School of Electrical Engineering

Quantum Coherence and Quantum-Vacuum Effects in Some Artificial Electromagnetic Media

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ABSTRACT

The author of this thesis concentrates his attention on quantum optical properties of some artificial electromagnetic media, such as quantum coherent atomic vapors (various multilevel electromagnetically induced transparency vapors) and negative refractive index materials, and suggests some possible ways to manipulate wave propagations inside the artificial electromagnetic materials based on quantum coherence and quantum vacuum effects.

In Chapters 1 and 2, the author reviews the previous papers on quantum coherence as well as the relevant work such as electromagnetically induced transparency (EIT), atomic population trapping and their various applications. The basic concepts of quantum coherence (atomic phase coherence, quantum interferences within atomic energy levels) and quantum vacuum are introduced, and the theoretical formulations for treating wave propagations in quantum coherent media are presented.

In Chapter 3, the author considers three topics on the manipulation of light propagations via quantum coherence and quantum interferences: i) the evolutional optical behaviors (turn-on dynamics) of a four-level N-configuration atomic system is studied and the tunable optical behavior that depends on the intensity ratio of the signal field to the control field is considered. Some typical photonic logic gates (e.g. NOT and NOR gates) are designed based on the tunable four-level optical responses of the N-configuration atomic system; ii) the destructive and constructive quantum interferences between two control transitions (driven by the control fields) in a tripod-type four-level system is suggested. The double-control quantum interferences can be utilized to realize some photonic devices such as the logic-gate devices, e.g., NOT, OR, NOR and EXNOR gates; iii) some new quantum coherent schemes (using EIT and dressed-state mixed-parity transitions) for realizing negative refractive indices are proposed. The most remarkable characteristic (and advantage) of the present scenarios is such that the isotropic left-handed media (with microscopic structure units at the atomic level) in the optical frequency band can be achieved.

Quantum vacuum (the ground state of quantized fields) can exhibit many interesting effects. In Chapter 4, we investigate two quantum-vacuum effects in artificial materials: i) the anisotropic distribution of quantum-vacuum momentum density in a moving electromagnetic medium; ii) the angular momentum transfer between quantum vacuum and anisotropic medium. Such quantum-vacuum macroscopic mechanical effects could be detected by current technology, e.g., the so-called fiber optical sensor that can measure motion with nanoscale sensitivity. We expect that these vacuum effects could be utilized to develop sensitive sensor techniques or to design new quantum optical and photonic devices.

In Chapter 5, the author suggests some interesting effects due to the combination of quantum coherence and quantum vacuum, i.e., the quantum coherent effects, in which the quantum-vacuum fluctuation field is involved. Two topics are addressed: i)
spontaneous emission inhibition due to quantum interference in a three-level system; ii) quantum light-induced guiding potentials for coherent manipulation of atomic matter waves (containing multilevel atoms). These quantum guiding potentials could be utilized to cool and trap atoms, and may be used for the development of new techniques of atom fibers and atom chips, where the coherent manipulation of atomic matter waves is needed.

In Chapter 6, we conclude this thesis with some remarks, briefly discuss new work that deserves further consideration in the future, and present a guide to the previously published papers by us.

**Keywords:** Quantum coherence, quantum vacuum, electromagnetically induced transparency, negative refractive index, artificial electromagnetic media, electric- and magnetic-dipole transitions, multilevel atomic vapors, transient evolution

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List of Papers

1) “Backward waves and negative refractive indices in gyrotropic chiral media”

2) “Negative refraction and quantum vacuum effects in gyroelectric chiral medium
and anisotropic magnetoelectric material”

3) “Dimension-sensitive optical responses of electromagnetically induced transparency
vapor in a waveguide”

4) “Negative permeability in a Lambda-type three-level atomic vapor”

5) “Local field contribution to the optical properties of multilevel coherent atomic
media”

6) “Influence of the signal light on the transient optical properties of a four-level EIT
medium”

7) “Negative refractive index in gyrotropically magnetoelectric media”

8) “Double-control quantum interferences in a four-level atomic system”

9) “Transient evolitional behaviours of double-control electromagnetically induced
transparency”

10) “Field quantization and vacuum effects in a chiral medium”
## Acronyms

<table>
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<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>1D</td>
<td>One-Dimensional</td>
</tr>
<tr>
<td>2D</td>
<td>Two-Dimensional</td>
</tr>
<tr>
<td>ALMWs</td>
<td>Array of Long Metallic Wires</td>
</tr>
<tr>
<td>BWM</td>
<td>Back Wave Media</td>
</tr>
<tr>
<td>CIM</td>
<td>Constructive-Interference Media</td>
</tr>
<tr>
<td>CPT</td>
<td>Coherent Population Trapping</td>
</tr>
<tr>
<td>DNM</td>
<td>Double Negative Media</td>
</tr>
<tr>
<td>DIM</td>
<td>Destructive-Interference Media</td>
</tr>
<tr>
<td>EIF</td>
<td>Electromagnetically Induced Focusing</td>
</tr>
<tr>
<td>EIT</td>
<td>Electromagnetically Induced Transparency</td>
</tr>
<tr>
<td>EXNOR gate</td>
<td>Exclusive-NOR gate</td>
</tr>
<tr>
<td>EXOR gate</td>
<td>Exclusive-OR gate</td>
</tr>
<tr>
<td>H.C.</td>
<td>Hermitian Conjugation</td>
</tr>
<tr>
<td>LFC</td>
<td>Local Field Correction</td>
</tr>
<tr>
<td>LHM</td>
<td>Left-Handed Media</td>
</tr>
<tr>
<td>LWI</td>
<td>Laser Without Inversion</td>
</tr>
<tr>
<td>NGV</td>
<td>Negative Group Velocity</td>
</tr>
<tr>
<td>NI</td>
<td>Negative Index</td>
</tr>
<tr>
<td>NIM</td>
<td>Negative Index Materials</td>
</tr>
<tr>
<td>QED</td>
<td>Quantum Electrodynamics</td>
</tr>
<tr>
<td>RHM</td>
<td>Right-Handed Media</td>
</tr>
<tr>
<td>RWA</td>
<td>Rotation Wave Approximation</td>
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<tr>
<td>SRRs</td>
<td>Split Ring Resonators</td>
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Last but not least, I thank my family members for their support, encouragement and patience during my doctoral work.

Jianqi Shen
January 7, 2009
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Chapter 1

Fundamentals: literature review and basic concepts

1.1 Background and motivation

Over the past 50 years, the design and fabrication of artificial materials has attracted considerable attention in a variety of scientific and technological areas [1]. In the 1950s and 1960s, for example, artificial dielectrics were exploited for the manufacture of lightweight microwave antenna lenses, and in the 1980s and 1990s the use of artificial chiral materials for microwave radar absorber applications was explored [2]. In recent decades, a type of materials termed photonic crystals, which are patterned so as to show a periodicity in the dielectric constant, allowing then to create a range of forbidden frequencies constituting what is called the photonic band gap, has attracted the attention of many researchers [3]. Recently, artificial composite metamaterials (so-called left-handed media or negative refractive index media) possessing frequency bands in which the electric permittivity and the magnetic permeability are negative simultaneously captured the attention of researchers in a variety of fields, such as those of material science, condensed matter physics, optics and classical applied electromagnetism. These are a new kind of materials that possess the ability to mold the flow of light inside media. With the development of photonics and quantum optics, considerable attention was directed by scientists in many different areas at new techniques [4, 5, 6] to manipulate light wave propagations by use of artificial electromagnetic materials [7, 8]. At present, a particularly flexible and promising approach to manipulating light propagation has been that of quantum coherence [7, 8, 9, 10]. During the past two decades, quantum coherence effects (atomic phase coherence and quantum interference within atomic energy levels) have been demonstrated in many interesting physical phenomena, such as electromagnetically induced transparency (EIT) [7], light amplification without inversion [8], spontaneous emission cancellation [9], multi-photon population trapping [10], coherent phase control [11, 12] and sensitive optical responses in EIT waveguides [13]. More recently, there has been increasing interest of quantum coherence in photonic applications such as EIT-based coherent information storage [14, 15] and quantum logic gates, which focus special attention among physicists in both fundamental and applied areas [4, 5, 6].

In this thesis, we study the subject of manipulating light propagations by using artificial electromagnetic materials (e.g., quantum coherent media and negative index materials). As is well known, in the 20th century researchers in solid state physics (condensed matter physics) concentrated their attention on how to control motion of electrons using e.g. semiconductor materials via quantum mechanical principle. Here, likewise, we try to suggest and investigate some possible ways to manipulate light wave propagations inside artificial electromagnetic materials based on quantum coherence and quantum-vacuum effects. In the sections that follow, we discuss the topics of quantum coherence, quantum vacuum and some relevant quantum optical properties and phenomena (including basic concepts in quantum optics and electromagnetism that would be used in this thesis).

1.2 Brief history of researches in quantum coherence

Recently, many theoretical and experimental investigations showed that the control of phase coherence in multilevel atomic ensembles give rise to a number of novel and striking quantum optical phenomena of near-resonant light [16], including atomic coherent population trapping (CPT) [17], laser without inversion
Chapter 1. Fundamentals: Literature Review and Basic Concepts

(LWI) [18] and electromagnetically induced transparency (EIT) [7]. In the literature, the idea of atomic CPT was first suggested by Orrriols et al. in 1976 [19], and was experimentally demonstrated by Gray et al. in 1978 [20] and by Alzetta et al. in 1979 [21]. The CPT opened up a possibility of investigation of other novel quantum optical effects, e.g., the EIT (and hence the LWI). In general, EIT arises from the quantum interference and coherence effect in the atomic transitions from the ground states to the excited ones. It is such a quantum optical phenomenon that if one resonant laser beam propagates through a medium, the beam will get absorbed at once; but if two resonant laser beams instead propagate through the same medium, neither would be absorbed. Thus the opaque medium is turned into a transparent one. The physical essence of EIT is just the CPT since EIT is essentially a subset of the coherent population trapping phenomenon [16, 17]. In CPT the two fields interacting with the atoms are close to the same strength and so that the interference effects arise from both fields. In EIT, however, one of the propagating laser beams is much weaker than the other. EIT is thus due to the interference effect driven by only the stronger of the two lasers [16]. This stronger field is called the coupling or control field, and the weaker field is referred to as the probe field. Historically, the foundations of EIT were laid by Kocharovskaya, Kharin (together) in 1988 [22] and Harris (independently) in 1989 [23]. The first experimental observation of EIT was performed by Harris et al. in 1991 [24]. Besides the CPT explanation (in which the concept of dark state is involved), EIT can also be interpreted by using the points of view of quantum interference between dressed states [25], the multi-pathway interferences model (multiple routes to excitation) [26], and some other explanations, e.g., the formulation of quantum field theory, where one should use the Feynman diagram to represent the interfering process occurring in EIT [8, 16].

As stated above, the requirement of occurrence of EIT is such that the intensity of the coupling (control) field is much stronger than that of probe field [7, 27] according to the theoretical analysis of multilevel atomic phase coherence. Under this condition, the EIT atomic vapor allows the probe field to propagate through the medium without dissipation. Due to the unusual quantum coherent character, the discovery of EIT has so far led to many new peculiar effects and phenomena [7], some of which are believed to be powerful for the development of new techniques in quantum optics. More recently, the physical effects associated with EIT observed experimentally included ultraslow light pulse propagation [28, 29] and light storage [30] in atomic vapors, and atomic ground state cooling [31].

Though it is now well known that the foundations of EIT were suggested by Harris et al., there were many evidences showing that the LWI and EIT would have been predicted even more than two decades earlier. Recently, a number of old papers that were written in Russian and published in Soviet journals in the late 1960s and the early 1970s have been found [16]. These papers proposed the above quantum coherence and interference effects in quite different ways compared with those suggested by the western scientists. However, these investigations had not captured any attention of the western scientists because of political reasons. These papers have now been translated into English and are available at e-preprint arXiv server (http://xxx.lanl.gov or http://arxiv.org): quant-ph/0005042 1, quant-ph/0005049 2, quant-ph/0005060 3, quant-ph/0005081 4, quant-ph/0005089 5, quant-ph/0005094 6, quant-ph/0005108 7, quant-ph/0005114 8 and quant-ph/0005118 9.

1.2.1 Atomic phase coherence

Here we briefly discuss the effects and phenomena associated with atomic coherence. It is well known that the quantum optical phenomena (including the semiclassical effects) are described and treated by the formalism of light-matter interactions. In this sense, the processes considered in this thesis may originate from the interference. It is, however, not the classical wave interference. Instead, it is the quantum
1.2. BRIEF HISTORY OF RESEARCHES IN QUANTUM COHERENCE

Figure 1.1: The Fano interference, where two paths destructively interfere, i.e., the phenomenon of cancellation in absorption arises due to the interference between the two excitation paths to the same ionizing state within a continuum.

interference occurring within atoms themselves in the presence of external light fields. In this process, the quantum coherence within energy levels of atoms will then arise.

Then what is the physical meaning of the concept of “coherence”? In general, the coherence is such that the matching of the amplitudes, phases or polarization vectors of waves. From many quantum optical experiments in which waves interfere, we can see that the quantum coherence plays an important role for realizing many intriguing quantum optical effects. As is well known, one should require that the light goes through a pair of slits to be coherent in order to see a well-defined interference pattern. For the similar reason, within the atom, the quantum wave functions (atomic levels) should be coherent (the atom retains a distinct phase relationship with the applied optical field) in order to realize a quantum coherent effects that can control the light propagations. In general, the atoms can retain such a relationship for a length of time approaching the atomic decay time (relaxation time), if the atoms are isolated (as in a vapor) [16]. The coherent phenomena can therefore be observed. The EIT is one such optical process, an effect resulting from quantum interference (i.e. destructive quantum interference), which will be interpreted in more details later.

In the history, the foundation of quantum coherence associated with EIT was laid in an experiment performed in 1961 by Fano [32], which might be the first verification of coherence interference observed experimentally. Fano’s interference (see Fig. 1.1) is such that two excitation paths to the same ionizing state within a continuum leads to a cancellation in absorption as the two paths destructively interfere: specifically, excitation takes place between some lower states and a continuum ionizing state, and it also occurs between the lower states and an auto-ionized state, and hence there are two routes to the final state, once the atom in the auto-ionizing state relaxes to the auto-ionized state. Fano found that the coherent interference between these two routes led to asymmetric peaks in the excitation spectra. Furthermore, it was found by Fano that the transition probability vanishes on one side of the resonance [32]. In a word, coherent interference has turned off the absorption in the medium, i.e., the destructive quantum interference took place in Fano’s experiment [16].

The ability to turn off the absorption can also be used to trap population in a particular level. The atomic levels can be viewed as being trapped, if no population is moving from one level to another (under normal circumstances it is easy for such a population moving to take place). The generalization of Fano’s finding led to the idea of coherent population trapping (CPT) [17, 20, 21]. This was predicted by Orriols and Arimondo in 1976, based on the experiments which had been performed by Orriols and his coworkers [33]. In these experiments, the elimination of fluorescence from an illuminated sodium cell was observed. The fluorescence was absent because of the fact that the population had been trapped into a lower state, unable to move into the upper state [16].

It is well known that the CPT can be easily realized in the Λ-configuration level system. Consider a three-level system in which there are two lower levels \( |b⟩, |c⟩ \) and one upper level \( |a⟩ \) (see Fig. 1.2). We assume that two resonant laser fields with the Rabi frequencies being \( \Omega_2 \) and \( \Omega_1 \) are coupled to the \( |b⟩-|a⟩ \) and \( |c⟩-|a⟩ \) transitions, respectively. If the initial atomic state is a superposition of the two lower levels, it is then possible to choose proper intensities of the fields applied to the system so that the probability...
Figure 1.2: The schematic diagram of a three-level Λ-type atomic system.

amplitude for being in the upper level is zero and hence the population remains trapped in the two lower
states (the coherent-superposition state, known as dark state or uncoupling state). This may be understood
from the following mathematical point of view: according to the probability amplitude equations of motion
\[
\begin{align*}
\dot{c}_a &= i \frac{2}{\hbar} (\Omega_1 c_c + \Omega_2 c_b), \\
\dot{c}_b &= i \frac{2}{\hbar} \Omega_2^* c_a, \\
\dot{c}_c &= i \frac{2}{\hbar} \Omega_1^* c_a,
\end{align*}
\]
(1.1)
one can see that one of the solution is
\[
\begin{align*}
c_a &= 0, \\
c_b &= -\frac{\Omega_2}{\Omega_1},
\end{align*}
\]
(1.2)
where \(c_i\)'s (\(i = a, b, c\)) denote the probability amplitudes of atomic levels \(|a\rangle\), \(|b\rangle\) and \(|c\rangle\). This means that the CPT can be interpreted by the point of view of dark state (uncoupling state), which is
\[
|\text{dark}\rangle = \frac{\Omega_1 |b\rangle - \Omega_2 |c\rangle}{\sqrt{\Omega_1^2 + \Omega_2^2}}.
\]
(1.3)
When the field strengths of the two laser beams are set in the appropriate ratios [as expressed by (1.2)], the
minus sign appearing in \(|\text{dark}\rangle\) will cause the vanishing transition dipole moment from \(|\text{dark}\rangle\) to the upper
state \(|c\rangle\): specifically, if the interaction Hamiltonian can be written as \(H_{\text{int}} = \Omega_1 |c\rangle\langle a| + \Omega_2 |b\rangle\langle a| + \text{H.C.}\) (H.C. stands for the Hermitian conjugation), the interaction matrix element can then be \(\langle \text{dark}|H_{\text{int}}|a\rangle = 0\). Apparently, the dark state remains uncoupled to the upper state \(|c\rangle\) through the two laser beams. If the initial state is just the dark state (1.3), the CPT phenomenon (no population in the upper level) will then be caused by the destructive quantum interference between the two routes allowed to get into the upper
level, e.g., \(|b\rangle-|a\rangle\) and \(|c\rangle-|a\rangle\) paths.

Since we have introduced the concept of CPT that is the physical essence of EIT, let us now review the
brief history in the late 1980s and early 1990s, which was an exciting period in the history of EIT studies.
As stated above, the foundations of EIT were laid by Kocharovskaya et al. [22] and independently by Harris
[23]. Both papers addressed a concept that is now known as lasing without inversion (or inversionless laser).
This is a process, in which a laser can, by means of atomic coherence, be made to operate without the usual necessary population inversion condition. This work was quickly followed by a paper on a similar concept proposed by Scully et al. [34]. In 1990 EIT was first called by Harris in a paper, where another effect based on EIT was addressed, i.e., the enhancement of nonlinear processes [34]. The first experimental demonstration of EIT was finished in 1991 again by Harris group [24]. This experiment was carried out in a Λ-configuration atomic system in a strontium vapor using pulsed lasers [16].

1.2.2 Quantum interferences between the light-induced transition paths
According to expression (1.3), if the condition \(\Omega_1 \gg \Omega_2\) is satisfied, level \(|b\rangle\) will become a dark state:
\[
|\text{dark}\rangle \rightarrow |b\rangle.
\]
(1.4)
This, therefore, means that level \(|b\rangle\) cannot be easily excited by the light \(\Omega_2\), and that the quantum
coherence and destructive quantum interference effect will arise (see Fig. 1.3).
1.3. QUANTUM VACUUM IN ELECTROMAGNETIC MEDIA

Three bare states |a>, |b>, |c> of atomic levels

One bare state |b> and two dressed states |a'>, |c'>

Figure 1.3: The interference between dressed states. In the presence of a strong coupling light, the two atomic bare states |a⟩ and |c⟩ form a coherent superposition of states and the two dressed states |a′⟩ and |c′⟩ result from such coherent superposition. This system exhibits a quantum destructive interference between the probe absorption amplitudes to these two dressed states. Such a destructive interference leads to EIT. Here, Ω_p = Ω_2, Ω_c = Ω_1.

The three-level system (Fig. 1.3) can enable us to understand the two concepts: quantum coherence and quantum interference. The dark state is a coherent superposition of two bare states |b⟩ and |c⟩. This can be considered the quantum coherent effect. The dark state can be referred to as the uncoupling state, since it cannot be coupled to the upper level |a⟩: specifically, the coupling coefficient of |c⟩-|a⟩ transition is Ω_1, and the probability amplitude of level |c⟩ is ~ −Ω_2. Thus, the driving contribution from level |c⟩ to |a⟩ is ~ −Ω_1Ω_2. Meanwhile, the coupling coefficient of |b⟩-|a⟩ transition is Ω_2, and the probability amplitude of level |b⟩ is ~ Ω_1. Thus, the driving contribution from level |b⟩ to |a⟩ is ~ Ω_1Ω_2. Thus, the total driving contribution of the two routes to level |a⟩ vanishes, i.e., (−Ω_1Ω_2) + (Ω_1Ω_2) = 0. This can be viewed as the destructive quantum interference effect that causes the EIT.

1.3 Quantum vacuum in electromagnetic media

From the point of view of quantum mechanics, there is infinite (divergent) energy density of quantum-vacuum fluctuation field (zero-point fluctuation field). In the literature, the quantum vacuum (ground state of quantum fields) has attracted attention of many physicists in various areas such as quantum field theory [35, 36], quantum optics [37, 38] and condensed matter physics [39]. Quantum vacuum can exhibit many interesting effects, including vacuum topological structures [35], vacuum polarization (leading to the Lamb shift and hence the hyperfine structure of Hydrogen atomic spectrum) [36], Casimir effect [40, 41] and dramatic change of spontaneous emission decay in QED cavity [13, 37]. Apart from the vacuum effects in quantum field theory, quantum vacuum in electromagnetic materials can also exhibit some novel effects such as vacuum magneto-electric birefringences [42], vacuum contribution to medium momentum [43, 44] and geometric phases at quantum vacuum level [45]. But most of these vacuum effects occur in microscopic domain, and in the literature the influence of the quantum vacuum on the mechanical properties (e.g., motion) of a macroscopically sized medium got less attention than it deserves. One of the reasons for such a situation is that the change in the vacuum mode distribution structure (particularly the breaking of universal symmetry of the quantum vacuum) inside conventional materials is not conspicuous enough to exhibit macroscopically observable mechanical effects. With the recent development of technology of artificial composite materials, however, the macroscopically observable phenomena caused by the anisotropic quantum vacuum could be possibly realized experimentally. In this thesis, we present an interesting effect of quantum-vacuum contribution to the macroscopic mechanical properties of an anisotropic material (Faraday chiral material), in which an anisotropic electromagnetic environment could be built up and hence the universal symmetry of the quantum vacuum can be broken. This means that one can controllably ma-
nipulate the quantum vacuum by choosing proper optical parameters of the anisotropic electromagnetic materials. It will be shown that the produced anisotropic quantum vacuum would possess a small but nonzero angular momentum density. Thus the anisotropy of the quantum vacuum may lead to a transfer of angular momentum from the quantum vacuum to the anisotropic materials (e.g., Faraday chiral material).

These vacuum effects are the phenomena taking place in the materials at rest, and less attention was paid to the vacuum effects inside moving materials. As an isotropic electromagnetic medium becomes anisotropic (with gyrotropy characterization in its constitutive relation [46]) when it moves, we will show that a moving isotropic (or uniaxial) medium can exhibit a physically interesting quantum-vacuum mechanical effect due to the universal symmetry breaking of vacuum field (created in an anisotropic electromagnetic environment) [43]. This can be viewed as a macroscopically observable mechanical effect, which may possibly lead to the transfer of quantum-vacuum electromagnetic momentum or angular momentum from the vacuum zero-point fluctuation field to the moving material.

1.4 Artificial electromagnetic Media

1.4.1 EIT and multilevel atomic vapors (quantum coherent atomic media)

1.4.1.1 Experimental work on EIT and related topics

The first observation of EIT phenomenon by Harris group in 1991 indicated the importance of the quantum interference [24]. It should be noted that such an experimental phenomenon is not due to the hole-burning or saturation effects [16]. One could for instance imagine that if all the populations were in some way removed from the lower probe level then obviously there would be a reduction in absorption. But this is not what happens because the probe field is kept sufficiently weak and cannot cause any significant population movements. This experiment was first performed in strontium, and then demonstrated in a cascade (ladder)-type atomic system in a lead vapor [47]. It is of interest that the work in lead gas has been expanded by Kasapi [48] as a technique for enhanced isotope discrimination. Such a work by using EIT to make one isotope of lead transparent to a probe field while another remains opaque. Kasapi showed that 0.03% of Pb207 could be clearly seen against a background of Pb-208 [48]. This was one of the interesting applications proposed in the early investigations of EIT.

Subsequent work by Harris’ group examined the dispersive properties of EIT [49]. They showed that since the absorption of the medium can be modified, the optical refractive index must be dramatically changed as well. They found that at the point where the absorption is swept through a zero the refractive index varies rapidly with the probe frequency and that this implies a significantly reduced group velocity near the zero probe detuning position. This work provided a foundation for subsequent ideas of slow light. In their experiment Harris’ group demonstrated a reduced group velocity of $c/250$ due to EIT. However, they did not measure the dispersion directly. Such an experiment was carried out by Xiao’s group who measured the dispersion of the medium [50] using a Mach-Zehnder interferometer technique [51]. They observed that the reduced group velocity of light is about $c/13.2$.

Other experimental work on EIT included further work by Xiao’s group, the behavior of continuous-wave EIT in rubidium vapor [52]. This has also been explored by Moseley et al. in which the technique is compared with two-photon spectroscopy [53]. Further studies have included general investigations of EIT in cascade-type [54, 55, 56] and Lambda atomic systems [57]. A novel effect called “electromagnetically induced grating” was observed by some researchers [16, 58]. In this work there is a strong coupling standing wave interacting with three-level Lambda-type (or cascade-type) atoms. This can diffract a weak probe field (propagating along a direction normal to the standing wave [15]) into high-order diffractions. By taking advantage of the absorption and dispersion properties of electromagnetically induced transparency, the authors have demonstrated an atomic grating that can fan effectively diffracted light into the first order direction. Work carried out in the St. Andrews group [16] has covered similar ground to that of Xiao’s group at similar times [26, 59, 60, 61].

1.4.1.2 Applications of quantum coherence

A dielectric medium can be used to manipulate propagations of light pulses. However, optical absorption limits the extent of possible control: this is especially significant for weak light pulses [62]. As stated above, electromagnetically induced transparency is known as an effect such that the absorption for a weak probe light in an opaque medium can be eliminated via quantum mechanical interference and coherent effects.
Besides, EIT has many other applications due to its inherently novel nature. In what follows we list the most important applications of EIT:

i) Lasing without inversion

It is well known that Einstein’s rate equation forbids inversionless lasers. A medium will ultimately become saturated when half of the population is in the upper level of the laser transition (and half is in the lower state). Since the medium suffers from stimulated emission as well as stimulated absorption then the medium can never experience laser action without a population inversion. If, however, the stimulated absorption (a laser requires population inversion in order to overcome the absorption from the lower level [63]) is turned off, or significantly decreased then it may be possible to have these strange upside-down lasers [16]. Such were the proposals by Kocharovskaya and Khanin [22], Harris [23] and Scully [18].

For many laser wavelengths the effort that would go into building an inversionless laser would be an attractive idea. The main area of interest in the inversionless laser is short-wavelength lasers. Since the Einstein A coefficient increases with frequency: \( A = B \hbar \omega / (\pi^2 c^3) \), the spontaneous emission on transitions with short wavelengths is very rapid. This means that the atoms undergoing excitation on such a transition will rapidly decay to a lower state. Hence a population inversion is increasingly difficult to establish as the transition wavelength gets shorter. Inversionless lasers, however, open up the possibility of circumventing this problem [16]. Since inversionless lasers themselves have problems of their own, in particular the difficulties involved in constructing them, no inversionless laser of technological importance has so far been demonstrated. The first proof of principle inversionless laser was built by the Scully group. The same group reported amplification without inversion in Rb and extended their set-up to demonstrate laser oscillation (experimental demonstration of laser oscillation without population-inversion via quantum interference in Rb) [64] (see Fig. 1.4). We hope that the laser equipment without population inversion could be finally realized and employed successfully for modern usage.

ii) Giant nonlinearities

The enhancement of nonlinear process is the one area of EIT where strong, potentially useful experiments have been carried out [65, 66]. The first experimental evidence for enhancement of nonlinearities was reported by Hakuta et al. [65]. They demonstrated that a dc field could be used instead of an ac coupling field in certain cases. The first experiment was three-wave mixing in hydrogen, which is normally forbidden. The four-wave mixing schemes in hydrogen, sodium and some crystals, which opens up possibilities of using EIT for applications such as optical data storage were also investigated [66, 67, 68]. For other work related to EIT nonlinearity, readers may be referred to Refs. [69, 70]. Another intriguing applications of EIT in the nonlinear regime is the possible development of a broad band optical parametric oscillator with a high efficiency. Such a device has been proposed by Harris and Jain [71]. Imamoğlu et al. considered the dynamics of single photons in a nonlinear optical cavity, the optical mode in which can be well described by a spinning 1/2 Hamiltonian. They showed that it is possible to achieve coherent control of the cavity-mode wave function using \( \pi \) pulses for single photons that switch the state of the cavity with very high accuracy [72]. Some groups investigated the giant Kerr nonlinearities obtained by electromagnetically induced transparency [73]. Imamoğlu et al., for example, proposed a method using the strong dispersive
interactions to realize a single-photon turnstile device [72]. Recently, Wang et al. measured the Kerr-
nonlinear index of refraction of a three-level Lambda-type atomic system inside an optical ring cavity [74].
The Kerr nonlinearity is modified and greatly enhanced near atomic resonant conditions for both probe and
coupling beams. The Kerr nonlinear coefficient changes sign when the coupling beam frequency detuning
switches sign, which can lead to interesting applications in optical devices such as all-optical switches [74].

\textbf{iii) Near-resonance behavior of refractive index for the probe light}

Nearly all the novel properties, effects, phenomena as well as applications of EIT arise from the near-
resonance behavior of refractive index for the probe light. In order to make this point clear, here we shall
analyze the problem by using the mathematical formulae rather than the descriptive statements. Consider
a three-level Λ-type atomic ensemble with one upper level |a\rangle and two lower levels |b\rangle and |c\rangle (see Fig. 1.5). Such an atomic system interacts with two optical fields, i.e., the coupling laser beam and the probe laser beam, which couple the level pairs |a\rangle-|c\rangle and |a\rangle-|b\rangle, respectively. Here we assume that the coupling laser is in resonance with the |a\rangle-|c\rangle transition, while the probe laser has a frequency detuning Δ that is defined by Δ = ω_{ab} - ω, where ω_{ab} and ω denote the |a\rangle-|b\rangle transition frequency and the probe mode frequency, respectively. The probe-resonance related density matrix element \tilde{ρ}_{ab} is given by

\[
\tilde{ρ}_{ab} = \frac{-i\hat{p}_{ab}\mathcal{E} e^{-iωa t} (γ_3 + iΔ)}{2\hbar (γ_1 + iΔ)(γ_3 + iΔ) + \frac{Ω_c^* Ω_c}{4}},
\]  

where \( γ_1 \) and \( γ_3 \) represent the spontaneous decay rate of level |a\rangle and the dephasing rate (nonradiative decay rate) of |c\rangle, respectively. \( \hat{p}_{ab} \), \( \mathcal{E} \) and \( Ω_c \) denote the electric dipole matrix element, the probe field envelope and the Rabi frequency of coupling laser (\( Ω_c = \hat{p}_{ac}E_c/\hbar \) with \( E_c \) being the electric field strength of the coupling laser). The electric susceptibility of the medium at the probe frequency is [63]

\[
χ = \frac{iN|\hat{p}_{ab}|^2}{\epsilon_0 \hbar} \frac{γ_3 + iΔ}{(γ_1 + iΔ)(γ_3 + iΔ) + \frac{Ω_c^* Ω_c}{4}}.
\]  

The susceptibility can be rewritten as \( χ = χ' + iχ'' \), where \( χ' \) and \( χ'' \) denote the real and imaginary parts of \( χ \), respectively. Further calculation yields [63]

\[
\left\{ \begin{array}{l}
χ' = \frac{N|\hat{p}_{ab}|^2 \Delta}{\epsilon_0 \hbar Z^2} \left[ γ_3 (γ_1 + γ_3) + \left( Δ^2 - γ_1 γ_3 - \frac{Ω_c^* Ω_c}{4}\right)\right], \\
χ'' = \frac{N|\hat{p}_{ab}|^2 \Delta^2}{\epsilon_0 \hbar Z^2} \left[ Δ^2 (γ_1 + γ_3) - γ_3 \left( Δ^2 - γ_1 γ_3 - \frac{Ω_c^* Ω_c}{4}\right)\right],
\end{array} \right.
\]  

where

\[
Z = \left( Δ^2 - γ_1 γ_3 - \frac{Ω_c^* Ω_c}{4}\right)^2 + Δ^2 (γ_1 + γ_3)^2.
\]

It follows that there are three main features of the near-resonant probe light in the three-level EIT medium:
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Figure 1.6: The general behavior of the electric susceptibility of an EIT vapor. Both the real and imaginary parts of susceptibility at resonant probe frequency (∆ = 0) approach zero. As the real part varies rapidly at ∆ = 0, the dispersion in the real part is significant. This leads to an effect of slow light.

a) nearly lossless. It follows from the expression for the imaginary part of the electric susceptibility that the absorption for a resonant light (∆ = 0) depends on the dephasing rate γ₃. If the dephasing rate is very small (or approaching zero), the imaginary part of electric susceptibility will be negligibly small (see Fig. 1.6). Fortunately, at least in a gas medium, the dephasing rate γ₃ is small compared with the spontaneous decay rate γ₁ (γ₃ is in general two or three orders of magnitude less than γ₁ in a vapor).

b) electric susceptibility χ' → 0 and the electric permittivity of the EIT gas at the resonant frequency (∆ = 0) is approximately equal to that of vacuum (see Fig. 1.6). Such a property can be apparently seen from the expression for the real part of the electric susceptibility.

c) strong dispersion and reduced speed of light (see Figs. 1.7 and 1.8). By using the formula of group velocity

\[ v_g = \frac{\Omega^2 \Omega_c}{\Omega^3 \Omega_c + 2\omega_p N|\mu_{ab} |^2 \epsilon_0 \hbar} \]  

(1.9)

In a typical EIT experiment, the parameters in the above expressions can be taken to be as follows: ωₚ = 10¹³ s⁻¹, Ω_c = 10⁸ s⁻¹, ωₚℏ = 1.0 × 10⁻³⁰ Coulomb-metre (C-m). Here c denotes the speed of light in vacuum). The concentration N can range from 10¹⁸ to 10²⁴ m⁻³. Other physical constants are ε₀ = 8.85 × 10⁻¹² F/m, ℏ = 1.05 × 10⁻³⁴ J-s. By using these parameters, one can show that the group velocity of the probe light is much less than the speed in vacuum, i.e., \( v_g \ll c \).

Apart from the ultraslow light, the effect of superluminal propagation can also be realized in the gas medium (see Ref. [75]). Fig. 1.8 shows the negative group velocity (NGV) in a three-level atomic vapor. Here the negative group velocity means the superluminal propagation.

iv) Coherent control of refractive index and electromagnetically induced focusing

According to Eq. (1.6), the electric susceptibility χ (and hence the effective refractive index \( \sqrt{1 + \chi} \)) of the EIT medium at the probe frequency is a function of atomic concentration N, probe detuning ∆, spontaneous decay and dephasing rates γ₁, γ₃ as well as Rabi frequency of coupling field. So, the refractive index of the EIT medium can be coherently controlled by these physical parameters (see Fig. 1.9). One of the most remarkable applications is the electromagnetically induced focusing (and hence EIT lenses): since laser beams have Gaussian intensity profiles a probe field that has its beam waist matched to the coupling field waist will experience stronger EIT at the centre of the beam than at the edge. The refractive index that the probe light experiences will therefore change with the radial intensity of the coupling field (see Fig. 1.10). Moseley et al. showed that such an effect implied that the probe field experiences the coupling field
Figure 1.7: The dispersion of a typical dimensionless susceptibility $\chi'$ (real part). The $\frac{d\chi'}{d\Delta}$ versus $\Delta$ is plotted to show the dispersion near/at resonant probe frequency ($\Delta = 0$).

Figure 1.8: The real part of the group velocity of the probe laser beam in a three-level atomic vapor. The typical atomic and optical parameters are chosen: $\omega_p = 10^{13}$ s$^{-1}$, $\Omega_c = 10^8$ s$^{-1}$, $\nu_{ab} = 1.0 \times 10^{-30}$ C·m, $N = 10^{21}$ m$^{-3}$, and $\gamma_1 = 10^8$s$^{-1}$. 
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The optical property of EIT depends on the intensity of the coupling laser beam. It is shown how the electric susceptibility (corresponding to the probe laser beam) is coherently controlled by the field strength of the coupling laser beam. $\chi'$ and $\chi''$ are the real and imaginary parts of the electric susceptibility, respectively.

v) Light storage in atomic vapor and information readout

As the probe field of EIT system can be coherently manipulated by the external field (control field), the EIT system may thus play a key role in the light switching technique (photon switching). Apparently, the probe field (and hence the information) can be stored in the atomic levels by turning off the control field, and again read out by switching on the control field. As for the investigation of light storage, there are many references in the literature [30]. Recently, Prof. Thylén’s group (at Laboratory of Optics, Photonics and Quantum Electronics) in Department of Microelectronics and Information Technology, Royal Institute of Technology (Sweden) has studied this subject and published some significant research results (see, for example, Refs. [15, 78, 79]).

In addition to the multilevel atomic vapors (e.g., alkali metallic vapors), the semiconductor quantum dot (SQD) materials can also exhibit the EIT and quantum coherent effects. Very recently, in the literature, there are a number of works in the SQD area, including the design and fabrication, the structure analysis and their photonic applications [15, 78, 79, 80, 81]. In order to achieve the practical technique of light storage and readout, the proper SQD materials (which have proper QD density, energy level structure, and electric-dipole response) for EIT are required.

1.4.2 Negative refractive index materials

This subsection is devoted to reviewing and studying the physically interesting optical and electromagnetic properties, phenomena as well as effects of wave propagations in negative refractive index materials in the literature (it can be referred to as left-handed media if the electric permittivity and the magnetic permeability are both negative). At present, the negative refraction concept has been widely extended because a variety of schemes such as photonic crystals, transmission line networks and chiral nihility, where neither electric permittivity nor magnetic permeability is involved, were proposed to realize the effective negative refractive indices.

Prior to the main subject of this section, let us consider some related topics referring to the brief history and some concepts relevant to optics and electromagnetism. This may be helpful for understanding the historic background in which the concept of negative index media (and left-handed media) results.

