Gigawatt Pulsed Power Technologies and Applications

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Abstract

This thesis summarizes work on electrical pulsed power technologies and applications of very high electric power, of gigawatt levels, and involving high explosives. One pulsed power technology studied utilizes high explosives to generate electromagnetic energy while one application studied uses electromagnetic energy to disrupt fast-moving metal jets created using high explosives.

The pulsed power source studied is a helical explosively driven magnetic flux compression generator. This kind of device converts the chemically stored energy in a high explosive into electromagnetic energy in the form of a powerful current pulse. Two generators were studied in order to investigate their performance and to understand their operation. An electrical circuit model was used to simulate the electrical behaviour and a hydrocode was used to simulate the explosion and mechanical deformation of the device. The experimental results obtained were peak currents of 269 kA and 436 kA corresponding to current amplification ratios of 47 and 39. The general shape of the measured and simulated current pulses was in good agreement and the simulated peak currents did not deviate more than 1% from the measured peak currents.

The other application studied is a protection technology against anti-tank warheads with shaped charges. A shaped charge is a device that creates a metal jet travelling at high velocity capable of penetrating several decimetres of steel armour. A powerful current pulse may be passed through the jet and due to
heating and magnetic forces, the jet may be disrupted and its penetrative capability significantly reduced. A series of experiments has been performed to investigate the physical phenomena and a disruption process is suggested.

**Descriptors**

Magnetic Flux Compression, Explosives, Pulsed Power, Shaped Charges, Electric Armour, Circuit Simulations, Hydrodynamic simulations
Preface

This work was carried out at the Swedish Defence Research Agency, FOI, Division of Defence & Security, Systems and Technology. Part of the work was performed within a research program on pulsed power technology for electromagnetic weapons, and part was performed within a research program on future armour protection. The work was funded by the Swedish Armed Forces.
Acknowledgements

First of all I would like to thank Gert Bjarnholt. He was the designer of the explosive flux compression generator and was managing the FOA (now FOI) project in this research area. Fortunately, for me, only one of the three generators manufactured was exploded. Two generators remained unused and during the last year before his retirement, Gert and I fired the two generators, covered with various measurement devices, very successfully.

The second part of my doctoral work was research on electric armour, a novel technology for protection of military vehicles against shaped charge warheads. Thank you, Melker Skoglund - our work on electric armour would not have been such a success without your technical skills, innovative ideas and support. Thank you, Dr. Patrik Lundberg for your support, encouragement and technical expertise.

I would like to thank Prof. Nils Brenning and Prof. Anders Larsson, my supervisors at KTH and FOI for their encouragement, support and help in analysing results and writing the papers. Thank you, Dr. Torgny Carlsson, FOI, for encouraging me to begin the PhD-studies, and for your support.

A special thanks to you Dr. Bucur Novac, Loughborough University, UK, for your help with the modelling, the analysis of the experiments, and the interpretation of the results from the flux compression generator experiments. Thank you, Dr. Andreas Helte for setting up and performing the hydrodynamic simulations of the explosion process in the flux compression generator, providing valuable input to the electric armour research, and for guiding me into the area of detonics.

Dr. Tomas Hurtig and Lars Westerling, it has always been a pleasure to discuss the complex physics observed in the experiments and in the output from your numerical calculations.

Other colleagues at FOI that have been supportive and contributed to this work in different ways are Dr. Mose Akyuz, Mattias Elfsberg, Cecilia Möller and Sten Nyholm.

My gratitude to the workshop and construction service at FOI Grindsjö for their assistance and help in manufacturing all the necessary auxiliary equipment required when performing these experiments.

Finally I would like to thank Anna, my parents Berit and Åke, family and friends for their support, encouragement and patience.
To Anna and my son Linus
List of papers

The work on explosively driven magnetic flux compression generators was thoroughly presented in the licentiate thesis


This thesis is based on the work presented in the following journal and conference papers.

On explosively driven magnetic flux compression generators:


On electric armour:


My contributions to the included publications:

Paper I-III: These papers present the experimental work with, and numerical simulations of, explosives driven flux compression generators. The generators were designed and constructed at FOA in 1994-1995. My contribution was to modify, plan and perform the experiments with two remaining generators and to adapt and implement a generator model, developed by Dr. Bucur Novac, into Matlab-Simulink and perform various simulations and parametric studies. I performed the analysis of the experimental data and simulation results and wrote the papers with the support by the co-authors.
Paper IV: This paper presents a 10 GW pulsed power supply designed to power a high power microwave (HPM) source. The system was developed by Loughborough University and in order to demonstrate the operation with a microwave source, FOI brought a vircator to Loughborough and mounted the systems together. My contribution was to adapt the FOI vircator to the 10 GW system, participate in the experiments and contribute in writing the paper.

Paper V-VII: These papers present the experimental work, results and analysis performed to study the various phenomena occurring when a high velocity metal jet is electrified when passing an electrode configuration connected to a pulsed power supply. The papers include descriptions of the experimental conditions and present various results and observations. The three papers are strongly related, with an introduction of the experiments in the first paper followed by extended analysis of the results in the two following papers. I planned and participated in the experiments, performed most of the analysis and wrote the papers with support by the co-authors.

Paper VIII: This paper presents a study of electrical explosion of copper rods (static experiments) performed in order to study disruption mechanisms of shaped charge jets passing an electrode configuration. I planned and participated in the experiments, performed the analysis and wrote the paper.

Paper IX: This conference paper presents hydrodynamic simulations of the electrification of high velocity jets using the hydrocode GRALE. The simulations are compared to the experiments presented in Paper V, VI and VII. My contribution was to provide input data to the hydrodynamic simulations, which were performed by Lars Westerling. I performed the analysis of the simulation results, compared it with the experimental data, and wrote most of the paper.

Paper X: This conference paper presents the pulsed power supply designed for the electric armour experiments. My contribution was to specify the system requirements, participate in the system tests and perform part of the analysis and the circuit simulations.

The work was also presented at conferences and contains similar material as in papers I - X.

EAPPC, Vilnius, Litauen, 22-26 September 2008. 
Note: Identical to paper VIII.


Journal Papers not included in the thesis


Conference Papers not included in the thesis


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1 Introduction

This thesis summarizes work on electrical pulsed power technology and applications of very high electric power and involving high explosives. One pulsed power technology studied utilizes high explosives to generate electromagnetic energy while the application studied uses electromagnetic energy to disrupt fast-moving metal jets created by means of high explosives.

Pulsed power is the commonly used term for technologies and systems that compress electric energy in time to form high power pulses. The applications are vast, both civilian and military, and the power levels and pulse durations differ widely. There are several conferences and symposiums on pulsed power technologies and applications with topics that range from X-ray generation in huge facilities to waste water treatment with pulsed electric fields [1-3]. The compression of energy in time can be illustrated by the transformation of the energy chemically stored in a battery into a high power pulse. Batteries cannot deliver the energy at high power but if the energy is amplified to higher voltage and used to charge a capacitor, the same energy can be released much faster.

One type of pulsed power source is the explosively driven magnetic flux compression generator [4,5]. This kind of device converts the chemically stored energy in a high explosive into electromagnetic energy in the form of a powerful current pulse. Different types of magnetic flux compression generators were originally used to generate extremely high currents or magnetic flux densities for research purposes. The development of capacitors has made them a better alternative as they are reusable and easier to operate. Today, the high energy density of the high explosives makes flux compression generators attractive as compact expendable power sources.

Electric armour is a protection technology against anti-tank warheads using shaped charges [6]. A shaped charge is a device that creates a very fast metal jet capable of penetrating several decimetres of steel armour. A powerful current pulse may be passed through the jet and due to heating and magnetic forces, the jet may be disrupted and its penetrative capability significantly reduced. In this application, a high current pulse is delivered by a capacitor bank at
amplitudes of several hundred kiloamperes and duration of a hundred microseconds. In experimental work in this field during the 1970’s, magnetic flux compression generators were used as a power source whereas today, as in the work at the Swedish Defence Research Agency, FOI, capacitor banks have been employed.

In both the explosive magnetic flux compression generator and in the electric armour application the current amplitudes are very high. The high current densities in the conductors will cause heating of the conductors, and the magnetic forces are very high and may deform the conductors. Furthermore, the operation of these devices is so short that the magnetic field diffusion phenomenon needs consideration. The thesis presents and discusses various phenomena that play important roles and will help the reader to understand my work better.

The purpose of the work on explosive flux compression generators was to evaluate the performance of a specific generator design and to determine its potential as power sources in compact pulsed power systems. This was achieved by performing experiments with generators fitted with various types of diagnostics to monitor the operation in detail. A circuit simulation model of the generator was implemented and since the simulations generated results with very good agreement with the experimental results, the simulations could be used in the analysis of the generator performance. The potential of the generator as a power source in a pulse-forming network was evaluated using simulations.

The general purpose with the work on electric armour was to evaluate a protective technology for military applications based on pulsed power technology. The scope for the doctoral work is however limited to the physical process and to investigate the disruption mechanisms when a metal jet is electrified. The work included experimental work, numerical simulations with circuit models and hydrocode simulations in order to establish a good understanding of the electrification- and disruption process. This work may promote the development and implementation of the protective technology.
2 Electromagnetic field phenomena and material properties

Maxwell’s equations relate the electric and magnetic fields and the magnetohydrodynamic model describes how matter respond to the electric and magnetic fields [7-9]. In the devices and applications studied in this doctoral work, the magnetic fields and current densities are very high.

Conductivity, ohmic heating and the magnetic field diffusion are important properties or phenomena encountered in this work, and are discussed to some extent. There are also some less familiar phenomena that are central to this work. With the aid of high explosives it is possible to generate electromagnetic energy by compressing magnetic flux. The high explosive accelerates conductors that perform work against the magnetic field and hence increases the energy of the magnetic field. If the conductors are moving at sufficiently high velocity, magnetic flux may be considered frozen in the conductor i.e. advection dominates over diffusion. This is a requirement for magnetic flux compression and has interesting implications in the case of electric armour. The high magnetic pressures associated with strong magnetic fields may also induce magnetohydrodynamic instabilities into a conductor carrying a high current.

Copper is of special interest in this thesis since it is used as a conductor in the flux compression generator and as liner material in the shaped charge device used to create the metal jet that is electrified in the electric armour experiments. Hence, the examples of data presented or calculations performed are for copper. Calculations are also performed at time scales, amplitudes and dimensions that are relevant when discussing the devices and applications.
2.1 Maxwell’s equations

Maxwell’s equations in vectorial form are [7]

\[ \nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \]  \hspace{1cm} (1)

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]  \hspace{1cm} (2)

\[ \nabla \cdot \mathbf{B} = 0 \]  \hspace{1cm} (3)

\[ \nabla \cdot \mathbf{D} = \rho_e \]  \hspace{1cm} (4)

where \( \mathbf{H} \) is the magnetic field strength (A/m), \( \mathbf{B} \) is the magnetic flux density (T), \( \mathbf{D} \) is the electric flux density (C/m\(^2\)), \( \mathbf{E} \) is the electric field strength (V/m) and \( \mathbf{j} \) is the current density (A/m\(^2\)). The scalar \( \rho_e \) (C/m\(^3\)) is the density of free electric charges and \( \nabla \) is the nabla operator defining the operations curl (\( \nabla \times \mathbf{A} \)) and divergence (\( \nabla \cdot \mathbf{A} \)) on a vector \( \mathbf{A} \).

One approximation can be made when the characteristic time of the field variation is much larger than the time for an electromagnetic wave to propagate a characteristic system dimension at the light velocity. We then neglect the displacement term \( \partial \mathbf{D}/\partial t \) and assume \( \rho_e = 0 \) and obtain the magnetoquasistationary equations where

\[ \nabla \times \mathbf{H} = \mathbf{j} \]  \hspace{1cm} (5)

replaces Eq. (1) and Eq. (4) is neglected. In addition the following relation is important

\[ \mathbf{j} = \sigma \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B}) \]  \hspace{1cm} (6)

which is Ohm’s law where \( \sigma \) is the electric conductivity and \( \mathbf{u} \) is the velocity of a medium moving through fixed \( \mathbf{E} \)- and \( \mathbf{B} \)-fields. Eq. (6) is more familiar in the static situation when \( \mathbf{u} = 0 \).

The magnetic field and magnetic flux density and electric field and electric flux density relate through the magnetic permeability \( \mu \), dielectric permittivity \( \varepsilon \) where \( \mu \) and \( \varepsilon \) are properties of the medium in which the medium is defined.
The continuity equation expresses the conservation of fluid mass \[ \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) \] (9) where \( \rho \) is the mass density and \( \mathbf{u} \) is the velocity.

The equation of motion expresses the conservation of momentum

\[ \frac{\partial \mathbf{u}}{\partial t} = (\nabla \times \mu \mathbf{H}) \times \mathbf{H} - \nabla P \] (10)

where the first term on the right hand side is the Lorentz force, the second term is the pressure gradient, and \( \mu = \mu_r \mu_0 \).

The diffusion equation expresses change in magnetic field due to magnetic diffusion and due to advection resulting from mass flow

\[ \frac{\partial \mathbf{H}}{\partial t} = \frac{1}{\sigma \mu} \nabla \times (\nabla \times \mathbf{H}) + \nabla \times (\mathbf{u} \times \mathbf{H}) \] (11)

The first term on the right hand side is the diffusion term whereas the second term is the advection term. With the vector relations and notations

\[ \nabla \times (\nabla \times \mathbf{H}) = \nabla (\nabla \cdot \mathbf{H}) - (\nabla \cdot \nabla) \mathbf{H} \] (12)

\[ \Delta \mathbf{H} = (\nabla \cdot \nabla) \mathbf{H} \] (13)

we can express the ratio of the advection and diffusion terms according to

\[ \frac{\sigma \mu \cdot |\nabla \times (\mathbf{u} \times \mathbf{H})|}{|\Delta \mathbf{H}|} \approx \sigma \mu \cdot L \cdot u = R_m \] (14)

where \( R_m \) is the magnetic Reynolds number, \( L \) is a characteristic length and \( u \) is the velocity. For large values of the magnetic Reynolds number \( (R_m \gg 1) \) the conducer would move significantly during the time a field diffuses into or out.
of it whereas for a small value of the magnetic Reynolds number \((R_m << 1)\) the diffusion would dominate.

