Degree Project in Theoretical Physics

Second cycle, 30 credits

Mathematical Modeling of Plasma Dynamics and Dielectric Recovery in Vacuum Interrupters for HVDC Circuit Breakers

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Mathematical Modeling of Plasma Dynamics and Dielectric Recovery in Vacuum Interrupters for HVDC Circuit Breakers

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Master's Programme, Engineering Physics, 120 credits
Date: April 13, 2024
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Host company: Scibreak AB
Swedish title: Matematisk Modellering av Plasmadynamik och Återhämtning av Dielektrisk Hållfasthet i Vakuumbrytare för HVDC-brytare
Abstract

To ensure the safe operation of high-voltage direct current grids, circuit breakers are used to disconnect a faulty link from the rest of the grid. Incorporating vacuum interrupters as a part of these circuit breakers constitutes an outstanding technology for such DC interruptions. However, testing the interrupters take a long time and can be very expensive. Hence, to reduce the time and cost of testing the interrupters, the purpose of this project was to find the most important parameters to test in a vacuum interrupter to evaluate it for use in a DC circuit breaker. This was done by modeling the particle density, before and after current-zero, and the post-arc current using a new model along with existing ones. Review of existing research was also included to support the models in order to draw conclusions regarding reignitions and restrikes. $dI/dt$ before current-zero, $dV/dt$ after current-zero, and contact gap length were found to be the key contributors for reignition, while temperature, contact surface condition and contact gap length were of great importance for restrikes. These breakdowns should occur around the center of the contact surface, or at surface protrusions. The following parameters should be varied when testing vacuum interrupters: $dI/dt$ before current-zero, ranging from $10 \text{ A \mu s}^{-1}$ to $800 \text{ A \mu s}^{-1}$; arcing current, from $1 \text{ kA}$ to $20 \text{ kA}$; arcing time, from $1 \text{ ms}$ to $4 \text{ ms}$ before current-injection; $dV/dt$ after current-zero, from $0.5 \text{ kV \mu s}^{-1}$ to $20 \text{ kV \mu s}^{-1}$; maximum TRV from $5 \text{ kV}$ to $25 \text{ kV}$, to find the threshold voltage for failed interruption; and gap length, from $1 \text{ mm}$ to $10 \text{ mm}$, to find the critical gap length. Furthermore, temperature should also be measured, though measuring post-arc current seems to be of lesser importance. To minimize damage to the interrupter, it was recommended to start with higher gap lengths with low values on everything else.
Sammanfattning

För att säkerställa ett stabilt kraftflöde för högspända likströmsnät används brytare för att isolera defekta delar från resten av nätet. Vakuumbrytare kan utgöra en viktig komponent i sådana likströmsbrytare. Att testa brytarna tar dock lång tid och kan bli mycket dyrt. För att minska på tiden och kostnaderna för att testa brytarna, var syftet med detta projekt att hitta de viktigaste parametrarna att testa i en vakuumbrytare för att utvärdera den för användning i en likströmsbrytare. Detta gjordes genom att modellera partikeldensiteten före och efter nollgenomgången i strömmen, och strömmen som uppstår efter den nollgenomgången genom att använda en ny modell med hjälp av äldre modeller. Granskning av befintlig forskning inkluderades också för att stödja modellerna och dra slutsatser om nytändning och återtändning. $\frac{dI}{dt}$ före strömnollgenomgången, $\frac{dV}{dt}$ efter strömnollgenomgången, bågströmmen och kontaktseparationen visade sig vara nyckelparametrar för nytändning. Temperaturen, kontaktytstillståndet och kontaktseparationen var av stor betydelse för återtändning. Dessa elektriska urladdningar hade en högre sannolikhet att ske närmare mitten av kontaktytan eller vid små spetsiga ojämlikheter. Följande parametrar bör varieras vid testning av vakuumbrytare: $\frac{dI}{dt}$ före strömnollgenomgången, från 10 A µs$^{-1}$ till 800 A µs$^{-1}$; strömmen före strömnollgenomgången, från 1 kA till 20 kA; ljusbågstiden, från 1 ms till 4 ms; $\frac{dV}{dt}$ efter strömnollgenomgången, från 0.5 kV µs$^{-1}$ till 20 kV µs$^{-1}$; maximala TRV, från 5 kV till 25 kV, för att hitta tröskelspänningen för misslyckat brytning; och kontaktseparationen, från 1 mm till 10 mm. Att mäta strömmen efter strömnollgenomgången verkade vara av mindre betydelse. Dock föreslogs det att mäta temperaturen på vakuumbrytaren. För att minimera skador på brytaren rekommenderades det att börja testandet med högre kontaktseparationer med låga värden på allt annat.
Acknowledgments

For starters, I would like to thank my supervisors at Scibreak, Simon Nee and Tomas Modéer, for giving me the opportunity to write my thesis with them and for providing me with their invaluable insights into circuit breaker technology. Thanks also to my colleagues — Rickard Granath, Gustav Bergquist, Staffan Norrga, Bence Hátsági, Makarand M. Kane, Alan Alhallak, and Alexander Leijonhielm — for their warm welcome and enjoyable conversations; our chats about music, food and fitness were just as entertaining as our captivating and thought-provoking discussions around science, history and futurology. I would also like to thank my supervisor, Hans-Peter Nee, and examiner, Jack Lidmar, from KTH for taking care of the managerial aspects of the thesis.

Thanks to all my friends, family, and loved ones for their support throughout my life. Whether we met just once or have known each other for years, I have learned something valuable from each of them.

Although I rarely met them, I’d like to give special thanks to my grandparents. The stories of their lives, and the few times we met, have left a strong positive impact on my own life. I distinctly remember eating dinner with my maternal grandparents as a child. My grandmother told me, ”That is not the right way to eat paratha; here, I’ll show you how to do it” then my grandfather interjected, ”Oh my God he is just a kid; let him eat however he wants.” As a result, to this day, I still don’t know what ”the right way” of eating paratha is — but every time I have it, I am reminded of that cherished moment with them, and I know that they would have been proud of me for becoming an engineer.

Lastly, I am forever grateful for my good friend Artaza Shahid — his grit, passion, and diligence, combined with his visionary worldview, not only make him fascinating to listen to, but also act as a profound inspiration. Our conversations always leave me motivated to work harder and never give up on my dreams. It is thanks to his influence that I have been able to push through difficult times to finally complete my thesis and attain a master’s degree. Thus, while I feel a tremendous amount of joy in accomplishing such a feat, I know that this is merely a stepping stone towards even greater heights — the best is yet to come!
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Chapter 1

Introduction

This chapter contains the following: the relevant background information to set the context for the thesis, the problem that is derived from the background, the purpose, the set of goals to achieve the purpose, the method to achieve the stated goals, the limitations regarding the execution and outcome, and finally, the structure of the thesis.

1.1 Background

The demand for sustainable energy such as solar, wind and hydropower is increasing [1]. However, the sources for this type of energy are rarely available locally, and can instead be found hundreds, or even thousands, of kilometers away from our homes. Thus, the energy produced must be transferred over long distances using an electrical grid. Electrical grids are made up of power stations near the energy sources, substations to step up/down the voltage, transmission networks to transfer the electrical power over long distances, and electric power distribution that distributes the power to customers.

High-voltage direct current (HVDC) grids are able to transfer large amounts of power over very long distances [2]. HVDC transmission is more power efficient than alternating current (AC) transmission over long distances, with considerably less reactive power losses. HVDC grids also allow for the interconnections between asynchronous independent AC networks, which has lead to the idea of the future European super grid. Since electrical grids are big networks of many different parts, failure in one part of the grid can lead to failure in other parts as well, leading to cascading failure and extensive power outages [3]. To prevent such an outcome, it is paramount to break the link between the faulty parts and the rest of the grid. This is
why circuit breakers are used.

The circuit breakers at Scibreak contain a vacuum interrupter. The vacuum interrupter consists of two disk-shaped metal contacts in a vacuum. Ideally, when the contacts are closed, there is current, and when opened, the current stops. But in reality, as the mechanical contacts open, an arc of plasma is formed that maintains the current \(^4\), p. 19. Plasma can be described as a hot, partially or fully ionized gas that exhibits collective behaviour, and is altogether neutral but consists of charged particles (quasi-neutrality) \(^5\). Thus, plasma conducts electricity. AC circuit breakers have a natural occurrence of zero current, termed current-zero, due to their oscillating voltage, which terminates the arc. DC circuit breakers, however, do not have such a current-zero. To extinguish the arc in DC systems, Scibreak has developed circuit breakers that periodically inject an opposing current to interrupt it, resulting in a current-zero. However, the arc may reappear after current-zero under certain circumstances, causing the current to return and thus failing the interruption \(^6\).

1.2 Problem

There are two types of interruption failures: reignition and restrike \(^7\), pp. 18–20. Immediately after current-zero, reignition may occur, while electric breakdown in the gap after some time is called a restrike. The conditions under which reignition and restrike occur during high-frequency current interruption are not sufficiently well understood. Hence, testing the vacuum interrupters under various conditions to get a better grasp of their ability to interrupt a current is required. However, that can often be very expensive and time consuming since many tests need to be performed to achieve reliable statistical data.

1.3 Purpose

To create an inexpensive and fast way to test vacuum interrupters, research and theoretical modeling regarding the most important factors for interruption success would be beneficial. Hence, the purpose of this thesis is to characterize the important stresses to test in a vacuum interrupter in order to evaluate it for use in a direct current (DC) circuit breaker.
1.4 Goals

The work on the modelling of the stresses on the vacuum interrupter has been based on the following three divisions of the interruption process:

In stage 1, arcing and conduction of fault current prior to interruption was examined. The establishment of an arc generates charged particles (ions and electrons) and zero-charged (neutral) particles in the gap from the surface of the electrical contacts. The first goal is, therefore, to model the dynamics of particles prior to current-zero.

In stage 2, right after current-zero, there is a certain probability of reignition to occur. To determine the cause for this, the result of the model from stage 1, together with post-arc particle density models and post-arc current models, was used to evaluate the conditions for which reignition occurs.

In stage 3, some time after current-zero when the plasma has been expelled from the gap, restrike may occur. The models and external data from the previous stages, as well as new relevant external data was used to find criteria for restrikes.

1.5 Research Methodology

To model stage 1, a mathematical model by Yanabu et al. [8] describing the characteristics of the particle density before current-zero was used and compared with experimental data from external sources.

For stage 2, a number of mathematical models and experimental data exist in an attempt to understand the reignition phenomenon, such as the Continuous Transition Model and the commonly used Post-Arc Model. These models, along with a derived particle density model, and data from external research was used, together with the result from stage 1, to evaluate the cause for reignitions.

For stage 3, the results from stage 1 combined with theoretical models and experimental data from present papers will allow for the evaluation of the cause for restrike.

1.6 Delimitations

Since the emphasis of this thesis is on the theoretical aspects of the interruption process, the obvious limitation of the project are the assumptions being made to simplify the problems, which restricts the applicability of the models. This leads to a risk of failing to understand the problem in a holistic manner. The result of the theoretical models
may lead to many parameters being left open, such that they may be fit into various experimental results, making it less generalizable and less reliable for predictions.

Experimental results from previous papers were used to find values of certain parameters and relationship between different phenomena, but no experiments were performed. This means that the conclusions drawn are not specific to Scibreak’s vacuum interrupters, but may apply to vacuum interrupters in general.

1.7 Structure

Chapter 2 covers the relevant parts of the literature study of the thesis project and contains information about the types of vacuum interrupters that are used and the physics behind the interruption process. Chapter 3 delves into the details of the mathematical models and contemporary research used in this thesis. Chapter 4 presents the results of the models and analyzes their validity and reliability for testing vacuum interrupters for DC circuits. Chapter 5 discusses the relevance of the results, and finally, chapter 6 summarizes the results and limitations, while also providing with suggested work for the future.
Chapter 2

Background

When the pair of current-carrying disc-shaped electrical contacts in a vacuum interrupter separate, current is not immediately interrupted but keeps on being present in a bright arc of plasma. This arc of plasma is called vacuum arc or metal-vapor arc. Interruption of the current will be successful when the current through the vacuum arc reaches and maintains 0 A over time. Hence, the extinguishing of the vacuum arc is an extremely important part of the current-interruption process. The background goes through two different types of interruptions, the physics of the interruption process and relevant theoretical models that exist in literature. Figure 2.1 shows the inside of a vacuum interrupter [7, p. 3].

2.1 Interruption

To quench the vacuum arc and interrupt the current, there has to be no current supplying the arc with energy. The arc is sustained by the current being conducted through charged particles from the cathode electrode, the electrical contact where electrons are emitted into the plasma. The sources of metal vapor and charged particles on the cathode are called cathode spots and appear as tiny bright spots that display seemingly random motion on the surface of the cathode. The metal vapor gets ionized by some of the electrons from the cathode spots, leading to a chain reaction of ionization. This results in the production of positively charged ions and negatively charged electrons, thus forming the vacuum arc. The other electrode is the anode, the side where electrons enter from the inter-electrode plasma [7, p. 9].

In AC systems, after current-zero, the voltage applied across the gap changes polarity due to its oscillating nature. The cathode becomes positively charged, and the anode
becomes negatively charged. This makes the electrons reverse their direction of travel, resulting in the anode becoming the new cathode and the cathode becoming the new anode. Since the ions are positively charged, a positive space-charge sheath is built up in front of the new cathode. The new cathode is bombarded with ions from the space-charge sheath, which are accelerated by the built up voltage of the sheath. This voltage is called the transient recovery voltage (TRV) and is determined by the circuit. Due to the electric field of the sheath contributing to electron emission and the bombardment of the ions emitting more metal vapor, more metal vapor gets ionized and thus a new cathode spot is formed which may result in the creation of another vacuum arc. This type of failed interruption is called reignition, which will be explored in more detail later [6]. For DC systems, however, there are no naturally occurring polarity changes, but the same principle applies since the sheath growth can occur on either electrode depending on the polarity of the applied TRV [9].

The main takeaway is that it is desirable to have as little current as possible, or ideally no current, to go through the vacuum interrupter when the contacts open since higher currents produce more cathode spots. As the current falls to zero, the cathode spots disappear and the arc is extinguished [4, p. 261]. The following subsections will go through two ways to drive the current down to current-zero.
2.1.1 Alternating Current

An alternating current is a current whose direction periodically reverses, which means that the current will periodically be at current-zero. Since the current is driven by the voltage, the voltage must also periodically change its direction, and is often called an AC voltage. For current-interruption in an AC circuit, the electrical contacts separate at a random point in the current cycle [4, p. 261].

2.1.2 Direct Current

A direct current does not change its direction, and is also driven by a DC voltage. Unlike AC circuits, DC circuits do not have periods of current-zeros, therefore, making it more difficult to interrupt the current, especially at very high currents/voltages. Normally, to interrupt the current, a branch in parallel with the circuit breaker injects a current in the opposite direction (see diagram). This, however, only works for currents up to 9 kA [9]. High-voltage direct current for transmission lines can often times contain even higher levels of current. For example, circuit breakers in the Zhangbei ± 500 kV DC transmission project in China, are used to interrupt 25 kA fault current within 3 ms [10].

To solve this, Scibreak uses a voltage source converter (VSC) to incrementally excite an oscillating current through the vacuum interrupter and a parallel branch in resonance, see figure 2.2.

Given a circuit inductance, $L$, and capacitance, $C$, the resonant circuit’s characteristic impedance is

$$Z_{LC} = \sqrt{\frac{L}{C}}$$ (2.1)
with a resonant frequency of

\[ f_{LC} = \frac{1}{2\pi \sqrt{\frac{L}{C}}} \quad (2.2) \]

The VSC voltage \( U_{DC} \) is 0 V at all times until the initial voltage step at \( t = 0 \) is applied to the resonant circuit. This creates a current pulse with amplitude \( I_0 = \frac{U_{DC}}{Z_{LC}} \) that peaks at \( t_n = \frac{n\pi}{\sqrt{LC}} \). A half-cycle later, the VSC applies a reversal step voltage of \(-2U_{DC}\) on the resonant circuit. After \( n \) reversals, the oscillating current amplitude becomes

\[ I_n = (2n + 1)I_0 \quad (2.3) \]

with a peak current at time

\[ t_n = (n + 1/2)\pi \sqrt{LC} \quad (2.4) \]

As more and more reversals are applied, and the amplitude keeps increasing, the current of the circuit breaker will eventually reach 0 A, current-zero, as can be seen in Fig. 2.3. This technology is called "VSC assisted resonant current" (VARC) [11], see figure 2.4.
Figure 2.4: Prototype of module for VARC DC circuit breaker [11]
2.2 Physics of the Interruption Process

In this section of the chapter, the physics of the interruption process is described in detail. Firstly, an overview about the relevant basics of plasma will be presented, then the physical phenomena that occurs during contact-separation as the current approaches current-zero, and finally, the time period of space-charge sheath growth after current-zero, also known as the post-arc period.