In the early 1930s, Dirac, a young theoretical physicist from U.K., predicted a new particle (positive electron) based on his relativistic wave equation (relativistic quantum mechanics). In Dirac’s theory the
energy eigenvalue of Fermions satisfies the equation \( E^2 = p^2c^2 + m_0^2c^4 \) with \( p \) and \( m_0 \) denoting the momentum and rest mass of the particle, respectively. It follows that the energy eigenvalues \( E = \pm \sqrt{p^2c^2 + m_0^2c^4} \). According to the convention of classical physics, the negative-energy eigenstate can be viewed as a solution that has no physical meanings and can therefore be abandoned without any hesitation. But in quantum mechanics, the completeness of the solutions to the wave equation is an essential subject. If we desert the negative energy solution, we cannot guarantee the completeness property of the solutions of the wave equation. Therefore, the negative energy solution should have physical meanings and deserve consideration in quantum mechanics. Thus, the positive electron that corresponds to the negative-energy solution was predicted by Dirac. In Maxwell electrodynamics, we would have experienced such things. It is well known that in the second-order Maxwellian equation, the relation between the square of refractive index \( n \) and the permittivity \( \epsilon \) and permeability \( \mu \) is \( n^2 = \epsilon \mu \). If both \( \epsilon \) and \( \mu \) are positive, it follows that one can arrive at the following cases of \( n \):

\[
\begin{align*}
  n &= +\sqrt{\epsilon\mu}, \quad n = -\sqrt{\epsilon\mu}, \quad n = +\sqrt{(-\epsilon)(-\mu)}, \quad n = -\sqrt{(-\epsilon)(-\mu)},
\end{align*}
\]

(1.10)

all of which agree with the second-order Maxwellian equation. But for a time-harmonic electromagnetic wave, not all of them satisfies the first-order Maxwellian equation. In order to clarify this point, let us first consider the two equations \( \mathbf{k} \times \mathbf{E} = \mu_0\epsilon_0\omega\mathbf{H} \), \( \mathbf{k} \times \mathbf{H} = -\epsilon_0\epsilon_0\mathbf{E} \) for a time-harmonic wave, where \( \mu_0 \) and \( \epsilon_0 \) stand for the electric permittivity and magnetic permeability in free space, respectively. Here the wave vector \( \mathbf{k} \) may be rewritten as \( \mathbf{k} = n\hat{\mathbf{k}} \), where the unit vector \( \hat{\mathbf{k}} \) is so defined that \( \hat{\mathbf{k}} \cdot \mathbf{E} = \mathbf{H} \) form a right-handed system. Thus, one can arrive at \( \frac{\mathbf{k}}{\sqrt{n^2}} \times \mathbf{E} = \mathbf{H} \). It is readily verified that if \( \mu > 0 \), then \( n \) is positive, and while if \( \mu < 0 \), then \( n \) is negative. In the same fashion, from the equation \( \mathbf{k} \times \mathbf{H} = -\epsilon_0\epsilon_0\omega\mathbf{E} \), one can obtain \( \frac{\mathbf{k}}{\sqrt{n^2}} \times \mathbf{H} = -\mathbf{E} \). Thus the similar result can also be obtained: if \( n > 0 \), then \( \epsilon \) should be positive, and while if \( n < 0 \), then \( \epsilon \) should be negative. This, therefore, means that in Eq. (1.10), only the following two possibilities \( n = +\sqrt{\epsilon\mu} \) and \( n = -\sqrt{(-\epsilon)(-\mu)} \) satisfy the first-order Maxwellian equation. These two situations correspond to two refractive indices of electromagnetic wave propagating inside right-handed media and left-handed media, respectively. Historically, in 1967\(^{10}\) Veselago first considered this peculiar medium having negative simultaneously negative \( \epsilon \) and \( \mu \), and showed from the Maxwellian equations that such media exhibit a negative index of refraction [82]. It follows from Maxwell’s curl equations that the phase velocity of light wave propagating inside this medium is pointed opposite to

---

\(^{10}\)Note that, in the literature, some authors mentioned the wrong year when Veselago suggested the left-handed media. They claimed that Veselago proposed or introduced the concept of left-handed media in 1968 or 1964. On the contrary, the true history is as follows: Veselago’s excellent paper was first published in Russian in July, 1967 [Usp. Fiz. Nauk 92, 517-526 (1967)]. This original paper was translated into English by W.H. Furry and published again in 1968 in the journal of Sov. Phys. Usp. [82]. Unfortunately, Furry stated erroneously in his English translation that the original Russian version of Veselago’s work was first published in 1964.
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Figure 1.11: Pendry et al. suggested a periodic structure that is composed of the infinite wires arranged in a simple cubic lattice to realize a negative permittivity [90]. The length scale \(a\) in the structure is chosen to be a few millimetres and \(r\) a few microns.

the direction of energy flow, that is, the Poynting vector and wave vector of electromagnetic wave would be antiparallel, \(i.e.,\) the vector \(\mathbf{k}\), electric field \(\mathbf{E}\) and magnetic field \(\mathbf{H}\) form a left-handed system; thus Veselago referred to such materials as “left-handed” media, and correspondingly, the ordinary/regular medium in which \(\mathbf{k}\), \(\mathbf{E}\) and \(\mathbf{H}\) form a right-handed system may be termed the “right-handed” one. Other authors call this class of materials “negative-index media (NIM)” [83], “backward wave media (BWM)” [84], “double negative media (DNM)” [85] and Veselago’s media. There exist a number of peculiar electromagnetic and optical properties, for instance, many dramatically different propagation characteristics stem from the sign change of the refractive index and the phase velocity, including reversal of both Doppler shift and Cerenkov radiation, anomalous refraction, amplification of evanescent waves [86], unusual photon tunnelling [87], modified spontaneous emission rates and even reversals of radiation pressure to radiation tension [88].

In experiments, such an artificial negative electric permittivity may be obtained by using the \textit{array of long metallic wires} (ALMWs) [89], which simulates the plasma behavior at microwave frequencies, and the artificial negative magnetic permeability may be built up by using small resonant metallic particles, \(e.g.,\) the \textit{split ring resonators} (SRRs), with very high magnetic polarizability [90, 91]. A combination of these two structures yields a left-handed medium. Recently, Shelby et al. reported their first experimental realization of this artificial composite medium, the permittivity and the permeability of which have negative real parts [92]. One of the potential applications of the negative refractive index materials is to fabricate so-called “superlenses” (perfect lenses): specifically, a slab of such materials may has the power to focus all Fourier components of a 2D image, even those that do not propagate in a radiative manner [86, 93].

1.4.2.1 Realizations of negative permittivity and permeability

Here we shall consider some schemes for achieving simultaneously negative permittivity and permeability (and hence the negative refractive index).

**Negative permittivity resulting from ALMWs structure**

Pendry et al. have considered the electromagnetic response of arrays of thin metallic wires. They pointed out that there is a single gap in the propagation up to a cutoff frequency for modes with the electric field polarized along the axis of the wire, and that a 2-layer periodic array of conducting straight wires gives rise to effects of negative permittivity [89, 90] (see for the ALMWs structure in Figs. 1.11, 1.12 and 1.13). As is well known, metals have a characteristic response to electromagnetic radiation due to the plasma resonance of the electron gas [89, 90].

The electron plasma is certainly different from a superconducting medium. However, the equation of motion of electrons in Londons’ superconductivity theory is similar to that of charged particles in plasmas, this means that there are some common characteristics that are shared by plasmas and superconductor. In order to understand the plasma-like effect in metallic structures (\textit{array of long metallic wires} (ALMWs) [89]),
Figure 1.12: The realization of a negative permittivity structure (Pendry et al.’s work [89]). The thin gold plated tungsten wires (with nominally 20 microns in diameter), which are laid in parallel rows onto polystyrene sheets, are built into the structure. The aperture is 200 mm×200 mm. The photograph is presented in Fig. 1.13.

Figure 1.13: The realization of a negative permittivity structure (Pendry et al.’s work [89]). This is a photograph of the structure shown in Fig. 1.12. The aperture is 200 mm×200 mm.
let us first consider the electromagnetic wave propagation in a superconducting medium (a superconducting electron gas). It is well known that the Lagrangian density of electromagnetics is \( \mathcal{L} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} \) and that the electromagnetic Lagrangian density can be rewritten as follows

\[
\mathcal{L} = \frac{1}{2} \epsilon \mathbf{E}^2 - \frac{1}{2} \mu \mathbf{B}^2 - \frac{1}{4} \epsilon \mu \mathbf{F}^2
\]

where \( \epsilon \) and \( \mu \) are the electric permittivity and magnetic permeability of the medium, respectively. The electric field and magnetic field are related by

\[
\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} + \mathbf{v} \times \mathbf{B}
\]

and

\[
\mathbf{B} = \nabla \times \mathbf{A}
\]

where \( \mathbf{v} \) is the velocity of the electron, \( \mathbf{A} \) is the vector potential, and \( \mathbf{E} \) and \( \mathbf{B} \) are the electric and magnetic fields, respectively. The electron current density is given by

\[
\mathbf{j} = e \mathbf{v} n_0
\]

where \( e \) is the charge of the electron, \( n_0 \) is the number density of electrons, and \( \mathbf{v} \) is the velocity of the electron. The effective mass of the electron is defined as

\[
m_{\text{eff}} = m_e c^2 \left( 1 - \frac{\omega_p^2}{\omega^2} \right)
\]

where \( m_e \) is the rest mass of the electron, \( c \) is the speed of light, \( \omega_p \) is the plasma frequency, and \( \omega \) is the frequency of the electromagnetic wave. The effective mass is given by

\[
m_{\text{eff}} = m_e \left( 1 + \frac{\omega_p^2}{\omega^2} \right)
\]

for highly absorptive media. The effective mass squared is given by

\[
m_{\text{eff}}^2 = m_e^2 + \frac{\omega_p^2}{\omega^2}
\]

for non-absorptive media. The phase velocity of light is given by

\[
v_p = \frac{c}{n_0}
\]

where \( n_0 \) is the refractive index of the medium. The frequency of light is given by

\[
f = \frac{c}{\lambda}
\]

where \( \lambda \) is the wavelength of light. The frequency of light is related to the refractive index by

\[
f = \frac{c}{n_0}
\]

The splitting of rings in the SRR structure can be used to manufacture negative magnetic permeability media and hence designing left-handed materials. The SRR structure is formed by two coupled conducting rings printed on a dielectric slab of thickness \( t \), with \( t = 0.216 \text{ mm} \) [94]. Readers may be referred to Figs. 1.14 and 1.15 or in Refs. [94, 95] for the structures of SRRs under consideration. The physical mechanism to obtain negative permeability may be as follows: a time-varying magnetic field applied parallel to the axis of the rings induces currents that, depending on the resonant properties of the unit, produce a magnetic field that may either oppose or enhance the incident field [95]. The associated magnetic field pattern from the SRR is dipolar. By having splits in the rings, the SRR unit can be made resonant at wavelengths much larger than the diameter of rings (the SRR particle size is about one tenth of incident wavelength [94]). The purpose of the second split ring, inside and whose split is oriented opposite to the first, is to generate a large capacitance in the small gap region between the rings, lowering the resonant frequency considerably and concentrating the electric field [95].

\[\text{Negative permeability resulting from SRRs structure}\]

The split ring resonator (SRR) can be used for manufacturing negative magnetic permeability media and hence designing left-handed materials. SRR is often formed by two coupled conducting rings printed on a dielectric slab of thickness \( t \), with \( t = 0.216 \text{ mm} \) [94]. Readers may be referred to Figs. 1.14 and 1.15 or in Refs. [94, 95] for the structures of SRRs under consideration. The physical mechanism to obtain negative permeability may be as follows: a time-varying magnetic field applied parallel to the axis of the rings induces currents that, depending on the resonant properties of the unit, produce a magnetic field that may either oppose or enhance the incident field [95]. The associated magnetic field pattern from the SRR is dipolar. By having splits in the rings, the SRR unit can be made resonant at wavelengths much larger than the diameter of rings (the SRR particle size is about one tenth of incident wavelength [94]). The purpose of the second split ring, inside and whose split is oriented opposite to the first, is to generate a large capacitance in the small gap region between the rings, lowering the resonant frequency considerably and concentrating the electric field [95].

\[\text{1.4. ARTIFICIAL ELECTROMAGNETIC MEDIA}\]

11 An alternative approach to deriving the expression for the permittivity of ALMWs structures is presented in Appendix A to this chapter.
Figure 1.14: The split ring resonator (SRR). It is formed by two coupled conducting rings printed on a dielectric slab of thickness $t$. The parameters in Ref. [94] were chosen as $c=0.8$ mm, $d=0.2$ mm, $r=2.3$ mm, and $t=0.216$ mm. This figure was originally in Ref. [94].

Figure 1.15: The resonance curve of an actual copper split ring resonator (SRR). In Ref. [95], the parameters were chosen as $c=0.8$ mm, $d=0.2$ mm, and $r=1.5$ mm. This figure was originally in Ref. [95].
1.4. ARTIFICIAL ELECTROMAGNETIC MEDIA

In what follows we derive the expression for the magnetic permeability of SRRs structures (see Fig. 1.14). Assume that the planes of SRRs are parallel to the \( \hat{x}-\hat{y} \) plane, and an external magnetic field \( \mathbf{B}_z = B_{z}^{\text{ext}} \exp[i\omega t] \hat{z} \) acting on the SRRs, so that an electromotive force \( \mathcal{E} = -i\omega \pi r_0^2 B_{z}^{\text{ext}} \) (with \( \pi r_0^2 \) being the plane area of an SRR particle) is induced along the rings. This electromotive force is responsible for creating a current flow which produces a total magnetic moment in the SRR particle. The slot between the rings acts as a distributed capacitance, which stores the same amount of charge (but of opposite sign) at both sides of the slot \([94]\). According to the charge conservation law, one can obtain

\[
\frac{d}{d\phi} I_i = -\frac{d}{dt} q_i = -i\omega C(V_i - V_o), \quad \frac{d}{d\phi} I_o = -\frac{d}{dt} q_o = -i\omega C(V_o - V_i),
\]

(1.15)

where \( \phi \) and \( C \) denote the angle displacement of 2D polar coordinate and the per unit length capacitance between the rings, respectively, and \( I_i \) and \( I_o \) the currents flowing on both rings (inner and outer). \( q_i \) and \( q_o \) are the per unit length charge at the inner and outer rings, respectively. Note that on the right-handed sides of Eq. (1.15), the operator \( \frac{d}{dt} \) can be replaced with \( i\omega \). It is assumed that in the ring the current can be considered to be homogeneous. This, therefore, implies that \( \frac{d}{d\phi} I_i = \frac{I_i}{2\pi r_0} \). It is apparent that \( (V_i - V_o) = 2i\omega (LI + \pi r_0^2 B_{z}^{\text{ext}}) \), where \( L \) is the total inductance of the SRR (in Fig. 1.14). Thus, we have

\[
I = 4\pi \omega^2 r_0 C \left( LI + \pi r_0^2 B_{z}^{\text{ext}} \right),
\]

(1.16)

and consequently

\[
I = \frac{4\pi^2 \omega^2 r_0^3 C}{1 - 4\pi \omega^2 LC r_0} B_{z}^{\text{ext}}.
\]

(1.17)

Then the moment per unit volume is

\[
M = NI \pi r_0^2 = N \frac{\pi r_0^2 \mu_0^{-1} \omega^2 (4\pi C r_0) \cdot \mu_0 \pi r_0^2}{1 - \frac{\omega^2}{\omega_0^2}} B_{z}^{\text{ext}},
\]

(1.18)

where \( \omega_0^2 = \frac{1}{4\pi LC r_0} \) and \( N \) is the SRR unit number per unit volume. It can be easily verified that the per unit length inductance of ring with radius \( r_0 \) is \( \frac{\mu_0 r_0^2}{2} \). So, the total inductance of the SRR is \( \mu_0 \pi r_0^2 \). It follows from Eq. (1.18) that

\[
M = \frac{N \pi r_0^2 \mu_0^{-1} \omega^2}{1 - \frac{\omega^2}{\omega_0^2}} B_{z}^{\text{ext}}.
\]

(1.19)

Let \( B^{\text{int}}, H^{\text{int}} \) and \( H^{\text{ext}} \) stand for the magnetic induction and the magnetic field strengths of the internal and the external fields, respectively. Since all the currents (such as free current, polarization current and

Figure 1.16: The real and imaginary parts of the magnetic permeability of the SRRs structures. The parameters are chosen with typical values \( F = 0.56 \), \( \omega_0 = 4.0 \) GHz and \( \Gamma = 0.12 \) GHz.
magnetization current) will contribute to magnetic induction $B$, while the magnetic field $H$ is produced only by the free current, we may have $B^{\text{int}} \neq B^{\text{ext}}$, $H^{\text{int}} = H^{\text{ext}}$. Since $B^{\text{int}} = \mu_0 \mu_r H^{\text{int}} = \mu_0 \mu_r H^{\text{ext}} = \mu_r B^{\text{ext}}$, we can obtain $B^{\text{ext}} = \frac{B^{\text{int}}}{\mu_r}$. It then follows that

$$M = \frac{N \pi r^2 \mu_0^{-1} \omega^2}{1 - \frac{\omega^2}{\omega_0^2}} B^{\text{int}} \mu_r.$$  \hfill (1.20)

Insertion of $M = \frac{1}{\mu_0} \frac{\mu_r^{-1}}{\mu_r} B^{\text{int}}$ yields

$$\mu_r = 1 + \frac{N \pi r^2 \omega^2}{1 - \frac{\omega^2}{\omega_0^2}} = 1 - \frac{F \omega^2}{\omega^2 - \omega_0^2}. \hfill (1.21)$$

In general, expression (1.21) can be rewritten as the form \(^{12}\)

$$\mu_r = 1 - \frac{F \omega^2}{\omega^2 - \omega_0^2 + i \omega \Gamma}, \hfill (1.22)$$

if the dissipation (loss) of SRR structures is taken into account. Appropriate choice for the frequency of applied electromagnetic wave will yield negative permeability (see Fig. 1.16).

**Other designs to obtain negative refractive indices**

Pendry et al. used the array of long metallic wires (ALMWs) \(^{89}\) (that simulates the plasma behavior at microwave frequencies) and the split ring resonators (SRRs) (that gives rise to the artificial negative magnetic permeability) to realize negative refractive index materials. This may be referred to as the scheme of artificial metamaterials. There are, however, alternative approaches to the realization of such media. Podolskiy et al., for example, investigated the electromagnetic field distribution for thin metal nanowires by using discrete dipole approximation, and considered the plasmon polariton modes in wires by using numerical simulation \(^{96}\). In Ref. \(^{96}\), they described a material comprising pairs of nanowires parallel to each other (see Fig. 1.17) and showed that such materials can have negative refractive indices in near IR and visible spectral ranges.

It should be noted that the structures of the combination of two lattices (i.e., a lattice of infinitely long parallel wires and a lattice of the split ring resonators) are not isotropic. Such structures can be described with negative permittivity and permeability only if the propagation direction is orthogonal to the axes of wires \(^{97}\). More recently, Simovski and He presented an analytical model for a rectangular lattice of isotropic scatterers with electric and magnetic resonances. Here each isotropic scatterer is formed

\(^{12}\)An alternative way, in which the SRR is viewed as an LC circuit, for deriving the expression for the permeability of SRR structures is presented in Appendix B to this chapter.
by putting appropriately 6 Ω-shaped perfectly conducting particles on the faces of a cubic unit cell [97] (see Fig. 1.18). They derived a self-consistent dispersion equation and used it to calculate correctly the effective permittivity and permeability in the frequency band where the lattice can be homogenized. The frequency range, in which both the effective permittivity and the effective permeability are negative (simultaneously), corresponds to the mini-band of backward waves within the resonant band of the individual isotropic scatterer. For the detailed analysis of the electromagnetic properties of materials formed by the rectangular lattice of isotropic cubic unit cells of Ω particles, authors can be referred to Simovski and He’s work [97].

In addition to the above schemes of artificial metamaterials, there are new approaches to achieving the negative refractive index materials, e.g., photonic crystals [98, 99], transmission line simulation [100] (see Fig. 1.19) as well as chiral nilility [101, 102]. All these are methods based on classical electromagnetic theory. Recently, Oktel et al. [103], we [104], and Thommen and Mandel [105, 106] proposed some new schemes, which are based on the quantum optical formulation (quantum coherence) to design left-handed atomic media. Such scenarios (electromagnetically induced negative refractive indices) can be used to realize isotropic left handedness in photonic-resonant materials (quantum coherent media, EIT vapors and so on) at atomic-scale level at optical frequencies [107]. Besides, we will suggest a new quantum coherent scheme (using the dressed-state mixed-parity transitions) for achieving the negative refractive index, where the simultaneously electric- and magnetic-dipole allowed transitions can occur at any frequencies (in some proper frequency bands, the system can exhibit simultaneously negative permittivity and permeability). All these scenarios will be addressed in Chapter 3.

In addition, we can generalize the above mixed-parity scheme to the entanglement scheme, in which two two-level atoms are entangled and can lead to simultaneously nonzero electric- and magnetic-dipole moments in the entangled-atom transition processes. Entanglement has a large number of applications in quantum information [108], but we hope we could find its new applications in other areas such as in photonics and applied electromagnetism for manipulating light propagations in electromagnetic materials. We expect that such entangled-atom transitions can also exhibit nontrivial electromagnetic responses, which can be used to realize the negative refractive index.

In general, the above electromagnetic responses occurring in multilevel atomic vapors would arise in solid such as the semiconductor quantum dot materials. Such electromagnetic properties in quantum-well systems may also deserve consideration [109].

1.4.2.2 Some unusual phenomena in negative index media

In what follows we consider unusual phenomena in left-handed media. Many optical and electromagnetic effects/phenomena in left-handed (LH) media are reversed or modified. For example, the reflection and refraction laws on the interface between LH and RH (right-handed) media are dramatically modified compared with those on the interface between regular media (RH media). The influence of the LH-RH interface on the radiation propagation is governed by the reflection and refraction laws, which differ substantially from the laws in the conventional (right-handed) media. When a wave in a RH material hits an interface with a LH medium, it has negative angle of refraction, i.e., the refracted wave is on the same side of the normal as the incident wave, which are peculiar to those in the usual media. This is due to the negative value of the refractive index in left-handed media. In particular, the refracted wave can be absent at all if the absolute values of the refraction indices of LH and RH media are equal [82, 88].
As stated before, the reversals of many optical and electromagnetic effects arise due to the negative index of refraction. Now we consider in detail the reversals of two effects (Doppler effect and Cerenkov radiation) in left-handed media.

The reversal of Doppler effect in left-handed media

Let us assume that the medium is fixed at the initial frame K and the light source is moving inside the medium at velocity \(-v\) with respect to the rest frame K. It follows that the frequency one measures (the observer is fixed at the origin of the K system) in the rest frame K is

\[
\omega' = \gamma \omega \left(1 - \frac{nv}{c}\right)
\]

if we consider a one-dimensional wave propagation in the medium with the relative refractive index \(n\). For convenience, we assume that \(n = 1\). Since the relativistic factor \(\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}\), we can have

\[
\omega' = \omega \sqrt{c - \frac{v}{c} + v},
\]

(1.23)

which is a well-known normal Doppler effect. If, however, the medium is a left-handed material with \(n = -1\), it follows from \(\omega' = \gamma \omega \left(1 - \frac{nv}{c}\right)\) that the frequency measured in K is \(\omega' = \gamma \omega \left(1 + \frac{v}{c}\right)\), which can be rewritten as

\[
\omega' = \omega \sqrt{c + \frac{v}{c} - v}.
\]

(1.24)

Apparently, it is an abnormal Doppler effect compared with Eq. (1.23).

The reversal of Cerenkov radiation in left-handed media

Now we discuss the Cerenkov radiation. Electromagnetic radiation, usually bluish light, can be emitted by a beam of high-energy charged particles passing through a transparent medium at a speed greater than the speed of light in that medium. This effect is similar to that of a sonic boom when an object moves faster than the speed of sound. In the theory of Cerenkov radiation, the Fourier component of the magnetic fields produced by a moving charged particle is

\[
B_\omega = \frac{i \omega ne}{4\pi \epsilon_0 c^3} \frac{\exp(ikR)}{R} \sin \theta \delta \left(\frac{\omega}{v} - \frac{n \omega c}{\omega n c} \cos \theta\right),
\]

(1.25)

where \(\theta\) is an angle between the particle speed \(v\) and the direction of radiation, and \(e\) denotes the particle charge. First let us assume that the refractive index \(n > 0\). It is well known that when the charged particle velocity \(v\) satisfies \(v > \frac{c}{n}\), the magnetic field \(B_\omega\) in the direction of \(\cos \theta = \frac{c}{n} v\) is nonvanishing. If, however, the refractive index \(n < 0\), the radiation direction may change, i.e., \(\theta \rightarrow \theta + \pi\). This is referred to as the “reversal of Cerenkov radiation”. Additionally, the reversal of the direction of \(B_\omega\) will also arise, which results from the coefficient \(\frac{i \omega ne}{4\pi \epsilon_0 c^3}\) of \(B_\omega\) (because of the coefficient that involves a negative refractive index \(n\)).
1.4. ARTIFICIAL ELECTROMAGNETIC MEDIA

Appendices: Electromagnetic responses of ALMWs and SRRs, and wave propagation in a dispersive medium

Appendix A. Electric response of ALMWs
Let us consider briefly the permittivity of the metal thin wire array medium. For a nearly free electron in the structures of array of long metallic wires (ALMWs) [89], the equation of motion is \( \frac{\partial \mathbf{v}}{\partial t} = \frac{e}{m} \mathbf{E} \) (\( m \) and \( e \) denotes the mass and the charge of electron, respectively), and the electric current density \( \mathbf{j} \) therefore satisfies the equation \( \frac{\partial \mathbf{j}}{\partial t} = ne \mathbf{E} \), where \( n \) being the electron number density in the present ALMWs medium. If the differential operator \( \frac{\partial}{\partial t} \) can be replaced with \( i\omega \), where \( \omega \) is the frequency of incident electromagnetic wave, we can obtain a relation \( \mathbf{j} = ne \mathbf{v} \). Thus with the help of \( \mathbf{j} = \frac{\partial \mathbf{P}}{\partial t} = i\omega \mathbf{P} \), one can arrive at \( \mathbf{P} = -\frac{ne^2}{2}\omega \mathbf{E} \) with the definition of \( \epsilon \), i.e. \( \mathbf{P} = (\epsilon - 1)\epsilon_0 \mathbf{E} \). According to the definition of \( \epsilon \), it follows that the effective relative permittivity of ALMWs structures can be expressed by

\[
\epsilon = 1 - \frac{\omega_p^2}{\omega^2}, \quad \text{with} \quad \omega_p^2 = \frac{ne^2}{m\epsilon_0}.
\]  

Appendix B. Magnetic response of SRRs
Essentially, an SRR behaves like an LC circuit, and the equation of the charge \( q \) in the LC circuit is as follows

\[
L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + q = -\frac{1}{C} B \pi r_0^2
\]

according to the electromagnetic induction law. By replacing the derivative operator \( \frac{d}{dt} \) with \( i\omega \), and using the electric current \( I = \frac{d}{dt} q = i\omega q \) (hence \( q = \frac{I}{i\omega} \)), one can arrive at

\[
I = -\frac{B \pi r_0^2}{\omega^2 - \frac{R}{L} \omega - \frac{1}{LC}}.
\]

Thus we have the magnetization (total moments per unit volume)

\[
M = NI\pi r_0^2 = \frac{-NB(\pi r_0^2)^2}{\omega^2 - \frac{R}{L} \omega - \frac{1}{LC}},
\]

which is equal to \( \frac{1}{\mu_0} \mu^{-1} B \). Thus, we can obtain an effective relative permeability of SRRs structures

\[
\mu = \frac{\frac{\omega^2}{\omega^2 - i\frac{R}{L} \omega - \frac{1}{LC}}}{\frac{\omega^2}{\omega^2 - i\frac{R}{L} \omega - \frac{1}{LC}} + \frac{N_{\mu_0}(\pi r_0^2)^2}{\omega^2} - \frac{1}{LC}} = 1 - \frac{F \omega^2}{\omega^2 + i\omega \Gamma - \omega_0^2},
\]

which can be rewritten as

\[
\mu = 1 - \frac{F \omega^2}{\omega^2 + i\omega \Gamma - \omega_0^2}.
\]

Appendix C. Wave propagation in a dispersive medium
Since the artificial electromagnetic materials are dispersive, we need to establish some relations for the wave propagation in the dispersive materials. As an illustrative example, we shall consider a simple wave propagation behavior in a dispersive material, where the electromagnetic field equation that governs the wave propagation is given by

\[
\nabla \times \mathbf{E} + \frac{\partial}{\partial t} (\mu_0 \mu \mathbf{H}) = 0, \quad \nabla \times \mathbf{H} - \frac{\partial}{\partial t} (\epsilon_0 \epsilon \mathbf{E}) = 0.
\]
By using the formulation in Prade and Vinet’s paper [B. Prade and J. Y. Vinet, Phys. Rev. B 44, 13556 (1991)], the electric and magnetic field strengths can be written in terms of the quasi-monochromatic field components:

\[ \mathcal{E} = \exp[-i(\omega_0 t - \beta_0 z)]\int \exp[-i\alpha(t - \beta' z)] \mathbf{E}(x, \omega_0 + \alpha) d\alpha, \]

\[ \mathcal{H} = \exp[-i(\omega_0 t - \beta_0 z)]\int \exp[-i\alpha(t - \beta' z)] \mathbf{H}(x, \omega_0 + \alpha) d\alpha, \]

(1.33)

where \( \omega = \omega_0 + \alpha, \beta' = \frac{d\beta}{d\omega}|_{\omega_0} \) with \( \omega_0 \) being the central frequency. For simplicity, we assume that the wave vector of the electromagnetic wave is parallel to the \( \hat{z} \)-direction. It follows that

\[ i\beta_0 \mathbf{e}_z \times \mathbf{E} + i\beta' \mathbf{e}_z \times \mathbf{E} + \nabla \times \mathbf{E} - i\mu_0 \omega_0 \mu(\omega_0) \mathbf{H} - i\mu_0 \alpha \frac{\partial}{\partial \omega}(\omega \mu) \mathbf{H} = 0, \]

\[ i\beta_0 \mathbf{e}_z \times \mathbf{H} + i\beta' \mathbf{e}_z \times \mathbf{H} + \nabla \times \mathbf{H} + i\epsilon_0 \omega_0 \epsilon(\omega_0) \mathbf{E} + i\epsilon_0 \alpha \frac{\partial}{\partial \omega}(\omega \epsilon) \mathbf{E} = 0, \]

(1.34)

and consequently (multiplying the above two equations by \( \mathbf{H} \) and \( \mathbf{E} \), respectively) one can obtain

\[ i\beta_0 \mathbf{H} \cdot (\mathbf{e}_z \times \mathbf{E}) + i\beta' \mathbf{H} \cdot (\mathbf{e}_z \times \mathbf{E}) + \mathbf{H} \cdot (\nabla \times \mathbf{E}) - i\mu_0 \omega_0 \mu(\omega_0) \mathbf{H}^2 - i\mu_0 \alpha \frac{\partial}{\partial \omega}(\omega \mu) \mathbf{H}^2 = 0, \]

\[ i\beta_0 \mathbf{E} \cdot (\mathbf{e}_z \times \mathbf{H}) + i\beta' \mathbf{E} \cdot (\mathbf{e}_z \times \mathbf{H}) + \mathbf{E} \cdot (\nabla \times \mathbf{H}) + i\epsilon_0 \omega_0 \epsilon(\omega_0) \mathbf{E}^2 + i\epsilon_0 \alpha \frac{\partial}{\partial \omega}(\omega \epsilon) \mathbf{E}^2 = 0. \]

(1.35)

Subtracting the second equation from the first one in Eq. (1.35), one can arrive at

\[ 2i \left[ \beta_0 \mathbf{H} \cdot (\mathbf{e}_z \times \mathbf{E}) - \epsilon_0 \omega_0 \epsilon(\omega_0) \mathbf{E}^2 \right] + \nabla \cdot (\mathbf{E} \times \mathbf{H}) \]

\[ + 2i\alpha \left[ \beta' \mathbf{H} \cdot (\mathbf{e}_z \times \mathbf{E}) - \frac{1}{2} \mu_0 \frac{\partial}{\partial \omega}(\omega \mu) \mathbf{H}^2 - \frac{1}{2} \epsilon_0 \frac{\partial}{\partial \omega}(\omega \epsilon) \mathbf{E}^2 \right] = 0. \]

(1.36)

With the help of the relations \( \beta_0 = \sqrt{\mu_0 \omega_0} \) and \( |\mathbf{H}| = \sqrt{\epsilon_0 \mu_0 |\mathbf{E}|} \), it is readily verified that the following relation

\[ \beta_0 \mathbf{H} \cdot (\mathbf{e}_z \times \mathbf{E}) - \epsilon_0 \omega_0 \epsilon(\omega_0) \mathbf{E}^2 = 0 \]

(1.37)

can be obtained. Thus, integrating Eq. (1.36) and using \( \int \nabla \cdot (\mathbf{E} \times \mathbf{H}) \, dx = 0 \), one can get

\[ \frac{1}{\beta'} = \frac{\int_{-\infty}^{\infty} S_z \, dz}{\int_{-\infty}^{\infty} \left[ \frac{1}{2} \mu_0 \frac{\partial}{\partial \omega}(\omega \mu) \mathbf{H}^2 + \frac{1}{2} \epsilon_0 \frac{\partial}{\partial \omega}(\omega \epsilon) \mathbf{E}^2 \right] \, dz}. \]

(1.38)

Hence we obtain the expression for the group velocity of wave propagation in the dispersive material, i.e.,

\[ v_g = \frac{1}{\beta'}. \]

(1.39)

Then, it follows from Eq. (1.38) that the electromagnetic energy density \( w \) in the dispersive medium takes the form

\[ w = \frac{1}{2} \epsilon_0 \frac{\partial}{\partial \omega}(\omega \epsilon) \mathbf{E}^2 + \frac{1}{2} \mu_0 \frac{\partial}{\partial \omega}(\omega \mu) \mathbf{H}^2. \]

(1.40)
Bibliography


It should be noted that the present scheme (inhibited spontaneous emission of atoms) is the spontaneous emission cancellation occurring inside two parallel mirrors (or two parallel metals), where the change of the mode structure of quantum-vacuum zero-point field dramatically modifies the atomic spontaneous emission decay rate. This is, however, different from the three-level spontaneous emission elimination presented in Chapter 5, where the inhibition of the spontaneous emission is caused by the quantum interferences within the atomic levels.


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Chapter 2

Theoretical treatments for wave propagations

2.1 Introduction: wave propagations in atomic media

In the preceding chapter, we have considered the novel optical properties in a three-level Λ-type atomic coherent medium, where the strong dispersion, transparency and unusual coherent effects arise at/near resonance [1, 2, 3]. This leads to many important applications in quantum optics (as has been stated in the preceding chapter). As the EIT technique may be used to create large population of coherently driven uniformly phased atoms, it thereby makes possible new types of optoelectronic devices and measurement apparatus [4, 5, 6, 7]. In addition, The EIT phenomenon can also have many applications to the quantum information processing, e.g., optical double-cavity resonator (used to achieve a large cross-Kerr modulation of the probe field), quantum logic gate based on photonic qubits, and creation of entangled coherent states. Recent evidences have shown that the giant non-linearities (e.g., the enhancement of nonlinear absorption for the probe light) exist in a four-level coherent atomic medium [8, 9]. The schematic diagram of a four-level N-type system is plotted in Fig. 2.1. The large nonlinear optical susceptibilities in a four-level atomic medium have some novel applications, e.g., the realization of an absorptive two-photon optical switch, in which a laser pulse controls the absorption of another laser field [8]. In this process, the atomic system absorbs two photons, but not one photon [8]. Due to their novel effects as well as the potential applications, Such four-level atomic media have attracted attention recently. In 1998, for example, Ling et al. considered the quantum coherence effect in a four-level N-type Doppler broadened media [10]; Harris and Yamamoto described a four-level atomic system that exhibits greatly enhanced third-order susceptibility, but has vanishing linear susceptibility [8]; Based on the suggestion of Harris and Yamamoto [8], Yan et al. reported in 2001 an experimental demonstration of absorptive two-photon switch by constructive quantum interference in a four-level atomic system (such as the cold $^{87}$Rb atoms) [9]. Compared with the three-level systems, the four-level systems have exhibited more intriguing optical properties [11, 12, 13, 14].

In this chapter, we present some quantum-optical theoretical formulations that are used for treating the wave propagations in quantum coherent atomic media and the relevant artificial electromagnetic materials. We first discuss the general optical properties of three- and four-level atomic coherent media, and then present the basic concepts in the theoretical description of light-atom interaction, such as Hamiltonian, decay rates, Rabi oscillation and Rabi frequency. The dynamical equations (Maxwell-Bloch equations) and the derivation methods (probability amplitude method and matrix density method) are also discussed. Three approximations that are often used in quantum optics are also introduced. These approximation methods include: rotation wave approximation, slowly varying envelope approximation, and adiabatic approximation. The electric- and magnetic-dipole allowed transitions that lead to the electric permittivity and the magnetic permeability, respectively, are considered. For the dense atomic media, the local field contribution should also been taken into account. For this reason, the local field correction and the Clausius-Mossotti relation for the atomic vapors are briefly addressed in this chapter.
CHAPTER 2. THEORETICAL TREATMENTS FOR WAVE PROPAGATIONS

2.2 Three- and four-level atomic coherent media

Now we consider the optical properties of some typical systems. From Fig. 2.1, one can see that once the signal laser beam is switched off, the four-level N-type system is reduced to a three-level Λ-type EIT system, which is transparent to a probe light field. However, once the signal laser is switched on, the four-level N-type system may exhibit a giant non-linear absorption at the probe frequency. If both the signal and coupling laser beams are turned off, the system would be reduced to a simple two-level system \{\ket{1}, \ket{3}\}.

The general behaviors of the electric permittivities of the two-level, three-level Λ-type, and four-level N-type atomic coherent media as \(\Delta p\) varies are plotted in Figs. 2.2 and 2.3. It follows from Fig. 2.3 that the probe light at the resonant frequency is absorbed in both the two- and four-level systems. In the three-level EIT system, however, it can propagate without loss at/near the resonant frequency.

It is of physical interest to study the regular pattern of absorption and transparency in a multilevel chain-type atomic coherent system. As is well known, a resonant light will be absorbed by a two-level system. However, the three-level atomic and molecular systems coupled to two laser fields exhibit destructive interference effects that can result in cancellation of absorption at the resonant transition frequency (as well as other modifications to the optical responses). As far as the case of four-level system is concerned, it can exhibit a large nonlinear two-photon absorption for the probe light. But what is about in a five-level and other multilevel atomic systems such as the multilevel chain-type system (see Fig. 2.4)? One can find out the regular pattern of the absorption and transparency for the probe field in multilevel systems (e.g., the \(n\)-level chain-type systems). Our tentative result is such that: the probe light may be (linearly and nonlinearly) absorbed by the even-level systems, whereas it may possibly propagate without dissipation through the odd-level systems.

