Electromagnetic transformer modelling including the ferromagnetic core

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Abstract

In order to design a power transformer it is important to understand its internal electromagnetic behaviour. That can be obtained by measurements on physical transformers, analytical expressions and computer simulations. One benefit with simulations is that the transformer can be studied before it is built physically and that the consequences of changing dimensions and parameters easily can be asessed.

In this thesis a time-domain transformer model is presented. The model includes core phenomena as magnetic static hysteresis, eddy current and excess losses. Moreover, the model comprises winding phenomena as eddy currents, capacitive effects and leakage flux. The core and windings are first modelled separately and then connected together in a composite transformer model. This results in a detailed transformer model.

One important result of the thesis is the feasibility to simulate dynamic magnetization including the inhomogeneous field distribution due to eddy currents in the magnetic core material. This is achieved by using a Cauer circuit combined with models for static and dynamic magnetization. Thereby, all magnetic loss components in the material can be simulated accurately. This composite dynamic magnetization model is verified through experiments showing very good correspondence with measurements.

Furthermore, the composite transformer model is verified through measurements. The model is shown to yield good correspondence with measurements in normal operation and non-normal operations like no-load, inrush current and DC-magnetization.

**Keywords:** Power transformer, hysteresis, dynamic hysteresis, dynamic magnetization, eddy currents, excess losses, leakage flux, winding model, core model, magnetic measurements, magnetic materials
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Chapter 1

Introduction

1.1 Background

Power transformers have been in use for over 120 years to date [1] and are key components for transmission and distribution of electrical power. In a transformer, energy is transferred between two electrical circuits by magnetic coupling. The working principle of transformers is well-known; nevertheless, research on and development of design and materials are still taking place. The reason for this is that there is a vast amount of transformers in the electrical networks over the world and decreasing losses in the transformers means less expense for the network owners.

Power transformers are very large in size and expensive to manufacture. Therefore, models and computer simulations of transformers are saving both time and assets for the developers and manufacturers. Furthermore, by using computer models the risk of destroying the transformer during tests is reduced. In order to substitute real tests for simulations, the simulation models have to be highly detailed. This is since the internal operation of the transformer includes very complex electromagnetic behaviours. The causes for this are, e.g., the large number of conductors, the need for insulation between conductors, the potential difference between conductors and grounded parts and the magnetic core, respectively, etc. Especially the magnetic core introduces complex properties that are not present in electrical components without magnetic parts.

The importance of the core is highest at low frequencies whereas the winding is the more important part at high frequencies. This fact together with the complexity of transformers has resulted in that the describing models often have
been simplified. Moreover, this has usually been done by rather detailed modelling of the windings and coarse description of the core or vice versa. Therefore, there exists a wide variety of transformer models. A review of transformer models intended for low- and mid-frequency ranges is found in ref [2]. Below is listed a selection of references using models with different resolutions of the core models:

- Including magnetic saturation: Narang and Brierley [3] and Mork [4].

- Including magnetic saturation and hysteresis: Annakage et al [5].

- Including magnetic saturation, hysteresis and eddy currents: Dolinar et al. [6], Dallago et al. [7], Tarasiewicz et al. [8], Archer et al. [9], Chen and Neudorfer [10].

- Including magnetic saturation, hysteresis, eddy currents and excess losses: Chandrasena et al. [11], [12].

Furthermore, de Leon and Semlyen takes eddy currents and the eddy current shielding effect into account, however, they do not include magnetic saturation and hysteresis [13], [14], [15], [16], [17], [18].

Nevertheless, there is still a need for exact transformer models with high resolution of the core and winding.

### 1.2 Aim

The aim of this project is to develop models and methods that can be used in simulations and in the assembly of entire transformer models. The models shall include loss phenomena in both the core and the windings. Examples of phenomena to be included are:

- Magnetic static hysteresis

- Saturation of magnetic materials
• Dynamic magnetization effects like eddy currents and excess loss in the core

• Eddy current shielding effect in both windings and core

• Eddy currents in the windings

• Capacitive effects in the windings

• Leakage flux

The goal is to create a transformer model with detailed descriptions of both the core and windings where all the above listed phenomena are included. Moreover, the model shall be built as a module and capable to be a part of a larger electrical network. The model itself shall be built up of smaller modules. Furthermore, these modules should be able to be used in other components and applications.

1.3 Outline of the thesis

Chapter 2 starts with a short summary of the operation of transformers. Then the origin of electromagnetic losses in the core and windings are described. Moreover, the magnetization processes and hysteresis in magnetic materials are described. Furthermore, the background to dynamic magnetization; eddy currents and excess losses are presented. Especially, Bertotti’s statistical loss model [19] for the excess losses is presented. Finally, loss phenomena in the windings are described.

Chapter 3 presents a winding model including an analytical expression for the eddy currents in the winding conductors. The approach originally derived by Perry [20], [21], [22] is described. The model also includes capacitive effects and leakage flux. The author’s contribution to this chapter is the incorporation of resistive and reactive components, comprising the eddy current losses, into an electrical network of the windings. Furthermore, the dividing of the winding space into different regions regarding the flux was performed by the author.
Chapter 4 introduces the core model, starting with the static magnetization model first proposed by Dr. Anders Bergqvist [23], [24], [25]. Then, some modifications of this static model are presented. Next, a dynamic magnetization model using Cauer circuits for modelling eddy currents and the statistical loss model for description of the excess losses is presented.

The author’s contribution to this chapter is the development of the logarithmical modification of mathematical function for creating anhysteretic curves. Furthermore, the variable pinning strength was developed by the author. The use of a variable degree of reversibility was proposed by the author; however, the mathematical derivations were developed by Prof. Engdahl. The measurement of an anhysteretic curve was initiated by Prof. Engdahl and performed by Dr. Sisir Nayak. The author contributed with support during measurements and data processing. The idea to use Bergqvist’s lag model together with the classical eddy current expression and Bertottis statistical loss theory was first initiated by Prof. Engdahl. The substitution of the classical eddy current expression for the Cauer circuits was initiated and performed by the author. The idea to optimize the regions in the Cauer circuits was first proposed by Prof. Engdahl. The parameter estimation procedure for Bergqvist’s lag model, which is presented in Appendix II, was developed by Dr. Bergqvist.

Chapter 5 demonstrates the composite one-phase and three-phase transformer models including the core and winding models.

Chapter 6 contains verifications of both the static and dynamic magnetization models for electrical steels. For the static condition, measured and simulated results of major and minor loops are shown. Moreover, the dynamic magnetization model is verified under two different magnetization conditions: with controlled $H$-field and controlled $B$-field, respectively. Also comparisons between the proposed Cauer circuit model and the classical eddy current model are shown.

In addition, the dynamic model is used for studying the frequency dependence of the excess losses.

This chapter also includes verification of the composite transformer model, including measurements and simulations. This model is verified under normal operation with different loads and non-normal operations like no-load, inrush current and DC-magnetization conditions.
All measurements and simulations, except the measurements of the minor loops, have been performed by the author. The cutting of the electrical steels for verification of the dynamic magnetization model with controlled $H$-field was done by the author together with Dr. Arvid Broddefalk at Cogent Power Ltd. The stress-relief annealing of theses materials were performed by Dr. Broddefalk.

Chapter 7 concludes the thesis and proposes topics for future work.

1.4 List of publications

Some of the results in this thesis have been published in the publications below:


II. Ribbenfjärd D., Engdahl G., *Time-domain transformer model in Dymola*, Proceedings of 2\textsuperscript{nd} International Conference “From Scientific Computing to Computational Engineering”, Athens, Greece, July 2006


Chapter 2

Transformer basics

2.1 Operation of the transformer

2.1.1 Basic principles

The operation of transformers bases on the electromagnetic induction principle. An electric current, \( I \), in a winding wound around a core, induces a magnetic field, \( H \), in that core. This is in accordance with the line integral form of Ampère’s law:

\[
\oint_C H \cdot dl = I
\]  

(2.1)

Here, \( C \) is a closed curve and \( dl \) an infinitesimal element of \( C \). The field gives rise to a magnetic flux, \( \phi \), and by use of magnetic materials, the flux is magnified. The line integral form of Faraday’s law states that

\[
\oint_C E \cdot dl = -\frac{d\phi}{dt}
\]  

(2.2)

Here, \( E \) is an electric field enclosing the flux. This means that an alternating flux creates electric fields surrounding it.
For example, in a two-winding transformer as depicted in Fig 2.1, a voltage is applied to the primary winding which causes a current to flow in this winding. The current creates a magnetic flux in the core, according to Ampère’s law. The flux, in turn, induces voltages in both of the surrounding windings, according to Faraday’s law. In the primary winding the induced voltage has opposite polarity compared to the applied voltage. However, due to the losses in the core the induced voltage is slightly smaller than the applied. Therefore, a current corresponding to the difference between the applied and induced voltage, will flow.

In the secondary winding the induced voltage drives a current with opposite polarity with respect to the current in the primary winding. These currents induce opposing fluxes in the core, which however, never will be exactly equal in magnitude. This is due to the losses in core and windings.

Fig 2.1. Cross-sectional view of one leg of a two-winding transformer.
Let us now consider the working principle of an ideal transformer, which would have no losses. If the secondary side is unloaded (no-load condition) there is no current in this winding and the ratio between the voltages in the windings, $U_1$ and $U_2$ respectively, is:

$$\frac{U_1}{U_2} = \frac{N_1}{N_2} \quad (2.3)$$

Here, $N_1$ and $N_2$ are the number of turns in the windings, respectively.

In a load condition the ratio between the currents in the windings, $I_1$ and $I_2$, respectively, is:

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} \quad (2.4)$$

In Fig 2.2 an ampère-turn diagram of an ideal two-winding transformer is shown. The height of the diagram represents the current integral along the radial axis. Traversing along this axis from the innermost part of the inner winding the ampère-turn value starts from zero and increases to the outermost part of the inner winding where it reaches the value $N_1I_1$. The distance between the windings is kept small so this value is maintained in the area between the windings. In the outer winding the value decreases to zero since the currents $I_1$ and $I_2$ has opposite signs and equation (2.4) is fulfilled. In this case the ampère-turns are in balance.

However, for a non-ideal transformer there will be a difference in the ampère-turns. This is called magnetizing current since it gives rise to the flux in the core. The magnetizing current is necessary for the operation of the transformer.
2.1.2 Losses in transformers

The losses in power transformer are so small such that these apparatus function close to ideal transformers. There are, however, still losses that can not be neglected. These losses can increase the temperature of the materials in the transformer in such extent that degrading due to aging occurs. Furthermore, they can also affect the ratio between the voltages in the windings [26]. These losses can be divided into:

- **Losses in the core.** These mainly consist of magnetic losses which can be divided into hysteresis, eddy current and excess losses.

- **Losses in the windings.** These mainly consist of losses in the conductors and losses due to stray capacitances and leakage flux. The major losses in
the conductors are ohmic losses and losses due to skin and proximity effects.

- **Losses in surrounding metallic parts.** The leakage flux can cause eddy current losses in the metallic structure of the transformer. This will make the temperature increase in them.

In no-load condition the core losses are the most important since the load currents in the windings are small. However, in load condition the losses in the windings and the surrounding metallic parts are more apparent since the load currents are high.

In the succeeding parts of this chapter the core and winding loss mechanisms will be dealt with more in detail.

### 2.2 Losses in the core

The core consists of magnetic materials that guide the flux. The intrinsic energy losses in the materials consist of three major kinds:

- Magnetic static hysteresis losses

- Eddy currents losses

- Excess or anomalous losses

These parts together are sometimes called dynamic hysteresis since the two last mentioned losses increase with frequency [19].
2.2.1 Properties of magnetic materials

2.2.1.1 Magnetic domains
In absence of an applied magnetic field, the magnetization in magnetic materials originates from the spin of unpaired electrons in the outer electron shells of the atoms. A spin gives rise to a magnetic moment. In ferromagnetic materials the moments interact through interchange coupling so that they align along the same direction. However, there is competing energies like magnetocrystalline, magnetoelastic and shape anisotropies [27]. Therefore, in soft magnetic materials, the alignment of the moments due to interchange coupling will only be dominating at microscopic scales. At larger scales the anisotropy energies will be large enough to break up the material into smaller domains with homogenous magnetization within each domain. This will close the magnetic flux lines in an energetically favourable manner.

Between two adjacent domains there is a domain wall. This is an area in which the magnetization changes fast in a short range. E.g. for a 180° domain wall the magnetization reverses direction from one domain to the adjacent domain. In Fig 2.3 are examples of domain walls shown. Moreover, the domain wall is much thinner than the thickness of the domains [19], [27].

![Fig 2.3. To the left a 90° domain wall and to the right a 180° domain wall. The arrows indicate the magnetization directions.](image)

If the material is subjected to a magnetic field, this field will give an additional competing energy that will try to align the moments along the applied field. In this process the domains are forced to change their magnetization and thereby adjacent domains will merge to one larger domain. In this process the domain wall will move.
2.2.1.2 Basic relations
Let us now consider the relations between the magnetic flux density (or magnetic induction) $B$, the magnetic field $H$ and the magnetization $M$. The relation between $B$, $H$ and $M$ is given by:

$$ B = \mu_0 (H + M), \quad (2.5) $$

where $\mu_0$ is the permeability in free space. $B$ and $H$ are related through the permeability $\mu$, as:

$$ B = \mu H \quad (2.6) $$

Moreover, the relation between $M$ and $H$, is given by susceptibility, $\chi$, as:

$$ M = \chi H \quad (2.7) $$

It is also possible to define the differential permeability and susceptibility, respectively through the following relations:

$$ dB = \mu' dH \quad (2.8) $$

and:

$$ dM = \chi' dH \quad (2.9) $$

Note that the permeabilities and susceptibilities may not be constant here, since $B$ and $M$ may be nonlinear.

Furthermore, it is common to use the term relative permeability $\mu_r$, which is defined as:

$$ \mu_r = \frac{\mu}{\mu_0} \quad (2.10) $$
and:

\[ \mu_r = 1 + \chi, \quad (2.11) \]

From equation (2.10) it is also possible to define the differential relative permeability as:

\[ \mu'_r = \frac{\mu'}{\mu_0} \quad (2.12) \]

The magnetic flux in a material is defined as:

\[ \varphi = \int_B \mathbf{B} \cdot d\mathbf{S}, \quad (2.13) \]

where \( A \) is the area the flux flows through.

### 2.2.1.3 Hysteresis

For ferromagnetic materials, \( M \) is not only a highly nonlinear function of \( H \); however it also depends on its past history. This phenomenon is called hysteresis. An example of that can be seen in Fig 2.4, where it is apparent that \( B (H) \) is nonlinear and hysteretic. If \( B \) is first monotonically increased from negative saturation to positive saturation (this path is called positive branch); and then monotonically decreased back to negative saturation (this path is called negative branch) a closed loop will be created. This loop is called a major loop and is depicted in Fig 2.4.
Fig 2.4. Typical major hysteresis loop for a soft magnetic material. The loop is traced counterclockwise.

If the material cycles through a loop with smaller area than the major loop it is called a minor loop. Examples of minor loops are shown in Fig 2.5.

Fig 2.5. Example of minor loops.

Magnetic hysteresis can be divided into two kinds: rate-dependent and rate-independent. The rate-independent or static part is supposed to origin from
material imperfections in the material [29]. These imperfections, e.g., dislocations and impurities impede the magnetization, thereby creating energy loss. This effect is often called pinning.

Electrical steels are often divided into two categories: oriented and non-oriented. The non-oriented steels are isotropic and used in e.g. electrical machines where the flux is required to flow in more than one dimensions.

Conversely, in transformers, a requirement is that the flux flows mainly in the direction along the leg. Therefore, oriented, i.e., anisotropic materials are used. One process to orient a material is called grain-orientation. In this process the crystals in the material are aligned so that the easy axes of the crystals point in the same direction. This makes it easier to magnetize the material along that easy axis than perpendicularly to that axis.

2.2.1.4 Reversible and irreversible processes

The static hysteresis magnetization comprises both reversible and irreversible processes. A reversible process is one in which the magnetization changes with an applied magnetic field and returns to its original value if the field is removed. Conversely, in an irreversible process the magnetization takes a new value after the removal of an applied field.

Fig 2.6 illustrates the magnetization processes for a soft magnetic material that is first brought from a demagnetized state up into saturation. Then, it is brought to a remanent state by removal of the field. Finally, it is brought back to a state where the resulting magnetization is zero. The final step is achieved by applying a reversed field. In that procedure the following magnetization processes [23], [29] are present:

1. In a demagnetized state the material consists of domains with different magnetization such that the total magnetization in the material is zero. When applying a low field the domain walls start to bulge like an elastic membrane, which is a reversible process.

2. At low to intermediate field strength, the domains with magnetization direction close to the applied field start to grow in size. Thereby, the domain walls move, which is an irreversible process.
3. Increasing the field, eventually one large domain is created with magnetization direction along the crystallographic easy axis* closest to the applied field. This is also an irreversible process and now the material is technically saturated.

4. If the field is increased even further, the magnetic moments will slowly rotate into the field direction. This is a reversible process.

5. Now, if the field is decreased the moments will rotate back to the easy axis closest to the applied field direction.

6. If the applied field changes direction the material is split up into domains with different magnetization directions again. Now, a state where the total magnetization is zero is reached although the applied field is nonzero. This field is called the coercive field.

* E.g., in iron, the atoms are bonded together in a cubic crystal. In this case the moments of the atoms can align easily along the sides of the cube. Thereby, these directions are easily magnetized and therefore called easy axes. Conversely, the diagonal directions in the cube are hard to magnetized and therefore called hard axes [27].
Fig 2.6. Magnetization process from a demagnetized state to saturation (virgin curve), and from saturation to the coercive field. Note that in state 1 and 6 the exact domain sizes need not be equal, however, the total magnetization is zero in both cases.
2.2.2 Dynamic properties of magnetic materials

When the rate of change (e.g., the frequency) of the flux is increased, the hysteresis loop increases in width. This effect is due to rate-dependent magnetization processes and is usually divided into classical eddy currents and excess loss. These rate-dependent effects are sometimes referred to as dynamic hysteresis.

In this section infinite long sheets of magnetic materials are considered. Thereby, the magnetization is restricted to one dimension, i.e., along the axis perpendicular to the cross-sectional plane of the sheet. Therefore, the scalar expressions are used for $B$, $H$ and $M$.

2.2.2.1 Classical eddy currents

Applying a magnetic field on a magnetic material affects the magnetization which, in turn, gives rise to a flux. This flux gives rise to eddy currents rotating around the flux. Those currents will, in turn, create a counter field encircled by them. This counter field opposes the originally applied field and has a dependency proportional to $dB/dt$. Moreover, it can be expressed [19] as:

$$H_{\text{eddy}} = \frac{\sigma d^2}{2\beta} \frac{dB}{dt}$$

(2.14)

Here, $\sigma$ is the electrical conductivity, $\beta$ is a geometrical factor and $d$ is the thickness of the material according to Fig 2.7. For a sheet $\beta = 6$ [46]. Furthermore, $B$ is the spatial average of the magnetic flux density in the material. Equation (2.14) can be derived from Ampère’s law and Faraday’s law and is valid only when the material is homogenously magnetized. Then the currents are called classical eddy current. For a sinusoidal applied field with frequency $f$ this is valid when $d$ is smaller than the depth of penetration [19], i.e.:

---

* To be strictly correct, $\mu_0M = (B - \mu_0H)$ should be used instead of $B$ in this equation, since it is the magnetization that creates the currents. However, for the field strengths treated in this thesis $\mu_0H \ll B$, therefore here and throughout this thesis $B$ is used.
If the condition in equation (2.15) is not satisfied, eddy current shielding will occur. This means that the inner regions of the material will exhibit greater opposing field caused by the eddy currents than the outer regions. Since $H_{\text{eddy}}$ increases with frequency, the shielding effect will also increase with frequency. Moreover, the field amplitude will increase with the distance from the midpoint of the material. Now the field distribution inside the material is inhomogeneous and the classical eddy current expression cannot be used directly. In this thesis another model using a ladder network called Cauer circuit is presented. That model divides the cross-section of the material into smaller regions where the classical eddy current formulation is valid. This technique is presented in section 4.2.1.

\[ d < \sqrt{\frac{1}{\pi \mu_r \mu_0 \sigma f}} \]  

\( (2.15) \)

**Fig 2.7. Eddy currents in a magnetic sheet.**

### 2.2.2.2 Excess loss

It is common to consider the difference between the total magnetic loss in a material and the contributions from static hysteresis and eddy current losses as excess or anomalous losses. The main part of the excess loss is considered to origin from the domain wall movements when magnetizing a material. These movements will introduce locally flux changes which, in turn, induce microscopic eddy currents [19]. This loss is rate-dependent.

