Aeroacoustic studies of duct branches – with application to silencers

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ABSTRACT

New methodologies and concepts for developing compact and energy efficient automotive exhaust systems have been studied. This originates in the growing concern for global warming, to which road transportation is a major contributor. The focus has been on commercial vehicles—most often powered by diesel engines—for which the emission legislation has been dramatically increased over the last decade. The emissions of particulates and nitrogen oxides have been successfully reduced by the introduction of filters and catalytic converters, but the fuel consumption, which basically determines the emissions of carbon dioxides, has not been improved accordingly. The potential reduction of fuel consumption by optimising the exhaust after-treatment system (assuming fixed after-treatment components) of a typical heavy-duty commercial vehicle is ~4%, which would have a significant impact on both the environment and the overall economy of the vehicle.

First, methodologies to efficiently model complex flow duct networks such as exhaust systems are investigated. The well-established linear multiport approach is extended to include flow-acoustic interaction effects. This introduces an effective way of quantifying amplification and attenuation of incident sound, and, perhaps more importantly, the possibility of predicting nonlinear phenomena such as self-sustained oscillations—whistling—using linear models. The methodology is demonstrated on T-junctions, which is a configuration well known to be prone to self-sustained oscillations for grazing flow past the side branch orifice. It is shown, and validated experimentally, that the existence and frequency of self-sustained oscillations can be predicted using linear theory.

Further, the aeroacoustics of T-junctions are studied. A test rig for the full determination of the scattering matrix defining the linear three-port representing the T-junction is developed, allowing for any combination of grazing-bias flow. It is shown that the constructive flow-acoustic coupling not only varies with the flow configuration but also with the incidence of the acoustic disturbance. Configurations where flow from the side branch joins the grazing flow are still prone to whistling, while flow bleeding off from the main branch effectively cancels any constructive flow-acoustic coupling.

Two silencer concepts are evaluated: first the classic Herschel-Quincke tube and second a novel modified flow reversal silencer. The Herschel-Quincke tube is capable of providing effective attenuation with very low pressure loss penalty. The attenuation conditions are derived and their sensitivity to mean flow explained. Two implementations have been modelled using the multiport methodology and then validated experimentally. The first configuration, where the nodal points are composed of T-junctions, proves to be an example where internal reflections in the system can provide sufficient feedback for self-sustained oscillation. Again, this is predicted accurately by the linear theory. The second implementation, with nodal points made from Y-junctions, was designed to allow for equal flow distribution between the two parallel ducts, thus allowing for the demonstration of the passive properties of the system. Experimental results presented for these two configurations correlate well with the derived theory.

The second silencer concept studied consists of a flow reversal chamber that is converted to a resonator by acoustically short-circuiting the inlet and outlet ducts. The eigenfrequency of the resonator is easily shifted by varying the geometry of the short circuit, thus making the proposed concept ideal for implementation as a semi-active device. Again the concept is modelled using the multiport approach and validated experimentally. It is shown to provide significant attenuation over a wide frequency range with a very compact design, while adding little or no pressure loss to the system.

Keywords: silencer, muffler, confined flows, flow duct, aeroacoustics, vortex sound, acoustic multiports, linear stability, self-sustained oscillations, whistling, Herschel-Quincke tube, acoustic resonator.
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This doctoral thesis consists of an introduction and summary of the research and the following appended papers:

**Paper I**

**Paper II**
Submitted to Journal of Sound and Vibration

**Paper III**
M. Karlsson and M. Åbom, On the Use of Linear Acoustic Multiports to Predict Whistling in Confined Flows,
Acta Acustica united with Acustica 97 (2011)

**Paper IV**
M. Karlsson, R. Glav and M. Åbom, The Herschel-Quincke: The attenuation conditions and their sensitivity to mean flow.

**Paper V**
**Division of work between the authors**

The formulations of the problems and the methodologies derived in this thesis have been developed in cooperation between Mikael Karlsson and the thesis supervisors, Mats Åbom and Ragnar Glav. Mats Åbom was the main supervisor for the thesis issues related to the studies of flow-acoustic coupling and the inclusion of such coupling in the multiport formalism (mainly described in Papers I-III), while Ragnar Glav was the main supervisor for the applications (Papers IV-V). All experiments and post-processing of data were designed and carried out by Mikael Karlsson, using specialized data acquisition software developed with support from Bill Halvorsen. Colleagues at Swenox AB have supported this work with CFD simulations. All review of test and analysis results, as well as document writing, was done by Mikael Karlsson.

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1. Introduction

This thesis originates in the general concern about global warming, in particular the concern about emissions from road transportation. The thesis is partly financed by government bodies interested in emissions and energy research in the automotive sector. The other interested parties are mainly manufacturers of commercial vehicles and diesel exhaust after-treatment systems. The overall aim of the thesis is to investigate new silencer concepts that can provide sufficient attenuation of exhaust noise in an internal combustion (IC) engine while, at the same time, being as compact as possible and having minimal energy losses. This research is interesting from both environmental and commercial perspectives, since fuel economy and load capacity are key issues for a buyer of commercial vehicles.

A typical exhaust after-treatment system for a heavy-duty commercial vehicle, including catalysts and particulate filters, results in a pressure loss of 20-50 kPa at full load operating conditions. This loss corresponds to approximately 30-80 units of horsepower that are simply converted into heat instead of useful mechanical work. Often more than half of this pressure loss is due to the after-treatment components—that is, the flow losses over the catalyst and filter substrates. These components are not dealt with specifically in this research although their correct installation in the system has great potential for minimising flow losses and optimising their contribution to noise attenuation. The focus of this research is instead directed towards the losses produced by flow expansions, contractions, and reversals. A flow loss reduction of 50%, for example, would reduce fuel consumption by 2-4% in the typical system discussed above. This may seem a small benefit, but for the average long haul truck such a reduction amounts to approximately 10-20 kSEK or, more importantly, 3-6 tonnes of CO₂ annually.

In the early days of the automotive industry the silencer was not much more than a pipe that routed the hot exhaust from the engine compartment. However, it soon became clear that the noise from the exhaust had to be attenuated. The quotation below is from the Ford Model T 1921 owner’s manual:

"Why is the muffler necessary?"

The exhaust as it comes out from the engine through the exhaust pipe would create a constant and distracting noise were it not for the muffler. From the comparatively small pipe, the exhaust is liberated into the larger chamber of the muffler, where the force of the exhaust is lessened by expansion and discharged out of the muffler with practically no noise. The Ford muffler construction is such that there is very little back pressure of the escaping gases, consequently there is nothing to be gained by putting a cut-out in the exhaust pipe between the engine and the muffler."

As the quotation indicates, the Ford Model T’s silencer was an expansion chamber type. This is still a common design element in automotive silencers. It is also clear that the backpressure produced by the silencer has always been an issue (here, probably from a performance perspective rather than a fuel consumption perspective). It may be noted that in US English the term ‘muffler’ (first used in 1895) is used rather than the preferred UK English term ‘silencer’. The term ‘silencer’ as a “mechanism that stifles the sound of a motor or firearm” was first recorded in 1898, although the sound attenuating mechanisms often used were known at a much earlier date. Most of these mechanisms are described in Lord Rayleigh’s classic book [1, 2].

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1 Oxford English Dictionary, 2nd ed., s.v. "muffler"
2 Oxford English Dictionary, 2nd ed., s.v. "silencer"
Figure 1. Ford Model T chassis. Classic exhaust configuration with long pipe routing the exhaust from the engine to the back of the car and into an expansion type silencer.

Although not high-performance systems, early exhaust systems were based on the same basic principles used for exhaust systems in modern vehicles. Most exhaust systems are passive—that is, no external excitation is used to cancel the noise. In a passive exhaust system there are essentially two ways to attenuate the noise of the engine.

- **Reflective systems.** The incident sound is scattered and cancelled by destructive interference. Examples are:
  - Expansions and contractions: common implementations are expansion chambers (as used in the Ford Model T), horns, or orifices.
  - Various types of resonators; for example, Helmholtz and quarter-wavelength resonators.
  - Wave interference; for example, the Herschel-Quincke tube.

- **Dissipative systems.** The incident sound is converted into heat. Examples are:
  - Silencers filled or lined with a material with high loss factor, such as mineral wool or glass fibres.
  - Through-flow perforates generating flow-related losses.

In any automotive exhaust system there is a combination of these sound-attenuating mechanisms. The level of sophistication differs with the vehicle segment. Exhaust systems in the budget car segment are only designed to meet legislative requirements while cars in the premium segment often have systems that also are designed to provide a sound character matching the vehicle’s profile.

In the 1970-1990s, with the successful implementation of active noise control in, for example, aeronautical applications, studies were performed on the use of forced actuators (essentially loudspeakers) to control automotive exhaust noise [3-6]. An actively controlled system offers new opportunities to adapt to a time varying source, such as the IC engine, and to control the character of the sound or to design flow optimised systems. However, although relatively straightforward to implement, both in theory and in laboratory conditions, the durability needed to withstand the harsh conditions and mileage required in an automotive application soon disqualified most attempts. Yet there is still some research activity in the area [7]. Another example of an active system, although not based on loudspeakers, is the concept studied by, for example, Harouin et al [8], Boonen [9], and Carme et al. [10]. An externally driven valve is placed...
in the flow duct and the valve is operated so as to provide an alternating resistance—that is, pressure loss—that regulates the pulsed flow to cancel the incoming acoustic disturbance. This works well even at low frequencies, which is problematic with conventional loudspeakers. The obvious disadvantage of this concept is that it not only requires an external driver but also results in an energy loss.

More realistic concepts are referred to as semi-active. That is, a passive property is actively controlled. The best known semi-active application is the use of spring-loaded valves that open at certain flow conditions, thus changing the tuning of a silencer. Hill [11] and Krause et al. [6] review the applications that have been made. Other semi-active concepts discussed are:

- Tuneable Helmholtz resonators [12, 13].
- Adaptive Herschel-Quincke tubes [14].
- Stretched membranes over expansion chambers [15-18].

Regardless of the concept chosen, it is increasingly important, due to more rapid product cycles, to predict the performance of the system at an early stage. Full numerical simulations, including both mean flow and acoustics, are still not viable for a complicated flow duct system such as an exhaust system. A common simplification is to reduce the system to a set of linear subcomponents that can be analyzed individually and then assembled into a network representing the system [19, 20]. The aeroacoustic subcomponents are often referred to as multiports; the subsystem is described as a black box with well-defined ports at which the state variables of choice are determined. The principle is diagrammed in Figure 2, illustrating an expansion chamber in the plane wave range using acoustic two-ports. For this case, where the elements are coupled in series, the transfer matrix representation shown is useful. However, for more complicated geometries and networks the more general description is the scattering matrix form, relating forward and backward travelling wave amplitudes at the ports for the chosen state variable [21].

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For developing new concepts, it is important to acquire some fundamental understanding of the interaction of the mean flow and the acoustics, especially in flow duct junctions. For example, the design of a system that is flow optimised and, at the same time, geometrically compact, could force solutions that are prone to constructive flow-acoustic coupling—that is, self-sustained oscillations or whistles. Such coupling is a non-linear phenomenon. As stated above, the usual way of studying an exhaust system is to use linear multiport models. Therefore, this work required characterising flow duct junctions, in the form of T-junctions, where flow-acoustic coupling is present, and trying to incorporate this effect into the linear analysis. This is an important issue, not only for the design of exhaust systems, but also for any complex flow duct network such as oil and gas pipelines, process industry ducting, or ventilation systems.
The analysis tools developed are then applied to two configurations. The first configuration is the classic Herschel-Quincke tube, which is basically two parallel ducts of different lengths connected at the same two nodes. This configuration, which results in attenuation by deconstructive interference at certain frequencies, is not a new concept. The concept was first presented in 1833 [22], but it is not commonly used. However, since in principle it should be possible to implement the Herschel-Quincke tube with a very low pressure loss penalty, the decision was made to assess its applicability for exhaust systems where the particular interest is the effect of mean flow. The second configuration is a new concept, now patented [23]. This configuration is a resonator created by acoustically short-circuiting a flow reversal chamber. Thus it is called the flow reversal resonator. This configuration is well suited for implementation as a semi-active device, since a small geometrical change to the short circuit has a large effect on the eigenfrequency of the resonator.