2.3 Hamiltonian, decay rates and Rabi oscillation

As we have reviewed the history of EIT research as well as the novel properties and remarkable applications, we are now in a position to introduce some fundamental concepts of quantum optics, which is essentially significant for the theoretical treatment of quantum coherence and EIT effects.

2.3.1 The Hamiltonian of a three-level system

In order to consider the dynamics of an atomic system, one should first have a dynamical equation, or instead, one should first have a Hamiltonian. The Hamiltonian of a Lambda-configuration three-level quantum system is \(\mathcal{H} = \mathcal{H}_0 + \mathcal{H}'\), where the free Hamiltonian is given by

\[
\mathcal{H}_0 = \hbar \omega_a |a\rangle \langle a| + \hbar \omega_b |b\rangle \langle b| + \hbar \omega_c |c\rangle \langle c|,
\]  

(2.1)
2.3. HAMILTONIAN, DECAY RATES AND RABI OSCILLATION

Figure 2.2: The general behaviors of the real parts of the electric permittivities in the two-level, three-level Λ-type, and four-level N-type atomic coherent media as $\Delta_p$ varies. $\Delta_p$ and $\Gamma_3$ denote the probe frequency detuning ($\Delta_p = \omega_3 - \omega_1 - \omega_p$) and the spontaneous decay rate of level $|3\rangle$, respectively. The typical parameters are chosen as follows: the Rabi frequencies $\Omega_s = \Omega_c = 1.0 \times 10^8$ s$^{-1}$, the spontaneous decay rates $\Gamma_3 = \Gamma_4 = 1.0 \times 10^8$ s$^{-1}$, the electric-dipole transition matrix element $\varphi_{31} = 1.0 \times 10^{-29}$ C·m, the concentration (total number of atoms per unit volume) $N = 0.5 \times 10^{21}$ m$^{-3}$, and the dephasing rate $\gamma_{21} = 0$.

Figure 2.3: The general behaviors of the imaginary parts of the electric permittivities in the two-level, three-level Λ-type, and four-level N-type atomic coherent media as $\Delta_p$ varies. The typical parameters are chosen the same as those in Fig. 2.2.
and the interaction Hamiltonian is

\[
\mathcal{H}' = -\frac{\hbar}{2} \begin{pmatrix}
-2\omega_c & \Omega_p e^{-i\phi} e^{-i\nu t} & \Omega_e e^{-i\phi} e^{-i\nu t} & \Omega_{kp} e^{-i\phi} e^{-i\nu t} \\
\Omega_p e^{i\phi} e^{i\nu t} & -2\omega_b & -2\omega_b & 0 \\
\Omega_e e^{i\phi} e^{i\nu t} & -2\omega_b & -2\omega_b & 0 \\
\Omega_{kp} e^{i\phi} e^{i\nu t} & 0 & 0 & -2\omega_c
\end{pmatrix}
\]

(2.3)

where the basis vector is \{\langle a|, \langle b|, \langle c|\}. \Omega_c and \Omega_p are the Rabi frequencies of the electric fields of the light waves, which couple the level pairs \langle a|c\rangle and \langle a|b\rangle, respectively, and \Omega_{kp} is the Rabi frequency of the magnetic field of the light interacting with the level pair \langle a|b\rangle. In general, the contribution of the magnetic-dipole allowed transition to the light-atom interaction can be neglected, so that one does not need to take into account the terms associated with \Omega_{kp} in the Hamiltonian (and hence in the dynamical equation). In a proper EIT medium, however, the magnetic response of the medium might give rise to significant influence on the light propagation under certain conditions, and this may result in a negative permeability. Therefore, the magnetic-dipole allowed transition should be addressed. For this reason, the magnetic-transition term that is included in the Hamiltonian (2.2) and (2.3) will also be discussed in this chapter.

### 2.3.2 Spontaneous decay rate and dephasing rate

It is possible in a semiclassical theory to add the spontaneous decay and the dephasing terms phenomenologically to the Hamiltonian. Decay terms are added such that they either remove or add population to the atomic levels. For example, the spontaneous emission decay from level \langle j| to level \langle i| is included by adding a term \(-\Gamma_{ji}\rho_{ij}\) to the density matrix equation for \rho_{ij}. A corresponding positive term will be added to the \rho_{ii} term to conserve population (assuming a closed system). Hence, we have an equation of motion of density matrix as follows

\[
\dot{\rho}_{jj} = -\frac{i}{\hbar} [\mathcal{H}, \rho]_{jj} - \Gamma_{jj} \rho_{jj}.
\]

(2.4)

Here, \Gamma_{jj} includes both the spontaneous emission decay rate and the dephasing rate (nonradiative decay rate or collisional dephasing). The dephasing rate depends on the pressure of the vapor (the dephasing rate becomes large if the pressure increases). In general, the dephasing rate is two or three orders of magnitude less than the spontaneous emission decay rate in a dilute gas medium (with the atomic number density \(N = 10^{18} - 10^{21} \text{ m}^{-3}\), at room temperature). The spontaneous emission decay rate does not change if the atomic vapor pressure varies, because the spontaneous emission decay is the result of the interaction between the atom and the quantum-vacuum zero-point energy. Thus, the change of the quantum vacuum mode structure would lead to a dramatic modification to the atomic spontaneous decay rate (and hence

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**Figure 2.4:** The schematic diagram of a multilevel chain-type atomic coherent system. The probe light can be absorbed (linearly and nonlinearly) by the \(n\)-level chain-type system if \(n\) is an even number. Whereas, the probe light can possibly propagate without any dissipations (or with small loss) through the \(n\)-level system if \(n\) is an odd number.
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to the optical properties of the atomic system). Since the quantum-vacuum zero-point energy distribution
depends on the geometric shape of a cavity or other finite space (say, waveguide) with boundary conditions,
this means that we can control the optical properties of the atomic vapor in a finite space through changing
the geometric shape (or the scale) of the finite space (cavity and waveguide). For example, we have
suggested the sensitive optical properties of atomic vapor in an EIT waveguide (Shen J Q and He S 2006,
“Dimension-sensitive optical responses of electromagnetically induced transparency vapor in a waveguide,”

2.3.3 Rabi oscillation and Rabi frequency

Rabi oscillation is a consequence of coherent excitation of an atom by a monochromatic (or near-monochromatic)
light sources, resonant with an atomic transition. Consider a two-level system that is coupled to a light
with a frequency $\omega$ and an electric field envelope $E_0$. In this two-level system, $|a\rangle$ is the upper level and
$|b\rangle$ the lower level. Define $\tilde{u} = \tilde{\rho}_{ab} + \tilde{\rho}_{ba}$, $\tilde{v} = i (\tilde{\rho}_{ab} - \tilde{\rho}_{ba})$ and $\tilde{w} = \tilde{\rho}_{aa} - \tilde{\rho}_{bb}$, where the superscript $\tilde{}$ denotes
the envelopes of the physical quantities. In any textbooks of quantum optics is found the following set of
equations

$$
\dot{\tilde{u}} = -\delta \tilde{v} - \gamma_\perp \tilde{u}, \quad \dot{\tilde{v}} = \delta \tilde{u} + \frac{\varphi_{ab} E_0}{\hbar} \tilde{w} - \gamma_\perp \tilde{v}, \quad \dot{\tilde{w}} = -\frac{\varphi_{ab} E_0}{\hbar} \tilde{v} - \gamma_\parallel (\tilde{w} - \tilde{w}_0),
$$

(2.5)

where $\delta$ denotes the frequency detuning. It is easily seen that here the physical meanings of $\tilde{u}$, $\tilde{v}$ and $\tilde{w}$
are as follows: $u$ and $v$ have close relation to the dispersion and absorption (amplification), respectively,
and $\tilde{w}$ is the population inversion. It is readily verified that the quantities $\tilde{u}$, $\tilde{v}$, $\tilde{w}$ can be written as a
three-dimensional “vector”, $X = (\tilde{u}, \tilde{v}, \tilde{w})$. If $\gamma_\perp = \gamma_\parallel$ is assumed, and the following “vector” is defined

$$
y = \left(-\frac{\varphi_{ab} E_0}{\hbar}, 0, \delta \right),
$$

(2.6)

we can rewrite Eq. (2.5) as a vector equation

$$
\dot{X} = -\gamma X + y \times X,
$$

(2.7)

which is the same as the equation of motion of a top inside a gravitational field. Such a “top” precedes at
an angular frequency (known as the Rabi frequency)

$$
\Omega_R = |y| = \sqrt{\left(\frac{\varphi_{ab} E_0}{\hbar}\right)^2 + \delta^2}.
$$

(2.8)

Note that the Rabi frequency increases and the period of Rabi oscillations hence decreases as the detuning
of the driving light is increased away from resonance. Thus a field detuned far from resonance will have
no effects on the atom as expected. If the light is resonant with the atomic transition, $\delta = 0$, the Rabi
frequency in expression (2.8) is

$$
\Omega_R = \left|\frac{\varphi_{ab} E_0}{\hbar}\right|.
$$

(2.9)

Though both of them [(2.8) and (2.9)] can be viewed as the Rabi frequency, when authors mention the
concept of the Rabi frequency in the literature, in general it refers to expression (2.9). In the present thesis
we adopt this convention.

The Rabi frequency has a ubiquitous presence in quantum optics. Different investigators have different
understandings of the Rabi frequency (due to different research aims). Shore [15] has outlined a few of its
meanings:

i) a measure of interaction strength [seen from the interaction Hamiltonian (2.2) and hence from the
consequent equation of motion];

ii) a frequency of population oscillations [seen from the point of view of the dynamical equations
governing the density matrix elements and the atomic-level probability amplitudes];

iii) a precessional frequency and an optical Larmor frequency among, e.g., magnetic dipoles in a magnetic
field [seen from Eq. (2.7)].
CHAPTER 2. THEORETICAL TREATMENTS FOR WAVE PROPAGATIONS

2.4 Theoretical treatments of quantum optical properties

2.4.1 Semiclassical treatment

Many quantum optical effects and phenomena can be treated by using the semiclassical formulation, where the atom is characterized by operators and governed by the Schrödinger equation, while the optical field is described by the classical quantities, which obey the classical Maxwellian equations. In general, if the electromagnetic field strengths are fairly strong, the field can be regarded as a classical field and such a quantum optical system can reasonably be dealt with by the semiclassical formulation. The semiclassical treatment is in general valid for the EIT in a typical experiment, where both the probe and coupling laser beams are strong (i.e., the photon fluctuation due to quantum character can be neglected). However, the semiclassical method is no long valid if one of the fields is very weak. Then a full quantum treatment is needed in such a situation. The electromagnetic field in a full quantum treatment is viewed as a photon field, the behavior of which is analogous to that of a quantum harmonic oscillator. The quantum harmonic oscillator possessing an infinite-dimensional number-state space (i.e., the maximum occupation number tends to infinity) can well simulate the Bosonic fields. Thus, the light field is often considered a set of infinite number of harmonic oscillators in the full quantum treatment.

The difference (or connection) between a classical field and a quantized field is as follows: the electric field strength corresponding to a single photon can be expressed by $\sqrt{\hbar \omega / \epsilon_0 V}$, where $\hbar$, $\omega$, $\epsilon_0$ and $V$ denote the Plank constant, the photon mode frequency, the vacuum permittivity and the volume of the system, respectively. If the field strength of an electric field is more than ten times $\sqrt{\hbar \omega / \epsilon_0 V}$, it is reasonably believed that such a field can be considered a classical field. But if the field strength of this electric field has the same order of magnitude as $\sqrt{\hbar \omega / \epsilon_0 V}$, the quantum fluctuation should be taken into consideration. It should be treated by the full quantum formulation.

2.4.2 Dynamical equation: Maxwell-Bloch equation

The semiclassical (quantum) optical system is required of a dynamical equation, in order that the interaction between the multilevel system and the classical fields can be treated. As for the CPT (coherent population trapping) and EIT (electromagnetically induced transparency), the dynamical equation is the Maxwell-Bloch equation. Here, the Bloch equation refers to the equation of motion for the density matrix of a multilevel atom, i.e., Eq. (2.4).

The source current density on the right-handed side of the Maxwellian equation is the polarization current density, which results from the atomic transition between levels, namely,

$$\left( \epsilon_0 \frac{\partial^2}{\partial t^2} - \frac{1}{\mu_0} \nabla^2 \right) A = 2Np_{ba} \frac{\partial}{\partial t} \rho_{ab}, \quad (2.10)$$

where $A$ denotes the three-dimensional magnetic vector potential. By using $E = -\partial A / \partial t$, Eq. (2.10) can be rewritten as

$$\left( \frac{1}{\mu_0} \nabla^2 - \epsilon_0 \frac{\partial^2}{\partial t^2} \right) E = 2Np_{ba} \frac{\partial^2}{\partial t^2} \rho_{ab}. \quad (2.11)$$

By using the Bloch equation, one can obtain the solution $\rho_{ab}$. It is then possible for us to investigate the behavior of $E$ fields that interact with the atoms.

2.4.3 Theoretical methods for treating multilevel atoms

There are two approaches to treating the multilevel atoms: probability amplitude method and density matrix method. In general, the probability amplitude method is used for a pure state, and the density matrix method can be used for both the pure and mixed states.

2.4.3.1 Probability amplitude method

In the probability amplitude method, the wavefunction $|\Psi(t)\rangle$ of atomic levels agrees with the Schrödinger equation (we use an illustrative example of a three-level atom to discuss the probability amplitude method)

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = H |\Psi(t)\rangle, \quad (2.12)$$
where \(|\Psi(t)\rangle\) can be constructed in terms of the energy eigenstates of the atom, i.e.,
\[
|\Psi(t)\rangle = c_a(t)|a\rangle + c_b(t)|b\rangle + c_c(t)|c\rangle.
\]
(2.13)
Here, \(c_a(t)\), \(c_b(t)\) and \(c_c(t)\) are the probability amplitudes corresponding to the three atomic levels \(|a\rangle\), \(|b\rangle\) and \(|c\rangle\), respectively.

### 2.4.3.2 Density matrix method

The density operator of a multilevel atom is defined to be \(\rho(t) = \langle \Psi(t) | \Psi(t) \rangle\). In the representation of a basis-vector set \(\{|a\rangle, |b\rangle, |c\rangle\}\), the density operator \(\rho\) can be written as a matrix, which is often referred to as the density matrix. The connection between the density matrix elements and the probability amplitudes is:
\[
\rho_{ij} = c_i^* c_j, \quad i, j = a, b, c.
\]
According to the Schrödinger equation (2.12), one can arrive at a Liouville-von Neumann equation
\[
\dot{\rho} = i\hbar \frac{\partial}{\partial t} \rho = [H, \rho],
\]
(2.14)
which is the density matrix equation of motion (regardless of the decay terms).

### 2.4.4 Approximation methods

The analytical solutions of the Maxwell-Bloch equation are rather complicated. Moreover, it is impossible to obtain the analytic solutions for many quantum optical systems. But thanks to some physically allowed conditions (and assumptions), which can simplify the Maxwell-Bloch equation, the analytical solutions can then be readily achieved under these conditions. Fortunately, these approximation conditions can often be fulfilled in experiments. The approximation methods which we often adopt in our treatments are as follows: the rotation wave approximation, the slowly varying envelope approximation as well as the adiabatic approximation. These methods will be used in the present thesis. In what follows, we briefly discuss these approximation methods.

#### 2.4.4.1 Rotation wave approximation

Consider the interaction of an electric dipole (caused by the atomic transition) with a polarized light, the interaction Hamiltonian of which is
\[
V = -\varphi_{ba}\rho_{ba}E.
\]
Here \(E = E_0 \cos \omega t\) and \(\rho_{ab} = \rho_{ba}^0 \exp \{-i(\bar{\omega} + \gamma)t\}\) (to the zeroth order), where \(\omega, \bar{\omega}\) and \(\gamma\) denote the mode frequency of light, the atomic transition frequency and the decay rate, respectively. Thus the interaction Hamiltonian reads
\[
V = \frac{1}{2} \varphi_{ba}\rho_{ba} E_0 \{\exp \{-i(\omega - \bar{\omega} + i\gamma)t\} + \exp \{i(\omega + \bar{\omega} - i\gamma)t\}\}.
\]
(2.15)
Note that the contribution of the high-frequency oscillation term \(\exp \{i(\omega + \bar{\omega} - i\gamma)t\}\) in the above equation is small compared with that of the low-frequency (detuning) oscillation term \(\exp \{-i(\omega - \bar{\omega} + i\gamma)t\}\). For this reason, the high-frequency oscillation term can be ignored. Such an approximation is referred to as the rotation wave approximation (RWA).

In some cases the high-frequency oscillation term should be taken into account if one considers the quantum optical effects associated with the vacuum fluctuation. The physical meanings of the high-frequency oscillation term is in connection with the quantum vacuum field, which can exhibit the virtual photon effect.

#### 2.4.4.2 Slowly varying envelope approximation

If the field amplitude has a slowly varying envelop, the second-order Maxwellian equation can be reduced to some first-order equations. In a three-level EIT system, for example, the following Maxwellian equations
\[
\begin{bmatrix}
\partial^2 \xi_c - \frac{\partial^2}{\partial t^2} \\
\partial^2 \xi_p - \frac{\partial^2}{\partial t^2}
\end{bmatrix}
\begin{bmatrix}
\xi_c(t, r) \\
\xi_p(t, r)
\end{bmatrix}
\]
\[
= \frac{N_{c} \varphi_{ba} \partial^2}{\epsilon_0} \xi_c(t, r) - \frac{N_{p} \varphi_{ba} \partial^2}{\epsilon_0} \xi_p(t, r),
\]
(2.16)
can be simplified into the form [18]
\[
\begin{bmatrix}
\partial \xi_c + \frac{\partial}{\partial t} \xi_c \\
\partial \xi_p + \frac{\partial}{\partial t} \xi_p
\end{bmatrix}
\]
\[
= i \frac{\omega}{\epsilon_0} N_{c} \varphi_{ba} \xi_c(t, r) - i \frac{\omega}{\epsilon_0} N_{p} \varphi_{ba} \xi_p(t, r),
\]
(2.17)
where $\tilde{\rho}_{ab}$ and $\tilde{\rho}_{ab}$ are the envelopes of the density matrix elements, and $\hat{k}_{c,p}$ denote the unit vectors of the wave vectors $k_{c,p}$, i.e., $\hat{k}_{c,p} = k_{c,p}/k_{c,p}$.

As to the case of atomic levels, the slowly varying envelopes of density matrix elements can be defined by the relation $\rho_{ab} = \rho_{ab}^0 \exp(-i\omega t)$, where $\rho_{ab}$ and $\omega$ represent the slowly varying envelope of $\rho_{ab}$ and the mode frequency of the driving field, respectively. Note that here the oscillating frequency of the density matrix element $\rho_{ab}$ is taken to be the mode frequency of optical field. This situation is quite different from the previous relation $\rho_{ab} = \rho_{ab}^0 \exp \left[-(i\tilde{\omega} + \gamma)t\right]$ (see in the discussion of rotation wave approximation), where the oscillating frequency $\tilde{\omega}$ of $\rho_{ab}$ is the transition frequency. Here, $\rho_{ab}^0$ is merely the envelope in $\rho_{ab}$ in the absence of the external driving field, and cannot be viewed as the slowly varying envelope of $\rho_{ab}$. Only the $\tilde{\rho}_{ab}$ is the slowly varying envelope of $\rho_{ab}$.

### 2.4.4.3 Adiabatic approximation

The adiabatic approximation has been discussed in Refs. [16, 17]. Because the coupling (control) field is much stronger than the probe field in EIT, i.e., the Rabi frequency, $\Omega_c$, of the coupling (control) laser beam is larger than that of the probe laser beam, $\Omega_p$, some contributions of the decay rates can be ignored. Moreover, thanks to a fact that the equality $d\tilde{\rho}_{ij}/dt \ll \Omega_c$ can be satisfied [18], the Bloch equation will be simplified. Thus, the solutions of the density matrix equation of motion will be obtained by taking into account the adiabatic approximation for treating the dynamical problems of EIT.

### 2.5 Electric permittivity and magnetic permeability

In this section, we consider the relationship between the microscopic parameters (the off-diagonal density matrix elements $\tilde{\rho}_{ab}$ and $\tilde{\rho}_{ab}$) and the macroscopic parameters (the electric permittivity and the magnetic permeability). As an illustrative example, we consider a two-level atomic system and show how the optical “constants” (susceptibility, permittivity and permeability) of the atomic medium is related to the density matrix elements of the atomic system.

For the first, we discuss the Maxwellian equations with a polarization and magnetization current due to atomic transitions in a two-level atomic system. In general, the electric-dipole and magnetic-dipole interaction potential reads

$$V_{\text{tot}} = qr \cdot E + \frac{q}{2m} (L + 2S) \cdot B,$$

where the electric field strength $E = -\partial A/\partial t$ and the magnetic field strength $B = \nabla \times A$. Thus, with the semiclassical framework, where the atom is characterized by the operators, while the electromagnetic field is described by the classical quantities, the total interaction potential $V_{\text{tot}}$ can be rewritten as

$$V_{\text{tot}} = \langle 2\phi_a^* q \phi_a \rho_{ab} \rangle \left( -\frac{\partial}{\partial t} A \right) + \left[ 2\phi_a^* q \frac{m}{2} (L + 2S) \phi_{ab} \rho_{ab} \right] \cdot (\nabla \times A),$$

where $\phi_a$ and $\phi_b$ denote the wavefunctions of the two atomic levels, and $q = -e$ with $e$ being the electron charge; $m$, $L$ and $S$ represent the electron mass, the orbital and spin angular momenta, respectively. The full Lagrangian density of such an electromagnetic system is given by

$$\mathcal{L} = -\frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} - \frac{V_{\text{tot}}}{V},$$

where $\mu_0$, $F^{\mu\nu}$ and $V$ stand for the vacuum permeability, the electromagnetic field tensor and the volume of the electromagnetic system, respectively. By using the Euler-Lagrange equation

$$\frac{\partial \mathcal{L}}{\partial A} - \nabla \cdot \frac{\partial \mathcal{L}}{\partial \nabla A} = 0,$$

where $A$ denotes the three-dimensional magnetic vector potential, one can obtain

$$\frac{1}{\mu_0} \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) A - \frac{2}{V} \left\{ \phi_a^* q r \phi_a \frac{\partial}{\partial t} \rho_{ab} + \nabla \times [\phi_a^* m \phi_{ab} \rho_{ab}] \right\} = 0,$$

where the magnetic moment is defined through

$$m = \frac{q}{2m} (L + 2S).$$
2.5. ELECTRIC PERMITTIVITY AND MAGNETIC PERMEABILITY

Note that in Eq. (2.22) we considered only the contribution of a single atom. In the following discussion, we will consider the contribution of $N\mathcal{V}$ atoms, where $N$ is the concentration of the atomic gas (atomic number per volume). Thus, the expression for the total electric dipole moments in the volume $\mathcal{V}$ reads

$$2\mathcal{V} \sum_{i}^{N} \phi_{i}^{*} (r_i) q r_{i} \phi_{a} (r_i) = 2N\mathcal{V} \int \phi_{i}^{*} (r) q r \phi_{a} (r) dr = 2N\mathcal{V} p_{ba}. \quad (2.24)$$

Here, $\phi_{i}^{*} (r_i) q r_{i} \phi_{a} (r_i)$ is the electric dipole density of the $i$th atom, and $p_{ba}$ is the electric-dipole transition matrix element (electric-dipole moment element). In the same fashion, the expression for the total magnetic dipole moments in the volume $\mathcal{V}$ reads

$$2\mathcal{V} \sum_{i}^{N} \phi_{i}^{*} (r_i) m_{i} \phi_{a} (r_i) = 2N\mathcal{V} \int \phi_{i}^{*} (r) m \phi_{a} (r) dr = 2N\mathcal{V} m_{ba}, \quad (2.25)$$

where $\phi_{i}^{*} (r_i) m_{i} \phi_{a} (r_i)$ is the magnetic dipole density of the $i$th atom, and $m_{ba}$ is the magnetic-dipole transition matrix element (magnetic-dipole moment element). Thus, Eq. (2.22) can be rewritten as

$$\frac{1}{\mu_0} \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) A - 2Np_{ba} \frac{\partial}{\partial t} \rho_{ab} - 2N \nabla \times (m_{ba} \rho_{ab}) = 0. \quad (2.26)$$

This is the Maxwellian equation with polarization and magnetization currents.

In what follows, we discuss two typical cases for the electromagnetic responses:

### 2.5.1 Levels $|a\rangle$, $|b\rangle$ with opposite parity (electric response)

If levels $|a\rangle$, $|b\rangle$ have the opposite parity, the magnetic dipole $m_{ba} = 0$, and this is an electric-dipole allowed transition. Therefore, one can have

$$\left( \epsilon_0 \frac{\partial^2}{\partial t^2} - \frac{1}{\mu_0} \nabla^2 \right) A - 2Np_{ba} \frac{\partial}{\partial t} \rho_{ab} = 0. \quad (2.27)$$

By using the definition of electric field, $\mathbf{E} = -\partial \mathbf{A} / \partial t$, Eq. (2.27) can be rewritten as

$$\left( \frac{1}{\mu_0} \nabla^2 - \epsilon_0 \frac{\partial^2}{\partial t^2} \right) \mathbf{E} - 2Np_{ba} \frac{\partial^2}{\partial t^2} \rho_{ab} = 0. \quad (2.28)$$

As the electric dipole $p_{ba}$ is induced by the electric field $\mathbf{E}$ interacting with the level pair $|a\rangle$-$|b\rangle$, the direction of $p_{ba}$ may be parallel to the external field $\mathbf{E}$ if the angular frequency of the external field is not very high. Under this assumption, one can obtain the relation $p_{ba} \rho_{ab} = (\varphi_{ba} \rho_{ab} / E) \mathbf{E}$, where $\varphi_{ba}$ and $E$ are the magnitudes of $p_{ba}$ and $\mathbf{E}$, respectively. Hence, Eq. (2.28) can be rewritten in the form

$$\left( \frac{1}{\mu_0} \nabla^2 - \epsilon_0 \frac{\partial^2}{\partial t^2} \right) \mathbf{E} - 2N\frac{\varphi_{ba} \rho_{ab}}{E} \frac{\partial^2}{\partial t^2} \mathbf{E} = 0. \quad (2.29)$$

and consequently,

$$\frac{1}{\mu_0} \nabla^2 \mathbf{E} - \epsilon_0 \left( 1 + \frac{2N \varphi_{ba} \rho_{ab}}{\epsilon_0 E} \right) \frac{\partial^2}{\partial t^2} \mathbf{E} = 0. \quad (2.30)$$

It follows from Eq. (2.30) that the effective relative electric permittivity of the present two-level atomic vapor reads

$$\epsilon_e = 1 + \frac{2N \varphi_{ba} \rho_{ab}}{\epsilon_0 E}. \quad (2.31)$$

It can be verified from the Bloch equation that the off-diagonal density matrix element $\rho_{ab}$ is in general proportional to the magnitude ($E$) of the external electric field acting on the atoms (under the condition $\Omega \ll \Gamma$, where $\Gamma$ and $\Omega$ denote the spontaneous decay rate and the Rabi frequency of the driving field, respectively). Thus, the term $\rho_{ab} / E$ in Eq. (2.31) is independent of the electric field strength $E$. 
2.5.2 Levels $|a⟩$, $|b⟩$ with same parity (magnetic response)

If levels $|a⟩$, $|b⟩$ have the same parity, the electric dipole $p_{ba} = 0$, and this is a magnetic-dipole allowed transition. Eq. (2.26) then becomes

$$\frac{1}{\mu_0} \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) A - 2N \nabla \times (m_{ba} \rho_{ab}) = 0. \quad (2.32)$$

By using the definition of the magnetic induction field, $B = \nabla \times A$, Eq. (2.32) is rewritten as

$$\frac{1}{\mu_0} \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) B - 2N \nabla \times \nabla \times (m_{ba} \rho_{ab}) = 0. \quad (2.33)$$

In consequence, we have

$$\frac{1}{\mu_0} \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) B + 2N \nabla^2 (m_{ba} \rho_{ab}) - 2N \nabla \cdot [(m_{ba} \rho_{ab})] = 0. \quad (2.34)$$

Since the magnetic dipole moment $m_{ba} \rho_{ab}$ is induced by the external magnetic field $B$, the direction of $m_{ba} \rho_{ab}$ (or $L + 2S$) will be parallel to $B$. If the external electromagnetic radiation is a time harmonic planar wave, the wave vector will be perpendicular to the magnetic field $B$. This, therefore, means that $k \perp (L + 2S)$, i.e., $\nabla \cdot (m_{ba} \rho_{ab}) = 0$. So, Eq. (2.34) reads

$$\frac{1}{\mu_0} \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) B + 2N \nabla^2 (m_{ba} \rho_{ab}) = 0. \quad (2.35)$$

Since $m_{ba} \rho_{ab}$ is parallel to $B$, the magnetic dipole moment can be rewritten as $m_{ba} = (m_{ba} / B) B$. Then, Eq. (2.35) takes the form

$$\frac{1}{\mu_0} \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) B + \frac{2N m_{ba} \rho_{ab}}{B} \nabla^2 B = 0, \quad (2.36)$$

or

$$\epsilon_0 \frac{\partial^2}{\partial t^2} B - \frac{1}{\mu_0} \left( 1 - \frac{2N \mu_0 m_{ba} \rho_{ab}}{B} \right) \nabla^2 B = 0. \quad (2.37)$$

It follows from Eq. (2.37) that the effective relative magnetic permeability is of the form

$$\mu_r = \left( 1 - \frac{2N \mu_0 m_{ba} \rho_{ab}}{B} \right)^{-1}. \quad (2.38)$$

An alternative approach to deriving Eq. (2.38) can be presented as follows: it follows from the two relations $B = \mu_0 (H + M) = \mu_0 \mu_r H$ and $M = (\mu_r - 1) H$ that the magnetic susceptibility is $\mu_r - 1 = M / H = \mu_0 \mu_r M / B$. Since the magnetization is defined by $M = 2N m_{ba} \rho_{ab}$, the susceptibility $\mu_r - 1 = \mu_0 \mu_r (2N m_{ba} \rho_{ab})$. Thus, the magnetic permeability is $\mu_r = (1 - 2N \mu_0 m_{ba} \rho_{ab} / B)^{-1}$, which has the same form as Eq. (2.38).

In the above discussion, we did not take into consideration the local field effect, which is caused by the dipole-dipole interaction between neighboring atoms. Usually the dimensionless ratio $|\vec{m} / (\vec{g} c)|$ is small (about $10^{-2}$) in an atomic system. For this reason, the magnetic-dipole transition is usually negligible in some conventional materials. However, there may be two cases, which need consideration of local field effect:

i) in dense gas;

ii) in off-resonant case of EIT. The resonant behavior of an EIT medium is transparency to a resonant probe light, and the real part of the susceptibility is nearly the same as vacuum. This does not need to take into account the local field effect. In an off-resonant case (but not very far away from the central frequency), the local field effect, however, may be significant and deserves consideration (particularly in a dense EIT gas).

In the section that follows, we shall analyze the local field correction (and hence the Clausius-Mossotti relation).
2.6 Local field correction and Clausius-Mossotti relation

Before examining how the detailed properties of the atomic transitions between levels are related to the electric susceptibility, we must make a distinction between the fields acting upon the atoms (molecules) in the (dense) gas medium and the applied fields. The electric susceptibility \( \chi_e \) is defined through the relation \( \mathbf{P} = \epsilon_0 \chi_e \mathbf{E} \), where \( \mathbf{P} \) is the electric polarization (per volume) and \( \mathbf{E} \) the macroscopic electric field. In a dilute gas medium where atomic (or molecular) separations are large, the dipole-dipole interaction is thus very weak and can be ignored, and there is little difference between the macroscopic field and that acting on any atoms (molecules or group of molecules). But in a dense medium with closely packed atoms (molecules) the polarization of neighboring atoms (molecules) gives rise to an internal field \( \mathbf{E}_i \) at any given atom (molecule) in addition to the average macroscopic field \( \mathbf{E} \), so that the total field at the atom (molecule) is \( \mathbf{E} + \mathbf{E}_i \) [19]. The internal field \( \mathbf{E}_i \) can be written as the difference of two terms, i.e., \( \mathbf{E}_i = \mathbf{E}_{\text{near}} - \mathbf{E}_p \), where \( \mathbf{E}_{\text{near}} \) is the actual contribution of the atoms (molecules) close to the given atom (molecule) and \( \mathbf{E}_p \) denotes the contribution from those atoms (molecules) treated in an average continuum approximation described by the polarization \( \mathbf{P} \). \( \mathbf{E}_p \) in a medium is \( \mathbf{E}_p = -\frac{\mathbf{P}}{\epsilon_0} \) [19]. Thus, the internal field is \( \mathbf{E}_i = \frac{\mathbf{P}}{\epsilon_0} + \mathbf{E}_{\text{near}} \).

Generally speaking, the field due to the atoms (molecules) nearby is more difficult to determine. Lorentz showed that for atoms in a simple cubic lattice \( \mathbf{E}_{\text{near}} \) vanishes at any lattice site. The argument depends on the symmetry of the problem [19]. If \( \mathbf{E}_{\text{near}} = 0 \) for a highly symmetric situation, it seems plausible that \( \mathbf{E}_{\text{near}} = 0 \) also for completely random situations. Hence we expect amorphous substances to have no internal field due to nearby atoms and molecules. For lattices other than simple cubic structure, the components of \( \mathbf{E}_{\text{near}} \) can be determined through a traceless tensor that has the symmetry properties of the lattice. Nevertheless, it is a good working assumption that \( \mathbf{E}_{\text{near}} = 0 \) for most materials [19]. If the atomic polarizability due to atomic transition is \( \gamma_e \) (the ratio of the average atomic dipole moment to \( \epsilon_0 \) times the applied field at the atom), the polarization vector is \( \mathbf{P} = N\gamma_e \epsilon_0 \left( \mathbf{E} + \frac{\mathbf{P}}{\epsilon_0} \right) \), where \( \mathbf{E} \) is the average macroscopic field strength and \( N \) the atomic number density per volume (concentration). With the help of \( \mathbf{P} = \epsilon_0 \chi_e \mathbf{E} \), one can arrive at \( \chi_e = \frac{N\gamma_e}{1 - \frac{2\epsilon}{\epsilon_0}} \), which is the relation between the electric susceptibility (the macroscopic parameter) and the atomic polarizability (the microscopic parameter). The relation holds best for dilute substances such as gases [19]. In the meanwhile, the Clausius-Mossotti relation is [20, 21]

\[
N\gamma_e = \frac{3(\epsilon_r - 1)}{\epsilon_r + 2},
\]

(2.39)

where \( \epsilon_r \) denotes the relative permittivity.

When considering the EIT-realization of negative refractive index materials (left-handed media), we shall use the Clausius-Mossotti relations for the electric and magnetic responses to calculate the electric permittivity and the magnetic permeability [22]. Fig. 2.5 shows the differences of both the electric permittivity and the magnetic permeability between the two cases with and without local field correction (LFC).

2.7 Dynamical equations with both electric and magnetic transitions in a three-level system

To the best of our knowledge, the magnetic transition in a three-level EIT system may have never been considered in the literature. Though the magnetic transition is in general negligibly small, it would be drastically enhanced under the condition with some proper atomic parameters (this may be significant for the realization of EIT-based negative refractive index). Here, we present the dynamical equations with both the electric and the magnetic transitions in a three-level \( \Lambda \)-type EIT atomic system (see Fig. 1.5 in Chapter 1 for the \( \Lambda \)-type schematic diagram), where the electric-dipole allowed transition \( (|a\rangle-|b\rangle) \) and the magnetic-dipole allowed transition \( (|c\rangle-|b\rangle) \) may give rise to the electric polarization and the magnetization, respectively. By using the equation of motion (the decay terms will be phenomenologically added later)

\[
\frac{\partial}{\partial t} \rho = -\frac{i}{\hbar} [\mathcal{H}, \rho]
\]

(2.40)

for the density operator (matrix), \( \rho \), of the three-level atomic system [the Hamiltonian \( \mathcal{H} \) is given by Eq. (2.3)], one can obtain the equations governing the off-diagonal density matrix elements

\[
\frac{\partial}{\partial t} \rho_{ab} = -i\omega_{ab}\rho_{ab} + i\frac{1}{2}[\Omega_{ac} e^{-i\phi_a} e^{-iv_a t}(\rho_{bb} - \rho_{aa}) + \Omega_{bc} e^{-i\phi_c} e^{-iv_c t}\rho_{ab} - \Omega_{ac} e^{-i\phi_a} e^{-iv_a t}\rho_{ac}],
\]
and the frequency detuning of the probe beam is \( \Delta \), which is defined by
\[
\Delta = \text{spontaneous decay and dephasing decay} \to \text{Eq. (2.42)},
\]
one can yield varying density operator) of the off-diagonal density matrix elements.

where dot denotes the time derivative. Thus we obtain the equations of motion of the envelopes (slowly treated using the above set of equations.

In the present EIT medium the coupling laser beam is in resonant with the \(|a\rangle\rightarrow|c\rangle\) transition (\(i.e., \omega_{ac} = \omega_c\)), and the frequency detuning of the probe beam is \(\Delta\), which is defined by \( \Delta = \omega_{ab} - \nu_p \) (\( \nu_p \) denotes the frequency of the probe beam). Substituting the relations \( \rho_{ab} = \tilde{\rho}_{ab} e^{-i\nu_p t}, \rho_{cb} = \tilde{\rho}_{cb} e^{-i(\nu_p + \omega_{ac}) t} \) and \( \rho_{ac} = \tilde{\rho}_{ac} e^{-i\omega_{ac} t} \) into the above equations, one can obtain

\[
\begin{align*}
\dot{\tilde{\rho}}_{ab} &= -i\Delta \tilde{\rho}_{ab} + \frac{i}{2} \Omega_b e^{-i\phi_b} (\rho_{ab} - \tilde{\rho}_{aa}) + \frac{i}{2} \Omega_c e^{i\phi_c} \tilde{\rho}_{cb} - \frac{i}{2} \Omega_b e^{-i\phi_b} \tilde{\rho}_{ac} e^{-i\omega_{ac} t}, \\
\dot{\tilde{\rho}}_{cb} &= -i\Delta \tilde{\rho}_{cb} + \frac{i}{2} \Omega_b e^{-i\phi_b} (\rho_{cb} - \tilde{\rho}_{cc}) + \frac{i}{2} \Omega_c e^{i\phi_c} \tilde{\rho}_{ab} - \frac{i}{2} \Omega_b e^{-i\phi_b} \tilde{\rho}_{ca}, \\
\dot{\tilde{\rho}}_{ac} &= \frac{i}{2} \Omega_c e^{-i\phi_c} (\rho_{ac} - \tilde{\rho}_{aa}) + \frac{i}{2} \Omega_b e^{-i\phi_b} \tilde{\rho}_{bc} - \frac{i}{2} \Omega_b e^{-i\phi_b} \tilde{\rho}_{ab} e^{i\omega_{ac} t},
\end{align*}
\]

where dot denotes the time derivative. Thus we obtain the equations of motion of the envelopes (slowly varying density operator) of the off-diagonal density matrix elements.

