Equilibrium design for multi-channel random access networks with selfish users

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Equilibrium design for multi-channel random access networks with selfish users

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Abstract

Recently, there has been increasing research interest in the inefficiency in spectrum utilization, which is mainly caused by fixed spectrum allocation policies. There are some proposed approaches to solve this inefficiency, like Dynamic Spectrum Access (DSA), which allows users to share spectrum resources. Implementing DSA in a distributed way can avoid problems with system complexity that can arise in centralized DSA systems; however, it can create incentives for the users to behave selfishly. Selfish behavior reduces the efficiency of the DSA system, and it causes the system to end up in one of many possible operating points, which makes the performance analysis difficult.

In this work, we study a multichannel random access system with selfish users and we propose two mechanisms in which the access point charges users for transmission. We analyze the performance of these mechanisms using Game Theory. Results show that by charging users for transmission, we can reduce the possible operating points of the system to a single one. Of the two proposed mechanisms, the per-channel cost mechanism performs rather well, bringing the system sum utility close to that of scheduling systems.
Acknowledgment

“Life is a spiritual journey.” Two and a half years living in Sweden is one little part of the whole life, but it gives me too much joyful and unforgettable memory. The completion of this thesis reminds me that the end of the student life in Sweden is approaching and I would like to express my appreciation to all the people who help me a lot.

First of all, to my supervisor Ali Nazmi Özyagci, I want to say that I couldn’t find anyone else better than you. When you said my working process was going well and my writing skill was getting better, I believe these wouldn’t happen without your help. Moreover, specially thank my examiner Prof. Jens Zander for providing so many important comments and guiding me to drive on the correct road. And I also thank Jiahong Wang to be my opponent during the thesis defense.

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Jian Zhang

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Introduction

Today, most governmental agencies use fixed spectrum allocation policies to regulate wireless networks. A huge demand on TV-broadcasting and mobile communication made some parts of the frequency spectrum rather crowded. Meanwhile, the allocated spectrum range is used temporally and geographically with high variance; occupancy varying from 85% to as low as 15% [1]. Therefore, there has been a lot of research on new approaches of spectrum allocation to address this inefficiency. One of these approaches is Dynamic Spectrum Access (DSA).

1.1 Dynamic Spectrum Access and cognitive radio

DSA allows users to share spectrum resources according to the users’ and channel information obtained from environment [2]. DSA schemes can be classified in different ways.

Cooperative/Non-cooperative: the users in the system could choose working cooperatively or non-cooperatively based on the system configuration. In cooperative system, instead of maximizing their own individual benefit as in a selfish system (one typical example of non-cooperative systems), all users will cooperate with each other to achieve some goal which is probably defined by the operator. For example, if the operator aims to provide good system sum utility, each user has to choose his transmission strategy to improve the sum utility which means some of them may sacrifice.

Centralized/Distributed: a typical wireless system cell contains one Access Point (AP) and several users with handsets. If the DSA scheme is centralized, the AP will play a role as a central control entity which actively assigns channels to the users, and users have to accept the assignment passively. We could gain the greatest benefit from a dynamic spectrum allocation centrally, since AP can calculate all possible allocation
scenarios and choose the best one (scheduling case). However, this processing costs a lot of time and adds much complexity and overhead to the system. So distributed DSA scheme has been looked for as a solution to this problem where all users choose transmission strategies (transmitting on which channel or waiting) on their own.

Although this thesis is not strictly about Cognitive Radio system, we briefly mention Cognitive Radio (CR) here because it has been proposed as a method to implement distributed DSA schemes. As a further step of software-defined radio [3], CR aims to bring efficiency to radio spectrum utilization and a more reliable communication [4]. In general, cognitive radio allows the secondary (unlicensed) users to sense and predict the primary (licensed) user’s behavior and channel information. Then secondary users can use the licensed bands by inserting their data into the unused spectrum or white space (like unused time slots) of the prime user. When prime user wants to take the white space, secondary users have to leave to make sure that prime user’s right is not infringed.

1.2 Motivation of this thesis

In [5], the authors investigate the behavior of selfish users in a multichannel random access network and they calculate how these selfish users’ benefit will be when they choose different transmission strategies. They analyze the behavior of the selfish users with help of game theory, and derive the users’ transmission strategies at all of the Nash equilibria of the system. As a conclusion, it is illustrated that the selfish system has slightly worse performance compared to cooperative system when the users’ Nash equilibrium transmission strategies are considered.

We call these Nash equilibria operating points. In each time slot of the multichannel random access system with selfish users (we use selfish system for short), depending on the specific transmission costs for each user on all channels, several possible operating points could be realized. Therefore, the results in [5] are obtained as a
probability distribution of utilities in all of these operating points. However, not all of them have good performance. For example although some users have bad channel condition, they could still occupy on the channels. Furthermore, when analyzing a system, we wish it could end up to a certain point but the authors don’t suggest any approaches to reach anyone of these operating points.