In ideal MHD the length scale is considered large and the diffusion term is neglected. This assumption also simplifies the energy equation.

### 2.3 Conductivity and resistivity

The electric conductivity relates the electric field \(\mathbf{E}\) to the current density \(\mathbf{j}\) according to Eq. (6) where we in this discussion assume the conductor is static \((\mathbf{u} = 0)\). The electric conductivity is given by the balance between the momentum gained by the electrons due to the electric field, and the momentum lost due to collisions with ions or neutral atoms in a plasma or, in the case of a solid conductor, due to interaction with the irregularities in the lattice and with thermal vibrations of ions in the lattice. The conductivity of a plasma is given by

\[
\sigma = \frac{1}{\eta} = \frac{n_e e^2}{m_e \nu}
\]  

(15)

where \(n_e\) is the electron density, \(e\) is the electron charge, \(m_e\) is the electron mass, \(\nu\) is the electrons’ momentum exchange collision frequency and \(\eta\) is the resistivity. In the case of fully ionized plasma, assuming a Maxwellian distribution of the thermal velocity, we arrive at an expression for the resistivity

\[
\eta = \frac{m_e^{1/2} \pi e^2}{(4\pi n_e)^{1/2} (kT_e)^{1/2}} \ln \Lambda
\]

(16)

where \(k\) is Boltzmann constant and \(T_e\) is the electron temperature and \(\ln \Lambda\) is a correction factor weakly dependent on the electron density and can be taken to be equal to 10 for most plasmas. This expression is called the Spitzer resistivity [10] but is only valid for fully ionized non-degenerate plasmas, i.e. for high temperatures and low densities. The presence of magnetic fields will also affect the resistivity, which becomes a tensor with different values along and across \(\mathbf{B}\). A more general conductivity model was derived by Lee-More [11] and improved by Desjarlais [12], and considers the dense plasma state as well as the liquid and solid state. Based on this model, the conductivity as function of temperature and density is available as tabulated equation of state data in the SESAME database [13], and is useful input to numerical simulation of for example electrically exploded wires.
The conductivity of a metal at normal density and room temperature is usually given in material handbooks together with a temperature coefficient describing the conductivity change with temperature. The conductivity is given by \[ \sigma = \frac{1}{\eta} \frac{\sigma_0}{1 + \beta c_p T} \quad (17) \]

where \( \sigma_0 \) is a reference conductivity at some reference temperature (e.g. 300 K), \( T \) is the temperature, \( \beta \) is a heat coefficient, \( c_v \) is the volume specific heat, \( c_p \) is the mass specific heat. The conductivity can be extended to include the effect of compression \[ \sigma = \frac{\sigma_0}{1 + \beta c_p T} \left( \frac{\rho}{\rho_0} \right)^\alpha \quad (18) \]

where \( \rho \) is the mass density at the reference temperature and the exponent \( \alpha \) is a pressure coefficient. For liquids, the conductivity takes the form \[ \sigma = \frac{1}{\eta} \frac{\sigma_0}{1 + \beta c_p (T - T_m)} \quad (19) \]

with values of the coefficients in the liquid phase. These and other parameter values for copper are given in Table 1 obtained from ref [7,9]. The conductivity and resistivity for copper are plotted in Figure 1 using Eq. (17) and Eq. (19) and the data in Table 1.

**Table 1. Data for copper obtained from ref [7].**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melting point</td>
<td>( T_m )</td>
<td>K</td>
</tr>
<tr>
<td>Boiling point</td>
<td>( T_b )</td>
<td>K</td>
</tr>
<tr>
<td>Mass density (293 K)</td>
<td>( \rho_0 )</td>
<td>10^3 kg/m^3</td>
</tr>
<tr>
<td>Mass density (1356 K)</td>
<td>( \rho_1 )</td>
<td>10^3 kg/m^3</td>
</tr>
<tr>
<td>Conductivity (293 K)</td>
<td>( \sigma_0 )</td>
<td>10^6 S/m</td>
</tr>
<tr>
<td>Conductivity solid (1356 K)</td>
<td>( \sigma_{s,1356} )</td>
<td>10^6 S/m</td>
</tr>
<tr>
<td>Conductivity liquid (1356 K)</td>
<td>( \sigma_{l,1356} )</td>
<td>10^6 S/m</td>
</tr>
<tr>
<td>Reduced temperature coefficient (273 - 1356 K)</td>
<td>( \beta_{c_l,1356} )</td>
<td>10^3 K^{-1}</td>
</tr>
<tr>
<td>Reduced temperature coefficient (1356 - 1773 K)</td>
<td>( \beta_{c_l,1356} )</td>
<td>10^3 K^{-1}</td>
</tr>
<tr>
<td>Volume specific heat (293 K)</td>
<td>( c_v )</td>
<td>10^3 J/m^3K^{-1}</td>
</tr>
<tr>
<td>Mass specific heat (273 - 1356 K)</td>
<td>( c_p_{s,273} )</td>
<td>10^3 J/kg^3K^{-1}</td>
</tr>
<tr>
<td>Mass specific heat (1356 - 1773 K)</td>
<td>( c_p_{s,1356} )</td>
<td>10^3 J/kg^3K^{-1}</td>
</tr>
<tr>
<td>Latent heat of melting</td>
<td></td>
<td>10^3 J/kg</td>
</tr>
<tr>
<td>Latent heat of vaporization</td>
<td></td>
<td>10^3 J/kg</td>
</tr>
<tr>
<td>Latent heat of sublimation (from 273 K)</td>
<td></td>
<td>10^3 J/kg</td>
</tr>
<tr>
<td>Pressure coefficient</td>
<td>( \alpha )</td>
<td></td>
</tr>
</tbody>
</table>
2.4 Magnetic field diffusion

Magnetic field diffusion is one important phenomenon in pulsed power. Due to the short pulses with high frequency content and high currents, the current will often flow in a thin surface layer of the conductor and with locally high current densities. We begin the derivation of the diffusion equation in stationary matter by putting Eq. (6) into Eq. (5), assuming \( u = 0 \) and taking the curl

\[
\nabla \times (\nabla \times \mathbf{H}) = \nabla \times (\sigma \mathbf{E}) = (\nabla \sigma) \times \mathbf{E} + \sigma (\nabla \times \mathbf{E}) = \sigma (\nabla \times \mathbf{E})
\]

where we assume that \( \sigma \) is constant. With Eq. (2), (3) and (7), with \( \mu \) time-independent, and the vector relations and notations of Eq. (12) and (13), Eq. (20) becomes

\[
\Delta \mathbf{H} - \frac{1}{\kappa} \frac{\partial \mathbf{H}}{\partial t} = 0
\]

where

\[
\kappa = \frac{1}{\sigma \mu}
\]

is the magnetic diffusion coefficient.

Solving the magnetic field diffusion equation [7] in the plane one-dimensional case, the coordinate system is defined such that \( \mathbf{H} \equiv (0, 0, H_z), \mathbf{E} \equiv (0, E_y, 0) \) and \( \mathbf{j} \equiv (0, j_y, 0) \). We have
\[ \frac{\partial H_z}{\partial x} = -j_x, \quad (23) \]

\[ \frac{\partial E_z}{\partial x} = -\frac{\partial (\mu H_z)}{\partial t}, \quad (24) \]

The magnetic field diffusion equation is then

\[ \frac{\partial^2 H_z}{\partial x^2} = \frac{1}{\kappa} \frac{\partial H_z}{\partial t} = 0, \quad (25) \]

By looking for a solution on the form

\[ H_z(x,t) = H(x) \sin(\omega t - \phi) \quad (26) \]

we obtain the solution for a temporal sinusoidal variation at the boundary

\[ H_z(x,t) = H_0 e^{-\frac{x}{\delta}} \sin \left(2\pi \frac{t}{T} - \frac{x}{\delta} \right), \quad (27) \]

where

\[ \delta = \sqrt{\frac{2}{\omega \sigma \mu}} \quad (28) \]

\( \delta \) is usually called the classical skin depth. At \( x = \delta \) the field amplitude has decayed to \( 1/e \) of the surface value. A plot of the magnetic field penetration into copper (\( \sigma = 63.3 \times 10^6 \) S/m, see Table 1) for a frequency of 20 kHz and \( H_0 = 1 \) A/m is shown in Figure 2.

A step-function boundary condition

\[ H_z(0,t) = \begin{cases} 0, & \text{for } -\infty < t < 0 \\ H_0(\text{const.}), & \text{for } 0 \leq t < \infty \end{cases} \quad (29) \]

will have a solution [7]

\[ H_z(x,t) = H_0 \cdot \text{erfc} \left( \frac{x}{2\sqrt{\kappa t}} \right), \quad (30) \]

plotted for copper in Figure 3.

In magnetic flux compression devices the field often increases exponentially and an exponential field at the boundary

\[ H_z(x,t) = H_0 e^{-\frac{x}{\delta}} \quad (31) \]
\[ H_z(0,t) = H_0 e^{\tau t}, \]  
(31)

has the solution [7]

\[ H_z(x,t) = H_0 \exp \left( \frac{t}{\tau} \frac{x}{\sqrt{\kappa \tau}} \right) \]  
(32)

where

\[ \tau = \frac{H}{dH/dt} \]  
(33)

is a time constant or at least approximately constant. In the magnetic flux compression generator used in experiments discussed later, the ratio between the current and the current time derivative is not constant and varies between 1 and 10 μs during compression. The exponential function is plotted in Figure 4 with \( \tau = 10 \mu s \).

**Figure 2.** The magnetic field penetration into copper for a sinusoidal field of 20 kHz.

**Figure 3.** The magnetic field penetration into copper for a step-function applied at time 0 ms.
2.5 Heating of conductors

In both the magnetic flux compression device and during the electrification of the metal jet in an electric armour application, ohmic heating due to the extremely high current densities will occur. This is a limitation in the case of magnetic flux compression but is a desired effect in the metal jet passing the electric armour, although it should be avoided in the other parts of the system.

Addition of Eq. (1) multiplied by $\mathbf{E}$, and Eq. (2) multiplied by $-\mathbf{H}$, gives [7]

$$\mathbf{E} \cdot \nabla \times \mathbf{H} - \mathbf{H} \cdot \nabla \times \mathbf{E} = \mathbf{E} \cdot \mathbf{j} + \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t}$$

(35)

which with vector identity

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{H})$$

(36)

gives the power equation

$$-\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \mathbf{E} \cdot \mathbf{j} + \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t}$$

(37)
In this equation $\mathbf{E} \times \mathbf{H}$ is the Poynting flux, the term $\mathbf{E} \cdot \mathbf{j}$ is the ohmic heating term, which with Eq. (6) and assuming $u = 0$ reduces to

$$\mathbf{E} \cdot \mathbf{j} = \frac{j^2}{\sigma} = \sigma \mathbf{E}^2$$  \hspace{1cm} (38)

The second and third term in Eq. (37) describe the time variation of electric and magnetic field energy densities.

The increase in internal thermal energy $Q$ in a conductor carrying a current density $\mathbf{j}$ is [7]

$$\frac{\partial Q}{\partial t} = \frac{j^2}{\sigma} + k \cdot \nabla T = \eta j^2 + k \cdot \nabla T$$  \hspace{1cm} (39)

where $k$ is the thermal conductivity but is for now ignored, $k = 0$. An increase in internal energy $dQ$ gives rise to a temperature increase $dT$ according to

$$dT = \frac{1}{c_v} \frac{dQ}{\eta}$$  \hspace{1cm} (40)

where $c_v$ is the specific heat at constant volume. During a time $dt$ the energy $dQ$ is deposited according to

$$dQ = c_v dT = \eta j^2 dt$$  \hspace{1cm} (41)

Integration and rearrangement give

$$J = \int_0^T j^2 dt = \int_0^T \frac{c_v}{\eta} dT$$  \hspace{1cm} (42)

where $J$ is called current action integral and $\beta$ is called current action [7]. Following [16], the resistance increment $d\eta$ due to a temperature increase $dT$ is derived from Eq. (17)

$$d\eta = \frac{\eta_0 \cdot \beta c_v}{\eta} \cdot dT$$  \hspace{1cm} (43)

which together with Eq.(42) gives

$$J = \frac{1}{\eta_0 \cdot \beta} \int \frac{1}{\eta} d\eta$$  \hspace{1cm} (44)

and after integration:
Rearrangement gives an expression for resistivity as function of current action integral $J$

$$\eta(J) = \eta_0 \cdot e^{\eta_s \cdot \beta \cdot J}$$  \hspace{1cm} (46)$$

Phase transitions from solid to liquid will begin for some critical value of $J_S$ when the resistivity has a value $\eta_S$

$$J_S = \frac{1}{\eta_0 \cdot \beta} \ln\left( \frac{\eta_S}{\eta_0} \right)$$  \hspace{1cm} (47)$$

Combining Eq. (46) and (47) an expression for the resistivity is obtained

$$\eta(J) = \eta_0 \cdot e^{\eta_s \cdot \beta \cdot J - \frac{J}{J_S}} \quad 0 \leq J \leq J_S$$  \hspace{1cm} (48)$$

Note that this expression assumes that the energy does not disappear due to heat conduction or radiation. Under the assumption that the conductor does not expand due to the energy deposition, i.e. that the heat deposition is very fast, this expression is also valid for the liquid phase. The resistivity during the transition between solid and liquid phase can be modelled as two parallel resistances where one represents the resistance of the solid phase (S) and the other the resistance of the molten phase (M). Without going into detail the resistivity during melting is described by [16]

$$\eta(J) = \frac{\eta_S}{\sqrt{\left( J - J_S \right) \frac{J}{J_S}}} \frac{\eta_M}{\sqrt{\left( J - J_S \right) \frac{J}{J_S} + 1}}$$  \hspace{1cm} (49)$$

where $J_M$ is the action integral and $\eta_S$ and $\eta_M$ are the resistivities for the solid and molten phases at the melting point. $J_M$ is the action integral when the conductor is fully melted.

