kNN)**

KNN makes its decision based on a majority vote of the \(k\) closest points to the desired point (Rogers and Girolami, 2020). Given this definition, training kNN involves selecting a distance measure and then the best number of neighbours, both of which can be chosen using cross-validation techniques. Therefore, the kNN surrogate model can be expressed as Eq.(2.37).

\[
\hat{M}(x) = \text{mode} \left[ \mathcal{V}_{N_{d k}}(x) \cap D \right]
\] (2.37)

where \(\mathcal{V}_{N_{d k}}(\cdot)\) is an \(N_d\)-dimension volume over the \(k\)-neighbourhood and \(D\) is the training DoE.

A variety of distance measures have been adopted in this context. One of their general forms is the Minkowski distance, which is shown in Eq.(2.38).

\[
d(x, x') = \left[ \sum_{i=1}^{N_{d}} |x_i - x'_i|^q \right]^{1/q}
\] (2.38)

where \(q\) is its order. It is worth mentioning that the distance measures with \(q < 1\) are known as fractional distances, \(q = 1\) gives the Manhattan (Taxicab) distance measure, \(q = 2\) represents the Euclidean distance and as \(q \to \infty\) it denotes the Chebyshev distance metric. In Aggarwal et al. (2001) it was discussed that the ability of higher order distance measures to distinguish the farthest points from the closest ones decreases with the dimensionality of the problem. Therefore, it is recommended to use fractional distance metrics for such problems.

It should be emphasized that choosing an even number of neighbours in binary classification problems, such as structural reliability evaluation, may result in the
inability to distinguish the majority class. This can be solved either by selecting an odd \( k \) value or by weighting the contribution of the neighbours based on their distance to the unseen point. While kNN is a non-probabilistic model, the latter approach can be used to estimate the likelihood of belonging to each class, as presented in Eq.(2.39). This probability can be considered as the posterior predictive probability of the trained kNN model.

\[
P[c_i|x', x, y, k] = \frac{\sum_{j=1}^{k} d(x', x_j) \mathbb{I}(y_j = c_i)}{\sum_{j=1}^{k} d(x', x_j)}, \quad c_i = \{-1, +1\} \tag{2.39}
\]

where \( x' \) is the new point. In addition, \( x \) and \( y \) represent the training DoE.

Considering the above, training a single kNN model does not involve significant computational cost. This aspect encouraged to investigate the applicability of training an ensemble (set) of kNN models for a given problem in the hope that their composition compensates for the weaknesses of the other models. Such ensemble kNN models are referred to as a random subspace, where each individual kNN model is trained with a randomly selected subset of features in the input vector (Ho, 1998a).

**Decision tree and random forest**

Decision trees are greedy top-down algorithms that classify each point based on a set of yes or no questions (according to a flowchart). Therefore, in their basic form, they partition the problem space into non-overlapping hypercubes using binary splits, i.e. the decision boundaries run parallel to the problem axis. The class of each new point is then determined based on the subregion it locates at the end of the conditional statements (James et al., 2013).

Training a decision tree therefore involves determining the correct conditional statement at each decision node and the depth at which it ends. The former is achieved by selecting the best splitting feature and the corresponding threshold at each node that maximizes the difference in Gini index (impurity - see Eq.(2.40)) or entropy (information gain - see Eq.(2.41)) as a measure of splitting quality before and after splitting the remaining training data. The presented measures are formulated for a binary classification problem. It should also be emphasized here that the simple decision trees of interest in this study select only a single feature at each decision node. However, it has been shown that the performance of decision trees can be improved by a combination of features, making the decision boundaries oblique to the problem axis.

\[
\mathcal{I} = 1 - \sum_{j=1}^{2} p_j^2 \tag{2.40}
\]
2.3. SIMULATION-BASED METHODS

\[ IG = - \sum_{j=1}^{2} p_j \log_2(p_j) \]  

(2.41)

where \( p_j \) is the probability of selecting an item in class \( j \) corresponding to the relative frequency of the remaining data points within this class in the subregion.

This procedure terminates when a predefined depth is reached or the number of remaining data falls below a certain value. This approach can lead to very deep decision trees that are likely to be susceptible to the problem of overfitting. This concern is addressed by pruning the trained tree to a smaller subtree that leads to a smaller cross-validation error than the original decision tree. After determining such a decision tree, the probability of belonging to each class can be estimated as the frequency of points with similar class at the terminal node where the new point ends.

Similar to the kNN models, the straightforward training algorithm makes the use of an ensemble of decision tree models a popular practice. In this context, a variety of approaches have been proposed, of this only random forests were used in the current study. In this approach, a random subset of all possible features is selected to train individual independent decision trees. Then, each new point is classified based on the majority vote of all these individual decision trees. In this approach, \( 2^{N_d} \) individual decision trees can be trained for a problem with a dimensionality of \( N_d \). However, it has been shown that the desired accuracy would most likely be achieved with a much smaller number of combinations (Ho, 1998b).

**Ensemble models**

The above sections have mainly discussed the application of individual models to surrogate the limit-state function. However, it has been shown that the combined (aggregated) models can have a mean square error that is lower than the expected mean error of an individual model (Breiman, 1996). Furthermore, it is believed that training a surrogate model is often less (or even much less) computationally intensive than a single evaluation of the limit-state function (i.e., calling the true computational model). This opens up the possibility of improving the performance of heterogeneous surrogate models by blending them into a committee, also known as an ensemble of surrogate models.

Ensemble learning, in its most basic formulation, can be achieved by averaging in regression problems or by majority voting in classification problems, as expressed in Eq. (2.42) and Eq. (2.43), respectively (Goel et al., 2007; Sammut and Webb, 2011).

\[ \hat{M}(x) = \sum_{j=1}^{m} w_j \hat{M}_j(x) \]  

(2.42)
\[ \hat{M}(x) = \text{sign} \left[ \sum_{j=1}^{m} w_j \hat{M}_j(x) \right] \] (2.43)

where \( m \) is the number of individual surrogate models and \( w \) is their weight in the combination. The latter can be chosen according to different approaches. In the winner-takes-all approach, the entire weight is assigned to the most accurate model, which results in the weights being formulated as Eq.(2.44)

\[
   w_j = \begin{cases} 
   1, & \epsilon_j = \min(\epsilon_{i=1},...,m) \\
   0, & \text{Otherwise}
   \end{cases}
\] (2.44)

where \( \epsilon_i \) is the leave-one-out cross-validation error of the \( i \)th individual surrogate model. An alternative approach distributes the weights between all models based on their accuracy and is formulated as Eq.(2.45) and Eq.(2.46) (Goel et al., 2007).

\[
   w_j = \frac{\sum_{i=1, i \neq j}^{m} \epsilon_i}{(m-1) \sum_{i=1}^{m} \epsilon_i} 
\] (2.45)

\[
   w_j = \frac{(\epsilon_j + a \epsilon_{\text{ave}})^b}{\sum_{i=1}^{m} (\epsilon_i + a \epsilon_{\text{ave}})^b} 
\] (2.46)

where \( \epsilon_{\text{ave}} \) is the mean value of the leave-one-out cross-validation error. In addition, \( a \) and \( b \) denote constant values that control the importance of the average model compared to the individual models. They can be optimized considering the prediction error of the ensemble model; however, it is suggested to set them to 0.05 and -1, respectively (Goel et al., 2007).

In addition to the above approaches, ensemble models can be constructed in their general format using a concept called the stack generalization concept (Wolpert, 1992). This concept is shown schematically in Figure 2.2. The training dataset is first divided into \( K \) folds, one part of which is used to train the individual surrogate models in the first layer (denoted as level-0) and the other part is used to create the training dataset of the model in the next layer (denoted as level-1) based on the predictions of the trained level-0 models. After repeating the above method for all folds, the level-1 models are trained (Sagi and Rokach, 2018).

**Active learning**

As mentioned earlier, the main objective of surrogate-assisted reliability assessment is to minimize the number of calls to the computational model while still meeting the desired level of accuracy. This is the topic of active learning, also known as adaptive sampling or sequential experimental design. A variety of methods have been proposed in this context; however, the general methodology of almost all of them is based on starting the training phase with a small DoE that is iteratively enriched by adding new samples chosen by assistance of the trained surrogate model.
2.3. SIMULATION-BASED METHODS

Figure 2.2: Ensemble of surrogate models using stack-generalization concept.

in the previous iteration. This procedure ends when a convergence (stopping) criterion is met. In this section, the above methodology is briefly described; however, the interested reader can find a comprehensive review of active learning for structural reliability applications in Moustapha et al. (2022); Teixeira et al. (2021).

As far as the author is aware, there is no specific rule for the initial size of the DoE; however, some recommendations have been proposed in previous studies that can be used as a rule of thumb. For example are $10N_d$, $(N_d + 1)(N_d + 2)/2$ and $\max (10, 2N_d)$ as proposed in (Bichon et al., 2008; Moustapha et al., 2022).

Then the above training dataset is adopted to train the desired surrogate model. If the trained model was not accurate enough, its knowledge of the problem space should be extended by adding new sample(s). This sample is often selected from a large number of realizations that fill the problem space and is called the sample pool. Obviously, generating such sample pool does not incur significant computational cost for the problem since they are unlabeled. The confidence of the surrogate model increases either by adding samples where the model is less confident in these regions (resulting in a large variance - often referred to as exploitation) or by considering points with a large contribution to the estimated probability of failure (i.e. regions close to the limit-state - often referred to as exploration) (Echard et al., 2011). These conditions are implemented in a function called the learning function (or acquisition function), which evaluates all points in the sample pool and selects the most informative point (or batch of points).

In previous studies, a variety of learning functions have been developed that appear
to be designed primarily for Kriging models. The reason for this is probably the ability to explicitly provide the prediction variance. In one of the earliest versions, the Expected Improvement (\(EI\)) learning function is proposed in Jones et al. (1998), which reads as Eq.(2.47). It should be noted that this learning function is proposed for optimization objectives in a framework known as Efficient Global Optimization (EGO) and is not intended for structural reliability applications.

\[
EI(x) = \mathbb{E} \left[ \max \left( 0, f_{\min} - \hat{M}(x) \right) \right] \tag{2.47}
\]

where \(f_{\min}\) is the minimum of the surrogate model trained so far.