The next section presents general theoretical background and discusses the main findings of the research. (Details of the research are presented in the papers in the Appendices). First, there is a brief introduction to vortex sound that basically describes the flow-acoustic interaction in T-junctions—an element used in both applications (the Herschel-Quincke tube and the flow reversal resonator). Then the possible inclusion of the flow-acoustic interaction into the linear acoustic multiport is described. Next, the system’s potential for forming a whistle is described along with a method for determining whether the potential whistle will actually be triggered. Then the theoretical basis for the two applications is given. The validation of the theoretical models is done experimentally, as described in the next section. Results based on the characterisation of a T-junction are then presented. These results include the effect of the flow configuration as well as an explanation of how the incidence of the acoustic field influences the results. It is shown that one can predict the existence of fluid-driven whistles applying the linear theory derived. Finally, the two application examples are validated—first, the Herschel-Quincke tube, and second, the flow reversal resonator.
2. Theory

2.1 Flow-acoustic coupling

This section provides background material on the interaction of the hydrodynamic and acoustic fields. Although not specifically used in the later original work of this thesis, this material is useful in interpreting the results. The background material is largely based on the textbook of Rienstra and Hirschberg [24].

2.1.1 Vortex sound in confined flows

The most famous equation in aeroacoustics is probably Lighthill’s analogy [25, 26], where analogy refers to the idea of representing a complex fluid mechanical process with acoustically equivalent source terms. Derived as a non-homogenous wave equation, the Lighthill analogy is useful when one can identify a well-defined compact region where the source terms can be determined. Outside this region the standard homogenous wave equation is used for the propagation of the generated sound to the observer position. The integral form of this equation is widely used to obtain order of magnitude estimates for important aerodynamic sources as for example, jet engines. However, Lighthill’s analogy does not include any feedback from the acoustic field to the hydrodynamic field (or if there is feedback, it is already included in the description of the hydrodynamic field). In applications where the flow-acoustic interaction is significant, the concept of vortex sound is more useful because, especially for low Mach number homentropic flow, vorticity \( \omega = \nabla \cdot \mathbf{v} \) is a convenient quantity to describe the flow field.

For a homentropic, inviscid fluid the following inhomogenous wave equation can be derived for low Mach number flows:

\[
\frac{1}{c_0^2} \frac{D_0^2 h'}{Dt^2} - \nabla^2 h' = \nabla \cdot (\omega \times \mathbf{v})
\] (1)

where \( D_0/ Dt = \partial / \partial t + U_0 \cdot \nabla \), \( h' = p'/\rho_0 + U_0 u' \) is the acoustic stagnation enthalpy, \( c_0 \) is the speed of sound and \( \mathbf{v} \) is the total velocity. The source term on the right clearly indicates that the vorticity is responsible for the generation of sound. Assuming a free field and a compact source region (as in Lighthill’s analogy), the vortex sound analogy can be interpreted as the Coriolis force \( \mathbf{f}_c = \rho_0 (\omega \times \mathbf{v}) \) acting on the acoustic field [27]. However, using a potential flow instead of a quiescent fluid (as in Lighthill’s analogy) as a reference frame, Howe [28, 29] showed that the analogy is applicable even if there are boundaries present and the source region is not compact. This will prove useful in applications with confined flows that this research studies. As a consequence of the choice of reference frame, Howe [30] suggested that the acoustic field is defined as the unsteady part of the scalar potential flow \( \mathbf{u}' = \nabla \varphi' \).

A useful application of the vortex sound analogy is then the energy corollary derived by Howe [31] for low Mach number flows with a compact vorticity distribution in free space and in the presence of solid surfaces:

\[
< P > = -\rho_0 \int_V (\omega \times \mathbf{v}) \cdot \mathbf{u}' \, dV
\] (2)
where $<P>$ is the time averaged acoustic power and $V$ is the region enclosing the vorticity. If the equation is positive, sound is generated; if the equation is negative, sound is dissipated. A main advantage of this model is that it is expressed in integral form, thus making it less sensitive to random errors. A common application of vortex sound is for flow configurations where simplified models of the vorticity distribution can be assumed [32, 33]. Otherwise, detailed knowledge of the flow field is required, either using numerical simulations or experimental data.

2.1.2 Aperture tones – Flow-acoustic interaction in T-junctions

One common application, and the one studied here, is T-junctions. The simplest case is when there is flow in the main duct only—that is, when the side branch is subjected to grazing flow only. The vorticity within the shear layer given by the grazing flow will interact with the acoustic field while it is convected across the orifice. This flow-acoustic interaction can be described by Howe’s energy corollary, Eq. (2), as presented above. Upon formation of a new vortex, the acoustic field is directed into the side branch. Hence there is an initial phase where the hydrodynamic field absorbs energy from the acoustic field since $-(\mathbf{\omega} \times \mathbf{v})$ is opposite to $\mathbf{u}'$; then for the second part of the acoustic oscillation period, the interaction becomes constructive. Assuming that the travel time of the hydrodynamic disturbance over the orifice equals the oscillation period of the acoustic field the net result is then positive since the hydrodynamic disturbance grows while being convected across the orifice [34]. Hence, the convection velocity, $U_C$, of the hydrodynamic disturbance across the orifice needs to match the oscillation period of the acoustic field for the flow-acoustic interaction in the T-junction to be constructive. If the hydrodynamic disturbance travels too quickly across the side branch, there will be no contribution from the constructive part of the cycle, and the net result will be negative. Actually, one can derive Strouhal number, $St_ow = 2\pi f d_{eff} / U_c$ (where $d_{eff} = ml/4$ is the effective length of the orifice), regions for each hydrodynamic mode $n$ in which the T-junction is generating or absorbing sound. An estimation of the constructive Strouhal number regions is given by [35]:

$$St_ow \tan^{-1}\left(\frac{1}{St_ow}\right) = 2n\pi \pm \frac{\pi}{2}$$

An important question in this research is whether linear models can be used to predict a non-linear phenomenon such as whistling. In the T-junction case, it is clear that the frequency at which the system whistles depends on the convection velocity of the hydrodynamic disturbance. In the experimental characterisation of the T-junction discussed later, the system is kept within the linear regime. This is usually valid up to ratios of the acoustic particle velocity to the mean...
flow velocity $V = \frac{|u|}{U_o}$ of approximately $10^{-2}$ \[36, 37\], but varies with the flow configuration. Passing the critical value of $V$, the system becomes non-linear and the shear layer tends to roll up into discrete vortices. However, as long as the relative acoustic amplitude $V'$ is moderate, on the order of $10^1$, the vortices travel with the same velocity as in the linear case \[38\]. At very high relative acoustic amplitudes ($V \rightarrow 1$), the trajectory of the vortex is affected and the travel time across the side branch changes \[39\]. There are studies on the prevention of self-sustained oscillations in apertures using the introduction of spoilers (passive: Bruggeman et al. \[39\] or active: Kook et al. \[40\]) at the leading edge where the hydrodynamic disturbances are shed.

The above description is based on an orifice subjected to grazing flow alone. Although a common configuration, there are also many applications with combinations of grazing and bias flow—that is, flow that combines or divides in the T-junction. The influence of these more complicated flow configurations is one topic in this thesis. In Figure 4 the two flow configurations are illustrated via a steady state computational fluid dynamics (CFD) simulation. The presented parameter is vorticity, which is just normalized by its maximum value within the simulation domain since the amplitude is of no interest here. The black line is the trace of a massless particle seeded at the leading edge. This simple simulation yields some basic insights. With grazing-bias inflow, the shear layer is still present and the vorticity passes over the orifice, although the trajectory is changed and the convection velocity is increased (the velocity was monitored along the streamline). In the grazing-bias outflow case, the shear layer containing the vorticity turns into the side branch. Hence, following the discussion above, the hydrodynamic disturbance does not have a well-defined travel time across the orifice, and the production phase is lost. That is, this configuration should not amplify sound.

![Figure 4. Steady state (RANS k-ε) simulation of the flow field \[41\]. The black line is a streamline of a massless inert particle seeded at the leading edge. a) Grazing-Bias inflow $M_g=0.1$, $M_{b,in}=0.025$. b) Grazing-Bias outflow $M_g=0.1$, $M_{b,out}=0.025$.]

### 2.2 Multiport formalism

As mentioned in the Introduction, a common way of representing a complex flow duct system is to break it down into a network of linear multiports \[19, 20\]. In practice this representation is mainly used with two-port elements representing the components of the system—say, a duct or a resonator—and one-port elements representing sources and terminations of the system. There are several commercially available frameworks for this type of modelling. Examples of such
frameworks are SIDLAB, LAMPS and modules within GT-Power and AVL BOOST. Choosing the scattering matrix form, any linear and time-invariant $N$-port can be described as [42]:

\[
\mathbf{x}_+ = \mathbf{S}\mathbf{x}_- + \mathbf{x}^{s+}
\]  

(4)

\[
\mathbf{S} = \begin{bmatrix}
R_i & T_{II,II} & T_{III,II} \\
T_{I,II} & R_{II} & T_{III,II} \\
T_{I,III} & T_{III,II} & R_{III}
\end{bmatrix}
\]  

(5)

Figure 5. Example of active acoustic multiport ($N=3$)

where $\mathbf{x}$ is the state variable of choice (the useful choice of state variable depends on the application and will be discussed later) and the plus and minus signs indicate travelling wave amplitudes out of (+) and into (-) the multiport. See Figure 5 for an example of an active acoustic three-port. The passive properties of the linear system—that is, the reflection, $R$, and the transmission, $T$, of sound—are now described by the scattering matrix $\mathbf{S}$ while the additional vector $\mathbf{x}^{s+}$ represents sources within the system that are independent of the incident sound. This representation is useful for studying fluid machines [43] or applications with flow-generated noise without feedback [44].

However, for applications with strong flow-acoustic coupling, such as the T-junction case, the use of an independent source term needs to be modified. To describe the physics involved, one must allow the source term to be modulated by the incident acoustic wave, which can be expressed as:

\[
\mathbf{x}^{s+}_{\text{mod}} = \mathbf{S}_m\mathbf{x}_-,
\]  

(6)

which can be used to modify Eq. (4) to:

\[
\mathbf{x}_+ = (\mathbf{S}_0 + \mathbf{S}_m)\mathbf{x}_- + \mathbf{x}^{s+}_0
\]  

(7)

where the subscript zero indicates the original—meaning unaffected by the flow-acoustic coupling—scattering matrix and source vector. Hence, as long as the flow-acoustic interaction is linear, it is included in the linear scattering properties of the system under study. From the previous section we know that this implies that the acoustic feedback to the system, which determines the ratio ($V$) of the acoustic particle velocity to the mean flow velocity, must be kept under a critical level. Successful characterisation of the linear acoustic scattering matrix including the flow-acoustic coupling has been done both numerically and experimentally for both T-junctions [45, 46] and orifices [47, 48].
2.3 Quasi-steady approximation of the acoustic scattering of a T-junction

For low Strouhal numbers, the scattering matrix $S$ for the passive properties of a multiport can be described using a quasi-steady model. The resulting scattering matrix is real valued and frequency independent and represents the resistive part of the scattering associated with area changes and flow related losses. The non-represented reactive part (which would have been represented by the imaginary part of the scattering matrix elements) describing acoustic near-field effects is negligible at the low frequencies studied.

Although only valid for low Strouhal numbers, quasi-steady models are useful in a range of engineering applications. One important application is the automotive silencer, studied in this research, where the frequency range of interest often is limited to the firing frequency range of the IC engine. A typical multiport modelling approach for a new silencer concept can be limited to $\sim 500$ Hz, making quasi-steady models viable. See, for example, Section 4.3, where the model derived is applied to the flow reversal resonator concept.

Quasi-steady models have been successfully derived for orifices [49, 50] and bends [51]. It is common to relate the flow losses to the contraction ratio of the flow when it passes the flow restriction, which is basically how both the orifice and the bend act. However, from a practical point of view, it is more convenient to characterise the flow losses by a pressure loss coefficient. For most practical configurations, these coefficients are readily found in handbooks [52, 53] or easily obtained from simple steady state simulation using CFD. An example of this approach is the bend model proposed by Nygård [54]:

$$\begin{bmatrix}
\rho_u^+ \\
\rho_d^+
\end{bmatrix} = \frac{1}{2 + M_u K_b} \begin{bmatrix} M_u K_b & 2 \\ 2 & M_u K_b \end{bmatrix} \begin{bmatrix}
\rho_u^- \\
\rho_d^-
\end{bmatrix}$$  

(8)

where $K_b$ is the pressure loss coefficient for the bend and $M_u$ is the Mach number upstream of the bend. This model is used both for the Herschel-Quincke tube discussed in Section 2.6 and in comparisons with the derived model for the T-junction.

Figure 6. Definition of the flow configurations.
While previously reported quasi-steady models have been two-ports, the T-junction is a three-port. This does not present a problem since the procedure presented below can be applied to any compact $N$-port as long as the flow loss coefficients between the $N$ ports are known. For the model to be useful, it should handle all possible flow configurations through the T-junction. See Figure 6. The variants are denoted “grazing” for flow in the main branch passing over the side branch, while flow through the side branch orifice is denoted “bias”. Flow that bleeds off the main branch into the side branch is denoted “bias outflow” while flow entering the main branch from the side branch is denoted “bias inflow”.

