Degree Project in Electrical Engineering

Second cycle, 30 credits

A Rolling Horizon Approach to Stochastic Hydropower Modeling and Equivalent Calculation

FABIAN PERSSON DJURHED

A Rolling Horizon Approach to Stochastic Hydropower Modeling and Equivalent Calculation

FABIAN PERSSON DJURHED

Master’s Programme, Electric Power Engineering, 120 credits
Date: October 31, 2024

Supervisor: Uli Max Rahmlow
Examiner: Mikael Amelin
School of Electrical Engineering and Computer Science
Swedish title: En rullande horisont-ansats för stokastisk modellering av vattenkraft och ekvivalentberäkning

Abstract

Mathematical models of hydropower stations can become exceedingly complex and hard to compute when modelling larger river systems. Consequently, methods to simplify the complexity have been developed, such as hydropower equivalents, which can reduce a system of stations into one equivalent station. However, these equivalents are calculated based on data from detailed models which have been made under the assumption that all information, such as the natural inflow of water and electricity prices, is certain. In reality, this is not the case. In order to include the uncertainty of the data, a new stochastic model was developed in this thesis. This model was based on the rolling horizon algorithm, which was combined with multiple possible future scenarios of inflows and electricity prices to represent the uncertainty. Two stochastic models were constructed: one aggregated model with a weekly future horizon, as well as a more detailed model with a daily future horizon. The results indicated that, in general, the reservoir levels were higher in the stochastic models than in the deterministic model. This was also reflected in the calculated hydropower equivalent parameters, where the minimum reservoir level was increased. Furthermore, the aggregated model showed a more realistic production pattern than both the deterministic and detailed stochastic model as it was following the electricity price variations more closely.

Keywords

Hydropower, Inflow, Rolling horizon, Stochastic optimization, Scenarios

Sammanfattning

Matematiska modeller av vattenkraftverk kan bli mycket komplexa och svåra att beräkna när man modellerar större flodsystem. Därför har metoder för att förenkla komplexiteten utvecklats, såsom vattenkraftekvivalenter, som kan reducera ett system av stationer till en ekvivalent station. Dessa ekvivalenter beräknas dock baserat på data från detaljerade modeller som har gjorts under antagandet att all information, såsom den naturliga tillrinningen av vatten och elpriser, är säker. I verkligheten är detta inte fallet. För att inkludera osäkerheten i data utvecklades en ny stokastisk modell i denna rapport. Denna modell var baserad på rullande horisont-algoritmen, som kombinerades med flera möjliga framtidsscenarier för tillrinningar och elpriser för att representera osäkerheten. Två stokastiska modeller konstruerades: en aggregerad modell med en veckovis framtidshorisont, samt en mer detaljerad modell med en daglig framtidshorisont. Resultaten visade att, generellt sett, var magasinnivåerna högre i de stokastiska modellerna än i den deterministiska modellen. Detta återspeglades även i de beräknade parametrarna för vattenkraftekvivalenten, där den lägsta magasinnivån ökades. Dessutom visade den aggregerade modellen ett mer realistiskt produktionsmönster än både den deterministiska och den detaljerade stokastiska modellen eftersom den följde elprisvariationerna närmre.

Nyckelord

Vattenkraft, Inflöden, Rullande horisont, Stokastisk optimering, Scenarion

Acknowledgments

I would like to express my gratitude to my supervisor Uli, for allowing me to have such an interesting master's thesis. I would also like to thank him for all the guidance and knowledge on the subject, as well as his dedication to answer all the questions I've had whenever it was needed. I would also like to thank my parents Mia and Johnny, and my partner Melinda, for their unwavering support and belief in me throughout this thesis.

Stockholm, October 2024
Fabian Persson Djurhed

Contents

List of Figures

List of Tables

Nomenclature

In this segment the nomenclature for the thesis is described.

Sets

Ω	Scenarios from 1 to 6, index ω
I_{down}	Set of all down-streams reservoirs
I_{up}	Set of all directly up-streams reservoirs
J	Production segments from 1 to 2, index j
R	Weeks in the rolling horizon algorithm from 1 to 52, index r
T	Hours in a week from 1 to 168, index t
V	Weeks in horizon from v to 52, index v

Variables

$\mathcal{M}_{r,v}^{\omega}$	Energy in reservoir rolling week r,week v, scenario ω in MWh
$\mathcal{P}_{r,v}^{\omega}$	Energy production in rolling week r, week v, scenario ω in MWh
$\mathcal{S}_{r,v}^{\omega}$	Energy spillage rolling week r, week v, scenario ω in MWh
$\mathcal{V}_{r,v}^{\omega}$	Energy inflow in rolling week r, week v, scenario ω in MWh
$M_{r,i,t}$	Reservoir level rolling week r, reservoir i, hour t in m^3
$P_{r,t}$	Total electricity production rolling week r, hour t in MWh/h
$Q_{i,j,t}^{E}$	Discharge in hydropower equivalent, reservoir i, segment j, time t
$Q_{r,i,j,t}$	Discharge rolling week r, reservoir i, segment j, hour t in m^3/s
$S_{r,i,t}$	Spillage rolling week r, reservoir i, hour t in m^3/s

$V_{r,i,t}$ Water inflow rolling week r, reservoir i, hour t in m^3/s

Parameters

$\lambda_{r,t}$ Electricity price rolling week r, hour t

$\lambda^{\omega}_{r,v}$ Average electricity price rolling week r, week v, scenario ω

$\mu^{E}_{i,j}$ Marginal production equivalent in hydropower equivalent, reservoir i, segment j

$\mu_{i,j}$ Marginal production equivalent reservoir i, segment j

$\overline{\mathcal{M}}$ Maximum energy in reservoir

$\overline{\mathcal{P}}$ Maximum energy production from reservoir

$\overline{\mathcal{S}}$ Maximum energy spillage from reservoir

$\overline{M}_i$ Maximum content in reservoir i

$\overline{Q}_{i,j}$ Maximum discharge from reservoir i, segment j

$\overline{S}_i$ Maximum spillage from reservoir i

π^{ω} Probability of scenario ω

τ_i Water delay time from reservoir i to down-streams reservoir in hours

$M_{0,i,0}$ Starting reservoir level rolling week 0, reservoir i, week 0

Chapter 1

Introduction

1.1 Background

In Sweden, there are approximately 1800 hydropower plants [1], which in 2023 produced nearly 40% of the country's total electricity production [2]. With goals to increase the renewable energy sources in the system as described in [3], hydropower is expected to continue its expansion and be the third largest electricity producer globally by 2050. Hydropower is a flexible source of electricity because of the ability to store energy in reservoirs, but also as kinetic energy in the inertia of turbines. Therefore hydropower can provide stability to a highly fluctuating system with higher shares of intermittent sources such as wind power and photovoltaics [4].

With rapid changes and increasing complexity in the electricity grid, the necessity to understand how hydropower works and behaves is an important factor to tackle the challenges of future development. One way to analyze electric grids and energy systems is by creating optimization problems of mathematical models where grid operation can be simulated and optimized. The emergence of energy models is associated with the oil crisis that happened in 1973, which highlighted the importance of planning for various future scenarios [5]. With energy models it is possible to create multiple forecasts and scenarios in order to plan for the future.

When creating accurate models of complex energy systems, the mathematical models can become quite complex as well and be very computationally heavy. Therefore it is useful to simplify models where it is possible while retaining accuracy. The complexity of hydropower modeling comes from the large amount of constraints needed to describe the system [6]. This can be for example water flow balance constraints that need to be applied for all the branches of the rivers. To mitigate this problem, ways of aggregating river systems over wide areas has been developed [7]. The aggregations are called hydro power area equivalents or just hydro power equivalents and are used to represent an area or a river of reservoirs and hydro power stations as one equivalent reservoir and station. With an aggregated model, large systems of rivers get a lot less computationally heavy which in turn make the models more usable and flexible.

The problem with some of the current hydropower equivalents are that the data they build upon is deterministic. This means that all information is perfect and forecasts are assumed to be accurate even for longer time spans up to multiple years. But this is not very realistic, the most commonly used electricity forecasts have a timespan of at most a couple of days into the future and have mean absolute errors ranging between 10-20% [8]. Hence it is not appropriate to assume that forecasts are accurate one year into the future. The uncertainty in forecasts is something that can be incorporated in mathematical models by using stochastic models and one way to represent the uncertainty is by creating multiple different scenarios of the uncertain variable. For stochastic hydro power modeling there are primarily two distinct variables which can be seen as uncertain which are electricity prices and natural inflow of water to the rivers. It is therefore of interest in this thesis to investigate how a stochastic model with uncertain prices and inflow of water would impact the calculation of hydropower equvalents.

1.2 Problem

Deterministic models are based on perfect information and perfect foresight which could in some cases be an appropriate way to handle short term problems. However, when dealing with long term problems such as hydropower optimization spanning a yearly timeframe, this approach becomes unrealistic. What this means for the deterministic hydropower models is that the most optimal solution is to deplete reservoirs before periods of high natural inflow or conserve water for longer periods, anticipating higher electricity prices in the future [9]. However, in reality, this operation scheduling is impractical since it does not take into account any of the uncertainty which exists in reality.

1.3 Purpose

With the electric grids becoming more and more interconnected the importance of large electricity models will also increase. In large electricity models hydropower equivalents are important since they have been proven to reduce the time to compute solutions to the problems drastically [3]. But if the equivalent models do not provide accurate results, it may not be worthwhile to use them. With a stochastic model, real uncertainties such as inflow and electricity prices could be reflected in the results and consequently be more accurate and realistic. With the ability to produce accurate equivalent hydropower models, cascade river system complexity would become less of an issue and the efficiency of large energy models would increase. The main goal of this thesis was therefore to create a stochastic model of a river system with uncertain inflows and electricity prices which could be used to calculate a new hydropower equivalent model. The main research questions to be answered in this thesis were:

- How could the issue with perfect foresight for this long term problem be handled?
- How can a stochastic model be developed to deal with the uncertainty of electricity prices and natural inflows?
- How does the stochastic model impact the calculation of the hydropower equivalent?

