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Non-Unitary Neutrino Mixing from an Extra-Dimensional Seesaw Model

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Abstract

We study the generation of light neutrino masses in an extra-dimensional model, where right-handed neutrinos are allowed to propagate in the extra dimension, while the Standard model (SM) particles are confined to a brane. Motivated by the fact that extra-dimensional models are non-renormalizable, we truncate the Kaluza–Klein (KK) towers at a maximal KK index. The structure of the bulk Majorana mass term, motivated by the Sherk–Schwarz mechanism, implies that the right-handed KK neutrinos pair to form Dirac neutrinos, except for a number of unpaired Majorana neutrinos at the top of each tower. These heavy Majorana neutrinos are the only sources of lepton number breaking in the model, and similarly to the type-I seesaw mechanism, they naturally generate small masses for the left-handed neutrinos. The lower KK modes mix with the light neutrinos, and the mixing effects are not suppressed with respect to the light neutrino masses. Compared to conventional fermionic seesaw models, the non-unitary effects induced by such mixing are quite significant. We study the signals of this model at the Large Hadron Collider (LHC), and find that the current bounds on the non-unitarity parameters are strong enough to exclude an observation.

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I. INTRODUCTION

Experimental studies of neutrino oscillations have provided us with compelling evidence that neutrinos have masses and lepton flavors mix. Among various theoretical attempts, the famous seesaw mechanism [1–3] provides us with a very natural way of understanding why the masses of the three known neutrinos are so small compared to the masses of the other Standard Model (SM) fermions. In the simplest type-I seesaw model, heavy right-handed neutrinos with a mass scale $M_{\rm R}$ are introduced in addition to the Standard Model (SM) particle content. In order to stabilize the light neutrino masses around the sub-eV scale, $M_{\rm R} \sim 10^{14}$ GeV is naturally expected, and the Dirac mass $m_{\rm D}$ between the left- and right-handed neutrinos is expected to be comparable with the mass of the top quark. The testability of such conventional seesaw models is therefore questionable. Also, the heavy right-handed neutrinos potentially contribute to the hierarchy problem through loop corrections to the Higgs potential, unless a supersymmetric framework is considered.

The Large Hadron Collider (LHC) will soon bring us to TeV scale physics, and the question of whether we can find hints on the neutrino mass generation mechanism at the LHC or not is relevant and interesting. Actually, there are several indications that new physics will show up at the TeV scale, in particular theories that are able to stabilize the Higgs mass and to solve the unnatural gauge hierarchy problem. The geometric mean of the Planck mass and the 2.7 K background temperature also suggests that 1 TeV is the maximum mass that any cosmologically stable perturbatively coupled elementary particle can have, otherwise the density of the Universe exceeds its critical value [4]. Within the seesaw framework, for the purpose of lowering the seesaw scale without spoiling the naturalness criterion, some underlying symmetry preserving the lepton number, L, is usually incorporated. For example, in the type-I seesaw with more than one heavy right-handed neutrino, contributions to the light neutrino masses from different right-handed neutrinos may cancel each other due to the conservation of L, which results in massless left-handed neutrinos after decoupling the heavy degrees of freedom from the theory [5]. Such a low-scale fermionic seesaw mechanism may not be able to stabilize the light neutrino masses, since loop corrections may be unacceptably large. A possible way to avoid this problem of the type-I seesaw model is given by the inverse seesaw model, which contains a Majorana insertion used to reduce the $B-L$ scale [6]. In the type-II seesaw model, extending the SM with an $SU(2)$ triplet Higgs scalar [6–8], the coupling between the triplet and the SM Higgs scalar breaks lepton number explicitly, and hence, it is expected to be very small. Hence, light-neutrino masses are suppressed through the seesaw mechanism. In general, the canonical leptogenesis mechanism [9], which provides a very attractive description of the origin of the observed baryon asymmetry of the Universe, does not work for the low-scale seesaw mechanisms, unless severe fine-tuning is invoked [10].

In this paper, we employ the alternative framework of extra spacetime dimensions, where the fundamental Grand Unified scale and the Plank scale are lowered in a natural way [11, 12]. In our higher-dimensional seesaw model, a truncating scale restoring the renormalizability of the theory plays the role of breaking $B - L$, so that the light neutrino masses are suppressed, while the lower Kaluza–Klein (KK) states can be searched for at the LHC. Significant non-unitarity effects, due to the mixing between left-handed neutrinos and heavy KK states, give observable phenomena in future neutrino oscillation experiments, in particular in a neutrino factory. Resonant leptogenesis could possibly be achieved in this model.

The remaining parts of the paper are organized as follows: First, in Sec. II, we present the general formalism of our model. Then, in Sec. III, we show explicitly how sizable non-unitarity effects emerge in the leptonic flavor mixing. Section IV is devoted to the collider signatures and the discovery potential of the heavy KK modes at the LHC. We comment on the origin of baryon number asymmetry in our model in Sec. V. Finally, a summary and our conclusions are given in Sec. VI.