The action integral expressions are useful when describing a conductor that is heated by a current pulse, melts and explodes electrically. The data obtained in exploding wire experiments performed by Tucker and Toth [16] is given in Table 2, where the explosion is assumed to occur at the vaporisation temperature. Figure 5 shows the resistivity increase as function of current
action integral. From Eq. (17) and (19) one may derive the temperature coefficients assuming no thermal expansion. In fact, pulse heating is used to determine material parameters in both solid and molten phase [13], and in the metal plasma with temperatures ranging from 10000 K to 30000 K [15], but the thermal expansion is considered in those cases. The data on resistivity and action integral by Tucker and Toth gives the value of the reduced temperature coefficient. Even if there is considerable thermal expansion, the resistance increase of the conductor as function of the current action integral calculated with nominal (initial) conductor area is considered sufficiently accurate here.

Figure 5. Resistivity increase (for conductor initial at room temperature) as function of action integral.

Table 2. Values of the action integral and copper resistivity [16].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dimension Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action integral at ~300 K</td>
<td>( J_0 ) ( 10^{17} \text{ A}^2\text{m}^{-4} ) 0</td>
</tr>
<tr>
<td>solid phase at 1356 K</td>
<td>( J_{1356K} ) &quot; 0.80492</td>
</tr>
<tr>
<td>molten phase at 1356 K</td>
<td>( J_{1356K} ) &quot; 0.94228</td>
</tr>
<tr>
<td>vaporized phase at 2855 K</td>
<td>( J_{2855K} ) &quot; 1.24008</td>
</tr>
<tr>
<td>Resistivity at ~300 K</td>
<td>( \rho ) ( 10^8 \text{ \Omega m} ) 1.77</td>
</tr>
<tr>
<td>solid phase at 1356 K</td>
<td>( \rho_{1356K} ) &quot; 9.9</td>
</tr>
<tr>
<td>molten phase at 1356 K</td>
<td>( \rho_{1356K} ) &quot; 18.9</td>
</tr>
<tr>
<td>molten phase at 2855 K</td>
<td>( \rho_{2855K} ) &quot; 26.3</td>
</tr>
<tr>
<td>vaporized phase at 2855 K</td>
<td>( \rho_{2855K} ) &quot; 62.0</td>
</tr>
<tr>
<td>Reduced temperature coefficient (273 - 1356 K)*</td>
<td>( \beta_{c,0} ) ( 10^{-3} \text{ K}^{-1} ) 4.35</td>
</tr>
<tr>
<td>Reduced temperature coefficient (1356 - 1773 K)*</td>
<td>( \beta_{c,1} ) ( 10^{-3} \text{ K}^{-1} ) 0.261</td>
</tr>
</tbody>
</table>

*Calculated using Tucker and Toth data with nominal conductor area
2.6 Magnetic flux compression

Magnetic flux compression is a process where the magnetic flux is concentrated by an external force to produce a higher magnetic field density while maintaining the initial flux. For a magnetic flux compression device to work, the flux must then not be lost through diffusion during the compression time. In other words, the advection should dominate over diffusion, i.e., the magnetic Reynolds number $R_m$, Eq (14) has to be sufficiently high.

Assuming that the conductivity is so high that the diffusion term in Eq. (11) can be neglected ($\sigma \to \infty$, i.e., an ideal conductor), we have

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$  \hspace{1cm} (50)

A consequence of Eq. (50) is that the magnetic flux through any surface $S$, bounded by a closed contour $C$ moving with a highly conductive fluid, is constant. Following ref. [8], the time derivative of the flux is equal to the change of flux due to the time rate of change of $\mathbf{B}$ and the change in surface area due to movement of the bounding contour $C$ according to

$$\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{S} = \int_C \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \oint_C \mathbf{B} \cdot \mathbf{u} \cdot d\mathbf{l}$$  \hspace{1cm} (51)

One may interchange the dot and cross in the contour integral according to

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$$  \hspace{1cm} (52)

With Stokes theorem

$$\int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S} = \oint_C \mathbf{A} \cdot d\mathbf{l}$$  \hspace{1cm} (53)

into Eq. (51) we obtain

$$\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{S} = \int_C \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} - \oint_S \left( \nabla \times (\mathbf{u} \times \mathbf{B}) \right) \cdot d\mathbf{S} =$$

$$= \int_C \left( \frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u} \times \mathbf{B}) \right) \cdot d\mathbf{l} = 0$$  \hspace{1cm} (54)

where the integrand is identified as the expression in Eq. (50). This means that the flux $\Phi$, enclosed by $C$, does not change with time and is conserved. If it is
assumed that magnetic flux density \( \mathbf{B} \) is homogeneous across the enclosed area \( S \) we simplify Eq. (54)

\[
\int \mathbf{B} \cdot d\mathbf{S} = BS
\]  

(55)

where the product of \( B \) and \( S \) is a constant at any time. Hence, we may write

\[
B_0S_0 = B_fS_f
\]  

(56)

where \( B_0 \) and \( S_0 \) are the initial magnetic flux density and the initial area and \( B_f \) and \( S_f \) are the final magnetic flux density and the final area.

The magnetic energy stored per unit length of enclosed area is

\[
W_\ell = \frac{1}{2\mu_0} \int \mathbf{B}^2 dS
\]  

(57)

If it is assumed that magnetic flux density \( \mathbf{B} \) is always homogeneous across the enclosed area \( S \) the expression for magnetic energy simplifies to

\[
W_\ell = \frac{1}{2\mu_0} B^2S
\]  

(58)

With the initial energy before and final energy per unit length after compression given by

\[
W_0 = \frac{1}{2\mu_0} S_0B_0^2
\]  

(59)

and

\[
W_f = \frac{1}{2\mu_0} S_fB_f^2
\]  

(60)

we write using Eq. (56) the relative magnetic energy increase as

\[
\frac{W_f}{W_0} = \frac{S_fB_f^2}{S_0B_0^2} = \frac{B_f}{B_0} = \frac{S_0}{S_f}
\]  

(61)

The relative energy increase is simply the relation between final and initial magnetic flux density or initial to final enclosed area.

Practical considerations for magnetic flux compression are given in Chapter 4.
2.7 Magnetohydrodynamic instabilities

Following [8], we begin the discussion on the instabilities by considering static equilibrium configurations and restrict the discussion to cylindrical configurations. The conducting fluid of interest here is the solid or melted copper wire. Before deformation, the inertial term in the momentum equation, Eq. (10), can be neglected, giving the magnetostatic equilibrium inside the wire

\[(\nabla \times \mu \mathbf{H}) \times \mathbf{H} = \nabla P\]  

or

\[\mathbf{j} \times \mathbf{B} = \nabla P\]  

In an equilibrium state the \(\mathbf{j} \times \mathbf{B}\) force has to balance the pressure gradient in the wire. Consider the radial equilibrium of a cylindrical conductor. Cylindrical symmetry is assumed (no dependence of \(\theta\) and \(z\)) and from Eq (5) and (7) we have

\[
\mu \mathbf{j} = \left[0, -\frac{dB_z(r)}{dr}, \frac{1}{r} \frac{d}{dr}(rB_r(r))\right]
\]

The radial component of Eq.(63) and Eq.(64) yields [8]

\[
\frac{d}{dr} \left[ P + \left(\frac{B_z^2 + B_r^2}{2\mu_0}\right) \right] = -\frac{B_z^2}{\mu_0 r}
\]

which expresses the balance between the fluid pressure gradient, the magnetic pressure gradient, and the magnetic tension due to magnetic field curvature. Consider the case \(B_z=0\). Integrating Eq.(65) with \(B_z=0\) yields a configuration known as a Z-pinch [8]. In this case an axial current and its azimuthal magnetic field will generate a \(\mathbf{j} \times \mathbf{B}\) force directed towards the z-axis. The equilibrium pressure gradient is then

\[
\frac{dP}{dr} = -\frac{B_z}{\mu_0 r} \frac{d}{dr}(rB_r)
\]

The axial current density \(j_z\) is from Eq.(64)

\[
j_z = \frac{1}{\mu_0 r} \frac{d}{dr}(rB_r)
\]

which integrates to
\[ B_0 = \frac{1}{r} \int_0^r \mu_0 j_z(r) \, dr \]  

(68)

and the pressure gradient can then be expressed

\[ \frac{dP}{dr} = -j_z \frac{1}{r} \int_0^r \mu_0 j_z(r) \, dr \]  

(69)

With the total current \( I \) expressed

\[ I(r) = \int_0^r 2\pi r j_z \, dr \]  

(70)

we obtain

\[ \frac{dP}{dr} = -\frac{\mu_0}{(2\pi)^2} \frac{dI}{dr} I(r) \]  

(71)

Assuming a homogeneous current distribution, i.e. \( j_z \) is independent of radius and a conductor radius \( a \) we may write Eq.(70)

\[ I(r) = j_z r^2 = \frac{I(a)}{\pi a^2} \pi r^2 \]  

(72)

Eq. (71) is then rewritten

\[ \frac{dP}{dr} = -\frac{\mu_0}{(2\pi)^2} \frac{I(a)}{\pi a^2} \pi r^2 \frac{I(a)}{\pi a^2} \pi r^2 = \]  

(73)

\[ = -\frac{\mu_0}{(2\pi)^2} \frac{2r \cdot I(a)^2}{(2\pi)^2} \]  

and integrated and with the boundary condition of \( P(a) = 0 \)

\[ P(r) = -\frac{\mu_0 r^2 \cdot I(a)^2}{(2\pi)^2} + \text{const} = \]  

(74)

\[ = -\frac{\mu_0 r^2 \cdot I(a)^2}{(2\pi)^2} + \frac{\mu_0 a^2 \cdot I(a)^2}{(2\pi)^2} = \]  

\[ = \frac{\mu_0 I(a)^2}{4\pi a^2} \left( 1 - \frac{r^2}{a^2} \right) \]

Outside the conductor the magnetic field the magnetic field follows
\[ B_s(r) = \frac{\mu_0 I(a)}{2\pi r} \quad r > a \] (75)

The magnetic field inside and outside a cylindrical conductor carrying 200 kA and with radii 0.5 and 1.0 mm is shown in Figure 6, using Eq.(68), Eq.(70) and Eq.(75). The magnetic pressure distribution, calculated using Eq.(74), is shown in Figure 7. The conductor with smaller radius has higher magnetic field density inside, and hence higher inward directed magnetic \( j \times B \) force compared to the larger diameters. The values of the two radii and the current are of relevant size for the electric armour application discussed in Chapter 5. In these graphs it is clear why a small radial variation along the cylinder tends to be enhanced i.e. an unstable configuration. Figure 8 shows the z-pinch configuration and another unstable configuration the kink instability. A kink on the cylindrical conductor will be enhanced due the higher magnetic field density on one side of it and lower magnetic field density on the other side. By applying perturbation theory to the MHD equations the development of small perturbations can be studied [8].

![Figure 6. The magnetic field density inside and outside the conductor for a given current (200 kA) evenly distributed in the conductor and for two radii, 0.5 mm and 1.0 mm.](image)
Figure 7. The magnetic pressure inside the conductor for a given current (200 kA) that is homogeneously distributed in the conductor and for two radii, 0.5 mm and 1.0 mm.

Figure 8. Two types of instabilities of a cylindrical conductor.
3 Pulsed power

The high power techniques and applications considered in this thesis generate or require very high electric power. The energy requirement is however relatively modest due to the short duration of the operation. Systems and applications of this kind are often referred to as pulsed power, a field of great scientific interest with numerous international conferences. Pulsed power is used in radars, high-energy lasers, and in facilities for material research or testing [1-3].

Within this work the pulses are short, of the order of 10 to 100 µs, with a power of the order of 1 GW. The power requirement is hence of the same order as the power delivered continuously by a single Swedish nuclear reactor at full operation. For a duration of 10 to 100 µs, the energy associated with 1 GW electric power range from 10 kJ to 100 kJ. For comparison a 60 W light bulb consumes 10 kJ in about 170 seconds whereas it consumes 100 kJ in about half an hour. The amount of energy is thus not a big challenge in these applications; the challenge is to compress the energy in time to raise the power to very high levels.

This chapter gives a brief introduction to pulsed power and the current and voltage levels that are associated with it. The issue of how to store and deliver the energy is discussed and how a system can be designed in order to deliver a high power pulse.

3.1 Power and impedance

Given the power of 1 GW to be delivered to a load, the voltage and current associated with the electric pulse is determined by the load impedance. The power $P$ relates to the voltage $U$ and current $I$ and load impedance $Z$ according to

$$ P = \frac{U^2}{Z} = ZI^2 = UI $$

(76)
or rearranged

\[ U = \sqrt{PZ} \]  

\[ I = \frac{P}{\sqrt{Z}} \]

For a load impedance of 1 kΩ the voltage across it is 1 MV and the current through it is 1 kA, whereas for a load impedance of 1 Ω the voltage is about 32 kV and the current 32 kA. If the load impedance is 1 mΩ the voltage is 1 kV and the current 1 MA. For an efficient power deposition the pulsed power system needs to have a system impedance similar to that of the load.