In the derivation of the quasi-steady model, low Mach numbers are assumed. Given the basic assumption of low Strouhal numbers, this implies a low Helmholtz number—that is, a compact acoustic problem. For both grazing bias inflow and grazing bias outflow, it is sufficient to study the conservation of volume flow over the junction and apply Bernoulli’s equation for each flow path. Starting with the grazing-bias outflow case:

\[
\begin{align*}
P_1 + \frac{1}{2} \rho_0 U_1^2 &= P_\Pi + \frac{1}{2} \rho_0 U_\Pi^2 + \frac{1}{2} \rho_0 U_1^2 K_{I-II}(U_1, U_\Pi) \quad (9) \\
P_1 + \frac{1}{2} \rho_0 U_1^2 &= P_\Pi + \frac{1}{2} \rho_0 U_\Pi^2 + \frac{1}{2} \rho_0 U_1^2 K_{I-III}(U_1, U_\Pi) \quad (10) \\
U_1 &= U_\Pi + U_\Pi \frac{A_{III}}{A_1} \quad (11)
\end{align*}
\]

where $A$ is the cross sectional area, $P$ is the steady flow pressure, $U$ is the steady flow velocity and $K$ the loss coefficients. To obtain the acoustic scattering matrix $S$, one then introduces an acoustic perturbation on the flow $P = P_0 + p'$ and $U = U_0 + u'$ which is then decomposed into positive and negative travelling waves. After some algebra (for more details see Appendix II), one then arrives at:

\[
S_{\beta, \beta'}^{out} = \begin{bmatrix}
1 - M_1 + R_{I-II} & -1 - M_\Pi & -R_{2,II-II} \\
1 - M_1 + R_{I-III} & 0 & -1 - M_{III} - R_{2,III-III} \\
1 & 1 & \alpha
\end{bmatrix}^{-1}
\]

\[
\begin{align*}
-1 - M_1 + R_{I-II} & \quad 1 - M_\Pi & \quad -R_{2,II-II} \\
-1 - M_1 + R_{I-III} & \quad 0 & \quad 1 - M_{III} - R_{2,III-III} \\
1 & 1 & \alpha
\end{align*}
\]

where $\alpha = A_{III}/A_1$ and $R_{I-n} = K_{I-n}M_1 + (\partial K_{I-n}/\partial U_1) \frac{1}{2} c_0^2 M_1^\beta$, $R_{2j-II} = (\partial K_{I-II}/\partial U_1) \frac{1}{2} c_0^2 M_1^\beta$, with $n=II$ or III. These expressions contain partial derivatives of the flow loss coefficients $\partial K$, which have to be determined from the flow data available. With the tabulated pressure loss coefficients from [53],

\[
K_{I-n}(U_{III} \alpha / U_1, \alpha) = K_{I-n}(\beta, \alpha) \quad (13)
\]

the derivatives can be estimated as:

\[
\begin{align*}
\partial_{U_1} K_{I-n} &= \partial_{\beta} K_{I-n} \hat{\partial}_{U_1} \beta = -\partial_{\beta} K_{I-n} U_{III} \alpha / U_1^2 = -\frac{\beta \Delta K_{I-n}}{U_1 \Delta \beta} \\
\partial_{U_\Pi} K_{I-n} &= \hat{\partial}_{U_\Pi} K_{I-n} / U_1 = \frac{\Delta K_{I-n}}{U_1 \Delta \beta}
\end{align*}
\]
The grazing-bias inflow case is derived similarly starting from:

\[ P_i + \frac{1}{2} \rho_o U_i^2 = P_{ii} + \frac{1}{2} \rho_0 U_{ii}^2 + \frac{1}{2} \rho_o U_{i}^2 K_{\text{I-II}} (U_{\text{II}}, U_{\text{III}}) \]  
(16)

\[ P_{\text{III}} + \frac{1}{2} \rho_0 U_{\text{II}}^2 = P_{\text{II}} + \frac{1}{2} \rho_0 U_{\text{II}}^2 + \frac{1}{2} \rho_o U_{\text{II}}^2 K_{\text{III-II}} (U_{\text{II}}, U_{\text{III}}) \]  
(17)

\[ U_i + U_{\text{III}} A_{ii} / A_i = U_{\text{II}} \]  
(18)

with the resulting scattering matrix given by:

\[
S_{\text{g,bm}} = \begin{bmatrix}
1 - M_1 & -1 - M_{\text{II}} & - R_{\text{I,II}} & R_{\text{II,II}} \\
0 & -1 - M_{\text{II}} & + R_{\text{I,III}} & - M_{\text{III}} + R_{\text{II,III}} \\
1 & 1 & \alpha & 0 \\
-1 - M_1 & 1 - M_{\text{II}} & - R_{\text{I,II}} & R_{\text{II,II}} \\
0 & 1 - M_{\text{II}} & - R_{\text{I,III}} & - M_{\text{III}} + R_{\text{II,III}} \\
1 & 1 & \alpha & 0
\end{bmatrix}^{-1}.
\]  
(19)

Where \( R_{\text{I,II}} = K_{\text{II-II}} M_{\text{II}} + (\partial_{U_0} K_{\text{II-II}}) \frac{1}{2} c_0 M_{\text{II}}^2 \) and \( R_{\text{II,III}} = (\partial_{U_0} K_{\text{II-III}}) \frac{1}{2} c_0 M_{\text{II}}^2 \), with \( n=\text{I or III} \). These models are validated against experimental data in Section 4.1.1 and applied in the modelling of the flow reversal resonator concept in Section 4.3.

2.4 Quantifying the flow-acoustic interaction

Having obtained the scattering matrix describing the passive acoustic properties of a multiport, the matrix can be used to derive various measures that provide further insight into its aeroacoustical characteristics. Two ways of describing the aeroacoustical response of the T-junction studied are discussed here: the acoustic impedance and a power balance. First, there is a short discussion on the appropriate choice of state variables.

2.4.1 State variables

The most common choice of state variables for describing an acoustic multiport is the acoustic pressure and volume velocity. This is the straightforward choice for applications without mean flow since the product of the state variables then represents power, and their ratio represents the acoustic impedance. For flow duct applications, the acoustic pressure and volume velocity are no longer an obvious choice since in the presence of mean flow these quantities no longer result in the appropriate expressions for acoustic power and impedance. An alternative set of state variables, whose product represents acoustic power in the presence of mean flow [55], is the acoustic stagnation enthalpy \( h' = p' / \rho_0 + U_0 u' \) and acoustic mass flow \( m' = (\rho_0 u' + \rho' U_0) A \) where \( p \) is pressure, \( \rho \) is density, \( U_0 \) is the mean flow velocity, \( u \) is the fluctuating velocity, \( A \) is the cross-sectional area of the duct, and the prime denotes an acoustic field quantity. Henceforth the scattering matrix is expressed in acoustic stagnation enthalpy (if not stated otherwise). However, this is merely a matter of convenience. As shown in [46], one could just as well use the scattering matrix expressed in terms of the acoustic pressure and apply the appropriate compensation terms for the convective effects when deriving the acoustic impedance and power balances.
2.4.2 Acoustic impedance

Perhaps the most common way of quantifying the aeroacoustical behaviour of a side branch orifice is the normalised acoustic impedance $Z = r + i\chi$, where $r$ is the acoustic resistance and $\chi$ is the acoustic reactance. The reactance can be seen as a change in the apparent mass of the orifice, often expressed as an end correction, while the resistive part represents the exchange of energy between the hydrodynamic and acoustic fields. A negative acoustic resistance then indicates that energy is transferred from the hydrodynamic field to the acoustic field—that is, sound is generated [56]. Systems with strong flow-acoustic interaction, such as the T-junction studied here, will certainly display regions of negative acoustic resistance. However, one has to be careful in the interpretation, which depends on the definition of the acoustic impedance. The general definition of impedance is the ratio of a potential quantity to a flow quantity, where the product of the two quantities represents power. Based on the discussion above on state variables, a suitable definition of the acoustic impedance in a confined flow is then the ratio of the acoustic stagnation enthalpy to the mass flow.

$$Z = \frac{h'}{m'}$$  \hspace{1cm} (20)

where the acoustic mass flow is positive going into the orifice. From here on it is assumed that the enthalpy formulation is used for the acoustic impedance if not otherwise stated.

As seen in Figure 7(a), the choice of impedance formulation greatly influences the result. The enthalpy formulation is compared to the more traditional impedance formulation based on acoustic pressure and volume velocity. Both formulations result in alternating regions of positive and negative acoustic resistance, but the traditional impedance formulation displays much larger regions with negative impedance. Further discussion about this is given in Section 4.1.2-3.
Both in theoretical attempts [31, 57] and in empirical models (e.g., [58, 59]) for the acoustic impedance of a single orifice subjected to grazing flow, it is assumed that the incident sound is in one given configuration. As shown in [37, 46, 60], this assumption does not hold generally. The interaction of the hydrodynamic and acoustic field varies with the incidence of the acoustic field, and consequently so does the acoustic impedance. This may be one explanation of the large variation in previously presented results and of the poor applicability of empirical impedance models to other configurations than the one it was derived from [61]. This problem can be overcome by having access to the full linear scattering matrix representing the T-junction. The decision in this research was to study the system with sound incident from one port at a time assuming the other two ports to be anechoic. Of course, that assumption rarely holds in practical applications, but yields some insights into the physics. The normalised acoustic impedance—expressed in “enthalpy form”—for sound incident from the upstream, downstream, and side branches then becomes:

\[ Z_{I-III} = \frac{h_I^1}{m_{III}^* Z_{char}} = \frac{1 + R_I}{T_{I-III}} \]  
\[ Z_{II-III} = \frac{h_{II}^1}{m_{III}^* Z_{char}} = \frac{1 + R_{II}}{T_{II-III}} \]  
\[ Z_{III} = \frac{h_{III}^1}{m_{III}^* Z_{char}} = \frac{1 + R_{III}}{1 - R_{III}} \]  

where \( R \) and \( T \) are coefficients from the scattering matrix expressed in enthalpy form, and the characteristic impedance is given by: \( Z_{char} = \frac{c_0}{\rho_0 A_{III}} \).

The strong influence of the incidence of the acoustic field is illustrated in Figure 7(b) where the resistance resulting from the three different impedance formulations of Eqs. (21-23) is shown. It is apparent that excitation from the upstream and side branches results in a stronger interactive behaviour between the hydrodynamic and acoustic fields than excitation from the downstream branch. See the Results/Validation section for further discussion.

Now, having derived the appropriate expression for the acoustic impedance, one can apply them to the different flow configurations of the T-junction. Most previous research has focused on grazing flow alone. See, for example, [36, 57-59, 61-64]. There are empirical models that include both grazing and bias flow; however, usually the grazing and bias flow cases are investigated separately and then combined into a model [65]. An experimental investigation of perforated plates including grazing-bias flow effects was performed by Sun et al. [66]. Since results are only available for a few perforated plates and low Strouhal numbers, it is difficult to draw general conclusions. However, it is evident that there is a difference between the grazing-bias inflow case and the grazing-bias outflow case. For larger single orifices, there is a numerical study by Belfroid et al. [67] that focuses on the pulsation amplitudes at various grazing-bias flow configurations.

### 2.4.3 Power balances

Perhaps the most intuitive way to study the flow-acoustic interaction in a multiport is with a power balance. By comparing the acoustic power leaving the multiport at a given incident acoustic power, one can determine whether the multiport acts as an attenuator or as an amplifier. This approach yields the same information as the vortex sound energy corollary derived by Howe, Eq. (2), only here the vorticity region is considered a black box with well-defined ports.
Like the acoustic impedance, the power balance is easily derived once the linear scattering matrix representing the multiport has been obtained. As for the acoustic impedance, it was chosen to excite the system from one port at a time, assuming the ports not currently excited to be anechoic. As discussed in Section 2.4.1, the product of the acoustic stagnation enthalpy and mass flow represent acoustic power in a flow duct. Hence the following three power balances can be derived for the three possible excitation cases in the T-junction (see Figure 5 for the coordinate system definition):

\[
\begin{align*}
\mathbf{P}_{\text{in}} & = h'_{II}^I m_i^II + h'_{III}^I m_i^III + A_{II} + h'_{III}^I m_i^III + A_{III} - |R_{II}^I|^2 A_{II} + |T_{I-III}^I|^2 A_{III} \\
\mathbf{P}_{\text{out}} & = h'_{II}^I m_i^II + h'_{III}^I m_i^III + A_{II} + h'_{III}^I m_i^III + A_{III} - |T_{II-I}^I|^2 A_{II} + |R_{II}^I|^2 A_{III} + |T_{I-II}^I|^2 A_{III} \\
\mathbf{P}_{\text{III}} & = h'_{III}^I m_i^III + A_{II} + h'_{III}^I m_i^III + A_{III} - |T_{III-I}^I|^2 A_{II} + |R_{III}^I|^2 A_{III} \\
\end{align*}
\]

where the reflection, \( R \), and transmission, \( T \), coefficients are taken from the linear scattering matrix in enthalpy form.