2.2.1 Plasma

99.9 % of the visible matter in the universe consists of plasma, mostly from stars but also from the intracluster medium and intergalactic medium [12] [13] [14] [15]. On Earth, plasma is not as common but do appear from time to time in our everyday life. Some examples of plasma on Earth are lightning, neon lights, and the plasma inside of human-made vacuum interrupters called vacuum arcs, see figure [16].

Plasma is a fluid that exhibit collective behaviour and consists of positively charged ions, negatively charged electrons and neutral atoms or molecules. Since the number of negatively charged electrons and positively charged ions in a plasma is roughly equal, plasma is considered as quasi-neutral. If there are no neutral particles in the plasma, it is fully ionized, otherwise, partially ionized. Since the plasma contains free charges, it conducts electricity. If a cloud of same-charged particles move to a certain area inside the plasma, a space-charge has been formed, resulting in the establishment of an electric field [5].

An electron, with charge $q = -1e$, where $e$ is the elementary charge, in a vacuum has
an electric field of indefinite length, as described by the equation

\[ E = \frac{q}{4\pi\epsilon_0 r^2} \]  (2.5)

where \( \epsilon_0 \) is the vacuum permittivity and \( r \) is the distance from the electron. However, in the presence of a positively charged particle with charge \( q = +1e \), the influence of the electric field is significantly reduced at certain lengths. In plasma, the distance from a charge where the electric potential decreases by a magnitude of \( 1/e \), where \( e \) in this case is Euler’s number, is called the Debye length:

\[ \lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{q^2 n}} \]  (2.6)

where \( k_B \) Boltzmann constant, \( T \) is the temperature and \( n \) is the particle density. A Debye Sphere is a volume with the radius of \( \lambda_D \). A plasma is typically considered to be neutral when it is several orders greater than its Debye length [17].

2.2.2 Vacuum Arc

Vacuum arcs are most commonly formed when the pair of electrodes separate while carrying a current, which occurs during the time period before current-zero. After current-zero, however, vacuum arcs may be formed through electrical breakdown of the contact gap. An electrical breakdown in a vacuum interrupter is when a conducting path is formed between the two electrode contacts. This occurs when the voltage across the contact gap exceeds a certain threshold, often called breakdown voltage \( U_B \), which creates a vacuum arc. Vacuum interrupters typically have very high dielectric strengths, which means that it can withstand high electric fields without undergoing electrical breakdown [6].

The vacuum arc in the contact gaps consists of three regions: the cathode spot region, in front of the cathode; the inter-electrode plasma; and the space-charge sheath, in front of the anode. In Fig. 2.6 these regions are labeled as I, II, and III, respectively. The figure, however, is not to scale since the cathode spot region and the anodic space-charge sheath are micrometers (\( 10^{-6} \) m) in thickness, while the inter-electrode plasma is usually several millimeters (\( 10^{-3} \) m) [7, p. 8].

2.2.3 Cathode Spot Region

As described in section ‘Interruption’, the vacuum arc is maintained from the particles (neutral metal vapor, electrons and positive ions) originating from hot tiny bright spots
Figure 2.6: Representation of vacuum arc [7, p. 8]

that move around in a random fashion on the surface of the cathode, called cathode spots.

Between the cathode and the inter-electrode plasma region exists an electric field that covers most of the vacuum arc’s voltage, see figure 2.6, and is a few micrometers in length. It is common for vacuum arcs to maintain approximately constant voltage across the gap, independent of the current. The vacuum arc’s voltage strongly depends on the material of the electrical contacts. The typical copper-based contacts have a voltage around $U_{arc} = 16$ V. The following equation is the electric field for two parallel conducting planes, with a potential difference, $U$, and distance between the planes, $l$

$$E = \frac{U}{l} \quad (2.7)$$

However, in the case for cathode spots on a finite-sized electrode, it is important to take into account the surface roughness that is believed to be the cause for the cathode spots and the non-uniformity of the field due to the geometry of the vacuum interrupter [4, p. 36]. Thus, an enhancement factor $\beta$ is included to the equation, where $l$ is the length between the cathode and the inter-electrode plasma,

$$E = \frac{\beta U}{l} \quad (2.8)$$
The electric field accelerates electrons emitted from the cathode, resulting in ionization of the metal vapor in the inter-electrode region. Sometimes, instead of ionizing the metal vapor, the electron may excite an atomically-bound electron. The excited electron will then relax back to its former state in less than a nanosecond, emitting radiation/light. This is why the cathode spots are bright.

In the cases of successful ionization, most of the ions move to the cathode, while some of them have attained ample kinetic energy to reach the anode. The highly energetic ions will be further discussed in subsection 'Inter-Electrode Plasma'. For the ions with insufficient energy, they are accelerated towards the cathode by the electric field. This mechanism, along with Joule heating from the current, produces enough heat to evaporate the metal cathode, leading to the emission of neutral metal vapor and liquid metal droplets into the gap.

2.2.3.1 Thermionic Emission & Fowler-Nordheim Tunneling

About 90% of the total vacuum arc current is due to the electron emission from the cathode. The cathode spots conduct electricity via the plasma generated as a consequence of the heat from two electron emission processes: Thermionic emission and Fowler-Nordheim Tunneling. These are also known as prebreakdown currents. Thermionic emission contributes less than 10% of the total electron emission before arc ignition [6]. Thermionic emission occurs when the electrode is heated, increasing the average kinetic energy of the electrons. This will increase the probability for the electrons to escape the electrode’s conduction band to then be accelerated away from the cathode by the electric field. Current density is of great interest, rather than total current, since higher current densities tend to be associated with higher temperatures [7, p. 18]. The current density for thermionic emission from wires, which in this case is considered to be larger areas, is described by the Richardson-Dushman Equation:

\[ j_e = AT^2 \exp \left( \frac{-\varphi}{k_BT} \right) \]  \hspace{1cm} (2.9)

where \( A \) is the Richardson constant, that is sometimes multiplied with a material-dependent correction factor, \( \lambda_R \), of about 0.5; \( T \) is the temperature of the cathode; \( \varphi \) is the work function, which is the minimum energy required to remove an electron from a solid; and \( k_B \) is the Boltzmann constant [6]. A metal with similar temperature to that of cathode spots, 4000 K, has a current density of roughly \( 10^7 \) A m\(^{-2} \) [7, p. 9].

It also seems like the electron emission stems from the microscopic projections on the cathode. Thanks to the invention of the field emission microscope by E. W. Müller [6], many experiments on closely spaced contacts have concluded that the
theory proposed by Ralph H. Fowler and Lothar Wolfgang Nordheim about quantum mechanical tunneling through a triangular potential barrier, called field emission, was responsible for the electron emission phenomena from the cathode spots [4, p. 44]. The equation for field emission is given by the Fowler-Nordheim equation:

\[
  j_e = 1.54 \times 10^{-6} \frac{E^2}{\varphi t^2(y)} \exp \left( -6.83 \times 10^7 \frac{\varphi^{1.5} v(y)}{E} \right)
\]

(2.10)

\[
y = \frac{3.79 \times 10^{-4} \sqrt{E}}{\varphi}
\]

(2.11)

where \( j_e \) is the current density in A cm\(^{-2} \), \( E \) is the electric field strength in V cm\(^{-1} \), and \( \varphi \) is the work function in eV, \( t(y) \approx 1 \) for electric fields at micro-projection tips \( (3 \times 10^9 < E < 10^{10} \text{ V cm}^{-1}) \), and \( v(y) = 0.956 - 1.062 y^2 \) is a correction for the potential barrier. Equation 2.10 is simplified by setting \( B_1 = \frac{1.54 \times 10^{-6}}{t^2(y)} \) and \( B_2 = 6.83 \times 10^7 v(y) \) to get

\[
  j_e = \frac{B_1 E^2}{\varphi} \exp \left( -\frac{B_2 \varphi^{1.5}}{E} \right)
\]

(2.12)

Substituting \( j_e \) with \( I_e/A_e \), where \( I_e \) is the electron emission current and \( A_e \) is the emission area, and \( E \) with \( \beta U/l \) from equation 2.8, then after rewriting and setting the equation into a logarithmic function, it can be expressed as

\[
  \frac{I_e}{A_e} = \frac{B_1 (\beta U)^2}{\varphi l^2} \exp \left( -\frac{B_2 \varphi^{1.5} l}{\beta U} \right)
\]

(2.13)

\[
  \log_{10}\left( \frac{I_e}{U^2} \right) = \log_{10}\left( \frac{A_e B_1 \beta^2}{\varphi l^2} \right) - \frac{B_2 \varphi^{1.5} l}{2.303 \beta} \left( \frac{1}{U} \right)
\]

(2.14)

By setting the equation up in this way, \( \beta \) and \( A_e \) can be calculated by using the slope and intercept of the straight-line plot of \( \log_{10}(I_e/U^2) \) vs. \( U^{-1} \). The Fowler-Nordheim equation is quite sensitive to small changes in electric field strength, \( E \), and the work function, \( \varphi \) [18]. This sensitivity is shown in Fig. 2.7.

Using the Fowler-Nordheim equation, Slade [4, p. 45] extrapolated plane-parallel copper electrode data from Davies and Biondini [19], see figure 2.8, to calculate the current density at a measured breakdown voltage of 50 kV. He found that the current density would be \( j_e \approx 2.3 \times 10^{13} \text{ A m}^{-2} \), which is in accordance with measured data [20]. However, despite alignment with experimental data, this is physically not possible with
Figure 2.7: Sensitivity of equation 2.13 [18]

Figure 2.8: Davies data of equation 2.14 [19]
Figure 2.9: Field enhancement at a cathode spot from locally ionized neutral vapor[22]

only Fowler-Nordheim Tunneling. This is because the current density would be limited by the space-charge above the cathode spots according to Child’s law, equation 2.20 [21]. Child’s law will be discussed later in section ‘Post-Arc Period’. Hence, there must be another reason for the high current density. It has been suggested by Schwirzke et al. [22] that as the field-emission current rises from tunneling, the ionization of neutral vapor, from the inter-electrode region and in the region close to the cathode spot due to Joule heating, leads to the release of electrons and ions that form the plasma of the cathode spot. Some of the ionized particles return to the cathode to bombard the electrode surface. This leads to further Joule heating and electric field enhancement, causing even more release of neutral vapor from the cathode electrode, as seen in Fig. 2.9, leading to the formation of a cathode spot. This process, along with the thermionic emission can explain the occurrences of high current densities in the range of $10^{12} - 10^{13} \text{ A m}^{-2}$. Hence, the combination of these phenomena has been named Thermal-Field (TF) Emission [7, p. 9]. Each cathode spot typically carries 50—100 A [4, p. 269]. As the current increases, so does the number of cathode spots and their current density [9].
2.2.4 Inter-Electrode Plasma

The electrons, ions and neutral vapor in the inter-electrode region mainly comes from the cathode spots, but also to an extent from the anode. For low currents, after an electron has ionized a neutral metal particle, it may sometimes have a very high kinetic energy, such that it can reach the anode electrode. The ion current is usually about 10% of the vacuum arc’s current. The current of electrons takes up the rest of the electric current. The kinetic energy of the ions can reach energies much higher than the vacuum arc voltage, \( U_{arc} = 16 \) V, up to 120 eV. There are several reasons for this.

The first is the potential hump theory, which assumes that ions are produced in a region of much greater potential difference, or voltage, than \( U_{arc} \) near the cathode. This high-potential region is believed to be caused by the space-charge that increases the potential in a narrow space, resulting in the strengthened electric field to accelerate the ions to the anode.

The second theory is the gas dynamic theory, that assumes that ions are accelerated by collisions with electron-flux of higher velocity. Thus, some kinetic energy from the electrons are transferred to the ions, resulting in greater ion velocity [23]. This is also believed to be the greatest contributor to the ion-flux phenomena [7, p. 11].

2.2.5 Anodic Space-Charge Sheath

The anode electrode is a passive collector, meaning that it receives the plasma in the contact gap [24]. The vapor shield also acts like a plasma sink. They do this by absorbing electrons into the metal or by neutralising ions with the surface electrons on the metal. In Fig. 2.6, there is a small drop in voltage in front of the anode from its space-charge sheath. This sheath arises because of the greater thermal velocity that electrons posses than that of ions, close to the electrodes. When the plasma interacts with the metal anode, there is a higher electron flux than ion flux into the anode, resulting in a build up of negative charge on the surface of the anode. This leads to an electric field in front of the anode, which repels the excess electrons, yielding a zero net charge flux. The electric field is what causes the slight voltage drop at the anode [7, p. 11].

2.2.6 Electrical Breakdown & Vacuum Arc Formation

As mentioned in subsection ’Vacuum Arc’, for an electrical breakdown to occur, there has to be a conducting path between the electrodes. This happens when the voltage across the gap exceed the breakdown voltage, \( U_B \). So far, it is understood that there are two important contributions to the formation of vacuum arcs. The first one is the existence of a cathode region that supplies the vacuum arc with a continuous stream
of electrons. The second is the ionization of neutral metal vapor in the gap, such that enough ions are in the plasma to maintain a quasi-neutral state. If either of these two processes are missing, then there will not be an electrical breakdown in the contact gap [4, p. 63].

2.2.6.1 Microparticles on Contact Surface

Kamikawaji et al. [25] has shown that particle size on polished contacts in a vacuum has an effect on $U_B$. Figure 2.10 shows that the smaller the particle size is, the lower the breakdown voltage.

Sato et al. [26] compared clean, polished Copper (Cu) contacts with the typical roughness level you see in vacuum interrupters on Copper-Chromium (Cu-Cr) contacts. The results are shown in Fig. 2.11. Here one can see that the larger the size of the microparticle, the greater the $U_B$ is for the rough cathode and anode surface and the polished anode surface, but decreases $U_B$ for the polished cathode. This suggests that microparticles could be beneficial for rough surfaces, but still detrimental for polished ones since $U_B$ is much higher for polished surfaces without microparticles. These microparticles tend to travel from the anode to the cathode [27], and it is known that anode material is present on the cathode after electrical breakdown in a vacuum [28].

2.2.6.2 Breakdown Voltage

Electric field enhancement from microscopic projections on the cathode surface has been discussed in subsection 'Thermionic Emission and Fowler-Nordheim Tunneling’. The designer of the vacuum interrupter has little control over this since it varies after...
each interruption due to surface variations and is also dependent on the contact material. However, the designer can control the internal geometry of the interrupter, such as the contact gap length during opening and shield dimensions, which will affect the electric field enhancement in a macroscopic scale, see figure 2.12. This is represented with another enhancement factor $\beta_g$, while the microscopic enhancement factor can be called $\beta_m$, and the total enhancement factor $\beta = \beta_g \beta_m$. The macroscopic field and microscopic field are the represented as,

$$E_g = \frac{\beta_g U}{l}$$  \hspace{1cm} (2.15)

and

$$E_m = \frac{\beta_m U}{l}$$  \hspace{1cm} (2.16)

respectively. Figure 2.13 shows how $\beta_g$ may vary for a pair of plane contacts with rounded edges [4, p. 40].

According to Farrall [29] the macroscopic field required for breakdown seems to be $E_g \approx 4-8 \times 10^7$ V m$^{-1}$; Chatterton [30] says that with a clean cathode surface in a good vacuum, field emission currents will occur at $E_g = 2 \times 10^7$ V m$^{-1}$; and Kranjec and Ruby [31] shows that breakdown will occur if the field exceeds $E_g = 1 \times 10^8$ V m$^{-1}$. The microscopic field required for breakdown for Cu-Cr contacts with $\beta = 100 - 300$ is $E_m \approx 8-11 \times 10^9$ V m$^{-1}$ [32]. The macroscopic field can be a result of either the potential drop across the whole contact gap, or the electric field between the cathode and the electrically charged particles near the cathode surface.
Figure 2.12: Finite element analysis (FEA) of equipotential lines in a vacuum interrupter with applied voltage across the electrical contacts [4, p. 40]

Figure 2.13: Geometric enhancement factor as a function of $d/r^*$ [4, p. 40]
Noe et al. [38] attempted to develop a design criteria involving macroscopic field strength in order to reduce the probability of electrical breakdown. This was done by using the data from figure 2.14 to establish an empirical expression for the breakdown voltage, $U(d)$, for a contact gap $d$,

$$U(d) = \sqrt{\frac{U(c)^2}{4} + \frac{U(c)E_0}{E_{g,max}/U}} - \frac{U(c)}{2}$$  \hspace{1cm} (2.17)

where $E_0$ is the initial slope in 2.14, $U(c)$ is the breakdown voltage at the field strength of $E_0/2$, $U$ is the voltage applied across the gap and $E_{g,max}$ is the maximum enhanced field strength in the cathode from macroscopic enhancements. Slade [4, p. 93] has shown with FEA analysis that, for a typical vacuum interrupter, the maximum $\beta_g$, hence $E_{g,max}$, occurs at the $90^\circ$ edges of the cylinder-shaped contacts.