With a high voltage, the problem of electrical breakdown arises which requires good insulation and careful control of the electric field. With a high current, the problems of heating, magnetic field diffusion, and magnetic forces arise.

3.2 Energy storage and power delivery

In a battery, stored chemical energy is converted into electrical energy. A commercial lead-acid 12 V battery with capacity 2 Ah, mass 1 kg, and volume 0.37 dm³ (Sonnenschein A512/2) [17] stores theoretically 86 kJ electric energy. The specific energy, for the battery in this example, is 86 kJ/kg or 230 MJ/m³. The power delivered at the rated capacity is only 0.6 W (50 mA) but for a short time about half a kilowatt (40 A) can be delivered by this battery. Although the energy of a single battery may be sufficient, the power rating is a millionth of what is required in the gigawatt application considered in this thesis. To connect the batteries in series or in parallel to reach those powers is practically impossible. The battery could, however, serve as a primary energy source.

In a capacitor the energy is stored in an electric field and hence can be delivered at a higher rate and hence generate higher power than batteries. However, the mass or volume specific energy storage capability is much lower than for chemical batteries. A simple capacitor may consist of two parallel conductive plates separated by a dielectric material. The electrostatic energy stored in the dielectric of the parallel plate capacitor is

\[ W_e = \frac{1}{2} \int \varepsilon_0 \varepsilon E^2 dv = \frac{1}{2} \varepsilon_0 \varepsilon E^2 Sd \]  

(79)
where $S$ is the area of each plate, $d$ is the thickness of and $\varepsilon_0\varepsilon_r$ is the permittivity of the dielectric, and $E$ is the electric field. The energy density is then

$$w_e = \frac{W_e}{Sd} = \frac{1}{2}\varepsilon_0\varepsilon_r E^2$$

(80)

A limitation in how much energy that can be stored is the electric breakdown strength $E_b$ of the material. For air the DC breakdown strength is 3 MV/m and hence, with $\varepsilon = 1.0$, the energy density is 40 J/m\(^3\). For most dielectric materials the relative permittivity is less than 10 and hence to reach high energy densities the dielectric strength has to be very large. Modern high power capacitors are constructed using metalized polymer films with relative permittivity between 2 and 3 and dielectric breakdown strengths of up to 600 MV/m [18]. The theoretical energy density is between 3 and 5 MJ/m\(^3\) or 3 to 5 kJ/dm\(^3\). In practice the energy density of high current discharge capacitors has been much lower, less than 1 MJ/m\(^3\), but due to recent research efforts, capacitors with energy densities of 2.5 MJ/m\(^3\) and higher have been developed. The energy density is at best a few percent of what can be stored in an ordinary chemical battery. The output power from a capacitor is much higher but is limited by its discharge current characteristics such as amplitude, oscillations, and oscillation frequency, and from the voltage characteristics such as reversals. These limitations are different for different types of capacitors due to different materials or configurations. The capacitors used in the pulsed power system for electric armour research at FOI, store 50 kJ each at nominal voltage corresponding to an energy density of 0.6 MJ/m\(^3\) and can deliver 150 kA each [19]. The pulse-forming network utilizing eight of these capacitors in four modules, delivers several gigawatts and the power density is in this case between 1 and 10 GW/m\(^3\).

In conclusion, ordinary batteries can store large amount of energy in a small volume while much more voluminous capacitors can deliver the same amount of energy at higher power.

### 3.3 Pulse-forming networks

By using capacitors together with inductors, it is possible to tailor a pulse to have a suitable pulse length, amplitude, and shape. The required discharge characteristics and load impedance determine the requirements on the capacitor and inductance values, how many of them and how they should be connected.

The design of such a pulse-forming network is exemplified by that developed for the electric armour experiments. It was required that the pulse should be square-shaped and have a variable pulse duration between 100 µs and 200 µs.
The energy during discharge should be several hundred kilojoules and the system should be matched to a load with a resistance of the order of 10 mΩ. The pulse-forming network chosen for this was a Guillemin type E network [20], illustrated in Figure 9. The pulse discharge time $T$, and system impedance $Z$ equal to the load resistance $R$, determine the required total capacitance $C$ and total inductance $L$ values of the system according to

\[ C = \frac{T}{2Z} \]  
\[ L = \frac{TZ}{2} \]

or rearranged

\[ Z = \sqrt{\frac{L}{C}} \]  
\[ T = 2\sqrt{LC} \]

For a square pulse the capacitance and inductance are evenly distributed on the number of capacitors and inductors used. Hence, in Figure 9 the capacitance $C_i$ equals $C/4$ and the inductance $L_i$ equals $L/4$. An increasing number of modules make the pulse become closer to a square shape. The energy of the capacitors is determined by the total capacitance $C$ and the charging voltage $U$ according to

\[ W = \frac{C \cdot U^2}{2} \]

For the values $T = 100 \mu s$, $Z = 20$ mΩ, and $W = 400$ kJ the total capacitance $C$ should be 2.5 mF, the total inductance $L = 1$ µF, and the charging voltage $U = 18$ kV. For practical reasons the system developed for electric armour research at FOI uses capacitors with a capacitance 206 µF and 22 kV nominal voltage, corresponding to 50 kJ per capacitor or 400 kJ for the whole system. In that system the inductance values can be changed from 100 nH to 2 µH to tailor the pulses length but as a consequence the system impedance will depend on the pulse length. Details on this system including the charging, control and dump system are given in Paper X.
Figure 9. Pulse-forming network with four capacitor modules.
4 Magnetic flux compression generators

It was shown in Section 2.6 that by compressing magnetic flux without flux losses through diffusion, it is possible to increase the magnetic energy of the field. The magnetic flux compression generator (MFCG or FCG) design studied in the doctoral work is explosives-driven. Only a part (at most a few percent) of the chemical energy stored in the explosives is converted into electrical energy. This is a two-step process where the explosion accelerates parts of the generator to perform work against a magnetic field. A requirement is that the compression takes place in a time shorter than the time it takes for the magnetic flux to diffuse out from the system. This requirement is met by the use of high explosives with which the conductor is accelerated to velocities of several kilometres per second, allowing compression times in the range of 1 to 100 μs.

In this chapter a brief overview of magnetic flux compression physics, various generator types, and methods for modelling is given. Experiments with a helical generator are presented and the generator performance is analysed and compared to simulations. The work on magnetic flux compression generators was presented in the licentiate thesis and in Papers I to IV. In the licentiate thesis, the work was presented in much more detail, while in this thesis the focus is on various electromagnetic phenomena that are present in these types of devices operating at very high electric powers.
4.1 Flux compression by plane conductors

The possibility to increase the energy of a magnetic field through compression was shown in Section 2.6. In this chapter the requirements are analysed. Consider two parallel, infinite plane conductors initially separated by a distance $x_0$ that move towards each other with velocity $v$, with the magnetic field parallel to the plates, Figure 10. Following [7], the initial kinetic energy per unit area of the plates is

$$W_{k0} = \frac{1}{2} \rho \cdot d \cdot v_0^2$$

(86)

and the initial magnetic energy per unit area between the plates is given by

$$W_{m0} = \frac{1}{2} \mu_0 H_0^2 x_0$$

(87)

Assuming that the conductor is ideal, the flux conservation relation is

$$\mu_0 H(t)x(t) = \mu_0 H_0 x_0$$

(88)

and the energy conservation relation is

$$W_{m0} + W_{k0} = \frac{1}{2} \mu_0 H(t)^2 x(t) + \frac{1}{2} \rho \cdot d \cdot v(t)^2$$

(89)

The plates will be slowed down by the magnetic pressure and finally the plates stop when all the kinetic energy has been transformed into magnetic energy. The ratio between final and initial field and between initial to final distance $x$ is

$$\frac{H_f}{H_0} = \frac{x_0}{x_f} = 1 + \frac{\rho \cdot d \cdot v_0^2}{\mu_0 H_0^2 x_0}$$

(90)

Due to the field diffusion some flux will diffuse into the conductor and by using the flux skin depth, defined in Eq.(34), the flux conservation relation takes the form

$$\mu_0 H(t)(x(t) + \delta x(t)) = \mu_0 H_0(x_0 + \delta x_0)$$

(91)
Here it is assumed that the skin depth is small compared to the thickness $d$ of the plate. The energy contained in the lost flux is converted to heat through ohmic heating. For an exponential field given by Eq. (31), with the diffusion into the plate given by Eq. (32) we may arrive, with some simplifications and assumptions, see ref [7], at an expression for the flux conservation factor

$$\frac{\phi}{\phi_0} \approx \exp \left( - \frac{2}{\sqrt{\sigma_0 \mu_0} \sqrt{\frac{x_0}{x}}} \right) = \exp \left( - \frac{2}{\sqrt{R_m}} \left( \sqrt{\frac{x_0}{x}} - 1 \right) \right)$$

(92)

where the plate is assumed to move at a constant velocity $v_0$, and have a constant conductivity $\sigma$. As discussed in Section 2.6 the magnetic Reynolds number has to be sufficiently high to compress the flux. If we require that the diffusion time should be a factor 10000 higher than the compression time i.e. $R_m = 10000$, for a device of with a length $x_0 = 0.1$ m, the compression velocity has to be about 1000 m/s. To achieve such velocities the plates have to be accelerated by high explosives. Apart from the requirement of a high magnetic Reynolds number there are other limitations to the operation of the device. The heating of the skin layer will increase the flux losses, and the field amplification is limited by compression of the plate itself due to the intense magnetic pressure.

Figure 10. Magnetic flux compression by two plates moving towards each other, left, and a representation of the magnetic field diffusion into the plate (from ref. [7]).
4.2 Generator types and applications

There are numerous types of flux compression generators. Complete systems may consist of several stages with different types of generators. They are categorised by their geometry and if they are imploding or exploding devices [4, 21-24]. Much work performed on explosive magnetic flux compression has been presented at Megagauss conferences, a conference series on high magnetic field generation [25 - 33].

Imploding devices consist of a metal cylinder in which an initial magnetic flux is generated [34 - 36]. The cylinder is covered with high explosives on the outside and is simultaneously initiated over the surface driving the cylinder inwards, Figure 11a, i.e. the same situation as in Section 4.1 but in cylindrical geometry. The peak field is generated in the centre of the cylinder, which is finally destroyed. This type of generator is used to generate ultra-high magnetic fields in a small volume.

In an exploding device, the flux is compressed into a load section that is not compressed by the explosion. A simple illustration is the plate generator of which a cross sectional view is shown in Figure 11b. Metal plates form a volume with a rectangular cross section, similar to the geometry in Section 4.1 and it is connected to a load into which the flux is to be compressed. Two high explosive blocks are placed on each side of the generator and are simultaneously initiated all over their outer surfaces. As the two sides start to move inwards, the capacitor bank used to supply the initial field is disconnected and the magnetic flux is trapped and compressed into the load coil. Other types of generators (e.g. strip-, loop-, coaxial-, and helical generators [37]) utilize different geometries to provide different characteristics.

![Figure 11. a) Imploding cylindrical generator and b) an exploding device called plate generator.](image)

All types of generators require an initial magnetic field to be present in the generator when the compression starts. In an imploding device, an external coil can be placed around or close to the imploding device to generate the magnetic field required. The coil is connected to a current source, which could be a capacitor bank or another flux compression generator. An exploding device can
be supplied by driving a current through the generator and then trap the flux at
the onset of the compression, or is supplied with by external coil.

4.3 Circuit representation

Instead of using the magnetic field and the enclosed area to describe the
operation of a magnetic flux compression generator one may describe the
operation in terms of inductance and current. The self-inductance \( L \) of a circuit
carrying a current \( I \) is the ratio of the flux \( \Phi \) linking the loop and the current,
\( L = \Phi / I \). Without losses the flux is conserved and a consequence of reducing
the inductance is an increase in current. The two expressions,

\[
\frac{d}{dt} \Phi = \frac{d}{dt} \int B \cdot dS = 0 \tag{93}
\]
\[
\frac{d}{dt} \Phi = \frac{d}{dt} (LI) = 0 \tag{94}
\]

are equivalent but Eq. (93) may be more appropriate to describe imploding
devices whereas Eq. (94) may be more useful to describe exploding devices.
Consider Figure 12 where \( L_{\text{gen}} \) is the time varying generator inductance, \( L_{\text{load}} \) is
the load inductance, \( R_{\text{gen}} \) is generator resistance, and \( R_{\text{load}} \) is the load resistance.
The circuit equation is

\[
L \frac{dI}{dt} + \frac{dL}{dt} I + RI = 0 \tag{95}
\]

where \( L \) is the total inductance and \( R \) is the total circuit resistance. The
resistance may represent ohmic losses and include effects of field diffusion,
heating, or other losses. Here, one can see that the term \( I \frac{dL}{dt} \) has to be larger
than the term \( RI \), otherwise the term \( L \frac{dI}{dt} \) will become negative i.e. there will
be no current amplification. The term \( I \frac{dL}{dt} \) is usually named internal voltage
or generator armature potential and is in fact the source voltage for the circuit.
Its value must be kept below a certain value to prevent electrical breakdown
inside the generator. Good models of coil inductance, inductance rate of
change, and resistive loss functions will predict the generator performance fairly
well.
The solution to the circuit equation is

\[
LI = L_0 I_0 \cdot \exp\left(-\int \frac{R}{L} \, dt\right) \tag{96}
\]

Two coefficients are useful and used as figures of merits of exploding generators. The flux compression coefficient is

\[
\lambda_k = \frac{LI}{L_0 I_0} = \exp\left(-\int \frac{R}{L} \, dt\right) \tag{97}
\]

and the inductive compression ratio is

\[
\gamma_L = \frac{L_0}{L} \tag{98}
\]

We may then rewrite Eq. (97) to obtain

\[
I = I_0 \gamma_L \lambda_k \tag{99}
\]

The energy amplification for the generator is then

\[
\frac{W}{W_0} = \gamma_L \lambda_k^2 \tag{100}
\]

### 4.4 Helical generators

A helical or spiral generator is a generator type that usually consists of a conducting cylindrical coil (stator) and a conducting cylindrical tube (armature) filled with high explosives, Figure 13 [21, 38 - 44]. It has a very high initial inductance compared to the final inductance. This makes it possible to amplify the initial magnetic energy hundredfold. The stator and armature are connected...
via, in most cases, an inductive load into which the magnetic flux is to be concentrated. The stator is usually magnetized by a seed current having its return path via the load and armature. The stator can also be magnetized by an external coil. When initiated, at one end, the high explosives in the armature will rapidly expand the armature in a conical fashion that will short out the seed current source and trap the magnetic flux in the volume between the stator and the armature. As the detonation moves forward, the stator coil is shorted out turn by turn by the armature, reducing the inductance of the circuit. Since the flux is conserved the current in the circuit will increase and thus also the magnetic energy which can be used for various applications. One of the best helical generators in terms of energy amplification was designed by Chernyshev et al with a two hundredfold energy amplification [38]. Helical generators of very different sizes have been built. Small generators of a few centimetres length can deliver a few kiloamperes while large generators with lengths of several meters can deliver hundreds of megaamperes. The inductance rate of change during the explosion can be tailored by dividing the stator coil into sections of different pitches that usually increase towards the end. The larger pitch then allows for a larger conducting area that can carry larger currents than the previous sections.