1.3 Problem definition and scope

The purpose of this thesis is to investigate mechanisms to reach an operating point (a unique Nash equilibrium) of a distributed multichannel random access system with selfish users and to analyze if they could provide good performance. In this paper, we assume all users have information about every other user, which includes channel conditions and thus associated transmission cost (see the transmission cost $e_{nk}$ function in Page 14). In another words they have complete information of the system. We propose two mechanisms to influence the behavior of the selfish users in the system in [5]. In the first mechanism, the AP determines an extra single cost which is paid by all users who transmit a packet. In the second mechanism the AP determines a different cost for each channel in the system.

We then analyze the performance of these two mechanisms and quantify the improvement in performance in terms of sum and individual utilities over the selfish system in [5]. Sum utility can be used to present system overall performance and reflects system total throughput. And individual utility shows how much benefit a user could get from the system. In particular, we find that these mechanisms can lead to unique Nash equilibria and calculate the sum utility difference compared to scheduling system and how much improvement to original selfish system [5]. (Scheduling system is the best case of all possible operating points and the upper bound on performance and the users are fully cooperative. In such system, the central control entity has to know all possible operating points, chooses the best one and tells every user how to behave. This procedure would significantly increase the
system complexity and sometimes is NP-hard.)
Previous work

Random access systems with selfish users have been studied previously. In this section we give an overview of the work in literature as well as we highlight the difference of our approach.

The authors in [6] present an example of single channel slotted ALOHA system analyzing with complete information by using game theory. It has been proved that a symmetric equilibrium exists in the slotted ALOHA game (proof can be found in [11]). Furthermore as a conclusion of this paper, compared to a centrally controlled slotted ALOHA system, selfish system is more reliable to prevent cheating. Selfish users, slotted ALOHA system with complete information and game theoretical analysis are the same components comparing with ours, but we will study multichannel system instead of single channel system.

In [7], the authors study another model slotted ALOHA system with incomplete information. In collision model, a pricing parameter is introduced as a network charge, which is monotonically decreasing with the number of users. As we know, if there is only one user whose transmission cost is smaller than a threshold, he will transmit alone in the channel. And the other users who have large transmission cost (greater than the threshold) will refrain from transmitting on this channel which means they have to look for other channels to transmit; otherwise they will wait. Then the authors find that this equilibrium threshold is equal to the user’s transmission probability and determined by the network charging price. This analysis based on one channel mode with incomplete information is not suitable for multichannel with complete information study.

Similar to [7], the authors in [8] defined a price-based algorithm but with unique payment which is charged on all users. An access point is needed to redistribute the
payment from transmitting users to everyone. To improve system sum utility, every user transmits on the channel where they have lowest transmission cost. Since all selfish users have incomplete information and some users could have the lowest cost on the same channel, collisions still would happen. The idea of using payment and redistribution in [8] is similar to this thesis but we assume users have complete information.

A Price-based Distributed Spectrum Sharing (PDSS) algorithm introduced in [9] is another solution for multichannel game but in an ad-hoc wireless system. User’s payoff is constructed by positive transmit and success utility and negative cost (due to interference) generated by the other users. Each user updates his utility and interference price sequentially and synchronously during the progress and broadcast this information to help others update theirs. Compare to iterative max SINR (a reference algorithm), the spectrum efficiency and power efficiency gain of the PDSS will increase with shared user number, and decrease with the shared channel number. In this thesis, our system has an Access Point (AP) and all users directly connect to this AP.
System model and assumptions

We consider a wireless network access scenario like Figure 1 shows. A set of users who want to transmit have to select their own transmission strategies in order to maximize their own utilities. In our assumption, we have N users and K channels. Every user behaves selfishly which means there isn’t a central control entity in the system. If there are several channels for a user to transmit, he will always choose the one where he transmits and would get better benefit. On the other hand, users communicate and transmit their packets to AP. During communication, AP distributes the information such as channel conditions and users’ transmission costs to all users. We assume if more than one user transmits on the same channel simultaneously, collision would happen and all packets will be lost and none will be received by receivers.

As we mentioned in Introduction, this paper is a further study of [5]. Hence, we use the same model presented in [5]. We assume this is a slotted-ALOHA system and in each time slot, every user takes one of the two actions: Transmit or Wait. When the packet is successfully transmitted, the user gets a positive payoff which is normalized as 1; if the transmission is failed, the user has zero payoffs. No matter if the transmission is successful or failed, user has to spend a cost \( e_{nk} \) on each transmission. So user’s transmission utility without introducing our mechanism could be written as follow.
\[ U_{nk} = \begin{cases} 
1 - e_{nk} & \text{Transmit and Succeed (TS)} \\
-e_{nk} & \text{Transmit but Fail (TF)} \\
0 & \text{Wait (W)} 
\end{cases} \tag{1} \]

