Reliability based design optimization for structural components

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Preface

The research presented in this thesis has been conducted between January 2007 and November 2009. The time spent has been divided between the Department of Solid Mechanics at the Royal Institute of Technology (KTH Hållfasthetslära) and the department for Dynamics and Strength Analysis, Chassis Development, at Scania CV AB. The research has been financially supported by Scania CV AB and the Swedish Governmental Agency for Innovation Systems (VINNOVA). The support is gratefully acknowledged.

The work has been highly rewarding from an engineering as well as a personal point of view. This is due to a number of people. It would lead to far to thank all in person. Thus; to those who have crossed my path I send my gratitude. However, a special thanks to those who have given significant contributions to this thesis is in order.

First and foremost, a sincere thank to my academic advisor Prof. Mårten Olsun. Without your encouragement, skillful advice and patience this thesis would not be. Also, the guidance and support from the steering committee of this project is appreciated.

Moreover, mental wellness is beneficial for successful results. Viewing, as I do, this thesis as a successful result, I wish to express my gratitude to my colleagues at both my work places. Going to work has been a pleasure during this time.

Finally, to those who matter the most, my friends and family: Your seemingly infinite support is invaluable to me.

Stockholm, November 2009
Tomas Dersjö
List of appended papers

**Paper A:** Reliability based design optimization with single point of constraint approximation  
Tomas Dersjö and Mårten Olsson  
*Report 483, Department of Solid Mechanics, Royal Institute of Technology (KTH), Stockholm, Sweden*

**Paper B:** Efficient design of experiments for reliability based design optimization using design variable screening  
Tomas Dersjö and Mårten Olsson  
*Report 484, Department of Solid Mechanics, Royal Institute of Technology (KTH), Stockholm, Sweden*
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Optimization is key to stay competitive in simulation-driven development of structural components. However, the definition of ‘optimal’ varies. Conventional deterministic optimization renders designs that are only optimal for a single input; the nominal conditions. In reality, manufacturing- and usage-induced variability in design variables and parameters cause variation in the structural integrity of components. In the case of trucks, the structural integrity varies from one component to the other due to variation in such design variables as material properties and geometrical dimensions, and such design parameters as driver behaviour and road profile. In such a situation, a deterministically optimal design is hazardous, since the variation in performance may render catastrophic consequences. Therefore, the field of stochastic optimization, which acknowledges the stochastic nature of design variables and parameters, have gained increasing attention. The computational effort can however be exhaustive and thus, there is a need for efficient algorithms. A successful stochastic optimization should include a) a choice of surrogate model, b) a problem-dependent permissible domain in design space and within that domain a recursive updating scheme for the surrogate model Region of Confidence (RoC), c) a Design of Experiments (DoE) and subsequent surrogate model fitting, d) a problem formulation, and e) an optimization strategy. This thesis focuses on c)-e) but the importance of a)-b) are recognized and shortly reviewed.

Surrogate models and design domains

Simulation-driven development of structural components involves evaluation of physics-motivated, or mechanistic, computer models, e.g. Finite Element (FE) models. An evaluation of a simulation model is, in line with physical testing, called an experiment. The design variable setting used for a specific experiment is called an experiment design. A thought-through combination of experiments is called a Design of Experiments (DoE). The computational effort associated with a single experiment may be large. Optimization and evaluation of stochastic constraints both requires a
significant amount of experiments. Thus, if the simulation model is to be used for each experiment, stochastic optimization is unfeasible. Instead, a surrogate model, which is an approximating functional expression that is not necessarily formulated with respect to the physics under study, is used to approximate the simulation model. The surrogate models are fitted to responses from experiments. A classification in interpolating and smoothing surrogate models can be made, where the former coincide with the simulation model at the experiment designs whereas the latter are not required to. Smoothing surrogate models are extensively used in the robust design methodology and the closely related response surface methodology which applies to physical tests, where the experiments are non-deterministic. In this work, it is assumed that computer codes render consistent, deterministic, results and that all input can be controlled. Therefore, interpolating functions are used rather than smoothing functions. The most commonly used surrogate model types are response surfaces, i.e. polynomial approximations, moving least squares, and kriging. If they are interpolating or not depend on the number of performed experiments and the basis functions used. For a concise review of surrogate models, see L¨ onn (2008). For more extensive studies of response surfaces, moving least squares models, and kriging models: see Myers and Montgomery (2002), Salkauskas and Lancaster (1986), and Martin and Craig (2005), respectively.