### 2.2.7 Diffuse and Constricted Vacuum Arc

For low currents the vacuum arc is at a diffused mode where the arcs are spread out over the anodic contact surface. However, for higher current, the vacuum arc is concentrated on the anode surface and forms an anode spot [7, p. 12], as seen in Fig. 2.15.

The anode spot is very similar a cathode spot since it generates its own vapor and plasma, going from a passive charge collector to a source of charge. The difference between the
Figure 2.15: Diffused mode and constricted mode of the vacuum arc [24]
anode spot and cathode spot is that all current is focused on the anode spot, resulting in a longer time to cool down after vacuum arc extinguishment. This exacerbate the electrode conditions, leading to a greater chance for reignition after current-zero [7, p. 12] [39].

An axial magnetic field (AMF), or axial magnetic flux density, can be applied between the electrodes to maintain the diffused mode of the vacuum arc at higher currents, resulting in less thermal stress on the anode [7, p. 13]. Since the magnetic field applies a force perpendicular to the velocity of the particles, the electrons and ions will move in a circular manner due the component of their velocity that is parallel to the electrodes:

\[ \mathbf{F} = q(\mathbf{v} \times \mathbf{B}) \]  

(2.18)

but since there is an electric field involved as well — mostly on the surface of the cathode spot — that accelerates the electrons perpendicular to the electrode plane, a more accurate description would be,

\[ \mathbf{F} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) \]  

(2.19)

which is the Lorentz force. This means that the charges will exhibit helical motion, therefore, so will the vacuum arcs. The radius of this helix expands as the electrons move across the gap [4, p. 160].

### 2.2.8 Post-Arc Period

As stated in Lanen [7, p. 8], current-zero occurs when the last cathode spot has vanished. The ions created from the last cathode spot keeps moving towards the anode due to their inertia. The total electrical current at this point is zero since the electrons match the velocity of the ions going from the cathode to the anode, since they are strongly influence by the electric field. After current-zero, the a new current appears. This current is the post-arc current. The post-arc current in a vacuum interrupter have three main causes: the movement of charged particles present in the space-charge sheath; the movement of charged particles by secondary emission at the electrodes or by collision of primary charged particles with neutral particles in the plasma; and field-emission currents or other transient emissions.

As can be see in Fig. 2.16, the post-arc period can be divided into three phases. Due to the cyclical nature of the current, it will reverse its polarity after current-zero. This will lead to the anode repelling electrons and attracting ions. Hence, the anode has become the cathode, and the cathode has become the anode. The anode and cathode during
arcing are referred to as the "new cathode" and "new anode", respectively, during the post-arc period.

In phase 1, the electrons will slow down their approach towards the new cathode due to the reversed polarity and there will be a net positive charge arriving. This positive charge is the beginning of the post-arc current, $i_{pa}$. However, the net charge in the plasma is still zero, so the voltage across the gap is zero in this phase.

After the electrons have slowed down to zero velocity, they then reverse their direction and accelerate towards the new anode. This is phase 2 of the post-arc period. As the electrons move towards the new anode, a positive space-charge sheath is built up on the new cathode, see figure 2.17. This sheath is caused by so-called transient recovery voltage (TRV), which is almost completely taken up by the sheath, while the post-arc current is caused by the sheath growth rate $dl/dt$.

As the sheath continues to grow, the inter-electrode plasma gets pushed away and diffused into the shields and new anode. This will continue until the sheath reaches the new anode. When that happens, the thickness of the sheath will be equal to that of the contact gap length. Phase 3 of the post-arc phase has now begun. Since all the electrons have been removed, the electrical current drops, while the electric field causes the ions in the sheath to move towards the cathode, resulting in a successfully interrupted current.
2.2.8.1 Sheath Growth: Child’s law, the Ion Matrix Model & the Continuous Transition Model

Andrews and Varey [41] showed that the ion sheath growth on an electrode, in the presence of low-pressure plasma, can be described by the transition between two models: Child’s law, for a static sheath; and the Ion Matrix Model, for a very fast-growing sheath. The model for the transition between the two limits is called the Continuous Transition Model (CTM). The sheath thickness $s$ for Child’s law and the Ion Matrix Model, respectively, are

$$ (s - x)^2 = \frac{4\epsilon_0}{9j} \sqrt{\frac{2e}{M}} V^{3/2} $$

$$ (s - x)^2 = \left(2\epsilon_0/n_0 e\right) V $$

where $V$ is the sheath voltage, $n_0$ is the ion density, $M$ is the ion mass, $u_0$ is the ion velocity at the sheath edge, $e$ is the elementary charge, $\epsilon_0$ is the vacuum permittivity, $x$ is the distance from the electrode, and $j$ is the ion current density.

Child’s law, also referred to as the Child-Langmuir law, describes a static sheath. It
can also describe a slowly growing sheath by representing multiple static sheaths added together. A slow-growing sheath allows the ions to adjust to electric field changes in the sheath. Child’s law is a great approximation to the ion sheath, given that the electric field changes very slowly. This usually occurs in the final stage of the post-arc sheath growth, when the sheath has almost covered the entire contact gap.

On the other hand, the Ion Matrix Model applies for very fast sheath growth, where the electric field is rapidly changing. This occurs in the initial stage of the post-arc sheath growth, when the electrode is suddenly given a large negative voltage, resulting in the electrons to quickly accelerate towards the new anode and leaving behind ions arranged as a matrix. Hence, the ion matrix model is a suitable approximation for the beginning of the post-arc sheath growth.

The rate of sheath growth tends to be between the limits of Child’s law, equation 2.20 and the Ion Matrix Limit, equation 2.21. The sheath thickness can be described by the CTM as

\[(s - x)^2 = \frac{4\epsilon_0 V_0}{9en_0}[(1 + \frac{V}{V_0})^{3/2} + \frac{3V}{V_0} - 1] \tag{2.22}\]

\[V_0 = \frac{M}{2e}(u_0 - \frac{ds}{dt})^2 \tag{2.23}\]

These are the assumptions made for the CTM model:

1. The sheath is planar with no electric field or ion velocity components parallel to the edge of the sheath.
2. No collisions or ionizations occur within the sheath.
3. No secondary emission or ion reflection occur within the sheath.
4. Ions arriving at the sheath are monoenergetic.
5. The sheath is completely free of electrons.
6. Electrons respond instantaneously to an applied field.
7. At the sheath edge \( V = dV/dx = 0 \)

The model matched well with their experimental data as shown in Fig. 2.18 and 2.19.
Failed Interruption

Right after current-zero, the gap is filled with charge and vapor from the vacuum arc, and the metal contacts are hot, sometimes with pools of hot liquid metal on the surface [7, p. 20]. Failure during this time is called reignition. It takes a couple of microseconds for the charge to be expelled from the gap by diffusion and sheath growth; however, it takes multiple milliseconds for the neutral vapor to diffuse and for the liquid metal to cool down [42, 43]. Failure in the presence of significant amount of neutral vapor, but without any charge present, is called dielectric reignition and failure with very low levels of neutral vapor and after the liquid metal has cooled downed is called dielectric restrike.

2.2.9.1 Townsend Breakdown

Due to high contact temperature, reignition may occur via TF emission, which is also the cause for dielectric restrikes, but due to the high pressure right after current-zero, Townsend breakdown may also occur [44–46]. Townsend breakdown is typical in air and other medium, but not in vacuums due to the negligible pressure. Townsend breakdown is when an electron that has been accelerated by the electric field ionizes a neutral particle via collision ionization, see figure 2.20. This lowers the kinetic energy of the initial electron, but now there are two electrons that are accelerated in the field. This
Figure 2.19: Sheath thickness and electrode potential over time [41]
Figure 2.20: Collisional ionization

causes a chain-reaction of collision ionization which causes an avalanche of electrons in the gap, resulting in the formation of an arc.

The likelihood of Townsend breakdown depends on the probability of an electron hitting a neutral vapor particle. To vary this probability, one can adjust the vapor pressure or contact gap length, therefore also varying the breakdown voltage as can be described by the Paschen curve in Fig. 2.21.

Glinkowski and Stoving [50] proposed certain breakdown/failure criteria indicating whether an interruption was successful or not.

\[
E = 2 \sqrt{\frac{Z q n_i}{\varepsilon_0} (u(t)V_0 + V_0^2 - V_0)}
\]  
\[
P = n_i M_i v_i \left( \frac{v_i^2}{2} + \frac{Z q u(t)}{M_i} \right)
\]

Where \( v_i \) is the sheath-edge velocity, and \( n_i \) is the ion density, \( M_i \) is the ion mass, \( V_0 \) is equation 2.23, \( Z \) is the average charge multiplicity number, \( u(t) \) is the transient recovery voltage, and \( q \) is the charge. Equation 2.24 describes the electric field at the new cathode. If this electric field exceeds a certain critical value, a breakdown may occur. Equation 2.25 is the power density at the new cathode. If this exceeds a certain specified value of power, then there may be an interruption failure, just like for equation 2.24.

These criteria stemmed from Glinkowski and Greenwood [51], where the authors concluded that, for high-current AC circuits, the rate of change of the current during arcing, \( dI/dt \), and the rate of change of the TRV, \( dV/dt \), are the most important factors for determining the likelihood for reignition. However, they also mentioned that breakdowns can occur for low values of \( dI/dt \) if the contact gap has been subjected
to a high current for a relatively long time (>100 µs), which fits the case for DC circuits.

### 2.2.9.2 Dielectric Strength With Metal Vapor Presence

Sandolache et al. [52] wanted to investigate the effect metal vapor had on breakdown voltage, separate from the effects of the plasma sheath and molten metal deformation, so they used a heated crucible with a few grams of copper in it and an electrode across it to do so. They used the Clausius-Clapeyron equation to find vapor density based on surface temperature of the metal,

\[
n_s = n_0 \left( \frac{T_0}{T_s} \right) \exp \left( -\frac{\Delta H_v (T_s^{-1} - T_0^{-1})}{R} \right)
\]

where \(T_s\) is the surface temperature and \(n_s\) density of the evaporated particle. For copper, \(\Delta H_v = 304.8 \text{ kJ mol}^{-1}\), \(T_0 = 2840 \text{ K}\), and \(n_0 = 2.55 \times 10^{24} \text{ m}^{-3}\). They performed an experiment applying up to 60 kV from a DC voltage source, with an electrode gap length of 10 mm and varied the temperature from 1750 K to 1950 K.

No breakdowns occurred for voltages up to 60 kV when an empty crucible was heated to 1900 K, indicating that breakdown would only occur in the presence of metal vapor. Based on equation 2.26, copper densities of about \(2 \times 10^{21} \text{ m}^{-3}\) and \(1 \times 10^{22} \text{ m}^{-3}\) were obtained for surface temperatures of 1750 K and 1950 K, respectively. For the 10 mm gap, this resulted in breakdown voltages of 29 kV and 34 kV, respectively.
2.2.9.3 Neutral Vapor Density

Sarrailh et al. [53] simulated the space-charge sheath evolution and found that the voltage drop across the sheath is higher with a background neutral vapor density in the residual plasma when compared to no neutral vapor density, 12 kV vs 30 kV, respectively. This is because the ion density increase with increased neutral vapor density due to the charge exchange collisions between ions and neutral particles. The ions are accelerated by the electric field in the sheath, such that high energy ions collide with low energy neutral particles, whose kinetic energy is only thermal. This results in the reduction of the mean ion velocity, hence, increasing the ion density. The time taken to expel the plasma is also three times greater with background neutral vapor density, 61 µs compared to 18 µs. This does not only depend on the applied voltage and the plasma bulk density, but also on the number of charge exchange collisions between ions and neutral particles in the sheath. The combination of the increased potential across the sheath and the slower rate of sheath growth for the plasma with background neutral vapor density led to the electric field to be $9 \times 10^6$ V m$^{-1}$, compared to $4 \times 10^6$ V m$^{-1}$ without neutral particles.
Chapter 3

Methodology

This chapter goes through the investigation done to develop the models for the interruption process, present the necessary studies that produced the data, and finally, walks through the steps taken to evaluate the interruption criteria.

3.1 Vacuum Arc Period: Models Before Current-Zero

The rate of current drop, $\frac{dI}{dt}$, before current-zero (during arcing) and the rate of rise in voltage, $\frac{dV}{dt}$, after current-zero (after arcing) are important factors for reignition and restrike in HVDC circuit breakers [51, 54]. Furthermore, Sarraïl [53] and Sandolache et al. [52] have highlighted the relationship between neutral metal vapor in the plasma with breakdown voltage and electric field strength of the post-arc sheath and the contact gap. It is, therefore, of interest to see the rate of change of particle density in the contact gap during the interruption process.

3.1.1 Characteristic Model of High DC Interruption

Yanabu et al. [8] developed a model to find a relationship between $\frac{dI}{dt}$ and the plasma density in the gap at current-zero. It is assumed that the plasma density of the arc is uniformly distributed between the electrodes; the generation of plasma density depends on the instantaneous current value; and that the size of the region does not change. From this, the following mathematical assumption will be made.

During arcing, it is assumed that the density of plasma is proportional to the vacuum arc current,
\[ n_0 = kI_0 \tag{3.1} \]

Where \( n_0 \) is the plasma density right before the injection of the opposing current to interrupt the DC fault current, which is also the vacuum arc current, \( I_0 \), and \( k \) is the proportionality constant. The attenuation of the plasma density in the absence of an arc is assumed to be proportional to the plasma density, and is described as

\[ \frac{dn}{dt} = -\alpha n \tag{3.2} \]

Where \( \alpha \) is the time constant of the plasma in the gap. In the presence of an arc current, the attenuation can be described by adding the contribution from the current,

\[ \frac{dn}{dt} = -\alpha n + \beta I \tag{3.3} \]

\( \beta \) is the proportionality constant between the arc current and its affect on the rate of plasma density. During arcing, the current tends to increase over time, but at some point when an attempt to interrupt it occurs, it has to remain steady. Therefore, at that time, \( \frac{dn}{dt} = 0 \), as shown in Fig. 3.1. This allows for the calculation of \( \beta \) using equation 3.1 and 3.3:

\[ \beta I = \alpha n \]

\[ \frac{\beta I}{\alpha} = n \]

\[ \frac{\beta I}{\alpha} = kI \]

\[ \beta = k\alpha \]

At \( 0 \leq t \leq T_1 \) in Fig. 3.1, \( \gamma = dI/dt \), such that \( I = I_0 - \gamma t \), where \( \gamma = I_0/T_1 \). It follows that,

\[ n_r(t) = -\frac{\beta \gamma}{\alpha^2} e^{-\alpha t} - \frac{\beta \gamma}{\alpha} t + \left( n_0 + \frac{\beta \gamma}{\alpha^2} \right) \tag{3.4} \]

Yanabu et al. \[8\] then proceeded to plot the densities. Figure 3.2 shows the time
Figure 3.1: Relationship between arc current and plasma density [8]

Figure 3.2: Temporal evolution of plasma density for different $dI/dt$ [8]
Figure 3.3: plasma density at current-zero over $dI/dt$ [8]

evolution of the plasma density after current injection for different $dI/dt$. The white circles at the end of the lines are the plasma density at current-zero. Figure 3.3 shows the trend of the plasma density at current-zero over $dI/dt$.

3.2 Post-Arc Period: Models After Current-Zero

After the vacuum arc has extinguished, the post-arc period begins, where the growth of the space-charge sheath in the new cathode begins, see subsection 2.2.8. The first thing to observe is the effect of the particle density during this phase to see how it affects the recovery of the gap. Then, a model for the post-arc current will be considered.

3.2.1 Particle Density Model (PDM)

One of the most common materials used for the contacts of vacuum interrupters is copper-chromium (CuCr) alloy. Assuming however, that there are two types of atoms present in the contact gap as a form of gas i.e., Cu and Cr. These atoms are assumed to have been ejected from the same contact surface and possess the same temperature. Due to the size of the contact gap and high temperature of the atoms, the mean-free path (average distance traveled before collision) is very large and the atoms are moving very fast, such that it is also assumed that there are no collisions made between the particles.