A statistical loss theory built on a phenomenological description of the magnetization in which there are a number of active correlation regions...
randomly distributed in the material has been proposed by Bertotti [19], [30]. The correlation regions are connected to the microstructure of the material like grain size, crystallographic textures and residual stresses. Hereby the magnetic field created by the excess losses can be expressed as:

$$H_{\text{excess}} = \frac{n_0 V_0}{2} \left( \sqrt{1 + \frac{4 \sigma G d w}{n_0^2 V_0} \left| \frac{dB}{dt} \right|} - 1 \right) \text{sign} \left( \frac{dB}{dt} \right)^* $$ (2.16)

In equation (2.16), $w$ is the width of the laminate according to Fig 2.7. Furthermore, $G$ is a parameter depending on the structure of the magnetic domains. $n_0$ is a phenomenological parameter related to the number of active correlation regions when $f \rightarrow 0$, whereas $V_0$ determines how much microstructural features affect the number of active correlation regions. The latter parameter also depends on the amplitude of the magnetization.

When

$$\frac{4 \sigma G d w}{n_0^2 V_0} >> 1 $$ (2.17)

equation (2.16) can be simplified to:

$$H_{\text{excess}} = \sqrt{4 \sigma G d w V_0} \left| \frac{dB}{dt} \right| \text{sign} \left( \frac{dB}{dt} \right) $$ (2.18)

This is usually the case for electrical steels.

### 2.2.2.3 Total field

The total magnetic field inside a material can be expressed as:

* See footnote to equation (2.14).
where $H_{\text{stat}}$ represents the field corresponding to the static hysteresis. Using equations (2.14) and (2.18) in equation (2.19) yields:

$$H_{\text{tot}} = H_{\text{stat}} + \frac{\sigma d^2}{2\beta} \frac{dB}{dt} + \sqrt{4\sigma GdwV_0} \left| \frac{dB}{dt} \right| \text{sign} \left( \frac{dB}{dt} \right)$$  \hspace{1cm} (2.20)

This results in a broadening of the B-H loop as seen in Fig 2.8. The static part gives rise to the rate-independent effect and $H_{\text{eddy}}$ and $H_{\text{excess}}$ give rise to the dynamic effects and thereby a broadening of the curve. From equation (2.20) we find that the relation between $H$ and $B$ is very complex, far from the linear approximation of the relative permeability that sometimes is used.

Fig 2.8. Composition of a dynamic magnetization curve at moderate frequencies for electrical steels. Solid line: $H_{\text{stat}}$. Dashed line: $H_{\text{stat}} + H_{\text{eddy}}$. Dotted line: $H_{\text{tot}} = H_{\text{stat}} + H_{\text{eddy}} + H_{\text{excess}}$. For higher frequencies the dashed and dotted lines can be rounded.
The total energy loss per volume of the magnetic material for one loop with period time $T$ can be written as:

$$W_{\text{tot}} = \int_0^T \left( H_{\text{stat}} \frac{dB}{dt} + \frac{\sigma d^2}{2\beta} \left( \frac{dB}{dt} \right)^2 + \sqrt{4\sigma GdW\nu} \left| \frac{dB}{dt} \right|^{3/2} \right) dt \quad (2.21)$$

where $T$ is the period time for one loop.

We can see from equation (2.21) that the static magnetization loss varies linearly with $dB/dt$, the classical eddy currents loss has a square dependency of $dB/dt$ and the excess loss varies with $(dB/dt)^{3/2}$. The parameter values will affect the absolute values of the terms; however, the following relation is usually valid: the static loss is dominant at low frequencies, whereas the excess loss is highest in the mid-frequency range and the classical eddy current loss is dominant for high frequencies. This is illustrated in Fig 2.9.
Fig 2.9. Loss separation: frequency dependency of the loss terms. In region I the static losses is dominant. In region II the excess losses are dominant whereas in regions III the eddy current losses are dominant. Note that for some materials region II do not exist since the excess losses do not reach the value of the eddy current losses at any frequency for these materials.

Examples of dynamic magnetization models including the classical eddy current expression and the statistical loss model are found in refs. [44], [45], [46]. Those models were applied under conditions where the classical eddy current expression was valid.
2.2.2.4 Another description of dynamic magnetization

There are also other dynamic magnetization models available in literature. One recent model that has attracted some attention is the viscosity-based magnetodynamic model suggested by Zirka et al. \[33\], \[34\], \[35\], \[36\]. In that model the dynamic effects are considered as a lag of $B$ behind $H$, i.e., magnetic viscosity. A semi empirical model has been developed which consist of one static and one dynamic term:

$$H_{\text{tot}} = H_{\text{stat}} + g(B) \frac{dB}{dt} \frac{1}{\alpha} \text{sign}(B)$$  \hspace{1cm} (2.22)

In the first approach, $\alpha$ is a material constant close to 2 for electrical steels and $g(B)$ is defined as:

$$g(B) = \frac{G_m}{1 - \frac{B^2}{B_s^2}}$$  \hspace{1cm} (2.23)

Here $B_s$ is the saturation flux density and $G_m$ is a material constant.

In ref. \[34\] it was showed that this model yields satisfying results up to 500 Hz for one grain oriented material. It was, however, later shown that in order to simulate other non oriented and grain oriented materials, the definitions of $g(B)$ and $\alpha$ needed to be updated \[33\], \[35\]. These were changed such that, e.g., $\alpha$ had a dependency of $B$ and the simple relation in equation (2.23) was changed to a 4:th and even 6:th degree polynomial of $B$. Furthermore, $\alpha$:s and $g(B)$:s dependency of $B$ differed for different values of $B$. It is obvious that this method requires much adapting of the parameters to fit the $B$-$H$ curves. Nevertheless, this method was as best able to replicate the measured curves up to around 1 kHz. However, the high number of adapting steps for each material makes the usability of this model questionable.

2.2.3 Additional loss sources in real cores

Until now the core has been considered to consist of one magnetic sheet. In reality, however, the core consists of a laminate of stacked sheets. Usually the
sheets are clamped together which can induce stresses in the material and lead to magnetostrictive losses [1]. Another source of increased losses can be “burrs” on the core laminations edges, i.e., rough areas which can create locally high eddy current losses [26]. Furthermore, aging of the material can also decrease the permeability and increase the magnetic losses in the core [78], [79], [80].

Another source of losses originates from the joints between the legs and yokes of the transformer. One reason for this is small air gaps in the joint. Another reason is that oriented materials, which have high anisotropy, are used. In the joints the flux direction is changed by 90°, however, by constructing a 45° mitre joint the losses are decreased. Nevertheless, they have to be considered in the calculation of the total losses of the core [1], [26].

### 2.3 Losses in the winding

#### 2.3.1 Losses in the winding conductors

The electromagnetic losses in the winding conductors consist of three major parts:

- Resistive or ohmic losses
- Skin effect losses
- Proximity effect losses

There has been some confusion about the term *eddy currents* in the literature. Some authors intend the proximity effect only and some others intend both the proximity effect and the skin effect [13], [20]. The latter meaning is used in this thesis.

The skin effect appears in a conductor during AC conditions when the current creates an alternating magnetic flux circulating around the current. This flux, in turn, creates an opposing current inside the conductor. With increasing frequency the opposing current will increase in amplitude and the current will
flow in a thin shell in the outer part of the conductor. This is in analogy with the magnetic eddy current effect.

Furthermore, when a number of conductors are placed close together, the $H$-field created by the current in one conductor will create eddy current losses in the nearby conductors. This is called proximity effect and occurs in the transformer windings where the conductors are exposed to $H$-fields created by the other conductors.

### 2.3.2 Other loss mechanisms in the windings

There are two other major loss mechanisms that are present in the winding window:

- **Dielectric loses due to capacitive effects.** The capacitive effects consist of, e.g., capacitances between individual turns, between discs or layers in a winding, between windings and between core and windings. The capacitances give rise to uneven voltage distribution in the windings, especially during transient conditions. At high frequencies, typically higher than 100 kHz, also dielectric losses in the insulating materials between conductors and discs or layers, become apparent [58], [69].

- **Leakage flux.** Since the magnetic core has finite permeability all of the magnetic flux created by the currents in the conductors will not be confined inside the core. Conversely, it will be present also in the winding window. In a no-load condition this is not a big issue, since only the magnetizing current is existent and this current is so small that the flux created in the winding window is negligible [26]. However, in a load condition the winding currents are much higher and therefore the flux through the conductors is significant. This flux can cause considerably eddy current losses in the winding conductors. Furthermore, the fraction of this flux that induces a voltage in only *one* of the windings and not in both gives rise to a reactive power loss. That part is here defined as leakage flux [1]. This flux is resident between the windings and inside the outer winding and corresponds to the short-circuit reactance of the transformer.
Chapter 3

Winding model

The winding types can be divided into four categories with regard to their mechanical design [1], [20]:

- **Layer winding.** The conductor is wound with succeeding turns along the axial direction of the core, constituting one layer. Thereby, the turns in one layer are connected in series. Multilayer windings consist of multiple numbers of layers connected in series with each other.

- **Helical winding.** This type consist of parallel wound conductors. The conductors are usually wound as a layer winding, however, with spacer between the turns.

- **Foil winding.** Consists of sheets of conducting materials with a width equal to the winding height and wound in a spiral around the core.

- **Disc winding.** Has two common winding techniques:
  - **Conventional.** The conductor is wound with succeeding turns outside each other in a spiral. When the space for one disc is filled up the conductor continues with turns below or above the first disc.
  
  - **Interleaved.** For a pair of discs, first half of the turns are wound in the first disc and then half of the turns in the next disc. Then the remaining turns are wound in the first disc and finally the last turns in the second disc. This procedure is continued for all pair of discs.
in the windings and gives a more linear voltage distribution along the winding.

In Fig 3.1 are the layouts of one disc winding and one layer winding compared.

*Fig 3.1. To the left is shown the layout of a disc winding and to the right the layout of a multilayer winding.*

### 3.1 Modelling of losses in conductors

As mentioned in Section 2.3.1, the losses in the conductors are mainly caused by resistive, skin and proximity effects. In a real transformer the field in the winding window will have both axial and radial components. However, the axial field is most apparent in the top and bottom of the windings. Power transformers usually have a high height to width ratio, which makes the axial field the most important. Therefore, in this thesis, an axial field is supposed in the winding model, however, a more exact model should also include a radial component.
3.1.1 Analytical expression for eddy currents

There exists, in literature, different analytical expressions for the eddy currents in coil conductors. Among these are the approaches of Dowell [65], Ferreira [66] and Perry [20], [21], [22]. Moreover, comparisons of these expressions are found in refs. [67], [68] and [72]. In ref. [68] two experimental comparisons are made on 1) a two layer air-core inductor and 2) a three layer iron core inductor. In these comparisons it is shown that Perry’s and Dowell’s expressions are in better agreement with measurements than Ferreira’s expression. It is also shown that for the iron core coil Perry’s expression is more correct than Dowell’s. Therefore, in this thesis Perry’s method is used.

Perry derives an analytical expression for the electromagnetic losses in a coil including resistive, skin effect and proximity effect losses [20], [21], [22]. Looking at the coil as a network of resistive and inductive elements excited through a single external terminal pair and comprising all electromagnetic properties in the coil, an equivalent impedance is derived, see Fig 3.2. The excitation source is assumed to be sinusoidal at a fixed angular frequency $\omega$. Since the material in the coil has a constant permeability the electromagnetic response in the network will also be sinusoidal with angular frequency $\omega$.

Here follows a short description of the procedure for calculation of the expression. For a more detailed derivation of the expression see Appendix I.

Starting with Maxwell’s equations and using the Poynting theorem, it is possible to express the input power through the terminals as a function of the current.
density, \( J \), and magnetic field \( H \). Moreover, one can derive an equivalent impedance of the coil expressed as a function of \( J \) and \( H \) in complex notation as:

\[
Z(j\omega) = \frac{1}{2\sigma|I|^2} \int \hat{J} \cdot \hat{J}^* dV - j\frac{\omega\mu}{2|I|^2} \int \hat{H} \cdot \hat{H}^* dV
\]  

(3.1)

where \( I \) is the current going through the terminal, \( J \) is the current density and \( H \) the magnetic field inside the coil. Moreover “\(^\wedge\)” denotes a complex amplitude.

Now, by finding \( J \) and \( H \) inside the coil it is possible to determine the impedance with use of equation (3.1). Consider the coil geometry as seen in Fig 3.3 with every turn having equal width and height. Solving the magnetic diffusion equation in rectangular coordinates (which is valid when the diameter of the coil is much larger than the thickness of one conductor), one gets the following expressions for \( H \) and \( J \) in the \( n \):th turn:

\[
\hat{H}_{n,z}(y) = \frac{I}{w} \left[ \frac{n\sinh(k(y-y_{n-1}))- (n-1)\sinh(k(y_{n}-y_{n-1}))}{\sinh(k(y_{n}-y_{n-1}))} \right]
\]  

(3.2)

\[
\hat{J}_{n,x}(y) = \frac{kI}{w} \left[ \frac{n\cosh(k(y-y_{n-1}))- (n-1)\cosh(k(y_{n}-y_{n-1}))}{\sinh(k(y_{n}-y_{n-1}))} \right]
\]  

(3.3)

where \( k \) is the complex wave number and \( n \) is the layer number counted inwards when \( H_z \) is increasing inwards and counted outwards when \( H_z \) is increasing outwards. E.g. for a single coil, \( n \) is counted inwards. Moreover, \( k \) is defined as:

\[
k = \frac{(j+1)}{\delta}
\]  

(3.4)
where $\delta$ is the *skin depth* or *depth of penetration* defined as:

$$
\delta = \sqrt{\frac{2}{\omega \mu \sigma}}
$$

(3.5)

*Fig 3.3. Cross-section of a portion of a multilayer coil with an arbitrary number of turns. The current is directed along the negative x-axis.*

* Compare with equation (2.15).
Furthermore, the whole part of the field in the coil is assumed to be generated in the coil itself, i.e., no field is penetrating the coil from outer sources. The field distribution when the field is increasing inwards is then as seen in Fig 3.4. Moreover, the field is assumed to only have an axial component and no radial component. This is an approximation; however, if the axial dimension is much larger than the radial, then the axial field is much larger than the radial field and the approximation is valid.

![Cross-sectional view of one side of a coil showing the field distribution in axial direction.](image)

Using equations (3.2) and (3.3) in equation (3.1) and integrating over one turn in layer $n$ with the circumference $l_n$ and height $w$ yields:

$$Z_n = \frac{l_n w}{2\sigma |\hat{I}|^2} \int_{y_{n-1}}^{y_n} \hat{J}_x(y) \hat{J}_x(y)^* dy - j \omega \mu l_n w \int_{y_{n-1}}^{y_n} \hat{H}_z(y) \hat{H}_z(y)^* dy \quad (3.6)$$
Setting in equations (3.2) and (3.3) in equation (3.6) and performing the calculations yields:

\[
Z_n = \frac{l_n}{2\sigma \delta_w} \left[ (2n^2 - 2n + 1) F_1(d_n') - 4n(n-1)F_2(d_n') \right] + \frac{j\omega \mu \delta l_n}{4w} \left[ (2n^2 - 2n + 1) F_3(d_n') - 4n(n-1)F_4(d_n') \right]
\]  

(3.7)

where

\[
d_n' = \frac{d_n}{\delta}
\]

(3.8)

and \(F_1, F_2, F_3\) and \(F_4\) are functions given as:

\[
F_1(d_n') = \frac{\sinh(2d_n') + \sin(2d_n')}{\cosh(2d_n') - \cos(2d_n')}
\]

(3.9)

\[
F_2(d_n') = \frac{\sinh(d_n')\cos(d_n') + \cosh(d_n')\sin(d_n')}{\cosh(2d_n') - \cos(2d_n')}
\]

(3.10)

\[
F_3(d_n') = \frac{\sinh(2d_n') - \sin(2d_n')}{\cosh(2d_n') - \cos(2d_n')}
\]

(3.11)

\[
F_4(d_n') = \frac{\sinh(d_n')\cos(d_n') - \cosh(d_n')\sin(d_n')}{\cosh(2d_n') - \cos(2d_n')}
\]

(3.12)

Now we have managed to find an analytical expression for the impedance of every single turn in a coil carrying a sinusoidal current. Identification of the terms in equation (3.7) shows that the first term on the right side corresponds to the resistive effect and the second term to the reactive.
In the transformer application with two windings (or coils) the layer number $n$ is counted inward for the outer winding and outward for the inner winding. In this case the axial component of the field is as shown in Fig 3.5.

![Diagram of transformer winding](image)

Fig 3.5. Axial field distribution in a winding window of a two-winding transformer.

### 3.1.1.1 Equivalent circuits

In order to use these expressions in a circuit simulation program, like Dymola*, an equivalent circuit has to be created. In this case it is easily done, since equation (3.7) consists of one resistive and one inductive part. Therefore, a resistance, $R$, is connected in series with an inductance, $L$, in order to simulate the equivalent impedance in every turn. The total impedance in one turn is given by:

$$Z_n = R_n + j\omega L_n$$  \hspace{1cm} (3.13)

---

* Dymola – Dynamic Modeling Laboratory, Dynasim AB, Sweden. www.dynasim.se
where

\[ R_n = \frac{l_n}{2\sigma\delta w} \left[ (2n^2 - 2n + 1) F_1(d_n') - 4n(n-1)F_2(d_n') \right] \]  \hspace{1cm} (3.14)

and

\[ L_n = \frac{\mu\delta l_n}{4w} \left[ (2n^2 - 2n + 1) F_3(d_n') - 4n(n-1)F_4(d_n') \right] \]  \hspace{1cm} (3.15)

### 3.1.1.2 Discussion

In the derivation of the eddy current expression it is assumed that the radial flux is negligible compared to the axial flux. It is also supposed that there are no capacitive effects between the turns. This is an approximation which is valid in a certain frequency range. It has been shown that the upper frequency limit is around 100 kHz for an inductor with iron core [68]. Moreover, one can understand that for higher frequencies both the radial flux and capacitive effects will affect the field distribution. Therefore, at higher frequencies, the linear increasing field distribution as shown in Fig 3.5 will not be valid and neither will equation (3.7) be.

### 3.2 Modelling of capacitive effects

The importance of the capacitive effects in transformer models is dependent on which frequency range it should be used for. E.g., at frequencies up to around 1 kHz the capacitances are negligible [1]. Above that the capacitances between winding and core/tank and between discs become important [1]. For frequencies above approximately 100 kHz also the capacitances between individual turns are essential [69].

A proposed disc winding model including capacitances between discs and discs and core/tank is shown in Fig 3.6. In this model the inter-turn capacitances are omitted. Moreover, the capacitances between turns in two adjacent discs are lumped to one capacitance.
• The impedance for each turn is calculated using equation (3.13). The turns in a disc are connected in series and we do not consider the capacitances between them. Therefore, the turn impedances in a disc can be connected together in series to constitute the total impedance for one disc. The total impedance of a disc is represented by the series connected $R_d$ and $L_d$ in the figure. This gives a resulting impedance for a disc with $M$ turns as:

$$Z_d = R_d + j\omega L_d = \sum_{n=1}^{M} R_n + j\omega \sum_{n=1}^{M} L_n$$

(3.16)

• The disc to disc capacitance is shunted with the impedance in the disc and denoted $C_d$ in the figure. Notice that for the discs in the ends of the windings the capacitance must be $C_d/2$. This gives the correct value of the total series disc capacitance of the winding.

• The capacitances between every disc in a winding to the closest disc in the other winding are named $C_{\text{wind-wind}}$. Furthermore, the capacitances between the windings and core, yoke and tank wall, respectively, are denoted $C_{\text{wind-core}}$, $C_{\text{wind-yoke}}$ and $C_{\text{wind-wall}}$, respectively. The core, yoke and wall are grounded.
Fig 3.6. Electrical network showing the impedances in the discs and the capacitances between discs and between discs and surrounding material. The impedances comprise the equivalent circuit for the ohmic and eddy current losses in the conductors as presented in the previous section.
3.2.1 Discussion

The model has a theoretical upper frequency limit around 100 kHz due to the earlier mentioned restrictions of the eddy current expression and the resolution of the capacitances. However, this should be verified through measurements.

Here, an electrical network for a disc winding has been used as example. It is, however, possible to derive the corresponding networks for the other type of windings.

3.3 Modelling of flux in the winding

In order to calculate the flux in the winding window let us consider the axial field distribution as shown in Fig 3.7.

Fig 3.7. Axial field distribution in a winding window also indicating the different flux regions.
According to Fig 3.7 it is possible to divide the core and winding area into five different regions:

Region 1: *core*

Region 2: *between core and inner winding*

Region 3: *inner winding*

Region 4: *between the windings*

Region 5: *outer winding*

Now, the flux in each region can be calculated using, the following expression, which originates from Ampère’s law (equation (2.1)):

\[
\psi_i = \frac{\mu A_i I_i}{l_i},
\]

(3.17)

where \(I_i\) is the sum of the current encircling the region \(i\), whereas \(l_i\) is the averaged magnetic path length of the flux and \(A_i\) is the cross-section area which the flux flows through in region \(i\).

