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Optimality Properties and Low-Complexity Solutions to Coordinated Multicell Transmission

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EMIL BJÖRNSON, MATS BENGTSSON,
AND BJÖRN OTTERSTEN

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KTH Royal Institute of Technology
ACCESS Linnaeus Center
Signal Processing Lab

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Optimality Properties and Low-Complexity Solutions to Coordinated Multicell Transmission

Emil Björnson
ACCESS Linnaeus Center
KTH Royal Institute of Technology
SE-100 44 Stockholm, Sweden
Email: emil.bjornson@ee.kth.se

Mats Bengtsson
ACCESS Linnaeus Center
KTH Royal Institute of Technology
SE-100 44 Stockholm, Sweden
Email: mats.bengtsson@ee.kth.se

Björn Ottersten
ACCESS Linnaeus Center
KTH Royal Institute of Technology
SE-100 44 Stockholm, Sweden
Email: bjorn.ottersten@ee.kth.se

Abstract—Base station cooperation can theoretically improve the throughput of multicell systems by coordinating interference and serving cell edge terminals through multiple base stations. In practice, the extent of cooperation is limited by the increase in backhaul signaling and computational demands. To address these concerns, we propose a novel distributed cooperation structure where each base station has responsibility for the interference towards a set of terminals, while only serving a subset of them with data. Weighted sum rate maximization is considered, and conditions for beamforming optimality and the optimal transmission structure are derived using Lagrange duality theory. This leads to distributed low-complexity transmission strategies, which are evaluated on measured multiantenna channels in a typical urban multicell environment.

I. INTRODUCTION

In conventional multicell systems, each terminal is allocated to a certain cell and served by its base station. There has been a tremendous amount of work on downlink multiple-input multiple-output (MIMO) techniques that can serve multiple terminals in each cell and control their co-terminal interference [1], but with only single-cell processing the performance will be fundamentally limited by interference from adjacent cells—especially for terminals close to cell edges.

Network MIMO is a recent base station cooperation concept, where the base stations coordinate the interference caused to adjacent cells and where cell edge terminals can be served through multiple base stations [2], [3]. The ideal capacity of these systems was given in [4] for unconstrained cooperation, and even with constrained backhaul signaling they provide major performance gains over conventional systems [5]–[7]. However, there is a large computational complexity involved in the transmission optimization that quickly becomes intractable for centralized implementations as the network grows [7].

Uplink-downlink duality is an attractive approach to optimize the multicell downlink (with single-antenna terminals), as the optimal beamforming vectors can be calculated separately in the dual uplink [8]. Lagrange duality theory has been exploited for iterative algorithms for minimizing the transmit power subject to individual rate constraints; [9] considered systems where all base stations serve all terminals and [10] where only one base station serves each terminal. In practical network MIMO, only a small subset of base stations will serve

each terminal (to limit the backhaul signaling and synchronization overhead). This was considered in [5] by defining fixed cooperation clusters where base stations iteratively coordinate transmissions to avoid interference. However, out-of-cluster interference still limits performance. An alternative is dynamic cooperation clusters where each base station shares the responsibility for a few terminals with adjacent base stations. An efficient suboptimal algorithm for iterative weighted sum rate optimization was proposed in [11] and extended for other utility functions in [12], while the impact of imperfect channel information and backhaul constraints was considered in [13].

Herein, we extend previous work on dynamic cooperation clusters in [11]–[13] by considering a multicell system where each base station has responsibility for the interference caused to a set of users, while only serving a subset of them with data (to limit backhaul signalling). The major contributions include:

- The relationship between maximizing the weighted sum rate (P1) and a convex problem formulation with individual rate constraints (P2) is analyzed. Under single user detection and per-base station power constraints, we prove that it is optimal for both (P1) and (P2) to perform single-stream beamforming and use full transmit power.
- A novel uplink-downlink duality is derived for (P2), which differs from [8]–[10] by guaranteeing solutions that satisfy fixed transmit power constraints. The duality shows that the optimal solutions to both (P1) and (P2) are given by a generalized Rayleigh quotient.
- Based on duality, we propose distributed low-complexity strategies suitable for systems with many subcarriers (where the overhead and computational power required for the iterative solutions of [9]–[12] are unavailable).
- The performance of any system depends on the channel where it operates. Thus, realistic channel models are necessary for reliable system simulations. Herein, the proposed strategies are evaluated on measured channel vectors in a typical urban macro-cell environment.

Notation: \mathbf{X}^T , \mathbf{X}^H , and \mathbf{X}^\dagger denote the transpose, the conjugate transpose, and the Moore-Penrose inverse of \mathbf{X} , respectively. \mathbf{I}_N and $\mathbf{0}_N$ are $N \times N$ identity and zero matrices, respectively. If \mathcal{S} is a set, then its members are $\mathcal{S}(1), \dots, \mathcal{S}(|\mathcal{S}|)$ where $|\mathcal{S}|$ is the cardinality.