Based on \(EI\), the Expected Feasibility Function (\(EFF\)) is proposed in Bichon et al. (2008), which is formulated as Eq.(2.48). It should be noted that the new candidate maximizes \(EFF\) to enrich the existing DoE.

\[
EFF(x) = \hat{\mu}(x) \left( 2\Phi\left( \frac{-\hat{\mu}(x)}{\hat{\sigma}(x)} \right) - \Phi\left( \frac{-2\hat{\sigma}(x) - \hat{\mu}(x)}{\hat{\sigma}(x)} \right) - \Phi\left( \frac{2\hat{\sigma}(x) - \hat{\mu}(x)}{\hat{\sigma}(x)} \right) \right) \\
- \hat{\sigma}(x) \left( 2\phi\left( \frac{-\hat{\mu}(x)}{\hat{\sigma}(x)} \right) - \phi\left( \frac{-2\hat{\sigma}(x) - \hat{\mu}(x)}{\hat{\sigma}(x)} \right) - \phi\left( \frac{2\hat{\sigma}(x) - \hat{\mu}(x)}{\hat{\sigma}(x)} \right) \right) \\
+ \left[ \Phi\left( \frac{2\hat{\sigma}(x) - \hat{\mu}(x)}{\hat{\sigma}(x)} \right) - \Phi\left( \frac{-2\hat{\sigma}(x) - \hat{\mu}(x)}{\hat{\sigma}(x)} \right) \right] \right] \tag{2.48}
\]

where \(\hat{\mu}(\cdot)\) and \(\hat{\sigma}(\cdot)\) denote the predictive mean and variance of the Kriging model trained in the previous iteration. In addition, \(\Phi(\cdot)\) and \(\phi(\cdot)\) are the Cumulative Distribution Function (CDF) and Probability Density Function (PDF) of the standard Gaussian distribution, respectively.

This learning function is further simplified by Dubourg et al. (2011) taking into account the exploration and exploitation objectives, which led to the proposal of the U-criterion (also known as the deviation number). This learning function is formulated as Eq.(2.49) and the best candidate minimizes it.

\[
U(x) = \left| \frac{\hat{\mu}(x)}{\hat{\sigma}(x)} \right| \tag{2.49}
\]

Similarly, other learning functions such as the H-Function (\(HF\)), the Potential Risk Function (PRF), the Kriging Classification Uncertainty Quantification (KCUQ) and the Expected Integrated Error Reduction (EIER) for Kriging (Hong et al., 2022; Li et al., 2022; Lv et al., 2015; Wei et al., 2023). Of these, only \(HF\) is adopted in this study as Eq.(2.50); therefore, the corresponding relationships for
2.4. IMPLEMENTING DEPENDENCY BETWEEN RANDOM VARIABLES

the others are not presented here for the sake of brevity.

\[
\mathcal{H}(x) = \ln(\sqrt{2\pi \hat{\sigma}(x)}) + \frac{1}{2} \left[ \Phi\left(\frac{2\hat{\sigma}(x) - \hat{\mu}(x)}{\hat{\sigma}(x)}\right) - \Phi\left(\frac{-2\hat{\sigma}(x) - \hat{\mu}(x)}{\hat{\sigma}(x)}\right)\right] - \left[\frac{2\hat{\sigma}(x) - \hat{\mu}(x)}{2}\phi\left(\frac{2\hat{\sigma}(x) - \hat{\mu}(x)}{\hat{\sigma}(x)}\right) + \frac{2\hat{\sigma}(x) + \hat{\mu}(x)}{2}\phi\left(\frac{-2\hat{\sigma}(x) - \hat{\mu}(x)}{\hat{\sigma}(x)}\right)\right]
\] (2.50)

The above learning functions were developed for the Kriging surrogate model; therefore, a "max-min" sampling scheme is considered in Basudhar and Missoum (2010), which is also applicable to other types of surrogate models. However, it should be mentioned that it was originally proposed for SVM models. It is assumed that the model is less reliable in regions far away from the existing DoE. Therefore, an optimization problem such as Eq.(2.51) is formulated to find the points in the sample pool farthest from the existing DoE that are very close to the approximated limit-state function. It is worth mentioning that the approach in Lacaze and Missoum (2014) is further generalized by transforming the optimization problem into an unconstrained one using the Chebychev distance and the joint probability distribution of the contributing basic random variables.

\[
\hat{x} = \arg\min_{x,z} -z
\]

\[
\text{s.t.} \quad \left\{\|x - x_i\| \geq z, \quad i = 1, 2, ..., N\right\}
\]

where \(z\) is an arbitrary parameter.

Each of the above learning functions has its own specific stopping criterion; however, some general recommendations have also been proposed. The latter mostly depend on the convergence of the estimated safety index and it is recommended to control it in several consecutive steps to prevent premature termination of the training. Therefore, some of the most commonly used stopping criteria are listed in Table 2.2.

2.4 Implementing dependency between random variables

The sampling methods discussed generate independent random numbers and neglect the correlation between the variables. Therefore, this section is dedicated to methods that take such dependencies into account in structural reliability assessment problems.

In practice, it is unlikely to have more information about the dependence between variables than their correlation obtained from statistical observations. However,
it should be noted that a high correlation between variables does not necessarily justify their dependence. In this context, the simplest approaches transform normally distributed, uncorrelated samples (denoted here as \( \mathbf{u} \) with a zero mean and a standard deviation of one) into normally distributed, correlated samples (denoted here as \( \mathbf{x} \) with a zero mean and a standard deviation of one) using \( \mathbf{T} \) as the orthogonal transformation matrix (i.e. i.e. \( \mathbf{T}^\top \mathbf{T} = \mathbf{I} \)). This approach is also known as Nataf Transformation. As it is shown in Eq.(2.52), such a transformation matrix is a lower triangular matrix obtained by Cholesky decomposition of the covariance matrix. For non-normally distributed variables, the above transformation can be used to first generate normally distributed correlated random variables and then transform them into non-normally distributed correlated random variables with \( \Phi(\mathbf{x}) = F_\mathbf{Y}(\mathbf{y}) \), where \( F_\mathbf{Y}(\mathbf{y}) \) is the CDF of the non-normally distributed variable (Sørensen, 2011). With the latter transformation, a direct transformation

---

**Table 2.2: Proposed stopping criteria to terminate active learning of surrogate models.**

<table>
<thead>
<tr>
<th>Method</th>
<th>Stopping Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{E}_F ) (Bichon et al., 2008)</td>
<td>( \max \left[ \mathcal{E}_F(\mathbf{x}) \right] \leq 10^{-3} )</td>
</tr>
<tr>
<td>( \mathcal{U} ) (Echard et al., 2011)</td>
<td>( \min [\mathcal{U}(\mathbf{x})] \geq 2.0 )</td>
</tr>
<tr>
<td>( \mathcal{H}_F ) (Lv et al., 2015)</td>
<td>( \max \left[ \mathcal{H}_F(\mathbf{x}) \right] \leq 0.5 )</td>
</tr>
<tr>
<td>( \hat{\beta} )-range(^\dagger) (Dubourg et al., 2011)</td>
<td>( \max (\hat{\beta}^+ - \hat{\beta}^0, \hat{\beta}^0 - \hat{\beta}^-) \leq {10^{-1}, 10^{-2}} )</td>
</tr>
<tr>
<td>( \hat{\beta} )-bound(^\dagger) (Moustapha et al., 2022)</td>
<td>(</td>
</tr>
<tr>
<td>( \hat{\beta} )-stability (Moustapha et al., 2022)</td>
<td>(</td>
</tr>
<tr>
<td>Failure domain stability(^*) (Moustapha and Sudret, 2019)</td>
<td>(</td>
</tr>
<tr>
<td>Failure/safe domain stability (Basudhar et al., 2008)</td>
<td>( \text{Card}[\tilde{\mathcal{M}}^{(i)}(\mathbf{x}).\tilde{\mathcal{M}}^{(i-1)}(\mathbf{x})^\top \leq 0]/N &lt; 10^{-3} )</td>
</tr>
<tr>
<td>Leave-one-out cross-validation error</td>
<td>( \epsilon_{\text{LOO}} \leq {10^{-2}, 10^{-3}} )</td>
</tr>
<tr>
<td>Generalization error of the model(^**) (Allahvirdizadeh et al., 2023)</td>
<td>( (N_{FS}^{(i)} + N_{FF}^{(i)})/N_{\text{DoE}}^{(i)} \leq 10^{-2} )</td>
</tr>
<tr>
<td>Failure probability estimation error(^**) (Allahvirdizadeh et al., 2023)</td>
<td>(</td>
</tr>
</tbody>
</table>

\(^\dagger\) \( \hat{F}_i = \{\mathbf{x} : \hat{\mu} - 1.96i\hat{\sigma} \leq 0\}, \quad i = \pm 1, 0 \rightarrow \hat{\beta}^i = -\Phi^{-1}(\hat{F}_i) \)

\(^*\) \( N_f^{(i)} \) corresponds to failed samples at \( i \)th iteration of active learning.

\(^**\) \( N_{FS}^{(i)}, N_{FF}^{(i)}, N_{TF}^{(i)} \) and \( N_{\text{DoE}}^{(i)} \) denote the number of false safe, false failure, true failure and DoE size at \( i \)th iteration of active learning.
2.4. IMPLEMENTING DEPENDENCY BETWEEN RANDOM VARIABLES

from independent normally distributed variables to the correlated non-normally distributed variables can be achieved if the joint probability distribution of the correlated variables was available. This approach is known as Rosenblatt transformation (Rosenblatt, 1952) and its application seems to be limited as the joint probability distribution is often not available.

\[ \Sigma_u \equiv \text{cov}(u, u^\top) = \text{cov}(Tx, x^\top T^\top) = T \Sigma_x T^\top \rightarrow \Sigma_x = T^\top \Sigma_u T = T^\top IT \] (2.52)

However, an alternative approach uses Sklar’s theorem, which guarantees the existence of a unique joint probability distribution defined by the copula function if the univariate CDFs of all variables are continuous (Sklar, 1959). Then, the joint CDF of the random variables can be expressed as Eq.(2.53) using the univariate CDFs of each random variable and the definition parameters of the copula function. Therefore, independent uniformly distributed samples can be fed into the inverse of the partial derivatives of the proper copula function to generate dependent random variables with the desired probability distribution function (Du et al., 2017; Lu and Zhu, 2018; Papaefthymiou and Kurowicka, 2008).

\[ F_X(x_1, x_2, \ldots, x_{N_d}) = C(F_{X_1}(x_1), F_{X_2}(x_2), \ldots, F_{X_{N_d}}(x_{N_d})|\theta) \] (2.53)

where \( C : [0, 1]^{N_d} \rightarrow [0, 1] \) is a non-decreasing function and \( \theta \) are its defining parameters.

In this context, a variety of copula families have been proposed in the literature, some of the most widely used ones are presented in Table 2.3 for a bivariant case. These copula functions are categorized into two groups, namely elliptical and Archimedean functions. The first group typically has no closed-form expression and the number of its parameters corresponds to the number of correlation coefficients between random variables. Archimedean copula functions, on the other hand, are developed using a generator function, which allows them to be expressed in closed-form expressions and generally have only one defining parameter (Durante and Sempi, 2010). It is also worth mentioning that the method described above, which is based on the Cholesky decomposition, basically assumes a Gaussian copula between random variables. This type of copula function is very popular in practical problems. However, it is very important to note that it does not model the dependence in the tails (Donnelly and Embrechts, 2010).