To make things mathematically simple, the particles in the gap are assumed to be uniformly distributed between the contacts in the shape of a cylinder, while their
velocities are isotropically distributed. Furthermore, assuming homogeneous initial conditions, elastic collisions at the contact surfaces, invariant dissipation of particles along the axis of the cylindrical shape, one can consider the case of one pair of contacts as having an infinite contact separation or consisting of an infinite amount of contact pairs stacked. This makes the problem axisymmetric with two variables: the only spatial variable being the radius and the other one being time. Let’s then say that the initial velocity distribution of the atoms follow a Maxwell-Boltzmann distribution:

\[
f_V(v)dv^3 = \left(\frac{m}{2\pi k_BT}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_BT}\right)dv^3
\]  

Equation 3.5 yields the probability density of the particles per infinitesimal element of the three-dimensional velocity space where \( dv^3 = dv_x dv_y dv_z \), centered on the velocity vector, \( v \), with particle mass, \( m \), temperate, \( T \), and Boltzmann constant \( k_B \). Let’s now integrate this equation to yield the distribution per velocity only, in an infinite cylinder.

First, let’s say that \( \beta = \left(\frac{m}{2\pi k_BT}\right)^{3/2} \), since it is a constant under the integral, and then making the following transformations from Cartesian into cylindrical coordinates \( v^2 = v_x^2 + v_y^2 + v_z^2 = v_r^2 + v_z^2 \), and \( dv^3 = dv_r dv_y dv_z = v_r dv_r dv_z d\theta \). This allows for integration with respect to \( v_r, v_z \), and \( \theta \). The term \( v_r \) comes from taking the determinant of the Jacobian for the transformation between the coordinate systems, and \( d\theta \) stems from the axisymmetry of the distribution [55]. Integrating \( d\theta \) from 0 to \( 2\pi \) simplifies the equation to,

\[
2\pi f_V(v) v_r dv_r dv_z = 2\pi \beta \exp\left(-\frac{m(v_r^2 + v_z^2)}{2k_BT}\right)v_r dv_r dv_z
\]  

Next, integrating with respect to \( dv_z \) from \(-\infty \) to \( \infty \),

\[
f(v_r)dv_r = 2\pi \int_{-\infty}^{+\infty} \beta \exp\left(-\frac{m(v_r^2 + v_z^2)}{2k_BT}\right)v_r dv_r dv_z
\]

\[
f(v_r)dv_r = 2 \times 2\pi \beta \int_{0}^{+\infty} \exp\left(-\frac{m(v_r^2 + v_z^2)}{2k_BT}\right)v_r dv_r dv_z
\]

\[
f(v_r)dv_r = \frac{\sqrt{\pi}}{2\sqrt{\frac{m}{2k_BT}}} \times 2 \times 2\pi \beta \times \exp\left(-\frac{mv_r^2}{2k_BT}\right)v_r dv_r
\]
\[ f(v_r) = \frac{\sqrt{\pi}}{2} \times 2 \times 2\pi \times \left( \frac{m}{2\pi k_B T} \right)^{3/2} \times \exp\left( -\frac{mv_r^2}{2k_B T} \right) v_r \]

which, before being multiplied with the initial particle density, \( n_0 \), finally simplifies to the following expression:

\[ f_r(v) = \frac{mv}{k_B T} \exp\left( -\frac{mv^2}{2k_B T} \right) \] (3.7)

Equation 3.7 gives the Maxwellian velocity distribution for atoms with a radial velocity component, \( v \), in an infinitely long cylinder. For the case of the vacuum interrupter, it is considered that there are an infinite number of cylinders within the gap, with each cylinder acting as a source point. This is because one can imagine the spread of particles with velocity, \( v \), originating from a point \( r \), at time \( t = 0 \) as circular ripples that move at constant velocities from their point of origin. The density at a measurement point, \( r' \), at time \( t \), is then given by the following integral over the circular cross section of the contacts:

\[ \int_C \frac{n_0}{2\pi t} f_r(v) r dr d\theta \] (3.8)

inserting the expression for the velocity distribution from equation 3.7 results in,

\[ \int_C \frac{n_0}{2\pi t} \left[ \frac{mv}{k_B T} \exp\left( -\frac{mv^2}{2k_B T} \right) \right] r dr d\theta \] (3.9)

setting \( v = \frac{|r - r'|}{t} = \frac{\sqrt{r^2 + r'^2 - 2rr'\cos(\theta - \theta')}}{t} \), the expression becomes:

\[ \int_C \frac{n_0 m}{2\pi k_B T t^2} \exp\left( -\frac{m(r^2 + r'^2 - 2rr'\cos(\theta - \theta'))}{2k_B T t^2} \right) r dr d\theta \] (3.10)

where \( \theta' \) is set to 0 due to axisymmetry, and \( r \) will be integrated from 0 to the radius of the cylinder, \( R \), and \( \theta \) integrated from 0 to \( 2\pi \).

Equation 3.10 was numerically solved with MATLAB 2023b using the \texttt{integral2} function. This was done for Cu and Cr atoms, showing the particle density as a function of the distance from the axis of symmetry in the contact gap. This was also plotted over a few time steps to show the temporal evolution of the distribution.
3.2.2 Post-Arc Decay Model of Metal Vapor

Rich and Farrall [56] wanted to discover the mechanism behind the rapid recovery of electrical strength of a vacuum gap after arcing. They developed two models: one to determine the initial particle density at current-zero, and the other to find the time-evolution of that density after current-zero. This subsection will only focus on the latter.

They assume that the recovery of dielectric strength is complete when the metal vapor in the gap becomes thin enough, such that the electrical breakdown mechanisms only depend on the surface properties of the electrode. This occurs when the mean free path of the atoms, \( \lambda_a \), is at least twice as long as the electrode gap length, \( L \). Hence, the critical number density is, \( n_c \) can be estimated by the following equation:

\[
\lambda_a = \left( \sqrt{2} n_c \pi d^2 \right)^{-1} = 2L
\]  

(3.11)

They then modelled the decay of metal vapor density after arc extinction using an expression by Molmud [57] as the starting point for the number density in an expanding cloud at point \( r \) and time at \( t > 0 \), in a vacuum:

\[
n(r,t) = \left( \frac{\beta}{\pi} \right)^{3/2} t^{-3} \int \int \int_{-\infty}^{+\infty} n(r',0) \exp \left[ -\frac{(r-r')^2 \beta}{t^2} \right] dx' dy' dz' \]  

(3.12)

Where \( n(r',0) \) is the initial density distribution and \( \beta = \frac{M}{2k_B T} \), with \( M \) being the atomic mass of the gas, \( k_B \) is the Boltzmann constant and \( T \) is the temperature of the gas. Assuming a Maxwellian velocity distribution and that the metal vapor condenses on the electrodes and arc chamber walls, the problem is equivalent to the expansion of a finite cylinder of vapor into a vacuum. The initial distribution is described as

\[
n(r',0) = \begin{cases} 
n_0 & \text{when } 0 \leq r' \leq a; \ 0 < z' < L \\ 
0 & \text{for } r' > a; \ z' \leq 0, \ z' \geq L 
\end{cases} \]  

(3.13)

Integrating equation equation 3.12 with the initial distribution from equation 3.13, the metal vapor density at the center of the electrode gap at a time \( t \) after vacuum arc extinction becomes
\[
    n \left( 0, \frac{L}{2}, t \right) \approx n_0 \left[ 1 - e^{-\frac{t}{\alpha}} \right] \times \text{erf} \left\{ \frac{1}{2} \left( \frac{L}{\alpha} \right) \left( \frac{1}{\alpha} \right) \right\} 
\]

(3.14)

where \( \alpha = \frac{t}{a \beta r^2} \) is a dimensionless variable.

### 3.2.3 Post-Arc CTM Model

The CTM, equation 2.22, proposed by Andrews and Varey [41] can be utilized to calculate the post-arc current,

\[
i_{pa}(t) = \pi R^2 e Z n_i \left( v_i - \frac{ds}{dt} \right)
\]

(3.15)

where \( R \) is the contact gap radius, \( e \) is the elementary charge, \( Z \) is the average charge number for ions, \( n_i \) is the ion density at the sheath edge, \( v_i \) is the velocity of ions at the sheath edge, and \( s \) is the sheath thickness. Here, \( \pi R^2 \) is the surface area of the contacts and \( (v_i - \frac{ds}{dt}) \) is the difference between the ion velocity and the growth rate of the sheath. \( n_i \) can be determined by the equation following equation:

\[
n_i = n_{i0} \times e^{-\frac{t-t_0}{\tau}} \times \left( \text{Amp} \frac{l^2}{d^2} + 1 \right)
\]

(3.16)

This equation takes into account the decay of density over time, represented by the exponential factor and the time constant, \( \tau \), which usually is chosen to be between 0.5\( \mu s \) to 10\( \mu s \). According to Glinkowski and Stoving [50], the \( \text{Amp} \) factor can be adjusted to match experimental results, but is typically in the range from 0 to 10. \( l \) is the sheath thickness, with \( l = 0 \) being at the new cathode, and \( l = d \) is at the new anode, where \( d \) is the contact gap length. \( n_{i0} \) is the initial ion density and can be determined from the initial post-arc current \( I_0 \), such that

\[
n_{i0} = \frac{4I_0}{v_i \pi R^2 Ze}
\]

(3.17)

However, there are multiple ways of finding \( I_0 \) according to literature. Shemshadi \textit{et al.} [58] recognized that the ion current is roughly 10% of the total arc current \( I_{arc} \), but this fraction is depended on the material, so a variable \( \alpha_i \) is used, such that \( I_0 = \alpha_i I_{arc} \). Zhang \textit{et al.} [59] assumed that the rate of current decline before current-zero, \( dI/dt \), would continue after current-zero until the electrons in the gap reversed direction, i.e. until the sheath appeared. This time tends to be between \( t_0 = 50 \) ns and \( t_0 = 100 \) ns.
for 10 mm gaps Shu et al. [60]. Hence, \( I_0 = \frac{dI}{dt} \times t_0 \). With these models, one can simulate how the sheath behaves for when varying input parameters to see how the ion density and post-arc current changes over time. This may tell provide with some information about the performance of the vacuum interrupter since the post-arc current is correlated with increased temperature, thus high post-arc current could result in reignition [61]. Furthermore, it may reflect the amount of residual plasma in the contact gap after current-zero and it is also largely affected by the last cathode spot before interruption, the latter giving the post-arc current its random nature.

The TRV, during a time \( t \), will be modeled by the following function:

\[
V = A_0 \times \left[ 1 - \exp\left(\frac{t}{\tau_0}\right) \right] + A_1 \times \left[ 1 - \exp\left(\frac{t}{\tau_1}\right) \right]
\]  

(3.18)

thus, the time derivative is

\[
\frac{dV}{dt} = \frac{A_0}{\tau_0} \times \left[ 1 - \exp\left(\frac{t}{\tau_0}\right) \right] + \frac{A_1}{\tau_1} \times \left[ 1 - \exp\left(\frac{t}{\tau_1}\right) \right]
\]  

(3.19)

where \( A_0 \) controls the amplitude of the waveform, \( A_1 = C - A_0 \), \( C \) is also a parameter controlling the amplitude, \( \tau_0 \) influences the initial slope and \( \tau_1 \) affects the second slope after the TRV minima. The post-arc CTM is solved using MATLAB 2023b with the \texttt{ode45} function. Ion density, frequency, arc current, gap length, and the TRV waveform will be varied.

### 3.3 External Research

In this section, the papers that will be analyzed along with the models are presented. They are all focused on the relationship between vacuum interrupter parameters and the effect on the recovery of dielectric strength / breakdown voltage.

#### 3.3.1 Temperature & Gap Length

Zhou et al. [62] performed a series of experiments with varying gap lengths (0.5, 1, 3, and 5 mm) for a pair of electrodes in a vacuum chamber. The cathode is a thin tip while the anode is a flat surface. The voltage pulse at the cathode provided up to \( V_{\text{max}} = -40 \) kV, which was sufficiently high to establish an arc in every single voltage pulse. The purpose was to find the effect of gap length on the arcing.
3.3.2 Frequency, Gap Length & Arc Duration

In a study by Qin et al. [63], an investigation on the influence of electrode separation, arc time, and frequency of the reverse current on the breaking capacity of a vacuum interrupter was done, with the dielectric strength being measured after the interruption. The breaker is rated at 40 kV and 4 kA, with a contact diameter of 5.08 cm.

3.3.3 Frequency & Gap Length

Gu et al. [64] studied how the commutation frequency and the mechanical characteristic of the high-speed switchgear for a 50 kV breaking unit as a building block of a HVDC circuit breaker. The gap distance of the circuit breaker contacts at an artificial current-zero and frequency of the injection-current was investigated for bi-directional DC interruption. The dimensions of the vacuum interrupter, commutation frequency and current was not specified.

3.3.4 Breakdown Voltage & Gap Length

Prebreakdown current is the low current that occurs right before the initiation of an arc. Ding et al. [65] divided the types of prebreakdown currents into three categories: In type I, the field-emission current/voltage was described by the Fowler-Nordheim equation 2.10. In type II, breakdown occurred without any measurable field-emission current. In type III, breakdown followed a sudden increase of prebreakdown current. The purpose of their paper was to measure this prebreakdown current given different gap distances in a vacuum gap using a vacuum interrupter rated at 40.5 kV, and a diameter of 65 mm.

3.3.5 Influence of dI/dt & dV/dt

Since post-arc current can reflect the properties of the plasma in the electrode gap and the current-zero characteristics $dI/dt$ and $dV/dt$, it may have a great influence on the interruption capability of vacuum interrupters. In the paper by Qin et al. [66], the post-arc current is studied for DC interruption using a synthetic test circuit. The influence of the frequency, from the injection current, and TRV on the interruption is investigated.
3.3.6 Electric Field Strength, Vapor Density & Temperature

Dullni [67] investigated the causes for breakdown in vacuum interrupters after current-zero. It compares the conditions of vacuum interrupters with Cu and CuCr contacts after high-current interruption using theoretical models and experimental measurements.

3.3.7 Re ignition & Restrikes

Tang et al. [68] examined the post-arc current to research the DC interruption characteristics of a vacuum interrupter. The characteristics include reignition, restrikes, temperature, current, plasma density, and arc memory. They used a test circuit to determine how the post-arc current affects the interruption characteristics of a DC vacuum interrupter. The experiment was divided into three cases, see table 3.1. The arcing time before current injection is 3 ms for all cases. For case I and case II, the gap distance is 7 mm, but case I has a positive breaking current, while case II has a negative one. The peak arc current in case II will be quite a lot higher than that of case I due to the HF injection current will superimpose with the arc current before driving it to current-zero, see figure 3.4. Case III is the same as case I, but there the gap distance is 4 mm.

Table 3.1: Cases of experiment [68]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breaking current</td>
<td>Positive</td>
<td>Negative</td>
<td>Positive</td>
</tr>
<tr>
<td>Electrode separation</td>
<td>7 mm</td>
<td>7 mm</td>
<td>4 mm</td>
</tr>
<tr>
<td>Arcing time before HF current</td>
<td>3 ms</td>
<td>3 ms</td>
<td>3 ms</td>
</tr>
</tbody>
</table>

3.3.8 Voltage-Zero Duration

Cheng et al. [69] used a follow current loop in their simulation of a HVDC circuit breaker that utilized both series connected vacuum interrupters and SF6 circuit breakers. They also used a mechanical switch gear to inject a current to bring the arc current down to zero in order to successfully interrupt the current. The purpose of the follow current is to create, what they called, a “voltage-zero duration”, which would enhance the circuit breakers ability to recover its dielectric strength. The voltage-zero duration is the time between the current-zero state and the initial TRV that is applied across the main interrupter / mechanical switchgear, which they called the “reverse TRV”.

After
current-zero, the reverse TRV is conducted by the thyristors, such that it is limited by the follow current loop. After some time, the positive TRV is applied to the mechanical switchgear. The structure of their circuit breaker is shown in Fig. 3.5.

The follow loop consists of SCR2, R2 and L2. The SCR2 are thyristors in series, while R2 is a resistor and L2 is an inductor. This model, however, only simulated a fault current in the positive direction. To take a fault current in the negative direction into account, another branch with the thyristors set in the opposite direction must be applied. The authors varied the values of R2 and L2 to see how the voltage-zero duration would change.
3.4 Method

First, the results from the research papers were presented. Then, the period before current-zero was investigated. This was done using the Characteristic Model by Yanabu et al. [8] from section Characteristic Model of High DC Interruption, along with data from relevant research. After that, the post-arc period was examined by using the PDM, the Post-Arc Decay Model by Rich and Farrall [56], and the Post-Arc CTM Model, along with relevant data from existing research to analyze the behaviour of the post-arc current and recovery of the dielectric strength. Finally, both periods were taken into consideration in order to evaluate what the most important stresses are for a successful interruption.
Chapter 4

Results & Analysis

In this chapter, the results of the external research and models are presented, analyzed and interpreted.