II. HIGHER-DIMENSIONAL SEESAW MODEL

We consider a brane world theory with a five-dimensional bulk, where the SM particles are confined to the brane. We also introduce three right-handed neutrinos Ψ_i $(i = 1, 2, 3)$ [13–17], which, unlike the SM fermions, are SM gauge singlets. Therefore, they are not restricted to the brane and can probe the extra spacetime dimensions. The action responsible for the neutrino masses is given by

$$\begin{aligned} S \; = \; & \int \mathrm{d}^4x\mathrm{d}y \left[\mathrm{i}\overline{\Psi}\not{D}\Psi - \frac{1}{2}\left(\overline{\Psi^c}M_{\mathrm{R}}\Psi + \mathrm{h.c.}\right)\right] \\ & + \int_{y=0} \mathrm{d}^4x \left(-\frac{1}{\sqrt{M_S}}\overline{\nu_{\mathrm{L}}}\hat{m}^c\Psi - \frac{1}{\sqrt{M_S}}\overline{\nu_{\mathrm{L}}^c}\hat{m}\Psi + \mathrm{h.c.}\right) , \end{aligned} \tag{1}$$

where y is the coordinate along the extra compactified dimension, M_S denotes the mass scale of the higher-dimensional theory, and a bulk Majorana mass term is assumed in the first line of Eq. (1). Due to the freedom in the choice of basis for the right-handed neutrino fields, one can always apply a unitary transformation in flavor space in order to diagonalize $M_{\rm R}$. Without loss of generality, we will therefore work in a basis in which $M_{\rm R} = {\rm diag}(M_1, M_2, M_3)$ is real and diagonal.

If the extra dimension is compactified on the $S^1/\mathbb{Z}_2$ orbifold with radius R, then the bulk right-handed neutrinos can be decomposed into two two-component Weyl spinors: $\Psi = (\xi\ \eta^c)^T$, where $\eta^c = {\rm i}\sigma^2\eta^*$. We take ξ to be even under the $\mathbb{Z}_2$ transformation $y \to -y$, while η is taken to be odd. Thus, in Eq. (1), the $\hat{m}^c$ term corresponding to the coupling between ν_L and η is not allowed. The orbifold relations allow us to expand ξ and η as

$$\begin{aligned} \xi(x,y) &= \frac{1}{\sqrt{\pi R}}\xi^{(0)}(x) + \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{N}\xi^{(n)}(x)\cos\left(\frac{ny}{R}\right), \\ \eta(x,y) &= \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{N}\eta^{(n)}(x)\sin\left(\frac{ny}{R}\right). \end{aligned} \tag{2}$$

In general, an extra-dimensional theory must be viewed as an effective theory, since it is non-renormalizable. This means that the KK towers are expected to be cut off at some energy scale Λ. The nature of the cutoff scale depends on the specific ultraviolet (UV) completion of the model. Here, we truncate the KK towers at a maximum KK index $n = N$. A cutoff of this kind arises, for example, in deconstructed extra dimensions models [18]. This truncation is equivalent to a cutoff in the extra-dimensional momentum at $p_y = \Lambda \sim N/R$.

Inserting the above expansion into Eq. (1) and integrating over the compactified dimension, we arrive at the following form for the four-dimensional action

$$\begin{aligned} S = \int {\rm d}^4x \Bigg\{ & \xi^{(0)\dagger}{\rm i}\overline{\sigma}^\mu\partial_\mu\xi^{(0)} + \sum_{n=1}^{N}\left(\xi^{(0)\dagger}{\rm i}\overline{\sigma}^\mu\partial_\mu\xi^{(n)} + \eta^{(n)\dagger}{\rm i}\overline{\sigma}^\mu\partial_\mu\eta^{(n)}\right) \\ & - \frac{\rm i}{2}\left[\xi^{(0)^T}\sigma^2 M_{\rm R}\xi^{(0)} + \sum_{n=1}^{N}\left(\xi^{(n)^T}\ \eta^{(n)^T}\right)\sigma^2\mathcal{M}_n\begin{pmatrix}\xi^{(n)}\\ \eta^{(n)}\end{pmatrix} + {\rm h.c.}\right] \\ & - {\rm i}\left[\nu_{\rm L}^T\sigma^2 m_{\rm D}\xi^{(0)} + \sqrt{2}\sum_{n=1}^{N}\nu_{\rm L}^T\sigma^2 m_{\rm D}\xi^{(n)} + {\rm h.c.}\right]\Bigg\}, \end{aligned} \tag{3}$$

where, written in block-form, the mass matrix $\mathcal{M}_n$ for the KK modes at the nth level takes the form

$$\mathcal{M}_n = \begin{pmatrix} M_{\rm R} & n/R \\ n/R & M_{\rm R}\end{pmatrix}. \tag{4}$$

The Dirac neutrino mass term is then given by $m_{\rm D} = \hat{m}_{\rm D}/\sqrt{2\pi M_S R}$.

For the purpose of simplicity in the following discussion, we define the linear combinations

$$X^{(n)} \equiv \frac{1}{\sqrt{2}}\left(\xi^{(n)} - \eta^{(n)}\right) \ ,$$
$$Y^{(n)} \equiv \frac{1}{\sqrt{2}}\left(\xi^{(n)} + \eta^{(n)}\right) \ , \tag{5}$$

for $n \geqslant 1$. The full neutrino mass matrix in the basis $\left\{\nu_{\rm L}, \xi^{(0)}, X^{(1)}, Y^{(1)}, \cdots, X^{(N)}, Y^{(N)}\right\}$ then reads

$$\mathcal{M} = \begin{pmatrix} 0 & m_{\rm D} & m_{\rm D} & m_{\rm D} & \cdots & m_{\rm D} & m_{\rm D} \\ m_{\rm D}^T & M_{\rm R} & 0 & 0 & \cdots & 0 & 0 \\ m_{\rm D}^T & 0 & M_{\rm R} - \frac{1}{R} & 0 & \cdots & 0 & 0 \\ m_{\rm D}^T & 0 & 0 & M_{\rm R} + \frac{1}{R} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 \\ m_{\rm D}^T & 0 & 0 & 0 & 0 & M_{\rm R} - \frac{N}{R} & 0 \\ m_{\rm D}^T & 0 & 0 & 0 & 0 & 0 & M_{\rm R} + \frac{N}{R} \end{pmatrix} . \tag{6}$$