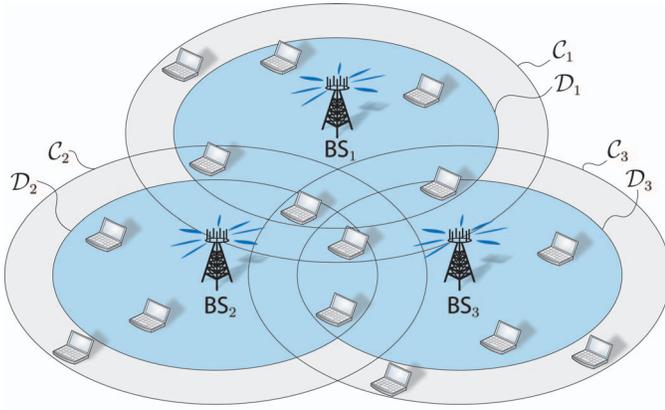


Fig. 1. Schematic intersection between three cells. BS_j serves terminals in the inner circle (\mathcal{D}_j) and controls interference within the outer circle (\mathcal{C}_j).

II. SYSTEM MODEL

We consider a downlink multicell scenario with K_t multi-antenna transmitters and K_r single-antenna receivers¹. The transmitters and receivers are denoted BS_j and MS_k , respectively, for $j \in \mathcal{J} = \{1, \dots, K_t\}$ and $k \in \mathcal{K} = \{1, \dots, K_r\}$. Transmitters serve different sets of receivers and may have different numbers of antennas. BS_j has N_j antennas, should control the interference caused to receivers in $\mathcal{C}_j \subseteq \mathcal{K}$, and should serve the subset of receivers in $\mathcal{D}_j \subseteq \mathcal{C}_j$ with data. The sets \mathcal{C}_j and \mathcal{D}_j are assumed to be provided by the scheduler² and are illustrated in Fig. 1.

Denote the flat fading channel between BS_j and MS_k by $\mathbf{h}_{jk} \in \mathbb{C}^{N_j}$, and assume that it is narrowband so that synchronous interference is achieved within each dynamic cooperation cluster [14]. The combined channel to MS_k is $\mathbf{h}_k = [\mathbf{h}_{1k}^T \dots \mathbf{h}_{K_t k}^T]^T$ and the received signal is modeled as

$$y_k = \mathbf{h}_k^H \mathbf{C}_k \sum_{\bar{k}=1}^{K_r} \mathbf{D}_{\bar{k}} \mathbf{s}_{\bar{k}} + n_k \quad (1)$$

where \mathbf{D}_k sorts out the base stations that transmit the signal $\mathbf{s}_k \in \mathbb{C}^{N \times 1}$ to MS_k (with $N = \sum_{j=1}^{K_t} N_j$). Formally, $\mathbf{D}_k \in \mathbb{C}^{N \times N}$ is block-diagonal with the block sizes N_1, \dots, N_{K_t} . It is defined as $\mathbf{D}_k = \sum_{j=1}^{K_t} \mathbf{D}_{jk}$, where \mathbf{D}_{jk} is zero except at the j th block which is \mathbf{I}_{N_j} if $k \in \mathcal{D}_j$ and $\mathbf{0}_{N_j}$ if $k \notin \mathcal{D}_j$.

Similarly, $\mathbf{C}_k \in \mathbb{C}^{N \times N}$ sorts out the signals from BS_j with $k \in \mathcal{C}_j$, while other signals are assumed to cause weak interference and are included in the additive white noise term $n_k \in \mathcal{CN}(0, \sigma_k^2)$. This limits the CSI required to model the transmission and is reasonable if transmitters coordinate the interference to all cell edge terminals of adjacent cells. In the analysis, BS_j is assumed to know the channels \mathbf{h}_{jk} perfectly to all MS_k with $k \in \mathcal{C}_j$. Formally, $\mathbf{C}_k \in \mathbb{C}^{N \times N}$ is block-diagonal and the j th block is \mathbf{I}_{N_j} if $k \in \mathcal{C}_j$ and $\mathbf{0}_{N_j}$ if $k \notin \mathcal{C}_j$.

¹This model also applies to simple multi-antenna receivers that fix a receive beamformer (e.g., antenna selection) prior to transmission optimization.

²How to select these sets efficiently, by scheduling spatially separated users and only accept the overhead involved with serving a terminal through multiple base stations if the performance gain is substantial, is a very interesting and important problem, but beyond the scope of this paper.

The power constraints are defined per base station as

$$\sum_{k=1}^{K_r} \text{tr}\{\mathbf{D}_{jk} \mathbf{S}_k \mathbf{D}_{jk}^H\} = \sum_{k \in \mathcal{D}_j} \text{tr}\{\mathbf{D}_{jk} \mathbf{S}_k \mathbf{D}_{jk}^H\} \leq P_j \quad (2)$$

where $\mathbf{S}_k = \mathbb{E}\{\mathbf{s}_k \mathbf{s}_k^H\}$ is the signal correlation matrix of MS_k . The effective signal correlation matrix is $\mathbf{D}_k \mathbf{S}_k \mathbf{D}_k^H$, but we keep \mathbf{D}_k and \mathbf{S}_k separated as we will prove properties of \mathbf{S}_k . Some work on network MIMO considers per-antenna constraints, with the motivation that each antenna has its own power amplifier [9]. However, having per-base station constraints makes sense from a regulatory perspective as it limits the radiated power per base station and subcarrier. In addition, it is possible to derive explicit transmission solutions.

A. Problem Formulations

Herein, we consider two different optimization problems: *weighted sum-rate maximization* (P1) and *successful communication with individual rate constraints* (P2). In both cases, we make the assumption of single-user detection (SUD) [15], which means that receivers treat co-terminal interference as noise (i.e., not attempting to decode and subtract interference). This assumption leads to suboptimal performance, but is important to achieve simple and practical receivers.