We write the transmission cost as \( e_{nk} \) which represents User n's transmission cost on Channel k and is a function of propagation loss on Channel k. We derive the normalized \( e_{nk} \) as a ratio of \( P_t \) and \( P_{max} \), where \( P_t \) is the transmit power that just satisfies SNR requirement for a successful transmission and \( P_{max} \) is the maximum transmit power of the user terminals, which just satisfies the SNR requirement for a successful transmission at the cell border if there were no shadow or fast fading. After simplification, we write transmission cost as below,

\[ e_{nk} = \frac{(r/r_0)^\alpha}{S \times R} \]

- \( r \): distance from user to access point
- \( r_0 \): cell radius
- \( \alpha \): pathloss exponent
- \( S \): shadow fading component, lognormally distributed with unit mean and standard deviation \( \sigma \)
- \( R \): fast fading component, exponentially distributed with unit mean

In order to model an urban area, we use \( \alpha = 3 \) and \( \sigma = 4 \).
Methodology

Aimed to mathematically analyze how individual players’ behaviors interact to each other by choosing different strategies, game theory is widely used in economics, biology and communication as well. When we talk about a multichannel random access game [5], all players are selfish and we assume everyone in the game is rational, chooses strategy independently in order to maximize his own utility. A multichannel random access system with selfish users can be formulated as a non-cooperative game.

Here, we first give a brief introduction about game theory. In game theory, there are three basic elements: a set of players \( i \in \{1, 2, ..., I\} \), their pure strategy space \( S_i \) and payoff function \( u_i \). In a game, there is competition relationship among players, and individual success (payoff) of choosing a strategy depends on other players’ choices as well. Nash equilibrium is a profile of strategies [10], whose definition is shown below.

**Definition:** A mixed-strategy profile \( \sigma^* \) is a Nash equilibrium if, for all players \( i \),

\[
    u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(s_i, \sigma_{-i}^*) \quad \text{for all } s_i \in S_i
\]

Therefore in Nash equilibrium, none of the players could change his strategy to increase his utility unilaterally.

A good tutorial of wireless game theoretical analysis can be found in [12] and the authors present several basic examples of different scenarios. In this paper, our analysis includes following points based on the knowledge from this tutorial.

- Prove the existence of Nash equilibrium
- Determine if the Nash equilibrium is unique (from [5], we know there are several Nash equilibria, however we deplete the Nash equilibria set by adding mechanism)
The first two points are presented in mechanisms design and analysis chapter, and the third point will be derived by using simulation.

Firstly, we analyze a (5, 3) system (5 users and 3 channels, we use $N$ to represent the number of users and $K$ to represent the number of channels) and make a PDF comparison of all four systems which are selfish, single cost mechanism, per-channel cost mechanism and the scheduling systems. The reasons that we choose (5, 3) system are:

- Take a system where $N>K$ is more likely to be the scenario where spectrum management will be needed
- Calculating all Nash equilibria is difficult for systems where $K>4$

To calculate transmission costs, all users are randomly located in the service area. The simulation is run 10,000 times and the confidence intervals of the sum utilities in all the four systems are presented as well.

Secondly, we also simulate both mechanisms with a set of combination of $N$ and $K$ (taking $K=3$, and $N=\{5, 9, 15, 30\}$) to see these mechanisms’ performance in crowded environment.
Mechanism design and analysis

In an N users and K channels system, we formulate an $N \times K$ matrix whose elements are every user’s transmission cost on each channel. (For abbreviation, we use $U$ to represent User and $L$ to represent Channel.)

![Table 5.1 System transmission cost matrix](image)

We announce that when a user’s successful transmission utility equals to his waiting utility, he might choose anyone of the both strategies. So in the following text, when we talk about the cost, it adds an infinity small value $\delta$ by default. This can remove the situation when successful transmission utility equals to (will be less than in reality) waiting utility.

5.1 Mechanism design: single Access Point charge cost

The notion when we create this mechanism is, when some users transmit, they “force” other users to wait, so they should pay some payment ($c$) to access point (AP), and AP distributes this as compensation ($c \times \theta$) to the waiting users. Hence, we have

$$U_{nk} = \begin{cases} 
1 - e_{nk} - c & \text{Transmit and Succeed (TS)} \\
- e_{nk} - c & \text{Transmit but Fail (TF)} \\
c \times \theta & \text{Wait (W)} 
\end{cases}$$

(3)
where $\theta = \frac{N_t}{N-N_t}$. $\theta$ can be treated as a scaling factor and $N - N_t$ is the number of waiting users. Then the compensation for waiting during one slot transmission comprises two components: the cost $(c)$, the numbers of transmitting users $(N_t)$. The idea of this mechanism is that, the more users are transmitting, the more benefit of waiting can be obtained due to the increase of $\theta$. However, the increase in the number of transmitting users may cause collision, and utility of a failed transmission is definitely less than the utility of waiting. So some users will wait instead of transmitting. Meanwhile, by choosing a suitable value of the cost $c$, we could control the mount of transmitting users and further reduce collisions.