![Figure 13. Helical generator with a high explosives filled armature inside a helical coil.](image)

When designing a helical generator there are several aspects that have to be considered. The desired current and energy amplifications as well as the final current and energy to be delivered to, or to be stored in, the load, determine the size of the generator. As described in section 4.3 the performance of a generator depends on the flux compression coefficient and the inductive compression ratio. The flux compression coefficient quantifies how much flux is lost during compression while the inductive compression ratio gives the ratio of initial to final inductance. A large inductance compression ratio and a large load inductance imply that the generator will need to have a large diameter and length. A large diameter allows for a large conducting area of the coil windings. This gives higher current handling capability. The compression time and hence the current pulse length is determined by the detonation velocity (6 to 9 km/s) of the high explosive used and the size of the generator. In a helical generator the armature diameter is usually half the diameter of the stator. The armature can expand twice its initial diameter without cracks developing in the expanding cone. There are several rules of thumb that can be used during the design process in order to limit magnetic flux losses and reduce high resistive losses.
However, in the design process it is useful to use computer codes to predict the performance of a generator and vary different parameters to meet the requirements.

One example of helical FCG is that used in experiments at FOI, and is described in detail in Paper I. Figure 14 shows the layout of the FCG and identifies some of the vital components of the generator. The generator is initiated on the left hand side and the detonation wave front moves from left to right. When the detonation front moves into the armature, the armature will be accelerated outwards and will form a cone moving forward shorting out the turns, as illustrated by hydrodynamic simulations using GRALE [47] in Figure 15. When this expanding cone (A) meets with the conical end (C) of the armature they are designed to form a more or less cylindrical part moving radially outwards towards the stator (D). The reduction in inductance is at this stage very rapid and so is also the rate of the increase in current. The expanding armature will make initial contact with the stator via a copper ring (crowbar ring) mounted on the stator.

![Figure 14. The FCG: 1 – Initiator, 2 – Seed current cables, 3 – Crowbar ring, 4 – Stator coil, 5 – Return conductor, 6 – Armature and 7 – High explosive PBXN-5.](image1)

![Figure 15. Hydrodynamic simulations of the armature movement as the detonation wave front (B; indicating the pressure) progresses to the right. The time between the frames is 4 µs.](image2)
4.5 Circuit simulation model

To simulate the FOI FCG behaviour a zero-dimensional (0D) code was developed and implemented in Matlab-Simulink. The code is based on the 0D-code developed by Novac et al at Loughborough University [43, 44], and has been modified to handle the geometry of the FOI FCG. The Loughborough code models helical FCGs with the stator coil made from wires with circular cross sections and cylindrical armatures. In this model, the inductance is only a function of geometry while the resistance depends on reduction of conductor lengths as well as heating of, and magnetic field penetration into, the conductor. This means that the inductance as function of time can be calculated only knowing the detonation velocity of the high explosive. The length of the conductor is also a function of geometry and can be calculated in advance before solving the circuit equation. The generator circuit is that of Figure 12 with the circuit equation according to Eq.(95). The implementation of the FCG as a component in Matlab-Simulink allows for system simulations with the generator connected to other components such as seed current sources and pulse forming networks. The simplicity of the implementation allows for a large number of simulations to perform parametric studies of various system components.

4.5.1. Simplifications

The circuit elements in the generator model are calculated for a geometry that has some simplifications compared to the physical generator (illustrated in Figure 16). The resistance and inductance of the small volumes formed near the crowbar ring and near the return conductor are ignored. Their values are very small compared to the inductance and resistance of the generator itself. The contact resistance of the conducting area between armature and stator is not modelled but in many MFCG codes the resistance of the contact point is adjusted to get an agreement with experiments.

Figure 16. a) The generator and b) the simplified representation.
4.5.2. Inductance calculations

The FOI FCG has of several sections with coils with different pitches. These sections have mutual inductances between each other and those have to be accounted for. The total inductance $L$ of a coil with $N_s$ sections is

$$L = \sum_{i=1}^{N_s} L_i + \sum_{i,j=1}^{N_s} M_{ij}(1-\delta(i,j)) \quad (101)$$

where $L_i$ is the self inductance of section $i$ and $M_{ij}$ is the mutual inductance between section $i$ and $j$ and $\delta(i,j)$ is the Kronecker delta function (i.e. $\delta = 0$ for $i \neq j$ and $\delta = 1$ for $i = j$). The method used to calculate the generator inductance was presented by Novac et al [43,44] and in the licentiate thesis the adaptations to the FOI generator geometry are given.

4.5.3. Resistance of stator and armature

The resistance of the FCG is calculated as the sum of the resistance of the helical stator windings, the resistance of the helical current path in the armature mirroring the current in the stator and the resistance of the current path in the expanding cone. The method was presented by Novac et al [43], but is modified to account for rectangular cross sections of the stator coil windings. Further, due to the rectangular shape of the windings the proximity effect is ignored.

The resistance of the stator is simply the resistance of a helically shaped conductor. The length of a helix is

$$l_{stator, i} = \frac{2\pi \cdot l_i}{p_i} \sqrt{r_i^2 + \left(\frac{p_i}{2\pi}\right)^2} \quad (102)$$

where $l_i$ is the axial length and $p_i$ is the pitch of the coil in section $i$ and $r_i$ is their inner radius. The resistance of the stator is then

$$R_{stator} = \sum_{i=1}^{N_s} \frac{l_{stator, i}}{\sigma_i \cdot d_i \cdot \delta_i} \quad (103)$$

where $\sigma_i$ is the conductivity, $d_i$ the width of the coil conductor and $\delta_i$ is the skin depth in which the current flows, see Figure 17.

The resistance of the armature is somewhat more complicated due to the cone expansion and the conical end, which interacts with the expanding cone when they meet towards the end of compression. It is assumed that the current in the
The armature is mirroring the current in the stator. This mirror current is set up by the magnetic field generated by the current in the stator, see Figure 17. Details on the expressions were presented in the licentiate thesis. To calculate the skin depth the current is assumed to rise exponentially. The skin depth is then given by

$$\delta = \sqrt{\frac{\kappa}{\tau}}$$

with \(\kappa\) and \(\tau\) defined by Eq. (22) and (33) [7]. The heating of the coil is calculated using the current action integral model for the conductor resistivity, see Section 2.5. The current action integral \(J\) is calculated from the instant current \(I\) and the instant conducting area \(\delta_{\text{skin}} \cdot d_i\)

$$J = \int \frac{I^2}{\delta_{\text{skin}} \cdot d_i} dt$$

Figure 17. The current in the armature is assumed to mirror that in the stator and that it floats in a layer with depth \(\delta_i\) and width \(d_i\). In reality the mirrored current in the armature would not be confined to the area projected onto the armature by the stator current.

4.5.4. Implementation of the model

The model for the FCG is implemented in Matlab-Simulink. The circuit scheme for the generator with seed current supply is given in Figure 18. During seeding phase the switch \(S\) is open. It closes at a time when the seed current peaks. The capacitor \(C_{PPS}\) will continue to discharge through \(R_{PPS}\) and \(L_{PPS}\) while the current and magnetic energy in the generator will increase as the magnetic flux is compressed. The seed current circuit is a straightforward implementation and the switch \(S\) is implemented as an ideal switch. The inductance of the generator and the conductor lengths are given by the geometry and the detonation velocity. The resistance of each section is calculated for each time step by calculation of the instant skin depth and conductor conductivity. The Matlab-Simulink model uses two look-up tables calculated in advance; one for the generator inductance as function of time and one with the conductor lengths of the sections as function of time. The inductance of the generator was calculated for the four cases where either the
cone-expansion was considered or not and the mutual inductance was considered or not. Figure 19a shows the four cases with an inductance measurement marked by circles. The agreement is good when both the mutual inductance and conical expansion are included in the inductance calculation (solid line in Figure 19a). The figure shows the importance of including both the mutual inductance and the conical expansion in the calculation of the generator inductance. The measured initial inductance is slightly lower than the calculated 23.4 µH. The calculated final inductance at end of compression is 170 nH. The total conducting length of the armature and the stator as function of time is calculated for each section. The skin depth and the conductivity will be the same within a section and thus the conducting lengths of the stator and the armature can be added. Figure 19b shows the lengths for the four sections (s1 - s4). The increase in length in section 1 (s1) during the first 10 µs is due to the conical expansion of the armature. As the stator turns are shorted out, the conducting length will reduce. At time 19.5 µs the contact has passed section 1 and enters section 2. In section 4 (s4) no turns will be shorted out and the conducting length will increase as the expanding cone enters the section. The end time of compression is defined as the time when the inductance reaches its final value and it occurs at 32.3 µs, indicated by the vertical line in Figure 19a and b.

Figure 18. The circuit scheme of the generator with seed current supply.

Figure 19. a) Calculation of generator inductance with different levels of accuracy. The solid line is the complete calculation and the circles mark the measured inductance. Time \( t = 0 \) corresponds to the time of crowbar. b) Total conducting lengths in the four sections.
4.6 Experiments and simulation

Two generators were used for experimental studies. It was decided that the generators were to be seeded with different currents, where the first generator (FCG #1) was to be seeded with modest current, while the second (FCG #2) was to be seeded with more current than it was designed for. Hence, in the first experiment the current amplification could be expected to be higher than in the second where higher resistive losses could be expected. The results were used to benchmark a code for simulation of the FCGs.

The generators were fitted with optical fibers to monitor the light inside the generators, and piezo gauges to monitor the arrival of the shock waves. The current was measured and together with the information obtained with the other diagnostics, good understanding of the generator performance could be obtained. The experimental results and analysis were presented in the licentiate thesis and hence the results are only briefly described here with focus on the generator performance in terms of current and energy amplification and with respect to the various loss mechanisms. In this chapter we discuss the results from both experiments and simulations to analyse the generator performance.

Figure 20 shows the generator currents and current amplifications for the two experiments. The peak current values are given in Table 3. The current amplification factor is 47 for FCG #1 and 39 for FCG #2. The current amplifications for the two generators are almost identical up to 4 µs before the current peaks, where the amplification rate of FCG #2 drops compared to FCG #1. The lower current amplification of FCG #2 is thought to be caused by resistive losses in the generator during the final compression. At crowbar time, approximately 0.4 kJ and 1.5 kJ are stored inductively in the two generators. The final inductance is difficult to estimate but it is approximately 0.2 µH. Using that value, the inductively stored energy at the time of peak current is 7.3 kJ and 19 kJ for FCG #1 and #2 respectively implying energy amplifications of 19 and 13 for FCG #1 and FCG #2.

Figure 21 shows the simulated current time derivatives together with corresponding experiments. The agreement is very good up to a few microseconds before the peaking of the current. The simulations give almost identical final currents that peak somewhat later than in the experiments, see Table 3. The simulated current amplification factors for the two generators are 48 and 40 compared to 47 and 39 in the experiments. FCG#1 has a peak energy of 7.2 kJ and FCG #2 has a peak energy of 20.8 kJ. This should be compared to the estimated 7.3 and 19 kJ from the experiments where the final inductance is however hard to estimate.
Figure 20. a) FCG currents. b) The current amplifications for the two generators obtained by dividing the FCG currents by the respective seed currents at time of crowbar. The common time reference is given by an optical fibre, mounted at the ends of the high explosives. The time of crowbar is at 0 µs.

![Figure 20](image1.png)

Figure 21. Experimental and simulated FCG current time derivatives. The circles mark when the contact point moves into a new stator section. The simulation is shifted +5 µs to distinguish it from measured curve.

![Figure 21](image2.png)

Table 3. Comparison of simulated and experimental peak currents and peak current time derivatives.