A surrogate model is however only valid in a sub-domain of the whole design domain. In this work, the surrogate model validity domain is called the Region of Confidence (RoC). First a global, permissible design space domain must be set. The permissible design domain is often set by constraints originating from requirements other than those under study. Examples of such aspects are geometrical restrictions, production-induced restrictions and performance requirements other than structural integrity. The design domain is often a hyperrectangle. In an optimization, the surrogate model is recursively updated as the design evolves in design space. An example of a recursively updated RoC is shown in Fig. 1. The first RoC is often set to a (subjective) uni-dimensionally scaled fraction of the design domain. In the simplest form, the RoC is only translated, or panned, throughout the design space as the design evolves. Another option is to use the entire design domain as the first RoC. The RoC is then updated through zooming. More sophisticated so-called pan-and-zoom algorithms have been proposed by Stander and Craig (2002).

Design of experiments

Design of experiments is a field of applied mathematics (particularly statistics) concerned with designing experiments that are in various senses optimal for experiments where scatter is present.
Figure 1: Design space and recursively updated Region of Confidence (RoC). The solid box is the permissible domain in design space and the dashed boxes are the RoCs. Red dots indicate experiment designs used to obtain responses to which the surrogate model coefficients are fitted.

For a thorough treatment of the subject, see Myers and Montgomery (2002). Design of experiments is considered an important part of the Japanese post-war industrial development. The design for six-sigma and robust design philosophies, see Taguchi (1993), are to a high extent based on design of experiments and the more wide-spanning response surface methodology. In the industrial development work where design of experiments have been used, the field of application has often been to enhance efficiency of production plants and similar. There are, however, some important differences between optimization of physical production processes and simulation-driven development of structural components. In most traditional applications, a vaguely understood process is studied. Furthermore, it is practically impossible to avoid scatter in physical experiments. If an experiment is repeated using, to the best of our knowledge, identical values for those inputs that can be controlled, the response will not be the same as for the prior experiments. Experiments are simply not deterministic and it is only possible to regard the expected response. Thus, the traditional "black-box" view, where a polynomial and a stochastic error term describe the response of the studied process in a given domain of the design space, is a reasonable approach, and one advocated by heuristics. The polynomial term is then assumed to describe the expected response and the stochastic error term accounts for the response noise caused by the scatter in uncontrollable parameters. One of the aims of design of experiments is to minimize the uncertainty in polynomial coefficient estimates. In simulation-driven development, to the best of our knowledge, the mechanistic model, e.g. the FE model, is an accurate description of the true, expected response of the phenomenon under study. Also, all parameters affecting the response can be controlled. Hence, there are reasons to use interpolating surrogate models. In a simulation context, the discrepancy between a smoothing surrogate model and the corresponding simulation model cannot be attributed to noise but rather shortcomings in the surrogate model.
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Figure 2: Examples of experiment designs in 3D: (a) Full factorial DoE (b) Koshal DoE (c) Reduced DoE

Consequently, the statistically founded experiment design optimality does not apply for simulation purposes. This does not mean that the scatter should be neglected but rather that there are more efficient ways to treat it. In the work presented in this thesis, an experiment design that require a minimum of simulations for fitting an interpolating surrogate model is used. Moreover, a novel approach to design of experiments which takes advantage of specific problem structure, called constraint-orthogonal experiment design, is presented in Paper B. Examples of experiment designs are given in Fig. 2 including an example of the reduced design presented in Paper B.