The rate $R_k(\mathbf{S}_1, \dots, \mathbf{S}_{K_r}, \sigma_k^2)$ at MS_k can be expressed

$$R_k = \log_2 \left(1 + \frac{\mathbf{h}_k^H \mathbf{D}_k \mathbf{S}_k \mathbf{D}_k^H \mathbf{h}_k}{\sigma_k^2 + \mathbf{h}_k^H \mathbf{C}_k \left(\sum_{\bar{k} \in \mathcal{I}_k} \mathbf{D}_{\bar{k}} \mathbf{S}_{\bar{k}} \mathbf{D}_{\bar{k}}^H \right) \mathbf{C}_k \mathbf{h}_k} \right) \quad (3)$$

since weak interference was assumed for all $\bar{k} \notin \mathcal{I}_k$, where

$$\mathcal{I}_k = \bigcup_{j \text{ with } k \in \mathcal{C}_j} \mathcal{D}_j \setminus \{k\}. \quad (4)$$

Using this rate notation, we define our optimization problems. The first one is weighted sum rate maximization, which corresponds to maximizing the instantaneous throughput with fairness/priority weights given by the scheduler. For any collection of positive weights $\boldsymbol{\mu} = [\mu_1, \dots, \mu_{K_r}]$, we have

$$\begin{aligned} & \text{maximize}_{\mathbf{S}_1, \dots, \mathbf{S}_{K_r}} \sum_{k=1}^{K_r} \mu_k R_k(\mathbf{S}_1, \dots, \mathbf{S}_{K_r}, \sigma_k^2) \\ & \text{subject to } \mathbf{S}_k \succeq \mathbf{0}, \sum_{\bar{k} \in \mathcal{D}_j} \text{tr}\{\mathbf{D}_{j\bar{k}} \mathbf{S}_{\bar{k}} \mathbf{D}_{j\bar{k}}^H\} \leq P_j \quad \forall j, k. \end{aligned} \quad (\text{P1})$$

All boundary points (R_1, \dots, R_{K_r}) of the achievable rate region are solutions to a weighted sum rate maximization for some $\boldsymbol{\mu}$ [16]. Thus, (P1) represents all reasonable performance measures, because all other feasible solutions can be improved in one of the rates without decreasing any other. Unfortunately (P1) is non-convex and therefore difficult to solve without performing an exhaustive search. The second problem is therefore designed to be convex. It is based upon satisfying predefined individual rate constraints; that is, $R_k \geq \gamma_k$ for some γ_k for each k . To achieve a feasible convex optimization problem, we multiply the noise with an artificial optimization variable α . In the following problem, all rate constraints are

satisfied if the solution gives $\alpha \geq 1$:

$$\begin{aligned} & \text{maximize } \alpha \\ & \mathbf{S}_1, \dots, \mathbf{S}_{K_r}, \alpha \\ & \text{subject to } R_k(\mathbf{S}_1, \dots, \mathbf{S}_{K_r}, \alpha^2 \sigma_k^2) \geq \gamma_k, \\ & \mathbf{S}_k \succeq \mathbf{0}, \sum_{\bar{k} \in \mathcal{D}_j} \text{tr}\{\mathbf{D}_{j\bar{k}} \mathbf{S}_{\bar{k}} \mathbf{D}_{j\bar{k}}^H\} \leq P_j \quad \forall j, k. \end{aligned} \quad (\text{P2})$$

This individual rate constraints problem is different from those in [8]–[10] as its solutions always satisfy the power constraints, instead of breaching them to support infeasible rates. There is an important connection between (P1) and (P2):

Lemma 1. If optimal rates R_k^* of (P1) are used as constraints in (P2), all optimal solutions to (P2) are also optimal for (P1).

Thus, the price of achieving a convex problem is that the system must propose the terminal rates. To move iteratively towards the optimal weighted sum rate, an outer control loop may be used to increase or decrease rate constraints if $\alpha > 1$ or $\alpha < 1$, respectively. Global convergence cannot be guaranteed, good performance was achieved by a similar approach in [11].

In the next sections, we derive general properties of (P2) and see how they also apply for the optimal solution to (P1).

III. BEAMFORMING OPTIMALITY & FULL POWER USAGE

In this section, we introduce a class of optimization problems that contains (P1) and (P2) as special cases. Similar to [15], we show that single-stream beamforming is optimal in this class, and that full transmit power always can be used. Each member of the class has a set of parameters $z_{k\bar{k}}, p_{jk} \geq 0$ and each \mathbf{S}_k is achieved by solving

$$\begin{aligned} & \text{maximize } \mathbf{h}_k^H \mathbf{D}_k \mathbf{S}_k \mathbf{D}_k^H \mathbf{h}_k \\ & \mathbf{S}_k \\ & \text{subject to } \mathbf{h}_{\bar{k}}^H \mathbf{D}_k \mathbf{S}_k \mathbf{D}_k^H \mathbf{h}_{\bar{k}} \leq z_{k\bar{k}}^2, \quad \forall \bar{k} \in \tilde{\mathcal{I}}_k, \\ & \mathbf{S}_k \succeq \mathbf{0}, \text{tr}\{\mathbf{D}_{jk} \mathbf{S}_k \mathbf{D}_{jk}^H\} \leq p_{jk} \quad \forall j \end{aligned} \quad (5)$$

where $\tilde{\mathcal{I}}_k$ is the set of terminals that base stations serving MS_k have responsibility for (i.e., terminals that might receive non-negligible co-terminal interference). This set is defined as

$$\tilde{\mathcal{I}}_k = \bigcup_{j \text{ with } k \in \mathcal{D}_j} \mathcal{C}_j \setminus \{k\}. \quad (6)$$

This class of optimization problems has the following relationship with (P1) and (P2).

