<http://www.diva-portal.org>

Postprint

This is the accepted version of a paper published in *Composites Science And Technology*. This paper has been peer-reviewed but does not include the final publisher proof-corrections or journal pagination.

Citation for the original published paper (version of record):

Kazemahvazi, S., Zenkert, D. (2009)

Corrugated all-composite sandwich structures. Part 1: Modeling.

Composites Science And Technology, 69(7-8): 913-919

<http://dx.doi.org/10.1016/j.compscitech.2008.11.030>

Access to the published version may require subscription.

N.B. When citing this work, cite the original published paper.

Permanent link to this version:

<http://urn.kb.se/resolve?urn=urn:nbn:se:kth:diva-18451>

Corrugated all-composite sandwich structures. Part 1: Analytical model

Sohrab Kazemahvazi* and Dan Zenkert

Kungliga Tekniska Högskolan, Department of Aeronautical and Vehicle Engineering,
Stockholm, SE-100 44, Sweden.

Abstract

An analytical model for the compressive and shear response of monolithic and hierarchical corrugated composite cores has been developed. The stiffness model considers the contribution in stiffness from the bending- and the shear deformations of the core members in addition to the stretching deformation. The strength model is based on the normal stress and shear stress distribution over each core member when subjected to a shear or compressive load condition. The strength model also accounts for initial imperfections. In part 1 of this series, the analytical model is described and the results are compared to finite element predictions. In part 2, the analytical model is compared to experimental results and the behaviour of the corrugated structures is investigated more thoroughly using failure mechanism maps.

Keywords: Sandwich structures, Fibre composites, Corrugated cores, Analytical model

* Corresponding author, Tel.: +46 (0)708 64 66 65; fax: +46 (0)8 20 78 65

Email address: sohrabk@kth.se

1. Introduction

Sandwich structures with cellular cores have proven superior weight specific stiffness and strength properties compared to its monolithic counterpart. Polymer foams and hexagonal honeycomb structures are commonly used as core materials. During the past years, numerous core designs with improved quasi-static and dynamic performance have been proposed. These comprise aluminium foams [1, 2] and various metallic core topologies such as truss- and plate-like configurations [3, 4]. Numerous metallic truss configurations have been proposed with competitive quasi-static properties [5, 6, 7] and dynamic properties [8, 9]. The metallic plate configurations include various honeycomb cores (square, hexagonal, triangular) [10, 11] and prismatic cores (diamond lattice and corrugations) [12, 13].

Using fibre composite materials to manufacture the aforementioned core topologies increases the design space additionally and consequently further optimisation of the structure can be done; this due to the anisotropic nature of the composite materials. Further, the weight specific stiffness and strength of the structures can be improved due to the superior weight specific performance of the constitutive material.

The existing analytical models [6, 7, 13] for the stiffness and strength prediction of corrugated, diamond lattice and truss cores have been used in numerous studies and have proven suitable for metallic cores with low relative density. The stiffness models are based on the assumption that each core element is pin-jointed to the faces. Pin-jointed boundary conditions imply that the core elements only stretch and the contribution in stiffness from bending and shear deformation is thus neglected. The strength prediction models are based on the elastic and plastic buckling behaviour of each core member. Hence, the strength prediction models specify the critical load for a

perfectly straight core member i.e. geometrical imperfections are neglected which results in an over estimation of the strength of the structure.

Composite corrugation cores introduce other failure modes compared to that of metallic structures. As an example, material rupture occurs instead of plastic buckling when the core members are subjected to large bending deformations. In addition to the differences in failure modes, it is also necessary to know in which lamina failure occurs. In order to predict such failure mechanisms, knowledge of the stress distribution over the entire core members is necessary.

Within this work a new analytical stiffness and strength model for monolithic (figure 1a) and hierarchical (figure 1b) all-composite corrugations is proposed. The stiffness model considers the contribution in stiffness from the bending- and the shear deformations of the core members in addition to the stretching deformation. The strength model is based on the shear stress and normal stress distribution over each core member when the sandwich core is subjected to a shear or compressive load condition. The strength model also accounts for initial imperfections that may arise during the manufacturing process giving a more accurate strength prediction.

Figure 1: (a) All-composite monolithic corrugation and (b) hierarchical corrugation.