Problem formulation

The overall aim in stochastic optimization is to find a design which, accounting for stochastics, is optimal. Common for all branches of stochastic optimization is to find a solution which avoids undesired performance for a large portion of a population of components. Following Taguchi (1993), an optimal design should minimize the societal cost. The societal cost includes all costs; the direct costumer cost, *e.g.* the purchasing and repair cost, the manufacturer cost, *e.g.* warranty cost, and the third party cost, *e.g.* costs related to failures, such as medical costs. Formulating a cost function which correctly incorporates these costs is a difficult task. Instead, requirements on stochastic properties of the performance, for instance the variance or failure probability, which are easier to quantify, are introduced. The assumption behind this appears to be that, within certain bounds, the societal, or at least customer and manufacturer costs, increases with these properties, even if the exact relation is unknown.

Two main branches of stochastic optimization can be distinguished; *robust design* and *reliability based design optimization*. Robust design aims at finding a design which, while meeting
optimality conditions, is insensitive to noise, that is to say variations in design variables and parameters. This approach may be beneficial for irregular functions, where, for a number of nominal designs, the expected responses are in the same order but variations in design variables and parameters cause largely different variations in performance. In contrast, reliability based design optimization (RBDO), has mostly been applied to problems with strictly decreasing or increasing performance. For this class of problems, a sufficient distance or safety factor, formulated with respect to probabilistics, is sought. The RBDO optimization problem is stated as

$$\min_{\mathbf{x}} \ C$$

subject to

$$p_{f,j}(\mathbf{x}) \leq \alpha_{req,j}, \ j = 1, 2, \ldots, N_C$$

$$x_i^L \leq x_i \leq x_i^U, \ i = 1, 2, \ldots, N_X$$

where $\mathbf{x}$ is a vector of design variables $x_i, \ i = 1, 2, \ldots, N_X$ and $C$ is the cost function. The symbol $p_{f,j}, \ j = 1, 2, \ldots, N_C$ is the probability of failure with respect to constraint $j$ and $\alpha_{req,j}$ is the required (maximum) probability of failure with respect to constraint $j$. Finally, $x_i^L$ and $x_i^U$ are stochastic design variable $x_i$’s lower and upper bound, respectively. In stochastic optimization, a distinction is often made between deterministic design variables, stochastic design variables and stochastic design parameters. In this work, the term design variable will be used for all variables without loss of generality. The design variables $x_i$ are continuous variables and each has an associated probability density function $f_{X_i}(x_i|\theta)$ and cumulative distribution function $P(X_i \leq x_i) = F_{X_i}(x_i|\theta)$, where $\theta$ are distribution coefficients. Examples of commonly used distribution types are the normal distribution, the log-normal distribution, and the Weibull distribution. For a normally distributed variable, the coefficients needed to describe the distribution is the mean, $\mu_i$, and the standard deviation, $\sigma_i$. An example of a normally distributed variable, $X \sim \mathcal{N}(0, 0.5)$ is shown in Fig. 3.

The constraints on $\mathbf{x}$ in Eq. (1) should be interpreted as constraints on the distribution coefficients since a design variable $X_i$ may in general take any value on the real axis, i.e. $x_i \in ]-\infty, \infty[$. In statistics, a distinction between aleatory and epistemic uncertainty, where the former means scatter or intrinsic variation and the latter refers to lack of information, is sometimes made. Assigning a normal distribution to a design variable is in this context a recognition of aleatory uncertainty. However, estimating the coefficients needed to describe a normal distribution is associated with epistemic uncertainty. This uncertainty may in general be significant. Only aleatory uncertainty is regarded in the work presented here but the difficulties adherent to epistemic un-
certainty is acknowledged. In this work, the cost function does not include the cost of failures. Instead, there is the constraint on \( p_f \). It can be said with some certainty that the total cost of the component would increase if the failure probability was higher than the required failure probability used in this work. However, in Paper A, a suggestion on how to include a larger part of a component’s production costs than just mass in the optimization is made. Moreover, it has been assumed that the manufacturing precision cannot be altered. Thus, those coefficients in \( \theta \) that are related to the spread of a design variable, \( e.g. \) the standard deviation for a normally distributed variable, are fixed. The mean, the median, and the mode of a stochastic variable are all measures of location. In this work, only the means are considered to be design variables. Also, the same required probability of failure has been used for all constraints. The RBDO formulation thus reduces to