4.1 External Research Data

4.1.1 Temperature & Gap Length: Metal Vapor at the Anode

After varying the gap lengths (0.5, 1, 3, and 5 mm) for a pair of electrodes in a vacuum chamber with voltage pulses up to \( V_{\text{max}} = -40 \) kV, Zhou et al. [62] found that the critical electric field strength for breakdown is 160 MV m\(^{-1}\), or \( 1.6 \times 10^8 \) V m\(^{-1}\), and that an anodic glow appeared by the electron from the cathode bombarding the anode. They also found that increasing the gap length decreased the rate of rise in temperature at the anode and increased the delay for the higher metal vapor pressure at the anode, see figure 4.1. The metal vapor pressure rise is in line with the anodic glow, suggesting that the electron bombardment colliding with the neutral atoms may be the cause for this glow. All gap lengths resulted in the temperature reaching the melting point of Cu, 1356 K, with a corresponding vapor pressure above 0.1 Pa, which is a neutral atom density between \( 1 \times 10^{18} \) m\(^{-3}\) to \( 1 \times 10^{19} \) m\(^{-3}\). The increase in metal vapor pressure is lead by the increase in temperature with a similar rate of growth, when looking at the initial slopes. This could be explained by the increased gap length, since a larger gap would allow for a broader spread of electron current from the cathode, thus reducing the heat per unit area. Hence, the anode needs a longer time to be heated to reach a temperature of intensive evaporation. This supports the idea that a shorter gap length
increases the risk of a failed interruption.

### 4.1.2 Frequency, Gap Length & Arc Duration: A Critical Gap Length

The results from Qin et al. [63] indicate that, for a breaker rated at 40 kV and 4 kA, with a contact diameter of 5.08 cm, there is a critical electrode separation. Within this separation, one can enhance the dielectric strength by increasing the separation or decreasing the arc time. Furthermore, a decrease in the reverse current frequency can also improve the dielectric recovery.

Figure 4.2 shows how the dielectric strength recovers under the gap lengths: 2 mm, 3 mm, 4 mm, 6 mm, with an arcing time for 2 mm and reverse current frequency of 666 Hz. One can see that the breakdown values are approximately the same for the first few microseconds after current-zero. Thereafter, there is a rapid rise in breakdown voltage until the inflexion point at around 15 µs, where the rate of increase slows down and eventually plateau at 22 kV. As the electrode separation increases from 2 mm to 4 mm, the breakdown voltage also increases. Notice also that the difference in breakdown voltage between 4 mm and 3 mm is greater than that of 4 mm and 2 mm. The reasoning for this is that a greater separation aids with the diffusion of the neutral metal vapor in the gap, thus allowing for quicker recovery. However, when the electrode gap length is at 6 mm, the breakdown voltage is lower than at 4 mm, before the plateau. It can be inferred that there exists a critical gap length for which the dielectric strength recovers the fastest, between 4 mm and 6 mm. As it exceeds the critical value, the breakdown voltage decreases due to the shrinking of the vacuum arc since the distance the arc must cover has increased, thus the cross-sectional area of the arc decreases in order to maintain a specific current density. This shrinking can then increase the surface energy...
flux at the electrodes. This causes greater rates of erosion, thus emitting greater amounts of metal vapor. When the rate of metal vapor gain is greater than the diffusion of metal vapor from the gap, there is a net increase in vapor, which leads to the reduction of the breakdown voltage.

This seems to contradict figure 2.14, which could indicate that this critical gap length varies for every vacuum interrupter due to their different designs. Thus, under the assumption that there is such a critical value for the gap length, perhaps there needs to be improvements to the models regarding particle density to take this into account. For example, one could add the parameter $\Lambda$ for the production of metal vapor, while the parameter $\eta$ could represent the diffusion of vapor as the gap length, $l_g$, increases. Adding this to equation 3.3 from Yanabu et al. [8] may look something like the following:

$$\frac{dn}{dt} = -\alpha n + \beta I + l_g \Lambda - l_g \eta$$  

(4.1)

With that said, more experiments on the gap lengths for DC interrupters should be done in order to fully understand this phenomena and how it could be used to improve the design of the vacuum interrupter.

The arc time was also varied, see figure 4.3. The breakdown voltage decreases as the arc time increases for a constant gap length. This is because the metal vapor particles increase with increasing arc time, while the diffusion of metal vapor is, for the most part, unchanged. Hence, prolonged arc times are detrimental for the recovery of dielectric strength after current-zero. An important observation is that the 2 mm2 ms (blue) line is higher than the 4 mm3 ms (black) line and much lower than the 4 mm2 ms (red) line,
indicating that the arc time has a greater impact on the speed of dielectric recovery than the electrode separation.

Lastly, the frequency of the reverse current was varied, see figure 4.4. The corresponding time for each frequency is shown in table 4.1. It is clear that as the frequency increases, the breakdown voltage decreases. There is also a linear relationship between the breakdown voltage and frequency. This is because the current decrease towards current-zero is determined by the frequency of the reverse current, given constant electrode separation and arc time. The reason is that the diffusion time of the metal vapor is shorter for a faster $dI/dt$, which leads to a higher density of vapor in the gap and results in a lower breakdown voltage. So a lower reverse current frequency is beneficial for the recovery of the dielectric strength.

It is important to understand that while these experiments were done by independently varying each parameter, there are cases where varying a parameter changes the another parameter. For example, unlike in this study, many vacuum interrupters, especially for high voltages, use an axial magnetic field (AMF). If you increase the gap length, then the magnetic field density weakens, decreasing the ability to control the arc, thus increasing the risk for the constriction to occur and may also increase the arc time [70].

Table 4.1: Time from the peak current to current zero [63]

<table>
<thead>
<tr>
<th>Commutation frequency (Hz)</th>
<th>Time ($\mu$s)</th>
</tr>
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<tbody>
<tr>
<td>375</td>
<td>282</td>
</tr>
<tr>
<td>666</td>
<td>198</td>
</tr>
<tr>
<td>1000</td>
<td>94</td>
</tr>
</tbody>
</table>
4.1.3 Frequency & Gap Length: Safe Interruption Zone

Figure 4.5 shows the results from the study by Gu et al. [64], where the gap distance of the circuit breaker contacts at an artificial current-zero and frequency of the injection-current was investigated for bi-directional DC interruption. The oscillation frequency is controlled by adjustments of the reactor and capacitor in the L-C commutation branch, while the contact gap distance in the vacuum interrupter is controlled by the trigger of the thyristor, at the moment of high-frequency current-injection. They found that for their HVDC circuit breaker test circuit, there exists an upper limit to the oscillation frequency for successful interruption, \( f_{\text{lim}} \). Above that limit, the risk for a failed interruption increases. They also found an economic frequency \( f_{\text{eco}} \), above which the system cost will significantly decrease.

The authors discovered a zone of safe interruption, see figure 4.5, which consists of the two frequency limits from \( f_{\text{eco}} \) to \( f_{\text{lim}} \), and a certain gap distance range for the specific DC current value tested. The values for the gap distance and frequency outside of the safe zone will result in a failed interruption without any voltage recovery or later restrike. The authors discussed that it was easy to understand why a smaller gap distance resulted in a failed interruption, due to the insufficient recovery of dielectric strength after current-zero, but suggested that the failure caused by larger gap distance needed further investigation.

Their conclusion suggests that this safe zone applies for reignition, and that the upper limit for the gap distance in their specific test conditions was the critical gap distance value that was discussed in subsection Frequency, Gap Length & Arc Duration: A Critical Gap Length.
4.1.4 Relationship Between Breakdown Voltage & Gap Length

Ding et al. [65] found that at a gap length of 1 mm, all breakdowns were of type I, suggesting that it was purely due to the field-emission current. At 2 mm, 70% of the breakdowns were of type I and 30% of type II, implying that the field-emission current still dominates during this gap length, while some breakdowns occurred from unknown mechanism. However, for a gap length of 4 mm, 0% of the breakdowns were from type I, 40% from type II, and 60% from type III. This implies that macroparticles travelling from the anode to the cathode were responsible for most of the breakdowns since the particles could cause the local electric field between the cathode and the particle to significantly increase, causing a spike in field-emission current. Figure 4.6 shows the breakdown type distribution for each gap length.

Ding et al. [65] finds the following relationship for their experiment: \( U_B = 70.12 \times d^{0.56} \). Therefore, based on their experimental results, as the contact gap increases, the probability for interruption failure decreases due to an increasing breakdown voltage at longer gap lengths. It is known from Slade [4, p. 31], seen in Fig. 2.14, that the breakdown voltage takes the form of \( U_B = K \times d^a \), where \( d \) is the contact gap length, \( 97 < K < 123 \) and \( 0.34 < a < 1 \). Huang et al. [71] has modified this to \( d = B_1 U_B + B_2 U_B^2 \), where for a 12 kV DC vacuum circuit breaker, \( B_1 = 0.283 \text{ mm kV}^{-1} \) and \( B_2 = -0.0274 \text{ mm kV}^{-2} \). This shows that gap distance has a characteristic effect described
by these formulas, but due to the inconsistencies in their parameter values means that other factors in the vacuum interrupter also play a role for the dielectric field strength. A problem with these equations is that they do not take the critical gap length into account since the experiments did not observe a decrease in breakdown voltage with an increase in gap length, as seen in the previous two subsections.

Before the experiment was conducted, the vacuum interrupter was set to a 2 mm gap length and conditioned with a standard lightning impulse voltage to replicate a breakdown. The time it took to reach the peak voltage was 1.2 µs and the time to decay to half the peak voltage was 50 µs. As the number of breakdowns increased, so did the breakdown voltage until it finally saturates, see figure 4.7. This is due to the elimination of microprojections and microparticles from the contact surface, thus reducing the field enhancement factor [4, p. 88].

4.1.5 Influence of $dI/dt$ & $dV/dt$: Threshold TRV

Qin et al. [66] finds that for their DC breakers, the interruption is not just affected by the $dI/dt$ and $dV/dt$, but failure of the interruption occurs when the TRV is near 2 800 V. They used a high frequency commutation current that injects a current in the opposite directions of the vacuum arc current when the gap reaches a certain distance. In the study, they refer to the commutation time as the time it takes for the peak current to reach current-zero. They state that, for DC interruption, the commutation time decreases with the rise in commutation frequency, which is very similar to the oscillation frequency for the VARC technology by Scibreak [72]. The high commutation frequency lead to a rise in post-arc current, which tends to be a disadvantage for DC interruption. The post-arc
current also seem to rise along the increase of the recovery voltage.

### 4.1.5.1 Influence of Commutation Current on the Post-Arc Current

The parameters for their vacuum interrupter is shown in table 4.2 and table 4.3 presents the commutation time for the corresponding commutation frequency. Figure 4.8 shows how the peak post-arc current is influenced by the commutation frequency, suggesting that the post-arc current is to an extent caused by the density of plasma left in the gap at current-zero.

#### Table 4.2: Vacuum Interrupter Parameters [66]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated value</td>
<td>12 kV/1250 A/25 kA</td>
</tr>
<tr>
<td>Diameter</td>
<td>5.08 cm</td>
</tr>
<tr>
<td>Electrode gap</td>
<td>10 mm</td>
</tr>
<tr>
<td>Stroke</td>
<td>13 mm</td>
</tr>
<tr>
<td>Material</td>
<td>Copper</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>Axial</td>
</tr>
</tbody>
</table>

### 4.1.5.2 Influence of Recovery Voltage on the Post-Arc Current

Figure 4.9 shows how the recovery voltage affects the post-arc current for various frequencies. Since it is the TRV than drives the sheath growth, an increase in TRV
increases the rate of sheath growth $dl/dt$, which, in turn, increases the post-arc current from equation 3.15. Hence, the post-arc current rise with the rise of the TRV. Moreover, secondary electron emission will occur if the TRV reaches a certain value, which may lead to a failed interruption due to the increased ion density from the TRV. This also happens to be one of the reasons that the post-arc current increases along with the recovery voltage. This is shown in table 4.4, where there seems to be a failed interruption at TRV values close to 2 800 V suggesting that the interruption failure is not only dependent on the $dI/dt$ and $dV/dt$, but also a threshold voltage.

<table>
<thead>
<tr>
<th>Commutation frequency (kHz)</th>
<th>Commutation time (µs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>79.56</td>
</tr>
<tr>
<td>2.0</td>
<td>23.84</td>
</tr>
<tr>
<td>4.0</td>
<td>1.24</td>
</tr>
</tbody>
</table>

Table 4.4: Interruption results for different TRVs [66]
4.1.6 Electric Field Strength, Vapor Density & Temperature: Townsend Discharge

Dullni [67] found that the dielectric strength of the gap after interruption at high current is determined by breakdown in metal vapor. In other words, the breakdown is probably caused by a Townsend discharge. A clear sign for this is an exponentially rising prebreakdown current developing from the post-arc current. This occurs with anode temperatures greater than 2000 K and metal vapor densities exceeding $1 \times 10^{22} \text{m}^{-3}$ with a line integral density of $4 \times 10^{19} \text{m}^{-2}$ across the contact gap.

When the surface temperature is below 2000 K, with metal vapor densities below a density of $6 \times 10^{21} \text{m}^{-3}$, the impact of larger droplets or liquid instability on the contact surface seems to play a significant role for breakdown. The large hot droplets could create an initiating plasma when leaving or approaching the high-voltage cathode. On the other hand, the liquid instability may cause liquid protrusions which magnifies the local electric field [73], thus generating an abrupt current. An electrical field strength of 8.3 kV mm$^{-1}$ is sufficient for a Cu surface. However, the dielectric strength for a CuCr surface is not well known since it is dependent on the surface tension at higher temperature. Either way, the metal vapor emitted from the hot surfaces may play an significant role in the discharge that could bridge the contact gap after the breakdown initiation.

Pu et al. [74], states that reignition of Cu occurs when the electric field strength exceeds the critical value $5 \times 10^9 \text{V m}^{-1}$, while other studies suggest that the critical value for breakdown is $1.6 \times 10^8 \text{V m}^{-1}$ [62], but for a solid surface. These values are much higher than the result from Dullni [67] in the presence of liquid metal, which highlights...
the severity that the formation of liquid has on the risk for breakdown.

### 4.1.7 Reignition & Restrikes: Case Comparisons

Figure 4.10 from Tang et al. [68] shows the portion of reignition and restrikes for this different experimental cases. Case I and III have positive breaking currents, while case II has a negative one. Case I and II have a gap length of 7 mm, while case III has a gap length of 4 mm. All three cases have a arcing time of 3 ms before current-injection. In each experimental case, reignitions are observed more often, indicating that it is more probable to occur before the post-arc current reaches its peak and the breakdown voltage is low. The smaller gap length in case III results in an clear increase in the number of restrikes when interrupting 5 kA and 10 kA; however, its effect on reignition is not as obvious, except for the 5 kA case which does not appear in case I but does so for case III.

When comparing case I and case II, both the reignition and restrikes increase significantly. More specifically, reignitions of 0.5 kA and 2 kA are observed in case II but not case I. Additionally, the probability to see reignition of 1.5 kA, and 5 kA in case II is similar to that of case I because low the breaking current has a comparatively smaller effect when there is a HF high amplitude injection current. For the restrikes the probability of failed interruption for 0.5 kA and 5 kA are higher in case II than in case I.

The authors concluded that the post-arc current does not reduce the interruption performance and that therecovery process is independent of the breaking current. Hence, reignitions and restrikes would probably be more dependent on the contact surface conditions during the post-arc period, such as temperature, microprotrusions and microparticles. The high temperature combined with the microprotusions would enhance TF emission, causing restrikes.

