Numerical study of boundary-layer receptivity on a swept wing

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Technical report

Direct numerical simulations (DNS) of the flow over a wing with 45° sweep and -4° angle-of-attack are presented. This flow configuration was investigated in a series of wind-tunnel experiments at the Arizona State University (ASU). On the upper wing side, the flow develops a substantial crossflow and is therefore ideally suited for a study of the receptivity mechanisms of crossflow vortices. Here, we examine the boundary-layer receptivity to surface roughness and to single vortical free-stream modes. The roughness is modeled by a shallow circular disk and is identical with one single element of the spanwise roughness array considered in the ASU experiments. The boundary layer develops a steady crossflow mode downstream of the roughness. The spatial evolution of the modal amplitude obtained by the DNS is in excellent agreement with a solution to the nonlinear parabolized stability equations (NPSE) while being lower than that measured in the experiments. The reasons for this discrepancy are yet to be determined. Possible explanations are the idealization of the roughness array by spanwise periodic boundary conditions in our simulations, or the presence of traveling crossflow waves due to background free-stream turbulence in the experiments. We demonstrate that the boundary-layer receptivity to roughness can be successfully predicted by a nonlocal, adjoint-based receptivity model. Stationary crossflow vortices can also be triggered by zero-frequency free-stream vortical modes. We consider two types of mode, carrying streamwise and chordwise vorticity. Both modes give rise to nonmodal disturbances near the leading edge, which soon evolve into a steady crossflow mode. The boundary layer is found to be somewhat more receptive to the streamwise-vorticity mode than to the chordwise vorticity.

1. Introduction

Long-range airliners are usually equipped with swept wings in order to avoid a drag enhancement due to shock waves at the wing leading edge. Swept-wing

flows are characterized by a misalignment between the streamlines and the direction of the pressure gradient. This gives rise to S-shaped streamlines and a secondary flow in the cross-stream direction, the crossflow. At large enough Reynolds numbers, the crossflow profiles are subject to inviscid inflectional instabilities, which become manifest in the form of crossflow vortices.

Crossflow instability is also observed in boundary layers on swept plates and cylinders, wedges, cones or rotating disks (Saric *et al.* 2003). These flows can therefore serve as model problems for flows over swept wings. The receptivity and instability of swept-plate flows were studied by Crouch (1993, 1994), Choudhari (1994) (finite Reynolds-number theory, FRNT); Högberg & Henningson (1998), Wassermann & Kloker (2002, 2003), Schrader *et al.* (2009) (DNS) and Tempelmann *et al.* (2010) (PSE-based optimization). The swept-plate configuration excludes the flow field around the leading edge. In contrast, the DNS studies of swept-cylinder flow by Collis & Lele (1999) (parabolic cylinder) and Piot *et al.* (2008) and Piot & Casalis (2009) (circular cylinder) also took the leading-edge receptivity into account.

However, the ultimate goal is to perform simulations of boundary layers on swept wings. Until recently, DNS of such flows were out of reach owing to the enormous computational cost. Therefore, Haynes & Reed (2000) used the more efficient methods of linearized and nonlinear parabolized stability equations (LPSE and NPSE) in order to address the crossflow instability of swept-wing boundary layers. Haynes & Reed (2000) considered the upper side of a 45° swept wing at an angle-of-attack of -4° , using the NLF(2)-0415 airfoil by Somers & Horstmann (1985). This configuration had been extensively studied in a number of wind-tunnel experiments at the ASU (Dagenhart 1992; Reibert *et al.* 1996; Saric *et al.* 1998*a,b*; Radeztsky *et al.* 1999; White *et al.* 2001). On the upper wing side – upstream of the pressure minimum – the flow is unstable to crossflow vortices while being stable to Tollmien-Schlichting (T-S) modes. This configuration is therefore ideally suited for an investigation of the receptivity mechanisms of crossflow instability. Recently, Nishino & Shariff (2009) presented a first attempt to simulate by DNS the amplitudes of roughness-induced crossflow vortices, using a compact finite difference scheme. The roughness array of the ASU experiments was idealized by inhomogeneous boundary conditions for the disturbance field. Nishino & Shariff (2009) considered cube-shaped functions for their roughness model instead of the circular disks of the experiments. The amplitudes of the excited crossflow vortices were more than one order-of-magnitude lower than those observed in the experiments. Nishino & Shariff (2009) conjectured that this disagreement was due either to their treatment of the roughness or to background noise in the experiments (free-stream turbulence), which was not captured by their simulations.