Lemma 2. Let $\mathbf{S}_1^*, \dots, \mathbf{S}_{K_r}^*$ be an optimal solution to (P1). For each j and $\bar{k} \in \tilde{\mathcal{I}}_k$, select $z_{k\bar{k}}^2 = \mathbf{h}_{\bar{k}}^H \mathbf{D}_k \mathbf{S}_k^* \mathbf{D}_k^H \mathbf{h}_{\bar{k}}$ and $p_{jk} = c_{jk} \text{tr}\{\mathbf{D}_{jk} \mathbf{S}_k^* \mathbf{D}_{jk}^H\}$ for $c_{jk} \geq 1$ such that $\sum_{\bar{k} \in \mathcal{D}_j} p_{jk} = P_j$.

With these parameters, all optimal $\tilde{\mathbf{S}}_1^*, \dots, \tilde{\mathbf{S}}_{K_r}^*$ to (5) are also optimal for (P1). The corresponding holds for (P2).

Proof: This is proved by contradiction. For (P1), suppose that $\tilde{\mathbf{S}}_k^*$ is not part of an optimal solution to (P1). As \mathbf{S}_k^* is a feasible solution to (5), this means that $\tilde{\mathbf{S}}_k^*$ achieves higher signal power for MS_k without increasing the interference or using too much power. Thus, by replacing \mathbf{S}_k^* with $\tilde{\mathbf{S}}_k^*$ in the solution $\mathbf{S}_1^*, \dots, \mathbf{S}_{K_r}^*$ the weighted sum rate will increase, which is a contradiction. A similar argument holds for (P2) as replacing \mathbf{S}_k^* with $\tilde{\mathbf{S}}_k^*$ can only increase $R_k - \gamma_k$ for all \bar{k} . ■

The relationship proved by Lemma 2 will not directly assist in solving (P1) or (P2) as the optimal parameters are unknown beforehand. However, all properties of the optimal solutions to (5) that hold for any parameters will also be properties of (P1) and (P2). The following theorem provides such properties.

Theorem 1. For some optimal solution $\tilde{\mathbf{S}}_k^*$ to (5) it holds that

- i) Beamforming is optimal, that is $\text{rank}(\tilde{\mathbf{S}}_k^*) \leq 1$.
- ii) Full power $\text{tr}\{\mathbf{D}_{jk} \tilde{\mathbf{S}}_k^* \mathbf{D}_{jk}^H\} = p_{jk}$ is used for all j with $k \in \mathcal{D}_j$ and $\mathbf{h}_{jk} \notin \text{span}(\bigcup_{\bar{k} \in \mathcal{C}_j \setminus \{k\}} \{\mathbf{h}_{j\bar{k}}\})$.

Proof: The first part is proved by maximizing $\mathbf{w}_k^H \mathbf{D}_k^H \mathbf{h}_k + \mathbf{h}_k^H \mathbf{D}_k \mathbf{w}_k$ under the constraints of (5) and showing that the solution satisfies the KKT conditions of (5) (with $\mathbf{S}_k = \mathbf{w}_k \mathbf{w}_k^H$). The second part is proved by contradiction. For space limitations, the proof is given in [17]. ■

The conclusion is that there exist optimal solutions to (P1) and (P2) that use single-stream beamforming and where all transmitters use full transmit power (however, other solutions may also exist). These properties greatly simplify the optimization by reducing the search space for optimal solutions. In prior work (e.g., [9]–[13]), beamforming is often assumed for single-antenna receivers without further discussion, although the optimality of beamforming under SUD and general utilities is non-trivial; see for example [15] and [17].

As a remark, Theorem 1 is based upon the condition $\mathbf{h}_{jk} \notin \text{span}(\bigcup_{\bar{k} \in \mathcal{C}_j \setminus \{k\}} \{\mathbf{h}_{j\bar{k}}\})$, which is fulfilled with probability one in practice if $|\mathcal{C}_j| \leq N_t$ and all \mathbf{h}_{jk} are modeled as independent random variables (with non-singular covariance matrices).

IV. BEAMFORMING PROPERTIES FROM DUALITY THEORY

In this section, we derive the Lagrange dual problem of (P2) and show that it can be interpreted as an uplink optimization with uncertain noise. The duality is used to obtain the optimal transmission structure for (P1) and (P2).

The next theorem provides the Lagrange dual problem to (P2). The duality result is different from the uplink-downlink dualities derived in [8]–[10] where the power constraints are scaled to satisfy infeasible rate constraints, whereas (P2) keeps them fixed and virtually scales the noise.