To be able to answer the questions, the resulting yearly production and reservoir level graphs will be compared with deterministic results to see how they differ. The parameters for the hydropower equivalent calculations for both deterministic and stochastic models will also be compared. The primary objective of this thesis is to address the research questions presented above. Furthermore, the results of this thesis should contribute with further knowledge of using stochastic models in hydropower optimization. Additionally, they should provide insight into the performance of a rolling horizon algorithm in this context.

1.4 Research methodology

To address the problems in this thesis, a stochastic mathematical model of the river system Luleälven was developed as an optimization problem.

The model was then implemented and solved in python, using the Gurobi optimizer alongside a rolling horizon algorithm. Python was chosen due to the availability of an existing codebase of the deterministic model described in [7]. The codebase could then be extended to incorporate a stochastic future information horizon with the rolling horizon algorithm. The results of the stochastic model were then used to create a hydropower equivalent which could be compared to the previous deterministic equivalent.

1.5 Delimitations

The thesis primarily focused on the uncertainty of natural inflow to reservoirs and electricity prices. Notably, the operation of the hydropower plants were solely based on electricity prices and did not take into account any electricity demand. Only one optimization method was investigated, the rolling horizon algorithm. There are other methods such as dynamic programming but a rolling horizon method was preferred due to new information being given to the model continuously much like in reality. Additionally, the model was scenario based which limited the number of possible states. To further limit the complexity of the model, no environmental constraints were taken into account. It was also made with the assumption of a perfect market with perfect competition. Furthermore this thesis was constrained to only simulating one river, Luleälven situated in Sweden in the electricity price area SE1 in Sweden due to its availability for historic data.

1.6 Report structure

Chapter 1 introduces the topic of hydropower modeling and equivalents as well as stating the problem in question. The primary research questions are also presented here. Chapter 2 provides background information of hydropower, optimization, the rolling horizon algorithm and also introduces the river used for the hydropower model. Chapter 3 describes the methodology used to create the stochastic model and what had to be done beforehand regarding data handling and preparation of parameters. Chapter 4 presents and discusses the acquired graphs and tables from the simulations of different versions of the stochastic model, a deterministic model and the equivalent model. The concluding chapter 5 revisits the research questions and provide answers to them.

Chapter 2

Theory

2.1 Hydropower

Hydropower is a highly efficient renewable energy source which can produce electricity from the potential and kinetic energy in the flow of water. Hydropower is also a very flexible energy source since it has the ability to store water in reservoirs which can then be used when needed. The electricity is produced when water flows through a turbine which in turn causes a generator to rotate as well which produces electricity, see Fig. 2.1. There are three main types of hydropower stations, impoundment, diversion and pumped hydro storage where the two first ones will be introduced in this thesis [10]. Impoundment is the most common type of hydropower station and uses reservoirs to store water, this makes it flexible and is usually found in larger hydro power stations. The release of water from the reservoirs can be controlled and can be in two different forms, discharge and spill. The term discharge is used to describe the release of water through the turbine, which generates electricity. In contrast, the term spill is used to describe the diversion of water past the turbines, which does not generate any electricity. The spillways of hydropower stations serve an essential function in ensuring the safety of reservoirs by preventing them from becoming overfilled [11]. In addition, there are cases where environmental constraints necessitate the utilization of non-zero minimum spillage levels in order to preserve biodiversity in and along the downstreams rivers [12]. The second type of hydropower station is the diversion station, also known as run-of-river station [10]. This type of station lacks any significant water reservoirs and is continuously generating electricity. In contrast to impoundment the diversion type diverts a portion of the water from the river through a turbine which then generates electricity.

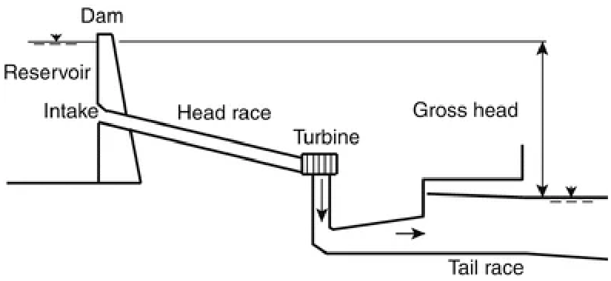


Figure 2.1: Layout of a hydropower station in [13]

2.2 Natural inflow of water

The natural inflow of water to reservoirs is depending on many different factors such as season, weather, climate, and topography [14]. In Sweden, the natural inflow is highly dependent on the season and the amount of snow and ice that has accumulated during the winter in upstream areas. As temperatures begin to rise in the spring, the snow and ice begin to melt, resulting in high natural inflows to nearby rivers and subsequently to the reservoirs in those rivers, as stated in [15]. This period of high inflow is referred to as the ”spring flood” or ”vårfloden” in Swedish and occurs annually around the transition from April to May. The magnitude of the spring flood also has a significant influence on the operational capacity of hydropower stations in affected rivers throughout the remainder of the year. During the spring flood, reservoirs are often filled and stored for the following winter, when electricity demand is at its highest. The magnitude of the spring flood thus determines to what extent hydropower stations can discharge throughout the remainder of the year without emptying their reservoirs in advance of the winter.

2.3 Luleälven

In Sweden close to 95% of the hydropower electricity production is made in less than 10% of the hydropower plants as stated in [16]. Luleälven is as mentioned in [17] and [18] one of the most water abundant and important rivers in Sweden where also the largest hydropower station in Sweden, Harsprånget is situated. Luleälven is 461 km long and stretches from Sulitelma by the border of Norway to Luleå and has 15 active hydropower stations along the river which can be seen in Fig. 2.2. As stated in [17] Luleälven has changed alot since the construction of the hydropower stations, but with preserved environmental value due to the constructed roads, making the mountains accessible for everyone.

The total hydropower production of the hydropower stations along the river is 4447 MW, which can be compared to the largest nuclear facility in Sweden, Forsmark, with an installed capacity of 3271 MW. In table 2.1 all the hydropower plants and their respective installed capacity and reservoir sizes can be seen.

Table 2.1: Hydropower station data in Luleälven

Station	**Capacity [MW]**	**Reservoir** [m^3]
Ritsem	320	177780
Vietas	320	1989000
Porjus	465	175500
Harsprånget	1001	1778
Ligga	367	1500
Messaure	442	14700
Seitevare	225	466000
Parki	20	131900
Randi	86	27200
Akkats	158	11500
Letsi	456	1900
Porsi	272	7500
Laxede	207	19165
Vittjärv	30	3610
Boden	78	220

2.4 Variable costs

In determining the optimal electricity source for production, the system operator considers the variable costs associated with each option. These costs include fuel, operation, and maintenance expenses, and are typically expressed in euros per megawatt-hour (EUR/MWh), as stated in [20]. Variable costs provide insight into the cost of producing a unit of electricity from a given source. For many different sources, this must to be taken into account, as some have relatively high variable costs. An example is electricity from biofuel or

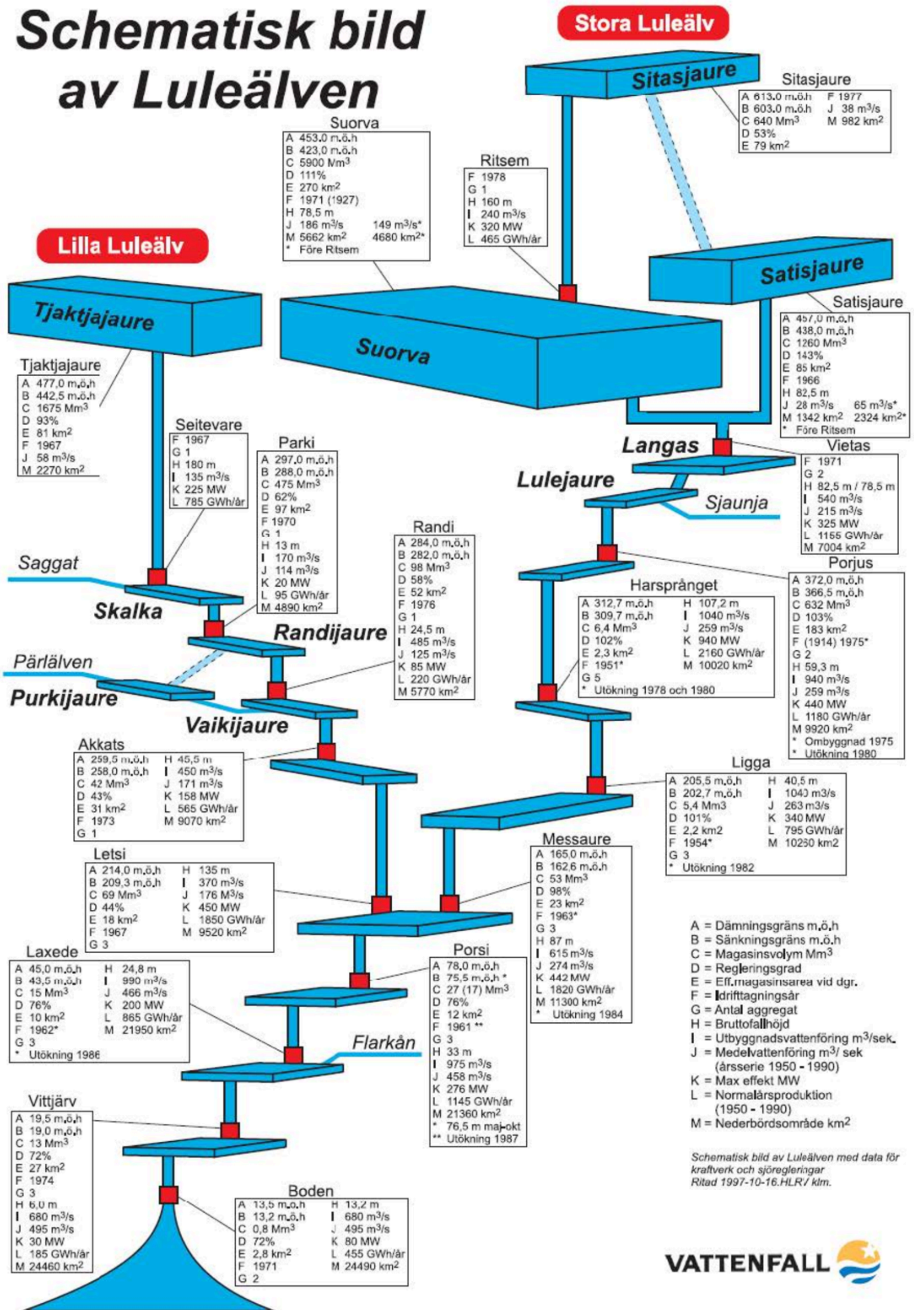


Figure 2.2: Overview of Luleälven from [19]

oil, which can have high fuel costs. Hydropower however has a comparatively low variable cost which is based on the opportunity cost, if wear and tear costs are neglected. This is the cost of saving water for the future, rather than utilizing it now as described in [21]. In this thesis, only the direct profit from selling electricity was analyzed and therefore the variable costs of hydropower were neglected.