A more generic approach has been presented by Aurégan and Starobinski [68]. Adopting yet another set of state variables \( a \) and \( b \), related to each other by a scattering matrix \( S_p \) such that \( a a^* = \text{inc} \) and \( b b^* = \text{refl} \), the time-averaged acoustic power output is

\[
\langle P_{\text{out}} \rangle = b^* b - a^* a = a^* \left( S_p^* S_p \right) a - a^* a
\]

This equation is positive for a system that generates sound. Normalizing the incident acoustic power to unity — that is, setting \( a^* a = 1 \) — the reflected acoustic power is given by the value of the first part of Eq. (27). This quadratic form, which is Hermitian and positive definite, can thus be transformed into a diagonal form:

\[
b^* b = a^* \left( S_p^* S_p \right) a = \sum_n \lambda_n |a_n|^2
\]

where the \( \lambda_n \) are the real and positive eigenvalues of \( S_p^* S_p \) and the vector \( a^* \) is given by \( a = T a^* \).

The transformation matrix \( T \) is a unitary matrix where the columns represent the eigenvectors of the Hermitian matrix \( (S_p^* S_p) \). Finally, inserting Eq. (28) into Eq. (27), the maximum and minimum time-averaged acoustic power outputs are expressed as [68]

\[
\begin{align*}
\langle P_{\text{max}} \rangle & = \lambda_{\text{max}} - 1 \\
\langle P_{\text{min}} \rangle & = \lambda_{\text{min}} - 1
\end{align*}
\]

That is, an eigenvalue larger than unity indicates sound production. This approach is a good way of investigating the extremes of the system under study including all possible excitation combinations. The relative excitation yielding a maximum or a minimum can be found by examining the corresponding eigenvector.
2.5 Linear stability

Self-sustained oscillation is a non-linear phenomenon, but, as has been discussed in Section 2.1.2, the frequency at which aperture tones occur is basically determined by the convection velocity of the hydrodynamic disturbances across the side branch. This convection velocity does not change significantly between the linear and non-linear regimes [38]. Thus it should be sufficient to study the linear system to identify the whistling frequencies [31]. Linear stability analysis is widely used in applied control theory such as in the design of microwave and radio frequency circuits [69]. It has also been studied in more closely related problems such as thermo-acoustic instabilities [70, 71].

Here linear stability is applied to the acoustic multiports directly. However, other representations of the problem are possible. For example, in musical acoustics the concept of impedance matching is often used [72], where the flow-acoustic interaction is reduced to an impedance $Z_R$. The frequency of the instabilities of the complete system are then approximated as the real part of the complex zeros of the total impedance:

$$Z = Z_R + Z_{ac}$$

where $Z_{ac}$ is the acoustic impedance of the system $Z_R$ is coupled to. However, for a geometry such as the T-junction, where the flow-acoustic interaction varies significantly with the incidence of the acoustic field, the reduction of the flow-acoustic interaction to a single impedance is not possible.

2.5.1 Response with a given termination

The acoustic power balances identifies Strouhal number regions with constructive or deconstructive flow-acoustic interaction for the various flow and acoustic incidence configurations. To assess whether a given configuration actually forms a self-sustained oscillation, the feedback given by the system attached to the multiport has to be included. The response to a given termination can be studied by introducing a reflection matrix $R$ whose elements represent the passive and time invariant terminations of the system seen at each port.

$$\mathbf{x}_- = \mathbf{R}\mathbf{x}_+$$

which, when inserted in the basic multiport description, Eq. (4), yields:

$$\mathbf{x}_+ = \mathbf{S}\mathbf{R}\mathbf{x}_+ + \mathbf{x}^{++}$$

The source vector is independent of the passive part of the multiport. Hence, eigenvalues of the system representing growing modes can be found studying:

$$(\mathbf{E} - \mathbf{SR})\mathbf{x}_+ = 0$$

where $\mathbf{E}$ is the unit matrix. This equation has non-trivial solutions (eigenfrequencies) when

$$D = \det(\mathbf{E} - \mathbf{SR}) = 0$$

If the system is reflection free ($\mathbf{R} = 0$), the function $D$ collapses to a single point (1,i0) in the complex $D(\omega)$-plane.
2.5.2 Nyquist stability criterion

There are various methods of identifying the critical zeros of Eq. (35) corresponding to exponentially growing instabilities. In the best of worlds, where one has analytical expressions describing the system, the complex poles and zeros can be found and both the frequency and the growth rate of the instability can be studied. However, for most realistic situations, where the system has been characterised numerically or experimentally, data exists only along the positive real axis. One way of dealing with this situation is to use a system identification approach (see, for example, [73]). However, this approach could be rather costly since normally there are many poles and zeros in the system, and in most cases only a few are critical.

An alternative approach, which only searches for the critical zeros, is the Nyquist stability criterion [74]. It is based on Cauchy’s principle of the argument that yields the number of poles and zeros of a function inside a closed contour in the complex plane. Assume that $D(\omega)$ is analytical (except at a finite number of points) in the complex plane $\omega$. Now, travelling around the closed contour (at which $D(\omega)$ must be analytical) in the clockwise direction, $D(\omega)$ encircles the origin in the complex $D(\omega)$ plane in the same direction $N$ times, where $N$ is given by:

$$N = Z - P \quad (36)$$

where $Z$ and $P$ stand for the zeros and poles of the function $D(\omega)$ inside the contour. If there are no poles in $D(\omega)$, then $N$ represents the number of zeros. This is the desired property, but knowledge of the function $D(\omega)$ for complex frequencies is still required. However, assuming that the system under study is causal, and that the acoustic reflections tend to zero for large $\omega$ implies that it is sufficient to study the variation of $D(\omega)$ along the real axis only. For large $\omega$, $D(\omega)$ approaches the same point $(1, i0)$ in the lower half-plane as for large $\omega$ along the real axis. Hence, closing the contour in the lower half-plane is not necessary since for large $\omega$ this simply corresponds to the point $(1, i0)$ already included. Also, since roots and poles appear in pairs (i.e., for each root/pole with a positive real part there is a corresponding part located at $-\omega$) it is sufficient to investigate only the positive real axis. This creates a clockwise contour covering the critical lower half-plane where zeros represent exponentially growing eigenfrequencies. This version of the argument principle (illustrated in Figure 8) can be seen as an extension of the so-called Nyquist stability criterion to an N-DOF system. Like the traditional Nyquist criterion, an estimate of the eigenfrequencies (the real part) is found when the function $D(\omega)$ crosses the negative real axis, thus completing one encirclement.

![Figure 8. Illustration of the Nyquist stability criterion.](image-url)
This procedure for identifying the critical zeros is simple to implement and has proven stable (see Appendix III [75]). A useful application of the linear stability procedure is to identify the stability limits—that is, the effect on the stability given by a change in the system—for example, when changing the reflection coefficients in the system attached to the multiport. A typical application is to determine the necessary amount of damping needed in a side branch to prevent it from whistling. This application can be seen as a variant of the root-locus diagram used in control theory where the trace of the root is plotted in the complex plane as some system parameter is changed.

2.6 The Herschel-Quincke tube

The Herschel-Quincke tube is the collective name given to configurations where a number of one-dimensional waveguides are coupled in parallel between two nodes, as illustrated in Figure 9. Being a parallel coupled system, it is convenient to characterise the Herschel-Quincke tube using the mobility matrix, \( Y \). The elements of the system matrix are then simply the sum of the corresponding elements in the matrices describing each branch:

\[
Y^{HQ} = \begin{bmatrix}
\sum_{n=1}^{N} Y_{11,n} & \sum_{n=1}^{N} Y_{12,n} \\
\sum_{n=1}^{N} Y_{21,n} & \sum_{n=1}^{N} Y_{22,n}
\end{bmatrix}
\]  

(37)

The Herschel-Quincke tube is named after J. F. W. Herschel who first introduced the idea [22] in another context—the absorption of light—and G. Quincke who validated the basic concept experimentally [76]. As their research took place in the mid-nineteenth century, the concept is far from new. Nevertheless, the concept is seldom used in practical applications, at least not in flow ducts. However, there have been attempts to use the concept in automotive applications. For example, Hwang et al. [14] allowed the lengths of the two paths to be changed actively to follow the firing frequency of an IC engine during operation. Trochon [77] and McLean [78] extended the Herschel-Quincke tube with quarter-wave resonators at the nodes to achieve a broader attenuation curve. Also the introduction of an active membrane in the side branch has been discussed [79].
The reason for studying the Herschel-Quincke tube in this thesis is its possible use as a low frequency silencing device with a low pressure loss penalty. To assess this possibility, the basic conditions that yield sound attenuation have to be understood, especially the influence of mean flow. Here a system with two parallel ducts will be studied; the extension to a $N$-duct system does not take the generic understanding of the attenuation conditions any further. As noted by Stewart [80, 81], there are two separate conditions that yield sound attenuation maxima. To derive them, one can start from the basic assumption for transmission loss of an anechoic termination downstream of the second node. This assumption implies that the acoustic intensity downstream of node two is zero—that is, both the acoustic pressure and volume velocity (which will be used as state variables here) are zero at node two. Now, to distinguish between the two attenuation conditions, it is sufficient to study the acoustic pressure in the upstream node. If the acoustic pressure in the upstream node is zero as well, the original destructive wave interference condition of Herschel [22] is obtained. In the more general case the state variables are non zero, and the maximum attenuation occurs when the state variables are out of phase by an odd integer multiple of $\pi/2$. From this basic interpretation of the involved acoustics the attenuation condition can be derived. For details see [82]. The two attenuation conditions are: 

**Type I**—the acoustic pressure is zero downstream but non-zero upstream.  

**Type II**—the acoustic pressure is zero at both nodes.

\[
\begin{align*}
\text{Type I} &: \quad \frac{1}{Z_1 e^{-ik_1 M_1 L_1}} \sin(k_1 L_1) + \frac{1}{Z_2 e^{-ik_2 M_2 L_2}} \sin(k_2 L_2) = 0 \\
\text{Type II} &: \quad k_1 L_1 = m * \pi, \quad m = 1,2,3,\ldots \\
& \quad k_2 L_2 = n * \pi, \quad n = 1,2,3,\ldots \\
& \quad \cos(k_1 L_1) \cos(k_2 L_2) - \cos(k_2 L_2 M_2 - k_1 L_1 M_1) \neq 0 
\end{align*}
\]

where $Z$ is the characteristic impedance, $k$ is the wave number, and $L$ the length of the ducts.

An interesting observation can be made from the derived expressions. The **Type I** condition depends on the ratio of the characteristic impedances in the ducts. Normally the condition of state is the same in all ducts, in which case the frequency where maximum attenuation occurs will shift with the area ratio between the ducts. As seen in Figure 10, the attenuation given by the **Type I** condition deteriorates significantly with mean flow. In the special case without mean flow and with the same characteristic impedance in all ducts, the **Type I** condition reduces to a function of the lengths of the ducts only. It then includes the solutions for the **Type II** condition as well [82].

The **Type II** condition is less sensitive to mean flow since each duct is described separately. As seen in Figure 10, the **Type II** condition yields attenuation at even as well as odd integral multiples of half a wavelength difference between the ducts when mean flow is present. This has been noted previously by Torregrossa et al. [83] and is explained by Eq. (41). Without mean flow the equation stipulates odd integral multiples of half a wavelength difference only (as in the original work by Herschel [22]), but with mean flow it allows for even multiples as well. Of course, with mean flow, the condition of Eq. (41) can disqualify other Mach number and frequency combinations. Examples are given in Appendix IV [82].
Figure 10. Transmission loss (TL) and acoustic pressure in the two nodes of a Herschel-Quincke arrangement where $L_2=2L_1$ and $Z_1=Z_2$. Without (upper) and with (bottom) mean flow. Simulations are made with straight duct two-port elements [82].