Furthermore, as one of the mechanism design requirement, we want the system could end up in a unique point which is complete collision-free. We can fulfill this requirement as well by selecting a suitable transmission cost $(c)$.

Here we show how to calculate this cost. At Nash equilibrium for those users who are waiting, their successful transmission utilities have to be less waiting utilities. Then we have a threshold for $e_{nk}$ as following, 

$$1 - e_{nk} - c \leq c \times \theta$$

$$e_{nk} \geq 1 - c \times \frac{N}{N-N_t} = \varepsilon$$

Every user whose transmission cost $e_{nk}$ is not lower than the threshold $\varepsilon$ will wait.

So far, we need to determine the value of $N_t$. This can be done by access point by going through the following steps.

- Find the lowest transmission costs in the transmission costs Matrix, check if it is the second lowest on its channel and the lowest for that user. If not, continue finding the next lowest one. This process will stop until it meets the requirements, and this cost is our reference transmission cost. An example shows below.
Table 5.2 Transmission cost matrix example

<table>
<thead>
<tr>
<th>Users</th>
<th>Channels</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.49</td>
<td>0.76</td>
<td>0.13</td>
<td>0.33</td>
</tr>
<tr>
<td>2</td>
<td>1.51</td>
<td>0.66</td>
<td>0.46</td>
<td>0.50</td>
</tr>
<tr>
<td>3</td>
<td>0.84</td>
<td>0.48</td>
<td>12.33</td>
<td>3.54</td>
</tr>
<tr>
<td>4</td>
<td>0.98</td>
<td>0.92</td>
<td>1.1</td>
<td>0.28</td>
</tr>
<tr>
<td>5</td>
<td>0.35</td>
<td>0.77</td>
<td>0.82</td>
<td>1.55</td>
</tr>
<tr>
<td>6</td>
<td>2.36</td>
<td>3.78</td>
<td>1.41</td>
<td>1.90</td>
</tr>
</tbody>
</table>

- $N_t$ is the count number of users **before** we stop. In this example, we mark the reference transmission cost with red rectangular. There are three costs less than it, so $N_t = 3$ which refers to the three of $U_1$, $U_4$ and $U_5$.
- $\varepsilon$ equals to the reference transmission cost. Then we could obtain the single cost $c$. If the reference transmission cost is greater than 1, $c = 0$.

This mechanism and the single cost $c$ determines strategy profile $S$ and we will prove $S$ is Nash equilibrium basing on the definition of Nash equilibrium given in (2).

**Proof:**

i. Nash equilibrium

1. Suppose $U_n$ transmits on $L_k$. If he wants to transmit on some other channel ($L_j$), he won’t have better utility, since $e_{nk}$ is the lowest transmission cost for $U_n$, so $e_{nk} < e_{nj}$ even if there is no collision on $L_j$.

2. Suppose $U_n$ transmits on $L_k$. If he wants to wait, he won’t have better utility. His successful transmission utility on $L_k$ is greater than waiting utility. When he waits, the number of transmitting users loses one so $\theta$ decreases. Hence waiting utility won’t increase and is less than his transmission utility on $L_k$.

3. Suppose $U_n$ waits. If he wants to transmit on some channel $L_j$,
   - If there is a user transmitting on $L_j$, $U_n$ will have negative utility due to collision.
• If $L_j$ is unused, by adding $\varepsilon$ $U_n$’s successful transmission utility will be less than waiting utility.

Since $U_n$ cannot improve its utility by unilaterally changing its strategy, this strategy profile is Nash equilibrium.

ii. Uniqueness

When we introduce this unique cost $c$ to the system transmission cost matrix, the new matrix is shown below.

<table>
<thead>
<tr>
<th>Channels</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Users</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$e_{11} + c$</td>
<td>$e_{12} + c$</td>
<td>$e_{1K} + c$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$e_{21} + c$</td>
<td>$e_{22} + c$</td>
<td>$e_{2K} + c$</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>$e_{n1} + c$</td>
<td>$e_{n2} + c$</td>
<td>$e_{nK} + c$</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.3 Transmission cost matrix with single cost

According to the definition of the single cost mechanism, each channel has at most one user who can transmit since anyone else’s successful transmission utility is less than waiting utility by spending an extra cost. Meanwhile, each user has at most one channel to transmit as well (it is not possible to transmit on two channels simultaneously). And at this moment, the strategy profile is a Nash Equilibrium, so user will always choose strategy for better utility from his successful transmission utility and waiting utility. Hence, finally every user has only one choice to make, so we prove that the Nash equilibrium is unique. To illustrate clearly, we again take Table 2 as an example. The new matrix is shown below. We can obtain that $c = 0.27$, which is calculated by substituting $N = 6$ $N_t = 3$ and $\varepsilon = 0.46$ to (5).
The transmission strategy profile is that $U_1$ transmits on $L_3$, $U_4$ transmits on $L_4$, $U_5$ transmits on $L_1$ and all the other users wait.