2.5 Electricity production

The power output of a hydropower station can be calculated using Eq, 2.1 in which P is the power, Q is the discharge, η is the efficiency, ρ is the water density, g is the gravitational acceleration, and h the difference in height between the entrance and exit of the water which can be seen in Fig 2.1.

$$P = Q\eta\rho gh \tag{2.1}$$

The issue with this formulation however is that it is a non-linear equation due to the efficiency being dependent on the discharge making it difficult to use in a linear optimisation problem. To simplify the equation, it can be rewritten as a piecewise linear concave function, as also done in [22] which yields Eq. 2.2

$$P_{i,t} = \sum_{j \in J} \mu_{i,j} Q_{i,j,t} \tag{2.2}$$

where $\mu_{i,j}$ is a marginal production equivalent with segments J, which represents the slope of the linear function. In order for the equation to be concave the slope of the segments must be decreasing which can be stated as Eq. 2.3.

$$\mu_{i,j} > \mu_{i,k} \; if \; j < k \tag{2.3}$$

2.6 Optimization

The term " optimization" is defined as the process to find the best solution to a problem according to [23]. Usually the problem can be described by an objective function and the most optimal solution to that would either be the maximal or minimal value of that function, depending on what type of problem it is. But for optimization problems there needs to be constraints to the solution which are in the form of inequality and equality expressions that must to be satisfied when determining the optimal solution. For this reason an optimization problem is said to maximize the objective function, f, while subject to some constraints, g, which can be seen in a mathematical formulation in Eq. 2.4. As explained in [23] some examples of constraints can be to preserve energy balances or environmental restrictions to set maximum values for pollutants. In the context of hydropower there are some technical aspects such as maximum discharge capacity which has to be taken into account as constraints or balance in water flows throughout the river.

$$\begin{aligned} \text{maximize} \quad & f(x_1, x_2, \dots, x_n) \\ \text{subject to} \quad & g_i(x_1, x_2, \dots, x_n) \leq b_i, \ i = 1, \dots, m. \end{aligned} \tag{2.4}$$

The objective function is dependent on the optimization variables or decision variables which are $(x_1, x_2, ..., x_n)$ in Eq. 2.4. The optimization variables can be seen as the parameters that can be manipulated and adjusted in order to find the best solution to the problem. They are also called decision variables because they can be controlled through decisions. In hydropower optimization, one of the decision variables is the discharge, because the hydropower station operators can decide on how much or little to discharge.

2.7 Deterministic and stochastic optimization

As previously stated in section 1.1 in reality there are a multitude of uncertainties. One method to handle uncertainty in problems is by using stochastic optimization [24]. The uncertainty in a stochastic optimization problem can be generated as scenario trees that provide different possible future values with some probability. As an example, a scenario tree could be constructed to represent the uncertainty associated with tomorrow's weather. The scenario tree could comprise three distinct future possibilities, such as sunny, rainy, or cloudy, with each scenario assigned a probability. By combining the future scenarios and their respective probabilities, it is possible to calculate the expected value, which is the value that is optimized in stochastic optimization. In this manner a wide range of uncertainties can be taken into account when optimizing the model.

In contrast, deterministic optimization does not model uncertainty. It is not necessary for a deterministic model to employ scenario trees, as all parameters are known and certain. A stochastic model that is equivalent to this would only allow for a single possible future scenario. Consequently, the probability of this scenario would be 100%.

2.8 Rolling horizon algorithm

For larger timespans, real world inflow of data in the context of hydropower is usually updated continuously. All data about the future is not known from the start, for instance, the price of electricity are highly volatile and is only predictable over a relatively short time horizon. Therefore data has to be updated continuously when new data from prices, inflows or weather forecasts are made. In mathematical modeling there are many different ways to optimize a formulated problem, but many do not utilize this constant inflow of new data [25]. The rolling horizon algorithm however is an optimization algorithm that can update the data while optimizing the model. The rolling horizon method is based on repeatedly solving a smaller timespan with parameters and data that get updated for every iteration. The total simulated timespan is divided into sections, see Fig. 2.3. The algorithm solves the divided timespans and a future horizon one at a time, where the horizon in each new period feeds the model with new information. For each solved period in the algorithm, only the solution to that period is retained, and the solution to the future horizon is discarded.

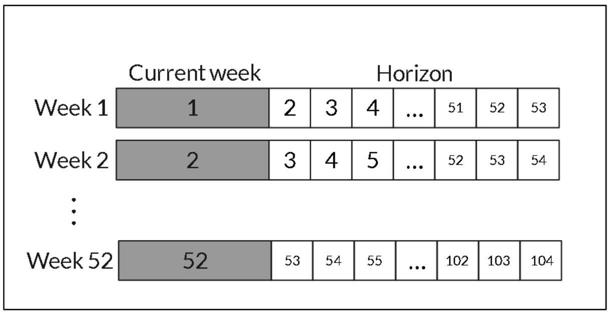


Figure 2.3: Rolling horizon algorithm

As an example, to optimize a full year with the rolling horizon algorithm the problem could be divided into solving the year one week at a time with a set horizon of some timespan into the future. Weather forecasts are about 80 % accurate for up to seven days into the future [26]. A weekly resolution in the rolling horizon algorithm could therefore be a suitable resolution. The algorithm optimizes the first week and the timespan of the horizon, but only saves the solution of the first week. For the next iteration the week to be optimized is the second rolling week together with the horizon, the algorithm would then continue in this manner until the last week had been optimized, see Fig. 2.3.

2.9 Hydropower aggregation

The complexity of large river systems renders them challenging to solve. It is therefore beneficial to reduce river systems by aggregating them. The aggregation in this context can be achieved through multiple different methods, but the main aspects of the aggregation is to represent a multi-station river system by a smaller river system that behaves in a similar manner. One of the most straightforward methods for creating an aggregated river system is just summing all the reservoir parameters such as production, inflows and reservoir levels [27]. However, when calculating hydropower equivalents, this approach is not precise enough. A more complex method is described in [7], where different parameters for the equivalent must be solved for to mimic the original system behavior.

2.10 Gurobi

In order to solve the optimization problem in this thesis the Gurobi optimizer was used. The Gurobi optimizer is a widely utilized optimizer in industry, suitable for a diverse range of businesses [28]. Additionally, the Python module gurobipy enables Gurobi to be utilized within the Python programming language.

2.11 Previous work

The work presented in [7] provided a deterministic base model that could be used for any river. The existing codebase used in this thesis was also based on this deterministic model. Furthermore, the method to calculate the hydropower equivalents for this thesis was also the same as presented in Blom's paper.

The paper in [29] presented a method to implement a rolling horizon algorithm on a river in Ethiopia with stochastic inflows. The paper described the implementation of the rolling horizon algorithm and the generation of scenarios in different ways. The problem presented was a single hydropower station with uncertain inflows and with certain electricity prices since they were set by the Ethiopian government. This paper inspired the utilization of the rolling horizon algorithm together with scenario based inflow and electricity prices in this thesis.

Chapter 3

Method

3.1 Code base

In this thesis, a Python code base that simulated the deterministic model created in [7] was used. The code base included both the deterministic model code and the code for calculating hydropower equivalents and was used as a starting point when implementing the stochastic rolling horizon model.

3.2 Deterministic model

The deterministic model made in [7] was in this thesis used as a base case in order to then be able to compare the stochastic models. The deterministic model was constructed as follows:

$$Maximize \sum_{t=1}^{T} P_t \lambda_t \tag{3.1}$$

$$P_t = \sum_{i=1}^{I} \sum_{j=1}^{J} Q_{i,j,t} \mu_{i,j} \qquad \forall t \in T \tag{3.2}$$

$$M_{i,t} = M_{i,t-1} + V_{i,t} - \sum_{j=1}^{J} Q_{i,j,t} - S_{i,t} + Q_{i,t}^{flow} + S_{i,t}^{flow} \qquad \forall i,t \in I,T \tag{3.3}$$

$$Q_{i,t}^{flow} = \sum_{i \in I_{up}} \sum_{j=1}^{J} Q_{i,j,t-\tau_i} \qquad \forall i,t \in I,T \tag{3.4}$$

$$S_{i,t}^{flow} = \sum_{i \in I_{up}} S_{i,t-\tau_i} \qquad \forall i,t \in I,T \tag{3.5}$$

$$Q_{i,j,t-\tau_i} = \frac{m_i}{60} Q_{i,j,t-h_i-1} + \frac{60-m_i}{60} Q_{i,j,t-h_i} \tag{3.6}$$

$$S_{i,t-\tau_i} = \frac{m_i}{60} S_{i,t-h_i-1} + \frac{60-m_i}{60} S_{i,t-h_i} \tag{3.7}$$

$$\sum_{i=1}^{I} \sum_{i \in I_{down}} \mu_{i,1} M_{i,8760} = \sum_{i=1}^{I} \sum_{i \in I_{down}} \mu_{i,1} M_{i,0} \tag{3.8}$$

$$0 \leq M_{i,t} \leq \overline{M}_i \qquad \forall i,t \in I,T \tag{3.9}$$

$$0 \leq Q_{i,j,t} \leq \overline{Q}_{i,j} \qquad \forall i,j,t \in I,J,T \tag{3.10}$$

$$0 \leq S_{i,t} \leq \overline{S_i} \qquad \forall i,t \in I,T \tag{3.11}$$

3.3 Problem and rolling horizon implementation

The rolling horizon method described in section 2.8 was adapted to address the specific characteristics of this hydropower problem. In this instance, a simulation with a timespan of one year was conducted for Luleälven. To implement the rolling horizon algorithm, the full year was divided into 52 weeks, which were then solved individually. Furthermore, the future horizon for every week was set to 52 weeks, ensuring that the model always had information one year into the future. Given the uncertainty of the future and the increase in computational time with complexity, the future horizon was chosen to not be fully detailed. This allowed for the use of an aggregated future horizon instead, where the reservoirs and stations could be reduced to a single reservoir with a single hydropower station.