These theoretical copula functions are all symmetric, which means that their lower-upper (see Eq.(2.54)) and upper-lower (see Eq.(2.55)) tail coefficients must be equal.

\[ \lambda_{l,u}(C) = \lim_{q \to 0^+} \mathbb{P}\left(x_1 \geq F_{X_1}^{-1}(1 - q)|x_2 \leq F_{X_2}^{-1}(q)\right) = \lim_{q \to 0^+} \frac{C(q, 1 - q)}{q} \] (2.54)

\[ \lambda_{u,l}(C) = \lim_{q \to 0^+} \mathbb{P}\left(x_1 \leq F_{X_1}^{-1}(q)|x_2 \geq F_{X_2}^{-1}(1 - q)\right) = \lim_{q \to 0^+} \frac{C(1 - q, q)}{q} \] (2.55)
CHAPTER 2. RELIABILITY-BASED METHODS FOR DESIGN IMPROVEMENT

Table 2.3: Some of the most widely used Copula functions (presented for a bivariant case).

| Copula   | \( C(u_1, u_2|\theta) \) |
|----------|---------------------------|
| Gaussian | \( \Phi_{\theta}\left[\Phi^{-1}(u_1), \Phi^{-1}(u_2)\right] \) |
| Student’s t | \( t_{\nu,\theta}\left[t^{-1}_\nu(u_1), t^{-1}_\nu(u_2)\right] \) |
| Clayton  | \( (u_1^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta} \) |
| Frank    | \( -\frac{1}{\theta} \ln \left[1 + \frac{(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)}{e^{-\theta} - 1}\right] \) |
| Gumbel   | \( \exp \left[-((\log u_1)^{\theta} + (\log u_2)^{\theta})^{1/\theta}\right]\) |

This aspect should therefore be checked before assigning a copula function, which is achieved by estimating the upper limit of exchangeability as formulated in Eq.(2.56). This aspect can be assessed using the available statistical data on the variables of interest and the assignment of an empirical copula function using Eq.(2.57) (Charpentier et al., 2007). Overall, a copula function would be symmetric if and only if \( \eta_{\infty}(C) = 0 \) (Klement and Mesiar, 2006). Otherwise, a product of the mentioned copula functions can be used to model non-symmetrical dependencies between random variables (Liebscher, 2008; Mesiar and Najjari, 2014).

\[
\eta_{\infty}(C) = \sup_{(u_1, u_2)\in[0,1]^2} \left\{ |C(u_1, u_2) - C(u_2, u_1)| \right\} \quad (2.56)
\]

\[
\hat{C}(\frac{i}{N}, \frac{j}{N}) = \frac{1}{N} \text{Card}\left[(u_1, u_2)|u_1 \leq u_{1i}, u_2 \leq u_{2j}\right], \quad 1 \leq i, j \leq N \quad (2.57)
\]

The quality assurance criteria, including the dissimilarity between the theoretical and empirical copula function (\( \bar{\Delta} \)), the Akaike Information Criterion (\( AIC \)) and the Bayesian Information Criterion (\( BIC \)), as defined in Eq.(2.58), Eq.(2.59) and Eq.(2.60) should be evaluated to select the best fitting theoretical copula function (Akaike, 1974; Michiels and De Schepper, 2013; Schwarz et al., 1978). It should be noted that such a theoretical copula function should minimize these quality assurance criteria.

\[
\bar{\Delta}(\theta) = \sum_{i,j=1}^{N} \left[ \hat{C}(\frac{i}{N}, \frac{j}{N}) - C(\frac{i}{N}, \frac{j}{N}) \right]^2 \quad (2.58)
\]
2.5. RELIABILITY-BASED DESIGN OPTIMIZATION (RBDO)

\[ AIC(\theta) = -2 \sum_{i=1}^{N} \ln[C(x_i, y_i|\theta)] + 2N\theta \] (2.59)

\[ BIC(\theta) = -2 \sum_{i=1}^{N} \ln[C(x_i, y_i|\theta)] + N\theta \ln(N) \] (2.60)

where \( N\theta \) is the number of parameters of the copula function and \( N \) is the sample size.

2.5 Reliability-based design optimization (RBDO)

The design of a system seems to be a compromise between the gains obtained from its operation (existence) and the sum of the induced direct/indirect costs due to its construction, inspection, maintenance and possible failures. The desired target safety aims to maintain the balance between these two situations, i.e. to create a sufficiently safe system that is feasible from an economic point of view, while the consequences of possible failures are tolerable for society. As mentioned above, this objective is implicitly achieved by applying partial safety factors through a semi-probabilistic approach. However, in light of the above, designing a system is a decision-making process involving an optimization problem. This opens up an alternative way of thinking about the design of a system.

It should be emphasized that such optimization problems should be formulated with simultaneous consideration of the associated costs and safeties (i.e. corresponding failure probabilities). This is the subject of Reliability-Based Design Optimization (RBDO), which can be expressed mathematically as Eq.(2.61).

\[ \hat{d} = \arg \min_{d \in D_d} \mathcal{J}(d) \]

\[ \text{s.t.} \left\{ \begin{array}{l} \mathbb{P}\left[g_i(d, \mathbf{x}) \leq 0\right] \leq p_{f,i}, \quad i = 1, 2, \ldots, n_p \\ S_j(d) \leq 0, \quad j = 1, 2, \ldots, n_d \end{array} \right. \] (2.61)

where \( d \) denotes the vector of design variables, \( D_d \) is their feasible domain, \( \mathcal{J}(\cdot) \) is the objective function, \( g_i(\cdot, \cdot) \) is the limit-state function corresponding to the \( i \)th failure mode with \( p_{f,i} \) as the target reliability level, \( n_p \) is the number of failure modes (hard constraints) considered, \( S_j(\cdot) \) represents the deterministic soft constraint and \( n_d \) is the number of deterministic constraints considered. It is worth noting that implementing the hard constraints as the associated risks (i.e. failure probabilities multiplied by the corresponding consequences) can reformulate the problem into a risk-based optimization (Beck et al., 2012).
CHAPTER 2. RELIABILITY-BASED METHODS FOR DESIGN IMPROVEMENT

Given the fundamental objective of RBDO, the objective function should be formulated as a function of the total associated cost (i.e. $C_t(\cdot)$), as presented in Eq.(2.62).

$$C_t(d) = C_0(d) + C_r(d) + C_f(d)p_f(d)$$ (2.62)

where $C_0(\cdot)$ is the initial cost including design and construction costs, $C_r(\cdot)$ is the inspection and repair cost and $C_f(\cdot)$ represents the cost for each failure mode with a probability of occurrence of $p_f(\cdot)$. It should be noted that $C_r(\cdot)$ is a time-dependent cost that requires the influence of the discount rate to be implemented (Ghasemi and Nowak, 2017). Although this formulation of the objective function is robust, it does not seem to be widely used in real-world problems. This is because the reliable information about the costs is often hardly available (Enevoldsen and Sørensen, 1994; Frangopol, 1985). In addition, its application is confronted with some ethical problems arising from the allocation of monetized losses to human life (Liu et al., 2021). Therefore, the objective function is often represented as a function of physical properties such as system geometry.

The introduction of the reliability problem as a hard constraint of the RBDO problem makes its solution a computationally prohibitive task. As a result, a variety of approaches have been proposed to solve this problem. The interested reader can find a detailed overview of these approaches in Aoues and Chateauneuf (2010); Li et al. (2021); Moustapha and Sudret (2019). The first attempts, including the Reliability Index Approach (RIA) and the Performance Measure Approach (PMA), were based on FORM, which led to these methods being referred to as double-loop (nested or bi-level) approaches (Nikolaidis and Burdisso, 1988; Tu et al., 1999). The latter is due to the fact that FORM itself is an optimization problem implemented within a deterministic optimization problem. In its special format, disregarding the cost function can also lead to it being called an inverse reliability problem (Der Kiureghian et al., 1994). This method searches for the corresponding MPP of the design parameter whose distance from the origin in the standard Normal space is equal to the target reliability level. It should be emphasized at this point that the double-loop approaches can still be computationally intensive, while they may suffer from the same accuracy concerns as the FORM.

These concerns have led to the reformulation of the RBDO problem into a single-loop (or decoupled) algorithm by combining the cost function with equivalent deterministic statements of the hard constraints (Chen et al., 1997). In this context, a variety of formulations have been proposed. Some of them are using MPP values as deterministic substitutions of the random variables (Wu and Wang, 1998), such as Sequential Approximate Programming (SAP) (Cheng et al., 2006), which approximates the reliability index with its first-order Taylor series expansion around MPP, and Sequential Optimization and Reliability Analysis (SORA), which replaces the probabilistic constraints with a percentile formulation (Du and Chen, 2004). These methods have proven to be very computationally efficient compared to the single-
2.5. **RELIABILITY-BASED DESIGN OPTIMIZATION (RBDO)**

loop approaches. However, their approximate nature raises concerns about their accuracy.

Given the shortcomings of traditional methods, recent studies in this field have been dedicated to the use of surrogate models to reduce the computational cost of RBDO. Traditionally, it has not been possible to use simulation-based techniques in the inner loop of the procedure. However, it seems to be applicable when a reliable surrogate model is available. The previous studies used polynomial response surface functions to approximate the limit-state function at each iteration (Agarwal and Renaud, 2004; Gasser and Schuëller, 1997). Since these approaches implement the training phase of the surrogate models within the inner loop of the procedure, a significant number of calls to the computational model would still be required. Later, a hyper-rectangular augmented space was introduced as a tensor product of the confidence intervals of the design variables and the basic random variables to support the decoupling of the training phase from the RBDO problem (Au, 2005). The interested reader can find some application examples of this approach in Kanakasabai and Dhingra (2014); Moustapha et al. (2016); Zhang et al. (2021).
Chapter 3

Summary of the appended papers

Design guidelines and regulations aim to address various limit-states by specifying criteria that, if met, can limit the probability of violating these limit-states (i.e., the probability of failure) to an acceptable level. Therefore, evaluating the applicability of conventional design methods, proposing a new design methodology, or improving a current design philosophy is inherently a reliability problem. However, performing such probabilistic analyses is often computationally intensive, especially for engineering problems with complex computational models and many variables involved. Therefore, design codes often use implicit deterministic approaches such as the introduction of partial safety factors applied to the nominal values of the design parameters. In this study, a similar approach is adopted to address the dynamic design limit-states of railway bridges by first evaluating the adequacy/appropriateness of the conventional design methods. The identification of possible shortcomings of the current normative design methods led to a proposal for possible improvements. These improvements include the determination of probabilistic design curves that provide minimum requirements for the target safety level and updated partial safety factors. Performing the associated reliability analyses to obtain such results is computationally intensive. Therefore, statistical learning concepts are used to surrogate the true computational models with approximation functions, also known as meta-models. A variety of such surrogate models have been introduced and their applicability to dynamic problems has first examined. Then, the influence of some neglected aspects or those with limited information on the derived conclusions is investigated to emphasize their importance for future studies. In this section, a brief overview of these studies is given as described in the appended articles. The main objective of each article and the link between them is shown in Figure 3.1.