Now, let us consider the sum of the encircling currents for the different regions, which will be as follows:

- Region 1 and 2: \(N_1 I_1 + N_2 I_2\), where \(N_1\) and \(N_2\) are the number of turns in the inner and outer winding, respectively and \(I_1\) and \(I_2\) are the currents in the corresponding winding.

- Region 3: \(N_1 I_1/2 + N_2 I_2\). From the ampère-turn diagram one can understand that the flux in a given turn in the inner winding will be proportional to the current in the outer winding plus the currents in all other turns with a larger radius than that turn in the inner winding. The average flux in the inner winding caused by the currents in the same
winding will then be proportional to \( N_1 I_1 / 2 \) since the flux increases approximately linearly.

- Region 4: \( N_2 I_2 \).

- Region 5: \( N_2 I_2 / 2 \). Same argument as for region 3.

Now, from Faraday’s law, equation (2.2), the induced voltage in the windings is given as:

\[
U_{\text{ind}} = -\frac{d\varphi}{dt}
\]  

(3.18)

where \( \varphi \) is the sum of all flux that is encircled by each winding, respectively. E.g., for the outer winding, combining equations (3.17) and (3.18), yields:

\[
U_{\text{ind,outer}} = -\frac{d\varphi}{dt} = -\frac{d\left(\varphi_{\text{core}} + \sum_{i=1}^{5} \varphi_i\right)}{dt} = \frac{d\left(\varphi_{\text{core}} + \mu \sum_{i=1}^{5} \frac{A_i}{l_i} I_i\right)}{dt}
\]  

(3.19)

where \( i = 1, 2, 3, 4, 5 \) corresponds to the each regions, respectively. \( I_i \) is calculated as previously shown and \( \varphi_{\text{core}} \) is given from a core model.

### 3.3.1 Discussion

The model for the flux in the winding is supposed to be valid in the same frequency region as the eddy current expression, i.e. up to around 100 kHz. This is since they both build on the field distribution as illustrated in Fig 3.7. This distribution does only include the axial direction of the field, however, for higher frequencies the radial direction has to be included.
Chapter 4

Core model

4.1 Static magnetization model

There exists a wide variety of static hysteresis models in literature today. The most common are the Jiles-Atherton model [29], [37], [38] and the Preisach model [39], [40], [41]. However, in this work the model originally proposed by Bergqvist [23], [24], [25] is used. This model has some similarities with the above mentioned models. E.g., the pinning of domain wall motion introduced in the Jiles-Atherton model is used. Moreover, the use of sets of simpler elements to build up a complex hysteretic relation is taken from the Preisach model. It also has similarities to mechanical hysteresis models. Benefits with the Bergqvist model is that it is easy to use and can be understood intuitively. Furthermore, it is intrinsic multidimensional and allows direct handling of magnetomechanical hysteresis.

In the next section a short survey of the model is presented, however more details are found in refs. [23], [24], [25], [42] and [43].

4.1.1 Bergqvist’s lag model

At a given time, a magnetic material consists of magnetic domains with different magnetization amplitude and direction. In Bergqvist’s lag model all domains with equal magnetization are lumped into volume fractions of the whole material. These volume fractions are called pseudo particles. Moreover, the total
magnetization in the material is a weighted sum of the individual magnetization of all pseudo particles.

### 4.1.1.1 One dimensional model

With no hysteresis present, the magnetization would follow an anhysteretic curve. The hysteresis is introduced by a play operator operating on the anhysteretic curve. Moreover, the pinning strength, or size, $k$, of the play operator, will determine the width of the hysteresis curve. This operation is shown in Fig 4.1 where $m$ is the magnetization of a pseudo particle and $H$ is the applied field on that pseudo particle. Moreover, $\eta$ is the magnetizing field, which in absence of pinning would produce the anhysteretic magnetization. Mathematically $\eta$, which also is called back field, can be written [42] as:

$$
\eta(t) = \begin{cases} 
\eta(t - \Delta t) & \text{if } |H(t) - \eta(t - \Delta t)| \leq k \\
H(t) - k & \text{if } H(t) - \eta(t - \Delta t) > k \\
H(t) + k & \text{if } H(t) - \eta(t - \Delta t) < -k 
\end{cases} 
\tag{4.1}
$$

The linear play operator, as depicted in Fig 4.1 is defined as:

$$
P_k[H] = \eta 
\tag{4.2}
$$

For a field change of $\Delta H$ the play operator yields:

$$
P_k[H + \Delta H] = \begin{cases} 
P_k[H] & \text{if } |H + \Delta H - P_k[H]| < k \\
\Delta H + H - k & \text{if } H + \Delta H - P_k[H] \geq k \\
\Delta H + H + k & \text{if } H + \Delta H - P_k[H] \leq -k 
\end{cases} 
\tag{4.3}
$$
Fig 4.1. To the left an anhysteretic curve is seen. In the middle is a play function which operates on the anhysteretic curve and generates the hysteresis curve seen to the right.

4.1.1.2 Multi dimensional model
In \( n \) dimensions \(|H - \eta|\) will be the driving quantity. Using the notation \( w = (H(t) - \eta(t - \Delta t)) \) the play-operator can be written:

\[
\eta(t) - \eta(t - \Delta t) = \begin{cases} 
0 & \text{if } |k^{-1}w| \leq 1 \\
(1 - |k^{-1}w|^{-1})w & \text{if } |k^{-1}w| > 1
\end{cases}
\] (4.4)

Here, \( k \) is a symmetric tensor that represents the pinning strength in \( n \) dimensions [23].

By using a finite number of pseudo particles, minor loops can be modelled as well. This is achieved by assigning an individual pinning strength, \( \lambda_i k \), to every pseudo particle, where \( k \) is the mean pinning strength of all pseudo particles and \( \lambda_i \) is a dimensionless multiplier for pseudo particle \( i \). Thereby, the total magnetization is given by a weighted superposition of the contributions from all pseudo particles, see Fig 4.2.
Fig 4.2. Weighted superposition of the contributions from the minor loops.

The total multi dimensional magnetization can be expressed:

\[
M = c M_{\text{an}}(H) + \int_0^\infty M_{\text{an}}(P_{\lambda k}[H])\zeta(\lambda)\,d\lambda
\]  

(4.5)

The first term on the right side of equation (4.5) represents the reversible part of the magnetization, referring to section 2.2.1.4. Here, \(M_{\text{an}}\) comprises the anhysteretic magnetization, which would be the magnetization if no hysteresis was present. The parameter \(c\) governs the degree of reversibility of the magnetization and defined as:

\[
c = \frac{\chi_0(0)}{\chi_{\text{an}}(0)}
\]  

(4.6)

where \(\chi_0(0)\) corresponds to the linear portion of the susceptibility of a virgin curve and \(\chi_{\text{an}}(0)\) is the susceptibility of the anhysteretic curve at \(H = 0\). This implies that \(c\) is set to a constant; however, this would not necessarily be true for a real material. This is discussed more in detail in section 4.1.2.5.

The second term of the right side in equation (4.5) constitutes the hysteretic behaviour (irreversible part). This part does also follow the anhysteretic path, however, with a hysteresis effect introduced by the play operator \(P_{\lambda k}\). The play operator has the pinning strength \(\lambda_{ik}\) for pseudo-particle \(i\). Moreover, \(\zeta(\lambda)\) is the density function which gives the weight of the pseudo particles and can e.g. be a Gaussian distribution:

\[
\zeta(\lambda) = Ae^{-(p\lambda-q)^2}
\]  

(4.7)
This expression contains three unknown parameters $A$, $p$ and $q$, governing the distribution. The following normalization conditions give two of the parameters:

$$\int_{0}^{\infty} \zeta(\lambda) d\lambda = 1 - c \quad (4.8)$$

$$\int_{0}^{\infty} \lambda \zeta(\lambda) d\lambda = 1 \quad (4.9)$$

The third parameter could be found by linearizing $M_{an}$ for small $H$. This would yield the second order term of the virgin curve [42], [43] as:

$$\kappa(0) = \frac{1}{2k} \zeta(0) \chi_0(0) = \frac{A}{2k} e^{-q^2} \chi_0(0) \quad (4.10)$$

The pinning strength, $k$, is given by the coercive field and by solving this equation system all parameters will be known. A complete walk-through of the parameter estimation procedure is found in Appendix II.

In the remaining parts of the thesis the one dimensional case will be treated.

### 4.1.2 Modifications

#### 4.1.2.1 Anhysteretic curve obtained from measurements

In order to generate a correct hysteresis model it is of importance that the anhysteretic curve is modelled correctly. The most correct approach is to find it through measurements on a material. This can be done by slowly demagnetizing the material from saturation using an AC-field with different superimposed DC-fields [19], [29]. This yields a set of $(B, H)$ - and $(M, H)$ - values that correspond to the anhysteretic curve. The material should be far into saturation when starting the demagnetization and the major loop should be traversed a number of times before minor loops are cycled.

The DC-fields should be applied as slowly increasing fields, thereby decreasing the risk of unwanted rate-dependent effects which could be the effects if the
fields were applied as step functions. Introducing a DC-field as a step in the start of the demagnetization process would also displace the demagnetization curve so that it may not reach negative saturation during the process.

Furthermore, the DC-fields should reach their final value before starting to traverse minor loops and keep those values until the applied AC-field diminishes. This is very important, since if the DC-fields are applied when the material does not reach saturation anymore, the reversal points of the demagnetization minor loops would be too low. This is due to the fact that the AC-field is continuously decreasing and even if the correct DC-fields are applied the magnetization would then not reach the correct values at the reversal points due to the hysteresis.

In Fig 4.3. - Fig 4.6 is shown an example of demagnetization of a sample of the non oriented electrical steel M700-50A*. Fig 4.3. and Fig 4.4 show demagnetization curves without and with a superimposed DC-field, respectively. Fig 4.5 shows the applied AC-field with a superimposed DC-field whereas Fig 4.6 shows the resulting $B$-field. The AC-field consists of an exponentially decreasing sinusoidal $H$-field at 0.04 Hz, which is considered as sufficiently slow not to introduce rate-dependent processes. However, the fast decrease at low amplitudes of $B$ may introduce errors. A better method would be to control the $B$-field instead.

The anhysteretic curve for M700-50A found with different applied DC-fields is shown in Fig 4.7.

* For the measurement and simulation results shown in this chapter, the rolling direction of M700-50A is considered.
Fig 4.3. Demagnetization $B$-$H$-curve for M700-50A with no DC-field applied. Note that this plot only shows the part of the demagnetization curve when leaving the saturation region.

Fig 4.4. Demagnetization $B$-$H$-curve for M700-50A with an applied DC-field of 5 A/m. Note that this plot only shows the part of the demagnetization curve when leaving the saturation region.
Fig 4.5. Applied AC-field with applied DC-field of 5 A/m giving rise to the demagnetization curve in Fig 4.4.

Fig 4.6. B-curve as result of the applied H-field shown in Fig 4.5.
Fig 4.7. Major loop (solid lines) and anhysteretic curve (dashed line) for M700-50A. Rings indicate measured points. Notice that the major loop closes at higher amplitude than shown in this plot.

The \((M, H)\)-values of the anhysteretic curve could be used directly in the static magnetization model as comprising the anhysteretic magnetization.

### 4.1.2.2 Anhysteretic curve as mathematical function

The drawback with achieving the anhysteretic curve from measurement is the amount of time it requires. Therefore, it is common to use mathematical functions that yield curve shapes that are similar to the real anhysteretic curve. Examples of adequate functions are \(\tanh()\), \(\arctan()\) [25], Langevin [56] and Sigmoid [42].

The anhysteretic curve based on the \(\arctan()\) function can be expressed as:

\[
M_{an}(H) = \frac{2}{\pi} M_s \arctan \left( \frac{H \chi_{an}(0)}{8M_s \cdot 10^{-7}} \right)
\]  

(4.11)
In equation (4.11), $M_s$ is the magnetization saturation and $\chi_{an}(0)$ is the susceptibility at $H = 0$, which determines the inclination of the anhysteretic curve.

Moreover, the anhysteretic curve based on the Langevin function could be expressed as:

$$\left\{ \frac{\cosh \left( \frac{H}{a} \right) - \left( \frac{a}{H} \right)}{} \right\} \quad (4.12)$$

In this function $a$ is a parameter determining the inclination of the anhysteretic curve.

Finally, the anhysteretic curve shape as the Sigmoid function could be written as:

$$\left\{ \frac{2}{H} - 1 \right\} \quad (4.13)$$

Examples of modelled anhysteretic curves using arctan and Langevin functions, respectively, are shown in Fig 4.8 and Fig 4.9. In order to yield modelled curves that have similar curve shapes as the measured ones, some of the parameters have to be adapted. In these cases the agreement is judged visually, however, it is possible to use e.g. the least-square method. Nevertheless, still it is not possible to achieve a good agreement at both low and high fields. A solution for this is proposed in the next section.
Fig 4.8. Anhysteretic curves for M700-50A given from the arctan function with values of $\chi_{an} (0)$ and $M_s$ adapted to yield good agreement of the slopes at low field and high field respectively. The measured anhysteretic curve (solid with rings) is given as comparison.

Fig 4.9. Anhysteretic curves for M700-50A given from the Langevin function with values of $a$ and $M_s$ adapted to yield good agreement of the slopes at low field and high field respectively. The measured anhysteretic curve (dotted with rings) is given as comparison.
**4.1.2.3 Improved curve modelling at high fields**

As seen in the previous section, one problem when modelling the anhysteretic curve as a mathematical function is the difficulty to generate a correct slope both at low fields and at high fields, where the magnetizations saturates. When the material starts to saturate, the magnetic moments will align along the crystallographic easy axis closest to the direction of the applied field. However, when increasing the field these moments will finally rotate into the direction of the applied field. The final, total saturation, will however, only be reached at extremely high fields (theoretically when every single moment is aligned along the applied field). This means that \( \mu' \) decreases slowly until it is close to one at very high fields. The problem with the mathematical functions presented in the previous section is the difficulty to handle the large difference in \( \mu' \). This is clearly seen in Fig 4.8 and Fig 4.9.

To solve this problem the mathematical functions can be modified. This has been done by Jiles and Atherton, with good results, in the case of the Langevin function [37], [64]:

\[
M_{an} (H_c) = M_s \left( \coth \left( \frac{H_c}{a} \right) - \left( \frac{a}{H_c} \right) \right) \tag{4.14}
\]

In this modified Langevin function, \( H_e = H + \alpha M_{an} \) is an effective field analog to the Weiss mean field (i.e. the field created by exchange interaction between the magnetic moment). Here, \( \alpha \) is a material parameter.

\[
M_{an} (H_e) = M_s \left( \coth \left( \frac{H + \alpha M_{an}}{a} \right) - \left( \frac{a}{H + \alpha M_{an}} \right) \right) \tag{4.15}
\]

Notice that \( M_{an} \) is on both sides of this equation. In order to use it in the Bergqvist model this equation has to be solved iteratively. On the contrary, here another modification is proposed which is less time-consuming. This modification consists of the introduction of an extra susceptibility term in equation (4.5), namely:

\[
\left( b - \ln \left( \frac{H}{H_c^b} \right) \right) \tag{4.16}
\]
Here, \( b \) is a dimensionless constant and \( b_c \) a constant with unit \((\ln (\text{A/m}))^{-1}\). The values of \( b \) and \( b_c \) can be fitted for different materials from a measured major loop. Notice, that for \( H = 0 \), the logarithm is omitted. This yields the total magnetization:

\[
M = \left( b - \ln\left( |H|^b_c \right) \right) H + c M_{an}(H) + \int_0^\infty M_{an}\left( P_{\lambda k}[H]\right) \zeta(\lambda) d\lambda \tag{4.17}
\]

The results using this logarithmic modification together with the arctan function and the Langevin function, respectively, for the anhysteretic curve are shown in Fig 4.10 and Fig 4.11. It can be seen in the figures that the logarithmic modification yields very good results at both low and high fields and, moreover, also at the saturation knee.

Fig 4.10. Anhysteretic curve (dashed) for M700-50A using the logarithmic modification together with the arctan function. The measured anhysteretic curve (solid with rings) is given as comparison.
Fig 4.11. Anhysteretic curve (dashed) for M700-50A using the logarithmic modification together with the Langevin function. The measured anhysteretic curve (solid with rings) is given as comparison.

4.1.2.4 Variable pinning strength
One problem with the original Bergqvist model is that the pinning strength is assumed to be constant for all magnetizations of the major loop. Hereby, the positive and negative branches of the loop will have the same shape and, moreover, the loop will be symmetric during saturation and desaturation. I.e., the bending of the curve when going in to and out of saturation will have the same slope. However, this is only an approximation for some materials, especially non oriented electrical steels. Consider again the major loop of the non oriented electrical steel M700-50A, shown in Fig 4.7. As it can be seen in the figures the curve shapes during saturation and desaturation, respectively, differ. Referring to the magnetization processes presented in section 2.2.1.4, we can conclude that this discrepancy is present mainly in the irreversible part of the hysteresis loop, i.e., during domain wall movements and irreversible domain rotation. Since the domain rotation dominates at the saturation knee, we refer to this phenomenon as asymmetric domain rotation, even if the asymmetry could be present in the domain wall movement process as well.
One method, proposed here, to model the asymmetric domain rotation is to use a variable pinning strength, $k$. I.e, a pinning strength that is dependent on the applied magnetic field, $H$. Thereby, the model can generate different slopes during saturation and desaturation. In this work, the following expression for the variable pinning strength was found to give adequate results:

$$k(H) = \max\left(k_0, t_0 + \text{abs}\left(\frac{H}{y}\right)\right)$$  \hspace{1cm} (4.18)

In this expression, $k_0$ is a constant positive value given by the pinning strength of the coercivity field. Furthermore, $t_0$ is a threshold variable smaller than $k_0$ and $y$ is a variable that govern the bending of the curve. When used in simulations, the expression works as follows:

- For small values of $H$, $k = k_0$ which is constant and generates the desaturation curve.
- For large values of $H$, $k = t_0 + \text{abs}\left(\frac{H}{y}\right)$, which varies with $H$. This expression yields the saturation curve where $y$ governs the sharpness.
- Large values of $y$ gives $k = k_0$.

Notice, that this modification will mainly affect the positive branch, as shown in Fig 4.12 when going into saturation. This means that the negative branch will not change when going out of saturation. Therefore, for an optimal fit of a major loop, $k$ and the parameter governing the inclination of the curve should be adapted in order to give the best fit of the negative branch. Thereafter, the parameters $t_0$ and $y$ can be adapted to give a good fit for the positive branch. This will result in an anhysteretic curve that has a slightly different slope than the measured. Nevertheless, this will yield a better fit than the original model for both the positive and negative branches of the major loop.
Fig 4.12. Upper half of major loop showing the effect of using a variable pinning strength. Dotted line shows a simulated curve with constant pinning strength and solid line a simulated curve with variable pinning strength. Notice that the negative branch is equal in both cases. Conversely, the positive branch differs. Also notice that the variable pinning strength function lowers the saturation value a little.

A comparison between a modelled curve using a constant and a variable pinning strength, $k$, respectively, for M700-50A is shown in Fig 4.13. For the case with constant $k$, the most realistic anhysteretic curve is used, i.e., the measured curve. However, for the case with variable $k$, an arctan function is used for the anhysteretic curve in which the parameters are adapted so that modelled major loop will fit the measured major loop best. This will yield a sharper anhysteretic curve which is shown in Fig 4.14. Therefore, this will not give the true anhysteretic curve and hence the results for minor loops should be investigated also. Nevertheless, the major loop modelling is in very good agreement with measurements.

More comparisons of modelling of major loops for other electrical steel grades are found in section 6.1.2.1.
Fig 4.13. Upper part of major loop for M700-50A. Solid line shows measured curve. Dashed line shows modelled curve with constant pinning strength using the measured anhysteretic curve, whereas dotted line modelled curve with variable pinning strength and an adapted anhysteretic curve. Notice that the major loop closes at higher amplitude than shown in this plot.

Fig 4.14. Dotted line shows the adapted anhysteretic curve for the major loop shown in Fig 4.13.
4.1.2.5 Variable degree of reversibility

The expression for the total magnetization in the original Bergqvist model, equation (4.5), assumes that the parameter $c$, which corresponds to the degree of reversibility of the magnetization, is constant. This means that the model assumes that the relation between the reversible and irreversible processes of the magnetization is constant for all magnetization levels. However, from the discussion in section 2.2.1.4 this assumption cannot be true since the degree of reversibility varies for different magnetization levels. The degree of reversibility should vary with the magnetization and/or the applied field [49], [50]. It has also been suggested that it could also be history-dependent [50].