The future horizon was used as an indicator of potential future outcomes, therefore the resolution was reduced to weekly values instead of the hourly values used in the code base. However, the detail in the initial week of each rolling period was retained, with the resolution left at the hourly level, in order for the total solution to still be fully detailed. It was also assumed that the forecasts for the initial week in each rolling period would be sufficiently accurate to be considered deterministic. The scenarios described in 3.4 were therefore used solely in the future horizon, resulting in a model with certain information for the initial week, followed by 52 weeks of uncertain information represented by the generated scenarios.

3.4 Scenario generation

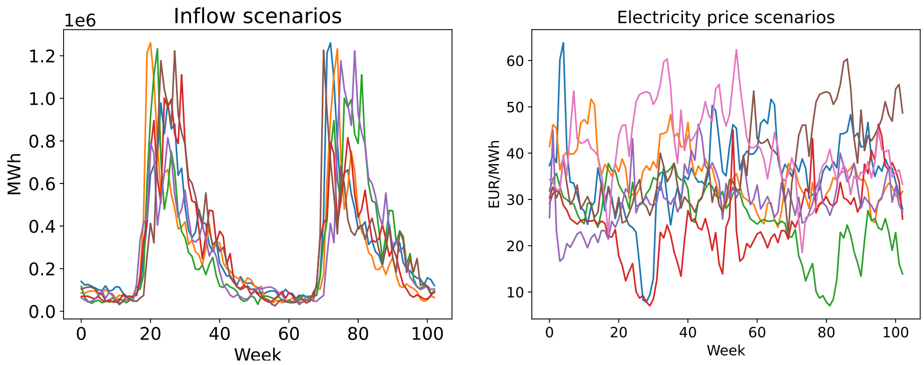
Previous hydropower equivalent models have had the problem of using perfect information and perfect foresight during the whole timespan. This thesis addressed this issue by creating multiple possible future scenarios of inflows and electricity prices from historical data. The data available for inflows spanned the years 2009 to 2018 and included daily inflow values of all the reservoirs in SE1 and SE2. To get scenarios from this data, each year in the data set could be seen as one possible future scenario. With this approach the preservation of the yearly natural periodic pattern in water inflows was ensured which was a crucial aspect in this context.

With the rolling horizon algorithm however, the horizon extended one year into the future,which necessitated the scenarios to be two years in length. With scenarios being two years, it was also necessary to avoid sampling two random years together as a scenario as this could introduce discrete jumps in the inflows between year one and year two. Consequently the two years in each scenario was chosen to be consecutive.

A slight problem in the scenario generation context with hydropower in Sweden was that hydropower produces a large share of the total electricity production as previously mentioned in section 1.1. This also implies that hydropower production influences the electricity prices by how much they are able to generate. A year with exceptionally high inflows of water will, in turn, result in a year with lower electricity prices, as the hydropower stations are able produce more electricity. Therefore a scenario with high inflows and high electricity prices is less probable. In order to obtain realistic scenarios it was therefore not appropriate to mix inflow data and price data from different years. The available electricity price data spanned the years 2012 to 2023 and inflow data from 2009 to 2018, to match years of both inflows and electricity prices the only remaining years were between 2012 and 2018 resulting in only six scenarios.

As the model was a weekly aggregation of the river system, the scenarios also had to be transformed into weekly data. As a result, for each scenario, the electricity prices were represented as the weekly average price as shown in

Fig. 3.1b and the inflows were represented by a weekly total inflow as shown in Fig. 3.1a.



(a) Inflow scenarios

(b) Price scenarios

Figure 3.1: Scenarios generated

3.5 Probability of scenarios

A scenario probability is necessary for all scenarios in order to calculate an expected value from them, however setting these probabilities is not straightforward. By looking at historical data over wet years and dry years in Sweden as shown in [30], it could be possible to draw some kind of conclusion to scenario probabilities. However when looking at this data it is evident that there is a large variation from year to year which can be between 50-80 TWh. Therefore with just a few scenarios a simple assumption would be that all scenario probabilities are equal. Another way to approach this however is to try and calculate the probabilities from historical data.

Therefore a test was conducted on existing inflow data to calculate the probabilities. The initial step involved the creation of a normal distribution of the available data, which was then divided into percentiles. The lower percentile represented a low inflow, the higher percentile a high inflow, and the middle percentile represented the average. The test was conducted on a weekly basis, examining the probability of a certain state occurring in the subsequent week given the state of the current week. By examining years of data and analyzing all state transitions between weeks, the probabilities could be calculated.

With three states low, medium and high, a 3x3 matrix representing the nine possible transitions between the three states could be constructed. The states was then translated to the numbers 1, 2, and 3 respectively, to be used as rows and columns. In this case, the row represented the current week, and the column the next week. Consequently, the probability of transitioning from a low inflow state in the current week to a medium inflow state in the following week could be determined by examining the probability in row 1, column 2 as shown in Eq. 3.12.

$$Probability = \begin{pmatrix} 0.907 & 0.093 & 0 \\ 0.093 & 0.808 & 0.099 \\ 0 & 0.096 & 0.904 \end{pmatrix} \tag{3.12}$$

This test was made on a week-to-week basis since the data needed to create probabilities between year-to-year was way more than what was available. However, the method above could be one way of calculating probabilities of scenarios given enough data or when working with another timeframe for the scenarios. In the continuation of this thesis however, the assumption of equal probabilities for all scenarios was used.

3.6 Weekly aggregated model

In the rolling horizon algorithm the full yearly problem was divided into weeks called rolling periods, and then solved one period at a time. For the aggregated model, the system modeled in the future horizon was aggregated, meaning that the initial hydropower stations were summed together with a simple aggregation. The future horizon was also made with a weekly resolution meaning that the scenarios were created as weekly inflows and weekly electricity prices.

3.6.1 Deterministic and future horizon variables

As described in section 3.3, the problem was solved for each week in a full year, with an hourly based deterministic week and a weekly based future horizon. To create the optimization problem from this, the variables were split into two categories: deterministic variables and future horizon variables. This was done firstly because the variables had different resolutions, hourly and weekly, and secondly because the deterministic variables were detailed with all reservoirs and the horizon variables were aggregated. Therefore, it was most straightforward to keep the variables separated and use constraints to connect them. The deterministic variables were $M_{r,i,t}$, $P_{r,t}$, $S_{r,i,t}$ and $Q_{r,i,j,t}$ for the reservoir, production, spillage and discharge respectively. To ensure clarity regarding the distinctions between and similarities among the variables, the future horizon variables, $\mathcal{M}^{\omega}_{r,v}$, $\mathcal{P}^{\omega}_{r,v}$ and $\mathcal{S}^{\omega}_{r,v}$, were highlighted with another font, distinct from the one used for the deterministic variables.

3.6.2 Calculation of parameters

For the aggregated model, the most straightforward approach was to convert all future horizon variables to energy in MWh. As a result the maximal values for reservoirs, production and spillage were not the same as the detailed variables. Therefore these variables had to be calculated and transformed into aggregated maximal values.

For the maximal aggregated production $\overline{\mathcal{P}}$ in Eq. 3.13 , it was calculated as the sum of the maximal production for all the reservoirs as the aggregation, and then multiplied by the total number of hours in a week which was the resolution for these variables. The aggregated maximum value was then multiplied with 0.8 which was chosen since the maximum sustained production is lower than the actual maximum [31].

The calculations for the future maximum values of reservoir and spillage were calculated similar to each other as can be seen in Eq. 3.14 and Eq. 3.15. The total energy from each reservoir was calculated as the amount of power produced by the water running through all down-streams hydropower stations and then summed for all reservoirs. The spillage was then multiplied by the hours of the week to get the maximal energy spillage for one week.

$$\overline{\mathcal{P}} = \sum_{i=1}^{I} \overline{P}_i \cdot 168 \cdot 0.8 \tag{3.13}$$

$$\overline{\mathcal{M}} = \sum_{i=1}^{I} \sum_{i \in I_{down}} \overline{M}_i \cdot \mu_{i,1} \tag{3.14}$$

$$\overline{\mathcal{S}} = \sum_{i=1}^{I} \sum_{i \in I_{down}} \overline{S}_i \cdot \mu_{i,1} \cdot 168 \tag{3.15}$$

3.6.3 Optimization problem formulation

The optimization problem was setup up to maximize profits for the hydropower station owners in the means of electricity sold to the market. The objective function for the weekly problem was set to maximise the direct profit from the current week and the expected profit from the future horizon as in Eq. 3.16.