Here, we report DNS results of crossflow receptivity and instability of the ASU swept-wing boundary layer. The triggers of instability considered are a shallow circular cylinder – in agreement with the roughness elements of the

FIGURE 1. A 45° swept wing at an angle-of-attack of -4° . (a) Dimensions of the computational domain and flow parameters considered. Lengths are normalized by the nose radius r_n , marked by the red circle. (b) The NLF(2)-0415 airfoil at -4° angle-of-attack.

ASU experiments – and free-stream vortical modes with different vorticity components. We used the spectral element method, which allows for accurate simulations of complex flow geometries.

2. Flow configuration

We studied the flow over a swept wing with a natural laminar-flow airfoil (NLF(2)-0415, Somers & Horstmann 1985). The flow configuration was identical to that of the ASU wind-tunnel experiments. The flow parameters are given in figure 1. The sweep angle was $\phi_0 = 45^\circ$; in addition, the wing was turned by an angle-of-attack of $\alpha_a = -4^\circ$. This produces a long region of chordwise flow acceleration due to a negative pressure gradient on the upper wing side. Such conditions are favorable in order to study crossflow instability. Lengths were normalized by the nose radius r_n of the wing and velocities by U_∞ , the chordwise component of the free stream Q_∞ . The flow conditions are defined by the Reynolds number

$$Re_{r_n} = \frac{U_\infty r_n}{\nu}, \quad (1)$$

where ν is the kinematic viscosity of the fluid. Flows around wings are also often characterized by a chord-based Reynolds number. The streamwise chord Reynolds number of the ASU experiments was $Re_C = Q_\infty C / \nu = 2.4 \times 10^6$, where C is the long (swept) chord of the wing. Instead of the streamwise and cross-stream coordinates adopted in the experiments, we used chordwise and spanwise coordinates (denoted by x and z), which are perpendicular and parallel to the wing leading edge, respectively (figure 1a). The chord Reynolds number in our coordinates is $Re_c = U_\infty c / \nu = 1.2 \times 10^6$, where $c = C \cos 45^\circ$ is the short (unswept) chord. The Reynolds number based on the nose radius is then $Re_{r_n} = 10010.9$.

FIGURE 2. Computational grids for the simulations with (a) free-stream vortical disturbances ('FSD mesh') and (c) surface roughness ('SR mesh'). (b) and (d) are close-ups of the finest mesh regions of (a) and (c), respectively, and also show the GLL nodes of the velocity grids.

3. Numerical method

The present results were obtained using the spectral element method (SEM). The SEM, introduced by Patera (1984), provides spectral accuracy and flexibility in space and hence is ideally suited for accurate simulations of flows around wings. The simulation code used is described in Fischer *et al.* (2008). The physical domain is decomposed into spectral elements. Figure 2 shows the spectral-element decomposition used for the simulations with free-stream disturbances ('FSD mesh', figure 2a) and with surface roughness ('SR mesh', figure 2c). The number of elements is 12009 for the FSD mesh and 18000 for the SR mesh. The elements are in turn subdivided into arrays of Gauss-Lobatto-Legendre (GLL) nodes for the velocity field and Gauss-Lobatto (GL) nodes for the pressure field, respectively. The solution to the Navier-Stokes equations is approximated element-wise as a sum of Legendre polynomials up to degree N , forming an orthogonal basis. Here, we chose $N = 11$ for the velocity (GLL) grids and $N = 9$ for the pressure (GL) grids. This setup – referred to as $\mathbb{P}_N$ - $\mathbb{P}_{N-2}$ discretization (Maday & Patera 1989) – is beneficial, because the

staggering of the pressure grid renders the specification of pressure boundary conditions unnecessary. The GLL nodes of the FSD mesh and the SR mesh are shown in figures 2(b) and 2(d), respectively. The present SEM code was optimized for the MPI based usage on supercomputers with thousands of processors (Tufo & Fischer 2001). Here, we performed parallel computations on either 512 or 1024 processors.