By using the EIT condition: \( \tilde{\rho}_{ab} \simeq 1, \tilde{\rho}_{aa} \simeq 0, \tilde{\rho}_{cc} \simeq 0 \) and \( \tilde{\rho}_{ac} \simeq 0 \), and adding the decay terms (spontaneous decay and dephasing decay) to Eq. (2.42), one can yield

\[
\begin{align*}
\dot{\tilde{\rho}}_{ab} &= -(i\Delta + \gamma_1) \tilde{\rho}_{ab} + \frac{i}{2} \Omega_b e^{-i\phi_b} + \frac{i}{2} \Omega_c e^{-i\phi_c} \tilde{\rho}_{cb}, \\
\dot{\tilde{\rho}}_{cb} &= -(i\Delta + \gamma_3) \tilde{\rho}_{cb} + \frac{i}{2} \Omega_b e^{-i\phi_b} e^{i\omega_{ac} t} + \frac{i}{2} \Omega_c e^{i\phi_c} \tilde{\rho}_{ab}, \\
\dot{\tilde{\rho}}_{ac} &= -i\gamma_2 \tilde{\rho}_{ac} + \frac{i}{2} \Omega_b e^{-i\phi_b} \tilde{\rho}_{bc} - \frac{i}{2} \Omega_b e^{i\phi_b} \tilde{\rho}_{ab} e^{i\omega_{ac} t}.
\end{align*}
\]

Most of the effects (including the transient evolution) that occur in the three-level EIT system can be treated using the above set of equations.
Bibliography


Chapter 3

Quantum coherence, interferences and their applications

3.1 Controlling light propagations via quantum coherence and quantum interferences

In this chapter, we consider three topics on the manipulation of light propagations via quantum coherence and quantum interferences:

i) The evolutional optical behavior (turn-on dynamics) of a four-level N-configuration atomic system is studied based on the transient solution to the equations of motion of atomic probability amplitudes. It is shown that the quantum interference between the signal and control fields can lead to the controllable absorption and transparency properties of the atomic vapor. One of the most remarkable properties is that the absorption (or transmittance) of the probe light in the atomic vapor depends on the intensity ratio of the signal field to the control field, and thus the tunable optical features (transparent and opaque to the probe light) can be realized by tuning the quantum interference between the signal and control fields. The present mechanism can be applicable to designs of some new photonic and quantum optical devices such as logic and functional devices as well as optical switches. Two typical photonic logic gates (NOT and NOR gates) designed based on the tunable four-level optical responses are presented as illustrative examples.

ii) We show that the destructive and constructive quantum interferences would be exhibited in a four-level double-control atomic systems, where the probe transition (driven by the probe field) can be manipulated by the quantum interferences between two control transitions (driven by the control fields) of the four-level system. The atomic vapor is opaque (or transparent) to the probe field if the destructive (or constructive) quantum interference between the control transitions emerges. The optically sensitive responses due to double-control quantum interferences can be utilized to realize some quantum optical and photonic devices such as the logic-gate devices, e.g., the NOT, OR, NOR and EXNOR gates.

iii) We suggest some quantum coherent schemes for realizing negative refractive indices (to realize simultaneously negative electric permittivity and magnetic permeability). In order to obtain a negative permeability, we choose a proper atomic configuration that can dramatically enhance the contribution of the magnetic-dipole allowed transition via the atomic phase coherence. We present two scenarios: a) a Lambda-type atomic system; b) a dressed-state mixed-parity system. Differing from the previous schemes of artificial composite metamaterials (based on the classical electromagnetic theory) to achieve the left-handed materials, which consist of anisotropic millimeter-scale composite structure units, the left-handed atomic vapor presented here is isotropic and homogeneous at the atomic-scale level. Such an advantage may be valuable for realizing superlens (and hence perfect image) with left-handed atomic vapors.

3.2 Tunable transient behavior of a four-level atomic vapor

With the development of photonics and quantum optics [1], new techniques to manipulate light wave propagations using artificial electromagnetic materials have captured a lot of attention of many scientists in various areas [2, 3]. One of the flexible and promising schemes to realize the manipulation of light propagations is quantum coherence [2, 3, 4, 5]. During the past two decades, quantum coherence (atomic
phase coherence) effect has exhibited many physically interesting phenomena such as electromagnetically induced transparency (EIT) [2], light amplification without inversion [3], spontaneous emission cancellation [4], multi-photon population trapping [5], coherent phase control [6, 7] and sensitive optical responses in EIT waveguides [8]. Recently, quantum coherence in photonic applications has received increasingly more attention from physicists in both fundamental and applied areas, e.g., EIT-based coherent information storage [9, 10] and quantum logic gates [1]. Since the transient evolution accompanies these physical processes (i.e., the information storage and the logic-gate operations) when switching on (and off) the control fields, the time-dependent behavior of atomic media deserves consideration. In the literature, Li et al. investigated the resonant transient properties induced by a quantum interference effect in a three-level EIT system [11]; Greentree et al. studied both the resonant and off-resonant transient behavior of a three-level EIT medium [12]. More recently, Yao et al. dealt with the problem of transient optical responses in a four-level N-configuration system [13] under certain approximation conditions (this is, however, not the exact solutions to the equation of motion of the atomic system) [14]. Besides the N-configuration system, there is another interesting four-level system (tripod configuration) [15] that can exhibit nontrivial double-control destructive and constructive quantum interferences [16, 17]. As the four-level EIT vapor exhibits a large dispersion in both the real and imaginary parts of optical ‘constants’, the optical properties would be more sensitive to the probe frequency as compared with a three-level EIT vapor. The quantum interference in the four-level systems can hence be applicable to many new techniques such as sensitive optical switches, optical magnetometers and wavelength sensors. For example, the optical magnetometers could be used to detect magnetic fields with very high sensitivity, and the multilevel EIT-based wavelength sensor can be utilized to measure the probe wavelength using the strong dispersion caused by atomic phase coherence. Such devices can have some practical applications like color matching and sorting, where precise measurements of light wavelengths or frequencies are required.

The transient evolutional behavior of the atomic vapors is one of the important physical phenomena and effects of the future technology of light storage (and hence quantum coherent information storage) with atomic levels [9, 10]. Obviously, the impact would be enormous if the transient evolution of both the atomic vapors and the probe fields can be tuned very conveniently using the quantum interferences between the external control fields (and signal fields). Here, we will show the tunable steady optical behavior (transparent and opaque to the probe laser beam) of an N-configuration atomic vapor, and then present the numerical solutions to the equation of motion of the atomic probability amplitudes, and consider the transient electric permittivity and medium absorption induced by the effects of quantum interference and phase coherence. We will analyze the transient evolutional behavior of the atomic electric polarizability, the permittivity and the refractive index (in the case when the external control and signal fields are switched on), and see how fast the atomic medium absorption and related optical responses respond to the switchings of the external optical fields. It is interesting that the quantum interference between the signal and control fields can lead to the controllable absorption and transparency properties of the atomic vapor, i.e., the tunable optical behavior (transparent and opaque to the probe laser beam) can be realized by tuning the intensity ratio of the signal field to the control field. The potential applications, such as photonic logic gates and optical switches, are suggested based on the steady and transient evolutional behaviors of the present multilevel atomic system.

### 3.2.1 N-configuration system and its steady optical behavior

We first present the steady solutions to the equation of motion of the four-level probability amplitudes, and show its steady optical behavior. Such steady optical properties depend on the intensities of the control and signal fields (particularly, it has a close relation to the Rabi-frequency ratio of the signal field to the control field). In other words, the optical properties of the present atomic vapor depend on the quantum interference between the signal and control fields.

Now we consider a four-level N-configuration atomic system interacting with three optical fields, i.e., the control beam, the signal beam and the probe field whose Rabi frequencies are denoted by $\Omega_c$, $\Omega_s$ and $\Omega_p$, respectively. The configuration of the four-level system is depicted in Fig. 3.1. In such a four-level atomic system of N-type, levels $|1\rangle$ and $|2\rangle$ are the ground states, and levels $|3\rangle$ and $|4\rangle$ are the excited states. The probe, control and signal fields couple the level pairs $|1\rangle$-$|3\rangle$, $|2\rangle$-$|3\rangle$, $|2\rangle$-$|4\rangle$, respectively. Such an atomic system $\{|1\rangle, |2\rangle, |3\rangle, |4\rangle\}$ can be found in some alkali metallic atoms, e.g. in the neutral Sodium $\{3^2S_{1/2}, 3^2S_{1/2}, 3^4P_{1/2}, 5^5P_{1/2}\}$, where each energy level is $0.000, 25739.991, 30266.990, 35040.380 \text{ cm}^{-1}$, respectively. Based on the Schrödinger equation, the equation of motion that governs the probability
3.2. TUNABLE TRANSIENT BEHAVIOR OF A FOUR-LEVEL ATOMIC VAPOR

Figure 3.1: The schematic diagram of a four-level N-type atomic coherent system. The atomic vapor is transparent to the probe light ($\Omega_p$) when switching off the signal light ($\Omega_s$). But when the signal beam is switched on, the giant absorption at the probe resonant frequency arises. The transmittance of the probe light depends on the intensity ratio of the signal beam to the control beam.

amplitudes of the above four-level atomic system is given by

\[
\begin{align*}
\hat{a}_1 &= \frac{i}{2} \Omega_s^* a_3, \\
\hat{a}_2 &= -i(\Delta_p - \Delta_c)a_2 + \frac{1}{2}(\Omega_s^* a_3 + \Omega_s^* a_4) - \frac{2}{\hbar} \omega_p a_2, \\
\hat{a}_3 &= -i\Delta_p a_3 + \frac{1}{2} (\Omega_c a_2 + \Omega_p a_1) - \frac{\Gamma_3}{2} a_3, \\
\hat{a}_4 &= -i(\Delta_s + \Delta_p - \Delta_c) a_4 + \frac{1}{2} \Omega_c a_2 - \frac{\Gamma_4}{2} a_4,
\end{align*}
\tag{3.1}
\]

where the atomic level decay terms have been added phenomenologically [13, 18]. Here, $\Gamma_3, \Gamma_4$ in Eq. (3.1) denote the spontaneous emission decay rates of levels $|3\rangle$ and $|4\rangle$, and $\gamma_{21}$ is the collisional dephasing rate (nonradiative decay rate). The frequency detunings of $\Omega_c, \Omega_p$ and $\Omega_s$ are $\Delta_c, \Delta_p$ and $\Delta_s$, respectively, which are defined by $\Delta_p = \omega_3 - \omega_1 - \omega_p$, $\Delta_c = \omega_4 - \omega_2 - \omega_c$, and $\Delta_s = \omega_4 - \omega_2 - \omega_s$.

By using the formula of the atomic electric polarizability $\gamma_e = 2\varphi_{13}^* a_3/(\epsilon_0 \mathcal{E}_p)$ (derived for the probe field) [19], one can arrive at

\[
\gamma_e = \frac{2\varphi_{13}^* a_3}{\epsilon_0 \hbar \Omega_p},
\tag{3.2}
\]

where the relation $\mathcal{E}_p = \hbar \Omega_p/\varphi_{31}$ has been inserted. As is well known, one must make a distinction between the applied fields (macroscopic fields) and the microscopic local fields acting upon the atoms in the vapor when discussing how the properties of the atomic transitions between the levels are related to the electric and magnetic susceptibilities. In a dilute vapor, there is little difference between the macroscopic fields and the local fields that act on any atoms (molecules or group of molecules) [20]. But in a dense medium with closely packed atoms (molecules), the polarization of neighboring atoms (molecules) would give rise to an internal field at any given atom in addition to the external macroscopic field, so that the total fields at the atom are very different from the external macroscopic fields [20]. This difference may lead to a local field correction to the macroscopic quantities (such as the electric permittivity) of the medium. The Clausius-Mossotti relation can reveal the connection between the macroscopic permittivity and the microscopic polarizability. The expression of this relation is of the form $\gamma_e = (3/N)(\epsilon_\tau - 1)/(\epsilon_\tau + 2)$ [20], where $N, \gamma_e, \epsilon_\tau$ denote the atomic concentration, the atomic polarizability and the relative permittivity, respectively, of the atomic vapor medium. Thus after the local field correction (LFC) the relative electric permittivity is given by

\[
\epsilon_\tau^{(\text{LFC})} = \frac{1 + \frac{2}{3} N \gamma_e}{1 - \frac{1}{3} N \gamma_e},
\tag{3.3}
\]

In addition to the linear term, there are the nonlinear terms $N^n$ in the Taylor expansion series of Eq. (3.3), because $\epsilon_\tau^{(\text{LFC})}$ contains the contribution of atomic dipole-dipole interactions.

In general, the intensity of the probe field is sufficiently weak and the Rabi frequency of the probe field is small compared with the Rabi frequencies of external control fields as well as some spontaneous emission decay rates. According to Eq. (3.1), the right-handed side of equation $\dot{a}_1 = (i/2)\Omega_s^* a_3$ is negligibly small.
since both $\Omega_s$ and $a_3$ are small compared with $\Omega_c$ and $a_1$, respectively. In what follows (for the steady case), we assume that $a_1$ is a constant number (this assumption can be verified using the numerical results for transient evolutions in the next subsection. See, for example, in Fig. 3.2), which would simplify the problem under consideration. The equations for the probability amplitudes $a_2$, $a_3$ and $a_4$ in (3.1) can be rewritten in the following matrix form

$$
\frac{\partial}{\partial t} \begin{pmatrix} a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} - \left[ \frac{\gamma_s}{2} + i(\Delta_p - \Delta_c) \right] & \frac{1}{2} \Omega_c^* & \frac{1}{2} \Omega_s^* \\ \frac{i}{2} \Omega_c & - \left( \frac{\gamma_s}{2} + i\Delta_p \right) & 0 \\ \frac{i}{2} \Omega_s & 0 & - \left[ \frac{\gamma_s}{2} + i(\Delta_s + \Delta_p - \Delta_c) \right] \end{pmatrix} \begin{pmatrix} a_2 \\ a_3 \\ a_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.
$$

(3.4)

The steady solution to the above matrix equation is of the form

$$
a_2 = \frac{1}{\mathcal{D}} \frac{\Omega_c^*}{4} \left\{ \frac{\Gamma_3}{2} + i(\Delta_s + \Delta_p - \Delta_c) \right\} \Omega_pa_1, $$

$$
a_3 = -\frac{i}{\mathcal{D}} \frac{1}{2} \left[ \left( \frac{\Gamma_4}{2} + i(\Delta_s + \Delta_p - \Delta_c) \right) \left[ \frac{\gamma_s}{2} + i(\Delta_p - \Delta_c) \right] + \frac{1}{4} \Omega_s^* \Omega_p \right] \Omega_pa_1, $$

$$
a_4 = \frac{1}{\mathcal{D}} \frac{i}{8} \Omega_s \Omega_c \Omega_pa_1, $$

(3.5)

where the denominator $\mathcal{D}$ is

$$
\mathcal{D} = -\left\{ \frac{\Omega_c^* \Omega_s}{4} + \left[ \frac{\gamma_s}{2} + i(\Delta_p - \Delta_c) \right] \left( \frac{\Gamma_3}{2} + i\Delta_p \right) \right\} \left[ \frac{\Gamma_4}{2} + i(\Delta_s + \Delta_p - \Delta_c) \right] - \frac{\Omega_s^* \Omega_s}{4} \left( \frac{\Gamma_4}{2} + i\Delta_p \right).
$$

(3.6)

By using the formula of the atomic electric polarizability (3.2), and substituting the steady solution (3.5) for $a_3$ and expression (3.6) for $\mathcal{D}$ into expression (3.2), one can yield

$$
\gamma_c = \frac{i|p_{31}|^2 a_1^* a_1}{\epsilon_0 \hbar} \frac{\Omega_s^* \Omega_c}{4} + \left[ \frac{\gamma_s}{2} + i(\Delta_p - \Delta_c) \right] \left( \frac{\Gamma_3}{2} + i\Delta_p \right) \left[ \frac{\gamma_s}{2} + i(\Delta_s + \Delta_p - \Delta_c) \right]
$$

(3.7)

The Kerr nonlinearity due to EIT could be studied based on Eq. (3.7) [19]. By using expressions (3.3) and (3.7) one can obtain the relative electric permittivity of the four-level atomic vapor at the probe frequency in the presence of both control and signal fields.

Now we consider the controllable optical properties of the present atomic vapor induced by the quantum interference between the signal and control fields. It follows that Eq. (3.7) would be reduced to the traditional three-level EIT atomic microscopic electric polarizability

$$
\gamma_c \rightarrow \frac{i|p_{31}|^2 a_1^* a_1}{\epsilon_0 \hbar} \frac{\Omega_s^* \Omega_s}{4} + \left[ \frac{\gamma_s}{2} + i(\Delta_p - \Delta_c) \right] \left( \frac{\Gamma_4}{2} + i\Delta_p \right) \left[ \frac{\gamma_s}{2} + i(\Delta_s + \Delta_p - \Delta_c) \right],
$$

(3.8)

if the ratio $\Omega_s/\Omega_c$ is small; whereas, if the ratio $\Omega_s/\Omega_c$ is very large, the atomic polarizability (3.7) can be reduced to that of a typical two-level resonant absorption

$$
\gamma_c \rightarrow \frac{i|p_{31}|^2 a_1^* a_1}{\epsilon_0 \hbar} \frac{\Gamma_4}{4} \left( \frac{1}{\Gamma_4} + i\Delta_p \right).
$$

(3.9)

This means that the medium absorption in the atomic vapor would increase as the ratio $\Omega_s/\Omega_c$ increases. In other words, if the control field is present, then once we switch off the signal field, the atomic vapor is transparent to the probe light, and once we switch on the signal field, the probe light would be stored in the atomic levels. Since the atomic microscopic electric polarizability (3.7) of the four-level system is more complicated than that of the three-level electric polarizability (3.8), there are more peaks and valleys in the dispersive curve of the four-level permittivity than that of the conventional three-level permittivity, and the dispersion in the four-level vapor is therefore more sensitive to the probe frequency than in the three-level vapor. Therefore, more intriguing optical properties would arise in the four-level atomic vapor, e.g., the effects of slow light and negative-group-velocity propagations.
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Figure 3.2: The transient behavior of the real and imaginary parts for the atomic probability amplitudes $a_1$, $a_2$, $a_3$ and $a_4$. The blue solid and red dashed curves represent the real and imaginary parts, respectively. The oscillatory probability amplitudes finally approach their respective steady values.

Figure 3.3: The transient behavior of the real and imaginary parts of the relative electric permittivity of the atomic vapor. The electric permittivity tends to the steady value after the relaxation process.
Figure 3.4: The transient behavior of the real and imaginary parts of the relative refractive index, $n_r$, of the atomic vapor. The real and imaginary parts of the refractive index tend to their own steady values after the relaxation process.

Figure 3.5: The transient behavior of the absorption coefficient, $\alpha$, of the atomic vapor.
3.2. TUNABLE TRANSIENT BEHAVIOR OF A FOUR-LEVEL ATOMIC VAPOR

3.2.2 The transient evolutionary behavior

As the atomic polarizability and the permittivity can be manipulated by changing the external fields, the transient evolutionary behavior of this multilevel atomic vapor deserves consideration. Here we present the transient case of optical responses and show how the four-level atomic probability amplitudes and the permittivity (and hence the refractive index and the absorption coefficient) evolve from the initial values of one state to another steady state when the external control and signal fields are switched on. The typical atomic and electrical parameters of the present atomic vapor are chosen as follows: the decay rates \( \Gamma_1 = 2.0 \times 10^6 \text{ s}^{-1} \), \( \Gamma_2 = 5.0 \times 10^7 \text{ s}^{-1} \), \( \gamma_21 = 0.01 \text{ ms}^{-1} \), and the electric dipole moment \( |\mu| = 1.0 \times 10^{-29} \text{ Cm} \).

The frequency detunings of the three optical fields are \( \Delta_2 = 0.1 \Gamma_3 \), \( \Delta_c = 0.01 \Gamma_3 \), and \( \Delta_s = 0.01 \Gamma_3 \), and the Rabi frequencies are \( \Omega_c = 1.0 \times 10^8 \text{ s}^{-1} \), \( \Omega_s = 3.0 \times 10^7 \text{ s}^{-1} \), and \( \Omega_p = 0.01 \Omega_c \). The atomic concentration (number density per volume) is \( N = 1.0 \times 10^{21} \text{ m}^{-3} \). The initial values of the probability amplitudes are chosen: \( a_1 = \sqrt{2}/4 \), \( a_2 = \sqrt{5}/4 \), \( a_3 = \sqrt{6}/4 \), and \( a_4 = \sqrt{2}/4 \), which satisfy the normalization relation \( \sum |a_i|^2 = 1 \).

The real and imaginary parts for the atomic probability amplitudes \( a_1 \), \( a_2 \), \( a_3 \), and \( a_4 \) are shown in Fig. 3.2. The atomic probability amplitude of each level evolves from the initial value and oscillatorily changes drastically, while the real and imaginary parts can be considered to change much (and its real and imaginary parts can be considered to be constant numbers) during the transient process. This is consistent with the analysis presented in the preceding section.

In Figs. 3.3 and 3.4, we plot the transient behaviors of the relative electric permittivity and the relative refractive index of the four-level atomic vapor. When the control and signal fields are turned on, the imaginary part of the refractive index \( n_i \) of the vapor is oscillatorily damped (exponentially) to its steady value, whereas the real part oscillatorily increases, and finally tends to its own steady value.

The dimensionless absorption coefficient \( \alpha \) is shown in Fig. 3.5 as a function of time \( t \). Here, the dimensionless \( \alpha \) is defined as \( 2\pi n''/n''_i \), where \( n''_i \) and \( n'' \) denote the real and imaginary parts of \( n_i \), respectively. For a time-harmonic plane wave, the oscillatory electric field amplitude \( \exp(ikx) = \exp(in''_i \omega x/c) \exp(-n'' \omega x/c) \), where the wave number \( k = (n'' + in''_i) \omega /c \).

In an absorptive medium, the light wavelength is \( \lambda = 2\pi n''_i \omega /c \). Thus, the oscillatory electric field amplitude can be rewritten as \( \exp(ikx) = \exp(2\pi x/\lambda) \exp(-a x/\lambda) \). It follows that when the ratio \( \Gamma/\Gamma_3 \) is large, the absorption coefficient \( \alpha \) increases rapidly with time \( t \rightarrow +\infty \). It can be seen that both the real and imaginary parts of \( a_2 \), \( a_3 \), and \( a_4 \) change drastically, while \( a_1 \) does not change much (and its real and imaginary parts can be considered to be constant numbers) during the transient process. This is consistent with the analysis presented in the preceding section.

The tunable optical property of the present four-level system is that the atomic vapor can be transparent or opaque to the probe field under proper density conditions of the external control and signal fields. We define the ratio \( R = \Omega_c/\Omega_p \) (describing the quantum interference between the signal and control fields), and show in Fig. 3.6 that the transient behavior of the absorption coefficient \( \alpha \) depends on \( R \). In Fig. 3.6 all the optical and atomic parameters (except for the two Rabi frequencies \( \Omega_s \) and \( \Omega_c \)) are chosen exactly the same as those used in the preceding section.

It follows that when the ratio \( R \) is greater than zero and less than (or equal to) two, \( \alpha \) increases rapidly while \( R \) is greater than two and less than five it increases with much slower rate. It reaches its saturation level for \( R \) greater than five, and from this point it is inversely time dependent, e.g., increasing time will decrease \( \alpha \). We have drawn two black solid lines to mark these borders in Fig. 3.6, where we have plotted three time series (1.5 \( \mu s \), 3 \( \mu s \), and 5 \( \mu s \)) of \( \alpha \) to illustrate this. The red solid, black dotted and blue dashed lines are extrapolating lines through the data points. When \( \alpha \) is large (\( R > 5 \)) the complete absorption of the probe light will occur (the probe light can propagate through the distance of only several wavelengths in the vapor). When \( \alpha \) is small (\( R < 5 \)) the probe light can propagate through the medium even without any dissipation (particularly for the case of switching off the signal field, i.e., \( R = 0 \), which corresponds to the case of EIT). This means that the probe light can be stored and released by the atoms when we adjust the value of \( R \). Obviously, the absorption and transparency of the atomic medium to the probe light is induced by the quantum interference between the signal and control fields. This property can be used to realize light storage, optical switches, and photonic logic gates, where the transient evolution (including the dispersion and dissipation of the stored light during the storage and readout processes) deserves consideration when

\[ \alpha = 2\pi n'' \]
we attempt to achieve such photonic devices with high sensitivity and efficiency. Since the atomic system has to experience a transient evolution once the control fields are switched on or off, the transient evolution is a very important physical process when one considers the mechanism of storage and readout of pulses for the future technology of quantum coherent information storage. In the literature, some authors treated the problem of light storage with atoms [10, 11], but did not consider the transient evolution of atomic polarizability and permittivity.

In the subsection that follows, we will give some illustrative examples to demonstrate the application of the four-level vapor to the photonic logic gates.

3.2.3 Illustrative example of photonic logic gates

Recently, ideas of realizing logic gates by using new optoelectronic materials have attracted attention of some researchers [1]. We believe that the tunable multilevel quantum interference effect can also be used to realize some logic and functional operations (e.g. the operations of NOT and NOR gates). In the preceding subsections, we have shown that the quantum interference between the signal and control fields can be used to realize the tunable absorption and transparency of the atomic vapor to the probe light. This implies that one could control the absorption coefficient of the medium by tuning the intensity of the signal field. Here we give some examples to show how the photonic logic gates, such as the NOT and NOR gates, work based on the four-level quantum interference effect, where the signal and probe fields behave like the input and output optical fields, respectively.

In Fig. 3.7, the NOT gate is designed using the tunable four-level optical responses. In this scheme, the probe and control fields are always present. The incident signal field and the transmitted probe field are viewed as the INPUT signal and the OUTPUT signal, respectively. We assume that the INPUT operation is IN = 1 when the signal field is switched on, and IN = 0 when the signal field is switched off. If the probe light can propagate through the atomic vapor (i.e., the absorption coefficient $\alpha$ is very small), then it implies that the OUTPUT operation result is OUT = 1; whereas, we have OUT = 0 if $\alpha$ is large and the probe light is absorbed by the atoms. Now consider the four-level N-type NOT logic gate. When the signal field is absent and $R$ is zero, we have IN = 0 (i.e., the INPUT is low). Thus, the N-type system will be reduced to the traditional three-level EIT system [as pointed out by Eq. (3.8)], and then the probe field can propagate through the atomic vapor, i.e., OUT = 1. However, the probe field will be stored in the vapor and OUT = 0 when the signal field is switched on, IN = 1 (i.e., the INPUT is high) and $R$ is
3.2. TUNABLE TRANSIENT BEHAVIOR OF A FOUR-LEVEL ATOMIC VAPOR

Figure 3.7: The schematic diagram of a NOT gate. When the signal field is switched on (IN = 1), the probe light is absorbed by the vapor, and hence OUT = 0. However, when the signal field is switched off (i.e. IN = 0), and then OUT = 1, since the absorption coefficient \( \alpha \) is very small or even zero.

Figure 3.8: The schematic diagram of a two-input NOR gate. If one of the signal fields is switched on, then the probe light would be stored, while the two signal fields are both switched off, the probe light can propagate through the two vapor media.

very large. This is the working mechanism of the photonic NOT logic gate, where the OUTPUT is always opposite to the INPUT.

In Fig. 3.8, we present another typical photonic logic gate, the two-input NOR (NOT-OR) gate that consists of two four-level N-type atomic vapors. The NOR gate produces a low output if any of the inputs are high (obviously, the output is high if all the inputs are low). According to the truth table of the two-input NOR gate, if \( \text{IN}_A = A \) and \( \text{IN}_B = B \), then \( \text{OUT} = \overline{A + B} \). If, for example, the two signal fields are both absent, i.e., \( \text{IN}_A = \text{IN}_B = 0 \), then each ratio \( R \) in the two vapors is zero (corresponding to EIT), the absorptions of the two atomic vapors to the probe light are small or zero, and the probe light can propagate through the two atomic vapors. Thus, we have \( \text{OUT} = 1 \). If, however, any of the two signal fields is switched on (corresponding to the cases: \( \text{IN}_A = \text{IN}_B = 1 \), or \( \text{IN}_A = 1, \text{IN}_B = 0 \), or \( \text{IN}_A = 0, \text{IN}_B = 1 \) ), at least one of the absorption coefficients \( \alpha \) is large, and therefore, the probe light would be absorbed in one of the vapors, i.e., \( \text{OUT} = 0 \). Thus, these processes yield the logic operations of the two-input NOR logic gate.

As we have pointed out, the response time (relaxation time) of such photonic logic gates is microseconds, this effect can be used in the technology of optical communications, where high-speed signal response is needed. Apart from the logic gates, the optical switches can also be designed by using the tunable optical responses of the present atomic vapor. Such switches might be a useful technique for future photonic microcircuits on silicon, in which light replaces electrons. At present, the all-optical switch on silicon where
CHAPTER 3. QUANTUM COHERENCE, INTERFERENCES AND THEIR APPLICATIONS

Figure 3.9: The schematic diagram of a Y-configuration double-control atomic system. The two control laser beams, $\Omega_c$ and $\Omega_{c'}$, drive the $|3\rangle-|2\rangle$ and $|3\rangle-|2'\rangle$ transitions, respectively. The probe transition $|1\rangle-|3\rangle$ can be tunable via the constructive (or destructive) quantum interference between the $|3\rangle-|2\rangle$ and $|3\rangle-|2'\rangle$ transitions. The atomic medium is transparent (opaque) to the probe field if the constructive (destructive) quantum interference between the $|3\rangle-|2\rangle$ and $|3\rangle-|2'\rangle$ transitions emerges.

one controls light with light on chips has been increasingly developed, and we hope that the present optical switches based on multilevel quantum interference would have potential applications in this field and other related areas, e.g., the integrated optical circuits.

3.2.4 Discussions

Compared with the conventional three-level EIT systems, the four-level systems have more flexible optical responses, and would have more applications for designing new photonic and quantum optical devices, which can be used to control the wave propagation of the probe light. We have considered the steady and transient optical behavior of a four-level N-type atomic vapor. It was shown that the absorption of the atomic vapor would increase when the intensity of the signal field becomes stronger. When the signal field is absent, the present atomic vapor is transparent to the probe light, and when the signal field is switched on, the atomic vapor would be opaque to the probe light (particularly, the optical property of the present four-level system will be reduced to that of a two-level resonant absorption, if the signal field is adequately strong). Thus, one can manipulate the optical properties of the atomic vapor by controlling the quantum interference between the signal and control fields. The tunable optical properties induced by the quantum interference can have some realistic applications, such as designs of some new photonic devices, e.g., photonic logic gates and optical switches. We have presented some examples of photonic devices, where the signal and probe fields act as the input and output optical fields, respectively.

The strong dispersion in the permittivity may lead to dramatic modification to the speed of light in a medium. In the literature, the ultraslow light and the superluminal propagation (negative group velocity) in three-level atomic media have received attention from many researchers [21, 22]. As the dispersion in both the real and imaginary parts of optical ‘constants’ in the four-level atomic medium is stronger than that in a three-level EIT medium, the tunable ultraslow and superluminal propagations of light induced by the quantum interference between the signal and control fields deserve consideration for the present four-level atomic vapor.

3.3 Y-type quantum interferences and photonic logic gates

We first consider the destructive and constructive quantum interference effects in a Y-configuration double-control atomic system (see Fig. 3.9), study the influence of quantum interferences between the two control transitions (driven by the two control fields) on the probe transition (and hence on the optical responses of the Y-configuration atomic medium), and then show how to use the double-control quantum interferences to realize some basic functional and logic devices, e.g., the NOT, OR, NOR and EXNOR gates.
The total Hamiltonian of the four-level Y-configuration double-control atomic system is $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$, where $\mathcal{H}_0$ and $\mathcal{H}_1$ denote the atomic free Hamiltonian and the interaction Hamiltonian, respectively. The explicit expressions for the Hamiltonians with RWA (rotating wave approximation) are given by

$$
\mathcal{H}_0 = \hbar \omega_1 |1\rangle \langle 1| + \hbar \omega_2 |2\rangle \langle 2| + \hbar \omega_2' |2'\rangle \langle 2'| + \hbar \omega_3 |3\rangle \langle 3|,
$$

$$
\mathcal{H}_1 = -\frac{\hbar}{2} (\Omega_p e^{-i\omega p t} |3\rangle \langle 1| + \Omega_c e^{-i\omega c t} |2\rangle \langle 2| + \Omega_c e^{-i\omega c t} |2'\rangle \langle 2'| + \hbar C, 
$$

where the term H.C. denotes the Hermitian conjugate term, and $\hbar \omega_i$ represents the energy eigenvalues of the atomic level $|i\rangle$ ($i = 1, 2, 2', 3$). The parameters $\Omega_p$, $\Omega_c$, $\omega_c$ and $\omega_p$ are the Rabi frequencies and the mode frequencies of the control and probe beams, respectively. The Rabi frequencies in Eq. (3.10) are defined through $\Omega_p = \varphi_3 \mathcal{E}_p/\hbar$, $\Omega_c = \varphi_2 \mathcal{E}_c/\hbar$, and $\Omega_c = \varphi_{2'3} \mathcal{E}_c'/\hbar$, respectively. Here $\mathcal{E}_p$, $\mathcal{E}_c$, and $\mathcal{E}_{c'}$ denote the probe and control field envelopes. From the Liouville equation of motion, we can obtain the equations for the off-diagonal density matrix elements of the four-level system:

$$
\begin{align*}
\dot{\rho}_{31} &= -\left( \frac{\Gamma_3}{2} + i \Delta_p \right) \rho_{31} + \frac{i}{2} \Omega_p (\rho_{11} - \rho_{33}) + \frac{i}{2} (\Omega_c^* \rho_{21} + \Omega_c' \rho_{2'1}), \\
\dot{\rho}_{32} &= -\left( \frac{\gamma_2 + \Gamma_3}{2} - i \Delta_c \right) \rho_{32} + \frac{i}{2} \Omega_c (\rho_{22} - \rho_{33}) + \frac{i}{2} (\Omega_p \rho_{12} + \Omega_c' \rho_{22'}), \\
\dot{\rho}_{32'} &= -\left( \frac{\gamma_2' + \Gamma_3}{2} - i \Delta_c \right) \rho_{32'} + \frac{i}{2} \Omega_c (\rho_{22'} - \rho_{33}) + \frac{i}{2} (\Omega_p \rho_{12'} + \Omega_c \rho_{22'}), \\
\dot{\rho}_{21} &= -\left( \frac{\gamma_2}{2} + i (\Delta_p + \Delta_c) \right) \rho_{21} - \frac{i}{2} \Omega_p \rho_{23} + \frac{i}{2} \Omega_c \rho_{31}, \\
\dot{\rho}_{21'} &= -\left( \frac{\gamma_2'}{2} + i (\Delta_p + \Delta_c) \right) \rho_{21'} - \frac{i}{2} \Omega_p \rho_{23'} + \frac{i}{2} \Omega_c \rho_{31}, \\
\dot{\rho}_{22'} &= -\left( \frac{\gamma_2 + \gamma_2'}{2} + i (\Delta_c - \Delta_c) \right) \rho_{22'} + \frac{i}{2} \Omega_c \rho_{32} - \frac{i}{2} \Omega_c \rho_{32}, \\
\end{align*}
$$

where $\Gamma_3$ and $\gamma_2, \gamma_2'$ denote the spontaneous emission decay rate and the dephasing (nonradiative decay) rates, respectively. For simplicity, we assume that the intensity of the probe beam is sufficiently weak ($i.e. \Omega_p$ is very small compared with $\Gamma_3, \Omega_c$ and $\Omega_c'$), and therefore nearly all the atoms remain in the ground state, i.e., the atomic population in level $|1\rangle$ is unity. On the other hand, the term $-i\Omega_p \rho_{33}/2, -i\Omega_p \rho_{23}/2$ and $-i\Omega_p \rho_{23'}/2$ in the equations of $\dot{\rho}_{21}, \dot{\rho}_{21'}, \dot{\rho}_{22'}$ could be negligibly small. Under these conditions, the equations of motion of the density matrix elements $\rho_{21}, \rho_{21'}, \rho_{31}$ in Eq. (3.11) form a closed set of equation, which can be rewritten in the matrix form:

$$
\begin{pmatrix}
\dot{\rho}_{21} \\
\dot{\rho}_{21'} \\
\dot{\rho}_{31}
\end{pmatrix} =
\begin{pmatrix}
\frac{\gamma_2}{2} + i (\Delta_p + \Delta_c) & 0 & \frac{i}{2} \Omega_c \\
0 & \frac{\gamma_2'}{2} + i (\Delta_p + \Delta_c') & -\frac{i}{2} \Omega_c \\
\frac{i}{2} \Omega_c & \frac{i}{2} \Omega_c' & \frac{\gamma_2}{2} + i \Delta_p
\end{pmatrix}
\begin{pmatrix}
\rho_{21} \\
\rho_{21'} \\
\rho_{31}
\end{pmatrix} +
\begin{pmatrix}
0 \\
0 \\
\frac{i}{4} \Omega_c \Omega_c'
\end{pmatrix}.
$$

The steady solution to Eq. (3.12) is given by

$$
\begin{align*}
\rho_{21} &= -\frac{1}{4D} \Omega_p \Omega_c \left[ \frac{\gamma_2}{2} + i (\Delta_p + \Delta_c) \right], \\
\rho_{21'} &= -\frac{1}{4D} \Omega_p \Omega_c' \left[ \frac{\gamma_2'}{2} + i (\Delta_p + \Delta_c) \right], \\
\rho_{31} &= \frac{i}{2D} \left[ \frac{\gamma_2}{2} + i (\Delta_p + \Delta_c) \right] \left[ \frac{\gamma_2'}{2} + i (\Delta_p + \Delta_c') \right],
\end{align*}
$$

where the denominator is

$$
D = \left( \frac{\Gamma_3}{2} + i \Delta_p \right) \left[ \frac{\gamma_2}{2} + i (\Delta_p + \Delta_c) \right] \left[ \frac{\gamma_2'}{2} + i (\Delta_p + \Delta_c') \right] + \frac{1}{4} \Omega_c \Omega_c' \left[ \frac{\gamma_2}{2} + i (\Delta_p + \Delta_c) \right].
$$
The relative electric permittivity is
\[ \varepsilon_r(\Delta p) = 1 + \frac{N\beta(\Delta p)}{1 - \frac{\gamma(\Delta p)}{2}}. \]

where \( N \) denotes the atomic concentration of the Y-configuration double-control atomic vapor. The dispersive behavior of the real and imaginary parts of the electric permittivity is plotted in Fig. 3.10, where the typical parameters of the atomic system are chosen as: \( \Gamma_s = 0.25 \text{ ms}^{-1} \), \( \Gamma_c = 2.5 \text{ ms}^{-1} \), \( \gamma_c = 5 \text{ ms}^{-1} \), \( \gamma_s = 2.5 \text{ ms}^{-1} \), \( \nu_{13} = 1.0 \times 10^{-8} \text{ cm}^{-1} \), \( \nu_{1} = 6.0 \times 10^7 \text{ s}^{-1} \), \( \Omega_c = 1.2 \times 10^7 \text{ s}^{-1} \), \( \Delta_c = 1.2 \times 10^7 \text{ s}^{-1} \), \( \Delta' = 5.4 \times 10^7 \text{ s}^{-1} \), and \( N = 7.0 \times 10^{20} \text{ m}^{-3} \). Note that in a conventional three-level EIT (electromagnetically induced transparency) system, there is only one resonant frequency for the atomic system to exhibit zero absorption. For the double-control system, however, there are two resonant frequencies, \( \Delta_p = -\Delta_c \) or \( \Delta_p = -\Delta' \), where the four-level vapor is transparent to the probe beam (zero or very small absorption). This can be referred to as "double-control EIT". Besides, we consider the coherences between levels \( |1\rangle \) and \( |2\rangle \) and levels \( |1\rangle \) and \( |3\rangle \). The behaviors of \( \rho_{21} \) and \( \rho_{23} \) are plotted in Figs. 3.11 and 3.12. From these two figures, one can see that there is a new property exhibited in the present double-control system: both the real and imaginary parts of the density matrix element \( \rho_{21} \) vanish as the probe frequency detuning \( \Delta_p \to -\Delta_c \) (i.e., \( \Delta_p/\Gamma_3 \to -0.9 \)), and both the real and imaginary parts of the density matrix element \( \rho_{23} \) vanish as the probe frequency detuning \( \Delta_p \to -\Delta' \) (i.e., \( \Delta_p/\Gamma_3 \to -0.2 \)). However, such a property is not included in the conventional three-level EIT system \( \{|1\rangle, \{2\rangle, |3\rangle \} \), where the density matrix element \( \rho_{21} \) is nonzero at or near the resonant frequency (i.e., \( \Delta_p \to -\Delta_c \)).