The rolling horizon algorithm solved the optimization problem one time for each week of a full year. Therefore the mathematical model below was solved 52 times with a rolling week index r. The index r was updated for each new week in the rolling horizon from 1 to 52.

$$\mathit{Maximize} \sum_{t=1}^{T} P_{r,t}\lambda_{r,t} + \sum_{\omega=1}^{\Omega} \pi^{\omega} \sum_{v=1}^{V} \mathcal{P}^{\omega}_{r,v}\lambda^{\omega}_{r,v} \tag{3.16}$$

The total production for every hour was calculated with Eq. 2.2 and set as a constraint as can be seen in Eq. 3.17 below where the production was summed over all the reservoirs in the river system.

$$P_{r,t} = \sum_{i=1}^{I}\sum_{j=1}^{J} Q_{r,i,j,t}\mu_{i,j} \qquad \forall t \in T \tag{3.17}$$

To deal with the cascaded rivers, hydrological constraints were needed to describe all the reservoir levels at any time which is shown in Eq. 3.18. For each reservoir the reservoir level at hour t could be described by the level obeserved the previous hour, the flow coming into the reservoir and the flow going out from the reservoir. The flow going in was from natural inflow of water $V_{r,i,t}$, and from water released from hydropower stations directly up-streams as discharge or from spillage which can be seen in Eq. 3.19 and 3.20. The outgoing water was then from the discharge or spillage in that reservoir.

$$M_{r,i,t} = M_{r,i,t-1} + V_{r,i,t} - \sum_{j=1}^{J} Q_{r,i,j,t} - S_{r,i,t} + Q^{flow}_{r,i,t} + S^{flow}_{r,i,t} \qquad \forall i,t \in I,T \tag{3.18}$$

The transfer of water from upstream reservoirs to directly down-streams reservoirs is not instantaneous. In reality, there is a time delay between the release of water from one reservoir and its arrival at the next. This time delay is not a constant and can vary depending on a number of factors. However, for the sake of simplicity, it is assumed to be a constant time delay between reservoirs. However, this delay must be accounted for, which was done with Eq. 3.21 and Eq. 3.22.

$$Q^{flow}_{r,i,t} = \sum_{i \in I_{up}}\sum_{j=1}^{J} Q_{r,i,j,t-\tau_i} \qquad \forall i,t \in I,T \tag{3.19}$$

$$S^{flow}_{r,i,t} = \sum_{i \in I_{up}} S_{r,i,t-\tau_i} \qquad \forall i,t \in I,T \tag{3.20}$$

$$Q_{r,i,j,t-\tau_i} = \frac{m_i}{60} Q_{r,i,j,t-h_i-1} + \frac{60-m_i}{60} Q_{r,i,j,t-h_i} \tag{3.21}$$

$$S_{r,i,t-\tau_i} = \frac{m_i}{60} S_{r,i,t-h_i-1} + \frac{60-m_i}{60} S_{r,i,t-h_i} \tag{3.22}$$

The future horizon variables also had to have a hydrological balance which can be seen in Eq. 3.23. This could also be seen as an energy balance since all units had been transformed into MWh.

$$\mathcal{M}^{\omega}_{r,v} = \mathcal{M}^{\omega}_{r,v-1} + \mathcal{V}^{\omega}_{r,v} - \mathcal{P}^{\omega}_{r,v} - \mathcal{S}^{\omega}_{r,v} \qquad \forall v,\omega \in V,\Omega \tag{3.23}$$

For each week in the rolling horizon algorithm, the starting reservoir content had to be updated since the optimization was made in 52 steps. The starting reservoir content was updated to equal the end reservoir content from the previous week which can be seen in Eq. 3.24.

$$M_{r,i,1} = M_{r-1,i,168} \tag{3.24}$$

To tie the future horizon variables to the deterministic variables, a constraint for the total energy in the reservoirs at the end of the deterministic week, was set to be equal to the starting energy for the future horizon reservoir, which can be seen in Eq. 3.25. In this equation it was also assumed that the water could be produced at the highest efficiency, therefore only using the first segment, $\mu_{i,1}$, of the marginal production.

$$\sum_{i=1}^{I} \sum_{i \in I_{down}} \mu_{i,1} M_{r,i,168} = \mathcal{M}^{\omega}_{r,0} \qquad \forall \omega \in \Omega \tag{3.25}$$

A constraint was needed to limit the last values of the future horizon reservoir levels. For deterministic models, end values can be set as constants since the last value is always at the end of the planning period. But in a rolling horizon algorithm, the end of the planning period is moving together with the rolling period and could therefore not be set to a constant value. For every week in the rolling horizon algorithm the end value had to be updated. Given the solution from the previous rolling week $\mathcal{M}^{\omega}_{r-1,v}$ the last value for the future reservoir level for the current week $\mathcal{M}^{\omega}_{r,51}$ was updated as Eq. 3.26 below. With this formulation the future reservoir levels are periodic in the way that the last value equals the beginning value in a yearly cycle.

$$\mathcal{M}^{\omega}_{r,51} = \mathcal{M}^{\omega}_{r-1,1} \qquad \forall \omega \in \Omega \tag{3.26}$$

In the deterministic model the reservoir levels at the end of the planning period were constrained to equal the starting values in the reservoirs. To constrain the reservoir value in this model in a similar manner, the following constraint had to be made which set the total energy in the reservoirs equal to the beginning energy in Eq. 3.27.

$$\mathcal{M}^{\omega}_{r,52-r} = \sum_{i=1}^{I} \sum_{i \in I_{down}} \mu_{i,1} M_{r,i,0} \qquad \forall \omega \in \Omega \tag{3.27}$$

Notice here the 52-r for the reservoir variable, which had to be used in order to move the week index for the end of the planning period along with each step in the rolling horizon algorithm.

All variables in the model had to be constrained to a minimum and maximum value and are shown in Eq's 3.28 - 3.33.

$$0 \leq M_{r,i,t} \leq \overline{M}_i \qquad \forall i, t \in I, T \tag{3.28}$$

$$0 \leq Q_{r,i,j,t} \leq \overline{Q}_{i,j} \qquad \forall i, j, t \in I, J, T \tag{3.29}$$

$$0 \leq S_{r,i,t} \leq \overline{S_i} \qquad \forall i, t \in I, T \tag{3.30}$$

$$0 \leq \mathcal{M}^{\omega}_{r,v} \leq \overline{\mathcal{M}} \qquad \forall v, \omega \in V, \Omega \tag{3.31}$$

$$0 \leq \mathcal{S}^{\omega}_{r,v} \leq \overline{\mathcal{S}} \qquad \forall v, \omega \in V, \Omega \tag{3.32}$$

$$0 \leq \mathcal{P}^{\omega}_{r,v} \leq \overline{\mathcal{P}} \qquad \forall v, \omega \in V, \Omega \tag{3.33}$$

3.7 Fully detailed daily model

A fully detailed stochastic model was developed to allow for comparison with the aggregated model. However, due to hardware limitations, it was not possible to simulate a fully detailed hourly-based future horizon. Consequently, the future horizon was constructed as a fully detailed model, but on a daily resolution. The existing similarities between the aggregated and detailed models meant that not everything required modification. The first modification was to the scenarios, which were changed from weekly values to daily values with inflows for each reservoir. Since the previous aggregation was only made on the future horizon, the deterministic week could be used as it was, consisting of equations 3.17 to 3.22. However, all the future horizon constraints had to be modified to work with all the reservoirs. The most heavily modified constraint was the hydrological balance shown in Eq. 3.35 to 3.37 for the future horizon since it had to incorporate the discharge from the stations much like the deterministic hydrological balance in equations 3.18 to 3.20. In the new, detailed model, this was expressed in terms of discharge rather than simply as power, as in the aggregated model. Therefore also the objective function had to be modified slightly to use the discharge instead of the power as shown in Eq. 3.34. In the detailed model, it was also more straightforward to refrain from transforming all values to energy and to utilize them in their original form. This approach permitted the daily discharge, for instance, to be regarded as a daily average discharge. Consequently, no transformation of the maximum values for the hydropower station parameters was required. The only modification needed was for the parameters $\overline{\mathcal{Q}}$ and $\overline{\mathcal{S}}$ which were multiplied by 24 to get the maximum discharge and spill in a day. Therefore Eq. 3.28 to 3.30 could be used as they were. The remaining constraints Eq. 3.26 and Eq. 3.27 could also be used by only changing the indexation from a weekly to daily resolution.

$$.Maximize \sum_{t=1}^{T} P_{r,t}\lambda_{r,t} + \sum_{\omega=1}^{\Omega} \pi^{\omega} \sum_{v=1}^{V} \lambda_{r,v}^{\omega} \sum_{i=1}^{I} \sum_{j=1}^{J} \mathcal{Q}_{r,i,j,v}^{\omega} \mu_{i,j} \tag{3.34}$$

$$\begin{aligned} \mathcal{M}_{r,i,v}^{\omega} = \mathcal{M}_{r,i,v-1}^{\omega} + \mathcal{V}_{r,i,v}^{\omega} - \sum_{j=1}^{J} \mathcal{Q}_{r,i,j,v}^{\omega} - \mathcal{S}_{r,i,v}^{\omega} + \\ + \mathcal{Q}_{r,i,v}^{flow,\omega} + \mathcal{S}_{r,i,v}^{flow,\omega} \qquad \forall i, v, \omega \in I, V, \Omega \end{aligned} \tag{3.35}$$

$$\mathcal{Q}_{r,i,v}^{flow,\omega} = \sum_{i \in I_{up}} \sum_{j=1}^{J} \mathcal{Q}_{r,i,j,v}^{\omega} \qquad \forall i, v, \omega \in I, V, \Omega \tag{3.36}$$

$$\mathcal{S}_{r,i,v}^{flow,\omega} = \sum_{i \in I_{up}} \mathcal{S}_{r,i,v}^{\omega} \qquad \forall i, v, \omega \in I, V, \Omega \tag{3.37}$$

3.8 Hydro power equivalent

The hydropower equivalent is more complex to calculate than the naive aggregation made for the future horizon 2.9. The hydropower equivalent can be represented as a single reservoir, single hydropower station reduction of the original system which simplifies the problem. In the equivalent system there are some unknown parameters such as the limits of the reservoir, discharge, spill and marginal production equivalent [7]. Some of the unknown parameters could be calculated directly such as the marginal production which was calculated as a weighted average of the marginal productions in the detailed model. However, the other parameters had to be solved in a bi-level optimization problem. In the lower-level optimization the problem is very similar to the deterministic presented in [7] with an objective function to maximise profits. For the upper-level problem the objective function is to minimize the difference between an obtained detailed production and the equivalent production, see Eq. 3.38.

$$\textit{Minimize} \sum_{t=1}^{T} (\sum_{i=1}^{I} P_{i,t} - \sum_{i=1}^{I} \sum_{j=1}^{J} Q^{E}_{i,j,t} \mu^{E}_{i,j})^2 \tag{3.38}$$

With $P_{i,t}$ here being the known production of the detailed model and $Q^{E}_{i,j,t}$ the discharge that has been optimized in the lower level problem. With an existing program which made these calculations the hydropower equivalent could be calculated with the input of any model.