3.1. Base flow

The base flow was computed by prescribing the usual zero-slip and zero-stress conditions at the wall and at the outflow boundaries, respectively. A spanwise invariant flow was obtained by using cyclic conditions at the lateral boundaries. At the inflow and free-stream boundaries, Dirichlet conditions were prescribed. These were extracted from a solution to the RANS equations obtained by a preliminary simulation of the entire wind-tunnel test section. Note that the free-stream boundaries of the FSD mesh are streamline shaped (cf. figure 2a), which is beneficial if vortical perturbations travel through the free stream. Although we are only interested in the flow field on the upper wing side, we retained parts of the flow over the lower side in order to account for the asymmetry of the flow configuration. It turned out that the flow separated from the lower wing surface slightly downstream of the leading edge. This separation was also observed by Nishino & Shariff (2010). The vortex shedding caused by the separation led to a local backflow at the lower outlet, destabilizing the simulations. Numerical tests revealed that the separation bubble did not affect the flow field on the upper side of the wing. We therefore eliminated the separation by using a sponge region, in which the flow was forced towards the time average of the separation bubble. This was accomplished by adding the forcing term

$$\mathbf{F}(\mathbf{x}, t) = A_{\max} \lambda_f(\mathbf{x}) [\mathbf{U}_f(\mathbf{x}) - \mathbf{U}(\mathbf{x}, t)] \quad (2)$$

on the right-hand side of the momentum equations. The field $\mathbf{U}_f$ denotes the enforced flow (containing the time averaged separation bubble), and $\mathbf{U}$ stands for the instantaneous separated flow field. The quantity λ_f takes the form of a step function (see Schrader *et al.* 2009) and varies smoothly between zero and one in the separated region while vanishing everywhere else. The coefficient $A_{\max}$ determines the maximal strength of the forcing and was set to $A_{\max} = 2.5$ here. Since the sponge forcing is proportional to the difference between the target and the actual flow, $\mathbf{F}$ was significant only in the separated boundary layer while decaying to zero in the free stream. We ensured that the insertion of the sponge region did not have measurable effects on the flow over the upper wing side.

The SR mesh was substantially truncated in comparison to the FSD mesh (figure 2c). The flow field in the stagnation region – including the leading edge – and over the lower wing side was removed. The inflow plane was placed about 10 roughness diameters upstream of the roughness element (see §3.2.1). Moreover, the free-stream boundary was shifted closer to the wing surface. The

FIGURE 3. Cylindrical roughness element near the leading edge of the swept wing. The roughness parameters are given in table 1. The bump height is scaled up by 500 in the figure.

Dirichlet conditions required at the inflow and free-stream boundaries were extracted from the baseflow obtained with the FSD mesh.

3.2. Perturbed flow

3.2.1. Roughness element

A single roughness element of the form of a shallow cylindrical disk was inserted near the leading edge of the swept wing (figure 3). The roughness was designed to match one of the circular cylinders of the spanwise roughness array used in the ASU experiments (Reibert *et al.* 1996). The circular disk was meshed in order to obtain a realistic representation of the roughness; however, we mimicked the roughness row by prescribing spanwise periodic boundary conditions. The diameter, height, spanwise spacing and chordwise placement of the roughness disk (normalized by the wing-nose radius) are listed in table 1, where the corresponding dimensional values of the experiments are also given.

TABLE 1. Dimensions and location of the circular roughness cylinder and spanwise spacing between adjacent elements of the roughness array. The values are given in the non-dimensional units of the present simulations and in terms of dimensional quantities pertaining to the experiments by Reibert *et al.* (1996).