<table>
<thead>
<tr>
<th></th>
<th>Peak current</th>
<th>Peak current time derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>Simulation</td>
</tr>
<tr>
<td>FCG #1</td>
<td>269 kA</td>
<td>266 kA</td>
</tr>
<tr>
<td></td>
<td>86.6 µs</td>
<td>87.2 µs</td>
</tr>
<tr>
<td>FCG #2</td>
<td>436 kA</td>
<td>434 kA</td>
</tr>
<tr>
<td></td>
<td>87.7 µs</td>
<td>87.9 µs</td>
</tr>
</tbody>
</table>
4.7 Analysis of generator performance

The good agreement between experimental results and simulations enables us to use the simulations to analyse the over-all performance of the generator. It should be noted that there is no non-physical tuning variable in the code that has been adjusted to obtain the excellent agreement. In the experiments, the two generators were seeded with different currents that would reveal effects of heating, more severe for the higher seed current than for lower seed current.

For lossless flux compression it is required that the flux does not diffuse out of the compression volume and that no magnetic energy is lost through ohmic heating. Figure 22a shows the simulated time development of the flux $L \cdot I$ inside the generator from time of crowbar. At crowbar time the flux in FCG #1 and #2 were 0.13 Wb and 0.26 Wb. The flux conservation factor during operation defined as the ratio of instant flux to initial flux, and is shown for the two generators in Figure 22b. The flux loss rate is constant during most part of the compression but increases just at the end. At the final compression when the current peaks the conservation factor are 0.34 and 0.28 for FCG #1 and #2 respectively.

The ratio of the simulated current amplifications for FCG #1 to FCG#2 is shown in Figure 23a indicating that during the last 5 µs FCG #2 has severe losses compared to FCG #1. The generator resistances for FCG #1 and FCG #2 are shown in Figure 23b. The resistance of FCG #2 increases during the last 5 µs and is some 34 % higher than in FCG #1 at the time of peak current.

The temperatures of the conductor in section 4 for the two generators are shown in Figure 24a. This is the average temperature of the conducting cross-section area. It clearly shows that the average temperature in FCG #2 is above the melting temperature of copper during the last microseconds and that it is most likely that the surface of the conductors in the physical generator melts towards the end of operation. This is probably not the case for FCG #1 were the temperature is far below the melting temperature. The conductivity in sections 3 and 4 of FCG #2, Figure 24b, drops fast during the last 5 µs due to heating and is accompanied by an increase in skin depth of the corresponding sections, Figure 25a.

The internal voltage of a generator is one of the design parameters that has to be considered. The internal voltage, $U = I \cdot dL/dt$, is plotted in Figure 25b for the two generators. The voltage inside FCG #2 is twice that of FCG #1 during most part of the operation. The current derivatives in Figure 21 indicate that electric breakdown occurred in FCG #2 several times and is probably due to
Figure 22. a) The magnetic flux in FCG #1 and #2 respectively and b) flux conservation factors of FCG #1 and #2 respectively.

Figure 23. a) Ratio of the simulated current amplification of FCG #2 to the simulated current amplification of FCG #1 and b) the simulated total FCG resistance of the two generators.

Figure 24. a) The temperature of the conductor in section 4 in FCG #1 and #2 respectively and b) the conductivity of the four sections in FCG #2.
the relatively high voltage, which is more than 20 kV during most part of the operation. The simulations provide some insight into the performance of this specific design. The FOI generator is considered a small generator with a stator coil diameter of 53 mm and a coil length of 237 mm. The armature with an outer diameter of 24 mm and a wall thickness of 2 mm contains some 270 g of high explosives and expands to more than twice its initial diameter. It is of limited use to compare the FOI generator to generators of significantly larger sizes in terms of energy and current output since a larger generator for obvious reasons can carry a larger current and has a higher initial inductance. A longer generator will have a longer compression time and hence its current pulse characteristics will differ. However, to compare generators a figure of merit can be calculated for any generator. The figure of merit, $\alpha$, is defined by

$$\alpha \left( \frac{I}{I_0} \right) = \left( \frac{L_0}{L} \right)$$

(106)

where $I_0$ and $L_0$ are the initial current and inductance and $I$ and $L$ are the final current and inductance. For FCG #1 and #2 the figure of merits are 0.81 and 0.77 using the estimated initial and final inductance values of the generators. These figure of merits are relatively high values for a small generator [48].

Another interesting value is to calculate the energy conversion efficiency of the detonation energy into magnetic energy. The energy density of high explosives is about 5700 MJ/kg and hence in the 270 g PBXN5 about 1.5 MJ detonation energy is stored. For FCG #1 and #2 the final magnetic energies were 7.3 and 19 kJ i.e. 0.5 % and 1.2 % of the detonation energy. The specific energy of the generator is the ratio of the final energy to the initial volume of the FCG. For FCG#1 and #2 the specific energy is 5.7 and 15 MJ/m³, using an initial volume as that of a cylinder with 70 mm diameter, i.e. the stator diameter including the glass fibre reinforcement, and the length 330 mm, i.e. the armature length.
4.8 System simulations

For efficient operation of a helical flux compression generator, the load has to have low inductance and low resistive losses. A transformer with a low impedance on the primary side and a high impedance on the secondary can be used between the generator and the load to match their impedances. An air core foil type transformer is usually required due to the high currents and short rise times of the pulse. A transformer will not increase the power of the pulse applied to the load. In order to increase the load power a pulse-conditioning system may be used. An example of such a system is shown in Figure 26. In this system the generator drives a current through an opening switch, e.g. an electrically exploded opening switch (EEOS), during the first phase of flux compression. A closing switch e.g. a spark gap (SG) and a high impedance load are connected in parallel with the opening switch. When the EEOS opens the current is interrupted and a high voltage developed that is applied to the load via the spark gap.

![Circuit scheme of the pulse-forming network used in system simulations.](image)

A common and simple type of opening switch is a switch consisting of thin metal wires. Once a sufficiently high current action integral is reached the wires explode with an instant 100-fold increase in resistance, see Section 2.5 and Figure 5. Simulations of this system were performed using the FOI flux compression generator model, an EEOS using the current action integral model, a spark gap modelled as a voltage controlled linear closing switch and a purely resistive load of 15 Ω, see Paper III. In the simulations, the capacitor was charged to 4.2 kV, 600 J and the current at crowbar was 5.7 kA, as was used in the simulations of the experiments above. The number of metal wires in the opening switch was varied in such a way that the optimum performance in terms of load voltage of the pulse-forming network was obtained. Figure 27 shows the generator current and load voltage for different number (20, 25,…60) of wires in the opening switch. The load voltage is 260 kV at optimal number of wires corresponding to 4.5 GW load power.

The circuit scheme of Figure 26 may also include a transformer with the EEOS on either side of the transformer. In Paper IV, a system incorporating a transformer was tested and used to power a microwave source. In that case the flux compression generator was replaced by a capacitor bank.
4.9 Concluding remarks

This MFCG work includes both experimental and numerical work. The two generators available for experiments were fitted with a variety of diagnostics; voltage- and current probes, piezoelectric gauges, and optic fibres looking into the generator. This gave good understanding of the generator operation. By operating one generator below and the other above its design specifications in terms of seed current it was possible to see the effect of excess heating. To successfully perform the two experiments that required accurate coordination of various systems such as that of timing the initiation of explosives with the discharge of the seeding pulsed power supply, and to record all data without problems, required careful planning and attention to detail.

The numerical work demonstrates that the relatively simple generator model, adapted to the geometry of the tested generators and implemented in a circuit simulation software, gives very good agreement with the experiments. Of special importance is that the loss mechanisms in terms of ohmic heating are accurately modelled. Using simulations it was possible to analyse the generator operation and investigate phenomena that could not be directly measured in the experiments.

The implementation of the generator into a circuit simulation software, in this case Matlab-Simulink, enables system simulation of complete systems where the generator serves as the energy source. One example is compact high power microwave radiating systems for use as single-shot electromagnetic weapons that will disrupt or destroy electronics. With good models of the pulse-forming network and of the radiation source, the performance may be evaluated. Given that the generator will not deviate too much in size or geometry from these studied here, it would also be possible to design a generator to meet the requirements of a specific radiation source.
5 Electric armour

The electric armour research presented here forms the second part of the doctoral work. Electric armour is a protection concept for military vehicles that utilizes electric energy to disrupt the threat penetrator before it reaches the vehicle hull. The disruption process involves many of the phenomena presented in Chapter 2.

5.1 Background

The protection concept considered in this study works against shaped charge warheads, commonly used in handheld anti-armour systems such as the Russian RPG-7. A shaped charge warhead creates a solid copper jet capable of penetrating several decimetres of armour steel. In the electric armour protection the jet bridges two electrodes connected to a current supply. A strong current flows through the jet that is heated and eventually disrupted, and looses its penetrative performance. The principle of direct electrification of the solid copper jet created by a shaped charge was proposed by Walker [6] in the 1970-ies. Experiments were reported by Pollock [49] in 1992 and an increasing number of papers related to electric armour have been published since [50-63]. In parallel with research on the electric armour system concept itself [64,65], there have been efforts aiming to improve pulsed power components such as capacitors, solid state switches, and diodes to meet the demands of a fieldable electric armour system. Demonstrations of this technology with live-firings on vehicles fitted with electric armour were reported by DSTL [66] in 2002 and by BAE Systems in 2005 [67]. A first European workshop on electric armour was hosted in 2009 by the French-German Research Institute, ISL, in Saint Louis, France [68]. Following this workshop, the European Defence Agency funded a study on Electric Armour for Armoured Vehicles (ELAV) conducted by BMT Defence Services [69-72]. The main objective was to develop a road map for a common European effort in development of electric armour.

The electric armour concept has been studied since late 1990-ies at the Swedish Defence Research Agency, FOI, and experimental work on disrupting shaped charge jets began in 2006. The studies performed at FOI serve to provide...
knowledge of the protection concept to the Swedish Armed Forces, including
the basic physics of the jet disruption process. FOI participated in the electric
armour workshop at ISL in Saint Louis, France, and contributed to the ELAV-
study. A minor part of the work performed at FOI on electric armour has been
released for publication and presented in a number of scientific papers and at
conferences [Paper V to X].

5.2 Shaped charges

A shaped charge consists of a metallic liner, usually copper, in the shape of a
cone surrounded by high explosives [73], Figure 28a. When initiated the
explosive deforms the metal cone into a metallic jet, having a velocity gradient,
Figure 28b. The tip of the jet reaches 7 to 8 km/s while the tail of the jet has a
velocity of about 2 km/s. The mass of the jet depends on the diameter and wall
thickness of the metal liner but is usually several tens of grams for a medium
size warhead. A slug of lower velocity, 0.5 km/s, follows the rear part of the jet,
and has a much higher mass but has limited penetrative capability. The shaped
charge jet will stretch with time up to some point when instabilities will cause
fragmentation, Figure 29, of the jet, which in turn reduces the penetrative
performance. The disruption of the jet by a current pulse serves to reduce the
penetration of the jet to such an extent that the hull of an armoured vehicle can
absorb the remainder of the penetrator. A simple expression for the
penetration $P$ of a jet segment with constant velocity is [73]

$$P = l \sqrt{\frac{\rho_j}{\rho_T}}$$  \hspace{1cm} (107)

where $l$ is the length of the jet segment and $\rho_j$ and $\rho_T$ are the mass densities of
the jet and target. As a steel target and the copper jet have about the same mass
density, the penetration will be approximately the cumulated length of the jet.

5.3 Electric armour principles

In its simplest design, the jet disruption system consists of a capacitor bank that
stores a sufficient amount of energy, a transmission line to transfer the energy,
and a pair of electrodes placed between the shaped charge and the protected
objects. In addition a charge-and-dump circuit and control units are required.
The shaped charge jet bridges the gap between the electrodes and the energy
stored in the capacitors is released in the form of a strong current pulse, Figure
30. The jet is heated by the current until it melts, vaporizes, or is disrupted by
magnetic forces. The electrodes could be placed some distance away from the
protected object and hence allow for the disruption to develop until the jet fragments have lost their penetrative capability.

Figure 28. a) Shaped charge with a conical metallic liner surrounded by high explosives, HE. b) Formation of a shaped charge jet simulated at FOI. The two top left pictures show the shaped charge with its cylindrical casing visible, while the following pictures only show the metal liner and its formation to a jet. Due to its velocity gradient the jet stretches accompanied by a decreasing diameter. The jet is followed by the slug of much higher mass but with lower velocity.

Figure 29. The fragmentation of a jet depicted about 200 µs after initiation.

Figure 30. The electric armour protection concept with two parallel electrodes connected to a capacitor bank. The circuit is closed by the jet itself and the current I and magnetic field B may affect the jet during its passage between the electrodes.
5.4 Experimental work

To perform systematic experiments a charge that generates reproducible jets is required. In the experiments, a shaped charge warhead with a straight conical liner made of copper was used. This charge generates a jet with tip velocity of 7.3 km/s and a total jet mass of 30 g. The interaction time between a current pulse and the jet is determined by the distance between the charge and the electrode pack and the electrode separation distance. For the given electrode setting and experimental setup, the rear part of the jet will leave the electrode region 100 µs after the tip reaches the second electrode and closes the circuit.

To perform experiments a suitable pulsed power supply is required. A 400 kJ pulsed power supply (PPS) was used. The design enables a wide range of output pulse shapes with minimal reassembly, see Section 3.3 and Paper X. The output pulse was transferred to the load via a flat transmission line with low inductance. The electric armour pack consisted of two aluminium plate electrodes with a separation distance of 150 mm. The shaped charge was placed on a typical test stand for shaped charge testing, Figure 31. A steel target made of square tiles was used to absorb the residual jet. The target was positioned at a large distance behind the back electrode of the armour pack to allow the jet to expand to enable the X-ray radiographing.

To calculate the dynamic impedance and resistance of the jet, the power and energy deposition, the PPS voltage, the voltage across the armour electrodes, and the current have to be monitored. Flash X-ray tubes can be used to depict the jet. If the jet characteristics are known, the mass and velocity distributions can be related to the current pulse and energy depositions.