In part 1 of this series, the analytical model is described and the results are compared to finite element predictions. Further, the effect of initial imperfections on the strength and stiffness of the structure is discussed. In part 2, the analytical model is compared to experimental results and the behaviour of the corrugated structures is investigated more thoroughly using failure mechanism maps.

2. Analytical model

2.1 Linear analysis

Consider a unit cell of a corrugated core with geometrical quantities as specified in figure 2. Assume that each core member is built in to the face sheets so that the ends of the core members have clamped boundary conditions.

Figure 2: Unit cell of a corrugated core. The left core member shows a monolithic version and the right core member shows a sandwich version.

Applying a unit deflection, δ_x , gives rise to a shear angle γ_{xz} and a resultant force F_x , see figure 3b. Assuming that the distance between the face sheets, i.e. the core thickness t_c , is constant the shear angle γ_{xz} can be expressed as,

$$\gamma_{xz} = \frac{\delta_x}{t_c} = \frac{\delta_x}{l_1 \sin \omega} \quad (1)$$

The resulting forces and deflections acting on the compression loaded core member are shown in figure 3a and the geometrical relations between them are found in equation 2 through 9.

Figure 3: (a) Free body diagram of a compression loaded core member. (b) A unit cell subjected to a unit shear deformation.

$$T = F_x \sin \omega - F_z \cos \omega \quad (2) \quad \left| \quad \delta_b = \delta_x \sin \omega - \delta_z \cos \omega \quad (6) \right.$$

$$N = F_x \cos \omega + F_z \sin \omega \quad (3) \quad \left| \quad \delta_s = \delta_x \cos \omega + \delta_z \sin \omega \quad (7) \right.$$

$$F_x = N \cos \omega + T \sin \omega \quad (4) \quad \left| \quad \delta_x = \delta_b \sin \omega + \delta_s \cos \omega \quad (8) \right.$$

$$F_z = N \sin \omega + T \cos \omega \quad (5) \quad \left| \quad \delta_z = \delta_b \cos \omega + \delta_s \sin \omega \quad (9) \right.$$

The relations between the internal forces that acts on a core member (T and N) and the global displacements of the core member can be found using linear beam equations.

These are,

$$T = \frac{\delta_b}{\frac{l_1^3}{12D_{11}} + \frac{l_1}{S_1}}, \quad (10)$$

$$N = A_{11} \frac{\delta_s}{l_1}, \quad (11)$$

where A_{11} is the extensional stiffness component of the laminate, D_{11} is the bending stiffness of the laminate and S_1 the shear stiffness. For isotropic materials A_{11} reduces to Et , where E is the Young's modulus of the material and t is the thickness of the core member. In the case of shear loading the displacement $\delta_z = 0$ and hence $\delta_b = \delta_x \sin \omega$ and $\delta_s = \delta_x \cos \omega$. However, the out-of-plane force $F_z \neq 0$. The shear modulus of the core is now expressed as,

$$G_{xz} = \frac{\tau_{xz}}{\gamma_{xz}} = \frac{\sin \omega}{(l_1 \cos \omega + l_2)} \left[A_{11} \cos^2 \omega + \frac{\sin^2 \omega}{\frac{l_1^2}{12D_{11}} + \frac{1}{S_1}} \right] = \frac{\sin \omega}{(l_1 \cos \omega + l_2)} [S_{G_{xz}} + B_{G_{xz}}] \quad (12)$$

The equation for the shear modulus is the same as for the previous developed pure stretching model [13] with the addition of the bending and shear part, B_{G13} . Shear deformations are negligible for monolithic cores members but needs to be accounted for when using sandwich core members.

In the case of out-of-plane compressive loading $\delta_x = 0$ and hence $\delta_b = -\delta_z \cos \omega$, $\delta_s = \delta_z \sin \omega$ and $F_z \neq 0$. This yields an expression for the out-of-plane modulus as in equation 13.

$$E_{zz} = \frac{\sigma_{zz}}{\varepsilon_{zz}} = \frac{\sin \omega}{(l_1 \cos \omega + l_2)} \left[A_{11} \sin^2 \omega + \frac{\cos^2 \omega}{\frac{l_1^2}{12D_{11}} + \frac{1}{S_1}} \right] = \frac{\sin \omega}{(l_1 \cos \omega + l_2)} [S_{Ezz} + B_{Ezz}] \quad (13)$$

Where S_{E33} is the stretching deformation term and B_{E33} the bending and shear deformation term.