	Height	Diameter	Spacing, wavenumber	Chordwise location
	ε_r	d_r	λ_z, β_r	$x_r, x_r/c$
Simulations	5.56×10^{-4}	0.34	1.11, 5.65	2.76, 0.023
Experiments	6×10^{-3} mm	3.7 mm	12 mm, 0.52 1/mm	29.76 mm, 0.023

3.2.2. Free-stream vorticity

In order to investigate the receptivity of swept-wing boundary layers to vortical free-stream disturbances, we specified at the inflow plane (subscript ‘in’) vortices modeled by Fourier modes with spatial periodicity,

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix}_{\text{in}} = \begin{pmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{pmatrix} e^{i(\alpha x + \gamma y + \beta z)}. \quad (3)$$

The chordwise, vertical and spanwise coordinates are denoted by x , y and z ; α , γ and β are the corresponding wavenumbers and u , v and w denote the chordwise, vertical and spanwise disturbance velocities, respectively. The amplitude functions $\hat{u}$, $\hat{v}$ and $\hat{w}$ were computed in a similar fashion as in Schrader *et al.* (2010). A streamwise-vorticity mode, for instance, is obtained from the definition of streamwise vorticity and the continuity equation, assuming a vanishing streamwise disturbance velocity. This assumption relies on an independent evolution of the streamwise components of vorticity and velocity, which only holds for a vortical mode aligned with a uniform free stream. Under this condition, the vortices are subject to decay only, but not to inviscid shearing, when traveling downstream. For swept free streams, the amplitude functions given by Schrader *et al.* (2010) must therefore be applied in a coordinate system aligned with the free stream. Denoting the streamwise, vertical and cross-stream coordinates by x_q , y and z_c , we write

$$\begin{pmatrix} q \\ v \\ c \end{pmatrix}_{\text{in}} = \begin{pmatrix} \hat{q} \\ \hat{v} \\ \hat{c} \end{pmatrix} e^{i(\alpha_q x_q + \gamma y + \beta_c z_c)}. \quad (4)$$

where α_q and β_c are the wavenumbers in the streamwise and crosswise directions, and q and c are the streamwise and cross-stream disturbance velocities, respectively. A mode with purely streamwise vorticity is obtained by setting α_q to zero (Schrader *et al.* 2010). In practice, we put the angular frequency ω to zero, invoking Taylor’s hypothesis. Using the amplitude functions of Schrader *et al.* (2010) for streamwise vorticity, we write $\hat{q}$, $\hat{v}$ and $\hat{c}$ as

$$\begin{pmatrix} \hat{q} \\ \hat{v} \\ \hat{c} \end{pmatrix} = \frac{i}{\sqrt{\gamma^2 + \beta_c^2}} \begin{pmatrix} 0 \\ \beta_c \\ -\gamma \end{pmatrix}. \quad (5)$$

The chordwise, vertical and spanwise disturbance velocities required in (3) are obtained by projecting the amplitude function $(\hat{q}, \hat{v}, \hat{c})$ onto the unswept coordinates,

$$\begin{pmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{pmatrix} = \begin{pmatrix} \cos \phi_0 & 0 & -\sin \phi_0 \\ 0 & 1 & 0 \\ \sin \phi_0 & 0 & \cos \phi_0 \end{pmatrix} \begin{pmatrix} \hat{q} \\ \hat{v} \\ \hat{c} \end{pmatrix}. \quad (6)$$

FIGURE 4. Swept-wing flow with 45-degree sweep angle. (a) Pressure coefficient versus normalized chordwise coordinate. (b) Comparison between the tangential velocity profiles from the DNS (solid lines) and those obtained by a boundary-layer solver (dashed lines).

Finally, we take the real part of (3), normalize the mode such that $\max_{(y,z)_{in}}[(u^2 + v^2 + w^2)/2] = 1$, and – after multiplying by a small amplitude ε_v – superimpose the steady streamwise-vorticity mode onto the mean inflow.

If the amplitude functions in (5) are replaced by those of the ‘ η -modes’ given in Schrader *et al.* (2010) and $\gamma = 0$ is chosen, a mode with vertical vorticity is obtained. Considering the ‘ ζ -modes’ of Schrader *et al.* (2010) and setting $\beta_c = 0$ yields a crosswise-vorticity mode. The streamwise, vertical and crosswise vortices can be combined in order to produce a free-stream mode with an arbitrary vorticity vector. In particular, the combination of streamwise and crosswise vortices with the same vorticity magnitude gives – for a sweep angle of 45 degrees – a disturbance with chordwise vorticity alone. Here, we shall consider two different incoming disturbances, namely steady streamwise and chordwise free-stream vorticity.