Chapter 4

Case study

4.1 Deterministic model

As a base solution to the problem, the model described in section 3.2 was simulated for 2017. The input data for this model was natural inflow data for all reservoirs and electricity prices in an hourly resolution for the full year.

4.2 Aggregated model

For the aggregated model with the weekly resolution in the future horizon, the equations and constraints described in section 3.6 were used. Many different years were simulated but some of them were infeasible without parameter tuning, therefore only the results from 2017 are shown which also was considered as the year with the most average conditions. With this setup there were four sets of input data, one set with inflows and one set with electricity prices for the deterministic week which were in an hourly resolution. Then there were the sets for inflows and price data for the future horizon which were in a weekly resolution, the inflow data for the future horizon was also aggregated for all reservoirs in the system as explained in section 3.6.2.

4.2.1 End constraints for the planning period

In section 3.6.3 two constraints for the future reservoirs levels were shown, see Eq. 3.26 and Eq. 3.27. The first equation always had to be used in order to constrain the end values of the future reservoirs. The second equation however was only used to constrain the reservoir values for the end of the planning period, therefore two simulations were made. The first simulation was made with both constraints and will be referred to as the constant end constraint solution. The other simulation only used Eq. 3.27 for r = 1 to ensure that the future reservoir levels were constrained for the first iteration. The solution to this solution is referred to as the moving end constraint solution.

4.2.2 Reservoir levels

As a first quality control of the aggregated model, the reservoir levels were compared to the deterministic solution in order to see if the rolling horizon algorithm could find a similar optimal solution which is shown in figures 4.1, 4.4a and 4.4b. In Fig. 4.1 the reservoir levels for the deterministic solution and two stochastic solutions is shown for 2017. As can be seen in the figure all of the graphs follow each other throughout the year. From approximately 2000 to 4000 hours, both stochastic models had higher reservoir levels than observed in the deterministic model. The greatest difference in reservoir levels is observed from around hour 6000 and onwards, as the end of the planning period approaches. The moving end constraint model retains more water in the reservoirs in anticipation for the next year, which is observed from the higher end values. The model with a constant end constraint also retains more water after the high inflow period but then produces at a higher rate to decrease reservoir levels to the set amount for the end of the planning period.

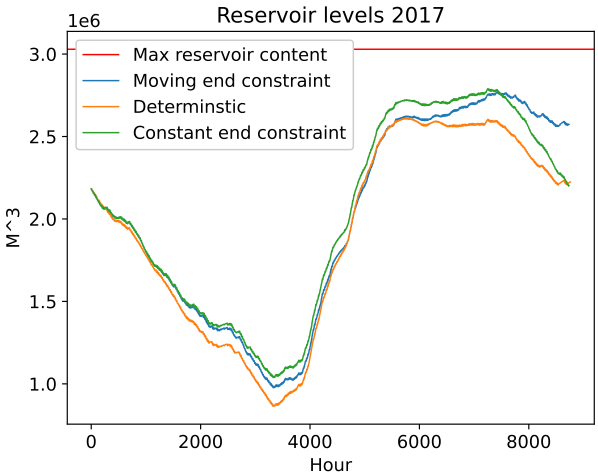


Figure 4.1: Moving and constant end constraint

4.2.3 Electricity production

After simulating 2017 with the aggregated and deterministic models, the results for the total production from all hydropower stations were obtained. In Fig. 4.2 the total production is shown plotted against the electricity price for the specified year. The plot shows week 26 during the high inflow period. This was plotted in order to see the impact of the electricity price on the operation of the hydropower stations in both the aggregated and deterministic model. What can be observed in this graph is that the production and electricity price follow a similar pattern, were if price goes up the production goes up and if the price goes down the production goes down as well.

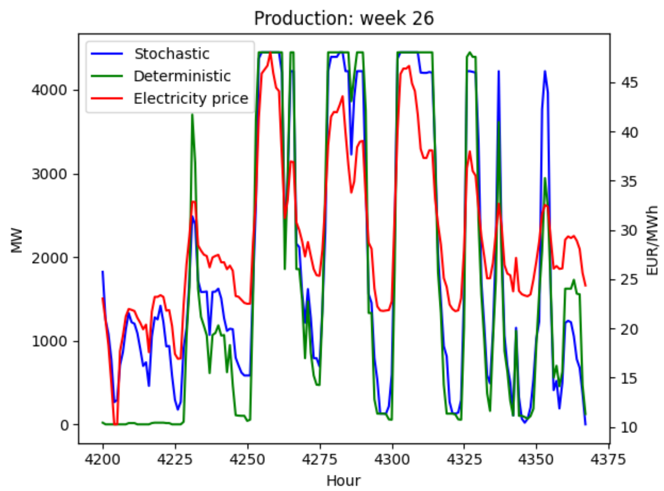


Figure 4.2: Electricity price and production, aggregated

The differences between the operation in the aggregated model and the deterministic model can be seen most clearly in the first 25 hours of this week. The deterministic operation demonstrates that all hydropower stations produce close to nothing for these hours. The aggregated model however continues to produce during these hours but to a lesser extent. During these hours the aggregated production closely follows the electricity price curve. Around hour 4315 there is also the opposite difference were the deterministic optimum solution was producing at maximum, whereas the aggregated model produced a bit less for a couple of hours following the shape of the electricity price. It is evident that, given the knowledge that electricity prices in the future would be higher than those of the current week, it would be better to store the water to produce at the higher price for more profit. This could lead to the behaviour in the first 25 hours where the deterministic solution has almost zero production.

The aggregated model was only certain of the prices of the current week with the subsequent weeks uncertain. Consequently, it would not be optimal to produce zero for low prices, since there was no certainty that the prices would be higher in the future.

4.2.4 Different scenarios

In the initial stages of the thesis, two sets of electricity price scenarios were created. One set included the most recent couple of years, during which electricity prices have been particularly high. The other set included earlier years, during which electricity prices were relatively low. Two simulations were run with high and low electricity price scenarios with the constant end constraint, see Fig. 4.3.

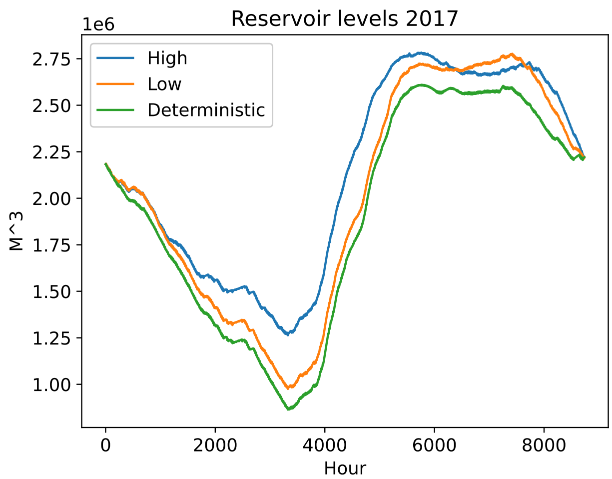


Figure 4.3: High and low electricity price scenarios

The figure shows that the set of scenarios with higher electricity prices have a larger amount of water in the reservoirs throughout the year, while the set of scenarios with lower prices have a lower amount of water throughout the year. Only at the end of the planning period does the reservoir levels for the low price scenarios go above the high price scenarios. The figure would indicate that the set of scenarios with high prices makes it profitable to save water during the deterministic week with 2017 prices, while the lower price scenarios suggests the opposite, that it is more profitable to produce more and take the current price. The problem of this large difference in the results however, could be due to the fact that all scenarios were treated with an equal probability of happening which is not fully realistic. For example the probability of a scenario happening with really high electricity prices should in practise be lower than a scenario with closer to average electricity prices. Considering only six scenarios were used for the models, the scenario outliers could also have a larger impact on the results than if more scenarios were used with equal probabilities.

4.2.5 Issue with infeasible solution

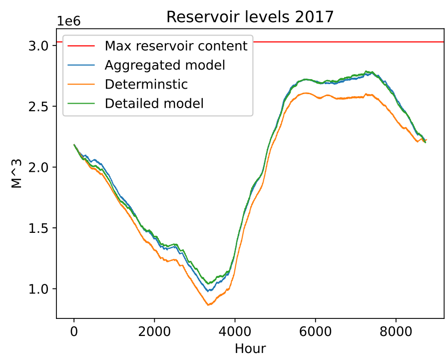
An issue that was observed during the simulation of different years was that the model was unable to identify a solution to the problem for some other years. The optimization was done iteratively for each week of the year since the rolling horizon algorithm was used. Therefore an infeasible solution could in theory be found at any of the weeks, however it was always around week 26. As shown in Fig 3.1a, this period coincides with the spring flood during which most of the reservoirs have very high natural inflows. One possible factor in making the problem unsolvable at this week was due to the naive aggregation made for the future horizon. When calculating the aggregated maximum parameters for the reservoir, production and spillage as described in section 3.6.2, the assumption was inherently made that all reservoirs could be at these maximum levels at the same time, which is not fully realistic. As an example, for the detailed system to reach the maximal total production calculated for the aggregation, it would require all hydropower stations to produce at maximum for a week straight. This would also necessitate that all reservoirs have sufficient water levels or that the smaller reservoirs have sufficient inflow to sustain their maximum discharge. If the detailed week optimization fail to produce the anticipated amounts in the future horizon the previous weeks, the reservoir levels may exceed the maximum levels, making the solution infeasible. In [27] an issue was presented that the parameters for simple aggregations could easily be overestimated. Which would explain the issue with the infeasible solution here quite well. If the maximum production has been overestimated for the aggregation, the detailed optimization would not be able to produce as much as what has been planned in the future horizon, leading to overfilled reservoirs. One straightforward solution to circumvent this issue was to simply set a factor alongside the aggregated parameters, which could be modified when needed. This approach made it possible to find a solution for all of the simulated years. However this shows why the naive aggregation is not the best in all cases and certainly not good enough for the hydropower equivalents.