Let us now discuss the conditions for realizing the constructive and destructive interferences. It follows from Eqs. (3.14) and (3.15) that if the following condition
\[ \Omega_c(\Delta_p + \Delta_c) \left[ \frac{\gamma_s}{2} + i \Delta_c \right] + \Omega_c(\Delta_p + \Delta') \left[ \frac{\gamma_s}{2} + i \Delta' \right] = 0 \]

is satisfied, then the atomic microscopic electric polarizability will be reduced to that of a two-level reso-
3.3. Y-TYPE QUANTUM INTERFERENCES AND PHOTONIC LOGIC GATES

Figure 3.11: The dispersive behavior of the real and imaginary parts of the density matrix element $\rho_{21}$ of the doubly-driven atomic vapor as the probe frequency detuning varies.

Figure 3.12: The dispersive behavior of the real and imaginary parts of the density matrix element $\rho_{2'1}$ of the doubly-driven atomic vapor as the probe frequency detuning varies.
Figure 3.13: The schematic diagram of the double-control OR gate. The Y-configuration atomic systems constitute a constructive-interference medium (CIM). The EIT effect arises (OUT = 1) if either, but not both, of its two control fields is switched off. The probe field will be resonantly absorbed (OUT = 0) if both of the two control fields are switched off (IN_A = IN_B = 0).

With the help of Eq. (3.17) and the expressions for $a_2$, $a_2'$ in solution (3.13), one can obtain the condition of destructive quantum interference

$$\Omega_c^* a_2 + \Omega_c'^* a_2' = 0. \quad (3.19)$$

This means that the driving contributions (defined as $\Omega_c^* a_2$ and $\Omega_c'^* a_2'$) of the $|3\rangle - |2\rangle$ and $|3\rangle - |2'\rangle$ transitions are opposite (i.e., they are different from each other by a phase $\pi$), and the four-level Y-configuration system will be reduced to an equivalent two-level resonant system $\{ |1\rangle, |3\rangle \}$. In other words, the $|3\rangle - |2\rangle$ and $|3\rangle - |2'\rangle$ transitions driven by the two control fields seem to be absent if the condition of destructive quantum interference (3.19) is fulfilled.

We have considered the double-control behavior of the present atomic system, and have shown that the constructive and destructive interferences can be tunable by the applied control fields. In a word, the present double-control Y-configuration gaseous medium can be transparent to the probe field if the $|3\rangle - |2\rangle$ and $|3\rangle - |2'\rangle$ transitions experience constructive quantum interference, and opaque to the probe field if the $|3\rangle - |2\rangle$ and $|3\rangle - |2'\rangle$ transitions experience destructive quantum interference. This interesting effect can be utilized to realize some photonic and quantum optical devices such as logic gates, functional-operation devices and optical switches. Here we suggest the working mechanisms of some typical logic gates based on the double-control quantum interferences:

i) NOT gate. The NOT gate can produce at its output an inverted version of the input. As we have pointed out, by choosing proper intensities of the two control fields (IN = 1, and hence the destructive interference condition (3.19) is satisfied), the probe field will be absorbed by the atomic medium (OUT = 0). However, once one of the control fields (say, $\Omega_c'$) is switched off (i.e., IN = 0), the probe field will propagate through the atomic medium (OUT = 1), since the destructive interference condition (3.19) no longer holds.

ii) OR gate: The operation of the OR gate can produce a high output (OUT = 1) if one or more of its inputs are high. The truth table of a two-input OR gate is given by

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>A + B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1. The truth table for the OR gate.
3.3. Y-TYPE QUANTUM INTERFERENCES AND PHOTONIC LOGIC GATES

Figure 3.14: The schematic diagram of the double-control NOR gate. The constructive-interference medium (CIM) and the destructive-interference medium (DIM) constitute a two-input NOR gate. The NOR gate is equivalent to an OR gate (realized by CIM) followed by a NOT gate (realized by DIM).

Differing from the NOT gate, here the constructive quantum interference is required to realize the OR operation: specifically, the present Y-configuration system can exhibit double-control EIT, i.e. \( \text{OUT} = 1 \), if both of the two control fields are switched on, i.e. \( \text{IN}_A = \text{IN}_B = 1 \), and it can also exhibit single-control EIT (\( \text{OUT} = 1 \)) if one of the control fields is switched off. But the atomic medium will be opaque (due to the \(|1\rangle\rightarrow|3\rangle\) resonant absorption), i.e. \( \text{OUT} = 0 \), when both of the two control fields are absent, i.e. \( \text{IN}_A = \text{IN}_B = 0 \). The working mechanism of the two-input OR gate is shown in Fig. 3.13.

iii) NOR gate: The NOR (NOT-OR) gate produces a low output if any of the inputs are high (the output is high if all the inputs are low). The truth table of a two-input NOR gate is

<table>
<thead>
<tr>
<th>( A )</th>
<th>( B )</th>
<th>( A + B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2. The truth table for the NOR gate.

The two-input NOR gate consists of one constructive-interference medium (CIM) and one destructive-interference medium (DIM) (see Fig. 3.14). The probe field \( \Omega_{p_1} \) transmitted from the CIM can be considered to be one of the incident control fields into the DIM. As the destructive quantum interference between \( \Omega_{p_1} \) and \( \Omega_{c''} \) can occur in the DIM, the DIM can lead to a NOT operation, and therefore the combination of CIM and DIM does the opposite to the OR gate, i.e., it yields an operation of the NOR (NOT-OR) gate.

iv) EXNOR gate: The EXNOR gate produces a low output if either (but not both) of its two inputs are high (otherwise, the output is high if the two outputs are simultaneously high or simultaneously low). It can be verified that the combination of two DIMs can yield a two-input EXNOR gate (see Fig. 3.15). If the two control fields \( \Omega_c, \Omega_{c'} \) are present simultaneously (\( \text{IN}_A = \text{IN}_B = 1 \)), the probe field \( \Omega_{p_1} \) will be absorbed in the first DIM because of the destructive quantum interference, and if both of the two control fields are absent (\( \text{IN}_A = \text{IN}_B = 0 \)), the probe field \( \Omega_{p_1} \) will also be absorbed due to the \(|1\rangle\rightarrow|3\rangle\) resonant absorption. Then in the second DIM, the single-control EIT will take place (leading to the transparency to the second probe field \( \Omega_{p_2} \), i.e. \( \text{OUT} = 1 \)). If, however, only one of the two control fields \( \Omega_c, \Omega_{c'} \) is present, then the single-control EIT to the probe field \( \Omega_{p_1} \) in the first DIM will emerge, and the transmitted probe field \( \Omega_{p_1} \) can then act as one of the two incident control fields into the second DIM. Therefore, the second probe field \( \Omega_{p_2} \) will be absorbed by the second DIM (i.e., \( \text{OUT} = 0 \)) because of the destructive interference between \( \Omega_{p_1} \) and \( \Omega_{c''} \). The working mechanism of the two-input OR gate is shown in Fig. 3.15. The truth
3.4 Double-control destructive and constructive interferences

In this section, we present the double-control quantum destructive and constructive interferences in a four-level tripod-configuration system. We first give the steady solutions to the equation of motion of the four-level probability amplitudes, and then show its steady optical behavior. Such steady optical properties depend on the intensities of the two control fields. In other words, the optical properties of the present atomic vapor depend on the quantum interferences (double-control destructive and constructive quantum interferences) between the two control fields.

Consider a four-level atomic ensemble with three lower levels $|1\rangle$, $|2\rangle$, $|2'\rangle$ and one upper level $|3\rangle$ (see Fig. 3.16). Such an atomic system interacts with three optical fields, i.e., the two control laser beams and one probe laser beam, which couple the level pairs $|2\rangle$-$|3\rangle$, $|2'\rangle$-$|3\rangle$ and $|1\rangle$-$|3\rangle$, respectively. The three frequency detunings $\Delta_c$, $\Delta_{c'}$ and $\Delta_p$ are defined as follows: $\Delta_c = \omega_{32} - \omega_c$, $\Delta_{c'} = \omega_{32'} - \omega_{c'}$, and $\Delta_p = \omega_{31} - \omega_p$, where $\omega_{32}$, $\omega_{32'}$ and $\omega_{31}$ denote the atomic transition frequencies, and $\omega_c$, $\omega_{c'}$, $\omega_p$ represent the mode frequencies of the control and probe beams, respectively. For the present atomic system, the equation of motion of the probability amplitudes in accordance with the Schrödinger equation is

\[
\dot{a}_1 = \frac{i}{2} \Omega_p^* a_3, \\
\dot{a}_2 = -\left[ \frac{\gamma_2}{2} + i (\Delta_p - \Delta_c) \right] a_2 + \frac{i}{2} \Omega_p^* a_3,
\]

Table 3. The truth table for the EXNOR gate.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$A \oplus B$</th>
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<td>1</td>
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Figure 3.15: The schematic diagram of the double-control EXNOR gate. The two destructive-interference media constitute a two-input EXNOR gate.
Figure 3.16: The schematic diagram of a double-control four-level system. The two control laser beams, $\Omega_\ld$ and $\Omega_c$, drive the $|2\rangle-|3\rangle$ and $|2'\rangle-|3\rangle$ transitions, respectively. The probe transition $|1\rangle-|3\rangle$ can be controllably manipulated via the destructive and constructive quantum interferences between the $|2\rangle-|3\rangle$ and $|2'\rangle-|3\rangle$ transitions. If levels $|1\rangle$, $|2\rangle$ and $|2'\rangle$ form a three-level dark state, then the atomic vapor is transparent to the probe field, whereas it is opaque to the probe field when levels $|2\rangle$ and $|2'\rangle$ form a two-level dark state.

$$\dot{a}_{2'} = - \left[ \frac{\gamma_2'}{2} + i (\Delta_p - \Delta_c) \right] a_{2'} + \frac{i}{2} \Omega_c' a_3,$$

$$\dot{a}_3 = - \left( \frac{\Gamma_3}{2} + i \Delta_p \right) a_3 + \frac{i}{2} (\Omega_p a_1 + \Omega_c a_2 + \Omega_c a_{2'}).$$

(3.20)

where the Rabi frequencies of the probe beam and the two control beams are defined through $\Omega_p = \varphi_{31} E_p / h$, $\Omega_c = \varphi_{32} E_c / h$, and $\Omega_c' = \varphi_{32} E_c'/h$, respectively. Here $E_p$, $E_c$, and $E_c'$ stand for the probe and control field envelopes (slowly-varying amplitudes). The $\gamma_2$ and $\gamma_2'$ are collisional dephasing (non-radiative decay rate), and $\Gamma_3$ is spontaneous emission decay rate. In general, such a double-control tripod-configuration system can be found in alkali metallic atoms. For example, according to the selection rule ($\Delta L = \pm 1$, $\Delta J = 0, \pm 1$, $\Delta m_J = 0, \pm 1$) for the electric-dipole allowed transition, the system $\{|1\rangle, |2\rangle, |2'\rangle, |3\rangle\}$ can be chosen as $\{5^2S_2, 4^2D_3, 6^2S_2, 6^2P_2\}$ of neutral Rubidium atom. If the energy level of the ground state $5^2S_2$ is assumed to be zero, the energies of the other three atomic levels $4^2D_3, 6^2S_2, 6^2P_2$ are 19355.649, 20132.510, 23715.081 cm$^{-1}$, respectively [23]. Such a tripod-configuration atomic system can also be found in neutral Lithium atom $\{2^2S_2, 3^2S_2, 3^2D_2, 4^2P_2\}$ with energy levels $\{0.000, 27260.066, 31283.018, 36469.714\}$ cm$^{-1}$ [24], and in neutral Sodium atom $\{3^2S_2, 4^2S_2, 3^2D_2, 4^2P_2\}$ with energy levels $\{0.000, 25739.991, 29172.889, 30266.699\}$ cm$^{-1}$ [25].

In general, the intensity of the probe field is sufficiently weak and the Rabi frequency of the probe field is small compared with the Rabi frequencies, $\Omega_p, \Omega_c$, of the external control fields as well as some spontaneous emission decay rates. According to Eq. (3.1), the right-handed side of equation $\dot{a}_1 = (i/2)\Omega_p^* a_3$ is negligibly small since both $\Omega_p$ and $a_3$ are small compared with $\Omega_c$ and $a_1$, respectively. In what follows (for the steady case), we assume that $a_1$ is a constant number (this assumption can be confirmed using the numerical results for transient evolutions). The equations for the probability amplitudes $a_2$, $a_2'$ and $a_3$ in (3.1) can be rewritten in the following matrix form

$$\frac{\partial}{\partial t} \begin{pmatrix} a_2 \\ a_{2'} \\ a_3 \end{pmatrix} = \begin{pmatrix} \frac{-\gamma_2'}{2} + i (\Delta_p - \Delta_c) \\ 0 \\ \frac{i}{2} \Omega_c \end{pmatrix} \begin{pmatrix} a_2 \\ a_{2'} \\ a_3 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{i}{2} \Omega_c' a_3 \\ 0 \end{pmatrix}.$$  

(3.21)

The steady solution of the four-level system is given by

$$a_2 = -\frac{a_1}{4D} \Omega_p \Omega_c' \left[ \frac{\gamma_2'}{2} + i (\Delta_p - \Delta_c) \right],$$

$$a_{2'} = -\frac{a_1}{4D} \Omega_p \Omega_c' \left[ \frac{\gamma_2}{2} + i (\Delta_p - \Delta_c) \right],$$

where $D = \frac{\gamma_2}{2} + \frac{\Gamma_3}{2}$. For the steady state, the Hamiltonian $H$ of the system has the form $H = \sum_{n=1}^{3} \epsilon_n |\psi_n\rangle \langle \psi_n|$, where $|\psi_n\rangle$ are the eigenvectors of $H$ and $\epsilon_n$ are the corresponding eigenvalues.
Figure 3.17: The dispersive behavior of the real and imaginary parts of the electric susceptibility as the probe frequency detuning varies. Both the real and imaginary parts of $\chi(\Delta_p)$ tend to zero at the probe frequency detunings $\Delta_p = 3.0 \times 10^7$ s$^{-1}$ and $\Delta_p = 8.0 \times 10^7$ s$^{-1}$.

$$a_3 = \frac{ia_1}{2D} \Omega_p \left[ \frac{\gamma_2}{2} + i(\Delta_p - \Delta_c) \right] \left[ \frac{\gamma_2'}{2} + i(\Delta_p - \Delta_c') \right],$$  \hspace{1cm} (3.22)

where the denominator

$$D = \left( \frac{\Gamma_3}{2} + i\Delta_p \right) \left[ \frac{\gamma_2}{2} + i(\Delta_p - \Delta_c) \right] \left[ \frac{\gamma_2'}{2} + i(\Delta_p - \Delta_c') \right] + \frac{1}{4} \Omega_c^* \Omega_c^' \left[ \frac{\gamma_2}{2} + i(\Delta_p - \Delta_c) \right] + \frac{1}{4} \Omega_c^* \Omega_c^' \left[ \frac{\gamma_2'}{2} + i(\Delta_p - \Delta_c') \right].$$ \hspace{1cm} (3.23)

As the atomic electric polarizability of the probe transition $|1\rangle - |3\rangle$ is $\beta(\Delta_p) = 2|\psi_{13}\rangle a_1^* a_3^*/(\epsilon_0 \mathcal{E}_p)$, by substituting the above results into $\beta(\Delta_p)$, one can obtain the explicit expression for the electric polarizability

$$\beta_e(\Delta_p) = |\psi_{13}\rangle |a_1^* a_3^*| \frac{i}{D} \left[ \frac{\gamma_2}{2} + i(\Delta_p - \Delta_c) \right] \left[ \frac{\gamma_2'}{2} + i(\Delta_p - \Delta_c') \right],$$ \hspace{1cm} (3.24)

where the relation $\mathcal{E}_p = \hbar \Omega_p / \gamma_{31}$ has been inserted. The relative electric susceptibility is $\chi(\Delta_p) = N\beta(\Delta_p)$, where $N$ denotes the atomic concentration of the EIT vapor (note: $\chi(\Delta_p) = N\beta(\Delta_p)/(1 - N\beta(\Delta_p)/3)$, if the local field correction is taken into account). The dispersive behavior of the real and imaginary parts of the electric susceptibility is plotted in Fig. 3.17.

Now we consider the quantum interferences in the four-level system. From the solution (3.22), one can obtain

$$\Omega_e a_2 + \Omega_e'^* a_2' = -\frac{a_1}{4D} \Omega_p \left\{ \Omega_c^* \Omega_c^' \left[ \frac{\gamma_2}{2} + i(\Delta_p - \Delta_c) \right] + \Omega_c^* \Omega_c^' \left[ \frac{\gamma_2'}{2} + i(\Delta_p - \Delta_c') \right] \right\}. \hspace{1cm} (3.25)$$

If the destructive quantum interference between the two control transitions ($|2\rangle - |3\rangle$ and $|2'\rangle - |3\rangle$) occurs, i.e., the following condition

$$\Omega_e a_2 + \Omega_e'^* a_2' = 0. \hspace{1cm} (3.26)$$

is fulfilled, the atomic electric polarizability can exhibit a two-level resonant absorption, i.e.,

$$\beta(\Delta_p) = i \frac{|\psi_{13}|^2 a_1^* a_3^*}{\epsilon_0 \hbar} \left( \frac{1}{\frac{\gamma_2}{2} + i\Delta_p} \right).$$ \hspace{1cm} (3.27)
This, therefore, means that the four-level EIT system will be reduced to an equivalent two-level system \{\{1, 3\}\}. From Eq. (3.26), we can have \(\Omega_2^c\Omega_3^c \{\gamma_2^c/2 + i (\Delta_p - \Delta_c)\} + \Omega_2^m \Omega_3^m \{\gamma_2^m/2 + i (\Delta_p - \Delta_c)\} = 0\). In general, the dephasing rate \(\gamma_2^c\) and \(\gamma_2^m\) in the dilute vapor is negligibly small (compared with the spontaneous decay rates). Thus, the present relation can lead to a probe frequency detuning

\[
\Delta_p = \frac{\Omega_2^c \Omega_3^c \Delta_c + \Omega_2^m \Omega_3^m \Delta_c}{\Omega_2^c \Omega_3^c + \Omega_2^m \Omega_3^m}.
\]

(3.28)

where the four-level EIT system is reduced to an equivalent two-level system, and the probe field will be absorbed by the atomic vapor. In other words, the present atomic vapor would be transparent to the probe field (or small loss), if we choose the probe frequency detuning far away from the value in (3.28). This quantum interference effect can be used to design some sensitive photonic devices (optical switches and logic gates).

3.5 Quantum coherent mechanism to realize negative indices

Quite recently, there have been some schemes to realize the negative refraction, including artificial composite metamaterial, photonic crystal structures, chiral nihility, and transmission line simulation. All these schemes were proposed within the framework of classical electromagnetic theory. Here, we suggest that some atomic systems with proper optical parameters can give rise to striking electromagnetic responses (leading to the negative refractive indices) and the atomic vapors can then become left-handed media in optical frequency bands. In what follows, we will consider two schemes for achieving the negative refractive indices:

i) **Lambda-type atomic vapor with simultaneous electric and magnetic transitions.** We choose an atomic system with proper configuration (Lambda-type), the electric- and magnetic-dipole allowed transitions of which can lead to the nontrivial electromagnetic responses that can dramatically modify the optical properties of the atomic vapor, even make the electric permittivity and the magnetic permeability simultaneously negative in certain frequency bands.

ii) **Dressed-state mixed-parity transitions.** The dressed states that are the linear combinations of two bare levels of an atom (e.g. alkali metallic atom) can be realized by a strong coupling laser beam. As the dressed states have mixed parities, both electric- and magnetic-dipole allowed transitions can occur between the dressed states and a third bare level with a definite (pure) parity. It is shown that such dressed-state mixed-parity transitions in an atomic vapor can give rise to a negative refractive index. The produced negative refractive index is isotropic with atomic-scale microscopic structure units, and the negative real part can emerge in the optical frequency band.

3.5.1 Lambda-type left-handed atomic vapor

The quantum optical mechanism to realize negative indices is to choose an atomic system with proper configuration (see, for example, Fig. 3.18), the electric- and magnetic-dipole allowed transitions of which can lead to the nontrivial electromagnetic responses that can dramatically modify the optical properties of the atomic vapor. Very recently, some authors suggested that the EIT (electromagnetically induced transparency) atomic system may exhibit simultaneously negative permittivity and permeability [26, 27]. Though such kind of schemes sounds reasonable, it requires sufficiently large magnetic-dipole transition moment to achieve significant magnetic responses. But in general, the dimensionless ratio of the magnetic dipole moment to the electric dipole moment is small in an atomic system (such a ratio is approximately equal to the fine structure constant, e.g., \(|m_{13}|/(\varphi_{23} c)| \approx 1/137\), where \(m_{13}, \varphi_{23}\) and \(c\) denote the magnetic- and electric-transition dipole matrix elements and the speed of light in vacuum, respectively). For this reason, the contribution of the magnetic-dipole allowed transition to the vapor is often negligibly small compared with that of the electric-dipole allowed transition, and therefore the magnetic-dipole transition should not be taken into account in the treatment for a wave propagation in an ordinary atomic vapor. In the atomic configuration presented here (see Fig. 3.18), however, the density matrix element \(\rho_{31}\) (related to the magnetic-dipole allowed transition) can be dramatically enhanced via the phase coherence (quantum coherence) between the two lower levels, and the atomic magnetic polarizability \(\beta_m\) would greatly increase and hence have the same order of magnitude as the atomic electric polarizability \(\beta_e\). In other words, in order to achieve the negative index of refraction in the atomic vapor, one should choose an atomic system with a configuration that can enhance the contribution of the magnetic-dipole allowed transition. Further
are assumed to be nearly stationary and hence any Doppler shifts are neglected. Under these conditions, the ground state, the intensity of the probe light is assumed to be sufficiently weak so that nearly all the atoms remain in the ground state, i.e., $|1\rangle$. The Rabi frequencies $\Omega_E$ and $\Delta_B$ (nonradiative decay rate) of level $|3\rangle$ are defined through $\Omega_E = |\gamma_E| \omega_p$, and magnetic permeability could possibly occur, the transition frequencies of $|1\rangle$ and $|3\rangle$ will have even parity. Obviously, in order that the resonances in both electric permittivity and magnetic fields of a probe light couple level pairs $|1\rangle-|3\rangle$ and $|2\rangle-|3\rangle$, respectively. Thus the parity of level $|2\rangle$ is different from those of both $|1\rangle$ and $|3\rangle$. If, for example, level $|2\rangle$ possess an odd parity, then levels $|1\rangle$ and $|3\rangle$ will have even parity. Consider a Lambda-type three-level atomic ensemble with one upper level $|3\rangle$ and two lower levels $|1\rangle$ and $|2\rangle$ (see Fig. 3.18). Such an atomic system interacts with a weak probe light: specifically, the electric and magnetic fields of the probe light couple level pairs $|2\rangle-|3\rangle$ and $|1\rangle-|3\rangle$, respectively. The analysis shows that such an atomic vapor (see Fig. 3.18) may exhibit the negative permittivity and the negative permeability simultaneously, and can therefore become an ideal candidate for realizing an isotropic left-handed vapor medium.

\[ \rho_{31} = -\left( i\Delta_B + \frac{\Gamma}{2} \right) \rho_{31} + \frac{i}{2} \Omega_B (\rho_{11} - \rho_{33}) + \frac{i}{2} \Omega_E \rho_{21}, \]
\[ \rho_{32} = -\left( i\Delta_E + \frac{\gamma_2}{2} \right) \rho_{32} + \frac{i}{2} \Omega_E (\rho_{22} - \rho_{33}) + \frac{i}{2} \Omega_B \rho_{12}, \]
\[ \rho_{21} = -\left[ i \left( \Delta_B - \Delta_E \right) + \frac{\gamma_2}{2} \right] \rho_{21} + \frac{i}{2} \Omega_E \rho_{31} - \frac{i}{2} \Omega_B \rho_{23}, \] (3.29)

where the decay rates are defined by $\Gamma = \gamma_{ph} + \gamma_3$ and $\gamma = \gamma_2 + \gamma_3 + \gamma_{ph}$. Here, $\gamma_2$ and $\gamma_3$ denote the spontaneous decay rates of levels $|2\rangle$ and $|3\rangle$, respectively, and $\gamma_{ph}$ stands for the dephasing rate (nonradiative decay rate) of level $|3\rangle$. The electric and magnetic frequency detunings are $\Delta_E = \omega_{32} - \omega_p$ and $\Delta_B = \omega_{31} - \omega_p$, where $\omega_p$ denotes the angular frequency of the probe light. The electric and magnetic Rabi frequencies $\Omega_E$ and $\Omega_B$ are defined through $\Omega_E = g_{32} e_p / \hbar$ and $\Omega_B = g_{31} B_p / \hbar$, where $e_p$ and $B_p$ represent the electric and magnetic field envelopes (slowly varying amplitudes) of the probe light. The intensity of the probe light is assumed to be sufficiently weak so that nearly all the atoms remain in the ground state, i.e., the atomic population in level $|1\rangle$ is close to unity. It should be noted that here the atoms are assumed to be nearly stationary and hence any Doppler shifts are neglected. Under these conditions, the steady values of the off-diagonal density matrix elements that have close relation to the magnetic and electric responses are given as follows.

\[ \rho_{31} = \frac{\Omega_B}{(i\Delta_B + \frac{\Gamma}{2})} \left[ i \left( \Delta_B - \Delta_E \right) + \frac{\gamma_2}{2} \right] + \frac{\Omega_E}{4} \rho_{31}, \]
\[ \rho_{32} = \frac{-\Omega_B}{(-i\Delta_B + \frac{\Gamma}{2})} \left[ -i \left( \Delta_B - \Delta_E \right) + \frac{\gamma_2}{2} \right] + \frac{\Omega_E}{4} \rho_{32}, \] (3.30)
3.5. QUANTUM COHERENT MECHANISM TO REALIZE NEGATIVE INDICES

The density matrix element $\rho_{12}$ that can characterize the atomic phase coherence of the present system is

$$\rho_{12} = -\frac{i}{2} \cdot \frac{\Omega_E \rho_{13} - \Omega_B^* \rho_{32}}{-i(\Delta_B - \Delta_E) + \frac{\Omega_E^* \Omega_B}{2}}. \quad (3.31)$$

The selection rule for the present electric-dipole allowed transition ([2]-[3]) is $\Delta l = \pm 1$, $\Delta m = 0, \pm 1$, and the selection rule for the magnetic-dipole allowed transition ([1]-[3]) is $\Delta l = 0$, $\Delta m = 0, \pm 1$. By using the definitions of atomic microscopic electric and magnetic polarizabilities

$$\beta_e = \frac{2\mu_2\rho_{32}}{\epsilon_0 \epsilon_p}, \quad \beta_m = \frac{2\mu_0 m_{13} \rho_{31}}{B_p}, \quad (3.32)$$

one can obtain the explicit expressions for the electric and magnetic polarizabilities of the above coherent atomic vapor as follows

$$\beta_e = \frac{|\gamma_{23}|^2}{4\epsilon_0 \hbar} \left( -\Omega_B^* \Omega_B \left( i(\Delta_E + \frac{\Omega_E^* \Omega_B}{2}) \right)^{-1} \right),$$

$$\beta_m = \frac{\mu_0 |m_{13}|^2}{\hbar} \left( \frac{-\Omega_B^* \Omega_B}{i(\Delta_B - \Delta_E) + \frac{\Omega_E^* \Omega_B}{2}} \right). \quad (3.33)$$

It should be noted that the above formulation can also be applicable to the case where both amplitudes of electric and magnetic fields are complex. In this case, the electric and magnetic induction field strengths can be written as $\vec{E}_p = \vec{E}_p e^{-i\omega t} + \vec{E}_p e^{i\omega t}$ and $\vec{B}_p = \vec{B}_p e^{-i\omega t} + \vec{B}_p e^{i\omega t}$, where $\vec{E}_p$ and $\vec{B}_p$ denote the slowly-varying amplitudes of the electric and magnetic induction fields, respectively. Assume the wavefunction of the three-level atomic system is $|\Psi\rangle$, which is a linear combination of $|1\rangle$, $|2\rangle$ and $|3\rangle$. Thus one can verify that the expectation values of the electric and magnetic dipoles induced by the applied light field are

$$\langle \Psi | e | \Psi \rangle = \gamma_{23} \rho_{32} e^{-i\omega t} + \gamma_{32} \rho_{23} e^{i\omega t} \equiv \vec{p} + \vec{p}_r,$$

$$\langle \Psi | (e/2m_e)(\mathbf{L} + 2\mathbf{S}) | \Psi \rangle = m_{13} \rho_{31} e^{-i\omega t} + m_{31} \rho_{13} e^{i\omega t} \equiv \vec{m} + \vec{m}_r. \quad (3.34)$$

According to the definition of the electric polarizability (microscopic quantity) $\beta_e$, we have

$$\vec{p} = \beta_e e_0 \left( \frac{\vec{E}_p e^{-i\omega t}}{2} \right). \quad (3.35)$$

It should be noted that here the electric field strength $\vec{E}_p$ is inserted by $\vec{E}_p e^{-i\omega t}$/2 (rather than by $(\vec{E}_p e^{-i\omega t} + \vec{E}_p e^{i\omega t})$/2). The reason for this choice is just the rotation wave approximation (the effect that is related to the high-frequency oscillating factor such as $e^{\pm i\omega t}$ could be neglected). In the similar fashion, according to the definition of the magnetic polarizability (microscopic quantity) $\beta_m$ [28], we have

$$\vec{m} = \frac{\beta_m}{\mu_0} \left( \frac{\vec{B}_p e^{-i\omega t}}{2} \right). \quad (3.36)$$

Thus, with the help of Eqs. (3.34), (3.35) and (3.36), one can obtain the atomic electric and magnetic polarizabilities (3.32).

These Clausius-Mossotti relations are of the form [28, 29]

$$\epsilon_e = 1 + \frac{2}{3} N \beta_e, \quad \mu_e = 1 + \frac{2}{3} N \beta_m, \quad (3.37)$$

where $N$ denotes the atomic concentration of the vapor. Thus we have the explicit expressions for the electric permittivity and magnetic permeability of the atomic vapor. Note that sometimes the Clausius-Mossotti relations would be invalid (or approximately valid) for the cases of large dielectric constants, especially in the polar liquids and solids with large $N \beta_e$ (and $N \beta_m$) [30].

Here we present a numerical example to demonstrate that the simultaneously negative permittivity and permeability can be truly achieved by the present quantum optical scheme. The typical parameters in expression (3.33) for the electric and magnetic polarizabilities can be chosen as: electric and magnetic
transition dipole moments $\varphi_{02} = 3.0 \times 10^{-29}$ C·m [31], $m_{31} = 9.4 \times 10^{-24}$ C·m²s⁻¹, decay rates $\gamma_2 = 0.4 \times 10^7$ s⁻¹, $\gamma_3 = 1.3 \times 10^7$ s⁻¹ [11, 32], dephasing rate $\gamma_{ph} = 0.3 \times 10^3$ s⁻¹, electric and magnetic Rabi frequencies of the applied light fields $\Omega_B = 3.3 \times 10^4$ s⁻¹, $\Omega_E = 0.9 \times 10^7$ s⁻¹ [33], and atomic concentration $N = 7.0 \times 10^{25}$ m⁻³. As assumed above, the electric and magnetic frequency detunings $\Delta_B = \Delta_E$ (expressed by $\Delta$ in the numerical example). The dispersive behaviors of the electric permittivity and the magnetic permeability of the present atomic vapor are plotted in Figs. 3.19 and 3.20. It follows from Fig. 3.19 that the relative permittivity has a negative real part $[−2.0, 0]$ in the probe frequency detuning range $[−4.0\gamma_3, +4.0\gamma_3]$, and the relative permeability has a negative real part $[−1.0, 0]$ in the probe frequency detuning range $[−2.5\gamma_3, +1.0\gamma_3]$. Therefore, the proper frequency detuning range, where the electric permittivity and the magnetic permeability simultaneously have negative real parts, is $[−2.5\gamma_3, +1.0\gamma_3]$. In this frequency band (i.e., negatively-refracting frequency range), the above atomic vapor would become an isotropic left-handed medium.

Since the atomic phase coherence (between levels $|1\rangle$ and $|2\rangle$) plays a key role in achieving simultaneously negative permittivity and permeability of the atomic vapor, we plot the evolution of the real and imaginary parts of the off-diagonal density matrix elements $\rho_{12}$ versus the frequency detuning (see Fig. 3.21). It follows from Fig. 3.21 that the real part of $\rho_{12}$ at the resonant frequency is taken to be a minimum value and the corresponding imaginary part is zero.

The dispersive behavior of the real and imaginary parts of the relative refractive index, $n_r$, of the coherent atomic vapor in a narrow negatively-refracting frequency detuning band is plotted in Fig. 3.22. It can be seen that the relative refractive index has a negative real part $[−2.2, −0.7]$ in this frequency detuning band. Though the electric permittivity has a negative imaginary part (see the solid line in Fig. 3.20), the imaginary part of the refractive index of the left-handed atomic vapor is positive (see Fig. 3.22). It can then be concluded that the present left-handed atomic vapor that exhibits a negative refractive index is an absorptive medium (it is the refractive index that is most crucial to the behavior of the wave propagation). The dimensionless absorption coefficient $\alpha$ (defined as $2\pi\Im\{n_r\}/|\Re\{n_r\}|$) is also plotted in Fig. 3.22 as a function of the frequency detuning of the probe light. From this curve one sees that the absorption increases as the frequency detuning increases in the present frequency range. The electric- and magnetic-dipole moments as well as the decay rates in the numerical example are the typical parameters for the multilevel metallic atomic system. At present, these parameters do not correspond to an existing atom. But the general features of the required atomic systems can be involved in the chosen typical parameters. The most principal feature of the present three-level atomic system is such that the energy separation $\omega_{21}$ should be much less than the transition frequencies $\omega_{31}$ and $\omega_{23}$ in order that the electric- and magnetic-dipole allowed transitions can be simultaneously driven by the applied light field. As an illustrative example,
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Figure 3.20: The imaginary parts of the relative permittivity and the relative permeability of the atomic vapor as the probe frequency detuning varies. The solid and the dash-dot lines represent the electric permittivity and the magnetic permeability, respectively. The typical parameters for the atomic system are chosen the same as in Fig. 3.19.

Figure 3.21: The real and imaginary parts of the off-diagonal density matrix element $\rho_{12}$ versus the frequency detuning. The solid and the dash-dot lines represent the real and the imaginary parts of $\rho_{12}$, respectively.
atomic system \{ |1\rangle, |2\rangle, |3\rangle \} that has such a configuration could be \{3d^8 \ ^3F_4, 3d^9 \ ^3D_3, 3d^8 \ ^5F_5 \} in neutral Nickel atom and \{4f(5d)^1 \ ^4G_4, 4f(5d)^3 \ ^4F_2, 4f(5d^2)^5 \ ^6G_2 \} in neutral Cerium atom, where \omega_{31} is about 0.8\% and 2.5\% of \omega_{31} (and \omega_{32}), respectively [34, 35]. The selection rules in Nickel atom for the electric-dipole transition between levels 3d^9 \ ^3D_3 and 3d^8 \ ^5F_5 and the magnetic-dipole transition between levels 3d^8 \ ^3F_4 and 3d^8 \ ^5F_5 are \Delta l = 1, \Delta m_1 = 1 and \Delta l = 0, \Delta m = 1, respectively, and the selection rules in Cerium atom for the electric-dipole transition between levels 4f(5d)^1 \ ^3F_2 and 4f(5d^2)^5 \ ^6G_2 and the magnetic-dipole transition between levels 4f(5d)^1 \ ^4G_4 and 4f(5d^2)^5 \ ^6G_2 are \Delta l = 1, \Delta m = 0 and \Delta l = 0, \Delta m = -1, respectively.