The scale of $M_{\rm R}$ is not governed by the electroweak symmetry breaking, and hence, one can expect that $M_{\rm R} \sim \mathcal{O}(1)\,{\rm TeV} \gg m_{\rm D}$ holds. Then, by approximately solving the eigenvalue equation of the matrix in Eq. (6) with respect to the small ratio $m_{\rm D}/M_{\rm R}$, the light neutrino mass matrix is found to be given by

$$m_\nu \simeq m_{\rm D}\left(\sum_{n=-N}^{N} \frac{1}{M_R + n/R}\right) m_{\rm D}^T = m_{\rm D}\left(M_{\rm R}^{-1} + \sum_{n=1}^{N} \frac{2M_{\rm R}}{M_{\rm R}^2 - n^2/R^2}\right) m_{\rm D}^T \ . \tag{7}$$

In Refs. [19, 20], the limit $N \to \infty$ is considered, and the light neutrino mass matrix is then given by

$$m_\nu \simeq m_{\rm D} \frac{\pi R}{\tan(\pi R M_{\rm R})} m_{\rm D}^T \ . \tag{8}$$

The light neutrino masses can be suppressed only in the presence of very large $\tan(\pi R M_{\rm R})$ in the denominator of Eq. (8). Therefore, a severe fine-tuning between $M_{\rm R}$ and R^{-1} has to be invoked, which appears quite unnatural. However, bare Majorana masses of the form $M_i = k^{(i)}/(2R)$, where $k^{(i)}$ is an odd integer, emerge naturally from the Sherk–Schwarz decomposition in string theory as a requirement of topological constraints, and hence, such relations do not suffer any fine-tuning problems (see detailed discussions in Ref. [13]). With our chosen cutoff, introducing a maximal KK number, together with the above condition on M_i, lepton number violation will be induced only at the top of the KK tower, as we will see shortly. There could, of course, be other lepton number violating processes at some intermediate point, but we choose to treat the simple scenario where the cutoff is the only source. One can easily prove that, in the simplest case $k^{(i)} = 1$, the light neutrino mass matrix is given by

$$m_\nu \simeq m_{\rm D} \left(M_{\rm R} + \frac{N}{R} \right)^{-1} m_{\rm D}^T \ . \tag{9}$$

Instead of heavy right-handed neutrino masses, the light neutrino masses are suppressed by the large cutoff scale N/R. We consider the interesting case that the scale of the UV completion is much larger than the scale of the extra dimension $1/R$ and the right-handed neutrino masses, i.e, we assume $N \gg k^{(i)}$ to hold. In such a limit, the neutrino mass matrix is simply given by $m_\nu \simeq (R/N) m_{\rm D} m_{\rm D}^T$, i.e., the scale of the neutrino masses is determined by a high-energy scale associated with the fundamental theory underlying the effective extra-dimensional theory. As for the heavy KK modes, from Eq. (6), the masses of the nth excited KK modes are given by

$$\begin{aligned} m_{X^{(n)}} &= M_{\rm R} - \frac{n}{R} \ , \\ m_{Y^{(n)}} &= M_{\rm R} + \frac{n}{R} \ . \end{aligned} \tag{10}$$

As we will discuss later, this implies that $X^{(n)}$ and $Y^{(n-1)}$ (as well as $X^{(1)}$ and $\xi^{(0)}$) form Dirac pairs. Thus, lepton number can be assigned to these pairs and the lepton number violating effects, such as neutrino masses, can only arise from the unpaired $Y^{(N)}$ at the top of the KK tower.

III. NON-UNITARY NEUTRINO MIXING

In order to figure out the generation of non-unitarity effects, we first consider the light neutrino mass matrix. Generally, m_ν is a complex symmetric matrix, and can be diagonalized by means of a unitary matrix U as

$$U^\dagger m_\nu U^* = D \ , \tag{11}$$

where $D = {\rm diag}(m_1, m_2, m_3)$, with m_i being the light neutrino masses. Note that, similarly to the ordinary fermionic seesaw mechanism, the light neutrinos mix with the heavy KK modes. Thus, U is not the exact leptonic mixing matrix entering into neutrino oscillations, even if one works in a basis where the charged-lepton mass matrix is diagonal. To see this point clearly, we can fully diagonalize Eq. (6) and then write down the neutrino flavor eigenstates in terms of the mass eigenstates

$$\nu_{\rm L} \simeq V\nu_{m{\rm L}} + K^{(0)}\xi^{(0)} + \sum_{n=1}^{N}\left[K^{(-n)}X^{(n)} + K^{(n)}Y^{(n)}\right] \ , \tag{12}$$

where $\nu_{m{\rm L}}$ denotes the mass eigenstates of the light neutrinos, and V is the upper-left 3×3 sub-matrix of the complete mixing matrix containing the full KK tower as well as the light neutrinos. Furthermore, we have introduced the quantities

$$K^{(n)} = m_{\rm D}(M_{\rm R} + n/R)^{-1} \ , \tag{13}$$

which represent the mixing between the light neutrinos and the KK modes. The charged-current Lagrangian in mass basis can be rewritten as

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}}\ell_{\rm L}^{\dagger}\bar{\sigma}^{\mu}\left[V\nu_{m{\rm L}} + K^{(0)}\xi^{(0)} + \sum_{n=1}^{N}\left(K^{(-n)}X^{(n)} + K^{(n)}Y^{(n)}\right)\right]W_{\mu}^{-} + {\rm h.c.} \tag{14}$$