4. Results

Before presenting the results, we introduce the coordinates s and n in the tangential and normal directions with respect to the wing surface. Most velocity profiles reported here pertain to these coordinates, as marked by an underline. When writing $\underline{U}$ and $\underline{V}$, for instance, we mean the velocity components in the body-fixed tangential and normal directions, respectively.

4.1. Validation of the baseflow

Figure 4(a) shows the chordwise pressure distribution along the upper wing surface. Most of the flow field (up to $x/c \approx 0.7$) is subject to a favorable pressure gradient, causing a substantial chordwise flow acceleration. Under these conditions, the flow is unstable to crossflow modes, whereas the negative

FIGURE 5. Swept-wing flow with 45-degree sweep angle. (a) Angle of the external streamlines. (b) Streamwise (Q) and crossflow (C) profiles at various chordwise locations. Note that the crossflow profiles are scaled up by five. (c) Boundary-layer displacement thicknesses based on the streamwise, chordwise and spanwise velocities, respectively.

pressure gradient has a stabilizing effect on the T-S instability mechanism. Figure 4(a) shows an excellent agreement with the pressure coefficient obtained by Haynes & Reed (2000) for the same flow configuration. This verifies the Dirichlet conditions used at the inflow and free-stream boundaries in order to compute the baseflow (cf. §3.1). Further validation is given in figure 4(b), where the present tangential velocity profiles are compared with those obtained by a boundary-layer solver based on a streamwise marching procedure. The edge conditions required for the boundary-layer solution were given in terms of the pressure distribution of Haynes & Reed (2000). Good agreement of the flow profiles is seen at all chordwise locations in figure 4(b).

FIGURE 6. Boundary-layer disturbance structure (tangential disturbance velocity) excited by a circular roughness disk (see table 1). (a) Near field and (b) far field (not to scale).

4.2. Mean-flow results

In figure 5(a), the angle of the external streamlines is depicted. The outer flow is incident at an angle of 45 degrees, subsequently turns into the spanwise direction due to the chordwise stagnation at the leading edge and finally turns back into the chordwise direction owing to the decreasing pressure. If the boundary-layer profiles are projected onto the direction of the external streamlines, the velocity profiles shown in figure 5(b) are obtained. This highlights the presence of a secondary flow in cross-stream direction inside the boundary layer, the crossflow. The crossflow profiles exhibit an inflection point between their maximum and the boundary-layer edge and are therefore potentially unstable to crossflow modes. Figure 5(c) indicates the thickness of the boundary layer. We computed the displacement thickness based on the streamwise, chordwise and spanwise mean profiles, respectively. It is noted that the downstream boundary-layer thickness is comparable to the wing nose radius, i.e. the layer is thin with respect to the dimensions of the wing.

4.3. Roughness-induced crossflow vortices

Figure 6 shows the structure of the disturbance excited by the cylindrical roughness element. In figure 6(a), the dominant contribution of the near field slightly downstream of the roughness cylinder is shown. This is a steady crossflow mode with the fundamental wavelength of the domain ('spacing between adjacent roughness cylinders'). Hence, the roughness element is an efficient generator of modal instability, whereas transient behavior due to nonmodal disturbances is not clearly visible in the near field. The far field is characterized by a large-amplitude crossflow vortex (figure 6b). In figure 7(a), the most energetic contributions of the boundary-layer disturbance are depicted. The steady crossflow mode is dominant both in the near field and the far field of the boundary-layer disturbance, where it starts to saturate. Figure 7(b) repeats

FIGURE 7. Receptivity and instability of the swept-wing boundary layer due to a cylindrical roughness element (see table 1). (a) Spanwise Fourier decomposition of the boundary-layer disturbance. The evolution of the tangential velocity component (spectral amplitude plus complex conjugate) is shown. (b) Comparison of the cross-flow modal amplitude as obtained from the DNS and from LPSE and NPSE calculations. (c) Comparison of the receptivity as predicted by DNS, a local receptivity model (FRNT) and the nonlocal, adjoint-based receptivity model by Tempelmann (2009).