Figure 4: Shear modulus as function of corrugation angle. (a) Monolithic core members and (b) sandwich core members.

The shear modulus of a monolithic corrugated core is presented in figure 4a as function of corrugation angle, ω , for several thickness to length ratios, t/l_1 . As stated in previous work [13], the contribution in stiffness from bending is very small for small values of t/l_1 . However, the pure stretching model underestimates the core stiffness as the bending stiffness of the core members increases, especially for large corrugation angles. The shear modulus of a corrugated core with sandwich core members is presented in figure 4b. The solid line shows the predicted modulus when shear

deformations are neglected. Neglecting the shear deformations, the shear modulus of the structure can be highly overestimated for corrugation angles larger than 45 degrees.

2.2 Non-linear analysis

In order to develop a strength model the shear and normal stresses acting on the core members have to be derived. The core members undergo bending and shear deformations simultaneously as they are loaded in compression. This typically results in a non-linear global force-deflection response. In order to predict both the elastic instability and the material rupture failure modes the previous elastic model has to be expanded so that it considers important secondary effects.

2.2.1 Sandwich core members – hierarchical structure

Consider the compression loaded part of a unit cell as described in figure 3a. Due to the presence of bending and shear deformations, the normal force N causes additional curvature of the core member as sketched in figure 5.

Figure 5: Beam problem that corresponds to a unit deformation of a core member.

The differential equation that describes the bending and shear deformation of a compression loaded core member is,

$$\frac{d^2 u_3(x_1)}{dx_1^2} \left(1 - \frac{N}{S_1} \right) = - (T_1 x_1 + N u_3 + M_1) \frac{1}{D_{11}} \quad (14)$$

with a general solution,

$$u_3(x_1) = A \sin(kx_1) + B \cos(kx_1) - \frac{1}{N_1}(T_1 x_1 + M_1), \quad (15)$$

where A and B are constants and $k = \sqrt{\frac{N_1}{D_{11}\left(1 - \frac{N_1}{S_1}\right)}}$. The boundary conditions are

$$u_3(0) = 0 \quad (16) \quad \left| \quad u_3(l_1) = \delta_b \quad (18)$$

$$\frac{du_3(0)}{dx_1} = \frac{T_1}{S_1 - N_1} \quad (17) \quad \left| \quad \frac{du_3(l_1)}{dx_1} = \frac{T_1}{S_1 - N_1} \quad (19)$$

Equation 15 together with the boundary conditions (16-19) now gives us a closed form expression for the deformation of a core member as function of applied global load (F_z or F_x). The expression for $u_3(x_1)$ has been excluded from the paper due to its length but can be determined easily using mathematical programs such as Maple.

The global force-deflection response of the structure can now be derived using equation 8 or equation 9. Thus, for a unit cell subjected to a shear load, the global force-deflection response is given by equation 8 where $\delta_b = u_3(l_1)$ and

$$\delta_s = \frac{N_1 l_1}{A_{11}} + \int_0^{l_1} \frac{1}{2} \left(\frac{du_3}{dx_1} \right)^2 dx_1. \text{ The second term in } \delta_s \text{ is the additional stretching caused by}$$

the bending deformation.

The bending moments and transverse forces that acts on a core member are,

$$M(x_1) = D_{11} \left(\left(1 - \frac{N}{S_1} \right) \frac{d^2 u_3}{dx_1^2} \right), \quad (20)$$

$$T(x_1) = \frac{dM}{dx_1}, \quad (21)$$

and the compressive normal stresses and shear stresses that acts on the face and the core of a sandwich core member are,

$$\sigma_f(x_1) = \frac{M(t_{cweb} + t_f)}{2D_{11}} + \frac{N}{2t_f}, \quad (22)$$

$$\tau_c(x_1) = \frac{T}{(t_{cweb} + t_f)} \quad (23)$$

2.2.2 Monolithic core members

The deformation behaviour of a monolithic core member is the same as for the sandwich core member with the exception of shear deformations. Assuming that the monolithic core members are relatively slender the shear deformations can be neglected.