It should be noted that the left-handed vapor medium produced is both isotropic and homogeneous due to the atomic-scale microscopic structure units, and the negative refraction could occur in the visible and infrared frequency bands since the typical transition frequency of atoms is in the frequency range \(10^{13} \sim 10^{15} \text{ s}^{-1}\). The left-handed medium that has negative permittivity and permeability (and hence the negative index of refraction) can cause the strong anti-shielding effects, which enable the probe light in the left-handed medium to exhibit many physically interesting optical and electromagnetic properties and effects. In what follows, we discuss the problem of polarization of light (including the orientation of electric and magnetic dipole moments matrix elements) in the left-handed atomic vapor, i.e., we consider the problem whether the orthogonal electric and magnetic dipoles of the atom could be induced by the light field with any given orientations of \(\mathbf{E}\) and \(\mathbf{B}\). If this is the true case, the optical “constants” such as the atomic microscopic polarizabilities (and hence the permittivity, the permeability and the refractive index) are independent of the polarizations of the applied light field, and so the vapor produced could be viewed as an isotropic left-handed medium. As is known, the magnetic field of a plane electromagnetic wave propagating inside an isotropic and homogeneous medium is perpendicular to its electric field. If the induced magnetic dipole of the atomic system is also perpendicular to the induced electric dipole, then we can conclude that such an atomic system is an isotropic one for the propagating wave. Indeed, the angular momentum \((\mathbf{r} \times \mathbf{p})\) of the electron is always orthogonal to the electric dipole \((\mathbf{cr})\), i.e., \(\mathbf{cr} \cdot (\mathbf{r} \times \mathbf{p}) \equiv 0\). This means that the induced electric dipole \(\vec{\mathbf{d}}_{23}\) and the induced magnetic dipole \(\vec{\mathbf{m}}_{13}\) are truly perpendicular to each other. Then the produced medium via the quantum optical mechanism (atomic phase coherence) suggested in the present scheme can be isotropically left-handed for both linear and circular polarizations.
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\[ H = \begin{pmatrix} \frac{1}{2} \hbar \delta & \frac{V}{\hbar} \\ \frac{V^*}{\hbar} & -\frac{1}{2} \hbar \delta \end{pmatrix}, \]  

(3.38)

where the interaction potential energy \( V = -\nu_{ab}E_c \), and \( \delta \) denotes the frequency detuning of the coupling field, i.e., \( \delta = \omega_{ab} - \omega_c \), where \( \omega_{ab} \) and \( \omega_c \) denotes the \( |b\rangle\langle a| \) transition frequency and the mode frequency of the coupling field, respectively. Here the spontaneous decay effect in the bare-state system \( \{ |a\rangle, |b\rangle \} \) can be neglected if the intensity of the applied coupling field \( E_c \) is very strong (i.e., \( |V| \gg \hbar \Gamma \) with \( \Gamma \) being the spontaneous emission decay rate). The eigenbasis (dressed basis) of the interaction Hamiltonian (3.38) can be of the form

\[
\begin{align*}
|+\rangle &= \cos \vartheta |a\rangle + e^{i\phi} \sin \vartheta |b\rangle, \\
|-\rangle &= -\sin \vartheta |a\rangle + e^{i\phi} \cos \vartheta |b\rangle,
\end{align*}
\]

(3.39)

where the coefficients \( \sin \vartheta \) and \( \cos \vartheta \) are so defined that

\[
\sin^2 \vartheta = \frac{1}{2} \left[ 1 - \frac{1}{\sqrt{\frac{4\Omega_c^2}{\sqrt{\pi}} + 1}} \right], \quad \cos^2 \vartheta = \frac{1}{2} \left[ 1 + \frac{1}{\sqrt{\frac{4\Omega_c^2}{\sqrt{\pi}} + 1}} \right].
\]

(3.40)

Here, the parameter \( \phi = \text{arg}[V] \) and the Rabi frequency, \( \Omega_c \), of the coupling field is \( V/\hbar \). For simplicity, we assume that \( \phi = 0 \) in what follows. Obviously, the frequencies, \( \omega_a \) and \( \omega_b \), corresponding to the

---

Figure 3.23: The schematic diagram for the dressed-state mixed-parity transitions. The electric-dipole allowed transition \( |a\rangle\langle b| \) is driven by a strong coupling laser beam, and two dressed states \( |+\rangle \) and \( |-\rangle \) will result from the linear combinations of the two bare levels \( |a\rangle \) and \( |b\rangle \). The energy level pair \( |g\rangle\langle -| \) is coupled to the probe electric and magnetic fields. Both electric- and magnetic-dipole allowed transitions between the ground level \( |g\rangle \) and the mixed-parity dressed level \( |-\rangle \) will take place.

3.5.2 Dressed-state mixed-parity transitions for realizing negative refractive index

In the present scheme, the dressed-state mixed-parity transitions that can give rise to both electric and magnetic responses are utilized to realize left-handedness of a probe light. In the schematic diagram depicted in Fig. 3.23, the electric-dipole allowed transition \( |a\rangle\langle b| \) is driven by a strong coupling laser beam, and this will lead to two orthogonal dressed states \( |+\rangle \) and \( |-\rangle \), which are the linear combinations of the two bare levels \( |a\rangle \) and \( |b\rangle \). As the lower dressed state \( |-\rangle \) possess a mixed parity, both electric- and magnetic-dipole allowed transitions between \( |g\rangle \) and \( |-\rangle \) will emerge, if the level pair \( |g\rangle\langle -| \) is coupled to the electric and magnetic fields of the probe beam.

Consider a three-level bare-state atomic system with two upper levels \( |a\rangle \), \( |b\rangle \) and one ground level \( |g\rangle \) (see Fig. 3.23). We assume that the two upper bare levels have opposite parities, and the parity of the ground level \( |g\rangle \) is even. For example, level \( |b\rangle \) possesses an even parity while level \( |a\rangle \) has an odd parity. In general, such an atomic system can be found in alkali metallic atoms. Note that the \( |b\rangle\langle a| \) transition can be driven by a strong coupling laser beam, and let us first consider the dressed states that contain the information on the interaction between the two-level system \( \{ |a\rangle, |b\rangle \} \) and the strong coupling field. In the interaction picture with the rotating wave approximation (RWA), the Hamiltonian of the system in the representation of bare-state basis vectors \( \{ |a\rangle, |b\rangle \} \) is given by
energy levels of \(|a\rangle\) and \(|b\rangle\) are \(\omega_a = \bar{\omega} + (\omega_c + \delta)/2\) and \(\omega_b = \bar{\omega} - (\omega_c + \delta)/2\), respectively. Here, \(\bar{\omega} = \omega_g + \omega_{by} + (\omega_c + \delta)/2\). The dressed-state \(|-\rangle\rangle\) transition frequency \(\omega_{+-} = \omega_c + \sqrt{\delta^2 + \Omega^2}\), which involves a modification compared with the bare-level transition frequency \(\omega_{ab} = \omega_c + \delta\). Obviously, the frequencies, \(\omega_a\) and \(\omega_b\), corresponding to the energy levels of \(|+\rangle\rangle\) and \(|-\rangle\rangle\) are \(\omega_+ = \bar{\omega} + (\omega_c + \sqrt{\delta^2 + \Omega^2})/2\) and \(\omega_- = \bar{\omega} - (\omega_c + \sqrt{\delta^2 + \Omega^2})/2\), respectively.

As is well known, the bare states have pure parities, but the dressed states have mixed parities. Thus, both the electric and magnetic fields of a weak probe light (with mode frequency \(\omega_t\)) can drive the transition between the ground level \(|g\rangle\) and the mixed-parity level \(|-\rangle\rangle\). The density of the interaction Hamiltonian of the system \(|-\rangle\rangle, |g\rangle\rangle\rangle\) that is coupled to an electromagnetic field (weak probe field) is given by

\[
v = -\left\{ \phi^* - (\mathbf{r} \cdot \mathbf{E} + \frac{e}{2m} (\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B}) \phi \right\} \rho_{-g} + \text{H.C.}, \tag{3.41}\]

where \(\phi_{-g}\) denote the wavefunctions of the two levels, and \(\rho_{-g}\) is one of the off-diagonal density matrix elements. \(q = -e\) with \(e\) being the electron charge. The H.C. represents the Hermitian conjugation term of the Hamiltonian. \(\mathbf{L}\) and \(\mathbf{S}\) stand for electron’s orbital and spinning angular momenta, respectively, and \(\mathbf{A}\) is a three-dimensional electromagnetic vector potential. The electric and magnetic field strengths of the weak probe light are \(\mathbf{E} = e\mathbf{r} \cdot \mathbf{E} \exp(-i\omega t), \mathbf{B} = e\mathbf{r} \cdot \mathbf{B} \exp(-i\omega t)\), and the density matrix element is \(\rho_{-g} = \tilde{\rho}_{-g} \exp(i\omega t)\), where \(E, B\) and \(\tilde{\rho}_{-g}\) represent the respective envelopes of these physical quantities.

Define the electric and magnetic dipole matrix elements as follows: \(\mathbf{p}_{-g} = \int_V \phi^*_\phi \mathbf{r} dV, \mathbf{m}_{-g} = \int_V \phi^*_\phi \mathbf{S} dV\), where \(dV\) denotes the volume element. Further calculation shows that the relations between the dressed-state electric and magnetic dipole moment elements \(\mathbf{p}_{-g}, \mathbf{m}_{-g}\) and the pure-parity counterparts \(\mathbf{p}_{ag}, \mathbf{m}_{ag}\) are

\[
\mathbf{p}_{-g} = (-|\mathbf{E}|g) = -\mathbf{p}_{ag} \sin \vartheta, \quad \mathbf{m}_{-g} = (-|\mathbf{E}/2m (\mathbf{L} + 2\mathbf{S})|g) = \mathbf{m}_{ag} \cos \vartheta. \tag{3.42}\]

Note that the length scale of the two-level system is much smaller than the wavelength of the electromagnetic wave. Thus the interaction Hamiltonian of the above system can be written in the form

\[
V_{\text{int}} = -\hbar \left( \frac{\mathbf{p}_{-g} \mathbf{E}}{\hbar} + \frac{\mathbf{m}_{-g} \mathbf{B}}{\hbar} \right) \tilde{\rho}_{-g} + \text{H.C.}.
= -\hbar (\Omega^E + \Omega^B) \tilde{\rho}_{-g} + \text{H.C.}, \tag{3.43}\]

where \(\Omega^E = \mathbf{p}_{-g} \mathbf{E}/\hbar, \Omega^B = \mathbf{m}_{-g} \mathbf{B}/\hbar\), and \(\Omega^E, \Omega^B\) denote the electric and magnetic Rabi frequencies, respectively. In the rotating wave approximation, the equation of motion can be written in the following
3.5. QUANTUM COHERENT MECHANISM TO REALIZE NEGATIVE INDICES

\[ \frac{d}{dt} \tilde{\rho}_{-g} = \frac{1}{2} \left( \tilde{\rho}_{gg} \Omega^* - \tilde{\rho}_{-g} \right) - \gamma \tilde{\rho}_{-g}, \]
\[ \frac{d}{dt} \tilde{\rho}_{gg} = \frac{1}{2} \Omega \left( \tilde{\rho}_{gg} - \tilde{\rho}_{-g}^* \right) - \frac{\gamma}{2} \tilde{\rho}_{gg}, \]
\[ \frac{d}{dt} \tilde{\rho}_{-g} = \frac{1}{2} \Omega \left( \tilde{\rho}_{gg} - \tilde{\rho}_{-g} \right) - \left( \frac{\gamma}{2} + i \Delta \right) \tilde{\rho}_{-g}, \]

where the frequency detuning of the weak probe light is defined by \( \Delta = \omega_{-g} - \omega \) (with \( \omega_{-g} \) being the transition frequency of the \( |g\rangle \rightarrow |\rangle \) transition), and the total Rabi frequency of the probe light is \( \Omega = \Omega^E + \Omega^B \). As the probe light is weak (\( \Omega \ll \gamma \)), the population in the ground level \( |g\rangle \) is close to unity, and \( \tilde{\rho}_{-g} \) is negligibly small. Thus, the steady expression for the off-diagonal density matrix element \( \tilde{\rho}_{-g} \) is given by

\[ \tilde{\rho}_{-g} = \frac{\Omega}{2} \left[ \frac{\Delta + 1}{\Delta^2 + (\frac{\gamma}{2})^2} \right]. \]

We shall give the explicit expressions for the atomic electric and magnetic polarizabilities of the \( |g\rangle \rightarrow |\rangle \) transition excited by the weak probe field. Using the formulation presented in the previous scheme (for the Lambda-type negative refractive index), we can obtain the electric permittivity and the magnetic permeability as well as the refractive index of the present atomic vapor. The atomic electric polarizability \( \beta_e \) (due to the \( |g\rangle \rightarrow |\rangle \) transition) and the relative electric permittivity \( \epsilon_r \) of the atomic vapor with concentration (atomic number density) \( N \) are given by

\[ \beta_e = \frac{\Delta + 1}{h} \frac{1}{\epsilon_0} \left( \frac{p_{gg} \Omega}{\epsilon_0} + \sqrt{\frac{\mu_0}{\epsilon_0} m_{gg} \Omega} \right), \quad \epsilon_r = 1 + \frac{N \beta_e}{1 - N \beta_e}, \]

where the local field effect (because of the dipole-dipole interactions between neighboring atoms) has been taken into account. In the similar fashion, the atomic magnetic polarizability \( \beta_m \) (due to \( |g\rangle \rightarrow |\rangle \) transition) and the relative magnetic permeability \( \mu_r \) of the atomic vapor are given by

\[ \beta_m = \frac{\Delta + 1}{h} \frac{1}{\epsilon_0} \left( \frac{m_{gg} \Omega}{\epsilon_0} + \sqrt{\frac{\mu_0}{\epsilon_0} p_{gg} \Omega} \right), \quad \mu_r = 1 + \frac{N \beta_m}{1 - N \beta_m}. \]

The typical dispersive behavior of the present mixed-parity atomic vapor is presented in Figs. 3.24 and 3.25.
A NOTE: The correct $\mu_r$ formula

In this chapter, we adopt the formula $\mu_r = 1 + \frac{N\beta_m}{1 - \frac{N\beta_m}{3}}$ for the magnetic permeability with local field correction. The derivation procedure for this formula can be found in the book: D. M. Cook, *The Theory of the Electromagnetic Field* (Prentice-Hall, Inc., New Jersey, 1975): Chapt. 11. However, in a recent reference: C. M. Krowne, *Phys. Lett.* A 372, 2304 (2008), the author states that the formula of permeability $\mu_r$ in Cook’s book may be not correct. In his opinion, the correct permeability $\mu_r$ should be $\mu_r = 1 + \frac{N\beta_m}{1 - \frac{N\beta_m}{3}}$.

We can show that Krowne’s derivations for $\epsilon_r$ and $\mu_r$ are as follows:

The electric susceptibility $\chi_e$ with local field correction is given by

$$\chi_e = \frac{N\beta_e}{1 - \frac{N\beta_e}{3}},$$  \hspace{1cm} (3.48)

where $\beta_e$ denotes the atomic electric polarizability. This is the Clausius-Mossotti relation. Since the relative electric permittivity $\epsilon_r$ is

$$\epsilon_r = 1 + \chi_e,$$  \hspace{1cm} (3.49)

this leads to $\epsilon_r = 1 + \frac{N\beta_e}{1 - \frac{N\beta_e}{3}}$. This is the result that is well known to us.

The magnetic susceptibility $\chi_m$ with local field correction is given by the magnetic Clausius-Mossotti relation:

$$\chi_m = \frac{N\beta_m}{1 - \frac{N\beta_m}{3}},$$  \hspace{1cm} (3.50)

where $\beta_m$ denotes the atomic magnetic polarizability. Since the relative magnetic permeability $\mu_r$ is

$$\mu_r = \frac{1}{1 - \chi_m},$$  \hspace{1cm} (3.51)

this leads to $\mu_r = 1 + \frac{N\beta_m}{1 - \frac{N\beta_m}{3}}$. This is the result obtained by Krowne. The present $\mu_r$ formula is different from that in Cook’s book.

Then who is right, Krowne or Cook? It can be seen that the difference between Krowne and Cook’s formulations is such that Krowne uses the formula $\mu_r = \frac{1}{1 - \chi_m}$ while Cook uses another form $\mu_r = 1 + \chi_m$.

M. Norgren in KTH thinks that Krowne’s formulation is wrong, since he has used a wrong local field. M. Norgren suggests a material that shows how to get a correct formula for the permeability (M. Norgren, “The Cook formula seems correct but not Krowns,” private communication, Jan. 17, 2009). This problem will be clarified in the future considerations.

In this thesis, we adopted Cook’s formula for $\mu_r$. 

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Chapter 4

Quantum-vacuum effects in artificial electromagnetic media

4.1 Introduction

Quantum vacuum (ground state of quantized fields) can exhibit many interesting effects, including the vacuum polarization (leading to the Lamb shift and hence the hyperfine structure of Hydrogen atomic spectrum), and the Casimir effect and dramatic change of spontaneous emission decay in QED cavity. Apart from the vacuum effects in quantum field theory, quantum vacuum in electromagnetic materials can also exhibit some novel effects such as vacuum magnetoelectric birefringences, vacuum contribution to medium momentum and geometric phases at quantum vacuum level (see the references in Sec. 1.3 of Chapter 1).

In this chapter, we propose two quantum-vacuum effects in artificial materials:

i) The anisotropic distribution of quantum-vacuum momentum density in a moving electromagnetic medium. An isotropic electromagnetic medium becomes gyrotropically anisotropic when it moves, and an anisotropic electromagnetic environment can then be created in this motion-induced-anisotropy medium. One of the most remarkable features is that the quantum vacuum in the anisotropic electromagnetic environment exhibits a nonzero electromagnetic momentum density, since the universal symmetry of the vacuum fluctuation field is broken, and the anisotropic quantum vacuum mode structure is produced because of the symmetry breaking. This would give rise to a noncompensation effect among the four vacuum eigenmodes (i.e., the forward and backward propagating modes as well as their respective mutually perpendicular polarized components), and lead to anisotropic correction to the vacuum momentum in the moving medium. The physical significance and the potential applications of the anisotropic quantum vacuum are discussed. This quantum-vacuum effect may be used to develop sensitive sensor techniques and to design new quantum optical and photonic devices.

ii) Angular momentum transfer between quantum vacuum and anisotropic medium. An anisotropic electromagnetic environment that can be created inside a Faraday chiral material may cause breaking of universal symmetry of the vacuum mode structure and hence leads to a nonzero electromagnetic angular momentum density of the quantum vacuum. A novel quantum-vacuum effect (i.e., the transfer of angular momentum from the anisotropic quantum-vacuum fluctuation field to the Faraday chiral material) is predicted. It can be verified that the Faraday chiral material may acquire a nanoscale velocity due to the angular momentum transfer at quantum vacuum level. Such a macroscopic mechanical effect could be detected by current technology, e.g., fiber optical sensor that can measure motion with nanoscale sensitivity.

Now we first consider the subject of anisotropic distribution of quantum-vacuum momentum density in a moving electromagnetic medium, and then discuss the possible transfer of angular momentum from the anisotropic quantum-vacuum fluctuation field to the materials.

4.2 Quantum-vacuum momentum density in a moving medium

As an isotropic electromagnetic medium becomes anisotropic (with gyrotropy characterization in its constitutive relation [1]) when it moves, here we show that a moving isotropic (or uniaxial) medium can exhibit a physically interesting quantum-vacuum mechanical effect due to the universal symmetry breaking of vacuum
field in an anisotropic electromagnetic environment. This can be viewed as a macroscopically observable mechanical effect, which may possibly lead to the transfer of quantum-vacuum electromagnetic momentum from the vacuum zero-point fluctuation field to the moving material [2].

For an isotropic material, where the temporal and spatial symmetries (including the rotational symmetry) are not broken in its constitutive relation, the summation of the four wave vectors of the counter-propagating eigenmodes (including their respective mutually perpendicular polarization components) is zero. It can then be readily verified that the total electromagnetic momentum density at quantum vacuum level in the isotropic material vanishes because the contribution of the forward and backward waves (including the left- and right-handed polarized modes) are exactly cancelled by each other. In other words, the isotropic vacuum has a zero momentum since the vacuum has a universal symmetry. But this result may no longer be valid for the vacuum in an anisotropic electromagnetic environment (such as in gyrotropic chiral media and magnetoelectric materials [2, 3, 4]), since the universal symmetry of vacuum in the anisotropic environment may be broken, and the noncompensation effect of a pair of counter-propagating vacuum modes arises due to the broken universal symmetry of the vacuum itself [2]. In a moving medium, likewise, the temporal and spatial symmetries are also broken, such a summation will no longer vanish, either. The nonzero total wave vectors of the eigenmodes may be one of the most remarkable features of a motion-induced-anisotropy material.

To consider the mechanical effect of the quantum vacuum modes, we first discuss the eigenmodes and wave propagation in a moving uniaxial medium, and then study the field quantization problem of the medium. Based on this, an explicit expression for the anisotropic distribution of vacuum momentum density in the moving medium is derived.

### 4.2.1 Eigenmodes in a moving medium

The electric permittivity and magnetic permeability tensors of a uniaxial medium at rest are \( \bar{\epsilon} = \text{diag}[\epsilon, \epsilon, \epsilon] \) and \( \bar{\mu} = \text{diag}[\mu, \mu, \mu_z] \), respectively. The present medium cannot exhibit a quantum-vacuum mechanical effect since there is a symmetry of spatial reflection in the quantum vacuum mode distribution structure inside the medium at rest. Once the medium moves, however, the universal symmetry (e.g. spatial reflection and rotation) would be broken, and the anisotropic quantum vacuum fluctuation field inside the material would have nonzero mechanical quantities (e.g. momentum density) and would give rise to new macroscopically observable mechanical effects at quantum vacuum level. In what follows, we analyze the anisotropic nature of the wave propagation (and hence the quantum vacuum modes) in the moving material.

In a \( kDB \) system, the constitutive relation of a moving uniaxial medium with velocity \( v \) in the \( \hat{z} \) direction takes the form [1]

\[
\bar{E} = \bar{k} \cdot \bar{D} + \bar{\gamma} \cdot \bar{B}, \quad \bar{H} = \bar{\xi} \cdot \bar{D} + \bar{v} \cdot \bar{B},
\]  

(4.1)

where the constitutive matrices are defined by

\[
\bar{k} = \begin{pmatrix}
\kappa & 0 & 0 \\
0 & \kappa & 0 \\
0 & 0 & \kappa_z
\end{pmatrix} \equiv \begin{pmatrix}
\frac{\mu(1-\beta^2)c^2}{n^2-\beta^2} & 0 & 0 \\
0 & \frac{\mu(1-\beta^2)c^2}{n^2-\beta^2} & 0 \\
0 & 0 & \frac{1}{\mu_z}
\end{pmatrix},
\]

(4.2)

\[
\bar{\gamma} = \bar{\gamma}^T = \begin{pmatrix}
0 & \chi & 0 \\
-\chi & 0 & 0 \\
0 & 0 & 0
\end{pmatrix} \equiv \begin{pmatrix}
0 & \frac{\beta(n^2-1)c}{n^2-\beta^2} & 0 \\
-\frac{\beta(n^2-1)c}{n^2-\beta^2} & 0 & 0 \\
0 & 0 & 0
\end{pmatrix},
\]

(4.3)

and the gyrotropy matrices are given by

\[
\bar{\gamma} = \bar{\xi}^T = \begin{pmatrix}
0 & \frac{1}{\mu} & 0 \\
0 & \frac{1}{\mu} & 0 \\
0 & 0 & \frac{1}{\mu_z}
\end{pmatrix}, \quad \bar{\nu} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & \frac{1}{\mu_z}
\end{pmatrix},
\]

(4.4)

Here, \( \beta = v/c \) and \( n^2 = \mu\epsilon c^2 \) with \( c \) the speed of light in vacuum. Obviously, the constitutive matrices are then
and \( \tilde{\chi} = \tilde{\gamma}^T = O \), when the medium velocity is zero (\( \beta = 0 \)). In this case, \( \mathbf{\kappa} = \tilde{\epsilon}^{-1} \), \( \mathbf{\nu} = \tilde{\mu}^{-1} \), and the constitutive relation (4.1) will be reduced to the usual one, \( \mathbf{E} = \mathbf{\tilde{\epsilon}}^{-1} \cdot \mathbf{D} \), \( \mathbf{H} = \mathbf{\tilde{\mu}}^{-1} \cdot \mathbf{B} \). However, the constitutive relation (4.1) is anisotropic in character with gyrotropic matrices \( \tilde{\chi} \) and \( \tilde{\gamma} \) when the medium is moving. This can be referred to as “motion-induced anisotropy”.

By using the Maxwell equations, one can find that there are four eigenmodes, \( i.e. \), the forward and backward modes and their respective mutually perpendicular polarization components corresponding to the monomode frequency \( \omega \). These four eigenmodes can be classified into two types of fields: specifically, one type includes the forward mode with left-handed circular polarization and the backward mode with right-handed circular polarization, the other type includes the forward mode with right-handed circular polarization and the backward mode with left-handed circular polarization. As is well known, with the help of the Maxwell equations and the isotropic constitutive relation, one can show that the eigenmode wave numbers, \( k \), in an isotropic medium agree with a quartic equation (4.7) that is moving. This can be referred to as “motion-induced anisotropy”.

Thus, we have the following relations (4.2) QUANTUM-VACUUM MOMENTUM DENSITY IN A MOVING MEDIUM

\[
\begin{align*}
\sin^2 \theta + \frac{\nu}{\nu_z} \cos^2 \theta \cos^2 \theta \right) k^2 - \left(\omega - \frac{k \chi \cos \theta}{\kappa \nu_z} \right)^2 &= 0, \\
\sin^2 \theta + \frac{\nu}{\nu_z} \cos^2 \theta - \frac{\nu^2 \cos^2 \theta}{\kappa \nu_z^2} k - \frac{2 \omega \chi \cos \theta}{\kappa \nu_z^2} &= 0,
\end{align*}
\]

where we have assumed that the wave vectors \( k \) in the spherical coordinate system is expressed by \( k = k (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \). It can be directly verified that the sum of \( k \) over the two modes of the first type is nonzero due to the motion-induced anisotropy, \( i.e. \),

\[
k^1_+ + k^1_- = - \frac{2 \omega \chi \cos \theta}{\kappa \nu_z} \left( \sin^2 \theta + \frac{\nu}{\nu_z} \cos^2 \theta - \frac{\nu^2 \cos^2 \theta}{\kappa \nu_z^2} \right),
\]

where the subscripts \( \pm \) in the wave vectors \( k^1_\pm \) denote the forward and backward modes, respectively. It should be noted that the nonzero summation of \( k \) presented here is not caused simply by the dragging of motion of the medium (though it has a close relation to the Fizeau dragging effect). Instead, it results mainly from the effect of motion-induced gyrotropy, because, as the relation (4.7) indicates, the summation of \( k \) depends on the off-diagonal element, \( \chi \), of the gyrotropy matrices \( \tilde{\chi}, \tilde{\gamma} \) (exhibiting the magnetoelectric coupling [5]).

In the same fashion, the wave numbers of the second type of eigenmodes in the present moving medium agree with

\[
\begin{align*}
\sin^2 \theta + \frac{\nu}{\nu_z} \cos^2 \theta \cos^2 \theta \right) k^2 - \left(\omega - \frac{k \chi \cos \theta}{\nu \kappa_z} \right)^2 &= 0, \\
\sin^2 \theta + \frac{\nu}{\nu_z} \cos^2 \theta - \frac{\nu^2 \cos^2 \theta}{\nu \kappa_z^2} k - \frac{2 \omega \chi \cos \theta}{\nu \kappa_z^2} &= 0,
\end{align*}
\]

Then one can obtain the sum of \( k \) over these two modes (forward and backward modes):

\[
k^H_+ + k^H_- = - \frac{2 \omega \chi \cos \theta}{\nu \kappa_z} \left( \sin^2 \theta + \frac{\nu}{\nu_z} \cos^2 \theta - \frac{\nu^2 \cos^2 \theta}{\nu \kappa_z^2} \right).
\]

In the above, we considered the wave propagation in a uniaxial medium, in which \( \epsilon_z \neq \epsilon, \mu_z \neq \mu \). For convenience, we can assume that the medium is an isotropic material when it is at rest (\( i.e. \), \( \epsilon_z = \epsilon, \mu_z = \mu \)). Thus, we have the following relations

\[
\begin{align*}
\kappa \nu_z &= \frac{\kappa}{\mu} = \frac{(1 - \beta^2) \epsilon_0^2}{n^2 - \beta^2}, \quad \nu \kappa_z &= \frac{\nu}{\epsilon} = \frac{(1 - \beta^2) \epsilon_0^2}{n^2 - \beta^2}, \\
\kappa &= \epsilon \kappa = \frac{n^2 (1 - \beta^2)}{n^2 - \beta^2}, \quad \nu \nu_z &= \mu \nu = \frac{n^2 (1 - \beta^2)}{n^2 - \beta^2}.
\end{align*}
\]
This means that \( \nu z = \nu z' \) and \( \nu / \kappa z = \nu / \kappa z' \) (and hence \( k^I_+ + k^I_- = k^I_+ + k^I_- \)). The products of \( k^I_+ \), \( k^I_- \) and \( k^I_+ \), \( k^I_- \) are exactly the same, i.e.,

\[
\begin{align*}
k^I_+ k^I_- &= k^I_+ k^I_- = -\frac{\omega^2}{\kappa z} \\
&= -\frac{\omega^2}{\sin^2 \theta + \frac{n^2 - \beta^2}{1 - \beta^2} \cos^2 \theta}
\end{align*}
\] (4.12)

Obviously, in an isotropic medium at rest, \( k^I_+ k^I_- = k^I_+ k^I_- = -\omega^2 n^2 / c^2 \). This means that Eq. (4.12) involves a modification to the products of \( k^I_+ \), \( k^I_- \) and \( k^I_+ \), \( k^I_- \) due to the medium motion. From Eq. (4.12), one can conclude that \( k^I_+ k^I_- = k^I_+ k^I_- > 0 \) once \( n^2 < 1 \) and \( \beta^2 > n^2 \). This implies that the backward modes can be reversed under certain conditions. This is a phenomenon of motion-induced negative refraction [6], another interesting effect caused by the medium motion.

In the study of anisotropic distribution of the quantum-vacuum momentum density, the most important quantity is the total wave numbers of the four eigenmodes. In the present moving medium, the total wave numbers is given by

\[
\begin{align*}
\sum_{\sigma=1}^{4} k_\sigma &= k^I_+ + k^I_- + k^I_+ + k^I_- \\
&= -\frac{4 \omega}{c} \frac{\beta(n^2-1) \cos \theta}{\sin^2 \theta + \frac{n^2(1-\beta^2)}{n^2-\beta^2} \cos^2 \theta - \frac{\beta^2(n^2-1)^2}{(n^2-\beta^2)(1-\beta^2)} \cos^2 \theta} \\
&= -\frac{4 \omega}{c} \frac{\beta(n^2-1) \cos \theta}{\sin^2 \theta + \frac{n^2-\beta^2}{1-\beta^2} \cos^2 \theta}.
\end{align*}
\] (4.13)

It follows that the sum of \( k_\sigma \) over the four eigenmodes is no longer zero as it is proportional to the velocity \( v \) of the medium. In other words, the universal symmetry in the moving medium is broken because of the medium motion, and the nonzero dynamical quantities (e.g. momentum) of the quantum vacuum will therefore result. For simplicity, we assume that the medium moves at a low velocity (i.e. non-relativistic case). Thus we have \( \beta^2 \ll 1 \) and \( \beta^2 \ll n^2 \), and so expression (4.13) for the total wave numbers of the eigenmodes is reduced to the simple form

\[
\begin{align*}
\sum_{\sigma=1}^{4} k_\sigma &= -\frac{4}{c} \frac{(n^2-1) \omega \beta \cos \theta}{c}.
\end{align*}
\] (4.14)

Consequently, the total wave vectors of the four eigenmodes corresponding to the mode frequency \( \omega \) is

\[
\begin{align*}
\sum_{\sigma=1}^{4} k_\sigma &= \sum_{\sigma=1}^{4} k_\sigma \mathbf{e}_k = -\frac{4}{c} \frac{(n^2-1) \omega \beta}{c} \cos \theta \mathbf{e}_k,
\end{align*}
\] (4.15)

where the unit vector \( \mathbf{e}_k \) in the three-dimensional spherical polar coordinate system is \( (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \).

It follows that the total wave vectors of the four eigenmodes is zero if the relative refractive index \( n = 1 \) (the case of free vacuum). This means that the isotropic free quantum vacuum mode structure cannot be altered by the relative motion of the inertial frames of reference. But the quantum vacuum in an isotropic medium \( (n \neq 1) \) moving at velocity \( v \) would be anisotropic in nature, and can exhibit the nontrivial quantum-vacuum mechanical effect.

### 4.2.2 Electromagnetic momentum density

We are now in a position to calculate the electromagnetic momentum density [7] at quantum vacuum level. The electromagnetic momentum density of all the \( \omega \)-frequency modes is \( p = e_0 \mathbf{E} \times \mathbf{B} \) [8], where \( \mathbf{B} = \mathbf{k} \times \mathbf{E} / \omega \) for the time harmonic modes in accordance with the Maxwell equations. Then the momentum density, \( \mathbf{p} \), of the electromagnetic field can be rewritten as \( \mathbf{p} = \frac{e_0}{2} \left[ \mathbf{k} \mathbf{E}^2 - \mathbf{E} \cdot \mathbf{k} \mathbf{E} \right] \), where \( e_0 \) denotes the vacuum permittivity. It is clearly seen that the total electromagnetic momentum density at quantum vacuum level
4.2. QUANTUM-VACUUM MOMENTUM DENSITY IN A MOVING MEDIUM

In an isotropic material vanishes, that is, the isotropic vacuum has a zero momentum since the vacuum exhibits a universal symmetry, i.e., the contributions of the forward and backward waves (including the left- and right-handed polarized modes) exactly cancel. This result, however, may no longer hold for the vacuum in an anisotropic electromagnetic environment created inside a moving medium. Here, the noncompensation effect of a pair of counter-propagating vacuum modes will arise [2] due to the broken universal symmetry of the quantum vacuum.

We shall calculate the nonzero momentum of the quantum vacuum. In general, the parameters such as $\chi$ (gyrotropy matrix element) and $\beta$ (normalized velocity) in the constitutive relation (4.1) are very small for a slow motion. This leads to the relation $|E(k \cdot \mathbf{E})| \ll |k|E^2$. Therefore, the expression for the electromagnetic momentum density in the medium is reduced to the form $p \simeq (\epsilon_0/\omega)kE^2$. Thus the total momenta of the quantum vacuum corresponding to the $\omega$-frequency modes (the counter-propagating modes and their respective mutually perpendicular polarization components) in the moving medium are given by

$$ P_\omega = \int_V \epsilon_0 E_0^2 \omega \sum_{\sigma=1}^4 k_\sigma dV, $$

(4.16)

where $E_0$ denotes the vacuum-fluctuation field strength and $V$ is the medium volume. In order that an explicit expression for the quantum-vacuum momentum can be derived, one should evaluate the quantized electromagnetic field energy in the moving material [9]. If the dispersion in the optical “constants” of the material is not significant, then one may have the relation $cE^2V \simeq (n_p + 1/2)\hbar\omega$ for a quantized electromagnetic field. Here $\hbar$ denotes the Planck constant and $n_p$ is the total number of the $\omega$-mode photons in the material. Therefore, the quantum vacuum fluctuation energy (when the photon number $n_p$ is taken to be zero) in the material is $cE_0^2V \simeq \hbar\omega/2$. By using the relation $P_\omega = \int_V P_\omega^{(vac)} dV$, one can obtain the momentum density $p_\omega^{(vac)}$ of the $\omega$-frequency modes at quantum vacuum level, i.e.,

$$ p_\omega^{(vac)} = \frac{\epsilon_0 \hbar}{2\epsilon_V} \sum_{\sigma=1}^4 k_\sigma. $$

(4.17)

Note that the total wave vectors in expression (4.17) is the summation of the wave vectors over the four eigenmodes (the counter-propagating modes and their respective polarizations) only corresponding to the mode frequency $\omega$. In the subsection that follows, we consider the quantum-vacuum contribution of all the vacuum eigenmodes by means of the phase-space integration, and then show the anisotropic distribution of the momentum density of quantum vacuum field in the moving electromagnetic medium.

4.2.3 Nonzero momentum density of anisotropic quantum vacuum

Let us now calculate the total contribution of all the vacuum modes (wave vector parallel to $\mathbf{e}_k$) with wave number $|k| \leq k_{cut}$, where $k_{cut}$ denotes the cutoff wave number in the material. The momentum density of all the vacuum modes (summation over all the eigenmodes corresponding to various mode frequencies) is

$$ p^{(vac)} = \frac{1}{2} \sum_{\omega} p_\omega^{(vac)}. $$

(4.18)

Here the factor 1/2 means that we have taken into account both forward and backward vacuum eigenmodes (counter-propagating vacuum modes) in expression (4.17) [also see Eqs. (4.14) and (4.15)]. In the following phase-space integration, where the integrating range of the azimuthal angle $\phi$ is $[0, 2\pi]$, the numerical factor 1/2 in (4.18) can avoid taking the backward modes into consideration for a second time. The dominant quantum vacuum contribution depends upon the eigenmodes with large wave numbers, since the density of states of the eigenmodes increases rapidly as the wavelengths of the eigenmodes become small. This, therefore, means that the procedure of continuity approximation in the phase space can be applicable to this problem, namely, the summation over the wave vectors can be replaced by a phase-space integral, i.e.,

$$ \sum_{k_\omega} \rightarrow (2\pi)^{-3} V \int d^3 k_\omega. $$

If the parameters such as $\chi$ and $\beta$ in the constitutive relation (4.1) is sufficiently small (this can be fulfilled in a slow motion) and hence the universal symmetry breaking of the vacuum modes is very weak, then the phase-space volume element $d^3 k_\omega = k_\omega^2 dk_\omega d\Omega$ with the solid angle element $d\Omega = \sin \theta d\theta d\phi$. Therefore, the expression for the quantum-vacuum momentum density can be rewritten as

$$ p^{(vac)} = \frac{1}{2} \frac{V}{(2\pi)^{3}} \int p_\omega^{(vac)} d^3 k_\omega. $$

(4.19)
Substitution of expressions (4.15) and (4.30) into (4.19) yields
\[
\mathbf{p}^{(\text{vac})} = -\frac{(n^2 - 1) \beta \epsilon_0 \hbar}{(2\pi)^3 \epsilon} \left[ \int \frac{\omega_{\text{cut}}}{c} \left( \frac{\omega}{c} \right)^3 \frac{d\omega}{c} \right] \mathbf{w},
\] (4.20)
where the three-dimensional vector \( \mathbf{w} \) is \( \mathbf{w} = \int_0^\beta \Omega d\Omega \) \((\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \). Thus, the angular distribution of the momentum density of the quantum vacuum is given by
\[
\frac{d\mathbf{p}^{(\text{vac})}}{d\Omega} = -\frac{(n^2 - 1) \beta \epsilon_0 \hbar}{4(2\pi)^3 \epsilon} \left( \frac{\omega_{\text{cut}}}{c} \right)^4 \cos \theta \left( \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta \right).
\] (4.21)

Here, the medium moves along the \( \hat{z} \) direction, and \( \theta \) and \( \phi \) are the polar and azimuthal angles, respectively. The wave vector of a given vacuum mode deviates from the \( \hat{z} \) axis by the polar angle \( \theta \). The vector \( \mathbf{w} \) is \( \mathbf{w} = \int_0^{2\pi} \int_0^\beta \sin \theta d\theta \cos \theta \left( \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta \right) \), which equals \((0, 0, 4\pi/3)\). Therefore, the explicit expression for the total electromagnetic momentum density at quantum vacuum level is given by
\[
\mathbf{p}^{(\text{vac})} = -\frac{(n^2 - 1) \beta \epsilon_0 \hbar}{3(2\pi)^3 \epsilon} \left( \frac{\omega_{\text{cut}}}{c} \right)^4 (0, 0, 1).
\] (4.22)

As an isotropic (or uniaxial) medium can be characterized by a constitutive relation with gyrotropy matrices that govern the magnetoelectric coupling resulting from the medium motion, the distinguished axis of the produced gyrotropy matrices in its constitutive relation is in the direction of motion, and the induced nonzero vacuum momentum density would also be parallel to that direction. It follows from the result (4.22) that the quantum-vacuum momentum density is really along the \( \hat{z} \) axis (the direction of motion of the medium). As \( \beta = v/c \), the vacuum momentum density is proportional to the velocity of the moving medium. Obviously, from (4.22) one can see that the vacuum momentum density vanishes if \( \beta = 0 \) (i.e., the medium is at rest) and \( n = 1 \) (the relative refractive index of the vacuum). This means that only a moving electromagnetic medium with the refractive index \( n \neq 1 \) can exhibit such a quantum-vacuum effect. The physical essence of such an effect is related close to the Fizeau effect whose Fresnel drag coefficient is \( 1 - 1/n^2 \). In this sense, we can refer to the present anisotropic quantum-vacuum momentum distribution as “quantum-vacuum Fizeau effect”. Since the quantum vacuum has the nonzero momentum, one can conjecture that this would therefore lead to the effect of transfer of some dynamical physical quantities (such as momentum, energy, and even angular momentum) between the quantum vacuum and the moving material. In the reference [2], Feigel has considered the problem of quantum vacuum contribution to the medium momentum, where he suggested that the momentum transfer between a magnetoelectric medium and the quantum vacuum can possibly take place. As there is also a magnetoelectric coupling in the constitutive relation of the moving medium, we think that such a quantum-vacuum mechanical effect would also emerge for the moving medium and the anisotropic vacuum.