The post-arc current for case I and case II are similar, despite case II having a much larger peak arcing current, see Fig. 4.11. The reason for this is due to an arcing memory of 4.5 µs, meaning that the properties of the vacuum arc that affects the post-arc current occurred within 4.5 µs before current-zero. They also mentioned that a higher gap length led to a higher post-arc current, but that it does not reduce the interruption performance of the vacuum interrupter. Instead, shortening the gap length increased the probability of interruption, especially restrikes. The authors emphasize on the $dI/dt$ before current-zero as having a remarkable influence on the residual charges.
Figure 4.10: Reignition and restrikes for the different cases [68]

Figure 4.11: Post-arc current between case I and II [68]
4.1.8 Voltage-Zero Duration: Dielectric Improvements

Cheng et al. [69] used series connected interrupters with a follow current loop to create a voltage-zero duration time. They achieved the following results when varying the resistor and inductor of the follow current loop:

<table>
<thead>
<tr>
<th>(L \ (\mu H))</th>
<th>(\Delta t_{0R} \ (\mu s))</th>
<th>(R \ (\Omega))</th>
<th>(\Delta t_{0L} \ (\mu s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>37.4</td>
<td>0</td>
<td>37.4</td>
</tr>
<tr>
<td>5</td>
<td>38.62</td>
<td>2</td>
<td>34.97</td>
</tr>
<tr>
<td>50</td>
<td>41.56</td>
<td>5</td>
<td>32.4</td>
</tr>
<tr>
<td>500</td>
<td>46.78</td>
<td>20</td>
<td>27.79</td>
</tr>
<tr>
<td>5000</td>
<td>48.13</td>
<td>100</td>
<td>24.74</td>
</tr>
</tbody>
</table>

For every case, they also measured a positive TRV of about 93 kV. The results show that as the resistance of the follow current loop increases, the voltage-zero duration decreases. An increasing inductance, on the other hand, shows an increasing voltage-zero duration. Hence, in order to increase the voltage-zero duration, the authors suggest that the current limiting resistors should have low resistance, while the inductors should have a high inductance. However, the problem with a high inductance is that it leads to a high reverse TRV, which is disadvantageous for the recovery of the gap. Therefore, in addition to a lower resistance and greater inductance, one should decrease the oscillating frequency to reduce the reverse TRV, since \(V_L = -L \frac{dI}{dt}\).

4.2 Investigation of the Arc Period

In this section, the Characteristic Model will be explored by varying some of its input parameters to find out how it affect the behaviour of the particles in the gap during arcing. It will then be compared with research from external sources to evaluate the models validity and to draw conclusions regarding current-zero characteristics.

4.2.1 The Characteristic Model

Using the Characteristic Model Yanabu et al. [8] calculated the particle density at current-zero to be \(n = n_r(T_1)/n_0 = 0.17\) at \(dI/dt = 170 \text{ A} \mu \text{s}^{-1}\). The authors did not mention the parameter \(\beta\) or the proportionality constant \(k\) in the study, but did include the values of \(\alpha = 1/20 \mu \text{s}^{-1}\) and \(I_0 = 20 \text{ kA}\). However, without \(\beta\) and \(k\), it will be difficult to replicate their results. Therefore, available research for particle densities in vacuum arcs have been utilized to find reasonable values for these parameters.
4.2.1.1 Particle Density & Current From Cathode Spot

Experiments by Robertson [75] has shown that, when the vacuum arc is in its diffused mode, electrons in the plasma will be neutralised by the ions from the cathode spots, traveling at a speed of $1 \times 10^4$ m s$^{-1}$, while the electrons travel at a speed of $1 \times 10^6$ m s$^{-1}$ [76]. For every ejected ion, ten neutral Cu atoms are also ejected at similar speeds to the ions. This results in a Cu-cathode erosion rate of about $80 \mu$g C - $100 \mu$g C. Reece [77] developed a model for the particle density in a plasma cone formed from the cathode spot, with the assumption that the metal vapor is emitted with the average velocity of $1 \times 10^4$ m s$^{-1}$ and that density of the metal vapor is uniform over circular planes normal to the axis of the plasma cone. The last assumption neglects the radial component of the velocity of metal vapor atoms, exactly like the PDM from equation 3.10. The particle density for a single cathode spot is described by the equation,

$$N_{nm} \simeq \frac{R_{nm} I}{\pi \bar{u} x^2 \tan^2 \phi}$$  \hspace{1cm} (4.2)

$R_{nm}$ is the number of atoms evaporated per coulomb, as given by Robertson [75] as $80 \mu$g C - $100 \mu$g C, $I$ is the current, $\bar{u}$ is the mean particle velocity, $x$ is the distance perpendicular to the cathode spot, and $\phi$ is the semi-angle of the plasma cone. The electron and ion densities, based on equation 4.2, are shown in Fig. 4.12.

Reece [77] states that the movement of ions reduces the total circuit current to 99% of the electron current. He also notes that for every positive ion emitted, so too are ten neutral metal atoms; and for every emitted neutral atom, ten electrons are emitted. Thus, the relative rates of the ion, neutral metal atoms and electrons entering the plasma will be about 1 : 10 : 100. This may, however, not reflect the scenario in all vacuum interrupters, since other studies have claimed that about 10% of the arc current stems from positive ions [58, 78, 79][7, p. 11]. Daalder [80] found that the cathode erosion production consists of predominantly ions and macro-particles, while Tuma et al. [81] discovered that the emission of neutral atoms correspond to less than 1% of the total flux from the cathode.

Mitchell [76] assumed a Cu cathode erosion rate of $87 \mu$g C$^{-1}$, based on Reece [77] and Robertson [75], and a vapor velocity of $1 \times 10^4$ m s$^{-1}$, to extend equation 4.2 for multiple cathode spots:

$$N_v = 8.30 \times \left( \frac{I}{A} \right) \times 10^{13}$$  \hspace{1cm} (4.3)
Figure 4.12: Electron and positive ion densities in front of a single cathode spot on copper [77]
\[ N_e = 6.30 \times \left( \frac{I}{A} \right) \times 10^{12} \]  \hspace{1cm} (4.4)

Where \( N_v \) and \( N_e \) are the average particle densities for metal vapor and electrons, respectively. \( A \) is the cross-section area of the electrode, and \( I \) is the current of the vacuum arc, assuming the discharges are uniform. Obviously, this does not consider any fluctuations across the plasma, but it may be quite realistic for the particles within a few millimeters of the cathode spots. Another limitation with this model is that it was based on experiments with currents less than 1.5 kA, so its application to the model proposed by Yanabu et al. [8] may not yield accurate results for \( I_0 = 20 \) kA. However, calculations were still performed to test this.

### 4.2.1.2 Replicating the Model

The equations 4.3 and 4.4 are proportional to the current, such that they can be used to calculate \( k \), and thus \( \beta \) from equations 3.1 and 3.4. Equation 3.1 states that \( n_0 = k I_0 \), and \( \beta = k \alpha \). Thus, for neutral vapor,

\[ N_v = \left( \frac{8.30 \times 10^{13}}{A} \right) \times I \]

\[ N_v = \left( \frac{8.30 \times 10^{13}}{\pi R^2} \right) \times I \]

and assuming contact radius of \( R = 37.5 \) mm,

\[ N_v = \left( \frac{8.30 \times 10^{13}}{\pi (37.5 \times 10^{-3})^2} \right) \times I \]

\[ N_v = 1.8787 \times 10^{16} \times I \]

thus,

\[ k = 1.8787 \times 10^{16} \text{ A}^{-1} \text{ m}^{-3} \]

\[ \beta = 2.3484 \times 10^{20} \text{ A}^{-1} \text{ m}^{-3} \text{ s}^{-1} \]

Now, applying \( \beta \) and \( \alpha = 1/20 \mu s^{-1} \) into equation 3.4 and \( I_0 = 20 \) kA into equation 4.3 to acquire \( n_0 = 3.7575 \times 10^{20} \text{ m}^{-3} \), resulting in the plots shown in Fig. 4.13 and Fig. 4.14.
Figure 4.13: Normalized particle density over time for different $dI/dt$

Figure 4.14: Normalized particle density at current-zero over $dI/dt$
Comparing Fig. 4.13 and Fig. 4.14, with Fig. 3.2 and Fig. 3.3, respectively, one can see that they are very similar to each other. This characteristic model computed a normalized current-zero density of \( n = 0.169586 \) at \( dI/dt = 170 \text{ A} \mu\text{s}^{-1} \), while Yanabu et al. [8] stated that they computed \( n = 0.17 \) at the same \( dI/dt \), yielding an error of 0.24%. Although this may add support to using equation (4.3) by Mitchell [76] for high current-levels, due to the highly debated composition ratio of cathode spot erosion, it may not be accurate for finding the neutral vapor density in the contact gap.

### 4.2.1.3 Parameter Dependency

The figures 4.13 and 4.14 show that a greater value of \( dI/dt \) leads to a greater proportion of vapor at current-zero and a shorter arcing time. Figure 4.15 plots the density at current-zero against \( dI/dt \), given various initial densities. The plot shows that an increase initial particle density leads to a proportionally smaller amount being left at current-zero. This seems counter-intuitive since a greater initial particle density would means that there are more particles in the gap, such that you would be left with more by the time you reach current-zero.

Figure 4.16 shows how the temporal evolution of various densities at a constant \( dI/dt = 200 \mu\text{s} \), according to the Characteristic Model. The values of the initial densities were \( n = 1 \times 10^{20}, 2 \times 10^{20}, 4 \times 10^{20} \text{ m}^{-3} \). and the final time and density-values
were \((t, n) = (26.61, 5.53 \times 10^{19}), (53.22, 6.99 \times 10^{19}), (106.45, 7.48 \times 10^{19})\). The model predicts that the relative change in vapor density in the gap is equal to the relative change in arcing time, since doubling the density doubles the time to reach current-zero. However, increasing the density seems to only marginally increase the final density, ceteris paribus. This indicates that the arcing time and initial density is strongly correlated with the current density of the arc, since the metal vapor density is proportional to the current density, while the final value of the vapor density may be more influenced by the slope of the current descent \(dI/dt\). The former shows similar results to a study by Slade [9], where a quadruple increase in current led to a doubling of arcing time of the initial phases of an arc.

Figure 4.17 shows how different values of the time constants affect the particle density at a constant \(dI/dt\), where \(\tau = 1/\alpha\). Changing \(\tau\) does not appear to change the arcing time, but it does have a significant impact on the final value of the particle density since decreasing its value by steps of 20\(\mu\)s leads to a incrementally larger change in the reduction of the current-zero density.

### 4.2.2 Model Validity & Assessment: Characteristic Model

Due to the uncertainty around the values for \(\beta\) and \(k\), the validity of the characteristic model by Yanabu et al. [8] is dependent on its effectiveness to predict accurate values.
of the decay of neutral vapor at current-zero. However, the characteristics of the decay seem to be somewhat similar to experimental data, so the model could perhaps be used to understand how changes in certain parameters may affect other arcing properties, such as arcing time. An improvement to the model could be the addition of the contact gap length into equation 3.3, as mentioned in subsection Frequency, Gap Length & Arc Duration: A Critical Gap Length, resulting in equation 4.1. This would perhaps give it more accurate results since the density at current-zero seems to be heavily influenced by the gap length. For the time being, using this model to predict accurate values of the neutral vapor density in the contact gap should be done with caution. Further experimental data showing vapor density at current-zero against $dI/dt$ would be beneficial in order to evaluate the accuracy of the model.

### 4.3 Investigation of the Post-Arc Period

#### 4.3.1 Particle Density Model (PDM)

Figures 4.18 and 4.19 shows the temporal evolution of particle density of Cu, 65 $u$, and Cr, 52 $u$, ($u = 1.66 \times 10^{-27}$ kg) with a particle density of $n = 1 \times 10^{22}$ m$^{-3}$, a temperature of $T = 2000$ K and contact surface radius of $R = 37.5$ mm, within 20 µs, and figures 4.20 and 4.21 extends this to 100 µs, based on equation 3.10. Figures 4.18 and 4.19 are
Figure 4.18: Particle density of neutral particles vs radial distance with varying time spans over a short time

Figure 4.19: Particle density of neutral particles vs time with varying radial distances over a short time
Figure 4.20: Particle density of neutral particles vs radial distance with varying time spans over a long time

Figure 4.21: Particle density of neutral particles vs time with varying radial distances over a long time
perhaps more relevant for reignition cases since that time span is very common for the duration of \(dI/dt\) and \(dV/dt\), while figures 4.20 and 4.21 could perhaps provide with some information relating to restrike, but also reignition in some cases.

### 4.3.2 Model Validity & Assessment: PDM

Sarrailh et al. [53], as mentioned in ‘Neutral vapor density’ found, through particle simulations, that the presence of a background neutral Cu vapor density of \(n = 1 \times 10^{22} \text{ m}^{-3}\) at 2000 K resulted in an increase in the time for complete interruption from 18 \(\mu\text{s}\) to 61 \(\mu\text{s}\) when compared to the case without any neutral particles. Anderson and Carroll [82] and Frind et al. [83] got similar results where Frind et al. [83] discovered that the residual plasma in the gap increased the recovery time of 100 \(\mu\text{s}\) at 8 kA to over 500 \(\mu\text{s}\) at 9 kA, for a recovery voltage of 35 kV with a 9.5 mm contact gap. According to the figures 4.20 and 4.21, the density has reduced to about 60\% at the center of the contact radius, after 51 \(\mu\text{s}\), so it is quite reasonable to think that the post-arc space-charge sheath could apply enough pressure to remove the remaining plasma around that time as well when a TRV is applied.

#### 4.3.2.1 Radial & Temporal Distribution of Particle Density

Another interesting observation is that the density seems to always be the highest in the center, which could indicate that the likelihood for reignition would be also be higher in that area, due to the chances of charge exchange collisions increasing from the increased density, along with the fact that the higher neutral density slows down sheath growth.

Mitchell [85] discovered that for a 3.7 kA copper vapor arc formed at a cathode 50 mm in diameter and extended 10 mm to a 7-element anode, see figure 4.23, had 30\% of
the current go through the center of the anode. However, as the voltage across the arc exceeded 40 V, the central current was reduced to 10 %, while 80 % of the current passed through the outer elements. This is not the anode spot phenomena, but it occurs between the current-levels of the diffuse vacuum arc and the constricted vacuum arc. Another study by Boxman [84] observed the same effect at the 40 V mark, where the 3 kA diffused vacuum arc was transitioning into a constricted one. During the transition, he noticed a drop in electron density from $1.7 \times 10^{21} \text{ m}^{-3}$ to $0.7 \times 10^{21} \text{ m}^{-3}$ at the centre of the discharge, see figure 4.22. The particle density model (PDM) shows a greater density at the center, which supports the idea that it could be used as a feasible model for diffuse vacuum arcs, but perhaps not for constricted vacuum arcs. However, it is worth noting that these experiments were done with AC, not DC, so the relevance of the experimental results to DC circuit breakers may be limited.

Data on the temporal evolution of plasma density is quite limited, but figure 4.24 from Schade [44], shows how the plasma density changes based on magneto-hydro-dynamics simulations of the plasma in a vacuum arc with time steps of 0.1 ms. The density decreases over time in a similar manner as the PDM. Based on the information from Sarraiilh et al. [53], the studies by Mitchell [85] and Boxman [84], and figure 4.24, the integral from equation 3.10 seems like a reasonable approximation to the neutral vapor in the plasma after arcing.

Jenkins et al. [86] postulated that it was the evaporation of the molten metal droplets in the gap that slowed down the decay of metal vapor when compared to the Post-Arc
Decay Model. Another difference between the models is that the PDM does not take variation in axial distribution of density into account, while the Post-Arc Decay model considers both the radial and the axial density-variation. Hence, some improvements to the PDM would be to also include axial variations in density, as well include some way to take the molten metal droplets into account.

### 4.3.3 Post-Arc Decay Model of Metal vapor

Rich and Farrall [56] plotted the results of the mode with parameters for Cu electrodes at 2,000 K with various electrode diameters and gap lengths, see figure 4.25. They discovered that the recovery of electric strength is the fastest for a large contact diameter that are closely spaced since that contributes with the lower initial and final neutral vapor density.

### 4.3.4 Model Validity & Assessment: Post-Arc Decay Model

They compared this with their measured data, see table 4.6, yielding fairly accurate results for two different electrode diameters and three different gap lengths. For the other model that Rich and Farrall [56] developed to calculate the initial density, Lins
Figure 4.25: Decay time of neutral particle density for various gap configurations. The dashed lines represent the neutral particle density averaged over the gap volume. The solid lines represent the particle density at the center of the gap from equation 3.14. The dashed horizontal lines represent the critical density at which full recovery should be achieved [56]

[87] found that the calculation was 25 times higher than their measured values for the initial neutral vapor density, i.e. only 4% of the eroded material appeared as Cu vapor, which is in agreement with Daalder [80] who discovered that mass loss from cathodes mainly stemmed from positive ions and macro-particles.