Theorem 2. Strong duality holds for (P2) and the Lagrange dual problem can be expressed as

$$\begin{aligned} & \text{minimize } \frac{1}{4 \sum_{k=1}^{K_r} q_k \sigma_k^2} + \sum_{j=1}^{K_t} \omega_j P_j \\ & \text{subject to } \max_{\bar{\mathbf{w}}_k} \bar{R}_k(\bar{\mathbf{w}}_k, \boldsymbol{\omega}, \mathbf{q}) = \gamma_k, \\ & q_k \geq 0, \omega_j \geq 0 \quad \forall k, j \end{aligned} \quad (\text{D2})$$

with $\boldsymbol{\omega} = [\omega_1, \dots, \omega_{K_t}]^T$, $\mathbf{q} = [q_1, \dots, q_{K_r}]^T$,

$$\bar{R}_k = \log_2 \left(1 + \frac{q_k \bar{\mathbf{w}}_k^H \mathbf{D}_k^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{D}_k \bar{\mathbf{w}}_k}{\bar{\mathbf{w}}_k^H \mathbf{D}_k^H (\boldsymbol{\Omega}_k + \sum_{\bar{k} \in \tilde{\mathcal{I}}_k} q_{\bar{k}} \mathbf{C}_{\bar{k}}^H \mathbf{h}_{\bar{k}} \mathbf{h}_{\bar{k}}^H \mathbf{C}_{\bar{k}}) \mathbf{D}_k \bar{\mathbf{w}}_k} \right) \quad (7)$$

and $\boldsymbol{\Omega}_k = \sum_{j=1}^{K_t} \omega_j \mathbf{D}_{jk}$. Strong duality means that the optimal utilities α and $1/(4 \sum_k q_k \sigma_k^2) + \sum_j \omega_j P_j$ are equal and that the optimal \mathbf{S}_k is equal to $\bar{\mathbf{w}}_k \bar{\mathbf{w}}_k^H$ up to a scaling factor.

Proof: From Theorem 1, we can take $\mathbf{S}_k = \mathbf{w}_k \mathbf{w}_k^H$ and select the (unconstrained) phase of \mathbf{w}_k such that $\mathbf{h}_k^H \mathbf{D}_k \mathbf{w}_k > 0$. Then, (P2) can be written as a second order cone program similarly to [18]. Thus strong duality holds, and the Lagrange dual problem can be obtained and rewritten in a similar way as in [9]. For space limitations, the proof is given in [17]. ■

The Lagrange dual problem (D2) can be interpreted as a virtual uplink from K_r single-antenna terminals to K_t multi-antenna base stations. The performance is optimized over the virtual transmit powers in \mathbf{q} for different terminals and the noise powers in $\boldsymbol{\omega}$ at different base stations, while $\bar{\mathbf{w}}_k$ represents the receive beamformer for MS_k . Thus, the utility provides a balance between increasing the transmit power to satisfy the uplink rate constraints and changing the noise.

The important duality result, for our purposes, is that for fixed $\boldsymbol{\omega}$ and \mathbf{q} , the receive beamformers $\bar{\mathbf{w}}_k$ can be obtained as separate rate optimizations—this is a well-known property of the uplink. Although Theorem 2 was derived for (P2), a main result herein is that it leads to a simple optimal beamforming structure for both (P1) and (P2):

Theorem 3. There exist optimal solutions $\mathbf{S}_k = \mathbf{w}_k \mathbf{w}_k^H \forall k$ to (P1) and (P2) with \mathbf{w}_k from the generalized Rayleigh quotient

$$\text{maximize}_{\mathbf{w}_k} \frac{\mathbf{w}_k^H \mathbf{D}_k^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{D}_k \mathbf{w}_k}{\mathbf{w}_k^H \mathbf{D}_k^H \left(\sum_j a_j \mathbf{D}_j + \sum_{\bar{k} \in \tilde{\mathcal{I}}_k} b_{\bar{k}} \mathbf{C}_{\bar{k}}^H \mathbf{h}_{\bar{k}} \mathbf{h}_{\bar{k}}^H \mathbf{C}_{\bar{k}} \right) \mathbf{D}_k \mathbf{w}_k} \quad (8)$$

for some parameters $a_j, b_{\bar{k}} \in [0, 1]$ (for all j, k, \bar{k}). For arbitrary $c_k \in \mathbb{C}$ satisfying the power constraints, \mathbf{w}_k becomes

$$\mathbf{w}_k = c_k \frac{\left(\sum_j a_j \mathbf{D}_j + \sum_{\bar{k} \in \tilde{\mathcal{I}}_k} b_{\bar{k}} \mathbf{D}_{\bar{k}}^H \mathbf{C}_{\bar{k}}^H \mathbf{h}_{\bar{k}} \mathbf{h}_{\bar{k}}^H \mathbf{C}_{\bar{k}} \mathbf{D}_{\bar{k}} \right)^\dagger \mathbf{D}_k^H \mathbf{h}_k}{\left\| \left(\sum_j a_j \mathbf{D}_j + \sum_{\bar{k} \in \tilde{\mathcal{I}}_k} b_{\bar{k}} \mathbf{D}_{\bar{k}}^H \mathbf{C}_{\bar{k}}^H \mathbf{h}_{\bar{k}} \mathbf{h}_{\bar{k}}^H \mathbf{C}_{\bar{k}} \mathbf{D}_{\bar{k}} \right)^\dagger \mathbf{D}_k^H \mathbf{h}_k \right\|} \quad (9)$$

Proof: For (P2), it follows from Theorem 2 and standard generalized eigenvalue techniques. Recall from Lemma 2 that (P1) can be written as (P2) using optimal rates in γ_k . ■

In other words, all boundary points of the achievable rate region (i.e., maximization of all weighted sum rates) can be reached by solving the generalized Rayleigh quotient in (8) for an appropriate choice of $K_t + 2K_r$ bounded parameters. Similar results were given in [19] for systems with only one transmitter per receiver and in [20] for interference channels. The beamforming vector in (9) is not unique, for example represented by the arbitrary phase of c_k . Parameters that solve (P2) can be found by solving the dual problem numerically. Heuristic values can be used to perform signal to leakage and noise ratio (SLNR) beamforming [14]. For (P1) it is generally hard to find optimal parameters, but next we propose low-complexity distributed solutions using heuristic parameters.