However, for some specific cases when we select a single $c$, there may be quite few transmitting users and many channels are unused on that point in a large system, which significantly decreases the sum utility. So we look for multi-threshold solution.

### 5.2 Mechanism design: per-channel Access Point charge costs

The advantage of introducing per-channel AP-charge costs is to improve channel utilization. By doing this we could utilize the channels as much as possible and each channel has its own cost which won’t influence each other. The following steps show this mechanism in detail.

- Locate the lowest transmission cost ($e_{np}$) in the matrix, and find the relevant user ($U_n$) and channel ($L_p$). So $e_{np} = \min(e_{ij}), i \in \{1, 2 \ldots N\}, j \in \{1, 2 \ldots K\}$; $e_{np} = \min(e_{ip}), i \in \{1, 2 \ldots N\}$
- Access point adds a cost $c_p$ to all transmissions on $L_p$ such that no user except for the one with the smallest transmission cost will have any incentive to transmit on this channel.

$$e_{mp} = \min(e_{ij}), i \in \{1, 2 \ldots N\}, i \neq n; \quad c_p = 1 - e_{mp}$$

Then the user with second lowest transmission cost ($e_{mp}$) has a successful transmission utility which is slightly less his waiting utility, he will wait during...
this time slot. The others with even greater than \( e_{mp} \) transmission costs will wait as well.

- After removing \( U_n \) and \( L_p \), the matrix degrades to an \((N - 1) \times (K - 1)\) sub-matrix. And we go back to the first step to find the lowest transmission cost in the reduced matrix of size \((N - 1) \times (K - 1)\). We note that if on some channel \( L_x \), all transmission costs are greater than 1, then \( c_x = 0 \) and this channel will be unused. The whole procedure will stop when the matrix is reduced to be empty.

- The compensation \( \omega = (\sum_{k=1}^{K} c_k)/N \) is distributed to all users in the target matrix.

Similar to what we have done in single cost mechanism, based on formulation (2) the proof is shown below.

**Proof:**

1. Nash equilibrium
   1. Suppose \( U_n \) transmits on \( L_k \). If he wants to transmit on some other channel \( (L_j) \), he won’t have better utility. Because it only happens that \( U_n \) transmits on \( L_k \) when this has already been his best choice. If there is no collision on \( L_j \), \( e_{nk} < e_{nj} \). Otherwise, \( L_j \) is occupied by some other user.
   2. Suppose \( U_n \) transmits on \( L_k \) and he wants to wait. Because AP will distribute the same compensation to all users and a successful transmission provides \( U_n \) at least a positive utility. If he waits, AP will gather less from the costs and compensation will be less and he gives up the utility from successful transmission. So \( U_n \) won’t have better utility.
   3. Suppose \( U_n \) waits. If he wants to transmit on some channel \( L_j \),
      - If there is a user transmitting on \( L_j \), \( U_n \) will have negative utility due to collision.
      - If \( L_j \) is unused, \( U_n \) will have less utility because the \( e_{nj} \) must be greater than 1 due to the third step in our proposed per-channel costs mechanism.
Since $U_n$ cannot improve its utility by unilaterally changing its strategy, this strategy profile is Nash equilibrium.

ii. Uniqueness

By adding these per-channel costs that we calculate, we show the new matrix below.

<table>
<thead>
<tr>
<th></th>
<th>Channels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Users</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$e_{11} + c_1$</td>
</tr>
<tr>
<td>2</td>
<td>$e_{21} + c_1$</td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>$e_{N1} + c_1$</td>
</tr>
</tbody>
</table>

Table 5.5 Transmission cost matrix with unique cost example

Similar to single cost mechanism, each channel has at most one user who can transmit and each user has at most one channel to transmit. Every user has only one strategy to choose in each time slot. So per-channel cost mechanism also leads the system to a unique Nash equilibrium.

We could see in both of the mechanisms, AP does not keep any utility for itself and distributes the utility to users.

5.3 Game theoretical analysis example

Here we present a $2 \times 2$ system (two users and two channels) as an example to show our game theoretical analysis. In this $2 \times 2$ matrix, columns represent two channels, rows represent two users. The figure with four small boxes, we call it transmission pattern. The white box represents the user doesn’t use that channel and the black box represents he uses. For example, if the first box on first row is black, this means
$U_1$ transmits on $L_1$.

<table>
<thead>
<tr>
<th>Channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

Table 5.6: $2 \times 2$ system transmission cost matrix

1. All wait. When all channel costs are greater than 1 ($e_{nk} > 1 \quad n, k \in \{1, 2\}$), none will transmit because they will always have negative utility.