Paper I: Reliability assessment of the dynamic behavior of high-speed railway bridges using first order reliability method

A significant increase in the operating speed of trains on modern networks can lead to the infrastructure being exposed to greater vibration, which in turn raises safety concerns that were not previously a problem. These include loss of contact between the wheel and the rail, destabilization of the ballast, or disturbing the comfort of passengers. In this context, the design regulations indirectly address the latter by limiting the deflection of the bridge deck, while the former (also known as running safety) is controlled by keeping the vertical accelerations of the bridge deck below...
CHAPTER 3. SUMMARY OF THE APPENDED PAPERS

Paper I

Preliminary probabilistic-based investigation on applicability of current dynamic design methods of railway bridges using FORM
✓ Highlighting deficiencies of the current normative design methods
✓ Computationally expensive reliability analyses using FORM

Paper II

Proposing probabilistic dynamic design curves for railway bridges to satisfy desired safety level with respect to running safety and passenger comfort criteria using FORM

Paper III

Investigating the applicability of approximating the computational models with regression-based surrogate models for reliability assessment objectives
✓ Selecting Kriging as the best choice for further studies

Paper IV

Using Kriging as surrogate model and performing reliability-based design optimization (RBDO) to propose minimum required mass and stiffness to satisfy safety objectives with respect to running safety criterion

Paper V

Using Kriging to surrogate the computational models and calibrate the partial safety factor with respect to running safety criterion
✓ Improve by neglecting some possibly important aspects
✓ Improve based on current knowledge

Paper VI

Evaluate the influence of soil-structure interaction on estimated failure probabilities with respect to running safety design criterion

Paper VII

Evaluate the influence of lack of knowledge about threshold of ballast instability in running safety design criterion by introducing epistemic uncertainties on estimated failure probabilities

Figure 3.1: The interconnectivity of the appended papers.
a threshold (EN 1990, 2002).

As already discussed, the control of wheel-rail contact loss requires the construction of complex computational models capable of accounting for the effects of TTBI (Cantero et al., 2016; Montenegro et al., 2021). The use of such models in the context of reliability analyses would therefore be associated with a considerable computational effort, which prevents the derivation of general conclusions from a large number of design scenarios. On the other hand, the previous deterministic/probabilistic comparative studies have shown that following the current regulations results in ballast instability almost always being the governing design criterion for high-speed railway bridges, compared to the wheel-rail contact loss and passenger comfort limit-states (Allahvirdizadeh et al., 2021; Arvidsson, 2018; Rocha, 2015).

Theoretically, the phenomenon of ballast destabilization occurs when the induced inertial forces due to the bridge vibrations caused by the passage of trains overcome the resistance forces, including friction and interlocking between the particles. As a result, the ballast particles begin to move laterally, causing the sleepers to shift and interrupting the load path from the rail level to the deck level. It is assumed that the occurrence of this phenomenon over a sufficiently long distance along the track can lead to train derailments, although its local occurrence can also increase the maintenance costs of the track. These assumptions were confirmed by a series of shaking table tests carried out by Zacher and Baeßler (2009). It was found that the lateral displacement of the sleepers increases significantly at vertical accelerations greater than 7.0 m/s$^2$. These conclusions seem to support the proposal of the design guidelines to limit the vertical acceleration of the bridge deck exposed to the passage of trains. In this context, a safety factor of 2.0 is applied, limiting the vertical acceleration of the bridge deck to 3.5 m/s$^2$ (EN 1990, 2002).

Partial safety factors should be calibrated using an optimization-based approach that relies on a higher order reliability analysis (Sørensen et al., 1994). However, no evidence was found to suggest the above safety factor related to running safety based on such calibration procedures. In view of this, it was necessary to assess the consistency and appropriateness of the current method before seeking possible improvements. This is the main objective of Paper I, which is achieved through a preliminary probabilistic approach.

Therefore, a series of generic single-span, simply supported reinforced concrete bridges with short to medium spans in the range of [5.0–30.0] m (with a resolution of 5.0 m) were considered. It is worth mentioning that 2D Euler-Bernoulli beams were used to model the bridges. The contribution of ballast and rails to the stiffness is neglected, while the mass of the ballast is implemented as an additional linear mass on the bridge. The passing trains are modelled as a series of equidistant moving loads and the distribution of their concentrated loads on the rail level within the
ballast is also implicitly considered by a reduction factor. In this study, the influence of rail irregularities on the calculated responses is neglected. At this point it should be emphasized that such a simplification may lead to an underestimation of the accelerations and consequently of the calculated failure probabilities. Nevertheless, the conclusions derived appear to be valid, as only relative comparisons were sought in this study.

In the Paper I phase, the author had limited access to information on the properties of the bridge cross-sections. The available data was obtained by surveying 62 single-span, single-track railway bridges in Sweden, which are not necessarily located on high-speed lines. Therefore, triangular and uniform probability distributions were assigned for the corresponding random variables to account for the lack of knowledge. It is important to note that mass and stiffness of the bridges are correlated; however, the direct implementation of this aspect was neglected in Paper I. Instead, a conditional probability distribution was assigned for both basic random variables as a function of the bridge span.

Subsequently, the permissible operating speeds of trains on such bridges were determined using both the conventional normative deterministic approach (here referred to as $v_{\text{det}}$) and the probabilistic method (here referred to as $v_{\text{prob}}$). The former corresponds to the first speed at which the maximum vertical acceleration of the bridge deck exceeds 3.5 m/s$^2$. To be conservative, the deterministic assessments were performed using mean values and mean values plus/minus one standard deviation of the basic random variables. On the other hand, $v_{\text{prob}}$ is calculated by approximating the multidimensional integral of the probability of failure using the FORM; however, it should be noted that the applicability of the FORM may be questionable for the problems of interest. Nevertheless, this study refrains from a detailed investigation of this aspect in order to make the intended calculations computationally affordable. Performing such analyses helps to estimate the average probability of failure as a function of the permissible operating speed of the trains. It is worth noting that calculating the upper bound of the probability of failure seems to be a more reliable estimate for improving the design philosophy of dynamic systems; however, the average probability of failure can be used for comparative studies such as those sought here. Thus, the intersection of the desired target reliability level (i.e. $\beta_t$, here considered as 3.1) with the reliability analysis results leads to the calculation of the allowable operating speed of the train based on a probabilistic approach (i.e. $v_{\text{prob}}$). Similarly, intersecting $v_{\text{det}}$ with the reliability analysis results can provide an approximate estimate of the corresponding system safety associated with the normative deterministic approach (denoted here as $\beta_{\text{det}}$). This approach is shown schematically as a flowchart in Figure 3.2.

In view of the simplifications described above, the absolute values of the estimated safety indices should not be regarded as the most important conclusions of the study. Rather, the difference between the deterministic results and the correspond-
• Select the bridge span length \((L)\)
• Assign \(v = 200\) km/h
• Assign \(v_{\text{max}} = 400\) km/h
• Set \(v_{\text{incr}} = 2.0\) km/h

- Determine conditional probability density functions
- Determine probability distributions of variables

**Initialization**

Perform dynamic analysis for the full speed range assigning mean values and mean±std for all variables

![Diagram of the initialization process](image)

Figure 3.2: The followed methodology in Paper I.

Probabilistic outcomes should be examined. In this context, it can be assumed that the estimated \(v_{\text{det}}\) values should be close to the corresponding \(v_{\text{prob}}\) values if the current normative design/assessment method corresponds to the desired safety level. Similarly, the estimated \(\beta_{\text{det}}\) values should be close to the target safety index. The relationship between these values is shown in Figure 3.3. As can be seen, the
current normative method does not necessarily fulfill the desired safety level and also does not lead to a consistent safety level for the different design scenarios. The latter would be the case if an identical safety factor were used for all design scenarios; however, the aim of calibration is to reduce possible deviations. This aspect will be addressed in further studies presented.

Figure 3.3: Estimated safety level based on current normative design methods regarding running safety criteria.

A sensitivity analysis was also performed to determine the importance of the contributing basic random variables based on the directional cosines determined in the FORM. It was found that the coach length, the flexural rigidity, the geometry of the bridge (or its mass per length) and the damping of the system are the most important variables.

Paper II: Probabilistic dynamic design curves optimized for high-speed reinforced concrete railway bridges using first order reliability method

As Paper I highlighted, the current normative dynamic design methods for high-speed railway bridges should be improved. This can be done in different ways, e.g. by changing the current design methods (i.e. the safety factors or design thresholds used) or by proposing a new design philosophy. The latter can be achieved by proposing minimum design requirements whose fulfillment ensures the achievement of the desired level of safety with respect to the intended dynamic design limit-states. It should be noted that the former still requires dynamic analyses to be
performed. It was therefore decided to explore the alternative approach at this stage and to keep the calibration of the safety factor for further investigation.

The response of a dynamic system can be controlled by adjusting its stiffness and mass. Therefore, the alternative design philosophy discussed can be based on proposing a minimum required mass and stiffness, which if exceeded will result in a system that is safer than the desired targets. It should be noted that the mass and stiffness of the designed bridge will be available after the conceptual and static design phases. Therefore, it would theoretically be possible to check the minimum requirements without performing a further dynamic analysis. This is the main objective of Paper II.

As already mentioned, mass and stiffness are dependent variables, as they both depend on the cross-sectional dimensions of the bridge. Different approaches can be used to implement this dependency. One example is the conditioning of both variables on the bridge span length, as pursued in Paper I. On the other hand, orthogonal, Nataf or Rosenblatt transformations can be used to transform the dependent variables into independent ones (Melchers and Beck, 2018). In addition, the copula concept can be used to model their joint probability distribution. However, in Paper II, general cross-section types such as slab, beam and box-girder bridges are considered to account for the dependency. Then, the desired cross-section type is first selected for each reliability analysis. Since dimensions other than the thickness of the cross-section are almost known, determining the proper cross-section would be reduced to an optimization problem with a single design variable. Once the correct thickness has been determined, the mass and stiffness can be calculated.