As illustrated in Figure 11, the Type I attenuation maxima change in frequency varying the impedance ratio between the ducts, while the Type II condition does not. Again, it is clear that the Type I condition is sensitive to mean flow and that there is an additional attenuation maxima of Type II with mean flow. In Figure 12 the influence of mean flow is studied further. First, with the same mean flow in both ducts, the Type II condition still holds, but the Type I condition deteriorates already at low Mach numbers. Then, with flow in only one duct, both attenuation conditions are significantly reduced at Mach numbers commonly seen in exhaust systems, say $M=0.2$. However, again the Type I condition is more sensitive than the Type II condition. In the case of equal Mach numbers in both ducts, Figure 12(a), an interesting observation can be made for the third attenuation maxima of Type II: at $M=0.33$ Eq. (41) is not fulfilled and the transmission loss is zero.
Figure 11. Influence of the ratio of the characteristic impedances between the two ducts on the transmission loss with and without mean flow. Simulations are made with straight duct two-port elements [82]. Note that the Helmholtz number based on the no flow case is used in both graphs.

Figure 12. Influence of mean flow on the transmission loss of the HQ tube. a) $M_1=M_2$, b) $M_1=0$, $M_2$ varies. Simulations are made with straight duct two-port elements [82].
2.7 The flow reversal resonator

Flow reversal resonators are commonly used elements in silencers, especially in single-unit systems such as are often used in commercial vehicles. In the concept presented here, a flow reversal chamber is turned into a flow-through resonator by acoustically short circuiting the inlet and outlet ducts (see Figure 13). In so doing, the acoustic behaviour is shifted from the quarter-wave resonator of the flow reversal chamber to the Helmholtz resonator given by the inlet and outlet ducts that act as neck pipes, and by the flow reversal chamber that acts as a volume. The lowest possible eigenfrequency of this system—the corresponding Helmholtz resonator—is given by a zero length short-circuit. The eigenfrequency of the system is easily shifted by changing the short circuit geometry. As shown later, a relatively small change in the geometry can result in a large shift in the eigenfrequency. This result makes the flow reversal resonator concept an ideal candidate for implementation as a semi-active device.

Two main questions arise in the practical implementation of the system. First, what is the eigenfrequency? Second, what is the influence of mean flow on a given configuration? The first question can be addressed using a simple lumped model where the system is represented as an equivalent circuit [84] (see Figure 14 in which the acoustic impedance $Z$ represents the apparent mass and spring characteristics of the components in the system). Keeping all parts of the system fixed, except for the short-circuit duct, the eigenfrequency of the system varies according to the changes in the element representing the short circuit, $Z_s$. Acting like a lumped mass, it is described as an inertance:

$$Z_s = \frac{i \omega \rho L_s}{S_s} \quad (42)$$

where $L_s$ and $S_s$ are the length and cross-sectional area of the short-circuit. Hence, the eigenfrequency of the system varies with the ratio of the length to the cross-sectional area of the short circuit duct.
Figure 14. Flow reversal resonator modelling: (Left) equivalent circuit; (Right) multiport approach.

Although the equivalent circuit approach yields some basic insights, the multiport modelling described in Section 2.2 is used here to also capture the influence of mean flow and higher-order effects such as wave propagation in the ducts. A multiport representation of the system is shown in Figure 14. The ducts in the system are simply represented by straight-duct two-port elements [20] where the influence of convective and visco-thermal effects given by the mean flow are compensated for using Documaci’s model [85]. The flow reversal chamber is built from two quarter-wave resonators. The first quarter-wave represents the volume from the bottom of the flow reversal to the apparent end of the inlet and outlet ducts given by end corrections, while the second quarter-wave resonator represents the remaining volume. The connection of the short-circuit duct with the inlet and outlet ducts is represented as three-ports (T-junctions). Flow related losses associated with the dividing and joining flow at the entrance and the exit of the short-circuit duct can then be included using the quasi-steady model derived in Section 2.3. It is assumed that these losses dominate over flow related losses given by the expansion and contraction of the flow in the flow reversal chamber.

The scattering matrix for the complete flow reversal resonator can then be derived. See Appendix V for the derivation and expressions for the individual elements.

\[
\begin{bmatrix}
\dot{\rho}_{in}^+ \\
\dot{\rho}_{out}^+
\end{bmatrix}
= S_{FRR}
\begin{bmatrix}
R_{in} & T_{out-in} \\
T_{in-out} & R_{out}
\end{bmatrix}
\begin{bmatrix}
\dot{\rho}_{in}^- \\
\dot{\rho}_{out}^-
\end{bmatrix}
\]  

(43)
3. Experimental techniques

The above derived theories and concepts have been validated experimentally. The techniques used are not new; rather, they are implementations of known methodologies. This section summarizes the techniques used and makes some general comments on lessons learned; further details of the instrumentation and geometry of each measurement are found in the individual papers (Appendices I-V).

The acoustic multiports discussed in the Theory section require knowledge of the state variables of choice at the ports. Acoustic pressure is used here since that is the quantity measured. However, it is easily transformed into other state variables—for example, acoustic stagnation enthalpy as proposed in the Theory section.

\[
\begin{bmatrix}
\hat{p}_{1+} \\
\hat{p}_{2+} \\
\vdots \\
\hat{p}_{N+}
\end{bmatrix}
= \begin{bmatrix}
R_1 & T_{I-I} & \cdots & T_{N-I} \\
T_{I-II} & R_2 & \cdots & T_{N-II} \\
\vdots & \vdots & \ddots & \vdots \\
T_{I-N} & T_{II-N} & \cdots & R_N
\end{bmatrix}
\begin{bmatrix}
\hat{p}_{1-} \\
\hat{p}_{2-} \\
\vdots \\
\hat{p}_{N-}
\end{bmatrix}
\]

(44)

A convenient tool for determining the travelling pressure amplitudes is the two-microphone wave decomposition methodology [86]. The acoustic field at each port is decomposed into forward and backward travelling plane wave pressure amplitudes using two microphones separated by a known distance \( s=x_2-x_1 \).

\[
\begin{bmatrix}
e^{-ikx_2} & e^{ikx_1} \\
e^{-ikx_1} & e^{ikx_2}
\end{bmatrix}
\begin{bmatrix}
\hat{p}_+ \\
\hat{p}_-
\end{bmatrix}
= \begin{bmatrix}
\hat{p}_1 \\
\hat{p}_1
\end{bmatrix}
\]

(45)

where \( k \) is the wave number and \( x_1 \) and \( x_2 \) are the distances from the two microphones to the port studied. To obtain good measurements, it is important to have a good model for the wave number. The importance of this increases with the microphone separation distance \( s \) and the distance from the first microphone to the test object. In this research the model proposed by Documaci [85] is used; this model accounts for convective as well as visco-thermal effects and is given by:

\[
k_\pm = \frac{\omega}{c_0} \frac{K_0}{1 \pm K_0 M}
\]

(46)

where \( K_0 \) is given by:

\[
K_0 = 1 + \bigg( (1-i)k_s \bigg( 1 + (\gamma-1)/\sqrt{Pr} \bigg) / \sqrt{2}
\]

(47)

where \( k_s = r \sqrt{\rho_0 \omega / \mu} \) is the shear wave number, \( \mu \) is the dynamic viscosity, \( r \) is the duct radius, \( \rho_0 \) is the ambient density, \( \gamma \) is the ratio of specific heats, and \( Pr \) is the Prandtl number.

In order to solve Eq. (45) for the forward and backward travelling plane wave pressure amplitudes, the two equations must be linearly independent. This restrains the useful frequency range of the measurement; when the microphone separation distance is near an integer multiple
of half a wavelength of the sound wave, the matrix inversion needed in Eq. (45) becomes sensitive to random errors. Since the bias error increases with frequency Åbom and Bodén [87] proposed the following criterion for determining the useful frequency range:

$$0.1 \pi < \frac{k_s s}{1 - M^2} < 0.8 \pi$$

(48)

This criterion usually implies that more than one microphone separation distance is required to cover the full frequency range of interest. For example, in this research an array of three microphones was used throughout. Random error may result from local turbulence at the microphones. This error can be suppressed by using a noise free reference signal such as the voltage to the loudspeaker [88]. That is, instead of using the pressure signals themselves, the transfer functions between the microphones and the reference signal are logged and used in the calculations.

With the wave decomposition methodology described above, each measurement yields \( N \) equations. Hence, to solve for the \( N \times N \) unknowns in the scattering matrix in Eq. (44), \( N \) linearly independent measurement cases are needed. There are two main ways of satisfying this requirement. Either the acoustic load on the system is varied \( N \) times or \( N \) different source configurations are applied. The multiple load [89] variant has the disadvantage of changing the base flow configuration of the system and is mainly used in situations where it is difficult to facilitate external sources, such as in the determination of the one-port representing an IC engine. The more common choice—and the one used here—is the source switching technique [90]. This technique is usually implemented using an external excitation source at each port and driving them one at a time. However, there are other ways of obtaining linearly independent cases. For example, combinations of sources can be used and for each combination the relative phase between the sources can be varied as well. The excitation of the system can—in theory—be made with either random noise or swept sine. Although equivalent in theory, swept sine is the more common choice for measurements with mean flow since the number of averages needed to obtain satisfactory results with random excitation is relatively large. Once the \( N \) linearly independent measurements have been made, one can solve for the scattering matrix:

$$S = \begin{bmatrix}
    H_{ep_x}^1 & H_{ep_x}^2 & \cdots & H_{ep_x}^N \\
    \vdots & \vdots & \ddots & \vdots \\
    H_{ep_x}^1 & H_{ep_x}^2 & \cdots & H_{ep_x}^N \\
\end{bmatrix}^{-1}
$$

(49)

where \( H_{ep_x}^y \) is the transfer function between the reference signal (here loudspeaker voltage) and the travelling plane wave pressure amplitude at port \( x \) for measurement case \( y \). This concludes the determination of the passive properties of the multiport, and, as described in the Theory section, the resulting scattering matrix will include useful information on the flow-acoustic coupling as long as the system remains linear.

The active part of the multiport, representing sources that are independent of the incident sound, is determined next. The source vector is defined as the generated sound that travels towards an anechoic termination. Of course, any practical test rig would not fulfil the conditions of anechoic terminations. Hence the effect of reflections at the system termination must be accounted for. The information about the system terminations, which is known from the determination of the
passive data [91], is simply the ratio of the incident to the outward travelling plane wave amplitude. However, the appropriate choice of excitation case must be made since any sound source beyond the chosen reference cross-section in the duct under study would corrupt the result, thus disqualifying the cases where the system has been excited from the same branch under study. But in the normal configuration, with possible excitation at each port, there will always be at least one appropriate measurement case available. Placing the reflection coefficient representing the system terminations along the leading diagonal in a matrix \( R \), the source vector can be estimated by [91]:

\[
\hat{p}^s = (E - SR)(E + R)^{-1} p = C\hat{p}
\]  

(50)

where \( E \) is the unit matrix and \( \hat{p} \) is the measured acoustic pressure at a reference position in each duct with any external excitation turned off. Normally the source is expressed in the form of a cross-spectrum matrix whose leading diagonal represents the amplitude of the source quantity while the cross terms contain the phase information.

\[
G^s = \hat{p}^s (\hat{p}^s)^* = \begin{bmatrix}
G_{p_1p_1} & G_{p_1p_1} & \cdots & G_{p_1p_1} \\
G_{p_1p_1} & G_{p_1p_1} & \cdots & G_{p_1p_1} \\
\vdots & \vdots & \ddots & \vdots \\
G_{p_1p_1} & G_{p_1p_1} & \cdots & G_{p_1p_1}
\end{bmatrix}
\]  

(51)

where * denotes Hermitian transpose and \( G_{xy} \) is the single-sided cross-spectrum of the signals \( x \) and \( y \). The leading diagonal of this matrix contains the desired source quantity. However, in this form, it is expressed in terms of autospectra, which are sensitive to uncorrelated sound such as local turbulence at the microphone positions. The estimates can be improved using more than one microphone position in each duct, where the microphones are sufficiently separated so that any local turbulence is uncorrelated. Eq. (51) could then be rewritten as

\[
G^s = T^{-1}C_2G^mC_1^s
\]  

(52)

where \( T \) is a matrix representing the transport of quantities between the two microphone positions in each duct (denoted by 1 and 2) and \( G^m \) is a matrix with the measured cross-spectra:

\[
G^m = \begin{bmatrix}
G_{p_1p_1}^s & G_{p_1p_1}^s & \cdots & G_{p_1p_1}^s \\
G_{p_1p_1}^s & G_{p_1p_1}^s & \cdots & G_{p_1p_1}^s \\
\vdots & \vdots & \ddots & \vdots \\
G_{p_1p_1}^s & G_{p_1p_1}^s & \cdots & G_{p_1p_1}^s
\end{bmatrix}
\]  

(53)

This is the procedure applied in this research to determine the active two- and three-ports. The estimation of both the passive and active parts could be improved by using over-determination techniques. See Holmberg et al. [92] for a discussion of techniques not used here.