4.3 Detailed model

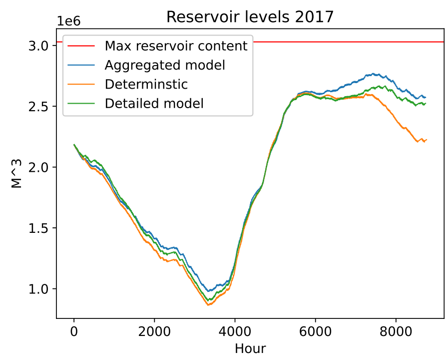
In section 3.7 the detailed daily model was described which unlike the aggregated model had a daily resolution in the future horizon. In the detailed model it was also possible to simulate all the reservoirs separately. This meant that the sets of data for the future horizon had to be made as daily inflows for all the reservoirs and daily electricity prices. As explained in section 4.2.1 two simulations were made for the aggregated model and these two simulations with and without Eq. 3.27 were made for the detailed model as well.

4.3.1 Reservoir levels

With results from both the detailed and aggregated model, the reservoir levels could be compared with both end constraints as well, see Fig. 4.4.



(a) Constant end constraint



(b) Moving end constraint

Figure 4.4: Detailed and aggregated model

With a constant end constraint as in Fig. 4.4a the difference between the aggregated and detailed model was very small, from hour 4000 and onwards the solutions were almost identical. The biggest difference occurs from the start point until hour 4000 were they have only small differences. Both the solutions have higher reservoir levels than the deterministic solution at all times though. Comparing this to the results in Fig. 4.4b there was a larger difference between the stochastic solutions, but closer to the deterministic solution throughout the year. The biggest differences in this figure also shows in the later parts of the year after the high inflow period. The detailed model follows the deterministic solution more closely than the aggregated model until both stochastic models plan to retain more water for next year. When looking at the reservoir levels in Fig. 4.4 the figures demonstrated that, for different end constraints, the models showed different behaviors. With a constant end constraint, the difference was almost negligible, and given the computational difference discussed in section 4.6, the aggregated model could be the preferable choice. However, the solutions from the aggregated and detailed model with a moving end constraint showed a more pronounced difference, suggesting that in certain instances, they may not be as similar in the solution. The moving end constraint was also dependent on the solution from the previous week which means that small changes in solution can make the difference larger at a later stage in the solution. The scenarios between the aggregated and the detailed model were also slightly different since the aggregated model had weekly values and the detailed model daily values. The slight increase in uncertainty from weekly scenarios together with the moving constraint, which does not quite limit the values for the end of the planning period could make the solutions differ more compared to with a constant end constraint.

4.3.2 Electricity production

For the detailed model, the production graph is shown in Fig. 4.5. The detailed model follows the deterministic model more closely than the aggregated model which is most clear in the first 25 hours of week 26.

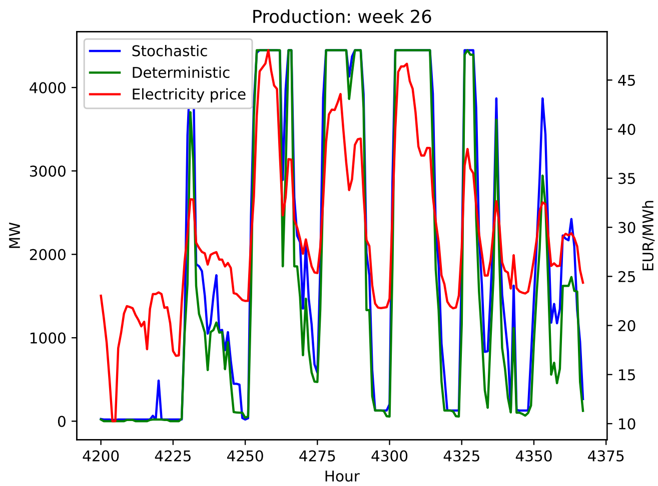


Figure 4.5: Electricity price and production, detailed

The production graph for the detailed model, see Fig. 4.5, was more similar to the deterministic model than the aggregated model. In the detailed models however there was a difference in maximum production where the weekly production had a factor 80 % applied since it is not realistic to produce at 100 % for a week straight. This factor was not used in the detailed model and could therefore explain the differences in production between the two. The scenarios also differed between the two models. For instance, the electricity prices for the aggregated model were weekly averages, whereas for the detailed, daily averages. Therefore some of the details in the scenarios is lost in the aggregated scenarios, which could introduce greater uncertainty to that model compared to the detailed model.

In Table. 4.1 some data for week 26 is shown for the different models. As can be seen, both stochastic models produce more during this week than the deterministic model, however they both spill more water as well. By looking at the start and end reservoir levels it is evident also that there is high inflows since the reservoir levels are increasing for all models even though they are producing and even spilling some of the water. This illustrates the

Table 4.1: Week 26 data for production, spillage and reservoir levels

Model	**Prod. [MWh]**	**Spill** $[m^3]$	**Res. start** $[m^3]$	**Res. end** $[m^3]$
Deterministic	288517	2572	1455133	1635401
Aggregated	341798	20160	1590496	1755588
Detailed	338850	15307	1498005	1668420

problem of the deterministic model, particularly in this extreme week in the middle of the spring flood. Since the deterministic model had all information from the beginning, it had good reservoir levels in all reservoirs during this week, allowing the model to produce less than the stochastic models while also avoiding the necessity to spill as much.

4.4 Future horizon solutions

For each simulation all the future horizon solutions were obtained. Since every week in the rolling horizon algorithm had a solution to the future horizon, 52 future horizon solutions were obtained in total. In Fig. 4.6 the optimal reservoir levels for the rest of the year is shown for every scenario. This solution is from the first rolling week and shows that the solutions from the different scenarios are vastly different. Some scenarios had very high inflows which can be seen in the scenarios that almost empty their reservoirs before the spring flood. Other years had a smaller amount of natural inflow and therefore

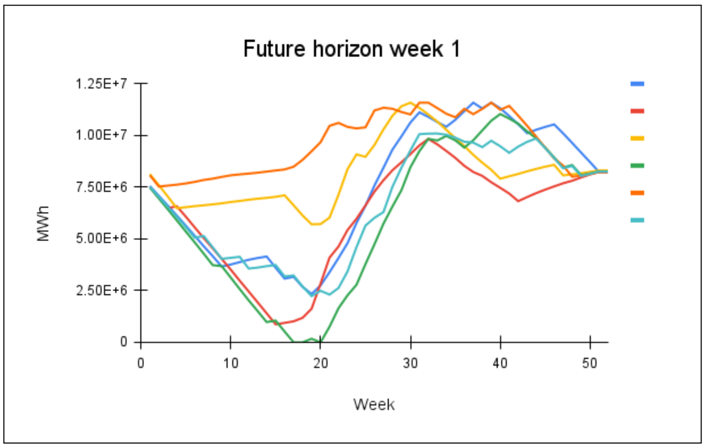


Figure 4.6: Reservoir levels for all scenarios week 1

had to store more of the water for the winter which can be seen in the scenarios with higher reservoir levels throughout the year.

4.5 Hydropower equivalent

A deterministic and stochastic hydropower equivalent was calculated as described briefly in section 3.8 and the most varying parameters are shown in Table 4.2. The majority of the parameters were very similar between the two equivalents and are therefore not shown in the table. However, the minimum reservoir levels for the two stand out as being notably different. As can be seen the minimum reservoir level made with stochastic data is considerably larger relative to the deterministic level. Furthermore the maximum reservoir level is slightly lower in the stochastic model than in the deterministic model. However, when looking at the absolute differences, the maximum reservoir level deviates more from the deterministic level (732 GWh) than the minimum level (92 GWh).

Table 4.2: Hydropower equivalent parameters

Model data	**Reservoir max [GWh]**	**Reservoir min [GWh]**
Deterministic	4536.7	23.61
Stochastic	3804.4	115.67

4.6 Difference in computation time

As mentioned slightly in section 1.1 the complexity of the system affects how difficult the problem is to solve and therefore also the time it takes to compute the solution. This is one of the major points in trying to create the hydropower equivalents to reduce the system complexity and therefore computation times. Throughout this thesis the issue with complexity has been a recurring theme, especially when trying to create a fully detailed hourly model which was not even possible to compute due to insufficient RAM. The fully detailed deterministic program that was used to create the deterministic solution had a computation time of roughly five minutes. The aggregated stochastic model was quite a bit faster, solving the problem in a mere one minute and 30 seconds. However, the fully detailed stochastic model on a daily-basis was by far the slowest with a computation time of one hour and 40 minutes. The computation time for the fully detailed stochastic model however seemed to slow down more as the available RAM was utilized more. One potential explanation for this is that when the RAM is fully utilized, the computer switches to utilize other memory in the computer, which operates at a slower pace as stated in [32]. Consequently, the total run time for the detailed model was not only influenced by the complexity of the problem but also by hardware limitations. The first 11 rolling periods were solved before the RAM was fully utilized in only five minutes. If the computer were to continue at that same pace, it would have a full solution in approximately 24 minutes, which is considerably less than the actual run time of one hour and 40 minutes. Even if the solution could be found in 24 minutes, this would still be way longer than one minute and 30 seconds, which shows the issue with increasingly complex models of hydropower systems which would have an increased solving time as well. If a very large model then would have to be simulated multiple times aswell this would become a large problem. If a large-scale model were to be subjected to multiple simulations, this would become a significant challenge. In contrast, with a hydropower equivalent, the parameters only need to be solved for once and then only a reduced single-station system would have to be solved which is more efficient.

Chapter 5

Conclusion

To conclude this thesis, the issue with perfect foresight presented in previous deterministic models was dealt with by generating multiple scenarios of natural inflows and electricity prices. These scenarios were then used as possible future scenarios in the model with a uniform probability. Two stochastic models were made with a rolling horizon algorithm which introduces new information to the model while optimizing much like in reality. The two models that were made had different amount of detail in the model, one was a fully detailed model with a daily resolution and the other was an aggregated weekly model.