The vacuum momentum transfer (should such exist) [2] can be viewed as a macroscopic quantum-vacuum effect that may be valuable in the development of new techniques for device designs in photonics, quantum electronics and other areas (e.g., high-sensitivity sensor). Besides, it seems that the anisotropic quantum vacuum would possess energy density lower than the isotropic free vacuum and that the sensitive devices whose working mechanisms are based on the above effect could extract some dynamical physical quantities such as energy and momentum from the quantum vacuum. In addition, we hope that the present quantum-vacuum effect could open up an area to study some fundamental physical problems such as the field quantization and some relevant quantum optical effects inside anisotropic electromagnetic materials.

4.3 Angular momentum transfer between vacuum and medium

Here we present an interesting effect of quantum-vacuum contribution to the macroscopic mechanical properties of an anisotropic material (Faraday chiral material), in which an anisotropic electromagnetic environment could be built up and hence the universal symmetry of the quantum vacuum can be broken. It will be shown that the produced anisotropic quantum vacuum would possess a small but nonzero angular momentum density. Thus the anisotropy of the quantum vacuum may lead to a transfer of angular momentum from the quantum vacuum to the anisotropic materials (e.g., Faraday chiral material).

The anisotropy of Faraday chiral material can lead to some intriguing effects of wave propagation such as negative refractions. Apart from the classical effects, the anisotropic quantum field effects in the Faraday chiral material also deserve consideration. In this section, we consider the anisotropic characteristics of the
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eigenmodes of the electromagnetic fields (including the breaking of the universal symmetry of the vacuum modes) inside the Faraday chiral material, and then treat the problem of the field quantization in the material. Based on this, we study the electromagnetic angular momentum of the anisotropic quantum vacuum, and evaluate the velocity acquired by the Faraday chiral material due to the angular momentum transfer at quantum vacuum level. The physical significance and potential applications of such quantum-vacuum mechanical effects are also discussed.

4.3.1 Wave propagation and eigenmodes in a Faraday chiral material

The Faraday chiral material can be viewed as a combination of natural optical activity (as realized in an isotropic chiral medium [10]) with Faraday rotation (as realized in a gyroelectric or gyromagnetic medium [11]). The frequency-domain constitutive relation of the Faraday chiral material is given by [12, 13]

\[
D(\mathbf{r}) = \varepsilon \cdot E(\mathbf{r}) + \varepsilon \cdot H(\mathbf{r}), \quad B(\mathbf{r}) = -\varepsilon \cdot E(\mathbf{r}) + \mu \cdot H(\mathbf{r})
\]

(4.23)

with constitutive dyadics [13]

\[
\varepsilon = \varepsilon_0 \left[ \varepsilon I - i \varepsilon \hat{z} \times I + (\varepsilon_z - \varepsilon) \hat{z} \hat{z} \right],
\]

\[
\zeta = i \sqrt{\varepsilon_0 \mu_0} \left[ \zeta I - i \zeta \hat{z} \times I + (\zeta_z - \zeta) \hat{z} \hat{z} \right],
\]

\[
\mu = \mu_0 \left[ \mu I - i \mu \hat{z} \times I + (\mu_z - \mu) \hat{z} \hat{z} \right].
\]

(4.24)

Such a material may be fabricated by the method of homogenized composite medium (HCM) [12, 13] arising from the blending together of an isotropic chiral medium with either a magnetically biased ferrite [14] or a magnetically biased plasma [15]. With the development of polymer synthesis techniques, intensive attention has been captured to both the fundamental theoretical problems (i.e., optical and electromagnetic properties) and the experimental realizations (including potential applications) of the Faraday (gyrotropic) chiral media [16, 17, 18, 19].

In this subsection we study the anisotropy feature involved in the eigenmodes inside the Faraday chiral material and see how the universal symmetry of the vacuum modes is broken. Consider a propagation of a plane wave with field amplitudes \( E(\mathbf{r}) = E_0 \exp(ik_0 \hat{k} \cdot \mathbf{r}) \), \( H(\mathbf{r}) = H_0 \exp(ik_0 \hat{k} \cdot \mathbf{r}) \), where the free-space wave number \( k_0 = \omega/c \) and the unit vector \( \hat{z} = \hat{x} \sin \theta + \hat{y} \sin \phi \cos \theta \). Here, \( \hat{x}, \hat{y}, \hat{z} \) denote the unit vectors of the coordinates. By using the Maxwell equations and constitutive relation (4.23), one can verify that the relative wave number \( k \) agrees with a quartic equation

\[
a_4 \hat{k}^4 + a_3 \hat{k}^3 + a_2 \hat{k}^2 + a_1 \hat{k} + a_0 = 0,
\]

(4.25)

where the four roots correspond to the relative wave numbers of the four eigenmodes (including the forward and backward wave modes as well as their respective left- and right-handed polarization components) in the anisotropic material. Obviously, the wave numbers of the eigenmodes in the Faraday chiral material exhibit a characterization of anisotropy, i.e., the magnitudes of the wave numbers depend upon the spherical angles \( \theta \) and \( \phi \). We point out that the coefficients in Eq. (4.25) have explicit physical meanings, which correspond to some nontrivial effects. For example, the nonzero \( a_4 \) in Eq. (4.25) will unavoidably give rise to a non-degenerate \( \hat{k}^2 \). This may lead to a possibility of backward wave propagation and negative refraction in the anisotropic material. In the literature, Mackay and Lakhtakia first considered such negative phase-velocity propagations with arbitrary directions in Faraday chiral media and obtained complete and general results [13]. The coefficient \( a_3 \) has a physical meaning related to the summation of the relative wave numbers over the four eigenmodes:

\[
\sum_{\sigma=1}^{4} \hat{k}_\sigma = -\frac{a_3}{a_4},
\]

(4.26)

where \( \sigma \) corresponds to the four roots of quartic equation (4.25), and the values of \( a_3 \) and \( a_4 \) are of the form [13]

\[
a_3 = 2 \left\{ \mu g (\varepsilon_\zeta - \varepsilon_\xi \xi + \varepsilon_\zeta (\mu_\zeta - \mu_\xi \xi) + \varepsilon_\zeta (\mu_\zeta + \varepsilon_\mu_\zeta - 2\xi_\zeta \sin^2 \theta + 2\xi_\zeta (\varepsilon_\zeta \mu_\zeta - \xi_\zeta^2) \cos^2 \theta) \cos \theta, \right.
\]

\[
a_4 = (\varepsilon \sin^2 \theta + \varepsilon_\mu \cos^2 \theta)(\mu \sin^2 \theta + \mu_\zeta \cos^2 \theta) - (\xi \sin^2 \theta + \xi_\zeta \cos^2 \theta)^2.
\]

(4.27)

In general, for the isotropic materials and most anisotropic media, where the time and spatial symmetries involved in constitutive relations are preserved, the summation of the wave numbers of the four eigenmodes
vanishes \( (i.e., \alpha_3 = 0) \). But for the Faraday chiral material characterized by constitutive relation (4.23), such a summation (4.26) no longer vanishes (because \( \alpha_3 \neq 0 \)). This means that there may be a breaking of universal symmetry of the four eigenmodes (the counter-propagating modes as well as their respective mutually perpendicular polarization components) and that this anisotropic material would exhibit some unusual classical and quantum optical effects (including the influence on the mechanical properties of the anisotropic material itself). One of the most remarkable effects is the macroscopic mechanical effect of the anisotropic quantum-vacuum eigenmodes on the rotational or linear motion of the material. Since now the dynamical contribution of all the eigenmodes (including the vacuum eigenmodes) cannot be eliminated by each other, the quantum vacuum in an anisotropic electromagnetic environment may possess a nonzero angular momentum density. It is thus possible for the angular momentum transfer between the quantum vacuum and the anisotropic electromagnetic material to take place. In the subsection that follows, we shall consider the angular momentum of the vacuum eigenmodes (zero-point fluctuation fields) in the Faraday chiral material.

### 4.3.2 Nonzero angular momentum of anisotropic quantum vacuum

In the literature, there was an Abraham-Minkowski controversy \([20, 21, 22]\) concerning the definition of the electromagnetic angular momentum inside materials \([23, 24]\). As a tentative study, here we adopt Nelson’s viewpoint \([23]\). Thus the electromagnetic angular momentum density of the \(\omega\)-mode is \(j = \epsilon_0 \mathbf{r} \times (\mathbf{E} \times \mathbf{B})\) \([23]\), where \(\mathbf{B} = \mathbf{k} \times \mathbf{E}/\omega\). The angular momentum density \(j\) can be rewritten as

\[
j = \frac{\epsilon_0}{\omega} \mathbf{r} \times [\mathbf{kE}^2 - \mathbf{E} (\mathbf{k} \cdot \mathbf{E})].
\]

(4.28)

It can be readily verified that the total electromagnetic angular momentum density at quantum vacuum level in an isotropic material vanishes because the contribution of the forward and backward waves (including the left- and right-handed polarized modes) are exactly cancelled completely by each other. In other words, the isotropic vacuum has exactly a zero angular momentum since the vacuum has a universal symmetry. But this result may no longer be valid for the vacuum in an anisotropic electromagnetic environment created inside an anisotropic medium such as Faraday chiral material. Here, the noncompensation effect of a pair of counter-propagating vacuum modes will arise \([2]\) due to the broken universal symmetry of the quantum vacuum.

In order to demonstrate that there is truly a quantum-vacuum contribution to the angular momentum of the anisotropic Faraday chiral material, we first calculate the nonzero angular momentum of the quantum vacuum in this material. For simplicity, we assume that the dimensionless gyrotropic parameters \(\epsilon_g, \mu_g\) and chirality admittances (chirality dyadic elements) \(\xi, \xi_z, \xi_g\) are small as compared with the relative permittivity and permeability. Indeed, such a condition can be fulfilled for a practical Faraday chiral material. This leads to a relation \(|\mathbf{E} (\mathbf{k} \cdot \mathbf{E})| \ll |\mathbf{kE}^2|\). Therefore, the expression for the electromagnetic angular momentum density in the material is reduced to the form \(j \approx (\epsilon_0/\omega) \mathbf{r} \times \mathbf{kE}^2\). Thus the total angular momenta of the quantum vacuum corresponding to all the \(\omega\)-frequency modes (including the counter-propagating modes and their respective mutually perpendicular polarization components) in the Faraday chiral material is

\[
\mathbf{J}_\omega = \int_V \frac{\epsilon_0 E_0^2}{\omega} \mathbf{r} \times \left( \sum_{\sigma=1}^4 \mathbf{k}_\sigma \right) \, dV,
\]

(4.29)

where \(E_0\) denotes the field strength of the quantum vacuum fluctuation and \(V\) the medium volume. In order to obtain an explicit expression for the quantum-vacuum angular momentum, we evaluate the quantized electromagnetic field energy in the material \([9]\). If the dispersion in the optical “constants” of the material is not significant, one may have \(\epsilon_0 E^2 V \simeq (n_p + 1/2) \hbar \omega\) for a quantized electromagnetic field. Here \(\hbar\) denotes the Planck constant and \(n_p\) the total number of \(\omega\)-mode photons in the material. Thus the quantum vacuum fluctuation energy (when photon number \(n_p = 0\)) in the material is \(\epsilon_0 E_0^2 V \simeq \hbar \omega/2\). We define \(\mathbf{J}_\omega = \int_V \mathbf{j}_\omega^{(\text{vac})} \, dV\) and obtain the nonzero total angular momentum density \(\mathbf{j}_\omega^{(\text{vac})}\) of the \(\omega\)-mode at quantum vacuum level:

\[
\mathbf{j}_\omega^{(\text{vac})} = \frac{\hbar}{2\epsilon_0 V} \mathbf{r} \times \left( \sum_{\sigma=1}^4 \mathbf{k}_\sigma \right).
\]

(4.30)

The total wave vectors in expression (4.30) is the summation of the wave vectors over the four eigenmodes (the counter-propagating modes and their respective polarizations) corresponding to the mode frequency
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\( \omega \). Apparently, the quantized electromagnetic angular momentum density (4.30) is not that of the vacuum eigenmodes of all frequencies, but only that of the \( \omega \)-mode. In the next section, we can take into account the quantum-vacuum contribution of all the vacuum eigenmodes by means of phase-space integral, and then show how the nontrivial macroscopically observable mechanical effect of the quantum vacuum field on the anisotropic material takes place.

4.3.3 Angular momentum transfer at quantum vacuum level

The total angular momentum density of an isolated electromagnetic system consisting of both the quantum vacuum and the anisotropic Faraday chiral material can be zero, i.e., \( \mathbf{J}_\omega \) + \( \mathbf{r} \times \rho \mathbf{v}_\omega = 0 \), where \( \rho \) is the medium mass density and \( \mathbf{v}_\omega \), the medium velocity resulting from the \( \omega \)-mode vacuum angular momentum transfer. Therefore, the Faraday chiral material will acquire an angular momentum density with the same magnitude (as in (4.36)) but an opposite sign. It follows that the velocity acquired by the material is

\[
\mathbf{v}_\omega = -\frac{\hbar}{2\epsilon\rho V} \sum_{\sigma=1}^{4} \mathbf{k}_\sigma.
\]  

(4.31)

Thus we obtained the \( \omega \)-mode quantum vacuum contribution to the Faraday chiral material. All the quantum vacuum modes will make contributions to the net velocity of the material.

It should be noted that in the Faraday (gyrotropic) chiral material, \( \epsilon_\omega \) and \( \mu_\omega \) are different from \( \epsilon \) and \( \mu \), respectively. But for convenience, we can assume \( \epsilon_\omega \approx \epsilon \) and \( \mu_\omega \approx \mu \). This would simplify the following calculation procedure and will not affect the order of magnitude of the numerical result of the net velocity acquired by the material. Thus, the coefficient \( a_4 \rightarrow \mu \) (the term \( -(\xi \cos^2 \theta + \xi_\sigma \cos^2 \theta) \) is negligibly small compared with \( \mu \)). The Faraday chiral material under consideration has a general constitutive relation (4.23). However, in most Faraday chiral materials considered in the literature, the parameter \( \xi_\sigma \) is often taken to be zero [16, 17, 18, 19]. Thus, the quantum-vacuum contribution due to the gyrotropy in chirality admittance \( \xi \) vanishes. It follows from Eq. (4.25) that the summation of the wave vectors over the four eigenmodes (corresponding to the mode frequency \( \omega \)) is

\[
\sum_{\sigma=1}^{4} \mathbf{k}_\sigma = -a_3 \frac{\omega}{a_4} \mathbf{e}_\mathbf{k} \approx -2(\mu \epsilon)^{-1}[(\mu_\epsilon + \epsilon_\gamma \mu)(\xi_\sigma - \xi)] \frac{\omega}{c} \sin^2 \theta \cos \theta \mathbf{e}_\mathbf{k},
\]  

(4.32)

where the unit vector \( \mathbf{e}_\mathbf{k} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \). As the result in (4.31) and (4.32) involves only the quantum-vacuum contribution of one frequency mode (i.e., \( \omega \)-mode), in what follows, we will calculate the total contribution of all the vacuum modes with wave number \( k \leq k_{\text{cut}} \), where \( k_{\text{cut}} \) indicates the cutoff wave number in the material. The totally net velocity acquired by the material due to the transfer of the quantum-vacuum angular momentum is defined by \( \mathbf{v} = (1/2) \Sigma_k \mathbf{v}_{\omega} \). Here the reason for the factor \( 1/2 \) is that we have taken account of both forward and backward vacuum eigenmodes when considering the quantum vacuum contribution to the medium velocity \( \mathbf{v}_\omega \) [see expression (4.31)]. With the help of expressions (4.31) and (4.32), one can obtain an expression

\[
\mathbf{v} = -\frac{\hbar}{4\epsilon\rho V} \sum_{\mathbf{k}} 2\gamma k \sin^2 \theta \cos \theta \mathbf{e}_\mathbf{k}
\]  

(4.33)

for the acquired net velocity, where the parameter \( \gamma \) is defined by

\[
\gamma = -(\mu \epsilon)^{-1}[(\mu_\epsilon + \epsilon_\gamma \mu)(\xi_\sigma - \xi)].
\]  

(4.34)

Here, the relation \( \omega \approx kc/\sqrt{\mu \epsilon} \) in the phase space has been inserted. Under the condition of continuity approximation, the summation over the wave vectors in expression (4.33) can be replaced with a phase-space integral, i.e., \( \Sigma_k \rightarrow (2\pi)^{-3} V \int d^3k \). The phase-space volume element \( d^3k \approx k^2 dk d\Omega \), where the solid angle element \( d\Omega = \sin \theta d\theta d\phi \). Therefore, the explicit expression for the velocity acquired by the material is

\[
\mathbf{v} = -\frac{\hbar \gamma k^4}{64\pi^2\epsilon\rho} \int \sin^2 \theta \cos \theta \mathbf{e}_\mathbf{k} d\Omega.
\]  

(4.35)

Substituting the expression for the unit vector \( \mathbf{e}_\mathbf{k} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \) into (4.35), one can now show that the \( x, y \)-component velocities \( v_x = v_y = 0 \) and that the \( z \)-component velocity (along the distinguished axis of the Faraday chiral material) is

\[
v_z = -\frac{\hbar \gamma k^4_{\text{cut}}}{120\pi^2\epsilon\rho}.
\]  

(4.36)
It can be easily seen that the acquired velocity depends on both the material parameters and the cutoff wave number. From expression (4.34) for the parameter $\gamma$, the physical origin of the angular momentum transfer at quantum vacuum level is the gyrotropy nature (nonzero $\epsilon_g$ and $\mu_g$) and uniaxial chirality admittance (nonzero $\xi_z - \xi$) of the material. In other words, an anisotropic quantum vacuum with a nonzero angular momentum would be built up inside a Faraday chiral material with such anisotropic characteristics.

### 4.3.4 Numerical evaluation and discussions

The numerical evaluation of the net velocity acquired by the Faraday chiral material would sound very exciting, since the order of magnitude of the acquired velocity can be compared to the degree of accuracy of the velocity measurement technology developed recently [25].

According to the models of magnetically biased chiroplasmas and chiroferrites [14, 15, 16, 17], we can choose the dimensionless parameters in constitutive relation (4.23) as follows: $\epsilon = 3.6$, $\mu = 2.5$, $\epsilon_g = 0.15$, $\mu_g = 0.10$, and $\xi_z - \xi = 0.18$. Since the anisotropic distribution structure (and the breaking of universal symmetry) of the quantum-vacuum modes with wavelength less than nanometer size cannot be built up by the anisotropic electromagnetic environment created inside the Faraday chiral material (even if the Faraday chiral material has nanoscale microscopic structure units [26]), the cutoff wave number, $k_{\text{cut}}$, of the quantum vacuum modes inside the material can be chosen to correspond to a nanometer-sized wavelength, $\lambda^{(\text{vac})}$. Here, we choose $\lambda^{(\text{vac})} = 10$ nm. It thus follows from expression (4.36) that the velocity acquired by the Faraday chiral material due to the angular momentum transfer at quantum vacuum level is

$$v_z \simeq 19 \text{ nm/s}.$$  (4.37)

Though the velocity has simply a nanoscale order of magnitude, it is a macroscopic mechanical effect of the anisotropic quantum vacuum on the electromagnetic material.

It should be noted that such a nanoscale velocity could be detected by current technology [25]. Very recently, Wang et al. developed an experimental technique to measure a generalized Sagnac effect [25, 27]. The experiments of the generalized Sagnac effect can provide a design principle for a new technology called fiber optical sensor, which has a high stability and a very high sensitivity. Wang et al. showed that the technology of fiber optical sensor can in principle detect a nanoscale velocity of $4.8 \text{ nm/s}$ (degree of accuracy) [25]. Since the sensitivity of this measurement technology is in the range of the velocity acquired by the Faraday chiral material, the angular momentum transfer between the quantum vacuum and the Faraday chiral material could be studied experimentally by means of such a technology. The angular momentum transfer caused by the universal symmetry breaking of the vacuum can be considered a new macroscopic quantum-vacuum effect. With the development of technology for fabricating artificially gyrotropic media [28], it may be valuable for developing new techniques for device designs in photonics, quantum electronics and other areas (e.g., the high-sensitivity sensor for navigation and seismology).

### 4.4 Concluding remarks

In an isotropic electromagnetic medium, the quantum vacuum inside has a universal symmetry and hence has no influence on the motion of the medium. As an isotropic medium can be anisotropic when it moves, the quantum vacuum in an anisotropic electromagnetic environment will exhibit a nonzero momentum density, and hence the mechanical effect of the quantum vacuum on the moving medium would emerge. This is a macroscopically observable mechanical effect due to the breaking of the universal symmetry of the quantum vacuum fluctuation field. The mechanical effects related to other dynamical quantities (e.g., energy, momentum, polarization and even spin [29]) of the anisotropic quantum fluctuation fields may no longer be exactly eliminated among the forward and backward wave modes as well as their respective polarization components in the moving medium. The schemes presented here may have a potential significance in both pure and applied physics. For example, it may provide us with a new insight into the electromagnetic structures of quantum vacuum fluctuation inside anisotropic materials (including artificial electromagnetic media), and we can utilize this mechanical effect to develop some sensitive, accurate measurement techniques, e.g., nanoscale-sensitivity motion sensor. We hope that all these effects (including the quantum-vacuum energy and momentum transfers) could be applied to designs of new quantum optical and photonic devices in the future.
Bibliography


Chapter 5

Quantum coherent effects at quantum-vacuum level

In the literature, the quantum vacuum (the ground state of quantum fields) has attracted attention of many physicists in various areas such as quantum field theory [1, 2], quantum optics [3, 4] and condensed matter physics [5]. Quantum vacuum can exhibit many interesting effects, including vacuum topological structures [1], vacuum polarization (leading to the Lamb shift and hence the hyperfine structure of Hydrogen atomic spectrum) [2], Casimir effect [6, 7] and dramatic change of spontaneous emission decay in QED cavity [3]. Apart from the vacuum effects in quantum field theory, quantum vacuum in electromagnetic materials can also exhibit some novel effects such as vacuum magnetooptic birefringences [8], vacuum contribution to medium momentum [9, 10] and geometric phases at quantum vacuum level [11]. In this chapter, we suggest some effects due to the combination of quantum coherence and quantum vacuum, i.e., the quantum coherent effects, in which the quantum-vacuum zero-point electromagnetic field is involved. Two topics are addressed:

i) Spontaneous emission inhibition due to quantum interference in a three-level system. Two dressed states (the eigen states of atom-field interaction Hamiltonian) that are the linear combinations of two bare levels of an atom can be realized by a strong coupling laser beam, and in turn, the two bare levels can be formed as the linear combinations of the two dressed states. We show that the atomic spontaneous emission elimination in a three-level Lambda-configuration system can be achieved via the quantum interference between the two dressed states. Both the transient and steady spontaneous emission spectra are considered for the dressed-state self-interference effect exhibited in the present three-level system.

ii) Quantum light-induced potentials for coherent manipulation of atomic matter waves. The propagation of atomic matter waves in weakly guiding fields are studied, and the eigen states of the atom-light interaction Hamiltonians for two- and three-level atoms are derived in the full-quantum framework. As the field envelopes of the quantized light fields interacting with the atoms vary in space, the eigen energies of the atom-light systems are the functions of the locations of atoms. This means that there are quantum light-induced potential fields that can guide the motions of atoms, and that the light-induced potentials depend on the internal energy level structures of the atoms (i.e., the atoms with different excitations would experience different quantum guiding forces, and this can be used to design new atomic matter-wave beam splitters). One of the most remarkable features of the present scheme is that the quantum vacuum fluctuation contributions can lead to the quantum-vacuum guiding potentials for the guided atoms. All these quantum guiding potentials could be used to cool and trap atoms, and may be utilized for the development of new techniques of atom fibers and atom chips, where the coherent manipulation of atomic matter waves is needed.

5.1 Spontaneous emission inhibition due to quantum interference

During the past decade, a number of theoretical and experimental investigations have shown that the control of phase coherence in multilevel atomic systems can exhibit many novel and striking quantum optical phenomena for both the atomic transition processes and the wave propagation of nearly resonant light in atomic media [12, 13, 14]. One of the most intriguing phenomena is electromagnetically induced transparency (EIT) [12]. More recently, some unusual physical effects associated with EIT observed exper-
immently include ultraslow light pulse propagation, superluminal light propagation, light storage in atomic vapor and atomic ground state cooling, some of which are believed to be useful for the development of new technologies in quantum optics [12, 13, 14]. Apart from the above quantum optical effects, quantum vacuum in artificial electromagnetic media can exhibit novel effects, such as atomic spontaneous emission decay modification in cavity and waveguide [15, 16], magnetoelctric birefringences of vacuum [8], vacuum-induced medium momentum transfer [9] and quantum-vacuum geometric phases [11]. Zhu et al. showed that the vacuum spontaneous emission can be cancelled in a four-level system via quantum interference effect [17] and later reported an experimental observation [18]. Here we consider the possibility of spontaneous emission elimination in a three-level atomic system (Fig. 5.1). We show that the present spontaneous emission elimination can be caused by the quantum self-interference effect that is exhibited between two dressed states.

The two upper level transition \(|c\rangle-|a\rangle\) in Fig. 5.1 is driven by a strong coupling laser beam (\(\Omega_c\)), which can lead to two dressed states \(|a'\rangle\) and \(|c'\rangle\) (the linear combinations of levels \(|a\rangle\) and \(|c\rangle\)). The dressed states are in fact the eigen states of the interaction Hamiltonian that rules the coupling of \(|a\rangle-|c\rangle\) level pair to the coupling laser beam (\(\Omega_c\)). On the other hand, the bare levels \(|a\rangle\) and \(|c\rangle\) can also be rewritten as the linear combinations of the two dressed states \(|a'\rangle\) and \(|c'\rangle\). The quantum interference between the two dressed states would have significant influence on the population of the ground level \(|b\rangle\). We refer to this effect as “quantum self-interference effect”. The wave function of the three-level system interacting with a strong coupling laser beam (classical field \(\Omega_c\), that excites the \(|c\rangle-|a\rangle\) transition) and a weak probe field (quantized field that excites the \(|b\rangle-|a\rangle\) transition) is

\[
|\psi(t)\rangle_m = |c_a(t)\rangle |a\rangle + |c_c(t)\rangle |c\rangle |ab\rangle + \sum_k B_k(t) \exp(-i\omega_k t) |nk + 1\rangle |b\rangle,
\]

and according to the Weisskopf-Wigner approximation [19], the equations of motion for the probability amplitudes are of the form

\[
\begin{align*}
\dot{c}_a &= \frac{i}{2} \left[ \Omega_c \exp(-i\delta_c t) c_a + \sum_k g_k \exp(-i\delta_k t) \sqrt{n_k + 1} B_k - \Gamma a, \\
\dot{c}_c &= \frac{i}{2} \Omega_c^* \exp(i\delta_c t) c_a - \frac{\gamma_{ph}}{2} c_c, \\
\dot{B}_k &= \frac{i}{2} g_k^* \exp(i\delta_k t) \sqrt{n_k + 1} c_a,
\end{align*}
\]

(5.1)

where \(g_k\) represents the Rabi frequency corresponding to a single probe photon, i.e., \(g_k = \gamma_{ph} \sqrt{\omega_k/\epsilon_0 h V}\), and \(n_k\) and \(V\) are the total number of probe photons and the quantization volume, respectively. The frequency detunings are defined through \(\delta_c = \omega_c - \omega_{ac}\) and \(\delta_k = \omega_k - \omega_{ab}\). Suppose that the probe photon number \(n_k = 0\) and the probability amplitude of ground level \(|b\rangle\) is zero at time \(t = 0\), namely, \(c_a(0)^* c_a(0) + c_c(0)^* c_c(0) = 1\) (the atom is in the excited state and the field modes are in the vacuum state). As the external probe field is absent (\(n_k = 0\)), the term \(g_k \exp(-i\delta_k t) \sqrt{n_k + 1} c_a\), in the first equation of (5.1) is actually negligibly small and can be ignored in the following calculations. Thus, \(c_c\) and \(c_a\) agree

Figure 5.1: The schematic diagram of a three-level Lambda-configuration atomic system.
with the following matrix equation

$$\frac{\partial}{\partial t} \begin{pmatrix} c_c \\ c_a \end{pmatrix} = \begin{pmatrix} -\frac{\gamma_p h}{2} & \frac{i}{2} \Omega^* \\ \frac{i}{2} \Omega & -\frac{\gamma_p h}{2} \end{pmatrix} \begin{pmatrix} c_c \\ c_a \end{pmatrix},$$  \hspace{1cm} (5.2)$$

where the detuning $\delta_c = 0$ has been inserted. The eigenvalues of the matrix in Eq. (5.2) are

$$\lambda_{\pm} = -\frac{(\gamma_p h + 1)}{2} \pm \sqrt{(\gamma_p h - 1)^2 - 4\Omega^2 \Omega_c}.$$

Thus, the solution to Eq. (5.2) is given by

$$\begin{pmatrix} c_c(t) \\ c_a(t) \end{pmatrix} = \begin{pmatrix} c^+_a \\ c^+_c \end{pmatrix} e^{\lambda_+ t} + \begin{pmatrix} c^-_a \\ c^-_c \end{pmatrix} e^{\lambda_- t},$$  \hspace{1cm} (5.4)$$

where $c^+_a$ and $c^-_a$ are the two time-independent coefficients, which can be determined by the initial conditions $[B_k(0) = 0$ and $c_a(0)^* c_a(0) + c_a(0)c_a(0) = 1].$ At $t = 0$, according to expression (5.4) we have

$$\begin{cases} c_c(0) = \frac{\gamma_p h}{2} \left[ c^+_a \left( \frac{\gamma_p h}{2} + \lambda_+ \right) + c^-_a \left( \frac{\gamma_p h}{2} + \lambda_- \right) \right], \\ c_a(0) = c^+_a + c^-_a. \end{cases}$$  \hspace{1cm} (5.5)$$

The explicit expressions for $c^+_a$ and $c^-_a$ will be presented below. We shall now substitute $c_a(t) = c^+_a e^{\lambda_+ t} + c^-_a e^{\lambda_- t}$ into the third equation of (5.1), and obtain

$$B_k = \frac{i}{2g_k} \left[ c^+_a e^{(\lambda_+ + i\delta_k) t} + c^-_a e^{(\lambda_- + i\delta_k) t} \right].$$  \hspace{1cm} (5.6)$$

We integrate Eq. (5.6) and get the expression for the transient $B_k(t)$:

$$B_k(t) = \frac{i}{2g_k} \left\{ c^+_a \left[ e^{(\lambda_+ + i\delta_k) t} - 1 \right] \frac{1}{\lambda_+ + i\delta_k} + c^-_a \left[ e^{(\lambda_- + i\delta_k) t} - 1 \right] \frac{1}{\lambda_- + i\delta_k} \right\}. \hspace{1cm} (5.7)$$

Note that here the physical meaning of $B_k(t)$ is the vacuum spontaneous emission spectrum of the atom (the usual spontaneous emission spectrum $S_0$ used in the literature is proportional to the square of the modulus of $B_k$). At time $t = 0$, the probability amplitude of the ground level is $B_k(0) = 0$, and at some later time, $B_k(t)$ has a nonvanishing value. Such a nonvanishing value is caused by the spontaneous emission of the three-level system (due to the interaction between the atoms and the quantum-vacuum field). The steady spontaneous emission spectrum corresponding to time $t \rightarrow +\infty$ is of particular interest. As the matrix eigenvalues $\lambda_{\pm}$ have negative real parts, the steady value of $B_k$ reads

$$B_k(+\infty) = -\frac{i}{2g_k} \left\{ \frac{c^+_a}{\lambda_+ + i\delta_k} + \frac{c^-_a}{\lambda_- + i\delta_k} \right\}.$$  \hspace{1cm} (5.8)$$

We are now in a position to exhibit the transient and steady behavior of the spontaneous emission spectra of the present atomic system. In Fig. 5.2, we plot the general behavior of the transient evolution of the real and imaginary parts of the probability amplitude $B_k$ of the ground level, where we choose the typical parameters as follows: the spontaneous decay rate $\Gamma = 2.0 \times 10^7$ s$^{-1}$, the collisional dephasing rate $\gamma_p h = 5.0 \times 10^5$ s$^{-1}$, the Rabi frequency of the coupling beam $\Omega_c = 4.0 \times 10^7$ s$^{-1}$, the Rabi frequency of the single-photon probe field $g_k = 6.7 \times 10^6$ s$^{-1}$, the initial probability amplitudes $c_a(0) = \sqrt{3}/2$, $c_c(0) = 1/2$, and the probe detuning frequency $\delta_k = -0.2 \Gamma = -4.0 \times 10^6$ s$^{-1}$. In Fig. 5.2, the initial probability amplitude of the ground level $|b\rangle$ is zero, and during the transient evolutional process, both the real and imaginary parts of the probability amplitude of level $|b\rangle$ are oscillating (particularly, the real part $Re\{B_k\}$ has a damped oscillating amplitude) and finally reach the steady-state values (the three-level steady values). The dispersive behavior of the spontaneous emission spectrum in the steady case is plotted in Fig. 5.3.

Now an interesting question is suggested: can the vacuum spontaneous emission be cancelled by the quantum interference effect exhibited by the two dressed states? If this is the true case, the spontaneous emission cancellation requires $B_k(+\infty) = 0$. A way to achieve $B_k(+\infty) = 0$ is such that we can choose appropriate initial populations of levels $|a\rangle$ and $|c\rangle$. The discussion is presented as follows: the solution of Eq. (5.5) is

$$\begin{cases} c^+_a = \frac{i}{2g_k} c_a(0) \left( \frac{\gamma_p h}{2} + \lambda_+ \right) c_a(0), \\ c^-_a = -\frac{i}{2g_k} c_a(0) \left( \frac{\gamma_p h}{2} + \lambda_- \right) c_a(0). \end{cases}$$  \hspace{1cm} (5.9)$$
CHAPTER 5. QUANTUM COHERENT EFFECTS AT QUANTUM-VACUUM LEVEL

Figure 5.2: A general transient evolution of the atomic spontaneous emission (the real and imaginary parts of the probability amplitude of the ground level). The real and imaginary parts of the probability amplitude oscillatorily decreases and increases, respectively, and both approach the respective steady values.

Figure 5.3: The dispersive behavior of the steady probability amplitude of level $|b\rangle$. The typical atomic and optical parameters $\Gamma, \gamma_{ph}, \Omega_c, g_k$ and the initial probability amplitudes $c_a(0), c_c(0)$ are chosen exactly the same as in Fig. 5.2.
5.2 QUANTUM LIGHT-INDUCED POTENTIALS FOR COHERENT MANIPULATION

Figure 5.4: The spontaneous emission cancellation for the case in which the initial probability amplitudes satisfy the constraint $c_c(0)/c_a(0) = i \gamma_{ph}/\Omega_c$. The typical atomic and optical parameters $\Gamma, \gamma_{ph}, \Omega_c, g_k$ are chosen exactly the same as in Fig. 5.2. The initial population in the ground level $|b\rangle$ vanishes, and during the evolutional process, the imaginary part of the probability amplitude $B_k(t)$ is always zero, and the real part of $B_k(t)$ is oscillatorily damped and finally tends to zero (the value corresponding to the case of spontaneous emission inhibition).

Inserting expression (5.9) into Eq. (5.8) and requiring $B_k(+\infty) = 0$, one can obtain

$$i\frac{\Omega_c}{2} c_c(0) \left( \frac{1}{\lambda_+ + i\delta_k} - \frac{1}{\lambda_- + i\delta_k} \right) = c_a(0) \left( \frac{\Gamma}{\lambda_- + i\delta_k} - \frac{\Gamma}{\lambda_+ + i\delta_k} \right).$$

By using the initial condition $c_a(0)^* c_a(0) + c_c(0)^* c_c(0) = 1$, one can obtain the appropriate values of $c_c(0)$ and $c_a(0)$, which can eliminate the spontaneous emission. We consider a particularly important case where the probe detuning $\delta_k = 0$ (this vacuum mode plays a key role in making contributions to the spontaneous emission). Thus one can get the restriction leading to the spontaneous emission elimination:

$$c_a(0) = -\frac{\Omega_c}{\sqrt{\gamma_{ph}^2 + \Omega_c^2}}, \quad c_c(0) = \frac{\gamma_{ph}}{\sqrt{\gamma_{ph}^2 + \Omega_c^2}}.$$ (5.11)

In general, the system would evolve into an EIT state if the above constraint cannot be fulfilled. This means that under certain proper conditions, the spontaneous emission inhibition can emerge due to the quantum self-interference between the two dressed states of the three-level system.