Due to the existence of the KK modes, the light neutrino mixing matrix is no longer unitary. To a very good precision, we have

$$V \ \simeq \ \left\{1 - \frac{1}{2}\left[K^{(0)}K^{(0)\dagger} + \sum_{n=1}^{N}\left(K^{(-n)}K^{(-n)^{\dagger}} + K^{(n)}K^{(n)^{\dagger}}\right)\right]\right\}U \ . \tag{15}$$

Assuming that $N \gg k^{(i)}$, Eq. (15) can be approximated by

$$V \simeq \left(1 - \frac{1}{2}\pi^2 R^2 m_{\rm D} m_{\rm D}^{\dagger}\right)U \ . \tag{16}$$

Compared to the conventional parametrization of non-unitarity effects $V = (1-\varepsilon)U$ [21], where ε is a Hermitian matrix, we obtain

$$\varepsilon \simeq \frac{1}{2}\pi^2 R^2 m_{\rm D} m_{\rm D}^{\dagger} \ . \tag{17}$$

An interesting feature of Eq. (17) arises immediately: the non-unitarity effects are dominated only by the combination $m_{\rm D}R$. As a rough estimate, if we keep $1/R$ at the TeV scale and $m_{\rm D} \sim 100$ GeV, $\varepsilon \sim 10^{-2}$ can be naturally expected. Another typical feature is that, if $N \gg k^{(i)}$ holds, then both the neutrino mixing and the non-unitarity effects are determined by a single Dirac mass matrix $m_{\rm D}$. To be concrete, we adopt a convenient parametrization [22], and rewrite $m_{\rm D}$ as

$$m_{\rm D} = U\sqrt{D}O\sqrt{N/R} \ , \tag{18}$$

with O being an arbitrary complex orthogonal matrix. In general, O can be parametrized by using two real matrices as [23]

$$O = e^{{\rm i}A}T \ , \tag{19}$$

where T is real and orthogonal and A is real and antisymmetric. We can then rewrite Eq. (17) as

$$\varepsilon = \frac{1}{2}N\pi^2 RU\sqrt{D}e^{2{\rm i}A}\sqrt{D}U^\dagger \ . \tag{20}$$

Therefore, in such a realistic low-scale extra-dimensional theory, the non-unitarity effects are strongly correlated with the neutrino mixing matrix and the radius of the extra spacetime dimension.

The present bounds at 90 % C.L. on the non-unitarity parameters are given by [24]

$$|\varepsilon| < \begin{pmatrix} 2.0\times10^{-3} & 6.0\times10^{-5} & 1.6\times10^{-3} \\ \sim & 8.0\times10^{-4} & 1.1\times10^{-3} \\ \sim & \sim & 2.7\times10^{-3} \end{pmatrix} \ , \tag{21}$$

where the most severe constraint is that on the $e\mu$ element, coming from the $\mu\to e\gamma$ decay. However, in case that $M_{\rm R}$ lies below the electroweak scale, but above a few GeV, the $\mu\to e\gamma$ constraint is lost due to the restoration of the GIM mechanism [24], and a more loose bound $|\varepsilon_{e\mu}| < 9\times10^{-4}$ should be used.

Now we consider the interesting case of CP-conservation. Then, $m_{\rm D}$ is a real matrix and Eq. (20) can be rewritten as

$$\varepsilon = \frac{1}{2}N\pi^2 RUDU^T = \frac{1}{2}N\pi^2 Rm_\nu \ . \tag{22}$$

In this special case, the structure of the non-unitarity parameters are determined only by the light neutrino masses and the leptonic mixing matrix. For a normal mass hierarchy ($m_1 < m_2 < m_3$), the generic bounds given in Eq. (21) are improved to

$$|\varepsilon| < \begin{pmatrix} 7.8\times10^{-4} & 6.0\times10^{-5} & 6.0\times10^{-5} \\ \sim & 8.0\times10^{-4} & 6.0\times10^{-4} \\ \sim & \sim & 8.0\times10^{-4} \end{pmatrix} \ . \tag{23}$$

Apart from resulting in non-unitarity effects in neutrino mixing, the heavy right-handed neutrinos in the bulk will also contribute to the lepton flavor violating (LFV) decays of charged leptons, e.g., $\mu \to e\gamma$ and $\tau \to \mu\gamma$, through the loop exchange of KK modes [25]. Different from the standard type-I seesaw mechanism, the corresponding branching ratios are not dramatically suppressed by the light neutrino masses, but only driven down by the factor $K^{(n)}$ defined in Eq. (13). Thus, appreciable LFV rates could be obtained.

IV. HADRON COLLIDER SIGNATURES

As shown in Eq. (14), the heavy singlets $\xi^{(0)}$, $X^{(n)}$, and $Y^{(n)}$ couple to the gauge sector of the SM, and thus, if kinematically accessible, they could be produced at hadron colliders. For a quantitative discussion, we now restrict ourselves to the simplest case $k^{(i)} = 1$. Note that, in neglecting the corrections from the cutoff scale, $\xi^{(0)}$ and $X^{(1)}$ are two-component Majorana fields with equal masses and opposite CP parities [26]. Thus, they are equivalent to a single Dirac field $P^{(0)}$ with $P_L^{(0)} = \frac{1}{\sqrt{2}}\left[\xi^{(0)} + X^{(1)}\right]$, $P_R^{(0)^c} = \frac{1}{\sqrt{2}}\left[\xi^{(0)} - X^{(1)}\right]$, and mass $M_{P_0} = M_{\rm R}$. Similarly, $X^{(2)}$ can be combined with $Y^{(1)}$, and hence, forms a higher KK Dirac mode with $P_L^{(1)} = \frac{1}{\sqrt{2}}\left[Y^{(1)} + X^{(2)}\right]$ and mass $M_{P_1} = M_{\rm R} + 1/R$. As a general result of the mass degeneracy, all the KK modes are paired together except for the highest mode $Y^{(N)}$ with mass $M_{\rm R} + N/R$. Actually, $Y^{(N)}$ is now the sole source of lepton violation, and thus, gives rise to the masses of the light neutrinos, which can also be seen from Eq. (9). The structure of the Dirac and Majorana neutrinos is schematically depicted in Fig. 1.