Transmission pattern:

2. One user transmits the other waits. 

   **Case 1** if $e_{2k} > 1 \quad k \in \{1, 2\}$ and $e_{11} > e_{12}, e_{11} < 1$

   a) Single cost mechanism

       Directly find the lowest transmission cost $e_{11}$ and let $U_1$ transmit on $L_1$. Since $e_{21} > 1, c = 0$ and the sum utility $\Delta = 1 - e_{11}$ which all comes from $U_1$.

   b) Per-channel costs mechanism

       $e_{11}$ is the lowest transmission cost in the matrix and refers to $U_1$ and $L_1$. Meanwhile, $e_{21} > 1$, so $c = 0$. After removing $U_1$ and $L_1$, the new matrix only has $e_{22}$ left and it is greater than 1. Finally, we have the sum utility $\Delta = 1 - e_{11}$ which all comes from $U_1$.

   **Case 2** if $e_{n2} > 1 \quad n \in \{1, 2\}$ and $e_{11} < e_{21}, e_{11} < 1$

   a) Single cost mechanism

       Directly find the lowest transmission cost $e_{11}$ and let $U_1$ transmit on $L_1$. If $e_{21} < 1$, the cost $c$ is calculated based on (5).
\[ e_{21} \geq 1 - c \times \frac{2}{2 - 1} = \varepsilon \]

Then, \( c = \frac{1 - e_{21}}{2} \) and the sum utility \( \Delta = 1 - e_{11} \). Utility of \( U_1 \) is \( 1 - e_{11} - \frac{1 - e_{21}}{2} \) and utility of \( U_2 \) is \( \frac{1 - e_{21}}{2} \). Otherwise, the result is same as Case 1 (a).

b) Per-channel costs mechanism

\( e_{11} \) is the lowest transmission cost in the matrix and refers to \( U_1 \) and \( L_1 \).

If \( e_{21} < 1 \), \( c_1 = 1 - e_{21} \). After removing \( U_1 \) and \( L_1 \), the new matrix only has \( e_{22} \) left and it is greater than 1. The sum utility is \( 1 - e_{11} \), utility of \( U_1 \) is \( 1 - e_{11} - \frac{1 - e_{21}}{2} \) and utility of \( U_2 \) is \( \frac{1 - e_{21}}{2} \).

(The other cases’ result could be derived by following the similar procedure.)

Possible transmission pattern: 

3. Both of users have one transmission cost greater than 1 and the other lower than 1. Furthermore, the lower costs are on different channels. The users will transmit on the relevant channels without colliding and cost will be zero for both mechanisms.

Possible transmission pattern: 

4. Other cases.

The cases we discuss on this stage refer to those where all transmission costs are lower than 1. In reality, because all users and channels are identical, anyone could be \( U_1 \) and any channel could be \( L_1 \) by definition. Therefore, we call this user/channel symmetry. There are 24 combinations for the four transmission costs. If we remove the symmetry situations, there are only 6 basic combinations.

1. \( e_{11} > e_{12} > e_{21} > e_{22} \)
2. \( e_{11} > e_{12} > e_{22} > e_{21} \)
3. $e_{11} > e_{21} > e_{12} > e_{22}$
4. $e_{11} > e_{21} > e_{22} > e_{12}$
5. $e_{11} > e_{22} > e_{21} > e_{12}$
6. $e_{11} > e_{22} > e_{12} > e_{21}$

(All other cases on this stage are symmetry to one of the six basic combinations and can be obtained by interchange users’ and/or channels’ sequences. Here we only show how to solve this problem by using Case 1 as an example.)

**Case 1** $e_{11} > e_{12} > e_{21} > e_{22}$

c) Single cost mechanism

Sequentially locate transmission costs $(e_{11}, e_{12}, e_{21})$, stop until find $e_{21}$.

Return 1 to $N_t$, since $U_2$ has negative utility by charging with the cost, he will wait. Base on (5),

$$e_{21} \geq 1 - c \times \frac{2}{2 - 1} = \varepsilon$$

Then, $c = \frac{1-e_{21}}{2}$. The sum utility is $1 - e_{11}$, utility of $U_1$ is $1 - e_{11} - \frac{1-e_{21}}{2}$ and utility of $U_2$ is $\frac{1-e_{21}}{2}$.

d) Per-channel costs mechanisms

$e_{11}$ is the lowest transmission cost in the matrix and refers to $U_1$ and $L_1$.

Set up the cost on $L_1$, $c_1 = 1 - e_{21}$. After removing $U_1$ and $L_1$, $U_2$ can only transmit on $L_2$ and $c_2 = 0$. Therefore, sum utility $\Delta = 2 - e_{11} - e_{22}$.

Utility of $U_1$ is $1 - e_{11} - \frac{e_{21}}{2}$ and utility of $U_2$ is $1 - e_{22} + \frac{e_{21}}{2}$.