Lins [87] compared the post-arc decay model, equation 3.14, with their experimental data. His experiment consisted of a pair of copper electrodes with an applied an alternating current of 500 A, gap of $L = 10$ mm, radius of $R = 30$ mm at a temperature of $T = 2000$ K, and the measurement was made at 7 mm from the center of the cathode. The post-arc decay model showed that the density after current-zero decayed much faster than the measured values. Jenkins et al. [86] found similar results when comparing his measured values for 2 kA to 11 kA rectangular current pulse to this decay model and suggested that evaporation of molten metal droplets from the cathode spots may be the cause for the slower rate of measured decay. Lins [87] concluded that the model results implied a faster dielectric strength recovery than that of the density decay, such
Table 4.6: Comparison of Calculated and Measured Recovery Time for Different Gap Proportions [56]

<table>
<thead>
<tr>
<th>Electrode diameter (cm)</th>
<th>Gap length (mm)</th>
<th>Calculated recovery time (µs)</th>
<th>Measured recovery time (µs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.08</td>
<td>0.76</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>5.08</td>
<td>2.3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5.08</td>
<td>4.6</td>
<td>17</td>
<td>12</td>
</tr>
<tr>
<td>1.27</td>
<td>0.76</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>1.27</td>
<td>2.3</td>
<td>18</td>
<td>11</td>
</tr>
<tr>
<td>1.27</td>
<td>4.6</td>
<td>28</td>
<td>20-30</td>
</tr>
</tbody>
</table>

that correlation between recovery and vapor density may be different from the criteria proposed by Rich and Farrall [56], given by equation 3.11. Given that the Post-Arc Decay Model seem to yield a much faster rate of decay than what one would expect from experiments, it is perhaps not suitable when determining the time evolution of neutral vapor in the gap.

### 4.3.5 Analysis of Neutral Vapor Decay

Figure 4.26 shows the comparison between the PDM, measurements from Lins [87], and the Post-Arc Decay model. We can see that PDM overestimates the data from Lins [87], especially in the early stages, while the Post-Arc Decay model by Rich and Farrall [56] underestimates it later on. The reasoning for this is probably due to the assumptions being made for each model. The PDM assumes elastic collisions at the electrodes, thus preserving more of the density, while Post-Arc Decay model assumes a perfect sink that absorbs the colliding particles, resulting in a more drastic reduction of density. Realistically, the electrodes would have properties of both, which is probably why the data from Lins [87] lies between these two models. In order to predict the values of the vapor density during the post-arc phase, it may be best to use both the PDM, as an upper-bound, and the Post-Arc Decay model as a lower bound. A limitation with both models is that they do not take the change in temperature into account. Temperature decreases over time, but these models assume a constant temperature, which may further contribute to their inaccuracy. The next subsection will only focus on the PDM because it seems to be more accurate for longer times than the Post-Arc Decay model.

#### 4.3.5.1 Density After a Long Time

Figure 4.27 shows how the density of Cu evolves over time in the center of the contact gap. For 28 kA, under test conditions by Dullni [67], one can expect initial densities up
Figure 4.26: Vapor decay from the particle density model (PDM), data from Lins [87], and the post-arc decay model by Rich and Farrall [56] (RF Vapor Decay)

Figure 4.27: Density of Cu in the center of the contact gap over a long period of time
Figure 4.28: Cu density over time for different temperatures

The density of copper decays to $5 \times 10^{22}$ m$^{-3}$. Applying this density to the PDM model, one can see that the density passes the critical density for reignition at the anode of $1 \times 10^{22}$ m$^{-3}$ after 88 µs. After about 1000 µs or 1 ms, the density has decayed to $8.6 \times 10^{19}$ m$^{-3}$, which is much lower than the critical density for reignition. This suggests that the overall particle density in the gap does not play a major role for restrikes. Thus, the causes for restrikes are probably related to something else.

4.3.5.2 Temperature & Atomic Mass Dependency

We can see that the density of chromium decays slightly faster than copper, indicating that a lower atomic mass results in a faster rate of decay, see figures 4.18, 4.19, 4.20, 4.21. This is to be expected since the two types of atoms possess the same amount of average kinetic energy, which means that the one with less mass has greater velocity, resulting in a faster rate of decay. Hence, the contact material, whether it is Cu or CuCr alloy, does not seem to have a significant impact on the rate of decay and may not be responsible for eventual restrikes. However, their melting point are different. Cu has a melting point of 1358 K, while CuCr has a higher melting point, such that there is less metal vapor in the gap from molten contacts when using CuCr than that of Cu, given the same current [4, p. 279]. The PDM does not take this into account.

Figures 4.28 and 4.29 shows how the density of Cu after arc extinction varies over time with different temperatures at $r = 0$, based on the PDM. According to the figures,
Figure 4.29: Log-scaled Cu density over time for different temperatures

varying the temperature by 300 K did make a noticeable change in the decay of the neutral vapor density. Starting at a density of $n_0 = 1 \times 10^{18} \text{ m}^{-3}$, for $T = 1700 \text{ K}$ at $t = 120 \mu\text{s}$, the density is $n = 1.339 \times 10^{17} \text{ m}^{-3}$, while for $T = 1700 \text{ K}$ at the same time, the density is $n = 1.008 \times 10^{17} \text{ m}^{-3}$. So, in this case, increasing the temperature by 35% resulted in a 25% decrease in density. However, if looking at the time-span between current-zero and $t = 20 \mu\text{s}$, there is virtually no difference in density. This suggests that temperature of the vapor does not play a significant role in the decay of the density of the neutral particles for the first few microseconds during the post-arc period, but may do so at later stages. This is not to say that temperature does not play a role in reignition, since the model does not show how the initial density is affected by temperature. However, assuming constant initial density, it seems like the likelihood for reignition is not significantly affected by a 600 K increase from 1700 K since the rate of decay of experience a minor change as a consequence. Figures 4.18, 4.19, 4.20, 4.21 shows how the rate of decay increases as one approaches the contact edge, indicating that the density is preserved for a longer period of time in the center.

The limitations of this result is that it does not take the cooling of vapor and the temperature of the contact surface into account. We know from section Temperature & Gap Length: The Effect on Metal vapor at the Anode, that the contact surface temperature and metal vapor density are correlated, but the PDM only takes the particles that are already in the gap into account, not how many particles that are produced from
The temperature of the contacts.

Overall, this model predicts that the density of particles in the gap is higher in the center of the contacts, which is aligned with current research. This tells us that the likelihood for reignition is greater at the center due to the increased likelihood for charge exchange and collision ionization, which leads to an increased density of ions. It also provides with rather good accuracy of the decay of the vapor density over time and shows that during the first few microseconds of the post-arc period, the density remains virtually unchanged at the center, which could explain the reignition phenomena. It also suggests that the cause for restrike may not significantly dependent on the average density in the gap since the density drastically decreases after 1 ms. However, the axial distribution is assumed to be uniform, which we know is not the case for a real vacuum interrupter; since the density may be higher at the electrodes [67], which may contribute to the restrike phenomena.

4.3.6 Post-Arc CTM Model

4.3.6.1 Ion Density

The post-arc current is formed from the ion current separating from the plasma into a space-charge sheath when the initial TRV is applied across the contacts. As shown in Fig. 4.30, a greater ion density leads to a greater magnitude of the post-arc current. Not
only does the post-arc current peak become larger, but it reaches it peak faster with high ion-density, and naturally, it takes a longer time for the current to reach zero, when all the plasma in the gap has been expelled.

### 4.3.6.2 Current Frequency

The commutation frequency, or the frequency of the injection current, has a significant impact on the post-arc current, see figure 4.31. As the frequency increases, so does the peak post-arc current value. This is because an increased frequency increases $dI/dt$, which means that the arc current reaches current-zero at a faster rate such that more neutral vapor density is left in the gap during the post-arc phase.

### 4.3.6.3 Arc Current

Figure 4.32 shows how different arc currents yield different post-arc current. Just like the previous figures, an increase in arc current leads to a greater post-arc current since arc current is proportional to neutral vapor density, equation 4.3.

### 4.3.6.4 Gap Length

A longer contact gap length during interruption yields greater plasma density and a larger post-arc current [88]. This can be seen when using the Post-arc CTM model and
Figure 4.32: Post-arc current vs times with varying arc current

Figure 4.33: Post-arc current vs time with varying gap lengths
Figure 4.34: Post-arc current peak vs time with varying gap lengths

varying the gap length, see figure 4.34. This suggests that a larger gap length could lead
to a greater likelihood for failed interruption. However, a larger gap seems to also lead
to a faster rate of current rise towards current-zero, see figure 4.34. This could be caused
by the increased rate of decay of the neutral metal vapor for a when the gap gets larger.
From section Breakdown Voltage and Gap Length, we have evidence that a larger gap
tends to lead to a greater breakdown voltage, so it stands to reason that the increased
ion density from the greater gap length is not sufficient for initiating a breakdown since
it is counteracted by the higher rate of metal vapor decay and improved electric field
strength, according to equation 2.7, of the space-charge sheath that is causing the post-
arc current.

4.3.6.5 Transient Recovery Voltage

The first section of the TRV in Fig. 4.35 will be considered as the Initial Transient
 Interruption Voltage (ITIV), and the second part is the Transient Interruption Voltage
(TIV). The greatest magnitude of the post-arc current and the ITIV will be referred to as
the peak of the post-arc current and the peak of the ITIV, respectively. Even though the
post-arc current is increasing towards zero, this will be referred to as a decrease in post-
arc current. Likewise, when the peak of the ITIV becomes larger, i.e. more negative, it
is numerically decreasing, but we will address it as an increase of the ITIV peak voltage.
The second section part with the TIV will be addressed as normal.
To gain some insight to the effect the TRV has on the post-arc current, equation 3.18 was compared to the post-arc current values. Each parameter of the TRV equation was varied one at a time, and the subsequent peak post-arc current, peak ITIV, maximum TIV, \(dV/dt\) of ITIV and \(dV/dt\) of TIV were set on the tables 4.7, 4.8, 4.9, 4.10, within the time span \(0 < t < 10\mu s\). \(A_0\) controls the voltage magnitude during for the entire TRV but more for the ITIV than the TIV, while \(C\) mainly affects the TIV. \(\tau_0\) primarily affects the rate of change of voltage for the ITIV, while \(\tau_1\) has more of an influence on the rate of voltage change in TIV. Table 4.11 shows the correlation of the post-arc current with the TRV values for each table.

Table 4.7: Post-arc properties with time-stamps using the TRV waveform from 3.18.

<table>
<thead>
<tr>
<th>(A_0)</th>
<th>[Ipa-peak]</th>
<th>ITIV-peak</th>
<th>TIV-max</th>
<th>dvdt ITIV</th>
<th>dvdt-max TIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V)</td>
<td>(A, \mu s)</td>
<td>(V, \mu s)</td>
<td>(V, \mu s)</td>
<td>(V/\mu s)</td>
<td>(V/\mu s)</td>
</tr>
<tr>
<td>-25e3</td>
<td>4.96, 0.018</td>
<td>9.05e+03, 1.34</td>
<td>2.24e+04, 10</td>
<td>1.73e+01, 0.008</td>
<td>4.57, 3.9</td>
</tr>
<tr>
<td>-45e3</td>
<td>7.69, 0.008</td>
<td>2.19e+04, 1.73</td>
<td>1.51e+04, 10</td>
<td>3.52e+01, 0.008</td>
<td>5.57, 4.29</td>
</tr>
<tr>
<td>-70e3</td>
<td>13.3, 0.008</td>
<td>3.88e+04, 1.96</td>
<td>5.86e+03, 10</td>
<td>5.75e+01, 0.008</td>
<td>6.88, 4.52</td>
</tr>
<tr>
<td>-95e3</td>
<td>18.9, 0.008</td>
<td>5.59e+04, 2.09</td>
<td>-3.33e+03, 10</td>
<td>7.98e+01, 0.008</td>
<td>8.20, 4.65</td>
</tr>
<tr>
<td>-120e3</td>
<td>24.4, 0.008</td>
<td>7.31e+04, 2.17</td>
<td>-1.25e+03, 10</td>
<td>1.02e+01, 0.008</td>
<td>9.53, 4.73</td>
</tr>
</tbody>
</table>

From table 4.11, it is clear that the peak post-arc current has a strong correlation with the
Table 4.8: Post-arc properties with time-stamps using the TRV waveform from 3.18.
A0: -95e3, τ₀: 1e-6, τ₁: 10e-6

<table>
<thead>
<tr>
<th>C V</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>35e3</td>
<td>23.3, 0.008</td>
<td>5.88e+04, 2.21</td>
<td>-1.28e+04, 10</td>
<td>8.13e+01, 0.008</td>
<td>7.26, 4.77</td>
</tr>
<tr>
<td>45e3</td>
<td>20.1, 0.008</td>
<td>5.69e+04, 2.13</td>
<td>-6.5e+03, 10</td>
<td>8.03e+01, 0.008</td>
<td>7.89, 4.69</td>
</tr>
<tr>
<td>55e3</td>
<td>17.8, 0.008</td>
<td>5.50e+04, 2.05</td>
<td>-1.78e+02, 10</td>
<td>7.93e+01, 0.008</td>
<td>8.51, 4.61</td>
</tr>
<tr>
<td>65e3</td>
<td>15.9, 0.008</td>
<td>5.31e+04, 1.98</td>
<td>6.14e+03, 10</td>
<td>7.83e+01, 0.008</td>
<td>9.15, 4.54</td>
</tr>
<tr>
<td>75e3</td>
<td>14.4, 0.008</td>
<td>5.14e+04, 1.91</td>
<td>1.25e+04, 10</td>
<td>7.73e+01, 0.008</td>
<td>9.78, 4.47</td>
</tr>
</tbody>
</table>

Table 4.9: Post-arc properties with time-stamps using the TRV waveform from 3.18.
A0: -95e3, C: 50e3, τ₁: 10e-6

<table>
<thead>
<tr>
<th>τ₀ µs</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>59.3, 0.008</td>
<td>6.4e+04, 1.68</td>
<td>-3.34e+03, 10</td>
<td>1.2e+02, 0.08</td>
<td>9.33, 3.69</td>
</tr>
<tr>
<td>1.4</td>
<td>10.1, 0.011</td>
<td>4.7e+04, 2.51</td>
<td>-3.3e+03, 10</td>
<td>5.3e+01, 0.008</td>
<td>7.04, 5.71</td>
</tr>
<tr>
<td>2.1</td>
<td>6.75, 0.018</td>
<td>3.5e+04, 3.02</td>
<td>-2.53e+03, 10</td>
<td>3.06e+01, 0.008</td>
<td>5.59, 7.17</td>
</tr>
<tr>
<td>2.8</td>
<td>4.83, 0.023</td>
<td>2.5e+04, 3.31</td>
<td>-0.67e+03, 10</td>
<td>1.93e+01, 0.008</td>
<td>4.57, 8.26</td>
</tr>
<tr>
<td>3.5</td>
<td>3.56, 0.031</td>
<td>1.72e+04, 3.38</td>
<td>2.11e+03, 10</td>
<td>1.26e+01, 0.008</td>
<td>3.82, 9.03</td>
</tr>
</tbody>
</table>

Table 4.10: Post-arc properties with time-stamps using the TRV waveform from 3.18.
A0: -95e3, C: 50e3, τ₀: 1e-6

<table>
<thead>
<tr>
<th>τ₁ µs</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>11.8, 0.011</td>
<td>4.64e+04, 1.78</td>
<td>1.53e+04, 10</td>
<td>7.36e+01, 0.008</td>
<td>9.96, 4.047</td>
</tr>
<tr>
<td>9</td>
<td>15.8, 0.008</td>
<td>5.33e+04, 2.00</td>
<td>2.27e+03, 10</td>
<td>7.81e+01, 0.008</td>
<td>8.72, 4.47</td>
</tr>
<tr>
<td>11</td>
<td>22.66, 0.008</td>
<td>5.82e+04, 2.17</td>
<td>-8.4e+03, 10</td>
<td>8.11e+01, 0.008</td>
<td>7.74, 4.81</td>
</tr>
<tr>
<td>13</td>
<td>34.3, 0.008</td>
<td>6.20e+04, 2.32</td>
<td>-1.72e+04, 10</td>
<td>8.31e+01, 0.008</td>
<td>6.96, 5.10</td>
</tr>
<tr>
<td>15</td>
<td>71.3, 0.008</td>
<td>6.50e+04, 2.45</td>
<td>-2.44e+04, 10</td>
<td>8.46e+01, 0.008</td>
<td>6.32, 5.35</td>
</tr>
</tbody>
</table>

Table 4.11: Correlation of post-arc current with TRV properties for changing A0, C, τ₁, and τ₀.

| CORREL(|Ipa|, ...) → | ITIV | TIV-max | dvdt ITIV | dvdt-max TIV |
|--------------------|------|--------|----------|------------|
| A0                 | 0.99823223 | -0.935629634 | 0.948080787 | 0.998192742 |
| C                  | 0.991414468 | -0.557994742 | 0.989226233 | -0.988859404 |
| τ₀                 | 0.854828621 | -0.514902972 | 0.967265521 | 0.88624987 |
| τ₁                 | 0.833502263 | -0.855639393 | 0.805340764 | -0.848918735 |
earlier parts of the TRV, i.e., ITIV and $dV/dt$ of the ITIV. Variations in $\tau_0$ and $\tau_1$ seems to significantly reduce this correlation, especially with ITIV. As for the correlation with the $dV/dt$ of the ITIV, $\tau_1$ has a much weaker correlation than $\tau_0$. This is probably because $\tau_1$ primarily affects the rate of rising voltage for the TIV. Overall, it seems like the change in voltage amplitudes, $A_0$ and $C$ have a stronger affect on both the peak post-arc current and the TRV values than the time constants. The maximum TIV has a very poor correlation with peak post-arc current, however, this is probably due to the time span only being limited to 10 µs, so it just shows that value of the TIV at $t = 10$ µs, when the post-arc current is gone. The maximum $dV/dt$ at TIV has similar correlations as ITIV, but when adjusting the parameters $C$ and $\tau_1$, which have a greater effect on the TIV, the correlations are negative. This indicates that changing $C$ and $\tau_1$ give the post-arc current and the maximum $dV/dt$ at TIV opposite changes, i.e., as the post-arc current increases, the maximum $dV/dt$ at TIV decreases.