Finally, to visualise the method described above, the three-port set up used to characterise the T-junctions is shown in Figure 15. The mean flow is provided from a pressurised anechoic chamber, which via a series of contracting sections is connected to the inlet of the test rig (at bottom right in the picture). The flow can then either enter the main branch or be routed to the
side branch. The mass flow in the ducts is monitored via Prandtl tubes in the upstream and downstream ducts. The duct length between the two monitoring positions, which is considerable, yields a backpressure that must be accounted for. This is done by calibrating the system for a given series of grazing mass flows and noting the corresponding flow velocity at the monitoring positions. Even when carefully done, the determination of the flow velocities is a source of error. This error can be reduced by using the microphones to determine the mean flow velocity [93], but this technique has not been applied here.

A straight duct is attached to each port. Three microphones are placed closest to the measurement object. An array of loudspeakers is placed next, at uneven separation distances, to ensure sufficient excitation at all frequencies, followed by a large resistive silencer. The silencer is not really needed for the wave decomposition technique described, but certainly does help since it reduces the problem of standing waves. However, for problems involving flow-acoustic coupling, good knowledge of the reflection of the system terminations is essential. With an unfortunate combination of reflections, the feedback needed to drive a self-sustained oscillation can be provided. As a non-linear phenomenon, the whole concept of determining the linear scattering matrix is disqualified. With the silencers used, the terminations are far from anechoic (cf. Figure 25 in the Results section) but the reflections are sufficiently low for the system to be linear. The linearity of the system was tested by increasing the acoustic excitation of the system at a given flow configuration with a gradual increase in the input power to the loudspeakers. As reported in [46], the aeroacoustical response did not vary with the excitation level, thus confirming the linearity of the system.

Another important practical issue in flow-acoustic problems is the geometry of the flow junctions. For example, both the flow profile approaching the junction and the geometric configuration of the edge of the orifice influence the flow-acoustic coupling in the T-junction [36]. In this research, the intention is always to have long, straight ducts before the test object to ensure a fully developed turbulent flow profile (in the configuration shown approximately 60d was achieved) and sharp, well-defined edges.

![Figure 15. Example of two-microphone method set up. Three-port configuration used for T-junctions.](image)
4. Results/Validation

In this section the key findings are reported and discussed. Again, more details are in the individual papers (Appendices I-V).

4.1 Aeroacoustics of T-junctions

Two different geometries were tested using the measurement setup in Figure 15. In the first geometry, the main duct and the side branch were of the same diameter, \( d = 0.057 \text{ m} \), while in the other, the side branch diameter was changed to 0.025 m. Those dimensions yield values for the area ratio \( \alpha \) of unity and 0.2, respectively. The first case was chosen since that is the most commonly reported case. The second case matches one short-circuit used in the flow reversal resonator concept. An extensive number of flow configurations were run for each geometry, including grazing and bias (in and out) flow alone as well as combinations thereof. For each configuration, the linear scattering matrix obtained can be used for further post processing to validate the models derived in the Theory section.

If not otherwise stated, the Strouhal number used in the presentation is defined as:

\[
St = \frac{fd_{\text{eff}}}{U_0}
\]

(54)

where \( d_{\text{eff}} = \pi d / 4 \) is the effective diameter of the side branch.

4.1.1 Validation of quasi-steady model

First, some typical examples of the measured scattering matrix elements are shown along with the quasi-steady approximations using Eq. (12) and Eq. (19) for the same cases. For the geometry with the smaller diameter side branch, \( \alpha = 0.2 \), flow configurations with relatively high grazing but moderate bias flow are chosen. For a fair comparison with the quasi-steady model, only the real part is presented. To isolate the influence of mean flow, the result without mean flow is subtracted from the result with mean flow. Examination of the scattering matrix reveals there are only slight changes with increasing Strouhal number in most elements for both flow configurations. The most obvious difference is for the elements \( S_{31} \) and \( S_{33} \). Here there are clear indications of flow-acoustic coupling for the grazing-bias inflow case since alternating positive and negative regions are seen. The two elements of interest, \( S_{31} \) and \( S_{33} \), represent the acoustic response in the side branch to an acoustic excitation in the upstream duct and the side branch, respectively.

The quasi-steady models agree well with the measured data for all scattering matrix elements in the quasi-steady limit. The useful Strouhal number range, however, is clearly limited by the flow-acoustic interaction in the case of grazing-bias inflow. That is, depending on the flow configuration, the upper Strouhal numbers at which the models are valid differ, but are at least 0.1-0.2—and are significantly higher for configurations with low constructive flow-acoustic interaction.
Figure 16. Scattering matrix elements. $\alpha=0.2$. Two flow configurations: Grazing-bias inflow ($M=0.05$, $M_{II}=1.05$, $M_{III}=0.025$), Grazing-bias outflow ($M=0.105$, $M_{II}=0.1$, $M_{III}=0.025$).

Normally the scattering matrix elements are not used individually. Instead the complete scattering matrix is used as a part in a network or to derive another quantity. Here, the maximum attenuation given by the power balance procedure suggested by Aurégan and Starobinski [68], presented in Section 2.4.3 Eq. (30), is used. Taking the geometry with a side branch of the same diameter as the main duct and maintaining a mean grazing flow velocity corresponding to $M=0.1$, the bias flow is then varied. In the grazing-bias inflow case, the bias flow is increased until it equals the flow in the main branch—that is, the downstream Mach number is 0.2. In the grazing-bias outflow case, the grazing flow is gradually bled off until it is all routed into the side branch—that is, bias outflow only occurs. This case resembles the case of a sharp 90 degree bend and can be compared with the quasi-steady bend model of Nygård [54], introduced in Section 2.3 Eq. (8).

Figure 17. Validation of the quasi-steady model for various flow configurations. $\alpha=1$ and the grazing mean flow Mach number in the upstream branch is always 0.1.
The agreement between the quasi-steady model and the measurements is generally very good. For the grazing-bias inflow case, the predictions seem to deteriorate slightly with increasing bias flow but always remain within 10% of the measured value. The quasi-steady model for grazing-bias outflow case yields excellent prediction for the entire range. In addition, the quasi-steady approximation of the bias outflow case as a sharp 90-degree bend also holds.

The validation of the quasi-steady model is satisfactory and proves the usefulness of the model in practical applications. Access to a model for such a common element as a T-junction that allows for any flow configuration is useful, not only when studying exhaust systems, but also when studying any flow duct application where the low Strouhal number regime is of interest. The implementation is straightforward; here, tabulated data were used for the pressure loss coefficients—and derivatives thereof—but it should be possible to base it on simple steady state CFD simulations instead.

4.1.2 Acoustic impedance

As discussed in the Theory section, the aeroacoustic response of the orifice varies with the incidence of the acoustic perturbation to the system. Hence, the results for each flow configuration are presented for the three cases separately. The main interest is the change in the acoustic impedance, as a measure of the aeroacoustic response to a given excitation, induced by mean flow. Therefore the results are presented as the deviation from the no flow case:

\[
\begin{align*}
  r_f &= Re(Z) - Re(Z_{U=0}) \\
  \chi_f &= Im(Z) - Im(Z_{U=0})
\end{align*}
\]  

First, the acoustic impedance for the grazing-bias flow configurations is compared with the limiting cases—that is, grazing flow or bias flow separately (see Figure 18). The studied geometric configuration has a side branch of the same diameter as the main duct, \(d=0.057\) m. With grazing flow alone, there are alternating regions of positive and negative resistance, especially when the system is excited from the upstream or side branch. This indicates flow-acoustic interaction, which, as expected [34, 94], decays with increasing Strouhal number. The first trough in the resistance is centred at a Strouhal number of 0.45, which agrees with previously published results [39, 95]. The smaller diameter side branch duct yields a first minimum at 0.37. Assuming that the approaching flow profiles are identical, this change corresponds well with the shift in convection velocity of the hydrodynamic disturbance with a change in the diameter seen by Golliard (reported in Kooijman et al. [36]) of \((d_1/d_2)^{0.2}\).

Bias flow, on the other hand, results in positive resistance—that is, sound attenuation—for all cases. When the T-junction is excited from the side branch, the attenuation with bias inflow and bias outflow is of similar moderate amplitude. If the T-junction is excited from the upstream duct, the acoustic field is more attenuated for bias outflow than for bias inflow. The opposite result is obtained when exciting the T-junction from the downstream duct. Then the bias inflow case yields a stronger attenuation than the bias outflow. Both these cases seem consistent with the idea that the incoming acoustic perturbation is attenuated more when passing the same edge where the mean flow turns.
Figure 18. Grazing-bias flow compared to the limiting cases, that is, grazing flow or bias flow separately. Varying acoustic incidence and impedance formulation: a) upstream $Z_{in}$, b) downstream $Z_{out}$, c) side branch $Z_{br}$. The Strouhal number is always based on the velocity in the upstream duct, except for bias inflow only where the velocity in the side branch is used.
With grazing-bias outflow, the constructive flow-acoustic interaction seen in the grazing flow case is completely cancelled. This can be understood from the simple flow simulations and discussion in Section 2.1.2. The hydrodynamic disturbance shed at the leading edge will not travel across the whole orifice but rather turns into the side branch, thus not completing the productive phase. One interesting observation is that for excitation from the downstream branch, one gets a small but increasingly negative resistance with decreasing Strouhal number.

With grazing-bias inflow, the alternating regions of constructive and destructive flow-acoustic interaction given by grazing flow are still seen, but they shift in frequency (here seen as a shift in the Strouhal number since the same reference velocity is used for all cases). Again, an intuitive interpretation can be made from the flow field simulation of Section 2.1.2: the hydrodynamic disturbances still travel across the orifice, but both the convection velocity and the trajectory change. Thus, to match the time it takes the hydrodynamic disturbance to pass the orifice with the acoustic oscillation period the frequency has to shift accordingly.

A more detailed study on the grazing-bias flow cases is shown in Figure 19. The upstream Mach number is always 0.1. Then flow either enters via the side branch or bleeds off into it; the acoustic excitation is from the side branch. Beginning with the grazing-bias outflow, it is clear that the constructive flow-acoustic interaction is cancelled already at the lowest bias outflow rate. With grazing-bias inflow, the shift in frequency/Strouhal number is large initially but gradually decreases with increasing bias Mach number. Also, with increasing bias inflow Mach number the amplitude of the oscillations decreases. For the case presented—the impedance seen from the side branch—the reactance changes sign at maximum negative resistance, which is consistent with a “resonant” system. Comparisons may be made to the zero crossing of the real axis seen in the Nyquist plot at the frequency of the critical zeros.

Figure 19. Resistance and reactance for the impedance given by acoustic excitation from the side branch Eq. (23) for grazing-bias inflow and grazing-bias outflow, respectively. The grazing mean flow Mach number is always 0.1 in the upstream duct, which is also the reference velocity for the Strouhal number used. The geometry with the side branch of the same diameter as the main branch is chosen, $\alpha=1$. 

---

31---
A clear trend is seen when the frequencies at which the maxima of negative resistance occur for all the grazing-bias inflow configurations available (for both geometries) are divided by the frequency at which the maximum in negative resistance occurs with grazing flow alone. Even though the actual geometric and flow configurations are rather different, the shift in frequency collapses with the bias inflow Mach number. The one exception is the second amplification mode for the smaller diameter side branch. The collapse of the data with bias inflow Mach number can be used to derive an empirical compensation term for the Strouhal number when comparing different flow configurations [41]. However, such empirical models are always application-dependent, and therefore are not given here.

A more generic approach is to predict the change in frequency by studying the mean flow field only. An efficient way of doing this is steady state CFD simulations as illustrated in Section 2.1.2. A massless particle is seeded at the leading edge of the T-junction for each grazing-bias inflow configuration. The velocity and position of the particle is monitored as it travels across the orifice. For each flow configuration one can then define a travel time across the orifice. For the configuration with a side branch of the same diameter as the main duct and grazing flow alone, the observed travel time corresponds to a Strouhal number of 0.45, which agrees with the measurements. The change in travel time of the seeded particle as a function of the bias inflow Mach number is then compared with the acoustic measurement for the same cases. The Strouhal number is a comparison of the time scales of the hydrodynamic and acoustic fields. Hence one expects the change in travel time across the orifice to correlate with the change in frequency, thus holding the Strouhal number constant. As seen in Figure 20, the CFD simulations yield a prediction of the change in frequency as a function of the bias inflow rate that is at least within the tolerances of the measurements. This type of simulation is computationally inexpensive and industry standard today. Hence, the presented approach offers a simple and effective engineering tool for obtaining an estimate of the critical frequencies for constructive flow-acoustic interaction with grazing-bias inflow.