The solution to equation 14 is then,

$$u_3(x_1) = A \sin(kx_1) + B \cos(kx_1) - \frac{1}{N_1}(T_1 x_1 + M_1), \quad (24)$$

where A and B are constants and $k = \sqrt{\frac{N_1}{D_{11}}}$. And the boundary conditions are,

$$u_3(0) = 0 \quad (25) \quad \left| \quad u_3(l_1) = \delta_b \quad (27)$$

$$\frac{du_3(0)}{dx_1} = 0 \quad (26) \quad \left| \quad \frac{du_3(l_1)}{dx_1} = 0 \quad (28)$$

The compressive normal stresses acting on the core member is,

$$\sigma_f(x_1) = \frac{Mt}{2D_{11}} + \frac{N_1}{t} \quad (29)$$

2.2.3 Strength model

A strength model based on five competing failure modes is now employed. The different failure modes are shown schematically in figure 6 and the failure criteria are described in subsequent sections.

Figure 6: Failure modes of composite corrugation with sandwich core members. (a) Face fracture, (b) core shear failure, (c) general buckling, (d) local buckling/wrinkling and (e) shear buckling.

2.2.3.1 Face fracture / micro buckling

In this model, face fracture (figure 6a) is assumed to occur when the compressive stress in a core member face reaches the ultimate compressive strength of the material, $\sigma_f^{\max} \geq \hat{\sigma}_f$. However, one can use lamina based Tsai-Hill or Tsai-Wu failure criteria instead.

2.2.3.2 Core shear failure

Shear failure of the core elements (figure 6b) is assumed to occur when the shear stress in the core member core reaches the ultimate shear strength of the core material,

$$\tau_c^{\max} \geq \hat{\tau}_c.$$

2.2.3.3 Face wrinkling/local buckling

Hoff's method [REF Zenkert] has been used as a failure criterion for local buckling, see figure 6d. Local buckling is assumed to occur when the compressive stress in a core member face reaches the critical local buckling stress as specified in equation 6.

$$\sigma_f^{\max} \geq \sigma_{cr}^{Localbuckling} = 0.5\sqrt[3]{E_f E_c G_c} \quad (30)$$

where E_c is the modulus and G_c the shear modulus of the core material and E_f is the modulus of the face material.

2.2.3.4 General Buckling and shear buckling

The non-linear model presented in section 2.2.1 captures general buckling instability modes. The instability “failure” is assumed to occur when the compressive normal load on the core member is within 2 percent of the theoretical bifurcation load. For stocky core members this bifurcation load is approximately the same as the shear stiffness of the core member, [REF Zenkert]. Hence, when the bifurcation load is within 2 percent of the shear stiffness, S_1 , we refer to the shear buckling failure mode and otherwise to the general buckling failure mode.

3. Finite element model

A numerical study has been performed in order to validate the kinematics of the core members as well as the stress predictions made by the analytical model. A non-linear geometry model of a single core member was made in the commercial finite element code ABAQUS. Each core member was modelled using 4-node shear deformable shell elements with reduced integration (S4R in the ABAQUS notation). All out-of-plane rotations and translations were constrained in order to obtain plain strain conditions.

The ends of the core members were tied to analytical rigids. All degrees of freedom of the analytical rigid at the bottom of the core member were fully constrained. At the top of the beam, translations in the z-direction and rotations around the y-axis were constrained for the case of shear loading. For the case of compressive loading translations in the x-direction and rotations around the y-axis were constrained.

4. Comparison between numerical and analytical predictions

Figure 7-9 shows the global load-deflection predictions (a) and the stress predictions (b) for three different core member configurations. Predictions for a typical sandwich core member, with high bending stiffness and low shear stiffness, is shown in figure 7. Figure 8 shows predictions for a sandwich core member with slightly lower bending stiffness and figure 9 shows predictions for a monolithic core member with low bending stiffness and high shear stiffness.

Figure 7: (a) Load-deflection predictions of a shear loaded corrugated core. (b) Compressive normal stress and shear stress acting on the core member. The core configuration has a high bending stiffness, D_{11} , and a low shear stiffness, S_1 .