The results suggests that the aggregated stochastic model followed a more realistic production pattern by following daily variations in electricity prices more closely than the deterministic model. However, the detailed stochastic model showed more similarity to the deterministic production which could be due to the fact that the future maximum production were different for the two models effecting how it could produce in the coming weeks.

The parameters derived from the hydropower equivalent were largely consistent with those derived from deterministic data. However, relative to the deterministic values, the minimum reservoir level was considerably higher. The reservoir graphs from the stochastic models indicated that the reservoir levels were generally higher than those derived from the deterministic model which was reflected in an increased minimum reservoir level for the hydropower equivalent.

It was evidently a difficult problem to create a good stochastic model for Luleälven by using the rolling horizon algorithm together with an aggregated future horizon. With the sometimes infeasible solutions for the aggregated model it showed that the aggregation was too simple to handle all types of cases. However, the results showed that the rolling horizon algorithm was applicable on this type of problem, but that it would need to be implemented with a fully detailed model instead of an aggregated model to show its full potential.

5.1 Future work

As the results has shown, different sets of scenarios produce different results, therefore further research in how generating more scenarios for the models would change the solutions would be of interest. The method to calculate scenario probabilities explained in section 3.5 could also be tested on problems with a shorter time frame or be investigated further. The different used end constraints for the future horizon would also need some further research, in this thesis the moving constraint was always dependent on the solution of the previous week which might not be the most optimal route. Therefore finding end constraints that give the model more freedom of setting reservoir levels for the end of the planning period would also be of interest. For the aggregated weekly model and detailed daily model some variation in the results were also seen, therefore with better hardware, research could be made on a fully detailed hourly stochastic model as well. Regarding the time to compute solutions, python was used with the Gurobi optimizer which could have slowed the model building and solving, therefore other coding languages and optimizers could be tested to see if that would change to time to compute.

References

[1] Swedish Agency Marine and Water Management. (2019) Towards sustainable hydropower in sweden. [Online]. Available: https://www.havochvatten.se/en/eu-and-international/towards-sustainable-hydropower-in-sweden.html [Page 1.]

[2] Energi företagen. (2023) Vattenkraft. [Online]. Available: https://www.energiforetagen.se/energifakta/elsystemet/produktion/vattenkraft/ [Page 1.]

[3] International Energy Agency, *Net Zero by 2050*, 2021. [Online]. Available: https://www.oecd-ilibrary.org/content/publication/c8328405-en [Pages 1 and 3.]

[4] A. Harby, L. E. Schäffer, F. Arnesen, P. Bauhofer, A. Beckitt, S. Bockenhauer, A. Botterud, T. Christensen, L. Middleton, N. Nielsen, A. Somani, E. Tavarez, and C. Østlie, "Flexible hydropower providing value to renewable energy integration," 07 2020. [Page 1.]

[5] S. Pfenninger, A. Hawkes, and J. Keirstead, "Energy systems modeling for twenty-first century energy challenges," *Renewable and Sustainable Energy Reviews*, vol. 33, pp. 74–86, 2014. doi: https://doi.org/10.1016/j.rser.2014.02.003. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1364032114000872 [Page 1.]

[6] B. Stoll, J. Andrade, S. Cohen, G. Brinkman, and C. Brancucci Martinez-Anido, "Hydropower modeling challenges," 4 2017. doi: 10.2172/1353003. [Online]. Available: https://www.osti.gov/biblio/1353003 [Page 1.]

[7] E. Blom, "Hydropower area equivalents : Reduced models for efficient simulation of large-scale hydropower systems," Ph.D. dissertation, KTH,

Electric Power and Energy Systems. ISBN 978-91-8040-523-2 2023, qC 20230403. [Pages 2, 4, 12, 14, and 24.]

[8] P. Gabrielli, M. Wüthrich, S. Blume, and G. Sansavini, "Data-driven modeling for long-term electricity price forecasting," *Energy*, vol. 244, p. 123107, 2022. doi: https://doi.org/10.1016/j.energy.2022.123107. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S036054422200010X [Page 2.]

[9] M. S. Zambelli, I. Luna, and S. Soares, "Long-term hydropower scheduling based on deterministic nonlinear optimization and annual inflow forecasting models," in *2009 IEEE Bucharest PowerTech*, 2009. doi: 10.1109/PTC.2009.5281782 pp. 1–8. [Page 2.]

[10] Office of Energy eficiency & Renewable Energy, "Types of hydropower plants." [Online]. Available: https://www.energy.gov/eere/water/types-hydropower-plants [Page 5.]

[11] U. Lundin, P. Jonsson, and L. Facciolo, "Using hydropower turbine discharge as a complementary spillway," *Journal of Applied Water Engineering and Research*, vol. 11, no. 1, pp. 117–126, 2023. doi: 10.1080/23249676.2022.2087774. [Online]. Available: https://doi.org/10.1080/23249676.2022.2087774 [Page 5.]

[12] I. Söderbergh. Environmental measures in regulated rivers with little loss in power production. [Online]. Available: https://www.umu.se/en/news/miljoatgarder-i-reglerade-alvar-med-liten-forlust-i-produktion-_11282770/ [Page 5.]

[13] H. Bergh, *Hydropower*. Dordrecht: Springer Netherlands, 2012, pp. 355–356. ISBN 978-1-4020-4410-6. [Online]. Available: https://doi.org/10.1007/978-1-4020-4410-6_206 [Pages ix and 6.]

[14] M. Yousefi, J. Wang, Ø. Fandrem Høivik, J. Rajasekharan, A. Hubert Wierling, H. Farahmand, and R. Arghandeh, "Short-term inflow forecasting in a dam-regulated river in southwest norway using causal variational mode decomposition," *Scientific Reports*, vol. 13, no. 1, p. 7016, Apr 2023. doi: 10.1038/s41598-023-34133-8. [Online]. Available: https://doi.org/10.1038/s41598-023-34133-8 [Page 6.]

[15] A. Wahlström. (2022) Därför påverkar vårfloden elpriserna. [Online]. Available: https://www.vattenfall.se/fokus/tips-rad/varfloden-paverkar-elpriset/ [Page 6.]

[16] M. Xylia, F. B. Ashraf, and K. Barquet, "Hydropower development in the energy transition: perspectives from northern sweden," 2023. [Online]. Available: https://doi.org/10.51414/sei2023.048 [Page 6.]

[17] L. Kuhlin. Luleälven. [Online]. Available: https://vattenkraft.info/?what=Luleälven [Pages 6 and 7.]

[18] Vattenfall. Harsprånget. [Online]. Available: https://powerplants.vattenfall.com/harspranget/ [Page 6.]

[19] G. R. Yeomans, "Modeling swedish hydropower and intermittent production variability in times," 2017. [Online]. Available: https://api.semanticscholar.org/CorpusID:115267666 [Pages ix and 8.]

[20] The cost of electricity. [Online]. Available: http://www.open-electricity-economics.org/book/text/03.html [Page 7.]

[21] M. Tinoco, S. Barbeau, F. Montoya, and D. Stoilov, "Use of opportunity cost of hydro for regulated prices in the balancing mechanism: a case study for bulgaria," 05 2004. [Page 9.]

[22] J. Funkquist, "An optimization model to analyze hydropower river systems," 2023. [Online]. Available: https://group.vattenfall.com/se/siteassets/documents/vrd-r01-2023-an-optimization-model-to-analyze-hydropower-river-systems.pdf [Page 9.]

[23] "Overview of optimization," in *Process Integration*, ser. Process Systems Engineering, M. M. El-Halwagi, Ed. Academic Press, 2006, vol. 7, pp. 285–314. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1874597006800123 [Pages 9 and 10.]

[24] C. Li and I. E. Grossmann, "A review of stochastic programming methods for optimization of process systems under uncertainty," *Frontiers in Chemical Engineering*, vol. 2, 2021. doi: 10.3389/fceng.2020.622241. [Online]. Available: https://www.frontiersin.org/articles/10.3389/fceng.2020.622241 [Page 10.]

[25] L. Glomb, F. Liers, and F. Rösel, "A rolling-horizon approach for multi-period optimization," *European Journal of*

Operational Research, vol. 300, no. 1, pp. 189–206, 2022. doi: https://doi.org/10.1016/j.ejor.2021.07.043. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0377221721006536 [Page 11.]

[26] M. Redmon. (2024) How accurate are weather forecasts? 5 reasons for inaccuracy. [Online]. Available: https://tempest.earth/resources/how-accurate-are-weather-forecasts/ [Page 11.]

[27] P. Härtel and M. Korpås, "Aggregation methods for modelling hydropower and its implications for a highly decarbonised energy system in europe," *Energies*, vol. 10, no. 11, 2017. doi: 10.3390/en10111841. [Online]. Available: https://www.mdpi.com/1996-1073/10/11/1841 [Pages 12 and 29.]

[28] Gurobi. Industries. [Online]. Available: https://www.gurobi.com/industry/ [Page 12.]

[29] F. G. Dires, M. Amelin, and G. Bekele, "Long-term hydropower planning for ethiopia: A rolling horizon stochastic programming approach with uncertain inflow," *Energies*, vol. 16, no. 21, 2023. doi: 10.3390/en16217399. [Online]. Available: https://www.mdpi.com/1996-1073/16/21/7399 [Page 12.]

[30] K. Lindholm. (2023) "våtår" och "torrår". [Online]. Available: https://www.energiforetagen.se/energifakta/elsystemet/produktion/vattenkraft/vatar-och-torrar/ [Page 17.]

[31] H. E. Fälth, F. Hedenus, L. Reichenberg, and N. Mattsson, "Through energy droughts: hydropower's ability to sustain a high output," 2024. [Page 19.]

[32] R. Brun. (2021) Investigating the impact of ram usage on the performance of windows computers. [Online]. Available: https://outbyte.com/blog/investigating-how-ram-usage-affects-windows-performance/ [Page 34.]

TRITA-EECS-EX-2024:888
Stockholm, Sweden 2024

www.kth.se