![Figure 20. Change in frequency and travel time across the orifice as a function of bias inflow Mach number at a constant grazing flow. m refers to hydrodynamic mode number.](image)

Only a few examples of the large set of data available are shown here. More examples can be found in Appendix I and [41].
4.1.3 Power balances—whistling potentiality

Although acoustic impedance is a common way of describing the acoustic behaviour of many configurations, it is not necessarily straightforward to interpret, especially when mean flow is present. For example, is negative resistance always an indication of sound generation? This question can be addressed by comparing the impedance results with a power balance over the system. The usefulness of the enthalpy impedance formulation is shown in Figure 21 for a T-junction that is excited from the side branch. The results for the time-averaged power should be interpreted such that a positive value indicates sound amplification and a negative value sound attenuation. Starting with grazing flow alone, the enthalpy and standard impedance formulations yield the same result since there is a stagnant uniform fluid in the side branch. Also, the Strouhal number regions of negative resistance correlate well with the Strouhal number regions of sound amplification. However, for the grazing-bias inflow case, the standard impedance formulation over-predicts the Strouhal number range where sound is amplified, while the enthalpy formulation compensates for the flow effects and predicts the amplification regions correctly.

![Figure 21](image1)

**Figure 21.** Resistance, when exciting the system from the side branch. Comparison of the enthalpy impedance formulation, Eq. (23), with the “standard” formulation, p/q, and a power balance according to Eq. (26). Two flow configurations: grazing ($M_g=0.1, M_b=0$) and grazing-bias in ($M_g=0.1, M_b=0.05$)

![Figure 22](image2)

**Figure 22.** Resistance, when exciting the system from the upstream branch. Comparison of the enthalpy impedance formulation, Eq. (21), with the “standard” formulation, p/q, and a power balance according to Eq. (24). Two flow configurations: grazing ($M_g=0.1, M_b=0$) and grazing-bias in ($M_g=0.1, M_b=0.05$)

Performing the same exercise for the case where the system is excited from the upstream branch a completely different result is seen (see Figure 22). The definition of impedance according to Eq. (21) yields insight into the response of the shear layer to an excitation from the upstream duct, but does not necessarily give an indication of the exchange of energy between the hydrodynamic
and acoustic fields. In the grazing flow case, it seems that the system amplifies incoming sound 
when the resistance in the side branch is high—that is, “blocked”. Hence, none of the impedance 
formulations is useful for predicting amplification regions by definition, although the enthalpy 
formulation at least corrects the results to the correct mean level to oscillate around. Generally it 
seems acoustic impedance is not a good tool to understand the flow-acoustic interaction for 
complex configurations such as the T-junctions.

Studying the individual parts that contribute to the total time-averaged power out provides an 
increased understanding of the two different excitation cases discussed above (see Figure 23). For 
the case with excitation from the upstream branch, the constructive interaction of the acoustic 
field with the hydrodynamic disturbances in the shear layer indicated by the negative resistance in 
$Z_{I-III}$ is seen at $St=0.45$. However, it does not result in a net power out exceeding unity since at 
the same time there is a reduction in the power transmitted downstream. Instead, as seen from 
the impedance plots, the maximum time-averaged acoustic power is obtained when the side 
branch is “blocked” and there is an increase in the power transmitted downstream.

When the T-junction is excited from the side branch, the alternating regions of positive and 
negative interaction between the two fields are clearly seen. At the maxima, the main contribution 
comes from an increase in the transmission from the side branch to the downstream duct as well 
as in the reflection into the side branch itself. Although not shown here, the power balance given 
by excitation from the downstream duct confirms the results from the impedance section (4.1.2) 
that the flow-acoustic interaction is weak. See Appendix I and [41].

Next, the power balance results are used to design a system, based on the T-junction that may 
whistle. The purpose here is to illustrate the usefulness of the linear stability analysis. For the 
given test setup for the T-junction, as seen in Figure 15, an adjustable resonator arrangement that 
could be implemented in the side branch was designed. It is basically a quarter wave resonator, of 
adjustable length, attached perpendicularly to a piece of pipe of the same diameter as the side 
branch (see Figure 24). When the appropriate amplification regions are identified, the 
arrangement is placed so that the orifice of the quarter-wave resonator is located at half a 
wavelength of the frequency yielding maximum amplification into the side branch. Now, 
adjusting the quarter-wave resonator to the same frequency, a system that provides acoustic 
reflection with a particle velocity maximum at the shear layer is provided (remember that the 
system is triggered at a certain critical ratio of the particle velocity to mean flow velocity).

To avoid any problems with flow-generated sound over the resonator arrangement, a flow 
configuration with only grazing flow ($M=0.15$) was chosen. In the resulting configuration— 
grazing flow with excitation from the side branch—it is clear from the power balance of Figure

![Figure 23. Power balances according to Eq. (24) for excitation from the upstream duct and Eq. (26) for excitation from the side branch. Grazing flow only: $M=0.1$.](image)
that there are distinct amplification regions centred around integer multiples of $St=0.45$. Hence, the resonator arrangement is tuned towards the first two amplification maxima, $St=0.45$ and 0.90, which at the given flow velocity correspond to approximately 510 and 1020 Hz, respectively.

4.1.4 Whistling

In the previous sections the linear scattering matrix, including flow-acoustic interaction effects, was obtained and post processed to show the Strouhal number regions where the T-junction acts as an amplifier. However, as discussed in the Theory section, for the system to whistle, appropriately reflecting boundaries must provide the required feedback. A first step is then to study the three components of the reflection matrix $R$ representing the reflection seen looking into each branch from the T-junction (see Figure 25). At $St=0.45$ the reflection coefficients in the upstream and downstream ducts, $R_I$ and $R_{II}$, are rather low while at $St=0.9$ they are considerably higher. As shown below, this will have a significant effect on the system’s ability to form a fluid-driven whistle. With the proposed resonator arrangement, the reflection coefficient seen in the side branch, $R_{III}$, approaches unity at the targeted Strouhal numbers, while it is low for the rest of the Strouhal number range.
Before applying the Nyquist criterion to the two resonator arrangements, the discrete frequencies at which the resonators are tuned may be studied to get an idea of the stability limits of the system. For the system to be unstable, the real part of the function \( D(\omega) = \text{det}(E - SR) \), Eq. (35), must be negative at these frequencies (N.B. This is a necessary but not sufficient condition). Thus, by varying the reflections in the ducts attached to the T-junction, indications of the stability limits of the system can be found. First, by applying the actual rig conditions—that is, the acoustic reflections in the upstream and downstream ducts (seen in Figure 25)—and by varying the reflection coefficient in the side branch, it is clear the resonator arrangement tuned towards \( St = 0.45 \) never can become unstable, while the other resonator arrangement tuned towards \( St = 0.9 \) may become unstable provided the reflection coefficient in the side branch is greater than -0.92. The reason that one arrangement may be unstable while the other is not, even though the same reflection coefficient is provided in the side branch, is due to the difference in reflection coefficients in the upstream and downstream ducts. To further illustrate this, the resonator tuned towards \( St = 0.9 \), but assuming reflection-free terminations in the upstream and downstream branches, is shown as well. Now the system cannot become unstable even with a perfect reflecting boundary in the side branch.

The above discussion is validated by the Nyquist stability plot presented in Figure 27. For both arrangements, there are many encirclements around the base point \((1, i0)\) given by anechoic terminations. For the arrangement tuned towards \( St = 0.45 \), none encircles the origin, and thus the arrangement is linearly stable. The other arrangement, tuned towards \( St = 0.9 \), displays one encirclement around the origin, and the Strouhal number at the crossing of the negative real axis is 0.9. The other encirclement, which is close to the origin but does not encircle it, is for a Strouhal number just below 0.9.

Finally, the stability analysis is validated by actual measurements. From the procedure described in the Experimental section, the source cross-spectrum matrix, whose leading diagonal yields the amplitude of the source terms, is obtained. The results are adjusted to account for rig reflections and represent the outward travelling wave amplitudes. In comparing the available cases (the original rig setup without a resonator arrangement and the two resonator arrangements), it is
clear that a distinct tone is created by the resonator arrangement tuned towards $St=0.9$ while the two other cases do not sustain an oscillation. It can be noted that the fact that there is a whistle—that is, a nonlinear phenomenon—means that the data obtained cannot be used to predict, for example, the whistling amplitude in another case.

Figure 27. Nyquist stability plot for the two resonator arrangements. The traces are rotating clockwise with increasing frequency/Strouhal number.

Figure 28. Normalised oscillation amplitude in the upstream and downstream ducts with and without the resonator arrangements.
4.2 The Herschel-Quincke tube

The Herschel-Quincke tube has been realised in two different configurations where the difference lies in the geometry of the nodal points connecting the parallel ducts. First, T-junctions are used. This is the most common configuration. However, as shown in the previous section, T-junctions may be prone to whistling. To avoid whistling, and to have an equal distribution of the mass flow between the ducts, Y-junctions were applied.

4.2.1 Implementation with T-junctions

![Figure 29. Schematic of the Herschel-Quincke setup.](image)

The Herschel-Quincke tube arrangement should be able to sustain an oscillation by internal oscillations, providing a good test of the stability analysis derived in previous sections. In order to use the linear scattering matrices already available, the Herschel-Quincke tube was built from the same T-junctions as described previously (the \( \alpha = 1 \) case). Next, to mimic the flow configuration used for the stability analysis of the T-junction alone—grazing flow of \( M = 0.15 \)—a plastic film was inserted in duct 2 to prevent any bias mean flow. The plastic film was characterised separately before the measurements to ensure that it was sufficiently acoustically “transparent” so as not to influence the analysis.

By studying the internal reflection only, one assumes the upstream and downstream terminations are anechoic in the analysis. This assumption reduces the scattering and reflection matrices used in the stability analysis to two-ports. Since the two nodes interact, the stability analysis may be applied to any of them. Choosing the upstream node, the updated matrices become:

\[
\begin{bmatrix}
R^A_{11} & T^A_{11-2} \\
T^A_{11-2} & R^A_{22}
\end{bmatrix},
\begin{bmatrix}
R^B_{11} & e^{-i(kL_1+kL_2)}T^B_{11-2} & e^{-i(kL_2)}R^B_{22}
\end{bmatrix}
\]

where the diagonal elements of \( R \) represent the reflection seen from the ports at Node A from the corresponding port at Node B, and the off-diagonal elements represent the transfer of a wave from one of the active ports of Node A, via Node B, to the other active port of Node A.

From the fundamental understanding of the Herschel-Quincke tube derived in the Theory section, one can now construct illustrative cases. First, as noted in the Theory section, the first attenuation condition is sensitive to differences in the mean flow between the ducts and
disqualifies in the present setup. The second attenuation condition is less sensitive to mean flow differences between the ducts and has the interesting property that additional damping maxima occur when mean flow is present. At the attenuation maxima, the acoustic pressure is zero in the nodal point—that is, in the T-junction. One can now construct a Herschel-Quincke tube (although the duct lengths themselves are well matched to provide the reflections needed to sustain an oscillations) that will not provide the required feedback due to the pressure minima given by the combinations of the ducts. Equally, one can construct a case with attenuation minima at the frequencies where the system amplifies sound, thus allowing maximum possible feedback. The two cases can be achieved by setting the length of duct 1 to half a wavelength of the targeted amplification frequency and then letting duct 2 first be approximately twice as long as duct 1. This yields a system with attenuation maxima at the nodes. A system with a length ratio of three instead would have attenuation minima at the same frequencies. That is, one expects the arrangement with \( L_2/L_1 = 2 \) not to whistle, but the arrangement with \( L_2/L_1 = 3 \) may whistle.

Targeting the first amplification mode of the T-junctions at \( St = 0.45 \), the length of duct 1 was always 0.335 m while the lengths of duct 2 were 0.62 m and 0.98 m for the two cases. This corresponds to length ratios \( L_2/L_1 \) of 1.85 and 2.92, respectively. The deviation from the targeted values of two and three is simply due to practical difficulties with the installation. In the analysis the actual length ratios are used, but for ease of notation 2 and 3 are still used in the presentation. Now, applying the Nyquist stability criterion (Figure 30), it is seen that, as predicted, the arrangement with \( L_2/L_1 = 3 \) should whistle at the targeted Strouhal number, while the arrangement with \( L_2/L_1 = 2 \) is stable. The two encirclements for the \( L_2/L_1 = 3 \) configuration that are close to the origin correspond to the multiples of the targeted Strouhal numbers—that is, \( St = 0.9 \) and 1.35.