From this simple case, we show that Mechanism-per channel AP-charge cost has better sum utility performance compared to Mechanism-unique AP-charge cost. Only on some ideal cases, they may have same results. The results of the other five cases will be easily derived through the procedure similar to the above.
Simulation results

In this part, firstly we assume $N=5$ and $K=3$ and show the PDF of sum utilities in selfish, single cost mechanism, per-channel costs mechanism and scheduling systems (Figure 6.1). After 10,000 simulations, there are 185,756 sum utilities results in selfish system, which contains all possible Nash equilibrium for each snapshot. It is very interesting to see that lots of data (around 60% from Figure 6.2) assemble on the region from 0.75 to 1 and another 25% concentrate on the region from 1.75 to 2 in the single cost mechanism system. This could be explained by that each user who transmits successfully has a very low transmission cost since the mechanism always finds the cost starting from the lowest. Therefore he contributes a utility which is close to one. If there are more than one user could transmit, the sum utility will be close to the number of transmitting users. The probability distribution curve of per-channel costs mechanism system is reasonable close to the scheduling’s as we expect. From Table 6.1 we could also see both mean and standard deviation of per-channel costs mechanism are only slight worse than the scheduling’s. However the performance of single cost mechanism is even worse than the selfish system.

Table 6.1 also includes the confidence interval. By doing 10,000 times simulation, each system’s population mean will lay within the region $\bar{X} \pm z \frac{\sigma}{\sqrt{n}}$ respectively.

<table>
<thead>
<tr>
<th></th>
<th>mean ($\bar{X}$)</th>
<th>std ($\sigma$)</th>
<th>$\bar{X} - z \frac{\sigma}{\sqrt{n}}$</th>
<th>$\bar{X} + z \frac{\sigma}{\sqrt{n}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>selfish</td>
<td>2.1251</td>
<td>0.4743</td>
<td>2.1158</td>
<td>2.1344</td>
</tr>
<tr>
<td>single cost</td>
<td>1.3016</td>
<td>0.4942</td>
<td>1.2919</td>
<td>1.3113</td>
</tr>
<tr>
<td>per-channel costs</td>
<td>2.3760</td>
<td>0.4459</td>
<td>2.3673</td>
<td>2.3847</td>
</tr>
<tr>
<td>scheduling</td>
<td>2.5146</td>
<td>0.3885</td>
<td>2.5070</td>
<td>2.5222</td>
</tr>
</tbody>
</table>

Table 6.1 means/standard deviations and confidence interval analysis $\alpha = 0.05$ of sum utilities in the four systems
Figure 6.1 PDF of sum utilities in selfish, single cost mechanism, per-channel costs and scheduling systems with N=5, K=3

Figure 6.2 CDF of sum utilities in selfish, single cost mechanism, per-channel costs and scheduling systems with N=5, K=3
We also take a look at how the two mechanisms perform in individual utilities. Roughly, a user in both selfish and scheduling systems will have zero utility in more than 40% of time which happens when all of his transmission costs are greater than 1 or there are always someone else who has lower transmission cost on each channel. And in a (5, 3) system, at least two users have to wait during each time slot, which amounts to the probability 40%. But this situation would rarely happen in the other two mechanism-implemented systems due to compensation. From an individual user’s view, his mean utility increase by applying per-channel costs mechanism. However, by applying both the mechanisms the individual utilities will have small standard deviation. And since in both of the two mechanisms, the compensation is from the users who have high own utility and given to the users who don’t. The utility gap among them is obviously getting narrow. When a user in per-channel costs mechanism system has good utility from his own transmission and gets compensations from the others as well, he could have an individual utility greater than one in about 4% of the time, which means the system would encourage those users whose transmission costs are much lower than the others to transmit.

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>std</th>
</tr>
</thead>
<tbody>
<tr>
<td>selfish</td>
<td>0.4245</td>
<td>0.4074</td>
</tr>
<tr>
<td>single cost</td>
<td>0.2629</td>
<td>0.1359</td>
</tr>
<tr>
<td>per-channel costs</td>
<td>0.4780</td>
<td>0.2405</td>
</tr>
<tr>
<td>scheduling</td>
<td>0.5080</td>
<td>0.4360</td>
</tr>
</tbody>
</table>

Table 6.2 means/standard deviations of individual utilities in the four systems
Figure 6.3 CDF of individual utilities in selfish, single cost mechanism, per-channel costs and scheduling systems with N=5, K=3
In order to present the improvement from using per-channel cost mechanism in other different ways, we define two parameters ratio ($\mu$) and difference ($\tau$).