Engdahl [57] proposes a modification to Bergqvist’s lag model where $c$ is dependent on the applied field. The static magnetization will then be expressed as:

$$M = c(H)M_{\text{an}}(H) + \int_{0}^{\infty} M_{\text{an}}(P_{\lambda k}[H])\zeta_{H}(\lambda)d\lambda \quad (4.19)$$

The expression for $c(H)$ is then changed to:

$$c(H) = \frac{\chi_{0}(H)}{\chi_{\text{an}}(H)} \quad (4.20)$$

where $\chi_{0}(H)$ is the linear portion of the susceptibility of a curve starting from an arbitrary point, $(M, H)$, on the anhysteretic curve and $\chi_{\text{an}}(H)$ is the susceptibility of the anhysteretic curve in that point. Now, the density function will be dependent on $H$:

$$\zeta_{H}(\lambda) = A(H)e^{-(p(H)\lambda-q(H))^{2}} \quad (4.21)$$

Furthermore, the parameter functions $A(H)$, $p(H)$ and $q(H)$ are now given through the extensions of equations (4.8) - (4.10) in order to be valid for different $H$: 
\[
\int_0^\infty \xi_H(\lambda) d\lambda = 1 - c(H) \quad (4.22)
\]

\[
\int_0^\infty \lambda \xi_H(\lambda) d\lambda = 1 \quad (4.23)
\]

\[
\kappa(H) = \frac{1}{2k} \xi_H(0) \chi_0(H) = \frac{A(H)}{2k} e^{-q(H)^2} \chi_0(H) \quad (4.24)
\]

If instead the parameter \( c = c(M) \), i.e., dependent on the magnetization, the expressions corresponding to equations (4.19) - (4.23) are obvious.

### 4.1.2.6 Modified play operator

Bormann [58] proposes the following modified play operator:

\[
P_{k,c}[H + \Delta H] = \begin{cases} 
P_k[H] + c\Delta H & \text{if } |H + \Delta H - P_k[H]| < k \\ 
\Delta H + H - k & \text{if } H + \Delta H - P_k[H] \geq k \\ 
\Delta H + H + k & \text{if } H + \Delta H - P_k[H] \leq -k 
\end{cases} \quad (4.25)
\]

This new play operator is depicted in Fig 4.15.

The contribution of the reversible part is here introduced directly into the play operator via the parameter \( c \). Moreover, \( c \) could in principle be an arbitrary function of \( M \) or \( H \).
Fig 4.15. Play-operators. Blue, solid line shows the original operator and red, dashed line shows the new operator.

If \( \varsigma(\lambda) \) is the density function which gives the weight of the pseudo particles the total magnetization can be expressed:

\[
M = \int_{0}^{\infty} M_{\text{an}} \left( P_{k,c} [H] \right) \varsigma(\lambda) d\lambda
\]  (4.26)

The density function is normalized through the following conditions:

\[
\int_{0}^{\infty} \varsigma(\lambda) d\lambda = 1
\]  (4.27)

\[
\int_{0}^{\infty} \varsigma(\lambda) \lambda d\lambda = k
\]  (4.28)
Here, $k$ is the mean pinning strength. Now, the play operator on the major loop is given by:

$$P_{k,c} [H]_\pm = H \mp k$$  \hspace{1cm} (4.29)

for the positive and negative branch, respectively. This gives the magnetization branches as:

$$M_\pm = \int_0^\infty M_{an} (H \mp k) \zeta (\lambda) d\lambda$$  \hspace{1cm} (4.30)

An interesting feature of equation (4.30) is that it is independent on the degree of reversibility, $c$. Since both functions $M_\pm$ and $M_{an}$ can be obtained from measurements it is possible to determine the distribution function, $\zeta (\lambda)$ from this equation. For a description of this determination procedure see Appendix III.

### 4.2 Dynamic magnetization model

#### 4.2.1 Cauer circuits

As mentioned in section 2.2.2.1 the classical eddy currents expression, equation (2.14), is valid only if the condition in equation (2.15) is satisfied. That condition is usually satisfied for materials with a thickness of 0.1-1.0 mm up to around 100 Hz. For thinner materials the condition is valid for higher frequencies.

The sheets in a transformer core laminate have typically a thickness of 0.2-1.0 mm. This means that the field distribution inside the material will be inhomogeneous and the use of equation (2.14) introduces errors for higher frequencies. The analytical solution of this problem becomes very complex; since one has to consider that the magnetic domains in the material can exhibit different magnetization at a certain time. Furthermore, the magnetization law for the typical material has to be known, and as a result it is very difficult to derive a general model.
Bertotti [19] derives the solutions for materials with linear or steplike magnetization laws. However the solutions are only valid under sinusoidal conditions and are therefore not useful for minor loops or other waveforms.

In this thesis a different approach called Cauer circuits [48] are used. These are circuits of ladder network type and are more commonly used in signal theory. However, they have been used earlier for modelling of magnetic cores in general [28], [52], [73], [74], [75], [76] and transformers cores in particular [8], [13], [28], [51]. Furthermore, a survey of the technique is presented in ref. [28]. More recent applications are found in refs. [53] and [54]. In this thesis the feasibility of using Cauer circuits as a part of a dynamic magnetization model will be studied.

A short resume of the Cauer circuit technique is presented below.

### 4.2.1.1 Modelling method

When a material is exposed to a varying magnetic field, the field distribution will vary over the cross-section thickness due to the eddy current shielding effect. The governing idea of Cauer circuits is to divide the cross-section into a finite number of sections, where every section is smaller than the total thickness. This is shown in Fig 4.16. Each section carries the average field in that section. If the sections are chosen sufficiently small the field inside them is approximately uniformly distributed and so are the magnetization. This technique is often called “lumping”.
Consider the case in Fig 4.17 where a sheet with a flux flowing into the cross-section has been divided into smaller sections. We can see that every section creates eddy currents surrounding themselves. This is consistent with Faraday’s law (equation (2.2)). As with the fields, the eddy currents are represented by lumping them in every part of the cross-section. Moreover, the eddy currents will create counter fields in the enclosed sections according to Ampère’s law (equation (2.1)). For the modelling approach we suppose that the eddy currents in a section do not affect the magnetic flux in the same section. Furthermore, the flux in a section does not create eddy currents in the same section. Therefore, no current is flowing in the innermost section.
Fig 4.17. A magnetic sheet divided into smaller sections. For electrical steels the width of the sheets is much larger than the thickness in order to reduce the eddy currents. This also means that the vertical parts of the currents paths are negligible.

Now, referring to Fig 4.17, the total flux in the material can be expressed as:

$$\phi_{\text{tot}} = \phi_1 + \phi_2 + \phi_3 + \ldots + \phi_n$$  \hspace{1cm} (4.31)

This can be represented with the circuit illustrated in Fig 4.18 where $R_i$ denote the reluctances of the section of the magnetic material that is given by:

$$\theta_i = R_i \phi_i = R_i B_i A_i = R_i \mu H_i A_i$$  \hspace{1cm} (4.32)

$$\theta_i = H_i l_i$$  \hspace{1cm} (4.33)

$$H_i l_i = R_i \mu H_i A_i$$  \hspace{1cm} (4.34)

Here $\theta_i$ is the applied MMF (magneto motive force) and $l_i$ is the magnetic path length over which $H_i$ is applied. This finally gives:
Equation (4.35) shows that $\mathcal{R}_i$ is inversely proportional to the magnetic permeability of the material.

$$\mathcal{R}_i = \frac{l_i}{\mu A_i} \quad (4.35)$$

Fig 4.18. Lumping of reluctances of different sections.

The circuit shown in Fig 4.18 represents the lumping of the magnetic fluxes in the different sections of the sheet. However, we also have to represent the eddy currents. This can be done by using a so called winding component seen in Fig 4.19. The winding component realizes the relation between the magnetic flux in a material and the currents surrounding the flux, e.g., in a winding or, as in this case, a surrounding material section. In the magnetic part of the winding component a flux flows and a MMF is induced. In the electric part an EMF (electro motive force) is induced and a current flows through the resistance, $R$. 

Using Ohm’s law in the electric part we can calculate $I$ as:

$$I = -\frac{N}{R} \frac{d\phi}{dt}$$

(4.36)

where $N$ is the number of turns, in this case $N = 1$. Now, for the magnetic part we get:

$$\theta = NI = -\frac{N^2}{R} \frac{d\phi}{dt} = -\frac{1}{R} \frac{d\phi}{dt}$$

(4.37)

Through the electric-magnetic analogy this equals a “magnetic” inductance with value $1/R$. Inserting the magnetic inductances corresponding to the winding components, and thereby the eddy currents, into the circuit in Fig 4.18 yields the circuit depicted in Fig 4.20.
4.2.1.2 **Optimization of Cauer circuit at high frequencies**

For correct representation of the magnetization of magnetic materials the individual size and number of sections in the Cauer circuit are important. It has been shown that using 10 sections yields good results when choosing the appropriate size of them [28], [51]. Moreover, in those models, the sizes of the subsections have been chosen to increase linearly with 30 % [51] and 40 % [28], respectively, for each section, beginning from the outmost section. For the frequency range in this work, approximately DC – 10 kHz, this approach yields satisfying results. However, one can understand that at higher frequencies, where the field will change more rapidly near the material surface, this method would be rather coarse, and can yield incorrect results.

Here a method is proposed in which the sizes of the subsections depend on the frequency of the applied field. The idea is to choose the sizes of the sections such that we get many sections in the region where the field changes fastest and thereby also is hardest to predict.

The procedure of this frequency-dependent method is as follows:

First we estimate the field distribution inside the material which is difficult to predict. It is, however, possible to derive an analytical expression for the field.
distribution in a material with constant permeability [19], [55]. Consider the cross-section of a magnetic sheet. If the sheet has thickness $d$ and is exposed to a time-varying magnetic field, the normalized field distribution along the sheet thickness will be:

\[ H(x) = \sqrt{\frac{\cosh(2\gamma x/d) + \cos(2\gamma x/d)}{\cosh(\gamma) + \cos(\gamma)}}, \tag{4.38} \]

where $x$ is the distance from the outer edge of the sheet thickness and $\gamma$ corresponds to the degree of penetration expressed as:

\[ \gamma = \sqrt{\pi \sigma \mu f d} \tag{4.39} \]

Here it is assumed that the permeability is constant even if it is not the case.

Next, for the frequency of interest, we pick a number of points (equal to the number of sections) of $H$ equally distributed between the maximum value and minimum value of $H$ over the sheet thickness. The corresponding value of $x$ for each $H$-value is then picked and used as the inner boundary for each section in the Cauer circuit.

There is a great advantage with this procedure especially at high frequencies where the field changes rapidly near the surface. In these cases this procedure will yield most of the sections near the surface. On the other hand, at lower frequencies where the field does not change so fast the sections will be more evenly distributed over the cross-section and the method using linearly increasing sizes can be used.

Comparisons of simulation results of the field distribution in a sheet of a typical electrical steel with $\mu_r = 5000$, $\sigma = 2$ MS/m and $d = 0.3$ mm are shown in Fig 4.21 - Fig 4.23. The simulations were performed using Cauer circuits with the linear method (LM), with 40% increase between each section and the frequency-dependent method (FDM), which optimizes the sizes of the sections. The applied field was sinusoidal and the amplitude of the magnetization was kept below saturation. It can be seen in the figures that at 10 kHz, LM and FDM yield approximately the same field distribution. However, when increasing frequency up to the MHz-range FDM yields much better result than LM. This is due to the fact that at these high frequencies, the eddy current shielding effect is...
very high. Nevertheless, at frequencies below 10 kHz, which is what electrical steels usually are exposed to, the difference between LM and FDM is very small.

Fig 4.21. Magnetic field distribution at 10 kHz along the cross-section of a sheet of a typical electrical steel. The curves show the following methods: analytical expression (solid) and simulations using LM (dashed) and FDM (dotted), respectively. The rings and stars show the middle point in each section, respectively.
Fig 4.22. Magnetic field distribution at 1 MHz along the cross-section of a sheet of a typical electrical steel. The curves show the following methods: analytical expression (solid) and simulations using LM (dashed) and FDM (dotted), respectively. The rings and stars show the middle point in each section, respectively.

Fig 4.23. Magnetic field distribution at 100 MHz along the cross-section of a sheet of a typical electrical steel. The curves show the following methods: analytical expression (solid) and simulations using LM (dashed) and FDM (dotted), respectively. The rings and stars show the middle point in each section, respectively.
4.2.1.3 The feasibility of studying the flux distribution along the cross-section of a magnetic material

The eddy current shielding effect causes the flux density to increase in the outer parts of the material and decrease in the inner parts. However, when the flux density increases in the outer parts of the material, these will be saturated. This will, in turn, increase the flux density in the inner parts of the material [32]. This means that the correct flux distribution will be dependent both on the frequency and the nonlinearity of the material. This effect is not possible to model using the classical eddy current expression.

On the other hand, using Cauer circuits, it is in fact possible to model this effect. Fig 4.24 and Fig 4.25 show the B-H curve and flux distributions, respectively, in a typical electrical steel (\(d = 0.20 \text{ mm}\)) simulated at 1 kHz. It can be seen that with increasing flux density the eddy current shielding forces flux into the outer regions of the material. However, increasing the flux density even further, the outer parts saturate and the flux is forced into the inner parts of the material again. This behaviour is also claimed by Zirka et al. in a recent study [32].

![Figure 4.24](image_url)

*Fig 4.24. B-H curves in different sections of the material simulated at different applied fields at 1 kHz. Solid lines show the outermost section, dashed the middle and dotted the innermost. Notice that in the leftmost plot all curves coincide.*
**4.3 Composite magnetization model**

The governing equation for the composite model is the expression for the total field, given in equation (2.19):

\[ H_{\text{tot}} = H_{\text{stat}} + H_{\text{eddy}} + H_{\text{excess}} \]  

The total field is applied by a MMF, driving a magnetic flux through the circuit. The field created by the eddy current, \( H_{\text{eddy}} \), gives a MMF drop as shown in Fig 4.20. Moreover, the field created by the excess losses, \( H_{\text{excess}} \), gives a MMF drop in every reluctance element in *each section* individually, according to equation (2.18):

\[ H_{\text{excess}} = \sqrt{4\sigma GdwV_0} \left| \frac{dB}{dt} \right| \text{sign} \left( \frac{dB}{dt} \right) \]  

Here, \( B \) is the actual flux density in the actual section considered.
This gives the static part of the magnetization field, $H_{\text{stat}}$ from equation (4.40). Finally, $B$ is calculated using Bergqvist’s lag model (section 4.1.1) in every section. This means that the reluctance components in Fig 4.20 now contain the static hysteresis and the excess loss parts of the total magnetization in each section. Furthermore, the eddy current part is given by the “magnetic” inductances.

One problem with the formulations is that the static model calculates $B$ as a function of $H$ while the excess loss expression calculates $H$ as a function of $dB/dt$. This can, however, be solved by using the value of $dB/dt$ at the nearest previous time point, as an approximation for $dB/dt$ at the desired time point. This requires, of course, that the time step is small enough so that the small errors introduced can be neglected.

### 4.3.1 Discussion

An alternative for the excess loss component is to realize it as one component for the whole circuit, and not in every section. However, that approach would only manage to account for the average flux density in the material. When simulating it in every section, it is possible to account for the difference in the flux distribution, through the different flux densities in different sections, which is more physically correct.
Chapter 5

Composite transformer model

5.1 One phase

The composite transformer model consists of a winding model and core model connected together. For a one phase transformer this is achieved by attaching a winding component, comprising the features presented in Chapter 3, to a core component, comprising the features presented in Chapter 4. The interaction between the core and winding components are presented schematically in Fig 5.1. The currents in the two windings, respectively, are given through the winding module and generating an MMF in the core module. The flux in the core module, in turn, induces EMF:s in the windings. The generality of the model gives the opportunity to vary the resolution of the core and winding models individually.

In section 6.2 an one phase three leg transformer model is verified by measurements. Also different resolutions of the core and winding models are tested for frequencies up to 1000 Hz.
Chapter 5  Composite transformer model

5.2  Three phases

It is possible to connect three different legs together to constitute a three phase three leg model. Furthermore, one can connect magnetic elements between the legs to model the magnetic characteristics of the yokes and joints. Fig 5.2 illustrates the schematics of a three phase three leg transformer. As seen in the figure, three leg components are coupled together via a reluctance network with yoke and joint components. Each leg component comprises the composite core

**Fig 5.1. Simplified diagram of the composite transformer model comprising two winding coils, linked by a winding module, which generate MMF:s in the core. Also seen is the core module and the, by the flux in the core, induced EMF:s in the windings.**
and winding model as shown in Fig 5.1. Furthermore, the yoke and joints include the core model as presented in Chapter 4.

Fig 5.2. Schematics of a three phase three leg transformer model. The magnetic core consists of a reluctance network of limb, yoke and joints components.

As for the case of a one phase model it is possible to choose the resolution for the leg components individually for each leg. It is also possible to choose the resolution of each yoke component individually. Furthermore, it is possible to attach more legs and yokes if that is required. Also additional reluctance elements describing three dimensional leakages and loss phenomena, e.g., in the tank and construction details can be added.
Chapter 6

Experimental verification

In this chapter verifications of the magnetic core model and the composite transformer model, presented in Chapter 4 and 5 respectively, are presented. This includes verifications of the static and composite magnetization models. It also includes the verification of normal operation of a one-phase transformer model under different load conditions. Furthermore, this model is verified in no-load and non-normal operations like inrush currents and DC-magnetization.

Unfortunately, there has not been enough time to verify the detailed winding model as presented in Chapter 3. Instead a simplified winding model is used in the verification of the composite transformer model.

6.1 Core model verification

In order to verify the static and dynamic hysteresis models, a number of measurements were performed. These included measurements of static major and minor loops on electrical steels. Furthermore, dynamic measurements were performed at different frequencies and amplitudes of applied fields. The dynamic measurements were done both with controlled $H$- and $B$-field, respectively.

Thereafter, the same cases were simulated in order to verify the models. In the following section the procedure and results of the measurements and simulations are presented.
6.1.1 Static measurements

6.1.1.1 Set-up
The set-up for the static measurements is as seen in Fig 6.1. The measurement is controlled from a computer, which, via a digital to analog converter (DAC), gives a voltage as input to an amplifier. The amplifier gives a current as output. The Epstein frame is a quadratic board with coils located on each side. Inside the coils flat strips of magnetic materials are placed. The current from the amplifier runs through the coils (connected in series) and creates magnetic fields along the axial direction of the strips according to Ampère’s law. There are also pickup coils (also connected in series) on each side of the frame. The pickup coils are connected to a fluxmeter, which takes the induced voltage in the coils as input and calculates the flux by Faraday’s law. Thereby, the flux density becomes:

\[
B = \frac{\int U dt}{NA}
\]

where \( U \) is the voltage over the coils, \( N \) is the number of turns of the coil and \( A \) is the cross-sectional area of the magnetic material.

![Fig 6.1. Block scheme of set-up for static measurements.](image)

In the static case the field changes so slowly that the signal from the computer can be used directly in order to give the value of the magnetic field. However, these values have to be calibrated against the real field. In order to do this the current that flows through the Epstein frames primary coil is measured. This is done with a current sensor in form of a current clamp placed around the cable.
between the amplifier and the Epstein frame. Knowing the number of turns in the coil the magnetic field is known by using Ampère’s law, i.e.:

\[ H = \frac{NI}{l}, \]

where \( l \) is the magnetic path length. In the Epstein frame used, the total number of turns is 700 both for the coils for applying the field and for the pickup coils. Moreover, the sides of the frame are 28 cm long. For the measurements, strips of magnetic materials are placed inside every coil. The strips are 28 cm long and 3 cm wide and are lapped into each other in the joints. Hereby, the effective magnetic length of each strip is only 25 cm. Furthermore, the magnetic flux will leak inside the strips in the joints and as a result the effective magnetic path length for the whole set-up is approximately 0.94 m [59].

The values of the \( B \)-field are returned back via the analog to digital converter (ADC) to the computer, where they are stored.

6.1.1.2 Major loops
Static major loops of three electrical steels with different properties were measured. The applied magnetic fields were sinusoidal with frequency 1 mHz, which for the thickness of these materials are low enough to be considered as static. The following materials were measured:

- M5. Thickness 0.30 mm, width 3.0 cm, grain oriented.

- M700-50A, rolling direction. Thickness 0.50 mm, width 3.0 cm, nonoriented.

- 23ZDKH. Thickness 0.23 mm, width 3.0 cm, strongly oriented, laser-scribed*.

* As the word laser-scribe implies, a laser is used to mark dotted lines transverse to the direction of the orientation. The lines lie at a distance of a couple of centimeters and hinder the movement of domain walls when the material is exposed to a fluctuating field. This decrease the excess losses.
The measured static major loops of those steels are shown in Fig 6.2.

Fig 6.2. Static major loops for different electrical steels. Dashed line shows 23ZDKH, solid line shows M5 whereas dotted line shows M700-50A.