![Figure 30. Nyquist stability plot for the two Herschel-Quincke tube configurations.](image)

Finally, the predictions are validated experimentally. As usual, the two-microphone wave decomposition method is used to determine the travelling wave amplitudes upstream and downstream of the Herschel-Quincke tube. In this experiment only the active properties were studied. Hence, no passive data were available to compensate for the rig reflections. However, this will only have a slight effect on the amplitudes of the oscillations. From the result seen in Figure 31, it is clear that the arrangement with \( L_2/L_1 = 3 \) whistles at the targeted frequency while the other arrangement does not sustain an oscillation.
It has been shown that with unfortunate flow and geometric configurations, one can make a Herschel-Quincke tube whistle if the nodes are implemented as T-junctions. This result has not been reported in previous research [14, 77, 96-98] on similar setups that have focused on the passive properties that typically counteract the feedback to the whistle. It can be noted that even though a configuration with only grazing flow was forced here, other combinations are possible. Allowing some of the flow to pass through duct 2, one T-junction with grazing-bias outflow and one T-junction with grazing-bias inflow are obtained. As seen the previous sections, the latter is still prone to whistling.

![Figure 31. Normalised oscillation amplitude travelling outward from the Herschel-Quincke tube in the upstream (Top) and downstream (Bottom) directions.](image)

### 4.2.2 Implementation with Y-junctions

The passive properties of the Herschel-Quincke tube were investigated using a setup where the nodes were implemented as Y-junctions (see Figure 32). This setup had the advantage of avoiding flow-acoustic coupling effects seen with the T-junctions. Also, the relative mass flow distribution in each duct could be close to 50%, thus allowing the attenuation conditions to be studied at relevant flow speeds. This setup is not a common way of implementing the H-Q tube, but it resembles the configuration studied by Fuller and Bies [99]. As before, the two-microphone wave decomposition technique was used to determine the linear properties of the system. For more details of the setup and instrumentation, see Appendix IV [82].
A rather surprising result was observed without flow. See Figure 33. The attenuation maxima associated with the Type I condition were significantly more damped than expected from the simulations. This result was studied in detail and was shown to be due to imperfections in the node geometry [100]. The Y-junctions were characterised experimentally as three-ports, as was done previously for the T-junctions. The two nodes were then connected with ideal straight ducts to reassemble the Herschel-Quincke tube; this yielded a result in good agreement with the experimental results for the complete Herschel-Quincke tube—including the unexpected damping of the Type I attenuation maxima—thus validating that the observed phenomenon was due to the node geometry. Generally, the simple modelling approach of having parallel-coupled straight duct elements works well without flow: it captures the frequencies and amplitudes of the attenuation maxima.

The measurements with flow validate the general trends discussed in the Theory section—that is, the Type I attenuation condition is more sensitive to mean flow than the Type II attenuation condition. Moreover, one gets new attenuation maxima of Type II at even multiples of half a wavelength difference between the ducts. However, the use of straight duct elements only when modelling is insufficient. The system then overestimates the Type II attenuation maxima but underestimates those for Type I. However, the results can be improved using the quasi-steady bend model of Nygård presented in Section 2.2 Eq. (8). Each of the two ducts is now represented as two straight-duct elements before and after the lumped bend model, placed at the ends of the bends. The appropriate position for implementing the bend model can be discussed; the choice made was based on finding the position where the vena-contracta effect was expected to be most accentuated. With the bend model, the agreement is much improved and is sufficient for engineering applications. It can be noted that the unexpected phenomenon seen without mean flow for the Type I attenuation condition is not experienced with flow. This is explained by the strong influence of mean flow on the Type I condition that masks the behaviour seen without flow.

The measurements validate the derived attenuation condition and the predicted influence of mean flow. Although attenuation is provided in a wider frequency range than, for example, a Helmholtz or quarter-wave resonator, the Herschel-Quincke tube is not suitable for implementation, at least in its base configuration, in an automotive exhaust system, because of the long duct length needed to provide damping at the low frequencies and high temperatures involved.
Figure 33. Measured and simulated transmission loss for the Herschel-Quincke tube implemented with Y-junctions. With mean flow ($M=0.15$) and without mean flow.

4.3 The flow reversal resonator

The flow reversal chamber of the flow reversal resonator was implemented using a steel cylindrical duct with a diameter of 0.206 m and a length of 0.207 m. The inlet and outlet ducts were of diameter 0.056 m. The centre of the short-circuit duct was 0.190 m from the bottom of the flow reversal chamber. The actual short-circuit ducts were implemented as stiff rubber hoses. Three diameters, 10, 16, and 25 mm, were tested and for each diameter three lengths, 30, 40, and 50 mm, were evaluated.

Figure 34. Shift in eigenfrequency normalised with the corresponding Helmholtz resonator ($f/f_0$) as a function of the ratio of the length to cross-sectional area of the short-circuit duct for the test setup. The solid line represents predictions and the markers represent measurements.

First, to validate that the change in eigenfrequency of the flow reversal resonator is determined by the ratio of the length to the cross-sectional area of the short-circuit duct (as long as it is acoustically compact and the rest of the system is unchanged), the multiport modelling approach was applied using two different short-circuit duct diameters. For both diameters, the length was varied within a given interval. For each simulation, the eigenfrequency was then noted. As
expected, the agreement was very good. The same eigenfrequency was predicted for both diameters as long as the $L/S$ ratio was constant. The obtained change in eigenfrequency normalised with the eigenfrequency of the corresponding idealised Helmholtz resonator of the system is shown in Figure 34. In addition, the measured eigenfrequency of the nine test configurations—under no flow conditions—are plotted. The values are within a few percent of the predicted eigenfrequency for the corresponding $L/S$ ratio, which is satisfactory considering the practical difficulties involved, such as the implementation of the short circuit.

The basic characteristics of the system and the potential for semi-active control are illustrated in Figure 35. The eigenfrequency of the flow reversal chamber, when the short circuit is closed, is approximately five times greater than for the corresponding Helmholtz resonator. This is a substantial change and clearly shows the potential for the flow reversal resonator as a semi-active device. For example, in the given test setup, this corresponds to a range of more than 500 Hz, which is more than enough to adapt to an engine order during transient operation. In studying the two extreme cases tested, it is clear that the characteristics of the resonator change depending on whether the system is closer to the resonator limit or the flow reversal (expansion chamber) limit. For the low $L/S$ ratio case, the acoustic response is typical for a resonator with high attenuation in a narrow bandwidth that is rather sensitive to mean flow. Also, the attenuation in the frequency range outside the attenuation maximum is low. The attenuation maximum for the high $L/S$ case, on the other hand, is not as high as for the low $L/S$ case. However, the peak is broader and less sensitive to mean flow. Also, there is significant attenuation in the remaining frequency range. For both cases, the broadband part of the attenuation is not sensitive to mean flow.

![Figure 35. Measured transmission loss for various configurations.](image)

Finally, the multiport modelling approach is compared to the experiments (see Figure 36). The geometric configuration with the widest short-circuit duct, $d_s=25$ mm, was chosen as the configuration that is most sensitive to mean flow effects. The coefficient used for the quasi-steady model was estimated by first performing a steady state incompressible CFD simulation for two inlet flow speeds (see Appendix V). Both cases indicated that the relative mass flow passing through the short circuit was $\sim 10\%$. It was then assumed that this is the case for all inlet flow speeds tested. The pressure loss coefficient was then extracted from tabulated data [53]. Since the coefficients are given as a function of the area and the volume flow ratio between the main duct
and the side branch, the same coefficient could be used for all simulations. As seen in Figure 36, the fit of the predictions to the measured data is excellent except for the highest Mach number tested. This exception is most likely due to losses in the flow reversal chamber not accounted for by the model. This is further indicated by the fact that the results at higher frequencies start to deviate from the results for the other flow configuration, which are similar to the no flow case.

The potential of the flow reversal resonator concept has been illustrated. If the systems target as low frequencies as possible, the short-circuit duct has to be short and wide. However, this configuration suffers from the same problems as other side branch resonators—the attenuation bandwidth is narrow and the sensitivity to mean flow is high. Having a higher L/S ratio, the character of the flow reversal resonator tends more towards that of the expansion chamber. Although the attenuation maxima have shifted considerably downwards in frequency from the original flow reversal, there is still significant broadband attenuation and the sensitivity to mean flow is much improved. Also, the smaller the dimension of the short circuit, the greater the influence of a given geometric change on the eigenfrequency. In a typical automotive application, the silencer seldom needs to provide more than 30 dB attenuation at any frequency. Hence if a component of the silencer, as shown here, can provide >20 dB attenuation in a wide frequency range and can also adapt to changes in the incoming acoustic disturbance, this silencer component should prove most useful.

Figure 36. Measured and simulated transmission loss at varying inlet mean flows. \( d_s=25 \text{ mm}, L_s=40 \text{ mm} \).
5. Summary and Outlook

This thesis provides a formalism for including flow-acoustic interaction effects in linear multiports with the aims of quantifying the amplification and attenuation of incident sound and, perhaps more importantly, predicting nonlinear phenomena such as whistling using linear models. This research is of practical importance for the design of complex flow duct networks.

The measurement results presented are the first experimental representation of the scattering of a T-junction subjected to grazing flow and the only linear data currently available for T-junctions with grazing-bias flow. These data should be useful as a reference case in, for example, numerical simulations [45]. It is shown that the interaction of the acoustic and hydrodynamic fields at a T-junction varies significantly, not only with the flow configuration but also with the incidence of the acoustic field.

T-junctions are characterized for the whole plane wave range. However, in practical applications, such as automotive exhaust systems, the low frequency limit is usually the main interest. Thus a quasi-steady approximation of the scattering matrix representing the T-junction is derived, applicable to all possible flow configurations up to approximately half the Strouhal number of the first amplification maximum given by the constructive flow-acoustic coupling, a range sufficient for most applications. The quasi-steady scattering matrix approximation is successfully applied in the flow reversal resonator concept introduced in Section 2.7. The derived formalism is based on a pressure loss coefficient between the branches, and could easily be extended to an N-port.

The two attenuation conditions of the Herschel-Quincke tube are explained in section 2.6, particularly with regard to their sensitivity to mean flow. The first attenuation condition is generally more sensitive to disturbances and cancels even at rather low Mach numbers. The second attenuation condition—the classic one given by multiples of half a wavelength length difference—is less sensitive to disturbances and introduces damping at additional frequencies when flow is added. The prediction of the passive properties of the Herschel Quincke tube with mean flow is shown to improve significantly using a quasi-steady model for the losses associated with flow separation in the ducts. When implemented with a T-junction, certain combinations of the duct lengths may result in sufficient acoustic feedback to cause the Herschel-Quincke tube to whistle. The whistling frequencies are successfully predicted using the derived multiport formalism. Because of the required lengths of the ducts, the Herschel-Quincke tube may be impractical for use in automotive exhaust systems. However, it may be useful in applications where the problem frequencies are higher and/or the propagation media have a lower phase velocity.

The thesis presents a new silencer concept—the flow reversal resonator. Taking advantage of a design element commonly used in automotive silencers, a flow reversal chamber, it requires no additional volume. In addition, since the eigenfrequency of the resonator can be easily varied with a small geometrical change of the short-circuit, the concept can provide attenuation over a wide frequency range with a compact implementation. The preferred implementation would be with a short-circuit tuning the system towards the expansion chamber limit. The sensitivity to mean flow is then low and the shift in eigenfrequency large for a given geometrical change of the short circuit. The flow reversal concept was successfully modelled using the multiport concept, both with and without mean flow. Plans are now in place to test a semi-active prototype of the flow reversal resonator on a vehicle in order to prove the practicality of the concept.
The thesis also investigates the use of steady-state CFD simulations for interpretation of aeroacoustical phenomena. Such simulations can be used to obtain pressure loss coefficients used in the quasi-steady model for the T-junction, or to determine the convection velocity of the vorticity in the shear layer across a side branch orifice. Where applicable, steady state CFD simulations are a computationally effective tool for rapid design cycles.

The thesis improves our understanding of the interaction of the hydrodynamic and acoustic fields in T-junctions. However, there remain a number of interesting research issues to explore. Research into the stability of the shear layer under various flow and acoustic configurations has begun in which a more defined geometry (rectangular ducts) is used. This research will involve acoustic measurements as well as characterization and numerical simulations of the flow fields. Other areas of interest in this research are the geometries of the orifice, including different edge geometries and perforated plates.

Whistling must be prevented if exhaust systems are to meet legal limits for noise emissions. Hence, the thesis has focused on identifying—and avoiding—fluid driven whistles, not quantifying their amplitude. However, in other applications it can be of interest to predict the oscillation amplitude. This capability could be incorporated in the derived formalism by forcing the system to become nonlinear (but not unstable) and determining an amplitude dependent scattering matrix. That matrix could then be balanced with the system losses to obtain an estimate of the oscillation amplitude.

References