\[
\min_{\mu} C \\
\text{subject to} \begin{cases}
 p_{f,j}(x) \leq \alpha_{\text{req}}, \quad j = 1,2,\ldots,N_C \\
 \mu_i^L \leq \mu_i \leq \mu_i^U, \quad i = 1,2,\ldots,N_X
\end{cases}
\]  

\hspace{1cm} (2)

**Solution strategy**

The probability of failure with respect to constraint \( j \), \( p_{f,j} \) can be stated as

\[
p_{f,j} = P(G_j(X) \leq 0) = \int_{G_j(X) \leq 0} f_X(x) dx, \quad (3)
\]
Failure region: $G \leq 0$

Safe region: $G > 0$

Figure 4: Graphic representation of the probability of failure. The failure probability is computed by integration of the multivariate probability distribution $f_X$ over the integration domain $G \leq 0$.

where $G_j$, $j = 1, 2, \ldots, N_C$ is a performance or failure function where $G_j \leq 0$ means failure. A graphic representation of the failure probability constraint is shown in Fig. 4.

In computational solid mechanics problems, $G_j$ is almost without exception constituted by an FE model, which is computationally demanding to evaluate. Therefore, a surrogate model $\hat{G}_j$ is used to approximate it. Since the optimization is performed with respect to the mean values $\mu$, a relation between the probability of failure and the means is needed. Two main approaches can be identified for evaluation of Eq. (3); sampling based algorithms, such as Monte Carlo simulation, see Rubinstein (1981), and developments of it, and semi-analytical evaluations, see Madsen et al. (1986). The computational effort associated with the sampling-based estimates of failure probability increases with failure probability. For low probabilities, the computational effort can be comparable to that of the FE model evaluation. Also, the error in probability estimate is random. For the semi-analytical evaluations of strictly increasing or decreasing functions, the possible error is more likely to be consistent and thus more beneficial for optimization convergence. Early works on analytical evaluations of failure probability introduced the first order second moment reliability index, see Cornell (1969) and Hasofer and Lind (1974). First order refers to the order of the Taylor approximation of the failure function whereas second moment refers to the statistical measure used to describe the stochastic variables. For linear failure functions and normally
distributed failure functions, the first order second moment approach is exact. A logical next step would be to use a complete description, i.e. to attribute a distribution function, for the stochastic variables. The method of approximating the failure probability using first order failure functions and complete distribution functions is called the First Order Reliability Method (FORM), see Madsen et al. (1986), and it is used in the overwhelming majority of RBDO algorithms presented. In FORM, an isoprobabilistic transformation of the stochastic variables $X_i$ to standard normally distributed variables $U_i$ is carried out as

$$u_i = \Phi^{-1}(F_{X_i}(x_i | \mu_i)). \quad (4)$$

The relation in Eq. (4) holds for independent variables. If the variables are not independent, the Rosenblatt transformation, see Rosenblatt (1952), can be used to obtain independent standard normally distributed variables. If the individual (marginal) distributions and the covariances of the design variables are known, the Nataf transformation, see Liu and Kiu-reghian (1986), can be used for the same purpose. The constraint in Eq. (3) can after the transformation equivalently be expressed as

$$p_{f,j} = P(\hat{G}_j(U) \leq 0) = \int_{\hat{G}_j(U,\mu) \leq 0} f_U(u) du, \quad (5)$$

where also the use of a surrogate model was introduced. The transformation from design space to standard normal space is graphically interpreted in Fig. 5. Due to the hyper-rotatability of the multivariate standard normal distribution, the shortest distance from the origin to the failure limit state $\hat{G} = 0$ determines the probability of failure for linear integration domains. The point $u^*$ on the limit state which is closest to the origin is also the point on the limit state function where the integrand $f_U(u)$ is largest. Thus, it is often referred to as the Most Probable Point (MPP).