6.1.1.3 Minor loops
In section 4.1.2.1 a demagnetization curve was shown for M700-50A, see Fig 4.3.. Many of the loops in that curve were closed or nearly closed. Some of these loops were extracted in order to use for verification of the core model. These loops are shown in Fig 6.3.
6.1.2 Static simulations

In this section results from simulations of major and minor loops of the material presented in the previous section are shown. These static simulations were executed in Matlab*. Firstly, simulations of major loops for three electrical steels are presented. Then, an example of simulations of minor loops is shown.

* [www.matlab.com](http://www.matlab.com)
6.1.2.1 Major loops

The major loops of the three materials shown in Fig 6.2 were used as a starting point. Thereafter, it was tried to replicate these major loops as good as possible by simulations. For best possible fitting of the shapes of the loops (judged visually), the logarithmic modified arctan function shown in equation (4.17) was used together with the expression for the variable pinning strength in equation (4.18). As shown in Fig 4.13 this technique is able to yield a better fit for the major loop than using the measured anhysteretic curve. The reason for this is that the variable pinning strength was used together with the arctan function, but not with the measured anhysteretic curve. Therefore, the arctan function yields better fit for the whole major loop and was therefore used in the simulations.

In order to replicate the measured curves as good as possible the parameters in equation (4.17) and (4.18) were adapted in the simulations. This was done manually. Results for the simulation of the major loops of the three materials are shown in Fig 6.4 - Fig 6.6. The values of the adapted parameters are shown in Table 6.1 Notice that the Langevin function and other appropriate mathematical functions using the same modifications as for the arctan function is considered to yield equal results.
Fig 6.4. Upper half of major loop for M700-50A. Solid line shows the measured curve and dashed line the best fitted curve from simulations.

Fig 6.5. Upper half of hysteresis loop for M5. Solid line shows the measured curve and dashed line the best fitted curve from simulations.
Fig 6.6. Upper half of hysteresis loop for 23ZDKH. Solid line shows the measured curve and dashed line the best fitted curve from simulations.

Table 6.1. Static hysteresis model parameters.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\chi^*\mu_0$</th>
<th>$M_s^*\mu_0$</th>
<th>$k_0$</th>
<th>$t_0$</th>
<th>$y$</th>
<th>$b$</th>
<th>$b_c$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M700-50A</td>
<td>0.3</td>
<td>1.54</td>
<td>69</td>
<td>8</td>
<td>1.16</td>
<td>345</td>
<td>33</td>
<td>0</td>
</tr>
<tr>
<td>M5</td>
<td>0.35</td>
<td>1.70</td>
<td>7</td>
<td>1</td>
<td>1.65</td>
<td>440</td>
<td>45</td>
<td>0</td>
</tr>
<tr>
<td>23ZDKH</td>
<td>0.36</td>
<td>1.77</td>
<td>9</td>
<td>1</td>
<td>1.55</td>
<td>740</td>
<td>90</td>
<td>0</td>
</tr>
</tbody>
</table>

Since only the major loop is considered, some simplifications in the adapting of parameters are made:
• $c$ is chosen to zero, which can be done since we do not consider the minor loops. The inclination at high values is given by the anhysteretic curve.

• There is only need for one pseudo particle; therefore the distribution function is unnecessary.

This simplified magnetization model can therefore be expressed as:

$$ M = \left( b - \ln \left( |H|^k \right) \right) H + M_{an} \left( P_k [H] \right) $$

(6.3)

However, more interesting is the full expression including the feasibility to model also minor loops. An example of this is shown in next section.

6.1.2.2 Minor loops

The minor loops for M700-50A already shown in Fig 6.3 were used as starting point for the simulations of minor loops. Both the measured anhysteretic curve and the modified arctan function, giving the anhysteretic magnetization, were simulated using 50 pseudo particles.

The parameter $c$ was given according to equation (4.6) using the anhysteretic curve and measuring the virgin curve:

$$ c = \frac{\chi_0(0)}{\chi_{an}(0)} $$

(6.4)

Moreover, $k$, was given by the coercive field for the major loop and $q$ was initially calculated using the parameter estimation procedure as presented in Appendix II. This parameter did not, however, give good agreement with measurements so the value of $q$ was adapted a little.

Using the measured anhysteretic curve there is no other parameter to adapt. On the contrary, using the arctan function the parameters were adapted in order to fit the measured minor loops. This was done so that the amplitudes of $B$ of the
minor loops agreed between the simulated and measured curves. Especially, the value of the parameter $q$ affects these amplitudes.

The results of the simulations are shown in Fig 6.7 and Fig 6.8. Moreover, the adapted parameters are shown in Table 6.2. We can see from the figures that the minor loops are not satisfactorily replicated by the measurements. The difference between the two methods are not very large, however, the measured anhysteretic curve yields smoother curves than the arctan function. On the other hand, the arctan function gives more correct amplitudes than the measured anhysteretic curve.

![Fig 6.7. Measured (solid) and simulated (dashed) minor loops using arctan function for M700-50A. The simulation parameters were adapted to give as good fit, regarding the amplitudes of the loops, as possible to the measured curves.](image-url)
Fig 6.8. Measured (solid) and simulated (dashed) minor loops using the measured anhysteretic curve for M700-50A. The simulation parameters were adapted to give as good fit, regarding the amplitudes of the loops, as possible to the measured curves.

Table 6.2. Adapted parameters for minor loops simulations of M700-50A.

<table>
<thead>
<tr>
<th>Anhysteretic curve</th>
<th>$\chi^*\mu_0$</th>
<th>$M_s^*\mu_0$</th>
<th>$k_0$</th>
<th>$t_0$</th>
<th>$y$</th>
<th>$b$</th>
<th>$b_e$</th>
<th>$c$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified arctan</td>
<td>0.8</td>
<td>1.64</td>
<td>69</td>
<td>0</td>
<td>1.5</td>
<td>200</td>
<td>15</td>
<td>0.01</td>
<td>1</td>
</tr>
<tr>
<td>Measured</td>
<td>-</td>
<td>-</td>
<td>69</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.01</td>
<td>0</td>
</tr>
</tbody>
</table>
In the simulations of the minor loops the parameters $c$ is very small, almost zero. Furthermore, setting it to zero did not change the results noticeably.

### 6.1.2.3 Discussion

In this thesis the parameter values of the static model has been adapted such that the simulated curves fit the measured curves. This raises the questions how to determine the parameter values and how to evaluate the best fit.

Three different methods to determine the parameter values were tested:

1) Using the procedure proposed in Appendix II.

2) Using an algorithm that iteratively adapts the values in order to minimize the area between simulated and measured curves of a branch of the major loop and the virgin curve, respectively. This algorithm was created by Dr. Julius Krah [86].

3) Adapting the values manually to fit the shapes of major loop and minor loops.

None of these methods were able to yield a good fit both for minor and major loops. This recalls the need of modifying the static model for minor loops. Especially, the description of the parameter $c$ should be modified, e.g., as proposed in section 4.1.2.5 and 4.1.2.6.

On the other hand, it was possible to yield a good fit for the major loop using the third of the above listed methods and fitting only the major loop.

In order to evaluate the best fit there are different possibilities. The fact that the materials saturate means that the slope of the curve changes rapidly. One possibility is to judge the fit visually. Another possibility is to use a least-square method to minimize the error. A third possibility is to minimize the area between the simulated and measured curves. In order to do this the total area has to be divided into smaller elements. A fourth possibility, if the magnetization curve follows a closed loop, is to fit the area of the loop.
Fitting visually requires careful examination of the simulated results not to introduce errors, however, can yield very good fit and be performed in a short time. A least-square method requires an adequate description of which residuals to minimize. If one can achieve that this method is fast and can be exact. The method of minimizing the areas between the simulated and measured curves requires an adequate weighting of the different area elements. Finally, fitting the area of the loop is easy, however, can yield simulated curve shapes that do not fit the measured curve shapes.

In this section the parameter values have been adapted manually in order to fit the shapes of simulated curves to the measured. This was done since this was a fast method and did yield good correspondence with measured curves especially for the major loops. For the simulations of the minor loops it was not possible to achieve good fit for all loops simultaneously. Therefore, the parameter values were chosen such that the amplitudes between simulations and measurements agreed as good as possible. The judging of the fitting was done visually.

6.1.3 Dynamic measurements with controlled H-field

6.1.3.1 Set-up

For the dynamic measurements with controlled H-field another set-up than in the static case was used. The problem with the Epstein frame is that the large number of coil turns and the amount of magnetic materials imply a large inductance. Moreover, this inductance is strongly nonlinear due to the hysteresis effect.

Therefore, a single sheet tester (SST) which has a much smaller inductance was used. The SST used was presented in ref. [42]. It consists of two equal U-shaped yokes placed above each other, as shown in Fig 6.9. It was originally designed as a two dimensional measurements set-up (Rotational SST) with another yoke pair placed perpendicularly to and inside the first two yokes. Each yoke is encircled by a coil and the coils of both halves are connected in series which creates a magnetic field in the yokes. The magnetic field will drive a flux through the yokes, which are made of laminated steel, and the directions of the fluxes from the two yokes are so that they oppose each other where the yokes meet. Thereby, the flux is forced to return through the air window between the yokes. By placing a magnetic sheet between the yokes running from one side to
the other side, the flux will mainly flow in the material, since its permeability is much higher there than in the air. The number of turns is 50 for each coil and the horizontal distance between the yokes is 9.0 cm. For more information about the SST see ref. [42].

Fig 6.9. Cross-sectional view of a SST.

The set-up used is shown in Fig 6.10. Again, the computer gives the output signal to a DAC, which in turn provides the amplifier with a voltage. The amplifier drives a current through the coils in the SST. Inside the SST a strip of a magnetic material is placed. For measurement of the flux density a coil is surrounding the strip. The voltage over the coil is measured by a fluxmeter and the flux density is calculated in the same way as for the static set-up. For measurement of the field a Hall probe is placed as close as possible to the surface of the material. The Hall probe measures the drift of the electrons caused by the magnetic field. This probe is attached to a gaussmeter which calculates the field. Using the ADC the values of the flux density and field are provided to the computer which stores them. More information about these measurements is found in Appendix IV.
6.1.3.2 Influence of cutting on the permeability

During the performance of the measurements it was found that the permeability of the samples decreased sharply with decreasing width of the samples. To verify that this phenomenon really was a material phenomenon and not caused by the measurement set-up a 2 cm wide sample was first measured with a frequency of 1 Hz. Then, that sample was cut into two halves and both halves were inserted into the SST and measured again. This procedure was repeated twice so that totally four measurements were performed with 1, 2, 4 and 8 samples with 2, 1, 0.5 and 0.25 cm width respectively. Thereby, it was made sure that the total volume of material was the same in all set-ups, and that leakage flux between the SST and sample did not differ between the measurements. This test was done for two different materials, M600-50A, which is a non-oriented electrical steel, and 23ZDKH, and they both yielded the same result. The results of the measurements of one of the materials, M600-50A, are shown in Fig 6.11 where it clearly can be seen that the permeability decreases with decreasing width of the samples. It was also controlled that the applied field was the same for all measurements, so the effect is due to decreasing flux and not different field in the material.

The magnetic samples were cut into smaller strips in order to give a small inductance. The cutting introduces some deterioration of the material which is shown in the next section.

Fig 6.10. Block scheme of set-up for dynamic measurements.
Fig 6.11. Major hysteresis loops for different sample width of M600-50A. The figure clearly shows that the permeability decreases with decreasing width.

The same phenomenon has been found for electrical steel in [61], [62] and [63] where it is shown that the cause of the decrease in permeability is due to the cutting of the material. In the cutting process the material is permanently deformed by mechanical stress close to the cutting edge in such a way that the magnetic properties are deteriorated. The deterioration can be present up to 10 mm from the edge [62]. One can easily understand that the effect of the deterioration on the measured permeability increases with decreasing width of the material.

There are at least two ways to decrease this deterioration:

1) heating of the material after the cutting to a certain temperature where recrystallization occurs [62], [63]. This is called stress-relief annealing.

2) use of waterjet cutting instead of mechanical [47]
Therefore, samples of materials were stress-relief annealed at Surahammars Bruk, a subsidiary of Cogent Power Ltd. Results of measurements of these materials are presented in the next section.

In Fig 6.12 the effect on the dynamic magnetization loops of stress-relief annealing is shown. Both an annealed and non annealed sample of the electrical steel M270-50A, for more data see next section, was measured on. The figure shows that the static properties are deteriorated, however, the dynamic characteristics seems to be nearly unchanged.

![Dynamic Magnetization Loops](image)

**Fig 6.12.** Comparison of measured dynamic magnetization loops for an annealed (solid lines) and non annealed (dashed lines) sample of M270-50A. The frequencies are 1, 500 and 1000 Hz.

### 6.1.3.3 Measurements of stress-relief annealed materials

In order to get rid of the deterioration introduced by cutting of materials a number of cut samples of electrical steels were stress-relief annealed. These materials were then measured on in the SST. Results from measurements on four of those steels are presented here, namely:
• M5, grain oriented. Thickness 0.30 mm, width 8.4 mm. Conductivity $2.08 \times 10^6$ S/m

• NO20, thin non-oriented. Thickness 0.20 mm, width 6.3 mm. Conductivity $1.96 \times 10^6$ S/m.

• M0-H, grain oriented. Thickness 0.27 mm, width 8.4 mm. Conductivity $2.09 \times 10^6$ S/m.

• M270-50A, non-oriented. Thickness 0.50 mm, width 6.3 mm. Conductivity $1.82 \times 10^6$ S/m.

Samples of the above listed materials were inserted into the SST and dynamic magnetization loops were measured at the following frequencies: 1, 50, 500, 1000 and 2000 Hz. The applied H-field was sinusoidal.

The magnetization loops of M5 at different frequencies are shown in Fig 6.13. Notice that the sampling frequency was limited. The effect of this is especially seen at 2000 Hz where the curve has sharp corners when going in and out of saturation, respectively. In reality, these corners are smooth. With at higher sampling frequency this would be seen in the measurements as well. More information about the sampling frequency is found in Appendix IV.

The loops of the other materials are shown in the verification section 6.1.5.
6.1.4 Dynamic measurements with controlled B-field

Measurements of power loss and magnetization loops of different electrical steels with controlled $B$-field were also performed. The $B$-field was controlled to be sinusoidal. This was done at ABB using Epstein frames with a PID-controller feedback system to ensure the accuracy of the measurements. The equipment is certified according to the IEC standard [81] for power loss measurement. The following materials were measured on:

- M5. Thickness 0.30 mm, width 3.0 cm, grain oriented.

- M700-50A, mixed rolling and transverse directions. Thickness 0.50 mm, width 3.0 cm, non oriented.

- 23ZDKH. Thickness 0.23 mm, width 3.0 cm, strongly oriented, laser-scribed.
The measurements were done in the frequency range $10 - 8000$ Hz and $B_{\text{max}}: 0.5 - 1.7$ T. Some of the magnetization loops for M5 is seen in Fig 6.14 - Fig 6.17. More results of the measurements on this material and of the other materials are presented in section 6.1.6, where also results from simulations are shown.

Notice especially the magnetization loops in Fig 6.15. At $B_{\text{max}} = 1.7$ T, there are several bulges in the curve. This is due to the controlling of $B$. For the positive branch the flux increases very fast, and in order to keep the flux sinusoidal the $H$-field has to be decreased. The same is true for the negative branch. This gives very different behaviour of the curves compared to the case with controlled $H$.

**Fig 6.14.** Dynamic magnetization curves of M5 at 50 Hz and $B_{\text{max}} = 0.5, 1.1, 1.4$ and 1.7 T, respectively.
Fig 6.15. Dynamic magnetization curves of M5 at 500 Hz and $B_{max} = 0.5$, 1.1, 1.4 and 1.7 T, respectively.

Fig 6.16. Dynamic magnetization curves of M5 at $B_{max} = 0.5$ T and frequencies 50, 200, 500, 1500, 3500 and 5000 Hz with increasing width of the curves respectively.
Fig 6.17. *Dynamic magnetization curves of M5 at $B_{\text{max}} = 1.7$ T and frequencies 50, 200, 500 and 800 Hz with increasing width of the curves respectively*

**6.1.5 Dynamic simulations with controlled H-field**

In order to verify the dynamic magnetization model presented in section 4.3, simulations of the measured materials were performed in Dymola*. First, the cases with controlled $H$-field were simulated.

Simulations using the Cauer circuit model were performed for the stress-relieved materials in section 6.1.3.3. The results of this are shown in section 6.1.5.2. In order to compare the results of the model the materials were also simulated using the classical eddy current expression instead of Cauer circuits. The results of this are presented in section 6.1.5.3.

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*Dymola – Dynamic Modeling Laboratory, Dynasim AB, Sweden. www.dynasim.se*
6.1.5.1 Simulation procedure
In the measurements sinusoidal $H$-fields were applied and to ensure that the same waveforms were used in the simulations, the measured values of $H$ were used as input in the simulations.

For the excess loss term (equation (2.18), all parameters are known except $G$ and $V_0$. The value of $G$ is dependent on the magnetic domain structure of the material. According to ref. [19] $G$ has a theoretically value of 0.1357 for the stripe domain structure present in electrical steels.

This leaves us with the phenomenological parameter $V_0$. Since this parameter is a statistical parameter it is difficult (or impossible) to measure directly for these materials. However, if the assumption in ref. [44] that $V_0$ is dependent only on the peak magnetization and not the waveform or frequency, holds, it is possible to adapt the simulated curves to the measured curves and thereby find its value.

Therefore, in this case, $V_0$ was adapted so that the simulated magnetization curve at 50 Hz and the measured curve at the same frequency had the same energy loss for one loop. The same value of $V_0$ was then used for the other frequencies.

For the static contribution to the magnetization, the static model is adapted to the measured curves at 1 Hz. This is an approximation, however, the dynamic contribution at this frequency is very small and therefore this approximation only introduces small errors.

6.1.5.2 Simulations using Cauer circuits
According to the discussion in section 4.2.1.2 it is necessary to decide how many sections to use in the Cauer circuit. In the case of simulations of sinusoidal $H$ the curves are quite smooth and therefore the linear approach can be used. Here, it is done with 10 section and 40 % increase between each section. Conversely, when simulating with sinusoidal $B$ the optimized approach may be needed as is shown in section 6.1.6.

The values of $V_0$ found by fitting the simulation curve at 50 Hz to the measured curve at 50 Hz were:

- M5: $V_0 = 0.9$ A/m
- NO20: $V_0 = 0.7$ A/m
- **M0-H**: $V_0 = 0.8 \text{ A/m}$

- **M270-50A**: $V_0 = 0.3 \text{ A/m}$

The dynamic magnetization loops of these materials are seen in Fig 6.18 - Fig 6.21. The simulated curves follow the measured curves satisfyingly near the knees of the loops. However, close to the magnetization knee the simulated curves are sharper than the measured. Nevertheless, they replicate the shapes of the measured curves very good.

**Fig 6.18.** Measured (solid lines) and simulated (dashed lines) dynamic magnetization loops for M5. For the simulated results the eddy currents have been simulated using Cauer circuits. Frequencies: 1, 50, 500, 1000 and 2000 Hz respectively with increasing width of the loops.
Fig 6.19. Measured (solid lines) and simulated (dashed lines) dynamic magnetization loops for NO20. For the simulated results the eddy currents have been simulated using Cauer circuits. Frequencies: 1, 50, 500, 1000 and 2000 Hz respectively with increasing width of the loops.

Fig 6.20. Measured (solid lines) and simulated (dashed lines) dynamic magnetization loops for M0-H. For the simulated results the eddy currents have been simulated using Cauer circuits. Frequencies: 1, 50, 500, 1000 and 2000 Hz respectively with increasing width of the loops.
Fig 6.21. Measured (solid lines) and simulated (dashed lines) dynamic magnetization loops for M270-50A. For the simulated results the eddy currents have been simulated using Cauer circuits. Frequencies: 1, 50, 500, 1000 and 2000 Hz respectively with increasing width of the loops.

6.1.5.3 Simulations using the classical eddy currents expression

The samples were also simulated using the classical eddy current expression instead of Cauer circuits as comparison. The values of $V_0$ found by fitting the simulation curve at 50 Hz to the measured curve at 50 Hz were in this case:

- M5: $V_0 = 0.6 \text{ A/m}$
- NO20: $V_0 = 0.4 \text{ A/m}$
- M0-H: $V_0 = 0.5 \text{ A/m}$
- M270-50A: $V_0 = 0.01 \text{ A/m}$

Studying the magnetizations curves for the materials, shown Fig 6.22 - Fig 6.25, it is seen that with increasing frequency the eddy current expression becomes erroneous. This method is not capable of following the shapes of the curves,
except maybe for N020, which also is the thinnest material and thereby has the least eddy current loss.

Notice also that the adapted values of $V_0$ become lower for the method using the classical eddy current expression compared to the Cauer circuit method. This is due to the fact that the classical expression overestimates the eddy current loss and in order to compensate for that, the calculated value of the excess loss must be lower.