The constraints of the discussed optimization problem would be to limit the failure probability of the system with respect to the dynamic design limit-states below the desired safety level. In Paper II both running safety and passenger comfort criteria are considered, which imposes multi-constraints on the problem. It should be noted that a similar modelling strategy as in Paper I is followed here. Therefore, the implicit criteria of these limit-states, i.e. the maximum vertical acceleration of the bridge deck and its deflection, were used to formulate the limit-states of the corresponding reliability problems. The only differences in the modeling are the consideration of the Equivalent Additional Damping Approach (EADA) to account for the possible increase in system damping due to TTBI effects (Yau et al., 2019b) and the implementation of an amplification factor as a basic random variable due to rail irregularities. In addition, the range of generic bridges considered is changed to [5.0–50.0] m to consider a broader design scenario. Furthermore, continuous bridges with two spans are also considered. In light of this background, a Reliability-Based
CHAPTER 3. SUMMARY OF THE APPENDED PAPERS

Design Optimization (RBDO) problem should be solved as formulated in Eq.(3.1).

\[ t^* = \arg \min_{t \in \mathcal{D}} t \]

s.t. \[
\begin{align*}
\beta_{\text{Acce}} & \geq \beta_{t_{\text{Acce}}} \\
\beta_{\text{Disp}} & \geq \beta_{t_{\text{Disp}}}
\end{align*}
\]  (3.1)

where \( \mathcal{D} = [t_{\text{min}}, \infty) \) is the design domain of the cross-section thickness defined on the basis of the cross-section type under consideration. Furthermore, \( \beta_{\text{Acce}} \) and \( \beta_{\text{Disp}} \) are safety indices associated with the running safety and passenger comfort limit-states, respectively. Similar to **Paper I**, FORM is performed to calculate the failure probabilities. The already mentioned limitations of FORM were not disregarded. However, the use of other methods to perform such extensive reliability analyses as those sought here would be computationally prohibitive. Moreover, \( \beta_{t_{\text{Acce}}} \) and \( \beta_{t_{\text{Disp}}} \) are the corresponding target reliability levels. As discussed, the cases considered do not appear to be ultimate design limit-states. This is especially true for the passenger comfort limit-state. Nevertheless, based on the recommendations of JCSS (2001) and for reasons of conservatism, an identical target reliability level of \( \beta_{t} = 3.7 \) is considered.

Since the formulated RBDO problem has only a single design parameter, a line search function such as Eq.(3.2) is used to find the optimal solution.

\[ t^{(i+1)} = t^{(i)} + \epsilon_t \text{sgn}(\beta_t - \beta^{(i)}) \left( \frac{\beta_t}{\beta^{(i)}} \right)^{1/3} \]  (3.2)

where \( \epsilon_t \) is a step size constant assumed to be 0.01 and \( \text{sgn}(\cdot) \) is the sign function. The search function is scaled based on the calculated safety index of the previous iteration and the power 1/3 is chosen because the stiffness is related to the third power of the thickness.

It should be noted that although the RBDO problem has two constraints, they do not have to be satisfied simultaneously. Therefore, the procedure was simplified by first calculating the cross-section thickness that satisfies the running safety condition and then continuing the procedure based on the obtained thickness to satisfy the passenger comfort criteria. The thickness values smaller than those obtained by satisfying the running safety condition are not used for the passenger comfort constraint. Therefore, once the passenger comfort condition has been satisfied, no additional check for running safety is required. In view of the above discussions, the method used in **Paper II** to calculate the optimal thickness and consequently the minimum required mass and stiffness of the bridges is presented as a flowchart in Figure 3.4. It should be noted that the passenger comfort threshold is related to the maximum permissible operating speed of the trains. Therefore, two different permissible speeds, namely 200 km/h and 400 km/h, was considered. In contrast
to the approach followed in Paper I, the upper bound of the failure probabilities was estimated here. This is achieved by calculating the failure probability corresponding to the critical speeds of each situation within the considered range of permissible operating train speeds.

Figure 3.4: The followed methodology in Paper II.

Following the discussed approaches, the minimum design requirements (i.e. linear mass and moment of inertia) are presented in Figure 3.5 as a function of the bridge cross-section types, the number of spans and the maximum permissible operating train speeds. In addition to the design requirements, the presented design curves also provide some practical guidelines for the selection of the appropriate cross-section type. In this context, it can be concluded that box-girder bridges appear to be unsuitable for single-span bridges with a length of less than 20 m and for continuous bridges with a length of less than 40 m. It can also be deduced that slab bridges only appear to be suitable for bridges shorter than 15 m.

Similar to Paper I, a sensitivity analysis was performed for each limit-state considered, where the absolute directional cosines and omission sensitivity factors were calculated using the mean of the directional cosines obtained for all scenarios. It should be emphasized that these importance measures were calculated for basic
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(a) Minimum required mass for single-span bridges.

(b) Minimum required moment of inertia for single-span bridges.

(c) Minimum required mass for continuous bridges.

(d) Minimum required moment of inertia for continuous bridges.

Figure 3.5: Minimum design requirements for single-span and continuous bridges with different cross-section types as a function of span length (the solid curves show the results for cases with a maximum permissible train speed of up to 400 km/h and the dashed lines for cases with a maximum permissible train speed of up to 200 km/h).

random variables and not for the cross-sectional dimensions and capacity-related variables. Furthermore, the calculated sensitivities seem to be highly dependent on the modelling approach and especially on the assigned probability distributions.
Considering this, it was found that in the case of the running safety limit-state, the fundamental frequency of the coach, the axle loads, the amplification factor for the rail irregularities and the mass density of the concrete are the less important variables. On the other hand, the model uncertainty can only be considered as a deterministic variable in the case of the passenger comfort limit-state.

**Paper III: Applicability of meta-model assisted reliability assessment for dynamic problems: a comparison between regression-based methods**

As mentioned above, FORM is used in Paper I and Paper II to estimate the failure probabilities of the dynamic systems of interest. Although FORM is computationally efficient for reliability problems with up to moderate dimensionality (denoted here as $N_d$), it may perform less acceptably from an accuracy point of view for highly concave, convex or nonlinear limit-state functions. This aspect has been neglected above in order to allow the derivation of the general conclusions presented.

In general, simulation-based reliability assessment methods are more robust than approximate methods such as FORM. However, they suffer from a very low convergence rate, which requires a significant number of calls to the associated computational model. Such models are often very complex for engineering problems, where even a single point-wise estimate of the response can be computationally prohibitive. Therefore, the approximation of the limit-state function by fast-to-evaluate surrogate functions has recently attracted much attention. The roots of these functions, also known as meta-models, lie in statistical learning theory.

Given the formulation of the crude Monte-Carlo simulations, only the sign of the limit-state evaluation is required at each realization. Therefore, the surrogate models used can be categorized into two general groups, namely regression-based models and classification-based models. The former mimic the true response and then define its sign, while the latter directly approximate the sign (class) of the response. These classes for reliability assessment as a binary classification problem determine the safe or failure domains.

Looking at previous studies on reliability assessment, regression-based surrogate models seem to be used much more frequently than classification-based models. The applicability of classification-based surrogate models therefore needs to be further investigated. Therefore, Paper III focuses on regression-based meta-models used for dynamic problems. In this context, various models such as Kriging, Polynomial Chaos Expansion (PCE), Artificial Neural Networks (ANN) and polynomial Response Surface (RS) were considered.

The dynamic problem of interest was modeled similarly to Paper II, but it should be mentioned that the study is limited to single-span bridges with spans of 10 m,
The applicability of these surrogate models was evaluated in terms of their accuracy. To this end, the failure probabilities predicted by these models are compared with those estimated by Subset Simulation (SS). In this context, the upper bound of the failure probabilities (determined from the critical speeds of the individual realizations) was estimated. For this purpose, three maximum permissible operating train speeds of up to 200 km/h, 250 km/h and 300 km/h were taken into account.

The main objective of using surrogate models for reliability assessment is to minimize the number of calls to the true computational models. To achieve this goal, an active learning strategy is often pursued. In this approach, training of the surrogate model is initiated with a small DoE and then a learning (acquisition) function is used to evaluate a set of unlabeled points within a pool (samples in the problem domain) and select the most informative point (or batch of points). The new point is then added to the existing DoE and the surrogate model is retrained. This procedure terminates when a stopping criterion is met. It should be emphasized that the adopted learning functions are often defined based on characteristics of the surrogate model. To take this into account, a generalized learning function is implemented in Paper III to make it applicable and unified for all considered meta-models.

The most informative point should have a high contribution to the estimated failure probability and the model trained in the previous iteration should be uncertain with respect to this point. Considering these two conditions, the defined learning function first selects the points within the sample pool that are closest to the approximated limit-state function in the current iteration. This is achieved by selecting the points within the 5% lower quantile of the absolute predictions of the meta-models using the points in the sample pool. It is evident that the trained model has a higher confidence for the points closer to the existing DoE. Therefore, of the points selected above, the point furthest away from the existing DoE would likely have higher prediction uncertainty. This point is then selected to extend the existing DoE. This procedure ends when the DoE size becomes greater than 200. This procedure is illustrated as a flowchart in Figure 3.6.

It should be noted that the mentioned stopping criterion was arbitrarily chosen by the author without setting a threshold for the accuracy of the trained meta-model. Therefore, the absolute accuracy of the estimated failure probabilities using each surrogate model was not emphasized and instead their relative accuracy was compared. This procedure was repeated 10 times to also examine the stability of each surrogate model. In addition, the initial DoE and sample pool at each iteration were kept identical for all surrogate models in that iteration.

The distribution of calculated failure probabilities using each trained surrogate model was presented in the form of whisker (box) diagrams in the appended paper.
These results are categorized according to the length of the bridge span and the maximum permissible operating speed of the trains. On the other hand, they are post-processed here to be presented as Root Mean Square Error (RMSE). In this context, the failure probabilities determined with SS are considered as their true estimates. It is worth noting that this assumption was previously validated by Allahvirdizadeh et al. (2021). The calculated RMSE values are then categorized in the same order as shown in Figure 3.7.

It can be seen that Kriging almost always had the superior performance. The second best meta-model for the problems of interest appears to be PCE. It should be noted that, as stated in previous studies, PCE suffers from the curse of dimensionality, which makes its application to high-dimensional problems a computationally intensive task. However, the investigation of this aspect was neglected in this study. Among these models, RS showed the least favorable performance. Moreover, it should be mentioned here that Paper III is restricted to nearly shallow neural networks with at most three hidden layers and neurons on each layer chosen between the dimensionality of the problem and at most 1.5 times its dimensionality. The best structure was determined in the first iteration of active learning using the cross-validation technique and remained unchanged during the learning phase. This approach could be a possible reason for the lower expected performance of
the ANN. Although based on the universal approximation theorem, ANNs were expected to have higher accuracy. Despite this fact, the application of deeper neural networks was not investigated. The main reason for this decision was the significant computational effort required to train such neural networks.