the chordwise evolution of the fundamental stationary crossflow mode shown in figure 7(a). The modal amplitude just downstream of the peak at the roughness site may be interpreted as a receptivity amplitude (A_R). The coefficient for linear roughness-induced receptivity is usually defined by

$$C_R = \frac{A_R}{\varepsilon_r H(\alpha, \beta)_{CF}}, \quad (7)$$

where $H(\alpha, \beta)_{CF}$ is the spectral amplitude of the roughness element pertaining to the wave vector of the excited crossflow instability (subscript ‘CF’), and ε_r is given in table 1. Here, we obtained a value of $C_R = 33.56$ when defining A_R as the amplitude of the tangential disturbance velocity (see figure 7, caption). Note that C_R is non-dimensional and thus depends on the reference scales adopted. Figure 7(b) also shows the crossflow modal amplitude evolution as

obtained from LPSE and NPSE calculations, using the baseflow profiles of the boundary-layer solution (cf. figure 4b, dashed lines) and the value of A_R from the DNS as initial amplitude. The LPSE prediction agrees favorably with the DNS data in the region of linear growth; in addition, the NPSE result perfectly reproduces the nonlinear saturation of the crossflow mode. However, the methods of LPSE and NPSE can only predict the modal growth, but not the receptivity pertaining to the crossflow vortex. A receptivity model must be employed in order to determine the receptivity amplitude A_R . A classical receptivity model is the FRNT method (e.g. Crouch 1993; Ng & Crouch 1999), which is an adaptation of local stability theory to the receptivity problem. Here, we computed A_R by a local approach, which is – for incompressible flows – based on the direct and adjoint solutions to the Orr-Sommerfeld/Squire system (Hill 1995) and may be viewed as an equivalent to the FRNT. The subsequent chordwise evolution of the crossflow vortex was calculated using the standard NPSE approach. The result is compared in figure 7(c) with the modal evolution extracted from the DNS data, indicating that the local method over-predicts the receptivity to roughness of the wing boundary layer (‘save’ prediction). This behavior was also observed in the FRNT results by Ng & Crouch (1999). Tempelmann (2009) considered a refined, nonlocal receptivity model combining the direct and adjoint LPSE (see also Airiau *et al.* 2002). This approach was combined here with a NPSE solution in order to compute the receptivity and growth of the crossflow vortex. Figure 7(c) shows a significantly better agreement with the DNS result than that obtained by the local method. Note that A_R is sensitive to the location of the receptivity site (the center of the roughness cylinder here). This sensitivity results, on the one hand, from the use of a local mode for the initialization of the direct LPSE. This mode is subject to an upstream transient adjustment to the non-parallel boundary layer. On the other hand, the receptivity in the region between the leading edge and the first point of marginal stability (‘branch I’) is sensitive by nature, which is manifested by a steep gradient of the receptivity coefficient (see e.g. Schrader *et al.* 2009).

A preliminary comparison of our results with the experimental findings of Reibert *et al.* (1996) revealed that the measured crossflow amplitude was larger than that obtained in our DNS; however, the order-of-magnitude was the same. In §5, we discuss potential explanations for this discrepancy and outline the measures planned in order to obtain a better agreement.

4.4. Crossflow vortices due to free-stream disturbances

Next, we removed the roughness element and inserted a vortical mode at the inflow plane. Two steady modes were considered, representing streamwise and chordwise vorticity. The computation of these modes is outlined in §3.2.2. The vertical and spanwise wavenumbers were chosen as $\gamma = -\beta = 5.65$ for both vortical modes – in agreement with the spanwise spacing of the roughness cylinders (cf. §3.2.1). For these values, the vortex size is comparable to the

FIGURE 8. Downstream evolution of a free-stream vortical disturbance (streamwise vorticity) with $(\gamma, \beta) = (5.65, -5.65)$ and $\varepsilon_v = 10^{-3}$. (a) Chordwise disturbance velocity. (b) Evolution of the chordwise, vertical and spanwise disturbance velocities along the streamlines marked in (a).

boundary-layer thickness near the upper outflow plane. Note that we prescribed a negative value for β such that the horizontal wave vector (α, β) of the free-stream wave points into the crossflow direction. The modal amplitude was chosen as $\varepsilon_v = 10^{-3}$.