To conclude, we have presented both the transient and steady behaviors of the atomic spontaneous emission spectra, and suggested a scheme to cancel the spontaneous emission by using the quantum interference effect in a three-level system. This means that the artificial electromagnetic media can be used to control the vacuum mode structures and to manipulate the quantum vacuum effects.

5.2 Quantum light-induced potentials for coherent manipulation

Now we shall consider the problem of quantum light-induced potentials for coherent manipulation of atomic matter waves. Here, we study the atomic matter waves in quantized light fields (including the quantum vacuum fluctuation fields), concentrating our attention on the quantum guiding potentials (and the guiding potentials at quantum-vacuum level) in coherent manipulation of atomic matter waves guided by external fields. We first consider the dressed states of a two-level system and the quantum guiding potentials induced in the light-atom interaction, and then study the guiding potentials (including the quantum-vacuum guiding potentials) of a three-level system. All these quantum light-induced potentials can be used for the coherent manipulation of atomic matter waves (and hence for the designs of devices of atom optics).
5.2.1 Quantum effects in guiding potentials for a two-level atom

The full Hamiltonian of a two-level atomic ensemble interacting with a photon field is \( H = p^2/2\mu + H_{A-F} \), where \( H_{A-F} \) denotes the total Hamiltonian that includes the free Hamiltonians of the atomic internal levels, the external photon field as well as their interaction Hamiltonian. \( H_{A-F} \) is written in the form

\[
H_{A-F} = \frac{1}{2} \hbar \omega_c (|e\rangle \langle e| - |g\rangle \langle g|) + \hbar [g(r, t) a |e\rangle \langle g| + g^*(r, t) a^\dagger |g\rangle \langle e|] + \hbar \omega_a a^\dagger a,
\]  

(5.12)

where \( a \) and \( a^\dagger \) stand for the annihilation and creation operators of the photons, respectively. The coupling coefficient in the atom-field Hamiltonian (5.12) is \( g(r, t) = -E_0 \mathbf{d} \cdot \mathbf{u}^*(r, t)/\hbar \), where \( E_0 = \sqrt{\hbar \omega/2\epsilon_0 F} \) standing for the electric field strength of a single photon, and \( \mathbf{d} \) and \( \mathbf{u} \) denote the electric-dipole transition matrix element and the electric-field normal mode, respectively. In the expression for \( E_0 \), the quantization volume \( V = \int \mathbf{u}^* \cdot \mathbf{d} \mathbf{r} \). The quantity \( \omega_c \) is the transition frequency between the ground level \(|g\rangle\) and the excited level \(|e\rangle\). According to the Born-Oppenheimer approximation, the state, \(|\Psi_{\eta,m}(r, t)\rangle\), of the system under consideration can be separable, i.e.,

\[
|\Psi_{\eta,m}(r, t)\rangle = \Phi_{\eta,m}(r, t) \psi_m(r, t; \eta),
\]  

(5.13)

where \( \Phi_{\eta,m}(r, t) \) and \( |\psi_m(r, t; \eta)\rangle \) denote the motional state of the atomic center of mass and the eigen states of the atomic internal levels, respectively [we use the wavefunction to stand for the atomic motional state, but adopt a ket vector \( |\rangle \) to represent the atomic internal energy levels, which are the eigen states of \( H_A(r, t) \) expressed in Eq. (5.17)]. Here, \( \eta = \pm \) corresponding to the two eigen states of the Hamiltonian [see below in Eq. (5.19)], and \( m \) is the total photon number. The time-dependent unitary transformation \( V_\omega(t) \) in (5.13) takes the form

\[
V_\omega(t) = \exp \left[ \frac{i}{\hbar} \omega \left( \frac{|e\rangle \langle e| - |g\rangle \langle g|}{2} + a^\dagger a \right) t \right].
\]  

(5.14)

It follows that the motional state \( \Phi_{\eta,m}(r, t) \) of the atomic center of mass and the state \(|\psi_m(r, t; \eta)\rangle\) of the atomic internal levels satisfy the following two time-dependent Schrödinger equations

\[
\left[ \frac{\hbar^2}{2\mu} + U_{\eta,m}(r, t) \right] \Phi_{\eta,m}(r, t) = i\hbar \frac{\partial}{\partial t} \Phi_{\eta,m}(r, t)
\]  

(5.15)

and

\[
[H_A(r, t) - U_{\eta,m}(r, t)] |\psi_m(r, t; \eta)\rangle = i\hbar \frac{\partial}{\partial t} |\psi_m(r, t; \eta)\rangle,
\]  

(5.16)

respectively. The Hamiltonian \( H_A(r, t) \) in Eq. (5.16) is given by

\[
H_A(r, t) = \frac{1}{2} \hbar \Delta (|e\rangle \langle e| - |g\rangle \langle g|) + \hbar [g(r, t) a |e\rangle \langle g| + g^*(r, t) a^\dagger |g\rangle \langle e|].
\]  

(5.17)

The present explicit expression for the Hamiltonian \( H_A(r, t) \) can be obtained in the sub-Hilbert space corresponding to the operator \( N' \) (see the Appendix for the discussion):

\[
N' = \begin{pmatrix}
    aa^\dagger & 0 \\
    0 & a^\dagger a
\end{pmatrix}.
\]  

(5.18)

It should be noted that \( U_{\eta,m}(r, t) \) in Eq. (5.15) is a guiding potential field for the motional state of the atomic center of mass. However, \( U_{\eta,m}(r, t) \) in Eq. (5.16) serves as an adiabatic energy eigenvalue corresponding to the atom-light states [see Eq. (5.19)]. In what follows, the explicit expression for the effective potential \( U_{\eta,m}(r, t) \) induced by the quantized light field will be obtained based on the concept of the adiabatic eigen states (atom-light states) of the Hamiltonian \( H_{A-F} \) in (5.12). The adiabatic eigen states of the atom-field Hamiltonian (5.12) are as follows

\[
\left\{ \begin{array}{c}
    \ket{r; m, +} = \cos \frac{\theta_m}{2} \ket{e, m} - \sin \frac{\theta_m}{2} \ket{g, m + 1}, \\
    \ket{r; m, -} = \cos \frac{\theta_m}{2} \ket{g, m + 1} + \sin \frac{\theta_m}{2} \ket{e, m}
\end{array} \right.
\]  

(5.19)

corresponding to the energy eigenvalues

\[
U_{\pm, m}(r) = \left( m + \frac{1}{2} \right) \hbar \omega \pm \sqrt{\frac{\hbar^2 \Delta^2}{4} + (m + 1)^2 |g(r)|^2},
\]  

(5.20)
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Figure 5.5: The occupation probabilities $|\cos(\theta_m/2)|^2$ of the bare states in the dressed states as the normalized spatial coordinate $\chi$ varies. The typical parameters are chosen as: $g = 0.05\Delta \sin \chi$, $\chi = x/(a/\pi)$, and $a = 100$ nm in the presence of the standing-wave optical field (optical lattice). The curve with the total photon number $m = 0$ corresponds to the quantum vacuum fluctuation contribution to the dressed states.

respectively. The parameter angle $\theta_m$ in the eigen states (5.19) is defined as $\theta_m(\mathbf{r}) = \arctan \left[ -2\sqrt{m + 1} g(\mathbf{r})/\Delta \right]$, where $g(\mathbf{r})$ is assumed to be a real function. As pointed out above, the energy eigenvalues $U_{\pm,m}(\mathbf{r})$ of the two adiabatic eigen states (5.19) are actually the effective potentials induced by the quantized light field, and then the center-of-mass motional states of the atomic matter waves in the adiabatic eigen states (5.19) would be influenced and guided by the present quantum light-induced guiding potentials. It is interesting that the light-induced potentials (both the quantum guiding potentials and their vacuum fluctuation contributions) depend on the energy level structures of the atom itself. This means that the atoms with different excitations would experience different quantum guiding forces: specifically, if the atom in the adiabatic eigen state $|\mathbf{r}; m, -\rangle$ experiences a repulsive force (i.e., deceleration directed towards the origin of the coordinate system), then the atom in the adiabatic eigen state $|\mathbf{r}; m, +\rangle$ would experience an attractive force (i.e., acceleration directed towards the origin of the coordinate system). Thus, if some atoms (in the eigen state $|\mathbf{r}; m, +\rangle$) can propagate through the potential region, then other atoms (in the eigen state $|\mathbf{r}; m, -\rangle$) would be recoiled. Therefore, one can use this mechanism to design and fabricate the so-called atomic matter-wave beam splitter. Besides, the motional state of the atomic center of mass (in the adiabatic and stationary states) satisfies the energy eigenvalue equation

$$\left[ \frac{\hbar^2}{2\mu} + U_{\pm,m}(\mathbf{r}) \right] \Phi_{\pm,m}(\mathbf{r}) = E\Phi_{\pm,m}(\mathbf{r}). \quad (5.21)$$

This implies that the present light-induced force (potential) could be used to guide the atomic matter waves for the development of atom fibers (atom guides) and atom chips [20, 21, 22].

It should be noted that the adiabatic eigen states (5.19) depend strongly on the spatial coordinate. For a 1D atomic matter wave propagation, the occupation probabilities of the bare states, $|e, m\rangle$ and $|g, m+1\rangle$, in the dressed states $|\mathbf{r}; m, \pm\rangle$ are plotted in Fig. 5.5, where we assume that the atoms move in a standing-wave optical field (optical lattice). It can be seen that the dressed states (5.19) still survive even when the external weak field is not present (i.e., the total photon number $m = 0$). Such dressed states are caused by the quantum vacuum fluctuation effect. In contrast, in the classical field theory, the mixing angle $\theta_m(\mathbf{r})$ in the eigen states (5.19) vanishes, if the light field is absent. In the realm of quantum theory, however, the vacuum fluctuation can lead to the quantum light-induced guiding potential $U_{\pm,m=0}(\mathbf{r})$ whose contribution cannot be neglected for the atomic center-of-mass motional states.
The full Hamiltonian is problems regarding the coherent manipulation of atomic matter waves), and can provide us with more light field. This make the light-induced guiding potential more flexible and open (in order to resolve more two external quantized light fields, and thus one can manipulate one quantized light field (to change, for system in quantized light fields. Here, we suggest a scheme to manipulate atomic matter waves by using two external quantized light fields, and thus one can manipulate one quantized light field (to change, for example, its intensity or mode frequency) to control the quantum guiding potential induced by another light field. This make the light-induced guiding potential more flexible and open (in order to resolve more problems regarding the coherent manipulation of atomic matter waves), and can provide us with more freedoms to control the propagations of matter waves.

In the preceding subsection, we studied the quantum guiding potentials for a two-level atomic system, where we can control the atomic center-of-mass motional state by changing some physical parameters of the system, such as the total photon number and the optical lattice scale or cavity scale (then the atom-field coupling coefficient \( g(\mathbf{r}) \) varies). However, as there is only a single mode of light field interacting with the atoms, it is impossible for us to use one light to control the guiding potential caused by another light, and many features (advantages in manipulation of atomic matter waves) of the guiding potentials would be limited. For this reason, now we consider the quantum guiding potentials for a three-level atomic system in quantized light fields. Here, we suggest a scheme to manipulate atomic matter waves by using two external quantized light fields, and thus one can manipulate one quantized light field (to change, for example, its intensity or mode frequency) to control the quantum guiding potential induced by another light field. This make the light-induced guiding potential more flexible and open (in order to resolve more problems regarding the coherent manipulation of atomic matter waves), and can provide us with more freedoms to control the propagations of matter waves.

Now we discuss the problem of the interaction of three-level atoms with two quantized light fields. The full Hamiltonian is \( H = p^2/2\mu + H_{A-F}(\mathbf{r}) \), where the atom-field Hamiltonian \( H_{A-F}(\mathbf{r}) \) reads

\[
H_{A-F}(\mathbf{r}) = \hbar \omega_1^e|\epsilon\rangle\langle\epsilon| + \hbar \omega_2^1|g_1\rangle\langle g_1| + \hbar \omega_2^2|g_2\rangle\langle g_2| + \hbar \omega_1 a_1^\dagger a_1 + \hbar \omega_2 a_2^\dagger a_2 \\
+ h g_1(\mathbf{r}) (a_1^\dagger |\epsilon\rangle\langle g_1| + a_1^\dagger |\epsilon\rangle\langle g_1|) + h g_2(\mathbf{r}) (a_2^\dagger |\epsilon\rangle\langle g_2| + a_2^\dagger |\epsilon\rangle\langle g_2|). \tag{5.22}
\]

Here we assume that the three-level atoms have one upper (excited) state and two lower (ground) states, which are denoted by |\epsilon\rangle, |g_1\rangle and |g_2\rangle, respectively (see Fig. 5.6). Levels |g_1\rangle and |g_2\rangle have the same parity. The corresponding energy eigenvalues of these three bare states are \( \hbar \omega_1^e \), \( \hbar \omega_1^e \), and \( \hbar \omega_2^e \), respectively. \( \omega_1 \) and \( \omega_2 \) in (5.22) denote the mode frequencies of the two photon fields coupled to the |g_1\rangle-|\epsilon\rangle and |g_2\rangle-|\epsilon\rangle transitions, the coupling coefficients of which are \( g_1(\mathbf{r}) \) and \( g_2(\mathbf{r}) \), respectively. Then we can obtain the three-level adiabatic dressed states as well as the energy eigenvalues that serve as the light-induced potentials guiding the motional state of the atoms moving in the two-mode standing-wave optical lattice fields.

It can be easy to get the equation governing the atomic center-of-mass motional state, where the light-induced guiding potential yields. By using the Born-Oppenheimer approximation, the separable state \( |\Psi_{\eta,n_1,n_2}(\mathbf{r})\rangle \) can be written as \( |\Psi_{\eta,n_1,n_2}(\mathbf{r})\rangle = \Phi_{\eta,n_1,n_2}(\mathbf{r}) |\mathbf{r}; n_1, n_2, \eta\rangle \), where \( \Phi_{\eta,n_1,n_2}(\mathbf{r}) \) stands for the motional state of atomic center of mass and \( |\mathbf{r}; n_1, n_2, \eta\rangle \) is the eigenstate of the interaction Hamiltonian between the atomic internal levels and the two radiation fields. The eigenstate of the interaction Hamiltonian agrees with the eigenvalue equation \( H_{A-F}(\mathbf{r}) |\mathbf{r}; n_1, n_2, \eta\rangle = E_{\eta,n_1,n_2}(r) |\mathbf{r}; n_1, n_2, \eta\rangle \) with \( \eta = 0, \pm \). The three eigen states of \( H_{A-F}(\mathbf{r}) \) are given as follows

\[
|\mathbf{r}; n_1, n_2, 0\rangle = N_0(g_2 \sqrt{n_2 + 1}|g_1, n_1 + 1, n_2\rangle - g_1 \sqrt{n_1 + 1}|g_2, n_1, n_2 + 1\rangle), \\
|\mathbf{r}; n_1, n_2, \pm\rangle = N_\pm([\epsilon, n_1, n_2] + k_\pm(g_1 \sqrt{n_1 + 1}|g_1, n_1 + 1, n_2\rangle + g_2 \sqrt{n_2 + 1}|g_2, n_1, n_2 + 1\rangle)), \tag{5.23}
\]

Figure 5.6: The schematic diagram of a three-level Λ-configuration atomic system. The \( \omega_1 \)-mode and \( \omega_2 \)-mode light fields drive the \(|g_1\rangle-|\epsilon\rangle \) and \(|g_2\rangle-|\epsilon\rangle \) transitions, respectively.
5.2. QUANTUM LIGHT-INDUCED POTENTIALS FOR COHERENT MANIPULATION

The normalized effective potential \( U_+ \) in the two optical lattice fields as the normalized spatial position \( \chi \) varies. The typical parameters are chosen as: \( \hbar g_1 = 7.0 \times 10^{-3} \mathcal{E}_{0,n_1,n_2} \sin \chi, \hbar g_2 = 5.0 \times 10^{-3} \mathcal{E}_{0,n_1,n_2} \sin 0.5\chi \). The \( \omega_1 \)-mode and \( \omega_2 \)-mode photon numbers \( n_1 = 100, n_2 = 200 \). The normalized spatial coordinate \( \chi \) is defined by \( \chi = x/(a/\pi) \) with \( a = 100 \text{ nm} \).

As the coupling coefficients \( g_1 \) and \( g_2 \) are the functions of the spatial coordinate \( r \), the energy eigenvalues \( \mathcal{E}_{\pm,n_1,n_2} \) of the atom-field adiabatic eigen states (5.23) depend on the locations of the atoms. Thus the state \( \Phi_{\eta,n_1,n_2}(r) \) of the atomic center of mass motion satisfies

\[
\left[ \frac{p^2}{2\mu} + \mathcal{E}_{\eta,n_1,n_2}(r) \right] \Phi_{\eta,n_1,n_2}(r) = E \Phi_{\eta,n_1,n_2}(r).
\]

Now the framework for studying the atomic center-of-mass motional state in the presence of the quantized guiding fields is established. The normalized effective potentials \( U_\pm = \mathcal{E}_{\pm,n_1,n_2}/\mathcal{E}_{0,n_1,n_2} \) with respect to the spatial coordinate are plotted in Figs. 5.7 and 5.8. It should be emphasized that the quantum vacuum fluctuation field can also make contribution to the induced effective potential. As an illustrative example, the comparison between two typical cases (i.e., the vacuum fluctuation and the single photon) is presented in Fig. 5.9. It follows that, under the conditions of proper parameters, the vacuum-fluctuation induced potential cannot be ignored, since it can be comparable with the contribution of the single-photon induced potential.

To summarize, we have plotted the behavior of the quantum light-induced potentials \( U_\pm = \mathcal{E}_{\pm,n_1,n_2}/\mathcal{E}_{0,n_1,n_2} \) for the atoms, which are in the two adiabatic eigen states \(|r;n_1,n_2,\pm\rangle\). It should be pointed out that the

\[
k_\pm = \frac{1}{2} \left[ \frac{N_0^2 \mathcal{E}_{0,n_1,n_2}}{\hbar} \pm \sqrt{\left( \frac{N_0^2 \mathcal{E}_{0,n_1,n_2}}{\hbar} \right)^2 + 4N_0^2} \right]. \tag{5.24}
\]

The normalized coefficients in (5.23) are

\[
N_0 = \frac{1}{\sqrt{g_1^2(n_1+1) + g_2^2(n_2+1)}}, \quad N_\pm = \frac{1}{\sqrt{1 + k_\pm^2/N_0^2}}. \tag{5.25}
\]

The corresponding energy eigenvalues of the three eigen states (5.23) are given by

\[
\mathcal{E}_{0,n_1,n_2} = \hbar \left( \omega_e + n_1\omega_1 + n_2\omega_2 \right),
\]

\[
\mathcal{E}_{\pm,n_1,n_2} = \frac{1}{2} \left( \mathcal{E}_{0,n_1,n_2} \pm \sqrt{\mathcal{E}_{0,n_1,n_2}^2 + 4\hbar^2 N_0^2} \right). \tag{5.26}
\]
CHAPTER 5. QUANTUM COHERENT EFFECTS AT QUANTUM-VACUUM LEVEL

quantum light-induced potential for the atoms in the adiabatic eigen state $|r; n_1, n_2, \eta = 0\rangle$ is a constant number $\hbar (\omega_e + n_1 \omega_1 + n_2 \omega_2)$. This means that the center-of-mass motion of the atoms in the eigen state $|r; n_1, n_2, \eta = 0\rangle$ will not be affected by the applied light fields, that is, the optical-lattice medium is "transparent" to such atoms. In a word, there are three kinds of atoms, which correspond to the three adiabatic eigen states expressed in (5.23), but only the two kinds of atoms corresponding to the two adiabatic eigen states $|r; n_1, n_2, \pm\rangle$ can be guided by the quantum light-induced potentials.

5.3 Discussions of the quantum light-induced guiding potentials

The quantum guiding potentials for the two- and three-level atoms in weakly optical fields were studied, and the eigen states of the atom-light interaction Hamiltonians were obtained using the full-quantum formalism. As the field envelopes of the quantized light fields interacting with the atoms vary in space, the adiabatic energy eigenvalues of the atom-light systems are the functions of the locations of atoms, and hence there are quantized light-induced potential fields guiding the motions of atoms. We have considered the quantum vacuum contribution to the light-induced potentials. It should be noted that the atoms can also be guided by the quantum vacuum fluctuation (particularly in a standing-wave optical lattice or in a cavity). In the classical field theory, obviously, there are no guiding potential fields for atoms, if the light field is absent. When it comes to the quantum mechanical effects, however, the vacuum fluctuation can lead to the quantum light-induced guiding potential for the atomic center-of-mass motional states. As is known, the artificial electromagnetic media can be used to control the vacuum mode structures and to manipulate the quantum vacuum effects. This can enable us to achieve the quantum-vacuum guiding potentials which are desirable in order to serve certain purposes, e.g., to realize certain flow (propagations) of atomic matter waves. All these schemes can be utilized in the manipulation of multilevel atomic matter waves in wave guides, in which the external optical fields guide the atoms. The carbon peapod, for example, can be considered to be such a kind of matter wave guide. Here, the atoms and molecules in the carbon peapod could be coherently manipulated using the quantized guiding fields, which may have some potential applications in the technique of information storage [23]. Besides, the presented formulation can be applied to the investigation of manipulation of tunable atomic matter-wave bandgap structure that is controlled by the external fields. It follows from Eq. (5.27) that the atomic matter-wave bandgap structure caused by the optical lattice created by one of the optical fields (say, the field coupled to the $|g_1\rangle-|e\rangle$ level pair), can be manipulated by the other optical fields (i.e., the optical field coupled to the $|g_2\rangle-|e\rangle$ pair). Thus, it is possible to use one light to control the interaction between the atoms and another light, and the tunable atomic matter-wave bandgap can yield.

The present scenario (the quantum light-induced potentials, including the quantum-vacuum guiding potentials)
5.3. DISCUSSIONS OF THE QUANTUM LIGHT-INDUCED GUIDING POTENTIALS

Figure 5.9: The normalized effective potentials $U_\pm$ in the two optical lattice fields (induced by the quantum vacuum fluctuation and the single photon, respectively). The typical parameters are chosen the same as in Fig. 5.7. In the case of vacuum fluctuation, the total photon numbers of the two driving light fields are $n_1 = n_2 = 0$, and in the case of single photon, $n_1 = n_2 = 1$.

potentials) might be applicable to designs of some photonic and quantum optical devices. Recently, the implementation of atom chips based on the microfabrication techniques [20, 21, 22] in atom optics can allow trapping and guiding of atoms with a high accuracy, and therefore has many applications in mesoscopic physics [24, 25, 26]. We believe that the quantum guiding potentials in atom guides (atom fibers) and atom traps deserve consideration. Besides, the quantized light-induced potentials (including the quantum-vacuum guiding potentials) would be useful in Bose-Einstein condensation (for cooling and trapping atoms) and atom interferometry (for manipulating the phase and momentum of the atomic matter waves). We hope that the work presented here could stimulate an interest in these areas.

Appendix

Deriving the Hamiltonian $H_A(r, t)$ in Eq. (5.17) using the sub-Hilbert space corresponding to the operator $N'$

According to Sec. 5.2, it follows that the motional state $\Phi_{\eta, m}(r, t)$ of the atomic centre of mass and the wavefunction $|\psi_m(r, t; \eta)\rangle$ of the atomic internal levels satisfy the following two time-dependent Schrödinger equations

$$\left[\frac{p^2}{2\mu} + U_{\eta, m}(r, t)\right] \Phi_{\eta, m}(r, t) = i\hbar \frac{\partial}{\partial t} \Phi_{\eta, m}(r, t) \quad (5.28)$$

and

$$[H_A(r, t) - U_{\eta, m}(r, t)]|\psi_m(r, t; \eta)\rangle = i\hbar \frac{\partial}{\partial t} |\psi_m(r, t; \eta)\rangle, \quad (5.29)$$

respectively. In what follows, the explicit expression for the effective potential $U_{\eta, m}(r, t)$ induced by the quantized light field will be obtained based on the concept of the adiabatic eigen states of the Hamiltonian (5.12). The Hamiltonian $H_A(r, t)$ in Eq. (5.29) is

$$H_A(r, t) = \frac{1}{2} \hbar \Delta (|e\rangle\langle e| - |g\rangle\langle g|) + \hbar \left[ g(r, t) a|e\rangle\langle g| + g^*(r, t) a^\dagger |g\rangle\langle e| \right]. \quad (5.30)$$

We should simplify Eq. (5.15). By inserting the relation

$$|\psi_m(r, t; \eta)\rangle = \exp \left[ i \frac{\hbar}{\lambda} \int_0^t U_{\eta, m}(r, t')dt' \right] |\psi_m(r, t; \eta)\rangle_U \quad (5.31)$$

into Eq. (5.15), one can yield

$$H_A(r, t)|\psi_m(r, t; \eta)\rangle_U = i\hbar \frac{\partial}{\partial t} |\psi_m(r, t; \eta)\rangle_U. \quad (5.32)$$
In the representation of the two-state base vectors \( \{|e\}, |g\rangle \), the operator \( |e\rangle \langle e| - |g\rangle \langle g| \) can be rewritten as a matrix form, i.e., \( |e\rangle \langle e| - |g\rangle \langle g| = \sigma_3 \). Thus, one can obtain the commutation relation

\[
[a|e\rangle \langle g|, a^\dagger |g\rangle \langle e|] = N' \sigma_3,
\]  

(5.33)

where the operator \( N' \) is

\[
N' = \begin{pmatrix} a a^\dagger & 0 \\ 0 & a^\dagger a \end{pmatrix}.
\]  

(5.34)

The eigenvalue equation of \( N' \) is as follows

\[
N' \begin{pmatrix} |m\rangle \\ |m+1\rangle \end{pmatrix} = (m+1) \begin{pmatrix} |m\rangle \\ |m+1\rangle \end{pmatrix}.
\]  

(5.35)

Note that the operator \( N' \) commutes with all the generators in the Hamiltonian (5.30). For this reason, the eigen states of \( N' \) have an essential significance for treating Eq. (5.15) or (5.32), namely, the solutions of Eq. (5.32) can be obtained in the sub-Hilbert space corresponding to the particular eigenvalue of the operator \( N' \). If we set the operators

\[
S_+ = (m+1)^{-\frac{1}{2}} a |e\rangle \langle g|, \quad S_- = (m+1)^{-\frac{1}{2}} a^\dagger |g\rangle \langle e|, \quad S_3 = \frac{1}{2} \sigma_3,
\]  

(5.36)

we can obtain the following commutation relations \([S_+, S_-] = 2S_3 \) and \([S_3, S_\pm] = \pm S_\pm \). Thus, the Hamiltonian (5.30) of the atomic internal levels (the transition is driven by an optical field) can be rewritten as

\[
H_A(r, t) = \hbar \Delta S_3 + \hbar \sqrt{m+1} \left[ g(r, t) S_+ + g^*(r, t) S_- \right].
\]  

(5.37)

This (and the sub-Hilbert space of the operator \( N' \)) is one of the intriguing results that are powerful for treating the light-atom interactions in the full-quantum framework.


Chapter 6

Conclusions and guide to the papers

6.1 Brief concluding remarks on the present thesis

We concentrated our attention on quantum optical properties of some artificial electromagnetic media (such as quantum coherent atomic vapors and negative refractive index materials), and suggested a few possible ways to manipulate wave propagations inside the artificial electromagnetic materials based on quantum coherence and quantum vacuum effects. We believe that, in general, most of the effects, phenomena, and properties presented here can be realized or observed in experiments.

In Chapt. 1 and 2, we reviewed the previous papers on quantum coherence as well as the related work such as electromagnetically induced transparency (EIT) and their various applications. The basic concepts of quantum coherence (atomic phase coherence, quantum interferences within atomic energy levels) and quantum vacuum are introduced, and the theoretical formulations for treating wave propagations in quantum coherent media are presented.

In Chapt. 3, we considered three topics on the manipulation of light propagations via quantum coherence and quantum interferences: i) the evolutional optical behavior (turn-on dynamics) of a four-level N-configuration atomic system; ii) the destructive and constructive quantum interferences between two control transitions (driven by the control fields) of a tripod-type four-level system, and some photonic devices such as the logic-gate devices, e.g., the NOT, OR, NOR and EXNOR gates designed based on the double-control quantum interferences; iii) some new quantum coherent schemes for realizing negative refractive indices (including the mechanism of dressed-state mixed-parity transitions). Since quantum vacuum (the ground state of quantized fields) can exhibit many interesting effects, in Chapt. 4, we investigated two quantum-vacuum effects in artificial materials: i) the anisotropic distribution of quantum-vacuum momentum density in a moving electromagnetic medium; ii) the angular momentum transfer between quantum vacuum and anisotropic medium. Such quantum-vacuum macroscopic mechanical effects could be detected by current technology, and they could be used to develop sensitive sensor techniques and to design new quantum optical and photonic devices. In Chapt. 5, we suggested some interesting effects resulting from the combination of quantum coherence and quantum vacuum: i) spontaneous emission inhibition due to quantum interference in a three-level system; ii) quantum light-induced guiding potentials for coherent manipulation of atomic matter waves.

Since the author (of the present thesis) began to do research work in the area of optics and electromagnetism, he has now published about 50 journal papers on optical physics, electromagnetic wave theory and quantum theory of light propagations in electromagnetic media (he was the first author in most of these papers). The present thesis is a part of his previous work in quantum optics and electromagnetism. The most important studies presented in this thesis include:

i) we suggested the double-control constructive and destructive quantum interference effects in multilevel atomic systems and used this effect to design some photonic logic gates and functional devices;

ii) we proposed a few quantum coherent schemes (including the mechanism based on the dressed-state mixed-parity transitions) for realizing simultaneously negative permittivity and permeability, which can lead to isotropic left-handed media with microscopic structure units at the atomic-scale level;

iii) we suggested and studied some quantum-vacuum effects such as the EIT waveguide whose optical responses are sensitive to its scale of cross section and the quantum light-induced potentials that can be used to control the atomic matter waves.

Besides the above work, there are some ideas that deserve consideration in the future. For example,
negative refractive index in quantum-dot EIT media, field quantization in anisotropic negative-index materials, and light storage and readout process due to double-control quantum destructive and constructive interferences. Obviously, the impact would be enormous if some of the quantum effects presented here can be observed experimentally. We hope that they could be realized in experiments and then utilized for designing new quantum optical and photonic devices in the near future.

6.2 Guide to the papers

We present the guide to the published papers on quantum coherence and quantum-vacuum effects in EIT atomic media and negative index materials:

1) “Backward waves and negative refractive indices in gyrotropic chiral media”
   The possibility of the backward waves and the negative refractive indices of a gyrotropic chiral material is studied, and the impedances of the eigenmodes are derived. Since the gyrotropic parameters in the permittivity and permeability tensors favor the realization of the negative refractive index in the gyrotropic chiral material, the negatively refracting medium can be achieved even far off the resonances of the permittivity and the permeability. A potential effect of the field quantization in a compact subwavelength cavity resonator containing the gyrotropic chiral material is suggested.
   The contribution of the author: Mathematical derivation and major part of writing

2) “Negative refraction and quantum vacuum effects in gyroelectric chiral medium and anisotropic magnetoelectric material”
   Some nontrivial effects (negative refraction and quantum vacuum effects) in gyroelectric chiral medium and magnetoelectric material are studied. It is shown that the refractive indices corresponding to some of the eigen modes in the gyroelectric chiral medium and the magnetoelectric material may have negative real parts since both the gyroelectric and the magnetoelectric parameters can dramatically reduce the refractive indices in certain frequency bands. As an anisotropic electromagnetic environment could be created due to the breaking of universal symmetry of vacuum mode distribution (and hence the noncompensation effect of a pair of counter-propagating vacuum modes arises) inside the magnetoelectric material, the quantum vacuum in such an anisotropic electromagnetic environment may have a nonzero angular momentum. A novel quantum vacuum effect (angular momentum transfer between the quantum vacuum and the anisotropic magnetoelectric material) that may accompany the effect of magnetoelectric negative refraction is suggested. Such a nontrivial effect can be utilized to design sensitive, accurate measurement techniques, e.g., nanoscale-sensitivity sensor.
   The contribution of the author of this thesis: Mathematical derivation, discussion of ideas, and major part of writing

3) “Dimension-sensitive optical responses of electromagnetically induced transparency vapor in a waveguide”
   Citation information: Jianqi Shen and Sailing He, Phys. Rev. A 74, 063831(1-6) (2006).
   A three-level EIT (electromagnetically induced transparency) vapor is used to manipulate the transparency and absorption properties of the probe light in a waveguide. The most remarkable feature of the present scheme is such that the optical responses resulting from both electromagnetically induced transparency and large spontaneous emission enhancement are very sensitive to the frequency detunings of the probe field as well as to the small changes of the waveguide dimension. The potential applications of the dimension- and dispersion-sensitive EIT responses are discussed, and the sensitivity limits of some waveguide-based sensors, including electric absorption modulator, optical switch, wavelength sensor, and sensitive magnetometer, are analyzed.
   The contribution of the author: Idea discussions and major part of writing

4) “Negative permeability in a Lambda-type three-level atomic vapor”
A new approach that follows directly from quantum mechanics is suggested to realize the negative magnetic permeability. It is shown that a Lambda-type three-level atomic system with proper atomic parameters can give rise to the striking magnetic responses, which could exhibit negative permeability in an optical frequency band. Both the steady and transient behaviors of the magnetic permeability in the atomic vapor are studied. The present negative-permeability vapor could be mixed with a quantum coherent vapor whose electric permittivity is negative. Such a mixed vapor may give an isotropic left-handed vapor medium at the atomic-scale level.

The contribution of the author: Idea discussions and major part of writing

5) “Local field contribution to the optical properties of multilevel coherent atomic media”
Citation information: Jianqi Shen and Sailing He, Ann. Phys. (Leipzig) 16, No. 5-6, 364-378 (2007).

The local field contribution to some three- and four-level coherent atomic vapors is considered, and a significant modification to optical properties due to dipole-dipole interaction between neighboring atoms is discussed. It is found that the local field effect can reduce loss of a four-level N-type vapor medium at the resonant frequency in both the steady and transient cases. In order to achieve a left-handed medium with a negative refractive index, a scenario to realize simultaneously negative permittivity and permeability inside a mixed coherent atomic vapor by using quantum coherence effect is suggested. One of the remarkable features of the present scheme is such that it can lead to a controllable manipulation of negative refractive index of an atomic vapor by using external coupling and signal fields. This may be a new scheme to fabricate the negatively refracting materials based on quantum optical approach (atomic phase coherence).

The contribution of the author: Idea discussions and major part of writing

6) “Influence of the signal light on the transient optical properties of a four-level EIT medium”

General formulae for the transient evolution of the susceptibility (absorption) induced by the quantum interference effect in a four-level N-type EIT medium is presented. The influence of the signal laser beam on the transient susceptibility for the probe beam is studied for two typical cases when the strength of the coupling beam is much greater or less than that of the signal field. An interesting level reciprocity relationship between these two cases is found.

The contribution of the author: Idea discussion, mathematical derivation, and major part of writing

7) “Negative refractive index in gyrotropically magnetoelectric media”
Citation information: Jianqi Shen, Phys. Rev. B 73, 045113(1-5) (2006).

The possibility to realize a negative refractive index in gyrotropically magnetoelectric materials is studied. Since the Tellegen non-reciprocity parameter may probably dramatically reduce the refractive indices, the negative refractive indices of a gyrotropic chiral material with positive permittivity and permeability can be achieved. By using the concept of equivalent isotropic medium, the respective negative equivalent permittivity and permeability corresponding to the eigenmodes inside the anisotropic material can be derived. This is a scheme to realize the negative refractive index (and hence the backward wave propagation in artificial composite materials). Besides, we consider an alternative way to realize the possible negative refraction by means of a magnetoelectrically anisotropic material.

8) “Double-control quantum interferences in a four-level atomic system”
Citation information: Jianqi Shen and Pu Zhang, Opt. Express 15, No. 10, 6484-6493 (2007).

A new scheme is suggested to manipulate the probe transitions (and hence the optical properties of atomic vapors) via double-control destructive and constructive quantum interferences. The influence of the phase coherence between the two control transitions on the probe transition is also studied. The most remarkable feature of the present scheme is that the optical properties (absorption, transparency and dispersion) of an atomic system can be manipulated using this double-control multi-pathway interferences (multiple routes to excitation). It is also shown that a four-level system will exhibit a two-level resonant absorption because the two control levels (driven by the two control fields) form a dark state (and hence a destructive quantum interference occurs between the two control transitions). However, the present four-level system will exhibit electromagnetically induced transparency to the probe field when the three lower levels (including the probe level and the two control levels) form a three-level dark state. The present scenario has potential applications in new devices (e.g. logic gates and sensitive optical switches) and new techniques (e.g. quantum coherent information storage).
The contribution of the author: Idea discussions and major part of writing

9) “Transient evolutorial behaviors of double-control electromagnetically induced transparency”

Citation information: Jianqi Shen, New J. Phys. 9, 374-388 (2007).

The evolutorial optical behavior (turn-on dynamics) of a four-level double-control tripod-configuration (electromagnetically induced transparency) system is considered based on the transient solution to the equation of motion of the probability amplitudes of the atomic levels. As the most remarkable property (quantum interference between the two control transitions) will arise in the present tripod-configuration system, the transient evolution of the permittivity in cases of both destructive and constructive quantum interferences is presented. It can be shown that the four-level double-control vapor can become a destructive-interference medium (exhibiting a two-level resonant absorption) and a constructive-interference medium (exhibiting transparency to the probe field), respectively, under certain conditions (related to the ratio of the two control field intensities). We hope that the present double-control scenario could be applicable to designs of photonic devices such as the logic and functional devices (logic gate circuits) and the key component of the technology of quantum coherent information storage.

10) “Field quantization and vacuum effects in a chiral medium”


A chiral medium can create an anisotropic electromagnetic environment, which leads to the anisotropic quantum-vacuum fields (and the observable quantum-vacuum effects). As the noncompensation effect of a pair of counterpropagating (and counterpolarized) vacuum modes will arise in the chiral medium, the physical effects resulting from the quantum-vacuum fluctuation of left- and right-handed polarized modes will no longer be exactly cancelled. This may lead to an observable quantum vacuum contribution to the Berry phases of circularly polarized modes in a time-dependent quantum system (e.g., a coiled light propagating in a noncoplanarly curved fiber). A scheme to separate the quantum-vacuum Berry phase of left-handed polarized mode from right-handed by using a chiral-medium fiber is suggested, and the time evolution of the vacuum zero-point energy in a coiled fiber is considered. We suggest an experimentally feasible scheme to test this vacuum effect by using a noncoplanar (coiled) chiral-medium fiber that has been used in the Tomita-Chiao experiment.