The interaction Lagrangian can be rewritten as

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}}\ell_{\rm L}^\dagger\bar{\sigma}^\mu\left[V\nu_{m\rm L} + \sqrt{2}\sum_{n=0}^{N-1}K^{(n)}P_L^{(n)} + K^{(N)}Y^{(N)}\right]W_\mu^- + \text{h.c.} \ , \tag{24}$$

$$\mathcal{L}_{\rm NC} = \frac{g}{2\cos\theta_W}\nu_{m\rm L}^\dagger\bar{\sigma}^\mu V^\dagger\left[\sqrt{2}\sum_{n=0}^{N-1}K^{(n)}P_L^{(n)} + K^{(N)}Y^{(N)}\right]Z_\mu + \text{h.c.} \ , \tag{25}$$

$$\mathcal{L}_h = \frac{-{\rm i}g}{\sqrt{2}M_W}\nu_{m\rm L}^T\sigma^2 V^T m_D\left[\sqrt{2}\sum_{n=0}^{N-1}P_L^{(n)} + Y^{(N)}\right]h + \text{h.c.} \ , \tag{26}$$

where θ_W denotes the weak mixing angle and M_W is the mass of the W boson. In the case $M_{\rm R} > M_h$ (where M_h denotes the Higgs mass), the heavy KK modes decay in the channels $P \to \ell^- + W^+$, $P \to \nu + Z$, and $P \to h + \nu$. The corresponding partial decay widths are

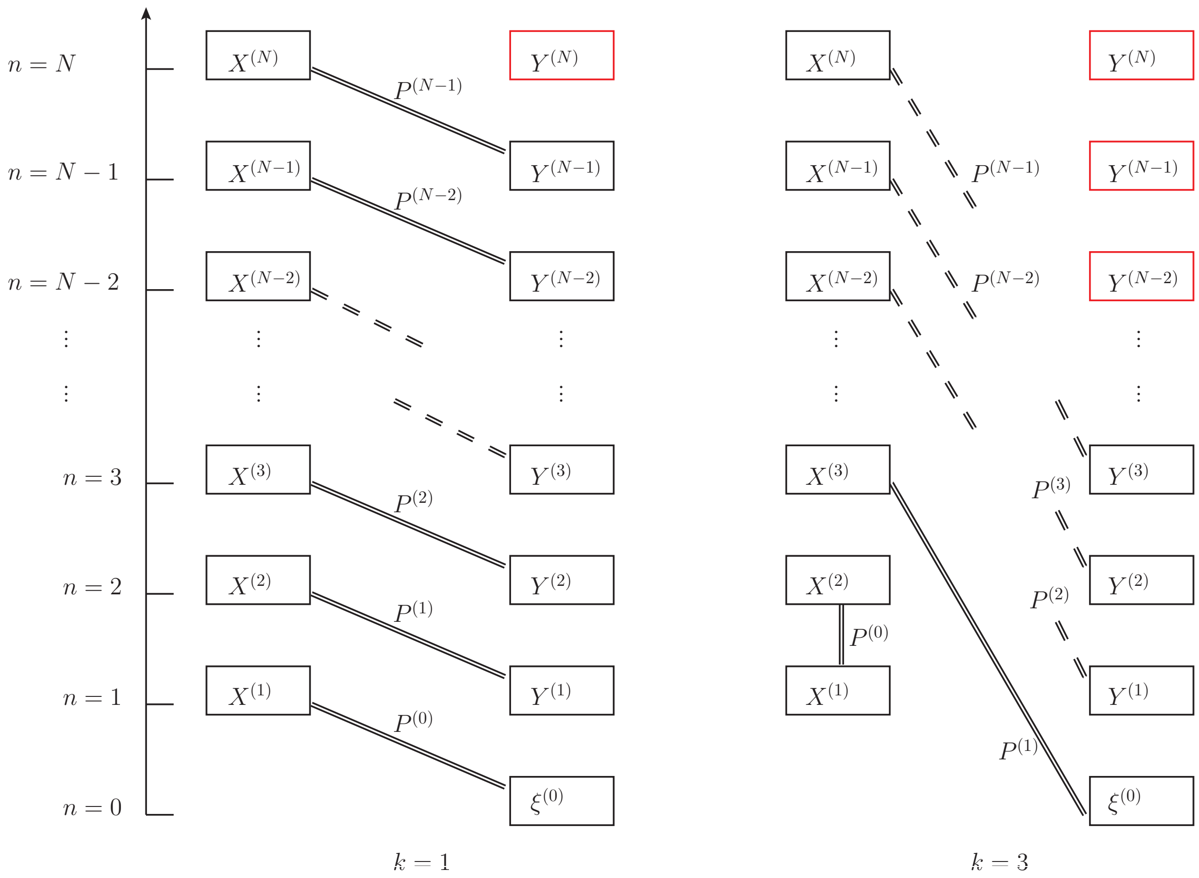

FIG. 1: Illustration of the construction of Dirac particles from pairs of modes in the KK tower. Two heavy KK Majorana modes with equal masses, but opposite CP parities, can be grouped together, as shown with double lines, in order to form a Dirac particle. In the case $k = 1$ (left column), the heaviest mode $Y^{(N)}$ is left, while for the case $k = 3$ (right column), there are three modes left: $Y^{(N-2)}$, $Y^{(N-1)}$, and $Y^{(N)}$.