Figure 8(a) shows the behavior of the streamwise-vorticity mode during its passage through the free stream. The mode is deformed when approaching the wing, and the chordwise disturbance velocity is seen to decay significantly in the stagnation region. Figure 8(b) depicts the evolution of the three velocity components of the free-stream disturbance along the two streamlines shown in figure 8(a). Far out in the free stream, some energy transfer from the spanwise to the vertical velocity component is observed, whereas the spanwise component is forced near the boundary-layer edge. This redistribution of disturbance energy is due to stretching and tilting of the free-stream vortical mode at the wing nose and the boundary-layer edge. Farther downstream, the free-stream disturbance is subject to decay.

Figure 9 illustrates the boundary-layer response to the streamwise free-stream vorticity. In figure 9(a), the disturbance structures in the leading-edge vicinity are shown; these are weak in comparison to the free-stream fluctuations. The free-stream mode is highly damped by the boundary layer ('shear sheltering', see figure 8a) and hence cannot penetrate deeply into the layer. Nonetheless, a crossflow mode arises farther downstream (figure 9b). In figure 10(a), we compare the amplitudes of the steady crossflow vortices excited by the streamwise and chordwise free-stream vorticity with that triggered by the roughness cylinder. The initial evolution of the boundary-layer disturbance due to free-stream vorticity is characterized by a significant transient amplification. This indicates that the upstream disturbance shown in figure 9(a) is of

FIGURE 9. Boundary-layer response (tangential disturbance velocity) to a streamwise vortical free-stream mode with wave vector $(\gamma, \beta) = (5.65, -5.65)$ and amplitude $\varepsilon_v = 10^{-3}$ at the computational inflow. (a) Boundary-layer disturbance structure near the leading edge and (b) far downstream (not to scale).

nonmodal nature. A connection between upstream nonmodal disturbances and downstream crossflow vortices was also established by Schrader *et al.* (2009) and Tempelmann *et al.* (2010) for swept-plate flows. It is seen in figure 10(a) that the boundary layer is more receptive to the streamwise vorticity than to the chordwise vorticity; however, the layer is most receptive to the roughness disk. Assuming a linear receptivity mechanism, we may determine threshold amplitudes $\varepsilon_v^{\text{thres}}$, for which the free-stream vortical modes excite steady crossflow modes with the same downstream amplitude as that due to the cylindrical roughness considered here (figure 10b). The thresholds are $\varepsilon_v^{\text{thres}} = 1.48 \times 10^{-2}$ for the streamwise-vorticity mode and $\varepsilon_v^{\text{thres}} = 4.44 \times 10^{-2}$ for the chordwise vorticity. These values are fairly large, indicating that the wing boundary layer is more receptive to the shallow roughness element than to the present free-stream vortical modes. Figure 10(b) also suggests that the disturbance growth is fully dominated by the excited crossflow vortex downstream of $x = 20$.

5. Summary and discussions

This report deals with the receptivity of the boundary layer over a 45° swept wing to a chordwise localized spanwise array of cylindrical roughness elements and to free-stream vortices. The wing configuration is identical to that studied by Saric and coworkers in the ASU wind tunnel during the 1990s. The roughness elements – incorporated in the computational mesh – feature the same shape as those used by Reibert *et al.* (1996). However, instead of considering a spanwise row of cylinders, we inserted one single element and modeled

FIGURE 10. Comparison of the boundary-layer response to free-stream vorticity with that to a roughness cylinder (see table 1). A streamwise-vorticity mode and a chordwise-vorticity mode with $(\gamma, \beta) = (5.65, -5.65)$ and $\varepsilon_v = 10^{-3}$ are considered. (a) Chordwise evolution of the amplitude of the tangential disturbance velocity. (b) Curves of (a) scaled in order to match the amplitudes in the region of linear modal growth.

the spanwise roughness distribution by cyclic boundary conditions. The free-stream vortices were represented by spatial Fourier modes and prescribed at the computational inlet. Specifically, we considered steady streamwise and chordwise free-stream vorticity. The results were obtained by spatial DNS, using a spectral element code. In addition, we performed calculations based on a direct and adjoint LPSE approach and the NPSE method in order to characterize the receptivity and instability of the swept-wing flow.