Finding the MPP is in itself an optimization problem. The MPP can be found by minimizing $u^T u$ subject to $\hat{G} = 0$, where $\hat{G}$ does not need to be linear. Obviously, for the optimality conditions to be satisfied, the MPP $u^*$ need to be proportional to the partial derivatives of the failure function at the MPP $\left. \frac{\partial \hat{G}}{\partial u} \right|_{u=u^*}$. In FORM, the failure probability is approximated by a linearization of the failure function at the MPP.

A number of solution strategies for RBDO have been presented. A review is given in Schueller and Jensen (2008). In the work presented in this thesis, a sequential approximation procedure involving two nested optimization loops are used to solve the optimization problem. The outer loop ($k$), is
related to updates of the surrogate model. The inner loop \((l)\) is related to linearizations of the failure function. In each iteration \((k, l)\), the MPP estimate is updated and, as the optimization converges, so does the MPP and the estimated failure probability. Also, a single point of multiple constraint approximation is proposed. The point of constraint approximation is formulated with respect to the probabilistic constraints and reduces the computational effort for a multiply constrained RBDO to that of a single constraint RBDO problem. For the examples used, only minor errors in constraint satisfaction are introduced.
Summary of appended papers

**Paper A:** *Reliability based design optimization with single point of multiple constraint approximation.*

The computational effort for Reliability Based Design Optimization (RBDO) is no longer prohibitive even for detailed studies of mechanical integrity. The sequential approximation RBDO formulation and the use of efficient surrogate models have greatly reduced the amount of computations necessary. However, for multiply constrained problems, such as fatigue design problems where each FE-model node constitute an additional constraint to consider, the computational effort may still be considerable. This paper present an RBDO-algorithm that use a single Point of Constraint Approximation (PCA), thus reducing the computational effort to that of a single constraint problem. Examples of different complexity from solid mechanics applications are used to present the accuracy and versatility of the proposed method. Furthermore, the PCA-based RBDO is shown to be capable of handling over 10,000 constraints and even an intermittent remeshing. Also, the benefits of considering other objectives than volume (mass) is shown through a cost optimization of a truck component where fatigue-specific procedures such a shot peening and machining to reduce surface roughness are included in both the cost and constraints.

**Paper B:** *Design of experiments for reliability based design optimization using near constraint-orthogonal design variable screening.*

In Reliability Based Design Optimization (RBDO) of large scale engineering problems, experiment design and subsequent surrogate model fitting are integral for keeping the computational effort at a reasonable level. However, most experiment designs are developed to deal with laboratory scatter, and thus not ideal for sampling of consistent computational results. In this paper, a novel screening algorithm for design of computer experiments is presented. The concept of nearly constraint-orthogonal design variables, *i.e.* variables that do not simultaneously have a significant influence on any of the constraints, is introduced. A correlation matrix, stipulating constraint orthogonality, is constructed from a complete experiment performed in a screening before the optimization. The correlation matrix is then used to construct a matrix of candidate experiment designs used as input to a binary integer problem and the number of required experiments at a given surrogate model significance is determined. Based on the relation between model significance and the necessary number of experiments, an *a priori* choice on acceptable accuracy loss can be made for each given problem. The efficiency and loss of accuracy for the proposed approach is demonstrated for a number of solid mechanics type problems. For the problems studied herein,
the necessary number of simulations can be reduced to half with only minor losses in accuracy.
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