Figure 3.7: RMSE of the different regression-based surrogate models under identical training situations (Krg, PCE, ANN and RS represent Kriging, Polynomial Chaos Expansion, Artificial Neural Network and Response Surface surrogate models, respectively).
Paper IV: Improved dynamic design method of ballasted high-speed railway bridges using surrogate-assisted reliability-based design optimization of dependent variables

Considering the discussed computational advantage of surrogate-assisted simulation-based reliability assessment, a similar methodology to Paper II is applied in Paper IV to improve the current dynamic design method of ballasted railway bridges. In this study, RBDO is performed to present design curves formulated as minimum required linear mass and moment of inertia, which providing them ensures satisfying desired safety level with respect to the running safety criterion.

It should be noted that a similar modelling strategy is followed here as Paper III. In this study, representative information on each contributing basic random variable was first collected and then a systematic approach was followed to assign an appropriate theoretical distribution. This approach involves narrowing down the possible distributions based on the nature of the random variables. For example, the damping ratio is positive in nature and the fundamental frequency of the car-body, which was obtained from an eigenvalue analysis, would probably be better described by distributions capable of modelling maximum/minimum values. The best distribution was then selected by performing statistical tests (e.g. $\chi^2$ and Anderson-Darling tests) and evaluating the goodness of fit between theoretical and empirical distributions. The parameters of the selected distributions are then calculated using the maximum likelihood method.

It is worth noting that the above approach was followed for all random variables except the vertical acceleration threshold for ballast instability. In this context, the author found only a few shaking table tests. Therefore, the sample population appears to be statistically insignificant. Therefore, an alternative approach was followed by restricting the potential distributions to truncated Gaussian, uniform and triangular distributions. The reason for selecting these distributions as potential candidates is that uniform and triangular distributions are often employed to describe situations with lack of knowledge. Furthermore, it is assumed that the resistance of the ballast particles against moving comes from various sources such as friction and interlocking. Therefore, a Gaussian distribution can be adopted based on the central limit theorem. The proper distribution (i.e. the truncated Gaussian distribution) is then selected by rejecting less proper distributions through hypothesis testing. The parameters of these distributions are then calculated using the least squares method. It is evident that the assigned distribution suffers from a considerable lack of knowledge, which increases the epistemic uncertainties of the problem. However, the investigation of the influence of this aspect on the derived conclusions will be left to a further study.

As already mentioned, the mass and the moment of inertia are dependent random variables. In contrast to Paper I and Paper II, where these variables were assigned
a conditional distribution or the dependence was implemented by restricting the problem to predefined cross-section types, a general approach using the copula concept is followed here. The evaluated exchangeability using the empirical copula CDF shows that the joint distribution between these variables can be considered symmetric. Therefore, their dependence can be modelled by choosing the right copula function among the conventional functions such as Gaussian, Student’s t, Clayton, Frank or Gumbel. Then, the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC) and the dissimilarity measure are computed to select the most appropriate theoretical copula function. In this context, the Student’s t copula function is selected, whose PDF versus the scatter of the collected data in the uniform space is shown in Figure 3.8.

![Figure 3.8: PDF of the assigned Copula function to linear mass and moment of inertia.](image)

Taking this approach into account, the RBDO problem with dependent variables can be reformulated as Eq.(3.3).

\[
[m^*, I^*] = \arg \min_{[m, I] \in D_d} J(m, I)
\]

\[
\begin{aligned}
\beta_i(m, I) &\geq \beta_{t,i} & i = 1, \ldots, n \\
Q_k(m, I) &\leq 0 & k = 1, \ldots, n_d \\
[u_m, u_I] &\in \text{CCR}_\alpha 
\end{aligned}
\]  

(3.3)

where \( J(\cdot, \cdot) \) is the cost function defined as Eq.(3.4). Moreover, \( \beta_i(m, I) \) is the estimated safety index corresponding to the \( i \)th limit-state function as a function of the
considered linear mass and stiffness, and $\beta_{t,i}$ represents the corresponding target reliability. **Paper IV** only considers the running safety design criterion assuming that its local occurrence does not cause safety concerns during train passage. Therefore, the considered limit-state is treated as a serviceability situation, resulting in the target reliability being assumed to be 2.9 based on the recommendations of (EN 1990, 2002; JCSS, 2001). Furthermore, $Q_k(\cdot, \cdot)$ is the $k$th deterministic constraint of the problem and $CCR_\alpha$ is the copula confidence region. The latter is defined as the region in which the defined joint probability density is greater than a prescribed threshold value of $\alpha$ (assumed here to be 0.01).

$$J(m, I) = \frac{m - m_{\min}}{m_{\max} - m_{\min}} + \frac{I - I_{\min}}{I_{\max} - I_{\min}}$$  \hspace{1cm} (3.4)

where $m_{\max, \min}$ and $I_{\max, \min}$ are the upper/lower bounds of the feasible domain of the design variables. It should be mentioned here that the feasible domain of each variable is defined as the region within the mean plus/minus five times its standard deviation.

It has already been mentioned that solving RBDO problems is generally a computationally intensive process. Among the variety of methods proposed for this objective, the current study facilitates the procedure by using surrogate-assisted simulation-based methods to calculate the failure probabilities at each iteration. For this objective, Kriging is selected to approximate the limit-state function. In this context, an active learning strategy was followed to train the surrogate models within the augmented space. The latter is a hypercube space created as a tensor product of the confidence interval of the design variables and other contributing basic random variables. As a result, the training of the surrogate models and their use in RBDO problem decouple into two non-intrusive blocks.

U-criterion ($U$ - see Eq.(2.49)) is used as a learning function within active learning. Traditionally, the training phase ends with this learning function when the minimum estimated U-criterion for the points in the sample pool becomes greater than 2.0. In addition, **Paper IV** introduces two new stopping criteria based on the misclassification ratio of the trained surrogate model. Taking the confusion matrix of the trained surrogate model in the iteration $i$ of active learning, as shown in Figure 3.9, the out-of-sample prediction of the trained model (see Eq.(3.5)) should be smaller than a given threshold. In addition, the estimation error of the calculated failure probability using the trained surrogate model should be acceptable enough (see Eq.(3.6)). It should be noted that the determination of the confusion matrix requires performing Cross-Validation (CV) and, in the most robust case, a Leave-One-Out (LOO) approach. However, its calculations would be computationally prohibitive, especially if the DoE size increases during active learning. Therefore, it is recommended to perform a regular CV.
Figure 3.9: Confusion matrix of the trained surrogate model.

\[
\begin{aligned}
\text{True Response} \\
\text{Safe} & \quad N^{(i)}_{TS} & \quad N^{(i)}_{FF} \\
\text{Failure} & \quad N^{(i)}_{FS} & \quad N^{(i)}_{TF}
\end{aligned}
\]

\[
\Delta p_f = \left| \frac{\hat{p}_f^{(i)} - p_f^{(i)}}{p_f^{(i)}} \right| = \left| \frac{N^{(i)}_{FF} - N^{(i)}_{FS}}{N^{(i)}_{FS} + N^{(i)}_{TF}} \right| \leq \epsilon_{pf}
\]

\[
\frac{N^{(i)}_{FS} + N^{(i)}_{FF}}{N^{(i)}} \leq \epsilon_{ge}
\]

where \(N^{(i)}_{TS}, N^{(i)}_{FS}, N^{(i)}_{FF}\) and \(N^{(i)}_{TF}\) represent the true safe, the false safe, the false failure and the true failure estimates at the \(i\)th iteration of active learning. Moreover, \(N^{(i)}\) is the DoE size, \(\epsilon_{ge}\) represents the threshold of the generalization error and \(\epsilon_{pf}\) denotes the threshold of the failure probability estimation error. Both thresholds are arbitrarily assumed to be \(10^{-2}\).

The RBDO problem is then solved after training the corresponding surrogate models. It is worth noting that the problem is restricted to single-span simply supported bridges with span lengths in the range of \([5–30]\) m. Therefore, regression analyses were performed with the obtained results, which led to propose the minimum required linear mass and moment of inertia of the considered bridges as Eq.\((3.7)\) and Eq.\((3.8)\), respectively.

\[
m_{\min} = 16030 \exp (0.026L) \quad \text{[kg/m]}
\]

\[
I_{\min} = 0.011L^2 - 0.14L + 0.6 \quad \text{[m}^4]\n\]

**Paper V: Partial safety factor calibration using surrogate models: an application for running safety of ballasted high-speed railway bridges**

The above studies have attempted to improve the conventional dynamic design methods for railway bridges by proposing minimum design requirements. However,
as mentioned above, the normative design methods often rely on partial safety factors. A valid partial safety factor should be determined by performing a higher-level reliability analysis for a large number of design scenarios and then minimizing the difference between the associated safety indices and the target reliability levels. This approach is then followed in Paper V to calibrate the safety factor corresponding to the ballast destabilization phenomenon.

The safety index (failure probability) associated with each applied safety factor can be obtained by linking the design equation to the limit-state function of the corresponding reliability problem. The former can be formulated in the context of ballast instability as Eq.(3.9), while the latter reads as Eq.(3.10).

\[
G(X_k, C, z, \gamma_a) = \frac{z_{a_{\text{lim},k}}}{\gamma_a} - \max \left( \left| a_{\text{comp}}(X_k, C) \right| \right) \chi_{M,k} \geq 0 \quad (3.9)
\]

\[
g(X, C, z) = z a_{\text{lim}} - \max \left( \left| a_{\text{comp}}(X, C) \right| \right) \chi_{M} \quad (3.10)
\]

where \(X\) and \(C\) represent the vector of basic random variables and deterministic variables respectively. In addition, \(a_{\text{lim}}\) denotes the threshold value for the vertical acceleration, \(a_{\text{comp}}(\cdot, \cdot)\) is the calculated vertical acceleration of the bridge deck using the constructed computational model and \(\chi_{M}\) is the model uncertainty (implemented here by assigning a lognormal distribution). The variables with the index \(k\) represent the characteristic value of each variable. In addition, \(\gamma_a\) is the safety factor for the vertical acceleration threshold and \(z\) is an arbitrary design variable.

The arbitrary design variable is introduced to link the above relationships. It acts as a scaling factor that shifts the resistance term (or conversely the load term) to find the situation in which each design scenario approaches the desired probability of failure. This concept is shown schematically in Figure 3.10.