Ion density, frequency and arc current amplitude seem to have the greatest effect on the post-arc current when using this model. Gap length causes marginal increases in post-arc current when varying the gap length. It also seems like, from the tables, that the post-arc current is strongly affected by the amplitude of the TRV, more so than the rate of voltage change, except for the earlier part of the TRV. However, post-arc current may not be a strong indicator of potential breakdown risk, since a greater gap length led to a higher post-arc current, even though longer gap lengths tend to suggest a lower risk of breakdown, which is inline with findings by Tang et al. [68]. Therefore, it may not be appropriate to draw conclusions of about the TRV’s effect on the vacuum interrupter’s interruption ability from the correlation data with the post-arc current and the TRV waveform-equation 3.18.

## 4.3.7 Model Validity & Assessment: Post-Arc CTM Model

This post-arc current model has been verified to fit experimental data by Glinkowski and Stoving [50] and has been used in multiple research papers to model post-arc current [50, 66, 89]. Hence, it is assumed to be a feasible model for this thesis as well. However, it is numerically unstable due to the non-linearity of the equations from Andrews and Varey [41], equations 2.22, 2.23 and equation 3.15 [50], and it does not predict prebreakdown current which may be the initiator for Townsend breakdown [67], see Electric Field Strength, Vapor Density & Temperature.
Chapter 5

Discussion

The Characteristic model by Yanabu et al. [8] highlights the strong effect that the \( dI / dt \) has on the neutral vapor density at current-zero. Fig. 4.14 shows that increasing the \( dI / dt \) yields a linear increase in particle density at current-zero. This effect has diminishing returns at around 400 A \( \mu \)s\(^{-1}\), for a time constant of 20 \( \mu \)s, \( \beta \) of 2.348 4 \( \times \) 10\(^{20}\) A\(^{-1}\) m\(^{-3}\) s\(^{-1}\) and initial current of 20 kA. One study by Tang et al. [68], showed show that arcing memory limits the effect that the arc current has on the post-arc current. In their case, the arcing memory was 4.5 \( \mu \)s. Qin et al. [63] and [66] shows that the primary parameter for determining the likelihood of reignition is the \( dI / dt \). This is supported by showing that the frequency of the injection-current, which affects \( dI / dt \), has a great influence on the post-arc current and breakdown voltage. As the \( dI / dt \) increased, so did the breakdown voltage and peak post-arc current. Given the values of \( dI / dt \) in the experiments, as well as the neutral vapor densities achieved from the model, a \( dI / dt \) in the range of 10 A \( \mu \)s\(^{-1}\) to up to 800 A \( \mu \)s\(^{-1}\) should be an appropriate starting point when testing vacuum interrupters. 10 A \( \mu \)s\(^{-1}\) for currents in the low hundreds, and 800 A \( \mu \)s\(^{-1}\) for higher currents in the thousands, and because any \( dI / dt \) higher than this would probably leave more than about 50% of the initial density in the gap by the time the current reaches 0 A.

The model also predicts that a greater initial vapor density in the gap has a strong positive correlation with arcing time, but has a relatively weak effect on the final density. This means that the initial density and arcing time is highly correlated with the current density of the arc, since vapor density is proportional to current density, but the final value of the vapor density is more affected by \( dI / dt \). The study by Qin et al. [63] shows that an increase in arcing time leads to a decrease in breakdown voltage, as seen in Fig. 4.3, which could be explained by the decreased vapor density from a longer time of
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arcing. According to the studies, an arcing time between 1 ms and 4 ms seems to be an appropriate range to test. The model itself does not consider the full arcing time from the time of opening the contacts, but from the moment the injection-current is applied. Furthermore, according to the model, the rate of decay of the vapor density does not appear to change the arcing time, see Fig. 4.17, but it does have a significant impact on the final value of the particle density since decreasing its value decreases the current-zero vapor density. The rate of particle decay increases from increased temperature, lower atomic mass, and perhaps also from a greater gap length.

However, the Characteristic model does not take gap length into account. Typically, an increase in gap length tends to show an increase in dielectric strength, see Fig. 2.14. This could be due to a greater rate of metal vapor decay from the larger volume, lower anodic temperature due to the increase spread of the current and weaker electric field between the electrodes. Based in Fig. 4.10, there is a clear increase in the number of restrikes from a smaller gap length, but this correlation is not obvious for reignition. However, the studies by Qin et al. [63] and Gu et al. [64] have found the presence of a critical gap distance. Qin et al. [63] found this critical length to be between 4 mm and 6 mm for their circuit. Going above or below this distance decreases the breakdown voltage. It is believed that this is caused by the weakened magnetic field density from the longer distance, which allows the Lorentz forces of the charges to take over, thus causing an early transition from a diffuse arc to a constricted one. This leads to a higher current density, resulting in a greater vapor density in the gap. Thus, equation 4.1 has been suggested as a way to improve the models accuracy. A constricted arc usually leads to an anode spot, which is a region of high current density and high temperature. This would increase the probability of reignition. Hence, the gap length has a significant effect for restrikes, but its influence on reignition is unclear unless it exceeds the critical gap length.

A study by Ding et al. [65], using a vacuum interrupter rated at 40.5 kV and diameter of 65 mm, found that breakdown at smaller gap lengths, 1 mm and 2 mm, were dominated by field-emission currents, while the larger one, 4 mm, were caused by macroparticles traveling from the anode to the cathode. This causes local spikes in electric field strength, which results in field emission, leading to Townsend breakdown when the electrons collide with the neutral vapor. This suggests that a larger gap distance decreases breakdown from field-emission, but increases breakdown from macroparticles, which could be an additional reason for the existence of a critical gap length. However, no such critical gap length was discovered, and a fit of $U_B = 70.12 \times d^{0.56}$ was made. It is worth noting that the study did not exceed 4 mm, so a critical gap length may have been found if that length was exceeded, as seen from the previous study by Qin et al. [63]. The study by Ding et al. [65] also shows the importance of
surface conditioning, as seen in Fig. 4.7, with standard lightning impulse voltage. The figure shows how the breakdown voltage increases with an increase in the number of breakdowns, highlighting the significance of the condition of the electrode surfaces on breakdown voltage. When testing a vacuum interrupter, it is recommended to test from 1 mm to 10 mm, and consider the critical gap length, especially if the vacuum interrupter has an axial magnetic field.

Along with a critical gap length, Gu et al. [64] also found a range for the commutation frequency where there was safe interruption, see Fig. 4.5. This suggests that a \( \frac{dI}{dt} \) that is too low may also cause failed interruption, most likely due to reignition. The experiments were done for a 50 kV breaking unit for a HVDC circuit breaker. The Characteristic model does not suggest that such a range of \( \frac{dI}{dt} \) exists, since a shorter \( \frac{dI}{dt} \) leads to a decrease in neutral vapor density. This indicates that other circuit breaker parameters may play a role for reignition to occur at low commutation frequencies.

A high temperature, above 2000 K for copper, may set the stage for reignition, and later restrikes, since it increases thermionic emission. If the metal vapor density exceed \( 1 \times 10^{22} \text{ m}^{-3} \), there seems to be an exponential rise in prebreakdown current stemming from the post-arc current. A high temperature with a vapor density less than \( 6 \times 10^{21} \text{ m}^{-3} \) could lead to the formation of liquid instabilities on the surface of the contacts, since copper has a melting point of 1356 K. This dramatically increases the chances of reignition and restrikes since it could lead to further microprotusions, thus magnifying the electric field in that area. Studies have shown that the critical macroscopic electric field strength for breakdown with copper is around \( 5 \times 10^9 \text{ V m}^{-1} \) and \( 1.6 \times 10^8 \text{ V m}^{-1} \), which is much higher than the macroscopic electric field strength in the gap when electric breakdown was observed with the formation of liquid instabilities, by Dullni [67]. Hence, the presence of liquid has a profound effect on the vacuum interrupter’s ability to successfully interrupt a current.

Zhou et al. [62] found that an increased gap length led to a decrease in the rate of rise in temperature, along with an increased delay for the metal vapor pressure to rise, at the anode. This probably happens because the increased gap length allow for a broader spread of electrons from the cathode due to the longer distance the electrons have to travel, thus reducing the heat per unit area on the anode, thus increasing the risk of liquid metal. This further supports the idea that a shorter gap length leads to an increased risk of breakdown, but from the temperature increase and not from the higher electric field strength. This also supports the claim that a smaller gap length leads to greater field emissions, since the increased temperature and metal vapor pressure would enhance TF emission and the likelihood for a Townsend breakdown. It is thus recommended to measure the temperature during vacuum interrupter testing.
It should be clear at this point that the neutral vapor density plays a crucial role for the breakdown phenomena. When comparing the PDM with the vapor decay model from 'Analysis of Neutral vapor decay', we see that the results from Lins [87] fits in between these two models. The vapor decay model significantly underestimates the density, while the PDM does a better job of modeling the decay of the neutral vapor density in the gap, but somewhat overestimates it due to its boundary conditions. Thus, the decay model provides a lower bound to the particle density, while the PDM gives an upper bound. The PDM shows that the neutral vapor has a higher density closer to the center of the contact gap and that the density remains very high for the first few microseconds after current-zero. This could indicate a greater likelihood for reignition to occur at the center of the contact gap. It also shows a significant decrease of vapor density after 1 ms, which suggests that the initial vapor density may not play a major role in the occurrence of restrikes since the density would be lower than the typical critical density of $1 \times 10^{22} \text{m}^{-3}$. The model could perhaps be improved if it took the temperature of the contact surface and the axial distribution of particles into account, since those factors would affect the production and decay of the metal vapor, as well as provide a more accurate picture of the density distribution in the contact gap.

The post-arc current was modeled with the CTM model. It was found, by the model and studied by Tang et al. [68], that the post-arc current may not be a reliable indicator of reignition, since it shows an increase in current when gap length increases, which typically decreases the breakdown voltage. Tang et al. [68] discovered that the post-arc current was independent of the peak arcing current. This is probably due to arcing memory, which limits the amount of information that affects the current-zero state. The model also showed that the post-arc current increases with increasing $dI/dt$ and arc current. However, due to arcing memory, the influence from the arc current may be limited, while enhancing the importance of the $dI/dt$ on the ion density and post-arc current. Although the significance of the post-arc current on reignition may be uncertain, it is clear that it has a strong correlation with the transient recovery voltage, which does have a causal effect on reignition. This is because the TRV is what is causing the space-charge sheath to grow, resulting in the post-arc current and changes in the electric field strength of the sheath. If the rate of TRV growth is large enough, reignition is more likely to occur, since the electric field across the space-charge sheath would rapidly increase as well. This suggests that both the $dI/dt$ and the TRV play a major role for reignition. Furthermore, a study by Qin et al. [66] shows that there may be a threshold voltage for the TRV, i.e. the likelihood for reignition depends on not just $dV/dt$, but also a certain critical TRV voltage, $TRV_c = 2800 \text{V}$. A study by Xiao et al. [90], achieved a safe area for successful interruption at recovery voltages up to about 35 kV for recovery times over 45 µs, with $dI/dt$ of 333 A µs$^{-1}$ and 666 A µs$^{-1}$. 
One should also consider implementing a follow current loop in order to establish a voltage-zero duration, as demonstrated by Cheng et al. [69]. This will reduce the likelihood of reignition and restrikes since delaying the time to apply the TRV will allow more neutral vapor to diffuse from the gap, effectively decreasing the particle density. After the neutral vapor has significantly decayed, typically several milliseconds after current-zero, the vacuum interrupter must still be able to withstand the voltage across the gap. During this time scale, the likelihood for breakdown clearly increases from a smaller gap length, most likely from the increased macroscopic electric field strength. However, due to the lower density of neutral vapor, the likelihood of breakdown should be significantly lower. This implies that restrikes are highly influenced by the surface conditions of the contacts. Also, if the temperature is still high, the likelihood for TF emission increases. This suggests that restrikes are primarily affected by the gap length, surface conditions and temperature.
Given the research papers and models presented in this thesis, the following parameters should be varied when testing vacuum interrupters: \( dI/dt \) before current-zero, in the range of \( 10 \text{ A} \mu\text{s}^{-1} \) to \( 800 \text{ A} \mu\text{s}^{-1} \), arcing time before current-injection from \( 1 \text{ ms} \) to \( 4 \text{ ms} \); \( dV/dt \) after current-zero, from \( 0.5 \text{ kV} \mu\text{s}^{-1} \) to \( 20 \text{ kV} \mu\text{s}^{-1} \); max TRV from \( 5 \text{ kV} \) to \( 25 \text{ kV} \), to find the threshold voltage for failed interruption; and gap length, from \( 1 \text{ mm} \) to \( 10 \text{ mm} \), to find the critical gap length. Testing post-arc current seems to be of less importance. Despite its potential limiting effects due to the arcing memory, arcing current is still important to test since it affects the temperature and vapor density in the gap. A higher arcing current also results in a greater number of cathode spots, which affects the surface condition of the contact surface. Thus, it should be tested in the range from \( 1 \text{ kA} \) to \( 20 \text{ kA} \). At the same time, temperature should also be measured. To minimize damage to the interrupter, it is recommended to start with higher gap lengths and longer arcing times after current-injection, but short arcing times before that, with low values on everything else.

The most important contributors for reignition are \( dI/dt \) before current-zero, \( dV/dt \) after current-zero, and contact gap length. The \( dI/dt \) determines how much metal vapor density is left in the gap after arc extinction. The higher the \( dI/dt \), the shorter the time it takes for the arc current to reach current-zero, and the less time there is for the metal vapor to diffuse from the gap, resulting in a higher vapor density during the post-arc period. The \( dV/dt \) after current-zero affects the electric field strength across the space-charge sheath during the post-arc period. The higher the \( dV/dt \) is, the stronger the electric field strength becomes, which results in enhanced TF emissions. The contact gap length affects the vapor density, temperature and electric field strength. A shorter contact gap tends to result in a higher anode temperature and vapor pressure,
while also increasing the macroscopic electric field strength in the gap. A higher vapor density, electric field strength, and temperature increase the probability for Townsend breakdown, which probably is the mechanism behind reignition. The effect arcing current has on reignition is not clear, since post-arc current seems to be independent of it due to arcing memory. There may also be a threshold value for the TRV which, if exceeded, will lead to a failed interruption. Qin et al. [66] found their threshold value to be 2 800 V.

According to the PDM, reignition would be more likely to occur at the center of the contact gap due to a higher vapor density in that region, while restrikes would probably occur at surface protrusions. Furthermore, there seems to be a critical gap length where any further increase leads to a decrease in breakdown voltage. This is probably due to a weakened axial magnetic field which allows the electrons to be attracted to each other due to their Lorentz forces. This increases the current density, resulting in a transition from a diffuse arc to a constricted one. A constricted arc tends to lead to an anode spot. Anode spots and liquid metal significantly increases the chances of electric breakdown.

As for restrikes, due to its occurrence during low vapor density, it is probably highly dependent on the surface conditions, temperature and gap length. The surface condition refers to microprotrusions on the contact surfaces. These microprotrusions enhance the microscopic electric field strength, resulting in TF emissions. The microprotrusions may occur more often when there is molten liquid metal on the contact surface, following high surface temperature. Experimental evidence also shows a clear increase in restrikes from a shorter gap length, see Fig. 4.10, which increases both temperature, see Fig. 4.1 and electric field strength, see equation 2.7.

Several models could be improved, such as the Characteristic model having a parameter for the contact gap length, or the PDM including contact temperature and axial distribution of the metal vapor. More research needs to be done on the critical gap length, the threshold TRV, and on the surface conditions of the contacts to advance our understanding of restrikes and reignitions. It would also be interesting to analyze the relationship between dielectric strength and arcing energy, as done by Xiao et al. [90].
References