In addition to the experiments performed with shaped charges, static experiments were performed with thin metal rods that were electrically exploded by a smaller PPS [Paper VIII].

![Figure 31. The experimental setup with the shaped charge being fired vertically down through the electrodes, into the target. Flash X-ray tubes project a shadow of the jet onto the X-ray film.](image-url)
5.5 Main results and analysis

Experiments were performed to study the effects of the current $j \times B$ force and the ohmic heating $j \cdot E$ on the jet disruption. In addition to the experiments, 2D-simulations were performed. An analytical description of the jet enables analysis of correlating the current pulse with the jet flow and also the jet heating. In the following sections, different aspects of the electrification are analysed and discussed.

5.5.1. Current and jet interaction and disruption

Figure 32 shows position-time diagrams for the jet in the experiments together with the traces of the measured current and deposited energy. A position-time diagram describes the position of different parts of a jet as function of time. The diagonal lines marked by numbers indicate the paths of jet parts with the velocity given by the number (in km/s). The electrode positions are marked by horizontal lines (the electrodes are numbered in the order they are hit) and the dashed vertical lines marks the time of the flash X-rays. The current is triggered by the arrival of the jet tip at Electrode 2. The current pulse from an experiment is included in Figure 32a, and shows that the current pulse length is sufficiently long to affect a large portion of the jet. Figure 32b includes the estimated energy deposition as function of time. The energy stored in the capacitor bank was 96 kJ and when the 2.1 km/s fragment leaves Electrode 2 more than 87 kJ have been deposited in the jet, in the electrodes, and in the surrounding plasma. The effect of the energy on the jet disruption is shown in Figure 33 and Figure 34 where energies 0, 39 and 96 kJ, were stored in the capacitor bank. Figure 33 shows the jets 31 or 36 µs after they made contact with Electrode 2 for three different capacitor bank energies (0, 39, and

![Figure 32. Position-time diagram for the jet with the (left) current pulse and (right) the deposited energy from a 96 kJ experiment. The vertical dashed lines marked X-ray A and X-ray B marks the times (36 and 86 µs) of the X-rays. At time 36 µs the part with velocity between 3 and 4.7 km/s is between the electrodes and at time 86 µs the part with velocity between 1.9 and 3 km/s is between the electrodes.](image-url)
The jet disruption clearly increases with energy and the disruption begins earlier. Figure 34 shows the jets 50 µs later. The jet disruption is evidently stronger for higher energies. The general shape of the current pulses in the different experiments is the same, and the pulse lengths are identical. The amount of energy deposited in the jet and electrode region is 80 - 90% in all experiments.

Figure 33. X-ray pictures of jet disruptions at different energies. From top 96, 39 and 0 kJ is stored in the capacitors. The pictures are taken 36 µs after contact is made by the tip with Electrode 2 (the right of the two shadowed areas), except for the lower picture taken 31 µs after contact. The jet tip has moved a distance of 260 mm from Electrode 2 after 36 µs.

Figure 34. X-ray pictures of jet disruptions at different energies. From top, 96, 39 and 0 kJ is stored in the capacitors. The pictures are taken 86 µs after contact is made by the tip with Electrode 2, except for the lower picture taken 81 µs after contact. The jet tip has moved a distance of 620 mm from the Electrode 2 after 86 µs.

5.5.2. Heating of the jet

We estimate the total energy deposition in the jet and under the assumptions of a constant inductance of the circuit, that all the current flows only through the jet, and that there are no losses in the contact between jet and electrodes. With the jet mass of about 30 g and assuming all energy in the 39 and 96 kJ is deposited in the jet, the average specific energy deposition is then 1.3 and 3.2 kJ/g respectively. The jet is already hot when it enters the electrode region, between 800 and 1000 K due to the plastic deformation by the detonation of the explosives. To raise the temperature from 800 K to the melting temperature requires 244 J/g and the heat of melting is 210 J/g [74]. The 1.3 and 3.2 kJ/g is hence more than sufficient to melt the jet.
Although the total energy deposition may be difficult to estimate one may look at the heating of cross sections of the jet. The jet geometry as function of time is given by analytical expressions obtained through experiments and hydrocode simulations. The analytical expressions do not include a description for the radial variations along the jet leading to its fragmentation. The current action and current action integral (see Section 2.5) may be calculated for the jet segments using this analytical jet flow description and the measured current in the 39 and 96 kJ experiment. Figure 35 shows the current action in the two experiments for segments with axial velocities of 3, 3.5,… 6 km/s where the traces with fat line is the 39 kJ experiment and the thin line is the corresponding traces for the 96 kJ experiment. The current action peak value is in the low-energy experiment about 40% of that of the high-energy experiment, which is reasonable since the energy has to be delivered within the same time frame by a similar pulse shape whereas the energies differ a factor two and a half. Integrating the current action over the interaction time yields the current action integrals for the segments. Figure 9 shows the current action integrals for the experiments as function of time. Included in the graphs are the current action integral values required to bring a copper conductor that has an initial temperature of 800 K to the melting temperature (1356 K) and to fully melt it. It is only in the 96 kJ experiment that the current action integral values reach above these values. In the 96 kJ experiment it was observed that the jet has begun to break up between the electrodes and that this occur about 25-30 mm from the second electrode. This is seen in the X-ray pictures in Figure 33 at 36 µs and in Figure 34 at time 86 µs. The values the segments have at the position 27 mm from the second electrode is indicated by the circles.

Figure 35. Current action and current action integral for segments with axial velocity 3, 3.5… till 6 km/s calculated for experiment with 39 and 96 kJ. The circles mark the times and values of the current action integral when the segments (axial velocity 3.5 to 5.5 km/s) are at the position (27 mm from Electrode 2) at which a break is observed in X-ray pictures, Figure 33 and Figure 34.
This method to analyse the heating is very crude since all effects of thermal expansion, deformation and diffusion are neglected. During the electrification the different parts of the copper jet appears in solid, molten, vapour and plasma phase simultaneously. Using a hydrocode with a detailed model of the conductivity as function of temperature and density would enable a better analysis of the heating process.

5.5.3. Enhanced jet fragmentation

A shaped-charge jet naturally breaks up into fragments due to inherent instabilities of the jet itself, see the 0 kJ case in Figure 33 and Figure 34. When an electric current passes through a jet, compressive magnetic forces will act on the jet and then enhance the natural radial variation, and accelerate the break up of the jet. This magnetohydrodynamic instability effect is discussed in Section 2.7. The growth rate of such instabilities in stretching jets subjected to a current was studied by Littlefield using perturbation theory [50-54]. Figure 36a shows the axial velocity versus fragment number for the 39 and 96 kJ experiment together with reference data from an experiment of a non-electrified jet [Paper VI]. The jet has divided into a similar number of fragments within the velocity range, indicating that the current interaction indeed enhances natural fragmentation. The velocity difference between two adjacent fragments is about 90 m/s on average. Figure 36b shows the X-ray pictures of the parts of the jets that are between the electrodes in experiments, at time 81 µs with energy 0 kJ, and at time 86 µs for energies 39 and 96 kJ, compare with Figure 34. With the aid of the position-time diagram in Figure 32 it can be seen that, at time 86 µs, the part with axial velocity 3.0 km/s is about to exit and the

![Figure 36. a) The axial velocity for individual fragments in the 39 (diamonds), 96 kJ (circles) and the reference data (squares). The jet has divided into a similar number of fragments within the velocity range. b) The part of the jets that is between the electrodes in the experiments with energies 0, 39 and 96 kJ stored in the capacitors (compare with Figure 34). The 0 kJ experiment is depicted at time 81 µs and the 39 and 96 kJ experiments are depicted at time 86 µs. The jet radius is scaled ×3.](image-url)
part with axial velocity 1.9 km/s has just entered the electrode region. The
radial variation is strong in the electrified jets and in the 96 kJ experiment the
jet is broken at two positions. In Figure 6 the magnetic flux density is shown
for 200 kA and 1 and 2 mm diameters. The magnetic flux density is several tens
of Tesla. The magnetic pressure inside the conductor, Figure 7, is up to 5 GPa
in the centre of the conductor. The yield strength of annealed copper at room
temperature is 70 MPa, which is much lower than the magnetic pressure [7].

5.5.4. Radial dispersion of the jet fragments

In Figure 33 and Figure 34 it is clearly seen that the disruption is dependent on
the energy deposited in the jet. The dispersion of the fragments follows a
pattern where the jet fragments are transformed from cylinders into rings. Figure 37 shows the same four jet fragments identified in three X-ray pictures
taken at different times. The left picture shows the jet fragments in a 58 kJ
experiment at time 36 µs. The centre and right pictures show the same
fragments at 86 µs from different angles. The higher contrast along the
circumference of the ring implies a ring shape rather than a disc, but that has to
be investigated further. The ring formation process can, at least qualitatively, be
observed in a single X-ray picture of the disturbed jet, where jet parts close to
the electrodes are still cylinders and become more and more ring shaped
towards the tip. By measuring the diameter of jet fragments identified in two X-
ray pictures taken at different times, the radial velocity can be estimated.

Figure 37. X-ray pictures of four jet fragments at 36 µs (a) and 86 µs (b, c) after the jet
made contact with Electrode 2. Figure (c) depicts the same four fragments as in (b) but from a
different angle.

The radial velocities of jet fragments in the 96 kJ experiment are plotted versus
their axial velocity in Figure 38a. There is a large spread in the data but the
fragments with axial velocities between velocities 5 and 6.6 km/s have radial
velocities of approximately 200 m/s. In the two X-ray pictures of the 96 kJ
experiment, Figure 33 and Figure 34, the radial expansion is observed to start
when a segment exits the electrode region. By assuming that all the jet segments
in the 96 kJ experiment, with axial velocities down to 3 km/s, instantly receive a
velocity of 200 m/s at exit of the electrode region, a simple estimate of the
radial dispersion can be obtained and compared to the experiment. The radius
of the jet before radial expansion is assumed to be 1 mm. Figure 38b shows the calculated radius of the jet segments versus distance from the SC cone base, 86 µs after the tip makes contact with the second electrode, i.e. the same time as in Figure 34. The X-ray picture of the jet in the 96 kJ experiment is resized ten times in the radial direction and inserted into the graph. There is a good agreement in radius along the jet axis, except for the tip, and this implies that the radial velocity for the parts with axial velocities below 5 km/s is also around 200 m/s.

![Graph showing radial velocity vs. axial velocity](image)

Figure 38. a) Radial velocity for dispersed fragments in the 96 kJ experiments versus axial velocity. b) The X-ray picture of the 96 kJ experiments stretched in the radial direction. The solid lines show estimates of the radius of the jet 86 µs after the tip makes contact with the second electrode (located at 0.4 m on the scale), assuming an instant radial velocity of 200 m/s of a segment at exit of the electrode region, for the two energies respectively. The estimates are made for jet segments with velocities of 3 to 7.3 km/s.

5.5.5. Mechanisms in the jet disruption

In addition to the shaped-charge experiments, static experiments have been performed to study mechanisms of the disruption process. The static experiments were performed with copper rods fitted with shallow notches to resemble the necks of a stretching jet [Paper VIII]. In the static experiments, a current pulse was passed through the rod causing the notches to explode and the full diameter parts to melt. These experiments were compared to the experiments with shaped charges and there are similarities in the disruption processes for the rod and for the jet respectively [Paper VII]. Figure 39a) shows the X-ray pictures of the segmented rod at times 50, 62.5 and 75 µs. The rod explodes in the notches and the material is ejected radially outwards. The rod segments are consumed from both ends and during 25 µs, the lengths of some of the segments have been reduced to less than half the initial lengths. The radial velocity of the material of exploding notches is of the same order as was observed in the experiments with the jet. Figure 39b) shows the X-ray picture of the 39 kJ SCJ-experiment at time 86 µs divided in portions with axial
velocities in the ranges 3.5 - 4.2, 4.2 - 4.9 and 4.9 - 5.7 km/s. The disruption process can qualitatively be observed along the jet as a longer time has passed since current interaction for a fast fragment compared to a slower. The degree of disruption of the slower jet part can hence be compared to the early picture of the rod disruption and the faster jet parts can be compared to the rod disruption at later times. There is a similarity in the way a rod segment and a jet fragment break up. A difference is however that the rod is constrained and that the segments do not move axially, whereas for the jet, the fragments separate due to a velocity difference of about 100 m/s between two neighbouring fragments.