V. LOW-COMPLEXITY MULTICELL BEAMFORMING

The optimality properties in Theorem 2 and 3 can be exploited for iterative transmission designs (e.g., [9]–[12]) that can be implemented in a partially distributed manner.

However, in practical systems with many subcarriers, limited computational resources, or tight delay constraints, it is necessary with truly distributed non-iterative beamforming [21].

For $|\mathcal{C}_j| \leq N_t$, we propose a heuristic solution to (P1) with low computational complexity. The beamforming strategy for BS_j only requires transmit synchronization between transmitters serving the same receivers—there is no exchange of CSI. BS_j knows \mathbf{h}_{jk} and σ_k^2 perfectly for all $k \in \mathcal{C}_j$ (see [7] and [17] for the case with CSI and synchronization uncertainty), retrieved through feedback or reverse-link estimation.

Let $\mathbf{w}_k = [\mathbf{w}_{1k}^T \dots \mathbf{w}_{K_t k}^T]^T$ be the beamforming vector for MS_k , where $\|\mathbf{w}_{jk}\|^2 = p_{jk}$. The transmit power p_{jk} is zero for all BS_j not serving MS_k , given as $j \notin \mathcal{S}_k$ with

$$\mathcal{S}_k = \{j; k \in \mathcal{D}_j\}. \quad (10)$$

The heuristic beamforming is divided into power allocation (among $p_{jk} \forall k \in \mathcal{D}_j$) and normalized beamforming. Starting with the former at BS_j , observe that interference coordination is mostly relevant for multicell systems with relatively high signal-to-noise ratios (SNRs). In the case of distributed zero-forcing beamforming, the part of the weighted sum rate in (P1) influenced by BS_j can be approximated as

$$\sum_{k \in \mathcal{D}_j} \mu_k \log_2 \left(\underbrace{\sqrt{p_{jk}} \frac{\mathbf{h}_{jk}^H \mathbf{w}_{jk}^{(\text{ZF})}}{\sigma_k \|\mathbf{w}_{jk}^{(\text{ZF})}\|}}_{=c_{jk}} + \underbrace{\sum_{\bar{j} \in \mathcal{S}_k \setminus \{j\}} \frac{\mathbf{h}_{\bar{j}k}^H \mathbf{w}_{\bar{j}k}^{(\text{ZF})}}{\sigma_k}}_{=d_{jk}} \right)^2 \quad (11)$$

where $\mathbf{w}_{jk}^{(\text{ZF})}$ is a distributed ZF vector satisfying $\mathbf{h}_{\bar{j}k}^H \mathbf{w}_{jk}^{(\text{ZF})} = 0$ for all $\bar{k} \in \mathcal{C}_j \setminus \{k\}$ [16]. There is a major difference from regular coherent zero-forcing (with $\sum_{\bar{k} \in \mathcal{C}_j \setminus \{k\}} \mathbf{h}_{\bar{j}k}^H \mathbf{w}_{jk}^{(\text{ZF})} = 0$) as the distributed version requires the contribution from each transmitter to be zero for robustness to synchronization errors³. For fixed c_{jk} and d_{jk} in (11), we solve the power allocation:

Lemma 3. For a given j and some positive constants c_{jk}, d_{jk} ,

$$\begin{aligned} & \text{maximize}_{p_{jk} \geq 0 \forall k \in \mathcal{D}_j} \sum_{k \in \mathcal{D}_j} \mu_k \log_2 (\sqrt{p_{jk}} c_{jk} + d_{jk})^2 \\ & \text{subject to} \quad \sum_{k \in \mathcal{D}_j} p_{jk} \leq P_j \end{aligned} \quad (12)$$

is solved by $p_{jk} = \left(\sqrt{(d_{jk}/(2c_{jk}))^2 + \mu_k \nu} - d_{jk}/(2c_{jk}) \right)^2$, where $\nu \geq 0$ is selected to satisfy the constraint with equality.

Proof: Similar to the proof of Lemma 1 in [16]. ■

Next, for given power allocation, the normalized beamforming vectors are given by Theorem 3 for unknown parameters a_j and $b_{\bar{k}}$. The generalized Rayleigh quotient in (8) becomes

$$\frac{\left| \sum_{\bar{j} \in \mathcal{S}_k} \mathbf{h}_{\bar{j}k}^H \mathbf{w}_{\bar{j}k} \right|^2}{\sum_{\bar{j} \in \mathcal{S}_k} a_{\bar{j}} p_{\bar{j}k} + \sum_{\bar{k} \in \tilde{\mathcal{I}}_k} b_{\bar{k}} \left| \sum_{\bar{j} \in \mathcal{S}_k} \mathbf{h}_{\bar{j}k}^H \mathbf{w}_{\bar{j}k} \right|^2} \approx \frac{|\mathbf{h}_{jk}^H \mathbf{w}_{jk}|^2}{\frac{a_j p_{jk}}{\delta_{jk}} + \sum_{\bar{k} \in \mathcal{C}_j \setminus \{k\}} b_{\bar{k}} |\mathbf{h}_{\bar{j}k}^H \mathbf{w}_{\bar{j}k}|^2} \quad (13)$$

where the approximation is due to replacing the impact from other transmitters with an (unknown) scaling factor δ_{jk} . This