Fig 6.22. Measured (solid lines) and simulated (dashed lines) dynamic magnetization loops using the classical eddy current expression for M5. Frequencies: 1, 50, 500, 1000 and 2000 Hz respectively with increasing width of the loops.
Fig 6.23. Measured (solid lines) and simulated (dashed lines) dynamic magnetization loops using the classical eddy current expression for NO20. Frequencies: 1, 50, 500, 1000 and 2000 Hz respectively with increasing width of the loops.

Fig 6.24. Measured (solid lines) and simulated (dashed lines) dynamic magnetization loops using the classical eddy current expression for M0-H. Frequencies: 1, 50, 500, 1000 and 2000 Hz respectively with increasing width of the loops.
6.1.5.4 Conclusions
Comparing the results using Cauer circuits and the classical eddy current expression, respectively, one can conclude that they can both be used at 50 Hz applied sinusoidal H-field. However, at higher frequencies the Cauer circuit method is superior. One can also conclude that using a frequency independent value of the excess loss parameter $V_0$ in the Cauer circuit model, replicates the simulated curves at 1 – 2000 Hz quite well. However, for more exact results it is necessary to study the frequency dependence of $V_0$.

6.1.6 Dynamic simulations with controlled B-field
In order to further verify the dynamic magnetization model, simulations of materials subjected to controlled B-fields were simulated. The simulated materials were the same as the measured materials presented in section 6.1.4. To ensure a sinusoidal B-field a feedback system with a P-controller was used in the simulation program, Dymola.
In order to get a good fit between simulated and measured curves the parameter $V_0$ was adapted. In the case with sinusoidal $H$-field it was shown that adapting $V_0$ at a low frequency yielded to low loss at higher frequencies. Therefore, in this case $V_0$ was adapted for every frequency and amplitude of $B$ so that the energy loss was the same in the simulated and measured case.

For the static part of the composite magnetization model, static major loops were measured for each material and the static model was adapted to these curves according to section 6.1.2. In order to model the minor loops correctly the parameters $q$ and $c$ in the static model were adapted to give as good agreement as possible with the measured minor loop with same amplitude.

However, since there can be a small difference between the simulated loops and the measured, this difference was compensated for. E.g. if the simulated static loss for one certain amplitude was higher than the measured loss for the same amplitude, the difference was subtracted from the simulated result of the total loss. This was done since the parameter $V_0$ was adapted to the measured loss value. Otherwise the adapted value of $V_0$ would be too high.

Three methods were compared:

- Linear Cauer circuit, 10 regions and 40 % increase in size between every region.
- Optimized Cauer circuit, according to section 4.2.1.2.
- Use of classical eddy current expression instead of Cauer circuits.

Furthermore, increasing the number of regions in the Cauer circuits were tested, however, the improvements were negligible. For the eddy current expression method the values of $V_0$ were found by using equation (2.20).

The optimization of the Cauer circuit requires determination of the degree of penetration, equation (4.39). For each material the highest degree of penetration was calculated, using the highest measured frequency and highest permeability in the static major loop in the materials, i.e. the mean value between the magnetization knees. This is an approximation, however, equation (4.39), is also
an approximation since it does not consider saturation and nonlinear magnetization.

In Fig 6.26 - Fig 6.29, examples of comparison between simulated and measured magnetization curves at different frequencies and $B_{\text{max}} = 1.7$ T are shown. These cases are very interesting since they include bulges in the curves, which makes them much more difficult to simulate than straight curves. The figures show that the model replicates the simulated curves quite well for all material. In particular, the simulations of M700-50A and M5 agree well with the measurements. Moreover, for these materials the linear and optimized Cauer circuits show almost the same results. Conversely, for 23ZDKH it is seen that the optimized circuit yields much better results than the linear circuit.

It is also seen in the figures that the classical eddy current method is inferior for these cases.
Fig 6.26. Measured and simulated dynamic magnetization curves for M5 at 800 Hz, $B_{\text{max}} = 1.7$ T. Solid line shows measured curve, dashed line simulated curve with linear Cauer circuit, dotted line simulated curve with optimized Cauer circuit and dotted-dashed line simulated curve with classical eddy current expression.
Fig 6.27. Measured and simulated dynamic magnetization curves for M700-50A at 800 Hz, $B_{\text{max}} = 1.7$ T. Solid line shows measured curve, dashed line simulated curve with linear Cauer circuit, dotted line simulated curve with optimized Cauer circuit and dotted-dashed line simulated curve with classical eddy current expression.
Fig 6.28. Measured and simulated dynamic magnetization curves for 23ZDKH at 500 Hz, $B_{\text{max}} = 1.7$ T. Solid line shows measured curve, dashed line simulated curve with linear Cauer circuit, dotted line simulated curve with optimized Cauer circuit and dotted-dashed line simulated curve with classical eddy current expression.
Fig 6.29. Measured and simulated dynamic magnetization curves for 23ZDKH at 2500 Hz, $B_{max} = 1.7$ T. Solid line shows measured curve, dashed line simulated curve with linear Cauer circuit, dotted line simulated curve with optimized Cauer circuit and dotted-dashed line simulated curve with classical eddy current expression.

Some examples of simulations at different frequencies with the optimized Cauer circuit are shown for M5 and 23ZDKH in Fig 6.30 and Fig 6.31, respectively. It is seen that the broadening of the curves are replicated in a satisfying way.
Fig 6.30. Measured (solid) and simulated (dotted) dynamic magnetization curves with optimized Cauer circuit for M5 at $B_{\text{max}} = 0.5 \ T$. Shown frequencies with increasing width: 50, 500, 1500, 3500 and 5000 Hz.
Fig 6.31. Measured (solid) and simulated (dotted) dynamic magnetization curves with optimized Cauer circuit for 23ZDKH at $B_{\text{max}} = 0.5$ T. Shown frequencies with increasing width: 50, 500, 2500, and 6500 Hz.

As previously mentioned, the value of $V_0$ was adapted for each simulation. Moreover, the values depended on both the amplitude and frequency of $B$. This is shown in Fig 6.32 - Fig 6.34. It can be seen in Fig 6.32 that $V_0$ increases with the amplitude for a fixed frequency. The increase is nearly linear until the material reaches saturation, where the its value increases faster. The same dependency has earlier been reported by other authors [44].

Furthermore, the results shown in Fig 6.33 and Fig 6.34 show that the value of $V_0$ also increases with the frequency. However, it seems to flatten out at around 1000 Hz. This implies that the formulation of this parameter needs to be modified.
Fig 6.32. Estimated values of the parameter $V_0$ at 50 Hz for M5 (solid) and M700-50A (dashed) at different amplitudes.

Fig 6.33. Estimated values of the parameter $V_0$ at $B_{\text{max}} = 0.5$ T for M5 (solid) and M700-50A (dashed) at different frequencies.
6.1.6.1 Conclusions and discussion

It has been shown in the previous sections that the dynamic magnetization model (Cauer circuit) is able to replicate measured curves in a satisfying way. This has been shown for both applied $H$-fields and $B$-fields for different electrical steels and for a wide frequency range.

It has also been shown that optimization of the regions in the Cauer circuit improves the results.

Furthermore, it has been shown that the excess loss parameter $V_0$ depends both on the frequency and the amplitude of $B$. It can also be dependent on the waveform of $B$.

It should be noticed that the only adapted parameter for dynamic magnetization was $V_0$. It is a great advantage that only one parameter needs to be adapted. This means, that if it is possible to find a frequency-dependency of $V_0$, e.g. for a class of materials, this would yield none parameter to adapt for the dynamic magnetization. However, the parameters of the static model still would be necessary to adapt.

It may be possible to achieve even better fit by adapting the value of $V_0$ individually for each section and/or the sizes of the sections. On the other hand,
that would make the adapting more complex and there is a trade-off between the simplicity of the model and the correspondence with measurements.

### 6.1.7 Estimation of the frequency dependence of excess losses

One application of the Cauer circuit model is to estimate the frequency dependence of the excess losses. It was seen in the previous section that the excess loss parameter $V_0$ was frequency dependent. However, the formulation of this parameter presumes that the field is homogenously distributed in the material. This is not the case in the considered situations and that could be one reason why $V_0$ did not deep a constant value with increasing frequency. It should be stressed that the values of $V_0$ were estimated. Therefore, if the variations of $V_0$ are due to the inhomogeneous field distribution and not due to frequency dependence of the parameter in itself, still the excess losses could have a square root dependency of frequency.

It was proven by Bertotti that the excess losses had a square root dependency of frequency, however, since he used the classical eddy current expression he could only prove it up to around 300 Hz [19]. On the contrary, with the use of the dynamic magnetization model presented in this thesis it is possible to investigated the excess loss characteristics even at higher frequencies. Therefore, the excess losses of the following materials were investigated:

- **M5.** Thickness 0.30 mm, width 3.0 cm, grain oriented.
- **M700-50A.** Mixed rolling and transverse directions. Thickness 0.50 mm, width 3.0 cm, nonoriented.
- **23ZDKH.** Thickness 0.23 mm, width 3.0 cm, strongly oriented, lasered.
- **M330-35A.** Mixed rolling and transverse directions. Thickness 0.35 mm, width 3.0 cm, nonoriented.

The total losses were measured for these materials according to the description in section 6.1.4. Then, the static hysteresis losses were measured for the
Experimental verification

Chapter 6

materials. Thereafter, the static and eddy current losses were simulated using optimized Cauer circuits. The difference between the measured total losses and the simulated static plus eddy current losses give the excess losses.

In Fig 6.35 – Fig 6.37 are shown the excess losses for those materials at $B_{\text{max}} = 0.5$, 1.1 and 1.7 T, respectively. The excess losses are plotted against the square-root of frequency, in order to facilitate the verification of a square-root dependency. I.e., a linear curve means a square-root dependency. The plots show that at low frequencies, the excess loss increase is faster than linear. However, for $B_{\text{max}} = 0.5$ T, when $f^{1/2}$ is greater than approximately 15 Hz$^{1/2}$ the increase is very close to linear for all materials. Furthermore, for $B_{\text{max}} = 1.1$ T, there is a close to linear increase for $f^{1/2}$ greater than approximately 20 Hz$^{1/2}$. For $B_{\text{max}} = 1.7$ T, the data at higher frequencies are limited, however, the increase seems to be slightly faster than linear. It should be noticed that in the latter case, the materials are run into saturation which could be the reason of the slightly different dependency.

![Graph of excess losses vs square-root of frequency](image)

**Fig 6.35.** Excess losses for different materials at $B_{\text{max}} = 0.5$ T plotted against the square-root of frequency. Solid line = M330-35A, dashed-dotted line = M700-50A, dashed line = M5 and dotted line = 23ZDKH.
Fig 6.36. Excess losses for different materials at $B_{\text{max}} = 1.1$ T plotted against the square-root of frequency. Solid line = M330-35A, dashed-dotted line = M700-50A, dashed line = M5 and dotted line = 23ZDKH.

Fig 6.37. Excess losses for different materials at $B_{\text{max}} = 1.7$ T plotted against the square-root of frequency. Solid line = M330-35A, dashed-dotted line = M700-50A, dashed line = M5 and dotted line = 23ZDKH.
6.1.7.1 Discussion
It should be stressed that the results in this section strongly depends on the models used. However, it has been shown in section 6.1.6 that the dynamic magnetization model using Cauer circuits is able to replicate measured magnetization curves in a satisfying manner. It has also been shown in section 4.2.1.3 that the inhomogeneous field distribution can be satisfactorily modelled. Therefore, these results are assumed to be correct.

It is interesting that the results indicate that the excess losses have a square-root dependency of the frequency in many cases. This is what Bertotti claimed [19], [30], however, was not able to prove for higher frequencies than approximately 300 Hz.

On the contrary, it seems like the assumption that the parameter $V_0$ is constant with frequency in ref [44] is incorrect. The reason to this incorrectness is that in that study the inhomogeneous field distributions and the saturation effects in the materials are not considered. This fact has also been pointed out by Zirka et al [31].

6.2 Transformer model verification

In order to verify the total transformer model a three-phase three-leg transformer was measured on and simulated. However, only one phase (middle leg) were measured on and simulated. The other terminals were left open during the measurements.

The transformer is at least 20 years old and some of its characteristics are unknown. The nameplate data is 4000 VA apparent power, it is D/Y – connected and the line-to-line voltage ratio is 380 / 750 V. The transformer is shown in Fig 6.38.
6.2.1 Measurements

In order to verify the model some of the unknown characteristics of the transformer were measured or estimated. These were:

- Number of turns. This was measured using a control winding with 20 turns and applying a voltage to the primary winding and measuring the induced voltages in the control winding and secondary winding. Since the number of turns in the control winding was known, it was possible to calculate the number of turns in the primary and secondary winding. This yielded 420 turns in the primary winding and 500 turns in the secondary winding.

- Resistance of each winding were measured to 1.67 $\Omega$ in the primary and 2.25 $\Omega$ in the secondary.
• Axial cross-section areas and heights of windings and space between windings and winding and core in order to calculate the leakage flux according to equation (3.17). The height of the regions was 15.3 cm and the radial cross-sectional areas approximately as listed below (also see the cross-sectional view of the transformer leg in Fig 6.39):

  – Between primary (inner) winding and core: 24 cm$^2$
  
  – Primary winding: 16 cm$^2$
  
  – Between windings: 7.1 cm$^2$
  
  – Secondary winding: 23 cm$^2$

*Fig 6.39. Cross-sectional view of one transformer leg from above.*

• Conductor diameter: 1.2 mm and insulation: 0.5 mm between the conductors.
• Core cross-section: 6.00 cm x 6.12 cm. Number of laminates in the core: 120. This gave a sheet thickness of 0.50 mm.

• It was not possible to measure the $B$-$H$ curves of the materials directly. Instead a static major loop was measured on the middle leg (see Fig 6.40). Moreover, this $B$-$H$ curve includes all effects of the transformer, e.g., losses in joints, leakage flux, air gap losses and magnetostrictive losses due to the bolt holes. There is one bolt hole with a diameter of 15 mm in every joint. As was shown in section 6.1.3.2 magnetic materials are deteriorated by cutting. Furthermore, since the transformer is old the magnetic material can have been subjected to aging [78]. These circumstances together with the fact that the joints are not 45° mitred but 90° joints, is probably the main reason that the saturation level is low and the coercivity high. The magnetic flux was measured with a control winding with 20 turns wound around the middle leg and the magnetic field with a Hall probe inserted between the inner winding and the core. The Hall probe cable was very flexible and a sample holder was used to get the probe element perpendicular to the axial direction of the leg. However, the measuring equipment is sensitive to noise, which also can be seen in the figure.

**Fig 6.40. Static B-H curve measured on the middle leg of the transformer.**
6.2.1.1 Normal operation
Measurements were done under normal operation at different frequencies and with different loads on the secondary side. The frequencies were 25, 50 and 250 Hz, respectively and the loads were in the range 1 – 4000 $\Omega$.

The measured quantities were primary and secondary voltages and currents, magnetic field strength and flux density in the core. The voltages and currents were measured using multimeters. Furthermore, the field strength and flux density were measured in the same way as described in the previous section.

The results of the measurements are shown together with the simulated results for the same operation modes in section 6.2.2.1.

6.2.1.2 Non normal operation
Measurements of four different non normal operation modes were performed:

- No-load operation.
- Inrush current
- DC-magnetization
- Short-circuit

The results of these measurements are presented in sections 6.2.2.3 - 6.2.2.6.

6.2.2 Simulations
A composite transformer model including a core and winding model were used in the simulations of the same operation modes as were measured.

Since the frequency was not higher than 250 Hz, a simplified winding model was used. At these low frequencies the capacitive and eddy current effects in the windings are small [1], [68], [69]. Therefore, those were omitted in the
simulations. This means that the winding model including capacitive and eddy current effects as presented in Chapter 3 has not been verified. Nevertheless, the leakage flux and resistances of the winding conductors were included in the model.

For modelling of the core it is not enough to lump all the magnetization characteristics into one component. On the contrary, a network of reluctance components has to be used. An example of such a network is shown in Fig 6.41. This type of networks can be made very detailed, however, in order to reduce the simulation time they can optionally be simplified. E.g., the yokes and the leg in one return path have usually the same properties and can therefore be lumped into one component instead of three.

![Reluctance network for the transformer core with two return paths.](image)

**Fig 6.41.** Reluctance network for the transformer core with two return paths. The left hysteresis component represents the reluctance in the middle leg. The two return paths to the right represents the reluctances in each of the other two legs. Moreover, the joints in each return path are represented by the two shunted hysteresis components. Finally, the air gaps are represented by the linear reluctance component in the middle of the circuit.

In the figure, the joints in each return path are represented by two shunted hysteresis components, since there are two flux paths around the bolt holes in each joint (see Fig 6.42).
Fig 6.42. Flux path in joints. The flux path is divided into two different paths in the core material. This is due to the bolt hole and gives rise to the two shunted hysteresis components representing the joints in the reluctance network.

In Fig 6.41 the reluctance network has two returns paths, representing the two outer legs of the transformer. However, since in this thesis the transformer is only run in one phase and the two legs have the same characteristics; it is possible to simplify this network. Therefore, the two paths can be represented by one path, with double cross-sectional area in order to yield the same flux density as in the two path case. This simplified network is shown in Fig 6.43.
In the hysteresis components in the network shown in Fig 6.43, the core model presented earlier in this thesis was implemented. However, one important task was to determine the parameters of the static hysteresis model. In order to do this the following approach was used:

Since the sheets in the core have a thickness of 0.5 mm they were supposed to be of grade M7, which is a commonly used grade and has this actual thickness. This material is grain oriented and has a magnetization loop similar to and a saturation level almost equal to M5 [85]. The latter material has been used in the core model verification. Therefore, for the static part of the magnetization model the parameters for M5 with an arctan anhysteretic curve were used. However, since the material has probably been aged, the losses are increased and thereby the coercivity is also increased [79], [80]. Therefore, parameters adapted for aging were used in the hysteresis components.

Moreover, the effect of cutting was implemented by adapting the parameters of the static part of the hysteresis components representing the joints. It was shown in section 6.1.3.2 that especially the permeability and saturation level were affected by cutting. It was found that the effect of cutting implemented in the
hysteresis components representing the joints, was highly affecting the $B-H$ curve of the transformer.

The parameters were adapted to fit a simulated static $B-H$ curve to the measured curve, shown in Fig 6.40. The parameter values are shown in Table 6.3 and the corresponding static $B-H$ curves in Fig 6.44. Note that the parameters for M7 governing the saturation regime is a little different from that for M5 presented in section 6.1.2. However, the resulting curve is almost the same, since different sets of parameters can yield similar curves. Also note that since a variable pinning strength is used both $t_0$, $y$ and $k_0$ affect the coercivity. The values of $q$, $c$ and the number of pseudo particles (25) were given by fitting of simulated minor loops to measured minor loops.

The total air gap was given by adapting the linear reluctance element with respect to the mean slope of the measured and simulated $B-H$ curves in Fig 6.45. This yielded a total air gap length of around 50 micrometers, which seems reasonable. In that figure is shown the resulting $B-H$ curve for the transformer when using the reluctance network with the adapted parameters due to aging and cutting.

Table 6.3. Parameter values for the different hysteresis components.

<table>
<thead>
<tr>
<th></th>
<th>$\chi^* \mu_0$</th>
<th>$M_s^* \mu_0$</th>
<th>$k_0$</th>
<th>$t_0$</th>
<th>$y$</th>
<th>$b$</th>
<th>$b_c$</th>
<th>$c$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M7</td>
<td>0.45</td>
<td>1.75</td>
<td>8</td>
<td>2</td>
<td>2.22</td>
<td>240</td>
<td>14</td>
<td>0.02</td>
<td>0</td>
</tr>
<tr>
<td>M7 aged</td>
<td>0.45</td>
<td>1.75</td>
<td>35</td>
<td>2</td>
<td>2.22</td>
<td>240</td>
<td>14</td>
<td>0.02</td>
<td>0</td>
</tr>
<tr>
<td>M7 aged and cut</td>
<td>0.35</td>
<td>1.65</td>
<td>47</td>
<td>2</td>
<td>1.2</td>
<td>650</td>
<td>14</td>
<td>0.02</td>
<td>0</td>
</tr>
</tbody>
</table>
Fig 6.44. Plot of static major loops for the different cases. Solid line shows simulated curve of M7, dotted line shows simulated curve of aged M7 whereas dashed-dotted shows simulated curve of aged and cut M7.

Fig 6.45. Measured static B-H curve of the whole transformer (solid line) compared with simulated curve (dashed line).
6.2.2.1 Normal operation

First the measured normal operation modes with different loads were simulated.

One crucial parameter in these simulations is the parameter $V_0$ of the excess loss term in the core model. It was shown in section 6.1.6 that this parameter increases with the amplitude of $B$. Moreover, it was reported in ref. [44] that this parameter is close to linear in the amplitude range 0.1 - 0.5 T. The values of $V_0$ as a function of $B_{\text{max}}$ were fitted at the operation mode with highest amount of core losses, i.e., 4000 $\Omega$ load at 25, 50 and 250 Hz, respectively. This gave $V_0 (B_{\text{max}} = 0.63) = 0.11$ A/m, $V_0 (B_{\text{max}} = 0.31) = 0.03$ A/m and $V_0 (B_{\text{max}} = 0.062) = 0.008$ A/m. In addition $V_0 (B_{\text{max}} = 0) = 0$ was used. The values of $V_0$ at other amplitudes of $B$ were then given through linear interpolation.