given by [27]

$$\Gamma\left(P_i^{(n)} \to \ell_\alpha W^+\right) = \frac{g^2}{32\pi}\left|K_{\alpha i}^n\right|^2 \frac{M_{P_n}^3}{M_W^2}\left(1-\frac{M_W^2}{M_{P_n}^2}\right)\left(1+\frac{M_W^2}{M_{P_n}^2}-2\frac{M_W^4}{M_{P_n}^4}\right), \tag{27}$$

$$\Gamma\left(P_i^{(n)} \to \nu_\alpha Z\right) = \frac{g^2}{64\pi c_W^2}\left|K_{\alpha i}^n\right|^2 \frac{M_{P_n}^3}{M_Z^2}\left(1-\frac{M_Z^2}{M_{P_n}^2}\right)\left(1+\frac{M_Z^2}{M_{P_n}^2}-2\frac{M_Z^4}{M_{P_n}^4}\right), \tag{28}$$

$$\Gamma\left(P_i^{(n)} \to \nu_\alpha h\right) = \frac{g^2}{64\pi}\left|K_{\alpha i}^n\right|^2 \frac{M_{P_n}^3}{M_W^2}\left(1-\frac{M_h^2}{M_{P_n}^2}\right)^2 . \tag{29}$$

Since the lower KK modes are Dirac particles, and lepton number breaking occurs only at the top of the KK towers, we focus our attention on lepton number conserving channels mediated by the lightest KK modes. For example, an interesting channel is the production

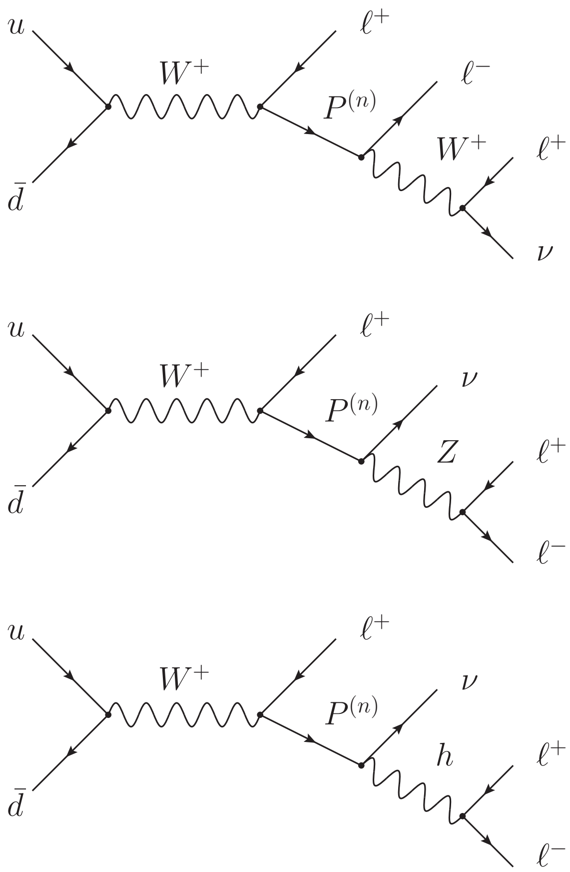

FIG. 2: Feynman diagrams for the potentially interesting LHC signatures with three charged leptons and missing energy in the model under consideration.

of three charged leptons and missing energy [28], i.e., $pp \to \ell_\alpha^\pm \ell_\beta^\pm \ell_\gamma^\mp \nu(\bar{\nu})$, which is depicted in Fig. 2. Another possible process is the pair production of charged leptons with different flavor and zero missing energy, i.e., $pp \to \ell_\alpha^\pm \ell_\beta^\mp$. However, it is difficult to make significant observations in this channel at the LHC, due to the large SM background [29].

An analysis of the collider signatures of an extra-dimensional model similar to the one that we consider was performed in Ref. [19]. It was found that the most promising channel for their model is three leptons and large missing energy (coming from a neutrino). Since tau leptons are difficult to detect, due to their short lifetime, only electrons and muons were considered, and the most promising combination of leptons was concluded to be the so-called two muon combination, given by the sum of the $e\mu\mu$ and $\mu\mu\mu$ signals, where e and μ denote leptons as well as antileptons of the indicated flavors. In order to reduce the SM background, which mainly comes from decays of Z bosons, the following kinematic cuts, taken from Ref. [30], were adopted: i) The two like-sign leptons must each have a transverse momentum larger than 30 GeV, and ii) The invariant masses from the two opposite-sign lepton pairs must each be separated from the mass of the Z boson by at least 10 GeV. Only the effects of the lowest KK level were considered, as it was concluded that the contributions from higher modes would be more than an order of magnitude smaller.