³Desired signals are comparably insensitive to synchronization errors [7].

approximation is necessary to achieve a distributed solution and is motivated by assuming that other transmitters create interference proportional to that from BS_j for each portion of added signal power. By heuristic selection of a_j/δ_{jk} and b_k , we achieve distributed virtual SINR beamforming (DVSINR):

Strategy 1. Distributed Virtual SINR Beamforming

Each BS_j selects its beamforming vectors \mathbf{w}_{jk} as follows:

- First, $p_{jk} = \|\mathbf{w}_{jk}\|^2$ is calculated as in Lemma 3 with $c_{jk} = \frac{|\mathbf{h}_{jk}^H \mathbf{w}_{jk}^{(ZF)}|}{\sigma_k \|\mathbf{w}_{jk}^{(ZF)}\|}$ and $d_{jk} = \sqrt{\frac{P_j}{N_t} \sum_{\bar{k} \in \mathcal{D}_j} \frac{c_{j\bar{k}}^2}{|\mathcal{D}_j|}}$ for $k \in \mathcal{D}_j$.
- Second, select $\mathbf{w}_{jk} = p_{jk} \mathbf{v}_{jk} / \|\mathbf{v}_{jk}\|$, where \mathbf{v}_{jk} maximizes the approximated virtual SINR in (13) for $a_j/\delta_{jk} = (\sum_{\bar{k} \in \mathcal{C}_j} \sigma_{\bar{k}}^2 / |\mathcal{C}_j|) / P_j$ and $b_k = K_r \mu_k / (\sigma_k^2 \sum_{\bar{k}} \mu_{\bar{k}})$:

$$\mathbf{v}_{jk} = \left(\frac{a_j}{\delta_{jk}} \mathbf{I}_{N_j} + \sum_{\bar{k} \in \mathcal{C}_j \setminus \{k\}} b_{\bar{k}} \mathbf{h}_{j\bar{k}} \mathbf{h}_{j\bar{k}}^H \right)^{-1} \mathbf{h}_{jk}. \quad (14)$$

This beamforming strategy is essentially a generalization of the distributed approach analyzed in [16]. The extended DVSINR beamforming herein can handle weighted sum rates and dynamic cooperation clusters. The power allocation in DVSINR considers separability and relative gain of terminals, while the beamforming directions balance signal power towards (weighted) interference to co-terminals in \mathcal{C}_j . Although heuristic assumptions were made, the next section shows that the approach performs well under realistic conditions.

VI. MEASUREMENT-BASED PERFORMANCE EVALUATION

The potential benefits of network MIMO over conventional single-cell processing has been studied extensively. Theoretical Rayleigh fading simulations have shown that the total throughput can be improved considerably by coordinating interference between cells and serving cell edge terminals through multiple coherent base stations (see e.g., [2], [12], [14], [16]). However, results obtained from simulations are highly dependent on the assumptions of the underlying wireless communication channel. In [22], it is shown that the channel characteristics between one mobile terminal and multiple base station sites are correlated. Such dependence between separate channels may affect the results of any coordinated multicell system.

Herein, we investigate the performance of network MIMO in a realistic multicell scenario using measured channels collected in Stockholm, Sweden. The MIMO channel data was collected using one mobile station and two base stations with four-element uniform linear arrays (ULAs) having 0.56λ antenna spacing. The system bandwidth was 9.6 kHz at a carrier frequency in the 1800 MHz band. The measurement environment can be characterized as typical European urban with four to six story high stone buildings. For further information on measurement details, see [22]. From the collected channel information, data representing four single-antenna user terminals moving around in the area covered by both transmitters was extracted, see Fig. 2.

The performance measure is the weighted sum rate with $\mu_k = c_w / \log_2(1 + \sum_{j \in \mathcal{S}_k} \frac{P_j}{\sigma_k^2 K_r} \max_j \mathbb{E}\{\|\mathbf{h}_{jk}\|^2\})$, where c_w is the scaling factor making $\sum_{k=1}^{K_r} \mu_k = K_r$. This represents

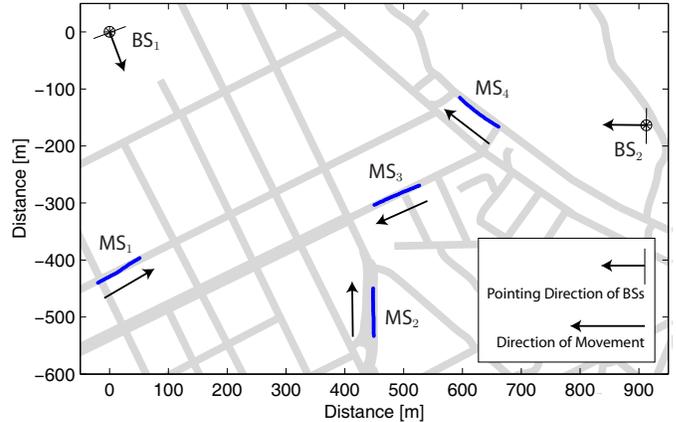


Fig. 2. Downlink scenario based on measurements in an urban environment. Two four-antenna base stations are serving four single-antenna terminals.