Substituting the design variable from the design equation into the limit-state function, provides a limit-state function that depends on the considered safety index. Then the safety level of the system can be estimated for different values of the safety factor. This requires estimating the multidimensional integral of the probability of failure for each variation of the safety index, which makes the procedure a computationally prohibitive task. However, from the above equations, it can be seen that changing a safety factor only shifts the capacity term up and down. This observation offers the possibility to decouple the demand determination for the considered design scenarios from any estimation of the failure probability. The number of calls to the computational model can be further reduced by approximating the computational model using surrogate models. Thus, the limit-state function can be reformulated as Eq.(3.11).

\[
g(X, C, \gamma_a) = \frac{\gamma_a \hat{M}(X_k, C)}{a_{\text{lim},k}} a_{\text{lim}} - \hat{M}(X, C) \quad (3.11)
\]
where \( \hat{M}(\cdot, \cdot) \) is the trained surrogate model.

It should be emphasized that Paper V uses Kriging as a surrogate model to approximate the computational model. In this context, active learning with the Expected Feasibility Function (EFF - see Eq.(2.48)) is used to train a surrogate model for each design scenario. These models estimate the maximum acceleration of the bridge deck at the critical speeds of each realization. Therefore, the calculated failure probabilities represent the corresponding upper bound so that the calibrated safety factors represent a conservative estimate. Figure 3.11 shows an example of the performance of such trained surrogate models on a test set after completion of the learning phase.

The mentioned design scenarios include bridges with beam, slab and box-girder cross-section types with span lengths in the range of [5.0–50.0] m, with 1-4 continuous spans and three maximum operating train speeds of up to 250 km/h, 320 km/h and 400 km/h. As discussed in Paper II, bridges with box-girder cross-section types are not suitable for shorter spans, so bridges with a single-span shorter than 10.0 m are disregarded for such cases. Thus, \( N' = 3 \times 4 \times (2 \times 46 + 41) = 1596 \)
design scenarios were taken into account for each permissible speed range of trains. It should be noted that slight variations were observed in the characteristics of the design scenarios with 3 or 4 spans, causing the cases with more than 3 spans to be disregarded in the calibration process. It should be noted that it was not possible to collect an extensive database of existing bridges corresponding to the design scenarios considered. Therefore, it was decided to calculate their properties by performing deterministic optimizations according to the current normative design methods.

A grid search optimization is then performed to find the optimal safety factor for each admissible speed range of the trains and also for all design scenarios considered together. This approach is illustrated in Figure 3.12. It is worth noting that the author did not have safety-related cost data available to calculate the target reliability level according to optimization-based approaches. Therefore, it was decided to consider a conservative target reliability index of $\beta_t = 3.7$ based on the recommendations of (JCSS, 2001). The considered target failure probability corresponds to a high risk to human life, although the author believes that a train derailment is very unlikely due to local instability of the ballast.

The results obtained are then summarized in Table 3.1. As can be seen, considering all design scenarios together leads to a vertical acceleration threshold of 4.40 m/s$^2$, which allows to accept vertical accelerations that are about 25% above the current normative threshold.
Figure 3.12: Calibrated partial safety factors regarding running safety criteria of ballasted railway bridges.

Table 3.1: Calibrated partial safety factors ($\gamma_a$) and design threshold ($a_{\text{lim},d}$) for running safety criterion of ballasted railway bridges.

<table>
<thead>
<tr>
<th>$V_{\text{max}}$ (km/h)</th>
<th>$\gamma_a$</th>
<th>$a_{\text{lim},d}$ (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\leq 250$</td>
<td>1.47</td>
<td>4.15</td>
</tr>
<tr>
<td>$\leq 320$</td>
<td>1.38</td>
<td>4.40</td>
</tr>
<tr>
<td>$\leq 400$</td>
<td>1.35</td>
<td>4.50</td>
</tr>
<tr>
<td>All Design Scenarios</td>
<td>1.38</td>
<td>4.40</td>
</tr>
</tbody>
</table>

Paper VI: Reliability assessment of running safety criteria of railway bridges considering soil-structure interaction effects using ensemble of classification-based surrogate models

The above studies have all modelled the boundary conditions of the bridges as simply-supported, neglecting the effects of Soil-Structure Interaction (SSI). This appears to be a common practice, even at the design stage, assuming that SSI effects are beneficial due to the increasing damping of the system (i.e. $\Delta \xi_{\text{SSI}} > 0$). However, the assumption that the contribution of SSI effects is on the conservative side does not seem to be a valid premise for all situations. This is because taking SSI effects into account, on the other hand, increases the flexibility of the system, i.e. the frequency of the system with SSI inclusion would often be lower than in the corresponding simply-supported case ($f_{\text{SSI}}/f_{\text{SS}} \leq 1.0$, also known as frequency shortening). The latter consequently reduces the critical speed of the system and increases the probability of exceeding vibration thresholds, including vertical acceleration. Therefore, there may be situations where the adverse contributions of SSI effects are governing. This aspect is then investigated for running safety criterion...
of high-speed railway bridges in Paper VI following a probabilistic framework.

In this context, three single-span bridges with span lengths of 10 m, 20 m and 30 m are considered. These bridges roughly represent bridges with short, medium and relatively long spans. The geometrical properties of the bridges are assigned based on data from the survey of existing bridges. Therefore, the aim here was not to investigate the absolute values of the estimated safety indices (failure probabilities). Rather, the relative comparison between the system safety considering SSI effects (denoted as $\beta_{SSI}$) and the corresponding simply-supported bridges (denoted as $\beta_{SS}$) were of particular interest.

The boundary conditions are modelled using a simplified approach that assigns lumped springs and dashpots to the supports and neglects the mass of the foundation. The stiffness and damping of the assigned springs and dashpots are frequency-dependent. However, it has already been shown that these properties can be approximated by a relative frequency parameter (denoted here as $\phi$), which depends on the fundamental frequency of the simply-supported bridges, the shear wave velocity of the soil, its Poisson’s ratio and the depth of the stratum. A constant stiffness and damping can then be assigned based on different values of $\phi$, leading to the same approach to be followed here.

The limit-state function of the reliability problem is thus formulated based on the vertical acceleration of the bridge deck. It should be noted that the underlying random variables of the bridges, passing trains, vertical acceleration threshold and model uncertainty are taken from previous studies. However, the considerable variability of the soil properties prevented the author from assigning a reliable probability distribution to them. This problem can be addressed with a variety of approaches, mostly related to the implementation of epistemic uncertainties in reliability assessments. Such methods are often very computationally intensive, which has led the author to isolate the soil parameters from other random variables by fixing them to certain feasible values. Therefore, a set of reasonable values for shear wave velocity, depth of stratum and width to length ratio were considered. Reliability analyses were then performed for each combination of these deterministic values, resulting in a total of 18 analyses for each operating train speed. Based on these results, the lower and upper bounds of the safety indices can be estimated by calculating their minimum and maximum, respectively. At this point, it should be emphasized that geotechnical reports will most likely be available for each individual bridge. Therefore, the associated bounds of the safety indices would probably be narrower. Nevertheless, the results presented can show the worst-case scenarios due to the non-inclusion of SSI effects.

In this study, a relatively wide permissible operating speed range, i.e. [200-300] km/h with a resolution of 10 km/h, is considered. Following the above discussions, a large number of reliability analyses should then be performed to obtain the stated
safety bounds. It is evident that the use of conventional simulation-based methods would require a considerable number of calls to the computational model, which the intended reliability assessments are computationally unmanageable. Therefore, a surrogate-assisted reliability assessment method is used in Paper VI. For this purpose, classification-based surrogate models are employed that directly classify each realization as safe or failure. The training of such surrogate models is often straightforward, but they seem to be less flexible than regression-based surrogate models and especially Kriging. Therefore, in this study, an ensemble of these models is trained using the concept of stack generalization. In this context, the prediction probabilities of the level-0 models are combined using Eq.(2.46) and then fed to the level-1 model. This surrogate model is shown in Figure 3.13.

![Figure 3.13: Ensemble of classification-based surrogate models used in Paper VI.](image)

The surrogate model discussed is trained according to an active learning strategy. However, the model under consideration does not directly provide the prediction uncertainty. Therefore, each new point is selected among the points within the sample pool furthest from the existing DoE that are close to the approximated limit-state function. This procedure terminates when the stopping criterion proposed in Eq.(3.6) is met.

After training the surrogate models, reliability analyses were performed to obtain the mentioned safety bounds. It was found that the safety index of the corresponding simply-supported bridges with shorter spans approached the lower bounds of the safety indices considering the SSI effects. This illustrates the greater contribution of the additional damping due to SSI compared to frequency shortening. In other words, it would probably be beneficial to consider SSI effects for bridges with shorter spans. However, these effects appear to disappear as the bridge span length increases. In this context, it was noted that neglecting SSI effects for longer span bridges may result in the system being less safe than it should be. Therefore,
the implementation of frequency shortening appears to be essential for longer span bridges. These conclusions are illustrated in Figure 3.14 by presenting the ratio of the lower bound of the safety indices to the safety indices of the corresponding simply-supported bridges. As mentioned above, these results should be interpreted as a rough estimate of the worst case, as SSI effects are neglected. Such situations can occur in soils with low shear wave velocity or very large depth of stratum. Therefore, it is recommended to evaluate the possible influence of SSI effects on such conditions.

![Figure 3.14: The ratio of the lower bound of safety considering SSI effects to the safety indices of the corresponding simply-supported bridges.](image)

**Paper VII: Surrogate-assisted investigation on influence of epistemic uncertainties on running safety of high-speed trains on bridges**

As discussed earlier, there are few experimental observations that support the considered threshold for the ballast destabilization phenomenon. It is therefore to be expected that the corresponding threshold (and thus the probability distribution function associated with it) will be changed if further supporting experimental studies are performed in the future. Nevertheless, the studies presented above have ignored this aspect by taking only the available information into account, i.e. assuming aleatory uncertainty for all basic random variables. Despite this simplification, the author believes that the results presented above and also the proposed improvements are valid until further evidence leads to a significant change in the current threshold. In this context, **Paper VII** investigates the possible influences of such variations by including epistemic uncertainties due to incomplete knowledge in the reliability problem of ballast instability. It is worth mentioning that
in addition to the threshold for vertical acceleration, model uncertainty was also
considered as a random variable with epistemic uncertainties.

The consideration of epistemic uncertainties results in the estimated probability of
failure itself being a random variable represented by a range of values instead of
being described by a deterministic single value, i.e. $p_f \in [p_f, \bar{p}_f]$ (where $p_f$ and $\bar{p}_f$
represent the lower and upper bounds of the estimated failure probability). The
estimation of these bounds requires the solution of optimization problems whose
cost function is the multidimensional integral of the failure probability. The feasible
region for finding the optimal solution is formed by lower and upper bounds on the
CDFs of the variables with epistemic uncertainties. These bounds are defined in
Paper VII with the help of parametric probability-boxes (p-boxes). It should be
emphasized that the parametric p-boxes assume that the type of distribution is
known, while the parameters of the distribution are unknown.