Measured and simulated voltages and currents are compared for different loads in Table 6.4 - Table 6.8. In the tables, index $p$ denotes primary winding and index $s$ secondary winding. Also the simulated value of the amplitude of $B$ for each case is shown in the tables. The importance of the core model increases with increasing load resistance and decreasing frequency, since the core losses are high in these cases. Conversely, the importance of the winding model increases with decreasing load resistance and increasing frequency. It can be seen in the tables that for the operations with 30 – 4000 $\Omega$ load the simulated results agree very well with the measured, especially for the secondary voltage and primary current. However, it can be seen that at 1 $\Omega$ load the simulated and measured results are diverging.
Table 6.4. Measured and simulated voltages and currents of the primary and secondary winding, respectively. Load on secondary winding is 4000 Ω. Shown values are peak values. The accuracy of the measured currents is ± 2 mA and of the measured voltages ± 1.6 V.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>$V_p$ (V)</th>
<th>$V_s$, meas (V)</th>
<th>$V_s$, sim (V)</th>
<th>$I_p$, meas (A)</th>
<th>$I_p$, sim (A)</th>
<th>$I_s$, meas (A)</th>
<th>$I_s$, sim (A)</th>
<th>$B_{max}$ (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>146.0</td>
<td>174.4</td>
<td>173.5</td>
<td>0.102</td>
<td>0.102</td>
<td>0.044</td>
<td>0.043</td>
<td>0.63</td>
</tr>
<tr>
<td>50</td>
<td>146.0</td>
<td>175.1</td>
<td>173.5</td>
<td>0.084</td>
<td>0.084</td>
<td>0.044</td>
<td>0.043</td>
<td>0.31</td>
</tr>
<tr>
<td>250</td>
<td>146.0</td>
<td>175.4</td>
<td>173.6</td>
<td>0.068</td>
<td>0.068</td>
<td>0.046</td>
<td>0.043</td>
<td>0.062</td>
</tr>
</tbody>
</table>

Table 6.5. Measured and simulated voltages and currents of the primary and secondary winding, respectively. Load on secondary winding is 2000 Ω. Shown values are peak values. The accuracy of the measured currents is ± 2 mA and of the measured voltages ± 1.2 V.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>$V_p$ (V)</th>
<th>$V_s$, meas (V)</th>
<th>$V_s$, sim (V)</th>
<th>$I_p$, meas (A)</th>
<th>$I_p$, sim (A)</th>
<th>$I_s$, meas (A)</th>
<th>$I_s$, sim (A)</th>
<th>$B_{max}$ (T)</th>
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<tr>
<td>25</td>
<td>120.7</td>
<td>143.3</td>
<td>143.3</td>
<td>0.126</td>
<td>0.124</td>
<td>0.071</td>
<td>0.072</td>
<td>0.50</td>
</tr>
<tr>
<td>50</td>
<td>121.2</td>
<td>144.0</td>
<td>143.9</td>
<td>0.111</td>
<td>0.114</td>
<td>0.072</td>
<td>0.072</td>
<td>0.26</td>
</tr>
<tr>
<td>250</td>
<td>121.4</td>
<td>144.2</td>
<td>144.2</td>
<td>0.095</td>
<td>0.100</td>
<td>0.072</td>
<td>0.072</td>
<td>0.052</td>
</tr>
</tbody>
</table>
Table 6.6. Measured and simulated voltages and currents of the primary and secondary winding, respectively. Load on secondary winding is 400 Ω. Shown values are peak values. The accuracy of the measured currents is ± 10 mA and of the measured voltages ± 0.9 V.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>$V_p$ (V)</th>
<th>$V_{s, \text{meas}}$ (V)</th>
<th>$V_{s, \text{sim}}$ (V)</th>
<th>$I_{p, \text{meas}}$ (A)</th>
<th>$I_{p, \text{sim}}$ (A)</th>
<th>$I_{s, \text{meas}}$ (A)</th>
<th>$I_{s, \text{sim}}$ (A)</th>
<th>$B_{\text{max}}$ (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>90.5</td>
<td>106.2</td>
<td>106.5</td>
<td>0.344</td>
<td>0.348</td>
<td>0.266</td>
<td>0.266</td>
<td>0.39</td>
</tr>
<tr>
<td>50</td>
<td>90.9</td>
<td>106.8</td>
<td>107.3</td>
<td>0.335</td>
<td>0.343</td>
<td>0.267</td>
<td>0.268</td>
<td>0.19</td>
</tr>
<tr>
<td>250</td>
<td>91.0</td>
<td>106.9</td>
<td>107.3</td>
<td>0.325</td>
<td>0.324</td>
<td>0.267</td>
<td>0.269</td>
<td>0.039</td>
</tr>
</tbody>
</table>

Table 6.7. Measured and simulated voltages and currents of the primary and secondary winding, respectively. Load on secondary winding is 30 Ω. Shown values are peak values. The accuracy of the measured currents is ± 30 mA and of the measured voltages ± 0.3 V.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>$V_p$ (V)</th>
<th>$V_{s, \text{meas}}$ (V)</th>
<th>$V_{s, \text{sim}}$ (V)</th>
<th>$I_{p, \text{meas}}$ (A)</th>
<th>$I_{p, \text{sim}}$ (A)</th>
<th>$I_{s, \text{meas}}$ (A)</th>
<th>$I_{s, \text{sim}}$ (A)</th>
<th>$B_{\text{max}}$ (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>25.1</td>
<td>25.6</td>
<td>25.9</td>
<td>1.02</td>
<td>1.02</td>
<td>0.856</td>
<td>0.864</td>
<td>0.097</td>
</tr>
<tr>
<td>50</td>
<td>25.2</td>
<td>25.7</td>
<td>26.0</td>
<td>1.02</td>
<td>1.02</td>
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<td>0.866</td>
<td>0.048</td>
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<tr>
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<td>25.4</td>
<td>25.6</td>
<td>1.01</td>
<td>1.02</td>
<td>0.847</td>
<td>0.854</td>
<td>0.010</td>
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</table>
Table 6.8. Measured and simulated voltages and currents of the primary and secondary winding, respectively. Load on secondary winding is 1 Ω. Shown values are peak values. The accuracies of the measured currents are ±50 mA at 25 and 50 Hz and ±30 mA at 250 Hz. The accuracies of the measured primary and secondary voltages are ±0.08 V and ±0.01 V, respectively.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>V_p (V)</th>
<th>V_s, meas (V)</th>
<th>V_s, sim (V)</th>
<th>I_p, meas (A)</th>
<th>I_p, sim (A)</th>
<th>I_s, meas (A)</th>
<th>I_s, sim (A)</th>
<th>B_max (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>8.94</td>
<td>1.78</td>
<td>1.88</td>
<td>2.11</td>
<td>2.24</td>
<td>1.78</td>
<td>1.88</td>
<td>0.02</td>
</tr>
<tr>
<td>50</td>
<td>8.98</td>
<td>1.77</td>
<td>1.86</td>
<td>2.08</td>
<td>2.22</td>
<td>1.77</td>
<td>1.86</td>
<td>0.012</td>
</tr>
<tr>
<td>250</td>
<td>9.04</td>
<td>1.32</td>
<td>1.33</td>
<td>1.58</td>
<td>1.56</td>
<td>1.34</td>
<td>1.33</td>
<td>0.004</td>
</tr>
</tbody>
</table>

6.2.2.2 Discussion
The obtained results indicate that the model with a detailed core model and a simple winding model yields good results when the core losses are dominating. However, when the winding losses and leakage inductance increase the simplified winding model has to be replaced with a more detailed model in order to correctly replicate the measured results. One obvious source of error is the modelling of the leakage flux; since the geometrical values needed to calculate those, were not known exactly. Conversely, they were roughly estimated by visual inspection.

Unfortunately, the time limit of the project was reached before the proposed, more detailed winding model was able to be verified.

6.2.2.3 No-load operation
With the secondary side unloaded, the magnetization of the core will easily go into saturation. This will affect the magnetization loop of the core and also the current in the primary winding, i.e., the magnetizing current. The shape of this
current will be non-sinusoidal due to the nonlinearity of the magnetization loop of the material.

No-load currents were measured at the primary side at four different frequencies: 5, 10, 20 and 50 Hz. The 5 and 10 Hz tests were measured with amplitude of 50 and 100 V, respectively, applied to the primary winding. The 25 and 50 Hz tests were measured at the maximum amplifier output voltage, i.e. with amplitude of 146 V.

The same cases were simulated with the transformer model. The results of these simulations are shown in Fig 6.46 - Fig 6.49. It can be seen that at 5 and 10 Hz the simulations are in very good agreement with the measured curves. At 50 Hz the simulations are also in good correspondence with the measurements, however, at 20 Hz the correspondence is slightly lower. The reason to the discrepancy at the latter frequency is not known. It shall be noted that at 50 Hz the amplitude of the applied field is not high enough to bring the material into saturation. In this case the importance of the minor loop model is higher than for the other simulated cases were the major loop model is more important.
Fig 6.46. Magnetizing current in no-load operation at 5 Hz. Solid line is measured curve and dashed line simulated curve.

Fig 6.47. Magnetizing current in no-load operation at 10 Hz. Solid line is measured curve and dashed line simulated curve.
Fig 6.48. Magnetizing current in no-load operation at 20 Hz. Solid line is measured curve and dashed line simulated curve.

Fig 6.49. Magnetizing current in no-load operation at 50 Hz. Solid line is measured curve and dashed line simulated curve.
6.2.2.4 Inrush current

When switching in a transformer to a net, a phenomenon called inrush current can occur. In normal operation and steady state condition the magnetization loop in the core will be symmetrical around origin. However, if the transformer has been in operation and switched off, there is usually a remanent flux in the core. Therefore, when the transformer is switched in, the core will be magnetized again and due to the remanence the magnetization loop may now not be symmetrical around origin [1], [26].

This phenomenon can occur even without remanence in the core, depending on where in the voltage cycle the transformer is switched in. The time integral of the voltage gives the flux so if the switching occurs in the early part of one of the two half periods of a cycle the flux will reach high values in the end of that half period. Theoretically, the flux can reach maximum twice the normal operation flux plus the remanent flux. Moreover, it may drive the core into saturation which will give a peak current much higher than the current in steady state condition. However, due to the losses in the transformer, the steady state condition is reached after a time. For power frequencies this time is in the range of 10 – 20 seconds for modern transformer [1], [26].

In order to test the transformer model one inrush current situation when the transformer core was put into the remanent state, $B_r = 0.72 \pm 0.008$, was measured. The primary current was measured in the following condition:

- $f = 50 \text{ Hz}$, secondary side unloaded, $V_{AC, \text{peak}} = 62.7 \pm 0.5 \text{ V}$. Supply switch-on angle = 0 °.

The same situation was then simulated. The primary currents found from the measurements and simulations, respectively are compared in Fig 6.50 - Fig 6.52. It can be seen that the simulated currents are in good correspondence with the measured values.
Fig 6.50. First 100 periods of primary winding current starting from the maximum remanent state. In this case the secondary side is unloaded. The upper figure shows measured values and the lower picture shows simulated values.
Fig 6.51. The first ten periods of the primary winding current for the same case as in Fig 6.50. Solid line shows measured current and dashed line simulated current.

Fig 6.52. The primary winding current after ca 50 periods for the same case as in Fig 6.50. Solid line shows measured current and dashed line simulated current.
The simulated traced minor loop for the first period is seen in Fig 6.53. In the figure are also seen the simulated major loops at DC (static condition) and 50 Hz, respectively. In Fig 6.54 the static and dynamic parts, respectively, of the traced minor loop from the simulation are plotted.

**Fig 6.53.** Simulated traced minor loop (solid line) for the first period. Simulated major loops at DC and 50 Hz shown by dashed and dashed-dotted lines, respectively.
It can be seen in Fig 6.51 and Fig 6.52 that the negative peak values of the measured curves is decreasing faster than the corresponding simulated curve values. It should be stressed that in this situation the magnetization is tracing minor loops as seen in Fig 6.53 and Fig 6.54. Moreover, the latter figure shows that the static part of the minor loops gives the major contribution to the losses of the dynamic minor loops when the material is close to saturation. In the showed example this is the case in the initial stage and is due to the fact that the rate of change is small there. Therefore, it is essential that the static minor loop model is correct.

Therefore, in order to improve the dynamic magnetization model for inrush currents, the static minor loop model should be improved.

### 6.2.2.5 DC-magnetization

There can be situations when a small DC-voltage or -current is added to the AC-voltage of one or both windings of the transformer. This DC-component can be due to non-balanced operation of DC-converters connected to the transformer or geomagnetically induced currents [1]. This DC-component will give an offset in the magnetization of the core and also to the currents in the windings. Furthermore, it can drive the core up into saturation and thereby increase the current dramatically.
Two measurements with DC-magnetization were performed:

- \( V_{AC, \text{peak}} = 10 \pm 0.08 \, \text{V}, \, V_{DC} = 0.3 \pm 0.001 \, \text{V}, \, f = 50 \, \text{Hz}, \, \text{load} = 100 \, \text{ohm} \)

- \( V_{AC, \text{peak}} = 25 \pm 0.3 \, \text{V}, \, V_{DC} = 0.3 \pm 0.001 \, \text{V}, \, f = 50 \, \text{Hz}, \, \text{load} = 400 \, \text{ohm} \)

Before the DC-offsets were applied the transformer was demagnetized. Then the primary current was measured and the same situations were simulated. Comparisons between the measurements and simulation are seen in Fig 6.55 and Fig 6.56. It can be seen in the figures that the simulated currents are in good correspondence with the measurements. It shall be noted that the mean value of the flux density was ca 0.95 T in both situations. Comparing this with the magnetization loop of the transformer core, seen in Fig 6.40, it can be understand that some parts of the core were in saturation. Thereby, it is essential that the model replicates the real object correctly. One reason to the small discrepancies between the simulated and measured results can be due to the fact that the characteristics of the measured object are not known exactly. Another reason can be that the model is not replicating the object perfectly due to the imperfections in the minor loop model as was the case for the inrush currents.
Fig 6.55. Current in primary winding for the 100 ohm load case. Solid line shows measured current and dashed line simulated current.

Fig 6.56. Current in primary winding for the 400 ohm load case. Solid line shows measured current and dashed line simulated current.
6.2.2.6 Short-circuit
Under short-circuit condition at 50 Hz the measured voltage was 11.7 ± 0.2 V (rms). Measured and simulated short-circuit current were 3.51 ± 0.08 A and 3.46 A, respectively.

6.2.3 Discussion
Most of the characteristics and parameters of the measured object, necessary for the model were unknown. This fact complicated the modelling process, since it required estimation and adaption of several parameters. Furthermore, it was found, when building the model, that the magnetic core of the transformer was highly degraded due to cutting and most likely also due to aging. This fact also complicated the modelling attempt, since the material parameters could not be obtained from measurements in e.g. an Epstein frame. Nevertheless, with use of a reluctance network for the core, with parameter adapted for aging and cutting of material, it was possible to achieve good correspondence between the measurements and simulations.

The best correspondence was achieved in conditions when the core losses were dominant, i.e., no-load, low load, inrush current and DC-magnetization. On the contrary, under the conditions when the winding losses, i.e., high load, were dominant, the correspondence was slightly lower. Therefore, the core model is considered to be satisfactory, even if the minor loop model can be improved further. However, the winding model should be improved. Nonetheless, with respect to the complexity of the measured object, the transformer model is considered to yield satisfactorily results.

An advantage with the proposed transformer model is the use of the detailed dynamic magnetization core model. In the core model verification section it was shown that this model yields correct results up into at least the kHz range. At low frequencies the static model is the most important, however, at higher frequencies the dynamic model becomes essential.

It is also interesting that it was possible to use a test object, with most of the parameters unknown and also highly degraded and, thereafter, build a model of that object that in high extent replicated the characteristics of the object.
Chapter 7

Conclusions and future work

7.1 Conclusions

In this thesis, a time-domain transformer model including loss phenomena in both the core and the windings has been presented. The model includes core phenomena as magnetic static hysteresis, eddy currents and excess losses. Moreover, the model includes winding phenomena as eddy currents, capacitive effects and leakage flux.

For modelling of the core losses, Bergqvist’s lag model has been used for the static magnetization part. Some modifications of this model, e.g. regarding the anhysteretic curve, have been proposed. This model has been shown to yield good correspondence with measurements, especially for major loops.

A dynamic magnetization model using Cauer circuits for modelling of eddy currents and Bertotti’s statistical loss model for excess losses has been assessed. One benefit with the Cauer circuit model is that it automatically includes the inhomogeneous field distribution due to eddy currents. Another benefit is that for adapting the simulated curves to measured curves it is only needed to fit one parameter, $V_0$, in the excess loss term in addition to the static magnetization model. The dynamic model has been verified to yield good agreement with measurements both for controlled sinusoidal $H$-fields and $B$-fields up to several kHz for electrical steels. Moreover, it has been shown that it is possible to investigate the inhomogeneous field distribution due to eddy current shielding in magnetic materials using this model.
Furthermore, the frequency dependence of excess losses has been studied using this model. The results indicate that the excess losses have a close to square-root dependency of frequency. Moreover, the statistical loss theory [19] implies that the parameter $V_0$ should be constant with the frequency; however, it has been shown in this thesis that that is not the case for electrical steels. One reason for this can be that the field distribution is varying with the frequency and since the value of $V_0$ is dependent on the peak magnetization, it will be affected by the field distribution.

A winding model including an analytical expression for the skin effect and proximity effect in the winding conductors has been used. Furthermore, the capacitive effects and leakage flux has been included in the model. This model has an upper theoretical frequency limit at 100 kHz.

Moreover, a composite three-phase transformer model containing the proposed core and winding models has been used. The frequency limit of this model is the same as for the winding model.

The composite one-phase transformer model containing the proposed core model and a simplified winding model has been verified through measurements on a real transformer. This has shown that the model shows good agreement with measurements in no-load conditions and in low load conditions up to 250 Hz. Furthermore, it has shown good correspondence with measurements in non-normal operation conditions like inrush current and DC-magnetization.

### 7.2 Future work

- The proposed winding model should be verified and its validity with respect to frequency and model parameter settings investigated.

- In the core model the static hysteresis model should be improved, especially regarding the modelling of minor loops. E.g., the distribution function of the pseudo particles and the relation between reversible and irreversible processes should be studied more in detail.
• In the dynamic magnetization model the optimization of the Cauer circuit sections and the frequency and amplitude dependency of the parameter \( V_0 \) could be studied. If it is possible to find that dependency for a class of material, this would make this model a very useful tool. Then the only necessary adapting step would be to fit the static curve to the measured static curve.

• The composite transformer model should also be validated in other operation modes like transient overvoltage.

• A more detailed reluctance network model of the transformer including three dimensional flux paths in the tank and construction details should be developed and verified.

• A static magnetization model that takes \( B \) as input and returns \( H \) should be developed.
Appendix I

Derivation of analytical expression for eddy currents

Here follows the derivation of the analytical expression of eddy currents as it is presented in ref. [20].