For each realization, there are several possible Nash equilibria in selfish system. Each Nash equilibrium yields one sum utility, and the one with the largest sum utility is the scheduling sum utility ($U_{sch}$). On the other hand, by implementing per-channel cost mechanism, we can obtain the per-channel cost sum utility ($U_{perch}$). Hence, our introduced ratio reflects how much better the per-channel cost sum utility is than the amount of all possible cases in selfish system and is shown by the equation below.

$$\mu = \frac{\text{the number of Nash equilibria whose sum utilities are not greater than } U_{perch}}{\text{total number of all Nash equilibria}}$$

(6)

In Figure 6.4, we can see almost 90% of the time, $U_{perch}$ is greater than or equals to 60% of all sum utilities in selfish system.

Meanwhile, we also analyze how much closely the per-channel cost sum utility approaches to the scheduling sum utility and introduce the second parameter utility difference.

$$\tau = U_{sch} - U_{perch}$$

(7)

Figure 6.5 shows that 90% of the time the difference between scheduling sum utility and per-channel cost sum utility is less than 0.5. And we can see that in this (5, 3) system even the worst case (the largest utility difference) is less 1.

We can see that 44.88% (1-55.12%) of the time $U_{perch}$ is greater or equals to all selfish system cases in Figure 6.4. This probability has been verified in Figure 6.5 where $x=0$. Because if $U_{perch}$ in a given realization is greater than or equals to all selfish system cases, it equals to $U_{sch}$. In that case, the difference between $U_{sch}$ and $U_{perch}$ is definitely zero.

When we consider a system with 4 users and 3 channels, we have a deterioration of the ratio performance as shown in Figure 6.6 and most of time its curve is above the (5, 3) system’s. On the other hand most of the time, 6 users and 3 channels system’s CDF curve is under the (5, 3) system’s. We can see that the more users in the system
will make the per-channel mechanism perform better.

Figure 6.4 CDF of ratio in a system with N=5, K=3

Figure 6.5 CDF of utility difference in a system with N=5, K=3
Figure 6.6 CDF of ratio in three systems with $K=3$ and $N=4, 5, 6$ respectively
We have addressed in the problem definition that we pay more concern about system sum utility. Therefore, Figure 6.7 and 6.8 show the CDF of sum utilities in both two mechanism-implemented systems when we increase the number of users and still keep the number of channels equal to 3, in another words we make the system more crowded which would obviously decrease the individual utility. In per-channel mechanism system, more users bring more choices for each channel. By selecting the better ones, the system sum utility is approaching 3 with the increase of user number. However, it is a different story in single cost mechanism. The other users with low transmission cost also raise the cost level which would probably block the users on the other channels. This is the reason that there is no significant improvement by increasing the number of users in a single cost mechanism system. However, this would lead the CDF to a step function, because with more users, the lowest transmission cost which is found by the single cost mechanism will be more probably close to zero so that user will contribute a utility which is slightly less than one. Hence, the amount of sum utility data is much close to 1, 2, 3...

More interesting observations in Figure 6.8 and 6.9 are that the number of steps equals to the number of channels and the ‘step height’ is rarely affected by the users and depends more on channel conditions.

![Figure 6.7 CDF of sum utilities in per-channel costs mechanism system with large N](image-url)
Figure 6.8 CDF of sum utilities in single cost mechanism system with large $N$

Figure 6.9 CDF of sum utilities in single cost mechanism system with large $N$

$(K=4)$
Conclusion

In this thesis, we design two AP charge cost mechanisms for multichannel random access system with selfish users and prove that by adding either of them, the system will end up to a unique operating point, although the performance of single cost mechanism is worse than the selfish system. As an improved version of single cost mechanism, per-channel costs mechanism has much better performance in providing good system sum utility which is also quite closed to the scheduling system’s sum utility. In spite of decreasing the individual utility, we could obtain more sum utility in a per-channel costs mechanism-implemented system when we increase the number of users. Furthermore, due to compensation, users in either of the two mechanism-implemented systems would rarely have zero individual utility and the individual utility difference among the users is getting smaller, which means everyone in the system will have a more equally individual utility. Especially, per-channel costs mechanism even gives user a chance to have an individual utility which is greater than one.

Because a selfish system can be treated as giving complete freedom to all users to choose their transmission strategies and in scheduling system the users are fully cooperative, our mechanisms put the users’ behavior in somewhere between complete selfish and fully cooperative. In either mechanism-implemented system, each user still makes his own decision, but he has to obey the rules which are set in the mechanism like being charged extra cost sometimes.
Future work

In our simulation, we only calculate pure strategy cases. From [5] formulation 8, 9 and 10, we know that in a (5,3) system each realization has 19 pure strategy cases on average which means the Fully Mixed Nash Equilibrium (FMNE) is only less than 1/20 of the total cases. However, some work on FMNE and Partially Mixed Nash Equilibria still can be done to derive more complete results. On the other hand, mechanism complexity analysis is also needed for both of the mechanisms to find what the behavior of the mechanism execution time is as a function of the number of inputs. Moreover, system robustness should be considered to defeat cheating (for example, a ‘cheater’ may never transmit in the system but get compensation).
References