Based on the central limit theorem, a Gaussian distribution is assigned to the
vertical acceleration threshold. Subsequently, the intervals of the distribution pa-
rameters (mean and standard deviation) are estimated by bootstrap sampling. As
a result, a uniform distribution is assigned for each of them based on the 99%
quantiles of the bootstrap sampling. The distribution is then truncated for values
less than normative threshold (i.e. 3.5 m/s$^2$) and for values above 1.0 g. On the
other hand, a Gaussian distribution based on the recommendations of JCSS (2001)
is assumed for the model uncertainty. The constructed model should be unbiased;
therefore, the mean of the model uncertainty is deterministically set to zero. In
addition, the standard deviation was assigned a uniform distribution by allowing a
variation of estimated responses between 5.0%– 30.0% based on the author’s per-
sonal judgment. These variations are converted into the standard deviation by
taking them as the 99% quantile of the assigned Gaussian distribution. This ap-
proach led to the definition of parametric p-boxes, whose CDF realizations and
upper/lower bounds are shown in Figure 3.15.

It is obvious that the estimation of the lower/upper bounds of the probability of
failure would not be computationally feasible with the conventional methods of
reliability assessment. However, by considering the formulated limit-state function
as Eq.(3.12), it can be noticed that the computational model only requires the
random variables with aleatory uncertainties, so that it can be approximated by
a surrogate model. Therefore, a single surrogate model can be trained for each
desired situation (considered bridges), which can later be retrieved for realizations
of the random variables with epistemic uncertainties without the need for further
calls to the computational model. It is obvious that this approach can significantly
reduce the computational cost of the procedure.

$$g(x) = a_{lim} - \max \left( |a_{comp}(x)| \right) (1 + \chi_M) \approx a_{lim} - \hat{M}(x)(1 + \chi_M) \quad (3.12)$$
It should be emphasized here that Paper VII uses active learning with H-Function ($\mathcal{H}F$ - see Eq.(2.50)) as learning function to actively train Kriging surrogate models. However, the training phase ends when both the stopping criterion of the H-Function and the U-criterion are met. In this study, three simply-supported single-span bridges with span lengths of 10 m, 20 m and 30 m are considered. In addition, the maximum allowable operating train speeds vary in the range of $[200–400]$ km/h with a resolution of $50$ km/h.

Then, a preliminary investigation was carried out to determine the required number of parameter realizations ($N_{\text{epistemic}}$) for variables with epistemic uncertainties. In this context, it was found that a stable estimate of the failure probability can be obtained for realizations of more than 8000, which leads to using the same value for further analyses.

Thus, the lower and upper bounds of system safety (denoted as $\beta$ and $\bar{\beta}$) are estimated as a function of the maximum allowable operating train speeds, taking into account the epistemic uncertainties. Furthermore, these safety indices are compared with the safety index of the corresponding system assuming that all random variables follow an aleatory uncertainty. In the latter case, the estimated safety index itself would be a deterministic value called $\beta_{\text{det}}$.

Obviously, from a design perspective, only the lower bound of safety is relevant. If, in this context and for a given system, the lower bound of safety achieved by implementing epistemic uncertainties in the problem is smaller than the corresponding...
safety if all variables have aleatory uncertainties, the permissible operating speed of
the train would consequently also be smaller. As can be seen from the relationship
between the estimated lower bounds of the safety indices and their corresponding
$\beta_{\text{det}}$ values shown in Figure 3.16, the discussed scenario applies to the benchmark
problems considered in Paper VII. At this point, it should be mentioned that
the estimated safety indices for bridges with a span length of 30 m and for higher
train operating speeds were unrealistically low, resulting in them being shown as
dashed lines. The reason for this situation is that the assigned properties of the
bridges come from a survey on existing bridges that are not necessarily designed
for high-speed networks.

Figure 3.16: The ratio of the lower bound of safety considering epistemic uncertain-
ities to the safety indices under the assumption that all variables follow an aleatory
uncertainty.

Considering the target reliability level of such systems in the range of 3.1-3.7, the
allowable speed of the bridges under consideration can be estimated from a prob-
abilistic point of view. It was found that the implementation of the epistemic
uncertainties can result in the allowable speed ratios of the trains being 0.87 on
average. In other words, neglecting the associated lack of knowledge about the
considered problem can lead to operating train speeds that are on average 13%
higher than the safety objectives.

To roughly investigate the influence of the lack of knowledge on the safety of the
system, the approximate relationship of the safety factor presented in Eq.(2.8) was
considered. The different combinations of the coefficients of variation for resistance
and demand terms in combination with the estimated values of $\bar{\beta}$ and $\beta_{\text{det}}$ were
considered. It was found that under the most severe circumstances, a reduction in system safety (estimated safety factor) of up to 30% can occur. It should be noted that the absolute values of the results presented here are of little significance. Nevertheless, they clearly show that the lack of sufficient experimental results on the ballast instability threshold can lead to systems being designed with less safety than expected.
Chapter 4

Concluding remarks

4.1 Discussion

A safe system is not necessarily a system that may not experience failure during its lifetime. On the contrary, it is said to be safe enough if the probability of such failures during a certain period of time does not exceed a desired value. In view of this, any study dedicated to improve/update the design philosophy of a system should evaluate the probability of violating associated limit-states and the corresponding risks, taking into account the associated uncertainties.

The main objective of the present work was to improve the design philosophy of dynamic systems, especially high-speed railway bridges, using such a probabilistic framework. Such an intended improvement should lead to both optimal (as light as possible) and safe design solutions. In this context, the investigations are limited to running safety and passenger comfort among a variety of possible failure modes. Running safety is a specific term in the design of high-speed railway bridges and refers to the prevention of derailments or other phenomena that jeopardize the safe passage of trains over the bridge. In addition, passenger comfort is aimed at limiting the vibrations perceived by passengers as the train passes over the bridge.

The study consists of different phases, including understanding the nature of the problem, collecting reliable information on the different aspects, assigning appropriate probability distribution functions, highlighting the potential aspects for further improvement, investigating the computational aspects of the problem, evaluating the impact of different parameters on the estimated safety, and proposing improved design methods. This path led to the studies listed below in chronological order:

1. Feasibility study to highlight the possible shortcomings of the current design methods as a motivation to carry out further detailed research on this topic.
2. Proposal of some preliminary improvements of the design methodology using simplified approaches.
3. Compare the performance of advanced reliability assessment methods in terms of accuracy to be used in the further phases of the study.
4. Use advanced reliability assessment methods, i.e. surrogate-assisted simulation-based methods, to propose minimum design requirements as a possible...
improvement to the current design methodology.

5. Updating the conventional design method, i.e. the safety factor based method, to meet the safety objectives using surrogate based methods.

6. Investigating the influence of soil-structure interaction on the estimated safety of the system

7. Quantification of the influence of the lack of knowledge (epistemic uncertainties) of the resistance term and model uncertainty on the estimated safety of the system.

4.2 Conclusions

The main conclusions that emerge from the appended papers are summarized below:

- Performing preliminary reliability assessments on generic short to medium span bridges using the FORM has shown that applying the current safety factor to the vertical acceleration threshold does not necessarily result in consistent designs.

- The results of FORM showed that the coach length, flexural rigidity, bridge geometry and system damping are the most important variables in evaluating the running safety of high-speed railway bridges.

- The proposed probabilistic design curves using the FORM showed that bridges with box-girder cross-section type are not feasible for span lengths shorter than 20 m. In addition, slab bridges appear to be inefficient for span lengths longer than 15 m.

- Comparing performance of widely used regression-based surrogate models, including Kriging, PCE, ANN and polynomial response surface, with the subset simulation method revealed that Kriging is superior from an accuracy perspective. This conclusion results from training these surrogate models under identical active learning strategy and for different operating train speeds and bridge span lengths.

- A new stopping criterion is defined to be used in active learning based on the misclassification error of the trained surrogate model. The proposed criterion approximately evaluates the failure probability estimation error using the surrogate model.

- The reformulation of the conventional reliability-based design optimization considering the dependency between the design variables led to a proposal for the minimum required linear mass and stiffness of single-span, simply-supported railway bridges as a function of the bridge span length.
4.3. FURTHER RESEARCH

- Surrogate-assisted partial safety calibration method led to an update of the conventional safety factor for the vertical acceleration threshold (i.e. $\gamma_a = 2.0$). The new safety factors are classified as a function of the maximum permissible operating train speed, which are consequently 1.47, 1.38 and 1.35 for allowable operating train speeds of up to 250 km/h, 320 km/h and 400 km/h, respectively. In general, a safety factor of $\gamma_a = 1.38$ can be used, which allows an increase of the current threshold value for vertical acceleration by about 25%.

- The impact of SSI on the estimated safety of the system is evaluated using an ensemble of classification-based surrogate models. It was found that neglecting SSI effects may underestimate the safety index for shorter span bridges; however, for longer span bridges, safety is most likely overestimated. Therefore, the additional damping due to SSI effects seems to be more important for shorter span bridges, while the consideration of frequency shortening is more important for longer span bridges.

- Implementing epistemic uncertainties in the reliability problem using parametric p-boxes revealed that neglecting epistemic uncertainties resulting from a lack of knowledge about the threshold of vertical acceleration can lead to an overestimation of allowable train speeds by about 13%. In addition, the rough estimates showed the possibility of reducing the safety factor of the system by up to 30%.

4.3 Further research

In this study, reliability-based methods have been employed to investigate the different aspects of dynamic design methods of high-speed railway bridges and to propose some potential improvements corresponding to these limit-states. Below are some suggestions for possible future studies on this topic.

- The collected information on the assignment of probability distributions, especially in connection with the geometrical properties of the bridges and the passing trains, should be expanded.

- The construction of simplified computational models and the application of modification factors to implement the effects of load distribution in the ballast, rail irregularities, and train-track-bridge interaction is computationally efficient. However, their approximate nature increases the epistemic uncertainties of the reliability problem. It is therefore recommended to construct complex finite element models to explicitly account for these parameters.

- Discrete element models of the track can also be constructed to numerically investigate the problem of ballast destabilization. This can lead to a better understanding of the nature of this phenomenon.
CHAPTER 4. CONCLUDING REMARKS

• The concepts presented in this study are limited to bridges with ballasted track; however, similar problems also exist for non-ballasted tracks. Therefore, a systematic approach should be adopted to investigate the different aspects of this problem for such bridges and to update the existing partial safety factors.

• The target reliability levels adopted in this study were based on the recommendations of the current guidelines. It is recommended that these be calculated using the optimization-based methods provided for this objective. Such methods require the collection of information on construction, inspection, repair and failure costs.

• Consideration of the consequences of each failure mode is very important when updating design codes, which requires the collection of cost-related information, similar to the previous recommendation.
References


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