When all the elements are in steady-state condition, the quasi-static form of Ampère’s law can be written as:

\[ \nabla \times \mathbf{H} = \mathbf{J} \]  \hspace{1cm} (8.1)

We also have

\[ \mathbf{J} = \sigma \mathbf{E} \]  \hspace{1cm} (8.2)

Since the fields in the coil vary sinusoidal we get the following relations in complex notation:

\[ \mathbf{H}(t) = \text{Re} \left( \hat{\mathbf{H}} e^{j\omega t} \right) \]  \hspace{1cm} (8.3)

\[ \mathbf{J}(t) = \text{Re} \left( \hat{\mathbf{J}} e^{j\omega t} \right) \]  \hspace{1cm} (8.4)

\[ \mathbf{E}(t) = \text{Re} \left( \hat{\mathbf{E}} e^{j\omega t} \right) \]  \hspace{1cm} (8.5)

\[ \mathbf{B}(t) = \text{Re} \left( \hat{\mathbf{B}} e^{j\omega t} \right) \]  \hspace{1cm} (8.6)

Where “^” denotes a complex amplitude and \text{Re} the real part of a complex number.
Introducing the complex conjugate, denoted with “*”, in equation (8.1) and multiplying with $E$ we get:

$$\hat{E} \cdot (\nabla \times \hat{H}^*) = \hat{E} \cdot \hat{J}^*$$  \hspace{1cm} (8.7)

Expanding the left term of this equation gives:

$$\hat{E} \cdot (\nabla \times \hat{H}^*) = -\nabla \cdot (\hat{E} \times \hat{H}^*) + \hat{H}^* \cdot (\nabla \times \hat{E})$$  \hspace{1cm} (8.8)

Faraday’s law written in complex form states:

$$\nabla \times \hat{E} = -j \omega \hat{B}$$  \hspace{1cm} (8.9)

Combining equations (8.7), (8.8) and (8.9) now yields:

$$-\nabla \cdot (\hat{E} \times \hat{H}^*) = \hat{E} \cdot \hat{J}^* - j \omega \hat{H}^* \cdot \hat{B}$$  \hspace{1cm} (8.10)

Using equations (8.2), $B = \mu H$, and the complex pointing vector

$$\hat{P} = \frac{1}{2} \hat{E} \times \hat{H}^*$$  \hspace{1cm} (8.11)

in equation (8.10) yields:

$$-2\nabla \cdot \hat{P} = \frac{1}{\sigma} \hat{J} \cdot \hat{J}^* - j \omega \mu \hat{H} \cdot \hat{H}^*$$  \hspace{1cm} (8.12)

Suppose that the network exist inside a volume $V$ enclosed by a surface $S$. Then we can integrate over $V$ to get the total power inside the network, referring to Fig 3.2. However, Gauss theorem yields:
Appendix

\[ \int_{V} \nabla \cdot \mathbf{P} dV = \int_{S} \mathbf{P} \cdot \hat{n} dS \tag{8.13} \]

where \( \hat{n} \) is a unit vector normal to \( S \) at each point. Assume that all power flow is confined to the cross sections of the input terminals. Then the complex power input is \( \nu_{t} \hat{i}_{t}^{*} \). Now the normal component of the Poynting vector can be written in terms of the complex power input [83], [84] as:

\[ -\int_{S} \mathbf{P} \cdot \hat{n} dS = \hat{V}_{t} \hat{I}_{t}^{*} \tag{8.14} \]

where \( \hat{V}_{t} \) and \( \hat{I}_{t} \) are the complex amplitudes of the voltage and current applied to the network through \( S \). This yields:

\[
\begin{align*}
\hat{V}_{t} \hat{I}_{t}^{*} &= -\int_{V} \nabla \cdot \mathbf{P} dV = \frac{1}{2} \int_{V} \left( \frac{1}{\sigma} \mathbf{J} \cdot \mathbf{J}^{*} - j \omega \mu \mathbf{H} \cdot \mathbf{H}^{*} \right) dV = \\
&= \int_{V} \frac{1}{2\sigma} \mathbf{J} \cdot \mathbf{J}^{*} dV - \int_{V} j \frac{\omega \mu}{2} \mathbf{H} \cdot \mathbf{H}^{*} dV 
\end{align*}
\tag{8.15} \]

Now the complex impedance \( Z \) at the terminal is

\[ Z(j \omega) = R + jX = \frac{\hat{V}_{t}}{\hat{I}_{t}} \tag{8.16} \]

Expanding the right term of equation (8.16) by \( \hat{I}_{t}^{*} \) gives

\[ Z = \frac{\hat{V}_{t} \hat{I}_{t}^{*}}{\hat{I}_{t} \hat{I}_{t}^{*}} \tag{8.17} \]

Using equation (8.15) in equation (8.17) and changing the notation of the current from \( I_{t} \) to \( I \) gives
\[
Z(j\omega) = \frac{1}{2\sigma\tilde{I}} \int_v \mathbf{J} \cdot \mathbf{J}^* dV - j\frac{\omega\mu}{2\tilde{I}^*} \int_v \mathbf{H} \cdot \mathbf{H}^* dV \quad (8.18)
\]

Realizing that \(\tilde{I}^* = |\tilde{I}|^2\) finally yields

\[
Z(j\omega) = \frac{1}{2\sigma|\tilde{I}|^2} \int_v \mathbf{J} \cdot \mathbf{J}^* dV - j\frac{\omega\mu}{2|\tilde{I}|^2} \int_v \mathbf{H} \cdot \mathbf{H}^* dV \quad (8.19)
\]

This equation is equal to equation (3.1).

Now we want to find \(\mathbf{J}\) and \(\mathbf{H}\) inside the coil. Start with the magnetic diffusion equation:

\[
\nabla^2 \mathbf{B} = \mu\sigma \frac{d\mathbf{B}}{dt} \quad (8.20)
\]

In complex form the magnetic diffusion equation is:

\[
\nabla^2 \mathbf{\hat{B}} = j\omega\mu\sigma \mathbf{\hat{B}} \quad (8.21)
\]
Fig 8.1. Cross-section of one-turn coil with linearly increasing flux from zero at the outer surface to $B$ at the inner surface.

Now consider the case with a one-turn coil as illustrated in Fig 8.1. The turn radial thickness is assumed to be much smaller than the total coil diameter. Thereby, a Cartesian coordinate system can be used instead of a cylindrical one. The $x$ direction corresponds to the $\varphi$ direction and the $y$ direction to the $r$ direction.

The flux will approximately only be present in the axial direction; thereby, the magnetic diffusion equation can be reduced to one dimension:

$$\frac{d^2 \hat{B}_z}{dy^2} = j\omega \mu \sigma \hat{B}_z$$  \hspace{1cm} (8.22)

The solution of equation (8.22) will be on the form

$$\hat{B}_z(y) = c_1 \sinh k(y + c_2)$$  \hspace{1cm} (8.23)

where $c_1$ and $c_2$ are constants that can be evaluated by the appropriate boundary conditions and

$$k = \frac{(j+1)}{\delta}$$  \hspace{1cm} (8.24)
where $\delta$ is the skin-depth defined as:

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}} \quad (8.25)$$

According to Ampère’s law and the ampere-turn diagram the boundary conditions are:

$$\hat{B}_z(y = d) = 0$$

$$\hat{B}_z(y = 0) = \frac{\mu_0 I}{l} \quad (8.26)$$

Here $I$ is the current in the turn, $l$ the axial length of the coil and $d$ is the thickness of the turn.

The boundary condition at $y = d$ gives:

$$\hat{B}_z(y = d) = c_1 \sinh k(d + c_2) = 0 \quad (8.27)$$

The nontrivial solution of this equation is:

$$c_2 = -d \quad (8.28)$$

Next, the boundary condition at $y = 0$ gives:

$$\hat{B}_z(y = 0) = c_1 \sinh k(-d) = -\sinh kd = \frac{\mu_0 I}{l} \quad (8.29)$$

This gives the constant $c_1$ as:

$$c_1 = -\frac{\mu_0 I}{l \sinh kd} \quad (8.30)$$
Inserting the constant $c_1$ into equation (8.23) finally gives:

$$
\hat{B}_z(y) = \frac{\mu_0 I \sinh k(d - y)}{\sinh kd}
$$

(8.31)

Fig 8.2. Cross-section of a portion of a multilayer coil with an arbitrary number of turns. The current is directed along the negative x-axis.

Now consider the coil with axial length $l$ and $M$ turns in vertical directions as seen in Fig 8.2. For a single turn in the layer $n$, the following boundary conditions hold with a linearly increasing flux:

$$
\hat{B}_{n,z}(y = y_{n-1}) = \frac{\mu_0 M I}{l} (n - 1)
$$

$$
\hat{B}_{n,z}(y = y_{n}) = \frac{\mu_0 M I}{l} n
$$

(8.32)
where $M$ is the number of turns in every layer, $I$ is the current in each turn and $l$ the axial length of the coil. In the same way as for the single turn case, the flux density is found from the new boundary conditions.

This yields with $d = y_n - y_{n-1}$:

$$
\hat{B}_{n,z}(y) = \frac{\mu_0 M I}{l} \frac{\sinh k(y_n - y_{n-1} - y)}{\sinh k(y_n - y_{n-1})} + \frac{\mu_0 M I}{l} (n-1) =
$$

$$
= \frac{\mu_0 M I}{l} \frac{\sinh k(y_n - y_{n-1} - y) + (n-1)\sinh k(y_n - y_{n-1})}{\sinh k(y_n - y_{n-1})}
\tag{8.33}
$$

This numerator can be expanded to contain $y$ in both the terms. However, this requires lengthy calculations and will not be shown here. The final expression will then be:

$$
\hat{B}_{n,z}(y) = \frac{\mu_0 M I}{l} \left[ \frac{n \sinh \left( k \left( y - y_{n-1} \right) \right) - (n-1) \sinh \left( k \left( y - y_n \right) \right)}{\sinh \left( k \left( y_n - y_{n-1} \right) \right)} \right]
\tag{8.34}
$$

From Fig 8.2 we realize that

$$
w = \frac{l}{M}
\tag{8.35}
$$

and by using the relation between field and flux density in free space we can express the field in layer $n$ as:

$$
\hat{H}_{n,z}(y) = \frac{\hat{I}}{w} \left[ \frac{n \sinh \left( k \left( y - y_{n-1} \right) \right) - (n-1) \sinh \left( k \left( y - y_n \right) \right)}{\sinh \left( k \left( y_n - y_{n-1} \right) \right)} \right]
\tag{8.36}
$$

According to Ampère’s law:

$$
\nabla \times \mathbf{H} = \mathbf{J}
\tag{8.37}
$$
The magnetic field has only a component in the $z$ direction which results in that the current density only has a $x$ component given by:

$$\hat{J}_x(y) = \frac{d\hat{H}_z(y)}{dy} \quad (8.38)$$

Using this expression with equation (8.36) gives:

$$\hat{J}_{n,x}(y) = \frac{k \hat{I}}{w} \left[ n \cosh(k(y - y_{n-1})) - (n-1) \cosh(k(y - y_{n})) \right] \frac{\sinh(k(y_n - y_{n-1}))}{\sinh(k(y_n - y_{n-1}))} \quad (8.39)$$

Now it is possible to use equations (8.36) and (8.39) in equation (8.19) and calculate the corresponding impedance in a turn with circumference $l_n$ in the n’th layer by the following integration:

$$Z_n = \frac{l_n w}{2 \sigma \left| \hat{I} \right|^2} \int_{y_{n-1}}^{y_n} \hat{J}_x(y) \hat{J}_x(y)^* dy - \frac{j \omega \mu l_n w}{2 \left| \hat{I} \right|^2} \int_{y_{n-1}}^{y_n} \hat{H}_z(y) \hat{H}_z(y)^* dy \quad (8.40)$$

Carrying out this integration yields the result in equation (3.7).
Appendix II

Determination of parameter values in Bergqvist’s lag model

Here follows a method to determine the parameter values in Bergqvist’s lag model from measured curves.

Start with the mean pinning strength \( k \). The value of this parameter can be found from the total loss of a static major loop in one dimensional excitation. This loss corresponds to the area of all individual pseudo particles. The area of the j:th pseudo particle with pinning \( \lambda_j k \) is \( 4M_s \lambda_j k \). If the loss is \( P_{sat} \), the following relation holds:

\[
P_{sat} = 4M_s k \int_{0}^{\infty} \lambda \xi(\lambda) d\lambda
\]

Now \( k \) can be found through the normalization condition:

\[
\int \lambda \xi(\lambda) d\lambda = 1
\]  

(9.2)

For anisotropic materials this procedure has to be repeated in the orthogonal directions.

The total magnetization including reversible parts is expressed as:

\[
\mathbf{M} = c \mathbf{M}_{an}(H) + \int_{0}^{\infty} M_{an} \left( P_{nk} \left[ \mathbf{H} \right] \right) \xi(\lambda) d\lambda
\]

(9.3)

Thereby, the normalization condition is changed to:
Moreover, the density function \( \varsigma(\lambda) \) which gives the weight of the pseudo particles can e.g. be a Gaussian distribution:

\[
\varsigma(\lambda) = A e^{-(\lambda - q)^2} \tag{9.5}
\]

This expression contains three unknown parameters \( A, p \) and \( q \), governing the distribution. However, this can be reduced to one through the aforementioned normalization conditions.

Inserting equation (9.5) into the second normalization condition, equation (9.4) gives:

\[
c + \frac{A}{2p} \sqrt{\pi} (1 + \text{erf}(q)) = 1 \tag{9.6}
\]

Here, \( \text{erf}(q) \) is the so called error function, defined as:

\[
\text{erf}(q) \equiv \frac{2}{\sqrt{\pi}} \int_0^q e^{-y^2} dy \tag{9.7}
\]

Inserting equation (9.5) into the first normalization condition in equation (9.2) gives:

\[
\frac{A}{2p^2} (\sqrt{\pi} q (1 + \text{erf}(q)) + e^{-q^2}) = 1 \tag{9.8}
\]

This has yielded two equations for the four unknown parameters \( c, q, p \) and \( A \). Therefore, two additional parameters are needed. These can be found e.g. from measurements of a virgin curve. At small magnetization levels the virgin curve has a parabolic form, approximately (here we suppose that the material is isotropic, so the vector notation is changed to scalar):
\[ M(H) = \chi_0(0)H + \kappa_0(0)H^2 \quad (9.9) \]

Where \( \chi_0(0) \) and \( \kappa_0(0) \) are the Rayleigh parameters found from the measurements of the virgin curve. These parameters can be used to give the additional two equations using the following method:

In the initial condition \( H = 0 \), the magnetization history is wiped out and \( P_{\lambda k}[H] = 0 \) for all \( \lambda \). Then when increasing \( H \) monotonically the following play operator is obtained:

\[
P_{\lambda k}[H] = \begin{cases} 
H - \lambda k & \text{if } H > \lambda k \\
0 & \text{if } H < \lambda k
\end{cases} \quad (9.10)
\]

Inserting this into equation (9.3) the model gives a virgin curve as:

\[
M = cM_{\text{an}}(H) + \int_0^{H/k} M_{\text{an}}(H - \lambda k)\xi(\lambda)d\lambda \quad (9.11)
\]

Supposing that \( H \) is small and linearizing \( M_{\text{an}} \) around \( H = 0 \) yields:

\[
M \approx cM'_{\text{an}}(0)H + \int_0^{H/k} M'_{\text{an}}(H - \lambda k)\xi(0)d\lambda \quad (9.12)
\]

Which gives:

\[
M \approx c\chi_{\text{an}}(0)H + \frac{1}{2k}\xi(0)\kappa_{\text{an}}(0)H^2 \quad (9.13)
\]

Here

\[
\kappa_{\text{an}}(0) = M'_{\text{an}}(0) \quad (9.14)
\]
Using equation (9.13) with equation (9.4) yields:

\[ M \approx c \chi_{an}(0)H + \frac{A}{2k} e^{-q^2} \chi_{an}(0)H^2 \]  

(9.15)

To get correspondence between the model and experiments we have to choose:

\[ c = \frac{\chi_0}{\chi_{an}(0)} \]  

(9.16)

and:

\[ \kappa(0) = \frac{1}{2k} \zeta(0) \chi_0(0) = \frac{A}{2k} e^{-q^2} \chi_0(0) \]  

(9.17)

Notice that this method also requires that the anhysteretic curve is known. Now, we have four equations for the four parameters \( q, c, p \) and \( A \), i.e. equations (9.6), (9.8), (9.16) and (9.17). This equation system can not be solved analytically, however, it can be simplified, with the following procedure:

The parameter \( c \) is trivially given through equation (9.16). The parameter \( p \) is given from equation (9.6) as:

\[ p = \frac{A}{2} \sqrt{\pi} \left(1 + \text{erf} \left( q \right) \right) \]  

\[ \frac{1}{1 - c} \]  

(9.18)

Setting in this expression into equation (9.8) yields:

\[ 1 = \frac{2(1-c)^2}{A \pi (1 + \text{erf} \left( q \right))^2} \left( \sqrt{\pi q (1 + \text{erf} \left( q \right)) + e^{-q^2}} \right) \]  

(9.19)

Moreover, the parameter \( A \) is given from equation (9.17) as:
Finally, inserting this expression for $A$ and the expression for $c$ into equation (9.19) gives:

\[
1 = \frac{\left(1 - \frac{\chi_0(0)}{\chi_{an}(0)}\right)^2 \chi_0(0)e^{-q^2}}{k\kappa(0)\pi(1 + \text{erf}(q))^2 \left(\sqrt{\pi}q(1 + \text{erf}(q)) + e^{-q^2}\right)}
\]  

The only unknown parameter is now $q$. To find the value of $q$ the equation has to be solved numerically.
Appendix III

**Determination of the pseudo particle distribution function from measurements**

In the proposed modification to Bergqvist’s lag model with a modified play operator, it is possible to determine the pseudo particle distribution from measured major loop and anhysteretic curve. The governing equation is:

\[
M_{\pm} = \int_{0}^{\infty} M_{\text{an}} \left( H \mp k \right) \varsigma(\lambda) d\lambda \quad (10.1)
\]

Now write this integral as a discrete sum with a set of discrete, equidistance field values \( h_i \), spanning the whole range of the major loop:

\[
M_{\pm}(h_i) = \sum_{j} \Delta h M_{\text{an}}(h_i \mp h_j) \varsigma(h_j) \quad (10.2)
\]

Let us consider the positive branch of the major loop, \( M_+ \). Then equation (10.2) can be solved for the distribution \( \varsigma(h_j) \) by numerical inversion of the matrix:

\[
m_{ij} = \Delta h M_{\text{an}}(h_i - h_j) \varsigma(h_j) \quad (10.3)
\]
Appendix IV

Performance of dynamic measurement with controlled H-field

Hall probe

A Hall probe uses two electrodes with an electrical conducting, non-ferrous material in between. Between the electrodes a current, with current density $J$, is forced to pass. When inserting the probe into a magnetic field, the charge carriers experience a force according to Lorentz’s law:

$$F = \mu_0 \int \int \int J \times H dV$$

This force creates a displacement of the charge carriers in the orthogonal direction to $J$ and $H$, and thereby an electric field in that direction. Two electrodes are positioned in this direction and the electric field gives a voltage, $U$, between these electrodes. $F$, $E$ and $U$ are related through the following expressions:

$$F = QE$$  \hspace{1cm} (10.4)$$

$$U = \int E \cdot dl$$  \hspace{1cm} (10.5)$$

Here $l$ is the length between the electrodes which the voltage is measured over. Furthermore, $Q$ is the total charge of the charge carriers with total force $F$. If $J$ and $V$ are known, it is thereby possible to calculate $H$. This is done in a gaussmeter connected to the Hall probe.

One problem with this technique is that the charge carriers scatter into empty places in the atoms in the material. This will of course affect the induced voltage of the electrodes. Therefore, to achieve high accuracy, a long Hall probe should be used.
In the measurements with the SST it was not possible to use a long Hall probe due to the limits of the SST:s dimensions. Instead a thin Hall probe that measures the field perpendicular to the probe plane, i.e. transverse, was used. It is then important that the plane of the probe is perpendicular to the applied field. Therefore, a holder was used for the probe to align it in the correct direction.

The probe had a low signal to noise ratio and a sensitivity of 10 A/m. Therefore, the $B$ - and $H$ - values were measured for at least 200 periods for every measurement. Thereafter, the mean value for every $(H, B)$ – point of one magnetizing loop was calculated. These mean values were then used for plotting of the hysteresis loops.

**Sampling frequency**

When running dynamic measurements the ADC:s and DAC:s sampling frequencies become important, since a too low sampling frequency can introduce aliasing effects

The sampling frequency depends on the computer program that governs the output signals of the computer and reads the input signals. The highest sampling frequency that was achieved in the program was 59 kHz, which corresponds to a sampling interval of 17 $\mu$s. The sampling frequency is 29 times the measured frequencies so the Nyquist theorem (which states that the sampling frequency must be at least two times the measured frequency to be able to recreate the analog signal correctly [60]) is fulfilled.

**Compensation for time lag**

It was found that the output from the gaussmeter had a time delay. The reason for this is that the corrected analog output of the gaussmeter is re-created with a DAC from the sampled ADC data from the Hall probe. This raw data is scaled and corrected for several types of error such as frequency response and linearity for both the probe and gaussmeter. This processing takes time and then the data is sent to the DAC for the analog output to create the corrected waveform, which again takes time. Furthermore, the time delay increases with the frequency [82].
In order to compensate for this time delay, a setup was created for measurement of the time lag between the fluxmeter and gaussmeter. A coil was placed on the surface of a magnetic sheet in the middle of the SST and connected to the fluxmeter. Then, the Hall probe was placed as close as possible to the coil and connected to the gaussmeter. Thereby, both the fluxmeter and the gaussmeter measured the field in the air and the time lag was found by comparing the signals. The following time lags for different frequencies of the applied field were found:

- 1 Hz: no measurable time lag
- 50 Hz: 10 μs
- 500 Hz: 22 μs
- 1 kHz: 37 μs
- 2 kHz: 44 μs

Then, in order to achieve the correct values during B - H curve measurements, the time lags were subtracted from the signal from the gaussmeter. This yielded synchronized signals for both the flux and field.
References


References


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References


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[85] Electrical Steel Grain Oriented, Unisil, Unisil-H, ASTM Grades, Cogent Power Ltd

[86] J. Krah, private communication
## List of Symbols

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