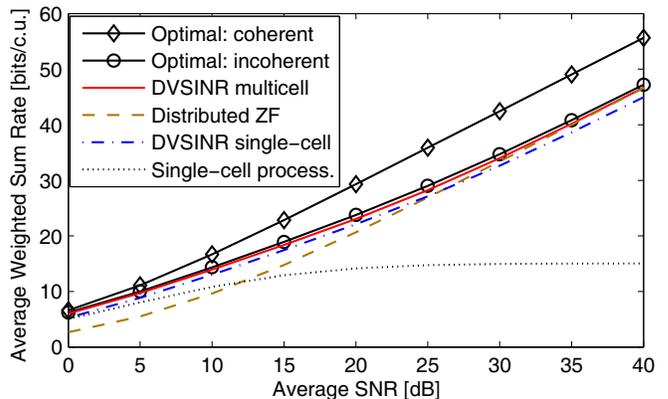


Fig. 3. Weighted sum rate with different beamforming schemes, including the proposed low-complexity distributed DVSINR beamforming scheme.

proportional fairness (with equal power allocation). The average SNR is defined as for transmission on one antenna with full power, averaged over terminals and BS antennas:

$$\text{SNR}^{\text{average}} = \frac{1}{K_t} \sum_{j=1}^{K_t} \frac{P_j \mathbb{E}\{\|\mathbf{h}_{jk}\|^2\}}{N_j}. \quad (15)$$

The analysis herein has assumed perfect base station synchronization, which cannot be guaranteed in practice due to estimation uncertainty, hardware delays, clock drifts, and minor channel changes. Due to space limitation, this is also assumed in the performance evaluation, but in [17] we show that DVSINR is robust to small synchronization errors.

Different beamforming strategies are compared. The optimal beamforming is derived numerically for (P1) and under the additional condition of incoherent interference reception⁴. The performance of the proposed DVSINR scheme is shown for the multicell case with $\mathcal{D}_1 = \mathcal{D}_2 = \mathcal{K} = \{1, 2, 3, 4\}$ and the single-cell case with $\mathcal{D}_1 = \{1, 2\}$ and $\mathcal{D}_2 = \{3, 4\}$ (in both cases, $\mathcal{C}_1 = \mathcal{C}_2 = \mathcal{K}$). As a benchmark, we also included the distributed ZF scheme and the single-cell processing case when out-of-cell interference is included in σ_k^2 -terms, see [16].

⁴That is, interference from different base stations is separated in the SINR to model that base stations cannot cancel out each other's interference.

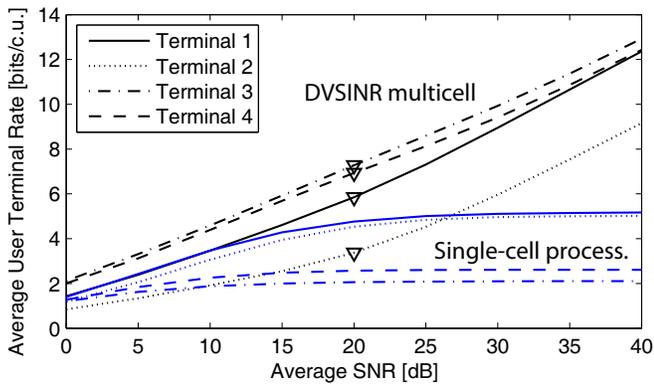


Fig. 4. Terminal rates with and without network MIMO. The proposed multicell DVSINR scheme (triangles) is compared with single-cell processing.

The average weighted sum rate (per channel use) over 750 channel realizations is given in Fig. 3. The difference between optimal beamforming and DVSINR increases with the SNR, but the latter is very close to optimum under the condition of incoherent interference, which might be the most robust [17] and reasonable case in practice [14]. Multicell DVSINR and distributed ZF are asymptotically equal at high SNR, while DVSINR outperforms the single-cell processing case which is bounded at high SNR. Observe that the gain of serving all users through both base station is rather small for the DVSINR scheme; thus, the major gain is from interference coordination.

The average individual user terminal rates are shown in Fig. 4 for multicell DVSINR (marked with triangles) and single-cell processing. Evidently, the large increase in weighted sum rate for network MIMO does not translate into a monotonic improvement in terminal rates. Terminals 3 and 4 have strong channels from both base stations and therefore experience large gains from base station coordination, while Terminal 2 which only has a strong link to BS₁ sees a decrease in performance at most SNRs (as power and beamforming efforts are concentrated on cell edge terminals). Thus, the common claim that network MIMO will improve both the total throughput and the fairness is not necessarily true in practice.

VII. CONCLUSION

Multicell transmission was considered with dynamic cooperation clusters, where each base station coordinates interference to a set of terminals and provides some of them with data. The relationship between weighted sum rate maximization and having individual rate constraints was analyzed and used to derive beamforming optimality conditions and the optimal transmission structure for both problems. These properties were used to propose low-complexity transmission strategies for distributed implementation. The performance was evaluated on measured multicell channels in an urban environment, which provides more reliable results than previous theoretical evaluations. The proposed strategy provides close to optimal performance and the major gain of multicell coordination seems to originate from interference coordination, while the gain of serving terminals through multiple base stations is small. While coordination improves performance for cell edge terminals, other terminals can experience degradations.

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