The circular roughness disk was found to be an efficient generator of steady crossflow vortices. The efficiency of the receptivity mechanism became manifest in a rapid excitation of the modal boundary-layer instability without a significant transient growth at the roughness site. The crossflow vortex started to saturate within the computational domain (about 50 % chord). We focused on the receptivity and verified a nonlocal receptivity model relying on the direct and adjoint LPSE. This model was combined with a NPSE approach in order to predict the boundary-layer response from the receptivity phase to the saturation phase of the steady crossflow vortex. We demonstrated that our nonlocal model is superior to a local receptivity approach, e.g. the FRNT. Local methods are known to over-predict the roughness-induced receptivity (Ng & Crouch 1999), and this was confirmed here.

The amplitude of the steady crossflow vortex extracted from our DNS data turned out to be somewhat lower than that observed in the ASU experiments. This discrepancy is yet to be explained. We conjecture that a refined treatment of the spanwise roughness array is required. The present roughness representation by one single cylinder in combination with spanwise periodicity only allows

for disturbances of odd symmetry with respect to the lateral boundaries. In the continuation of this research, we will carry out spectral element simulations with two roughness disks. This will also allow for the development of disturbances with even symmetry or of asymmetric disturbances. Moreover, the presence of weak natural free-stream turbulence in the ASU wind tunnel may have altered the streamwise evolution of the steady crossflow mode, because the swept-wing boundary layer is also receptive to unsteady free-stream disturbances. These may have excited weak unsteady crossflow waves inside the boundary layer, modifying the evolution of the roughness-induced crossflow vortex. Also note that the wing surface in the experiments – although hand-polished – featured natural roughness in addition to the artificial roughness array. The receptivity due to the rough leading-edge region upstream of the roughness cylinders may have contributed to the crossflow modal amplitude. This potential receptivity source was not captured by our simulations. Moreover, there might be some uncertainty in the height of the roughness cylinders used in the experiments. The roughness height was specified as $6\ \mu\text{m}$. When assuming a somewhat larger value, we were able to obtain a better agreement with the measured amplitude of the crossflow mode, using the NPSE method. This would suggest that the actual height of the roughness cylinders of Reibert *et al.* (1996) was larger than $6\ \mu\text{m}$. However, further investigation is needed before drawing a final conclusion. It is noted that the present simulations are an improvement with regard to the preliminary DNS presented by Nishino & Shariff (2009), who report an order-of-magnitude discrepancy between the crossflow modal amplitude from their data and that from the ASU experiments.

The swept-wing boundary layer is also receptive to steady free-stream vortical modes, although these are highly damped in the stagnation region and inside the boundary layer ('shear sheltering'). Near the leading edge, weak nonmodal disturbances are identified, evolving farther downstream into a steady crossflow mode. The boundary-layer receptivity to streamwise free-stream vorticity was found to be somewhat larger than that to chordwise vorticity; however, the receptivity was lower than that to roughness. It is known that the penetration of vortical modes into the shear is crucial for the receptivity of three-dimensional boundary layers. The penetration depth is influenced by the spatial scales of the free-stream vortices, with small scales being favored (Schrader *et al.* 2009). The magnitude of the wavenumber vector of the present vortices was comparable to the downstream boundary-layer thickness. We conjecture that the layer might be more receptive to vortices with smaller scales. This will be investigated in the future.

We also plan a DNS study of laminar-turbulent transition of the swept-wing boundary layer. Wassermann & Kloker (2002) demonstrated for steady crossflow vortices that vortex packets with various length scales are more susceptible to secondary instabilities than single crossflow vortices – and thus undergo a faster transition. Therefore, we will consider free-stream turbulence, known to excite broadband wave packets of traveling crossflow vortices.

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