![X-ray pictures of the segmented rod at times 50, 62.5 and 75 µs and the X-ray picture of the 39 kJ SCJ-experiment at time 86 µs divided in portions with axial velocities in the ranges 3.5 - 4.2, 4.2 - 4.9 and 4.9 - 5.7 km/s. Note that in the jet experiment, the flash X-ray is placed in line with Electrode 2 and hence depicts the 3.5 km/s fragment from the side at right angle while the fragments close to the tip depict at an angle from above, revealing the ring shape.](image)

5.5.6. Diffusion and advection

When the jet enters and exits the electrode region, the time for the current to diffuse into and out of the jet is comparable to the jet passage time between the electrodes. The high velocity of the jet (up to 7 km/s), the characteristic sizes (jet radius of 1 to 2 mm, and electrode separation of 150 mm) and a conductivity that varies with temperature (63·10⁶ S/m at 293 K and 9.8·10⁶ S/m at 1356 K in solid phase) give a magnetic Reynolds number, see Sections 2.2 and 2.6.
\[ R_n = \sigma \mu \cdot L \cdot u \]  

(108)

of the order of 100 or higher depending on the values used. This means that advection dominates over diffusion and that the jet has moved a significant distance before the magnetic field has diffused to the centre of the jet. A jet segment that enters the jet region is instantly subjected to a high current, like a step-function and the field penetration into the jet would be similar to that of Figure 3 in Section 2.4. It can be seen that it takes about 90 µs for the field to reach half the surface value at 1 mm depth in copper at 300 K. In a jet the temperature is much higher, about 800 K, and for that temperature the field reaches half the surface value in about 25 µs. A segment with an axial velocity between 2 and 7 km/s respectively would move between 50 and 175 mm i.e. a significant part of the electrode separation distance. Note however that the heating will reduce the conductivity further and that the diffusion is in cylindrical geometry, which will reduce the penetration time. Another consequence of the high magnetic Reynolds number is that at exit the field has to diffuse out of the jet for some time during which the jet section has moved out from the electrode region. A 2D hydrocode, GRALE, modified to include the MHD equations, was used to simulate the interaction between a metal jet created by a shaped charge and a current pulse [Paper IX]. Figure 40 shows the geometry of the simulation. The jet moves in the upward direction and the current enters the jet at the axial position 0.395 to 0.400 mm and exits at the axial position 0.240 to 0.245 mm (dashed lines). The colour of the jet indicates temperature and the lines indicate the magnetic field contour lines. Figure 41 shows area around the second electrode at times 10, 18 and 26 µs. It can be seen that the magnetic field is pulled along with the jet at the second electrode. In a real situation the contact is not perfect; instead the current between jet and electrode probably passes via an arc and there is then no 2D-symmetry. An interesting and open question is how far the current can be pulled along outside the electrode region and how it affects the disruption of the jet.
Figure 40. Geometry of 2D-simulation with the hydrocode GRALE.

Figure 41. The portion of the jet which is in contact with the second electrode shown at times 10, 18 and 26 µs. The jet moves upwards. The magnetic field is represented by the contour curves and the temperature by the colours. The temperature scale ranges from 770 K (blue) to 2900 K (red).
5.6 Concluding remarks

This work has arrived at some important conclusions concerning the physics of the jet disruption due to strong currents. From a physical point of view, many interesting phenomena are present that are not encountered elsewhere.

Some of the more important findings of this work are

- The energy requirement for disrupting a jet from a medium size shaped charge is less than 100 kJ stored in the capacitor bank.
- Given the shaped charge used and its stand-off distance, and the electrode configuration, the 100 kJ energy has to be delivered within the interaction time of some 100 µs corresponding to an average power of 1 GW.
- The pulsed power supply has to be matched electrically to the load (jet and electrodes) in order to deliver its energy within the interaction time, but this may be difficult due to the varying impedance of the load.
- At least for these experimental conditions, the magnetic forces and the ohmic heating will enhance the natural necking phenomenon leading to fragmentation of jets. A stronger effect is obtained for higher currents.
- The necks of the jet will eventually explode electrically due to heating and as neighbouring explosions interact, expanding rings with a expansion velocity of the order of 100 m/s are formed.
- Due to the high velocity of the jet the magnetic field will be advected i.e. the current will flow through the jet also outside the electrode region.

From an electric armour application point of view the work enables the establishment of the requirements on a fieldable system. Such a system would utilize pulsed power components capable of storing hundreds of kilojoules to be delivered to a load within a tenth of a millisecond in the form of a current pulse of several hundred kiloamperes. The components of such a system would have to withstand the high voltage of several tenths of kilovolts, very high current densities due to the finite time of the magnetic field penetration into the current carrying conductors, heating, and the huge magnetic forces associated with the current. Furthermore, the system would require rapid charging of the capacitors and charging times of a tenth of a second means charging rate of several megajoules per second. These are severe requirements considering that it in a fieldable system probably would rely on solid state switching devices to connect and disconnect capacitors or armour packs. Finally the size and weight constraints place high demands on the energy storage capability.
6 Conclusions

The work on magnetic flux compression generators showed that with a good diagnostics it is possible to analyse a generator in detail. Together with a circuit simulation model using relatively simple analytical descriptions of the inductance and resistivity change, one may analyse the generator even more. With a good model of the generator, one may predict the performance of a system powered by the same generator or even design new generators with good accuracy.

The work on electrification of shaped charge jets has provided some insight on how the jet disruption develops and information on required energy and current levels. The physics of electrified and exploded conductors in strong magnetic fields is of academic interest. From an applications point of view this work has provided information about the possibilities with this technology and if it has potential to serve as a protection system in military vehicles.
7 Suggestion for future work

The work on flux compression generators includes both experiments with and modelling of a specific type of generator, supported by hydrodynamic simulations of the detonation process. This has provided a good understanding of the generator performance and the modelling method that was used. There are some areas that have not been studied and hence would be of interest for future work.

From the experimental point of view, only two generators were tested providing limited experimental data and no data on the statistical variation between identical experiments. To fully explore the generator design itself, a larger numbers of generators should be tested and parametric studies of the generator should be performed e.g. by varying the seed current or even some of the generator dimensions. Another approach would be to use the existing generator design, with some improvements, to power a pulse-forming network (PFN) and perform some parametric studies of one or two PFN parameters. In either case it would require manufacturing of a number of new generators, which would be very expensive. The question is whether the present design, using a stator coil machined from a tube of copper, is worth the extra cost compared to the cheaper method of constructing the stator from cables. The work showed that it is possible to obtain good agreement between experiments and simulations. Since this type of generator has been considered for single shot systems that generates High Power Microwave pulses, e.g. munitions, a continued work could focus on system modelling and optimisation of such cheaper systems.

The work on electric armour has provided some insight into the physical process when a metal jet is subjected to a high current and a high magnetic field. It is clear from these experiments that it is difficult to extract detailed data on the heating, the electrical explosions and the disruption process. The experiments are expensive to perform and the amount of diagnostics limited (e.g. three flash X-ray tubes). A method not yet tested is to photograph the jet during and after electrification using high-speed photography. This could reveal the path of the current and indicate if and how far the current is pulled along the jet due to advection. Time resolved temperature measurements would be
very interesting in order to compare with the crude temperature estimates. Small magnetic field probes could reveal a lot about the path of the current. In addition to shaped charge jet experiments, static experiments should be performed. Static experiments are easier and faster to perform and could provide a lot of data to benchmark simulation tools.

One important step forward would be to simulate the whole process using a hydrocode. The hydrocode GRALE have been used to some extent but needs better material models of the conductivity. Although GRALE itself works with ALE coordinates, of which both Lagrange- and Euler-coordinates are special cases, only Lagrange-coordinates are so far implemented in the MHD module. Simulation of the explosion phase has to be performed in Euler-coordinates and remains to be implemented into GRALE.
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9 Summary of papers

9.1 Summary of Paper I

The paper presents the design of and experiments with two small high explosive-driven helical magnetic flux-compression generators. The generator was designed at FOI and has an overall length of 300 mm and a diameter of 70 mm. It could serve as the energy source in a pulse-forming network to generate high power pulses for various loads. The generator had an initial inductance of 23 μH and was operated into a load of 0.2 μH. The generator is charged with 0.27 kg of high explosives (PBXN-5). Various types of diagnostics were used to monitor the operation of the generator, including current probes, optical fibres, and piezo gauges. With seed currents of 5.7 and 11.2 kA, final currents of 269 and 436 kA were obtained, corresponding to current amplification factors of 47 and 39. The peak of the current was reached about 30 μs after the time of crowbar. The two generators showed only small losses in terms of $2\pi$-clocking. Using signals from optical fibers, the deflection angle of the armature could be determined to be in good agreement with hydrodynamic simulations of the detonation process and the detonation velocity to be 8.7 km/s in agreement with tabulated value.


9.2 Summary of Paper II

This paper presents a simulation model of the helical magnetic flux-compression generator presented in Paper I. The model, which was implemented in Matlab–Simulink, uses analytical expressions for the generator inductance. The model of resistive losses takes into account the heating of the conductors and the diffusion of the magnetic field into the conductors. The
simulation results are compared with experimental data from two experiments with identical generators but with different seed currents, influencing the resistive losses. The model is used to analyze the performance of the generator.


9.3 Summary of Paper III

This paper presents simulation results of high voltage systems that is powered by the helical flux compression generator. The generator design and simulation model was presented in Papers I and II. The design of this specific generator allows for a very fast inductance reduction in the final stages of the compression. The armature is not a straight cylinder as on most generators of this type but has a conical end with an angle of $8^\circ$. To study the benefits of the armature conical end design, simulations were also performed with the armature cone angle set to zero, i.e., a straight cylindrical armature. These simulations confirm the anticipated effect of the conical end cone. The flux compression generator, with and without armature end cone, is used to power a pulse-forming network using electrically exploded opening switches and spark gap closing switches. The simulations are performed in Matlab-Simulink, in which the generator and the other pulse shaping components are represented by blocks that can easily be reconfigured to form new system layouts. The batch feature in Matlab-Simulink allows for large parametric studies of various components. The paper was presented at the IET 10th Pulsed Power Symposium, 17-19 September 2007, Oxfordshire, UK.

9.4 Summary of Paper IV

The flux compression generator presented in Paper I cannot generate high voltage pulses directly but requires a pulse-forming network. This paper presents a 10 GW pulsed power supply designed to power a microwave source. In this system the current is delivered by a capacitor bank. However, a flux compression generator could replace the capacitor as current source in a single shot application. The system was developed by Loughborough University and to demonstrate the operation with a microwave source, FOI brought a vircator to Loughborough and mounted the systems together. The system proved that it could deliver the power and that microwaves could be generated by the microwave source when powered with this system.
9.5 Summary of paper V

This paper presents experimental work on electric armour. Electric armour is a protection concept for military vehicles that utilizes electric energy to disrupt the threat penetrator, e.g. a shaped charge jet, before it reaches the vehicle hull. The concept was studied experimentally using a system consisting of a pulse-forming network, transmission lines and an electrode pack and various diagnostic tools such as current and voltage probes and X-ray radiographing equipment. When the shaped charge jet bridges the gap between the electrodes, the energy stored in the capacitors of the pulse-forming network is released in the form of a strong current pulse. The jet is heated by the current until it melts, vaporizes or is disrupted by magnetic forces. The effect of energy available in the capacitors on the jet disruption was studied, and the disruption process could be studied to some extent. Of special interest is the radial velocity of the dispersed fragments when they are transformed into ring-shaped structures that expand in the radial direction.

9.6 Summary of Paper VI

This paper continues the analysis of results obtained in experiments reported in Paper V. The paper discusses the energy and power deposition in the jet and the development of magnetohydrodynamic instabilities. A shaped charge jet has a natural radial variation along the jet and the jet eventually breaks up into fragments. Due to the high magnetic pressure associated with the high current densities, the small variations in the jet radius along the jet will be enhanced, i.e. the narrow parts will decrease in diameter. This type of instability growth was studied in the experiments presented in Paper V. By digital processing of the X-ray pictures of the electrode region, the instability growth rate could be
obtained. It was shown that the electrification enhances the instability growth rate, a rate which increases with the amount of energy used to disrupt the jet. It was also shown that the number of fragments that an unelectrified jet breaks up into, agrees with the number of dispersed fragments of electrified jets, confirming that the instability growth is one of the mechanisms responsible for disrupting the jet.


### 9.7 Summary of paper VII

This paper continues the analysis of results obtained in experiments reported in Paper V and Paper VI. It was observed in the previous papers that the rate at which the diameter of the necks of a jet is reduced, increases with deposited energy and current. This paper discusses the disruption of the jet that is observed after electrification and beyond the electrode region. A mechanism for the formation of ring-shaped structures, looking like smoke rings, is suggested. Due to the magnetic pressure, the conducting area of the necks of the jet is reduced and the heating of the neck is intensified. The neck eventually explodes and expands spherically compressing the unexploded jet material in the jets axial direction. Neighbouring exploding necks act together to compress the unexploded material and eject it in the transverse direction. The mechanism is also observed in static experiments where metal rods fitted with notches are subjected to a current pulse.


### 9.8 Summary of paper VIII

This paper presents work on electrical explosion of copper rods i.e. static experiments in support for the study of disruption mechanisms of shaped charge jets in electric armour, Papers V-VII. The copper rods were fitted with five equally spaced notches having a cross section area of 75% of the rod cross section area. The rods were subjected to a current pulse and the explosion of the jet depicted using X-ray radiography. The disruption of rods fitted with notches is different from that when a smooth rod is subjected to a similar
current pulse. The disruption mechanism is similar to that observed in Paper VII.


9.9 Summary of paper IX

This paper was presented at the International Ballistics Symposium in New Orleans in 2008. It compares experimental results to hydrodynamic simulations using a 2D hydrocode called GRALE developed at FOI. An MHD module has been developed to include electromagnetic effects. Simulation of a jet passing two electrodes shows the effect of a high magnetic Reynolds number where the magnetic field lines are pulled along the jet out of the electrode region. The heating of the jet, the temperature distribution and the magnitude of the volume forces acting on the jet could be obtained. Evaporation temperatures are reached after a jet segment has passed about 2/3 of the electrode distance and in corresponding experiments, it can be seen that the jet has begun to disintegrate at about the same position. It was also shown that the compressive forces acting on the jet between the electrodes are tensile just outside the electrode region at the exit.

9.10 Summary of paper X

This paper was presented at the Megagauss XI conference in London 2006. The paper presents the pulsed power supply designed for the electric armour experiments. In order to deliver pulses of different pulse lengths and with the possibility to generate a square shaped pulse a four module system was chosen. Each module consists of two 206 μF capacitors, 22 kV. The modules are connected via inductors that can be varied in order to change pulse duration and impedance. The 1.7 mF bank stores 400 kJ at full charge and can deliver pulses of several hundred kiloamperes and between 100 and 2000 μs into a load of several tens of milliohms. The system is disconnected from the grid when operated and is fully battery powered and controlled via optic fibres.