The stress predictions are normalised with values for the shear and compressive strength of the constitutive material, $\hat{\sigma}_f$ and $\hat{\tau}_c$. All configurations have the same material properties while the core member core thickness is altered ($t_{cweb} = 5$ mm, 2.5 mm and 0 mm). Very good correlation is found between the analytical model and the finite element model.

Figure 8: (a) Load-deflection predictions of a shear loaded corrugated core. (b) Compressive normal stress and shear stress acting on the core member. The core configuration has an intermediate bending stiffness, D_{11} , and an intermediate shear stiffness, S_1 .

The two sandwich core member configurations fail through shear buckling giving very high shear stresses in the core member core. The monolithic core member, on contrary, is subjected to a low shear stress and a high compressive normal stress. Further, the monolithic core deforms in a mode two buckling shape giving the maximum

compressive normal stress at $x_1 = l_1 / 4$. The global load, F_x , asymptotically approaches the corresponding mode 2 critical buckling load for a clamped strut.

Figure 9: (a) Load-deflection predictions of a shear loaded corrugated core. (b) Compressive normal stress and shear stress acting on the core member. The core configuration has a low bending stiffness, D_{11} , and a high shear stiffness, S_1 .

4. Effect of initial imperfection

Most manufacturing routes give rise to some imperfections of the final structure. Models that do not account for structural imperfections tend to overestimate the peak strength of the structure. Initial imperfection can be included in the non-linear model by simply adding an initial deformed shape to the differential equation 14. Here, we have assumed the imperfection shape of the first buckling mode of a clamped strut. The initial imperfection, a , is described according to equation 31 where a_0 is a dimensionless imperfection parameter.

$$a = \frac{t_f a_0}{2} \left(1 - \cos \left(\frac{2\pi x_1}{l_1} \right) \right) \quad (31)$$

Equation 14 is now rewritten as,

$$\frac{d^2 u_3(x_1)}{dx_1^2} \left(1 - \frac{N}{S_1}\right) = - \left(\frac{T_1 x_1}{D_{11}} + \frac{N u_3}{D_{11}} + \frac{M_1}{D_{11}} + N a \right) + \frac{N}{S_1} \frac{d^2 a}{dx_1^2} \quad (32)$$

and is solved using the boundary conditions 25-28 .

The bending moments and transverse forces that acts on a core member are now,

$$M(x_1) = -D_{11} \left(\frac{N}{S_1} \frac{d^2 a}{dx_1^2} - \left(1 - \frac{N}{S_1}\right) \frac{d^2 u_3}{dx_1^2} \right), \quad (33)$$

$$T(x_1) = \frac{dM}{dx_1}. \quad (34)$$

Recall that monolithic corrugations without structural imperfections deformed in the same shape as the mode 2 buckling of a clamped strut due to the bending moments at the boundaries. Further, the critical load asymptotically reached the theoretical Euler mode 2 buckling load of a clamped strut. Introducing structural imperfections, however, changes the mode of deformation and consequently changes the critical load. Figure 10a and figure 10b shows the predicted deformation shape of a compression loaded monolithic core member in a corrugated core with $\omega = 70$ degree and $t/l_1 = 0.01$. For small initial imperfections (10a) the deformation shape is reminiscent of a mode 2 buckling. As the amplitude of the imperfection increases the deformation shape transforms into a mode 1 buckling shape (10b). The cause of the transition in deformation shape can be described as follows. Each core member is subjected to a transverse force and a normal force; the transverse force give rise to a bending moment couple at the boundaries that forces the core member into a mode 2 buckling shape. On the other hand, the imperfection gives rise to a bending moment that forces the core member into a mode 1 buckling shape. The division between the amplitude of these two bending moment couples decides the shape of the core member deformation.

Figure 10: (a) Deformation mode prediction for FE- and analytical model for $a_0=0.001$. (b) Deformation mode prediction for FE- and analytical model for $a_0=0.1$. (c) Applied force versus moment plot for the two initial imperfections ($\omega = 70$ degree). Dashed line in (c) shows critical Euler buckling load of a clamped strut (mode 1).

Figure 10c shows the bending moment at $x = l_1/4$ as function of applied force for a corrugation with a small imperfection and one with a larger imperfection. Both corrugation types asymptotically reach the critical buckling load of a clamped strut (mode 1). However, the corrugation with larger imperfections is induced to substantially larger bending moments and will thus fail at a lower load.