We can use the results of Ref. [19] in order to determine the signals of our model. The model-dependent input determining the signal is the mixing matrix $K^{(0)}$ given in Eq. (13), and the masses of the Dirac neutrinos, which were described above. Written in our notation, the matrix $M_{\rm R}$ in that model is given by

$$M_{\rm R} = \frac{1-\delta}{2R}, \tag{30}$$

where δ parametrizes a small deviation from the Majorana mass condition $M_{\rm R} = 1/(2R)$. In addition, the Dirac mass matrix is given by

$$m_{\rm D} = U\sqrt{D}O\sqrt{\frac{8}{\delta\pi^2}}\sqrt{M_{\rm R}} \simeq U\sqrt{D}O\sqrt{\frac{4}{\pi^2\delta R}}, \tag{31}$$

which should be compared to our expression, given in Eq. (18). From the collider signal point of view, the only difference between the two models is the proportionality constant in the expression for $m_{\rm D}$. The parameter N in our model corresponds to $4/(\delta\pi^2)$ in Ref. [19]. Since the cross sections are proportional to the fourth power of the elements of the mixing matrix $K^{(0)}$, the ratio of the cross section in our model, denoted σ, to that in Ref. [19], denoted $\tilde{\sigma}$, is given by

$$\frac{\sigma}{\tilde{\sigma}} = \left[\frac{NR}{(4/\pi^2)/(\delta/R)}\right]^2. \tag{32}$$

For the calculations in Ref. [19], the value $\delta/R = 10\,{\rm eV}$ was used.[1] Given a value for R, the maximum KK number N in our model is bounded from above by the contributions to the non-unitarity parameters given in Eq. (23), and hence, the LHC signals are correlated with the non-unitarity bounds. We obtain the upper limit

$$\sigma \leq 3\cdot 10^{-3}\,\tilde{\sigma} \tag{33}$$

on the cross section in our model.

[1] In Ref. [19], the following values for the neutrino parameters were used: $m_1 = 0$, $\Delta m^2_{21} = 8\cdot 10^{-5}\,{\rm eV}^2$, $\Delta m^2_{32} = 2.5\cdot 10^{-3}\,{\rm eV}^2$, $\sin\theta_{12} = 0.56$, $\sin\theta_{13} = 0.07$, $\sin\theta_{23} = 0.71$, and $\phi_{\rm D} = \pi$, where $\phi_{\rm D}$ is the CP violating Dirac phase. It was also assumed that the matrix O in Eq. (31) is real and orthogonal.

The authors of Ref. [19] found that, for a luminosity of $30\,\text{fb}^{-1}$, the maximum number of expected events for the 2μ signal, after the kinematic cuts have been taken into account, is of the order of 100. The corresponding maximum number of events in our model would be less than one. This should be compared to the background, which was calculated in Ref. [30] to be of the order of 100 events. We conclude that, for our model, the non-unitarity bounds are strong enough to rule out the part of the parameter space that could possibly be probed by the LHC.

V. COMMENTS ON RESONANT LEPTOGENESIS

Baryogenesis via leptogenesis is one of the main candidates for being the theory appropriately describing the production of a baryon asymmetry in the early Universe, which is measured to be $\eta_B = (6.2 \pm 0.15) \times 10^{-10}$ [31]. In its most basic form, leptogenesis occurs in a type-I seesaw scenario, where a net lepton asymmetry is produced through the out-of-equilibrium decay of the heavy neutrinos and then partially converted to a baryon asymmetry through sphaleron processes. The Sakharov conditions [32] are fulfilled by the decays occurring out of equilibrium, the CP-violation of the decays through complex Yukawa couplings, and the baryon number violation of the sphalerons, respectively.

Usually, the net lepton number is produced by the decays of the lightest right-handed neutrinos, since asymmetries produced by the heavier neutrinos will be washed out. However, in our scenario, the tower of Dirac neutrinos can be given definite lepton number assignments and lepton number violation only occurs at the top of the tower through the unpaired Y-states, which could take on the role of the right-handed neutrinos in the basic scenario. It is important to note that for $k_i = 1$ $(i = 1, 2, 3)$, there will be no net lepton number violation, since all of the three unpaired states will be degenerate in mass. However, if the k_i are different, e.g., $k_1 = 3$ and $k_{2,3} = 1$, then $Y_1^{(N-2)}$ (see Fig. 1) will be the unique lightest Majorana state and a net lepton asymmetry could be produced. Since the mass splitting of $1/R$ between the Y-states is expected to be very small compared to the masses, the model would have to be treated within the framework of resonant leptogenesis [33]. Furthermore, to accurately examine the prospects for leptogenesis in this model, one would have to properly take into account the effect of the Dirac tower. Even if the Dirac neutrinos in the tower preserve lepton number, they do not participate in the sphaleron processes, since they are

SM singlets, which could hide some part of the produced lepton number from the sphalerons if all Dirac neutrinos do not decay before sphaleron processes become inactive. Thus, a detailed analysis, which is beyond the scope of this paper, would be required to properly analyze the prospects for leptogenesis in this model.

VI. SUMMARY AND CONCLUSIONS

In this work, we have studied a possible mechanism for generating light neutrino masses in the context of extra dimensions. In the model that we consider, the SM particles are confined to a four-dimensional brane, while three right-handed neutrinos are allowed to propagate in an extra dimension, compactified on the $S^1/\mathbb{Z}_2$ orbifold. Since extra-dimensional models are generally non-renormalizable, and can only be considered as effective theories, the KK expansions of the higher-dimensional fields are expected to be truncated at some cutoff scale. We have imposed a cut on the KK number, truncating the towers at $n = N$.