Figure 11 shows the effect of initial imperfections on a corrugated core with sandwich core members. As seen in figure 11b, an increase of the imperfection amplitude results in a substantially larger shear stress in the core member core. The structure will thus fail at a lower global load.

Figure 11: (a) Force deflection response of a corrugated core with sandwich core members for two different initial imperfections. (b) Stress predictions at two different initial imperfections.

6. Concluding remarks

An analytical stiffness and strength model for corrugated composite sandwich cores has been developed. The stiffness model includes bending- and shear deformations in addition to stretching deformations. Including bending and shear deformations yields a more accurate stiffness prediction for corrugations with sandwich core members (hierarchical structures).

A strength model has been developed based on the stress distribution over the core members when the core is subjected to a shear or compressive load condition. Five different failure criteria have been employed to predict the core strength. The strength model also accounts for initial imperfections of the core members giving a more accurate strength and stiffness prediction. The following observations have been made.

- (i) The amplitude of an initial imperfection affects the deformation shape of a core member. The deformation shape may alter from a mode 2 buckling shape to a mode 1 buckling shape as the amplitude of the imperfection increases.
- (ii) The amplitude of the initial imperfections affects the strength and stiffness of the structure. For imperfection amplitudes that are similar to those obtained during the manufacturing process, the strength of the core can typically decrease by 20 percent compared to a perfectly straight core member.
- (iii) Results from the proposed analytical model are in excellent agreement with the finite element predictions for the corrugation configurations examined within this study.

Acknowledgements

The financial support for this investigation has been provided by The Office of Naval Research (ONR) through programme officer Dr. Yapa D.S. Rajapakse (Grant No. N00014-07-1-0344).

References

- [1] Radford DD, McShane GJ, Deshpande VS, Fleck NA. The response of clamped sandwich plates with metallic foam cores to simulated blast loading. *Int J Solid Struct* 2006;43,2243-2259.
- [2] Tagarielli VL, Fleck NA, Deshpande VS. *Advanced engineering materials* 2004;6,440-443.
- [3] Wadley HNG. Multifunctional periodic cellular metals. *Philos T R Soc A* 2006;364:31-68.
- [4] Kooistra GW, Wadley HNG. Lattice truss structures from expanded metal sheet. *Mater Design* 2007;28:507-514.
- [5] Wadley HNG, Fleck NA, Evans AG. Fabrication and structural performance of periodic cellular metal sandwich structures. *Compos Sci Technol* 2003;63:2331-2343.
- [6] Kooistra GW, Deshpande VS, Wadley HNG. Compressive behavior of age hardenable tetrahedral lattice truss structures made from aluminium. *Acta Mater* 2004;52:4229-4237.
- [7] Deshpande VS, Fleck NA. Collapse of truss core sandwich beams in 3-point bending. *Int J Solid Struct* 2001;38,6275-6305.
- [8] McShane GJ, Radford DD, Deshpande VS, Fleck NA. The response of clamped sandwich plates with lattice cores subjected to shock loading. *European Journal of Mechanics - A/Solids* 2006; 25, 2, 215-229
- [9] Lee S, Barthelat F, Hutchinson JW, Espinosa HD. Dynamic failure of metallic pyramidal truss core materials, Experiments and modeling. *International Journal of Plasticity* 2006; 22, 11, 2118-2145

- [10] Côté F, Deshpande VS, Fleck NA, AG Evans. The out-of-plane compressive behaviour of metallic honeycombs. *Material Science and Engineering A* 2004; 380, 272-280.
- [11] Rathbun HJ, Radford DD, Xue Z, He MY, Yang J, Deshpande VS, Fleck NA, Hutchinson JW, Zok FW, Evans AG. Performance of metallic honeycomb-core sandwich beams under shock loading. *Int J Solids and Struct* 2006;43,6,1746-1763.
- [12] Tilbrook MT, Radford DD, Deshpande VS, Fleck NA. Dynamic crushing of sandwich panels with prismatic lattice cores. *Int J Solids and Struct* 2007;44,18 6101-6123.
- [13] Côté F, Deshpande VS, Fleck NA, AG Evans. The compressive and shear responses of corrugated and diamond lattice materials. *Int J Solid Struct* 2006;43,6220-6242.