In the case that the bulk Majorana mass term for the right-handed neutrinos has the form $M_R = k/(2R)$, where k is an odd integer, the KK modes of the right-handed neutrinos pair to form Dirac neutrinos. Such a form for a Majorana mass is motivated by the Scherk–Schwarz mechanism. In the model that we consider, due to the truncation of the KK towers, a number of unpaired Majorana neutrinos remain at the top of each KK tower, and these are the only sources of lepton number violation in this model. If the cutoff scale is large, small masses for the left-handed neutrinos are naturally generated.

Due to mixing between the light neutrinos and the KK modes of the right-handed neutrinos, large non-unitarity effects can be induced. Since the light neutrino masses are generated by the top of each tower, these non-unitarity effects are not suppressed by the light neutrino masses. Current bounds on the non-unitarity parameters have constrained the parameter space of the model.

Finally, we have considered the prospects of observing the effects of the lowest KK modes of the right-handed neutrinos at the LHC. Specifically, we have considered the three leptons and large missing energy signal, which has previously been found to be promising for a similar model. We have found that, in contrast to the previous results in the literature, the potential of carrying out a discovery search of such a model at the LHC is actually pessimistic. In particular, most parts of the parameter space that could be probed at the LHC are ruled out by the bounds imposed by the stringent non-unitarity constraints. On the other hand, the non-unitarity effects in neutrino oscillations could be well searched for at future neutrino factory experiments [34]. Therefore, future long baseline neutrino oscillation experiments could play a very complementary role in searching for the clue of extra dimensions.

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[1] P. Minkowski, Phys. Lett. **B67**, 421 (1977).

[2] T. Yanagida, in *Proc. Workshop on the Baryon Number of the Universe and Unified Theories*, edited by O. Sawada and A. Sugamoto (1979), p. 95.

[3] R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).

[4] S. Dimopoulos, Phys. Lett. **B246**, 347 (1990).

[5] J. Kersten and A. Y. Smirnov, Phys. Rev. **D76**, 073005 (2007).

[6] J. Schechter and J. W. F. Valle, Phys. Rev. **D22**, 2227 (1980).

[7] G. Lazarides, Q. Shafi, and C. Wetterich, Nucl. Phys. **B181**, 287 (1981).

[8] R. N. Mohapatra and G. Senjanović, Phys. Rev. **D23**, 165 (1981).

[9] M. Fukugita and T. Yanagida, Phys. Lett. **B174**, 45 (1986).

[10] A. Pilaftsis and T. E. J. Underwood, Phys. Rev. **D72**, 113001 (2005), hep-ph/0506107.

[11] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. **B429**, 263 (1998), hep-ph/9803315.

[12] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999), hep-ph/9905221.

[13] K. R. Dienes, E. Dudas, and T. Gherghetta, Nucl. Phys. **B557**, 25 (1999), hep-ph/9811428.

[14] N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali, and J. March-Russell, Phys. Rev. **D65**, 024032 (2002), hep-ph/9811448.

[15] G. R. Dvali and A. Y. Smirnov, Nucl. Phys. **B563**, 63 (1999), hep-ph/9904211.

[16] R. Barbieri, P. Creminelli, and A. Strumia, Nucl. Phys. **B585**, 28 (2000), hep-ph/0002199.

[17] A. Lukas, P. Ramond, A. Romanino, and G. G. Ross, JHEP **04**, 010 (2001), hep-ph/0011295.

[18] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, Phys. Rev. Lett. **86**, 4757 (2001), hep-th/0104005.

[19] N. Haba, S. Matsumoto, and K. Yoshioka, Phys. Lett. **B677**, 291 (2009), arXiv:0901.4596.

[20] S. Bhattacharya, P. Dey, and B. Mukhopadhyaya, Phys. Rev. **D80**, 075013 (2009), arXiv:0907.0099.

[21] S. Antusch, C. Biggio, E. Fernández-Martínez, M. B. Gavela, and J. López-Pavón, JHEP **10**, 084 (2006), hep-ph/0607020.

[22] J. A. Casas and A. Ibarra, Nucl. Phys. **B618**, 171 (2001), hep-ph/0103065.

[23] S. Pascoli, S. T. Petcov, and C. E. Yaguna, Phys. Lett. **B564**, 241 (2003), hep-ph/0301095.

[24] S. Antusch, J. P. Baumann, and E. Fernández-Martínez, Nucl. Phys. **B810**, 369 (2009), arXiv:0807.1003.

[25] A. De Gouvêa, G. F. Giudice, A. Strumia, and K. Tobe, Nucl. Phys. **B623**, 395 (2002), hep-ph/0107156.

[26] S. M. Bilenky and S. T. Petcov, Rev. Mod. Phys. **59**, 671 (1987).

[27] W. Buchmüller and C. Greub, Phys. Lett. **B256**, 465 (1991).

[28] F. del Aguila and J. A. Aguilar-Saavedra, Phys. Lett. **B672**, 158 (2009), arXiv:0809.2096.

[29] F. del Aguila, J. A. Aguilar-Saavedra, and R. Pittau, JHEP **10**, 047 (2007), hep-ph/0703261.

[30] F. del Aguila and J. A. Aguilar-Saavedra, Nucl. Phys. **B813**, 22 (2009), arXiv:0808.2468.

[31] E. Komatsu et al. (WMAP), Astrophys. J. Suppl. **180**, 330 (2009), arXiv:0803.0547.

[32] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. **5**, 32 (1967).

[33] A. Pilaftsis and T. E. J. Underwood, Nucl. Phys. **B692**, 303 (2004), hep-ph/0309342.

[34] S. Antusch, M. Blennow, E. Fernández-Martínez, and J. Lopez-Pavon, Phys. Rev. **D80**, 033002 (2009), arXiv:0903.3986.